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Optimal Petri Net Supervisors of Discrete Event Systems via Weighted and Data Inhibitor Arcs

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ABSTRACT This paper handles the optimal supervisory control problem of Petri nets (PNs) via two PN structures, namely, weighted and data inhibitor arcs. It is a two-stage method. In the first stage, for each transition that may lead to illegal markings, a set of observer places with weighted inhibitor arcs is used to optimally control the maximal number of marking/transition separation instances (MTSIs) through the proposed integer linear program. Then, the controlled MTSIs are removed from the set of MTSIs. In the second stage, at each iteration, for an MTSI that cannot be controlled at the first control stage, we design an optimal observer place with a data inhibitor arc. This process terminates after all the MTSIs are controlled. The first-stage can sharply lower the computational burden compared with the method by using data inhibitor arcs alone. Finally, a typical example is presented to shed light on this technique. The proposed control strategy can definitely yield an optimal supervisor for any bounded PN on the premise that such a supervisor exists.

INDEX TERMS Petri net, weighted inhibitor arc, data inhibitor arc, observer place, supervisory control.

I. INTRODUCTION

A discrete event system (DES) is a dynamic system with discrete states and its transitions are triggered by events. The DES is said to be controlled if its evolution is supervised to satisfy a set of required specifications such as deadlock avoidance and liveness enforcement. The most significant and comprehensive control theory for discrete event system is referred as to supervisory control theory (SCT) [37], [38] that is extensively used in the synthesis of controllers of DESs [15], [28], [45], [49], [51].

In general, according to SCT, given some qualitative control specifications and objectives, the behavior of a system cannot violate them by adding suitable controllers to the system. Since deadlock or blockingness can cause disastrous results, maximal permissiveness or non-blockingness is usually selected as a significantly safe property for many DESs. In this paper, maximal permissiveness is considered as the supervisory control specifications of a system. Moreover, the proposed work not only achieves the optimal deadlock control requirement for a given system but also satisfies any supervisory control requirement to a system. In the following, when we say "optimal supervisor", it means "maximally permissive supervisor".

In a DES, the utilization of limited system resources by different processes can yield deadlocks [10]. It can be usually solved by three crucial mathematical tools such as Petri nets (PNs) [5], [39], [40], [46], [50], automata [16], [17], and graph theory [14], [53]. Among them, PNs are compact in structure and widely used for modeling, analysis, fault diagnosis and scheduling [12], [46], [54]–[61]. Hence, a number of methods have been developed to handle deadlocks by using PNs [9], [13], [22], [23], [25], [29], [34], [35], [47].

There are three important criteria to evaluate a liveness-enforcing PN supervisor, namely, behavioral permissiveness, structural complexity, and computational complexity in synthesizing a supervisor. An optimal supervisor ensures the highest utilization of system resources. Therefore, a lot of work focuses on designing optimal PN supervisors [4], [8], [27], [36], [41]. Reachability graph (RG) analysis is usually taken as an effective technique to achieve such a control purpose in PNs [19], [24]. However, it is subject to the notorious state explosion problem. Feng [16], [17], and Nazeem [31]–[33] have made contributions to solve deadlock problems by using automata and the obtained optimal deadlock avoidance strategies are optimal. Different from their work, in this paper, the deadlock problem is addressed in the PN framework by using RG analysis.

An RG can be divided into two zones: a live-zone (LZ) which contains all legal markings (LMs) and a deadlock-zone (DZ) which includes all illegal markings. The theory of regions [19], [41] that can derive PNs from a transition system is a useful methodology to find an optimal pure net supervisor. A pair of a marking M and a transition t forms a marking/transition separation instance (MTSI) denoted by (M, t), where M is an LM and t's firing at M yields an illegal marking. In an MTSI (M, t), M is called a dangerous marking and t is called a dangerous transition. The main idea of the approach in [19] and [41] is to separate transitions from dangerous markings. However, in this case, the final supervisor may have too many control places (monitors).

Uzam and Zhou [42], [43] present a representative iterative deadlock prevention approach by prohibiting first-met bad markings (FBMs). Specifically, let M_1 be a marking in the LZ, once a transition t fires at M_1 and it yields an illegal marking M_2, then M_2 is called an FBM. This method is computationally efficient since there is no need to compute monitors by solving integer linear program (ILP). However, this method cannot ensure the optimality of the final supervisor. Inspired by [42] and [43], Chen *et al.* [1] propose an efficient iterative approach to avoid deadlocks, which has two advantages compared with the work in [42] and [43]. One is that it can obtain an optimal supervisor if such a supervisor exists by solving an ILP at each iteration. The other is that only a subset of FBMs and LMs are considered, which is achieved by employing a vector covering approach which can sharply decrease the number of constraints in an ILP. In [2], a non-iteration method is presented to obtain a compressed control structure, namely minimal number of monitors, by solving only one ILP. However, it cannot be applied to large-scale systems because of too many constraints in the ILP. Later, Huang *et al.* [21] alleviate the computational burden by eliminating redundant constraints of the ILP in [2] while preserving the behavioral optimality and the compressed structure of the controlled net.

Actually, there exist net systems such that no pure net supervisor can optimally control them. To solve this problem, self-loops are introduced in [3] to design optimal supervisors by solving ILPs. It needs to solve an ILP to derive an optimal monitor with a self-loop connected to a dangerous transition at each iteration. An MTSI (M, t) is said to be controlled if t is disabled at M. The proposed ILP named MMP (the maximal number of MTSI problem) can control the maximal number of t-dangerous MTSIs associated with a dangerous transition t. However, the work cannot guarantee that MMP always has a solution with a positive objective value. When there is no solution with a positive objective value for MMP, it cannot obtain an optimal net supervisor for a system. Chen *et al.* [6] improve their previous work in [3] by assuming that a monitor can associate with multiple self-loops. Compared with [3], the work in [6] can find optimal supervisors with fewer monitors and even the fewest monitors through two proposed ILPs, namely the implementation of maximal number of MTSIs (IMNM) and the minimization of the number of control places (MNCP), respectively. However, the ILPs may have no solution with a positive objective value for some net models as the case in [3]. In [4], a novel net structure named interval inhibitor arcs is proposed for the design of optimal monitors. Via such arcs, the method can derive an optimal net supervisor that is structurally simple. Similarly, this method needs to solve an ILP to obtain a monitor too, and it does not ensure that the ILP can obtain a solution with a positive objective value for any bounded net model either. To solve this problem, in [7], a more general net structure named data inhibitor arcs is developed to design an optimal supervisor for any bounded net model if such a supervisor exists. More importantly, each formed ILP has a solution for any bounded net model [7], in other words, it can definitely obtain an optimal supervisor for any bounded net model. However, the drawback of this method is that it is inapplicable for complex systems due to too many constraints in the ILPs. Our previous work [11] proposes a novel optimal supervisor structure composed of a set of observer places with weighted inhibitor arcs, which can sharply reduce the number of constraints and variables in the ILPs compared with the work in [3], [4], and [6]. The drawback of the work in [11] is that there is no guarantee for finding an optimal supervisor for any bounded PN.

This work presents a two-stage method for designing an optimal supervisor. Although it does not lead to the simplest supervisory structure, it is an efficient method to lower the computational burden of the approach in [7]. In summary, the main contributions of this work are as follows:

1) A two-stage method is presented to obtain an optimal supervisor via both weighted and data inhibitor arcs. The presented control strategy is applicable to any bounded net system.
2) In the first stage, an ILP, namely the maximal number of t-dangerous MTSIs problem (MNMP(t)), is proposed to design an observer place with a weighted inhibitor arc associated with t to control as many t-dangerous MTSIs as possible. This process terminates after all t-dangerous MTSIs are controlled or the ILP has no optimal solution with a positive objective value.

3) In the second stage, another ILP, namely the controlled t-dangerous MTSI problem (CTMP), is proposed to find an observer place with a data inhibitor arc for each t-dangerous MTSI that cannot be optimally controlled in the first stage. It is significant that CTMP has a solution for any bounded PN. Since we deal with one MTSI at each iteration only, the computational burden is greatly alleviated compared with the approach in [7].

The rest of the paper is organized as follows. Section II exposes the preliminaries of PNs. Weighted inhibitor arcs are formally defined in Section III. The computation of optimal observer places with weighted inhibitor arcs is reported in Section IV. In Section V, an ILP-based method is developed as the first control stage to simplify the supervisory structures. Section VI presents another ILP-based method as the second control stage to ensure that an optimal supervisor can be definitely obtained for any bounded PN model. A control strategy is presented in Section VII. Experimental results show the performance of the proposed control strategy in Section VIII. Finally, Section IX concludes this paper.

II. PRELIMINARIES

For the sake of simplicity, the reader is referred to [30] for some basics of PNs.

A. ANALYSIS OF RGs

Given deadlock-free control specifications, all of the markings in RG of a net (N, M_0) are partitioned into two categories: legal and illegal. In general, the set of LMs is defined as

$$\mathcal{M}_L = \{M | M \in R(N, M_0) \wedge M_0 \in R(N, M)\}. \quad (1)$$

The set of illegal markings is denoted by $\mathcal{M}_{\overline{L}}$ defined as $\mathcal{M}_{\overline{L}} = R(N, M_0) \backslash \mathcal{M}_L$. An MTSI is a pair of a marking M and a transition t where t's firing at M yields an illegal marking. Thus, the set of MTSIs can be defined as

$$\Omega = \{(M, t) | M[t\rangle M' \wedge M \in \mathcal{M}_L \wedge M' \in \mathcal{M}_{\overline{L}}\} \quad (2)$$

where M is called a dangerous marking, whose set is denoted by $\mathcal{M}_D$. Deleting all dangerous markings in $\mathcal{M}_L$, the remaining ones are good markings, whose set is defined as

$$\mathcal{M}_G = \mathcal{M}_L \backslash \mathcal{M}_D. \quad (3)$$

Let us consider a net example in Fig. 1(a) and its RG in Fig. 1(b), where we have $\mathcal{M}_G = \{M_0, M_4, M_7, M_8, M_{11}, M_{12}, M_{14}\}$, $\mathcal{M}_D = \{M_1, M_2, M_3, M_6\}$, and $\Omega = \{(M_1, t_4), (M_2, t_1), (M_3, t_4), (M_6, t_1)\}$.

According to the definition of MTSIs, transitions can be divided into two categories: dangerous and good ones, whose sets are denoted by T_D and $T_{\overline{D}}$, respectively, as follows:

$$T_D = \{t \in T | \exists M \in R(N, M_0), \quad s.t. (M, t) \in \Omega\} \quad (4)$$

$$T_{\overline{D}} = \{t \in T | \nexists M \in R(N, M_0), \quad s.t. (M, t) \in \Omega\}. \quad (5)$$

In Fig. 1(a), the net has two dangerous transitions and four good transitions, i.e., $T_D = \{t_1, t_4\}$ and $T_{\overline{D}} = \{t_2, t_3, t_5, t_6\}$.

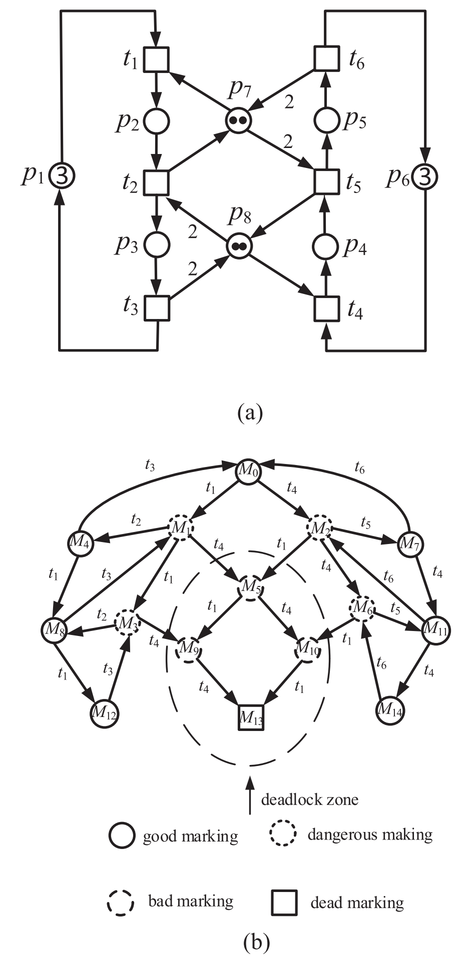

FIGURE 1. (a) PN model and (b) its RG.

For a transition t, it can be enabled at some LMs and its firing may yield legal or illegal ones. Hence, all LMs are divided into two groups in [3].

Definition 1 [3]: Let M be an LM and t be a transition. M is called a t-good marking if t is disabled at M or if M' in $M[t\rangle M'$ is legal. M is called a t-dangerous marking if M' in $M[t\rangle M'$ is illegal, whose sets are denoted by $\mathcal{G}_t$ and $\mathcal{D}_t$, respectively.

For a set $\mathcal{G}_t$, t can be fired at some t-good markings. Thus, all of these markings can be divided into two categories: t-enabled and t-disabled ones [3].

Definition 2 [3]: Let t be a transition and M be a t-good marking. M is called a t-enabled good marking if $M[t\rangle$; otherwise, it is a t-disabled one, whose sets are denoted by $\mathcal{E}_t$ and $\mathcal{E}_{\overline{t}}$, respectively.

It is obvious that $\mathcal{G}_t = \mathcal{E}_t \cup \mathcal{E}_{\overline{t}}$ and $\mathcal{M}_L = \mathcal{G}_t \cup \mathcal{D}_t$.

Definition 3 [3]: Let t be a dangerous transition and $\mathcal{D}_t$ be the set of t-dangerous markings. (M, t) is called a t-dangerous MTSI. The set of t-dangerous MTSIs is denoted by Ω_t.

B. OBSERVER PLACE COMPUTATION VIA PLACE INVARIANT

In [48], a computationally efficient control place technique by place invariant (PI) is presented. Motivated by [48], our previous work [11] introduces a method to compute an observer place which can be used to present the weighted sum of tokens in some places at a specific marking. This section recalls the synthesis of an observer place via PI.

Let $[N_p]$ be the incidence matrix of a plant with n places and m transitions. The observer places can be described as a matrix $[N_o]$ that includes the arcs connecting observer places and transitions of the plant. The net model with incidence matrix $[N]$ contains both plant and observer places can be shown as follows:

$$[N] = \begin{bmatrix} N_p \\ N_o \end{bmatrix}.$$

Suppose that the following constraint is added to the plant:

$$\sum_{i=1}^{n} l_i \cdot \mu_i \geq 0 \tag{6}$$

where μ_i denotes the marking of place p_i and l_i is a nonnegative integer.

Obviously, this inequality constraint can be modified as the following equation:

$$\sum_{i=1}^{n} l_i \cdot \mu_i - \mu_o = 0 \tag{7}$$

where μ_o is a nonnegative variable and can represent the marking of an observer place p_o. In general, the set of constraints in Eq. (6) can be combined into the following matrix format:

$$[L] \cdot \overrightarrow{\mu_p} \geq \mathbf{0} \tag{8}$$

where $\overrightarrow{\mu_p}$ is the marking vector of the PN model, $[L]$ is an $n_o \times n$ nonnegative integer matrix, and n_o is the number of constraints and can also represent the number of the observer places which are different from control places since they do not prevent any marking from being reached. Generally, all PIs of Eq. (7) can be combined into the following matrix format:

$$[L] \cdot \overrightarrow{\mu_p} - \overrightarrow{\mu_o} = \mathbf{0} \tag{9}$$

where $\overrightarrow{\mu_o}$ is an $n_o \times 1$ vector that can represent the marking of the observer places. Based on PI equation $I^T[N] = \mathbf{0}^T$, we have

$$[N_o] = [L] \cdot [N_p]. \tag{10}$$

Eq. (9) can also be satisfied at the initial marking $\overrightarrow{\mu_0}$ of a net. Hence, the initial marking $\overrightarrow{\mu_{o_0}}$ of the observer places is as follows:

$$\overrightarrow{\mu_{o_0}} = [L] \cdot \overrightarrow{\mu_{p_0}} \tag{11}$$

where $\overrightarrow{\mu_{p_0}}$ is the initial marking of the plant.

In particular, when we compute only one of the observer places, we have:

$$[N_{p_o}] = [l_1, l_2, \ldots, l_n] \cdot [N_p]. \tag{12}$$

According to Eq. (11), its initial marking is as follows:

$$\mu_{o_0} = [l_1, l_2, \ldots, l_n] \cdot \overrightarrow{\mu_{p_0}}. \tag{13}$$

Here an example is used to show the computation of the observer place. Assume that the total number of tokens in p_2 and p_3 in Fig. 1(a) is expected to be known. We have $l_2 = l_3 = 1$. Then, an observer place p_o can be designed for PI: $\mu_2 + \mu_3 - \mu_o = 0$ by the aforementioned method. Hence, we have $M_0(p_o) = 0$, ${}^\bullet p_o = \{t_1\}$, and $p_o^\bullet = \{t_3\}$.

Definition 4 [11]: An observer place p_o is said to be optimal if it does not prevent the reachability of any LM.

III. WEIGHTED INHIBITOR ARC

In this section, we review some basics of the weighted inhibitor arcs. Due to that a weighted inhibitor arc is more general than an inhibitor arc, it is more powerful to control a net model.

Definition 5 [18]: A weighted inhibitor arc is an arc from a place p to a transition t labeled by an integer γ, denoted by $H(p, t) = \gamma$, where γ is a nonnegative integer with $\gamma \geq 1$. It can be graphically described as an inhibitor arc from p to t with a label γ on it, as shown in Fig. 2.

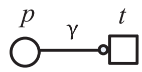

FIGURE 2. Weighted inhibitor arc.

The transition enabling and firing rules of a net with weighted inhibitor arcs are defined below.

Definition 6 [18]: Let p be a place and t be a transition with $H(p, t) = \gamma$. Transition t is enabled by $H(p, t)$ at marking M if $M(p) < \gamma$; otherwise, it is disabled. Once t is enabled, its firing does not remove the tokens in p.

For a PN model with weighted inhibitor arcs, a transition can be inhibited from firing by a weighted inhibitor arc or a normal (regular) arc. Hence, we have the following transition enabling and firing rules to complement Definition 6.

Definition 7 [11]: Let $N = (P, T, F, W, \mathcal{I})$ be a PN with weighted inhibitor arcs, where $\mathcal{I}$ represents a set of weighted inhibitor arcs. A transition t is enabled at marking M if $\forall p' \in {}^\bullet t$, $M(p') \geq W(p', t)$ and $\forall H(p, t) \in \mathcal{I}$, such that $M(p) < \gamma$; otherwise, it is disabled. Once a transition t is enabled at M, its firing leads to M' such that $M'(p) = M(p) - W(p, t) + W(t, p)$, $\forall p \in P$.

An inhibitor arc from a place p to a transition t can be regarded as a specific case of a weighted inhibitor arc with $H(p, t) = 1$.

Property 1 [11]: Suppose that the marking of a place is upper bounded by d ($d \in \mathbb{N}^+$), a weighted inhibitor arc from this place with a weight $\gamma > d$ is redundant.

A transition t can be associated with both a normal (regular) arc $W(p, t) = w$ or $W(t, p) = w$ and a weighted inhibitor arc $H(p, t) = \gamma$, as shown in Fig. 3. We have:

1) Fig. 3(a): a) t is enabled if $M(p) \geq w$ and $M(p) < \gamma$. Once t is enabled and fires, it leads to M' with

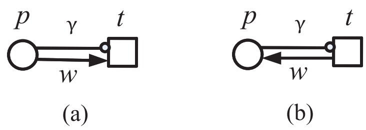

FIGURE 3. (a) $W(p, t) = w$ and $H(p, t) = \gamma$, and (b) $W(t, p) = w$ and $H(p, t) = \gamma$.

$M'(p) = M(p) - w$. b) t is disabled if $M(p) < w$ or $M(p) \geq \gamma$; and

2) Fig. 3(b): a) t is enabled if $M(p) < \gamma$. Once t is enabled and fires, it leads to M' with $M' = M(p) + w$. b) t is disabled if $M(p) \geq \gamma$.

IV. OPTIMAL OBSERVER PLACES WITH WEIGHTED INHIBITOR ARCS

This section first recalls the technique proposed in [11] to compute observer places with weighted inhibitor arcs for the design of an optimal supervisor. In this section, an MTSI (M, t) is said to be controlled by an observer place p_o if its transition is disabled by p_o with $H(p_o, t)$.

Let $G(N, M_0)$ be the RG of a PN model, $\mathcal{M}_L$ the set of LMs, and Ω the set of MTSIs. In the following, we show how to find optimal supervisors by designing observer places with weighted inhibitor arcs in order to ensure that, $\forall (M, t) \in \Omega$, t can be inhibited from firing at M while all LMs are reachable. Let (M, t) be an MTSI, p_o be an observer place satisfying Eq. (7), and $H(p_o, t)$ be a weighted inhibitor arc with $H(p_o, t) = \gamma$. Then, a technique is developed to compute p_o that can optimally control this MTSI, i.e., obtain the values of l_i's and γ such that the derived p_o with $H(p_o, t)$ can inhibit the firing of t at M with all LMs preserved. First, p_o does not prevent any LM, i.e.,

$$\sum_{i=1}^{n} l_i \cdot M_l(p_i) \geq 0, \quad \forall M_l \in \mathcal{M}_L. \tag{14}$$

Eq. (14) is said to be the reachability condition. Then, from Proposition 1 in [11], we do not need to consider Eq. (14) to guarantee that all LMs are remained after adding an observer place.

Proposition 1 [11]: Eq. (14) is always true and it does not give any restriction on l_i's.

Proof: Since $\forall M_l \in \mathcal{M}_L$, $M_l(p_i) \geq 0$ and l_i's $(i = 1, 2, \ldots, n)$ are nonnegative integers. ■

For an MTSI (M, t), t should be disabled by p_o with $H(p_o, t)$ at M. Hence, we have $M(p_o) < W(p_o, t)$ or $M(p_o) \geq \gamma$. According to Eq. (7), we have $M(p_o) = \sum_{i=1}^{n} l_i \cdot M(p_i)$ and $W(p_o, t) = -\sum_{i=1}^{n} l_i \cdot [N](p_i, t)$. Next, we show that only the weighted inhibitor arc $H(p_o, t)$ can make sense in disabling t at marking M.

Since t is enabled at M in a plant net, we have, $\forall p \in {}^\bullet t$, $M(p) \geq W(p, t)$, i.e., $\forall p \in {}^\bullet t$, $M(p) \geq -[N](p, t)$. Consequently, $\forall p_i \in {}^\bullet t$, $M(p_i) \geq -[N](p_i, t) > 0$, and $\forall p_i \notin {}^\bullet t$, $M(p_i) \geq 0 \geq -[N](p_i, t)$. Hence, we have $\sum_{i=1}^{n} l_i \cdot M(p_i) \geq -\sum_{i=1}^{n} l_i \cdot [N](p_i, t)$, i.e., $M(p_o) \geq W(p_o, t)$.

In this case, the disabling condition can be shown as

$$\sum_{i=1}^{n} l_i \cdot M(p_i) \geq \gamma. \tag{15}$$

At any t-enabled good marking $M_j \in \mathcal{E}_t$, the weighted inhibitor arc $H(p_o, t)$ associated with p_o should enable t, i.e., $M_j(p_o) \leq \gamma - 1$. By Eq. (7), we have $M_j(p_o) = \sum_{i=1}^{n} l_i \cdot M_j(p_i)$, i.e.,

$$\sum_{i=1}^{n} l_i \cdot M_j(p_i) \leq \gamma - 1, \quad \forall M_j \in \mathcal{E}_t. \tag{16}$$

Eq. (16) is called the enabling condition.

We can group Eqs. (15) and (16) into an ILP to obtain l_i's and γ. Accordingly, p_o can be designed by Eq. (7) with a weighted inhibitor arc $H(p_o, t) = \gamma$. On this occasion, (M, t) can be optimally controlled by the derived observer place p_o with $H(p_o, t)$, i.e., t is disabled at M with all LMs remained. If we optimally control all MTSIs by a number of observer places with weighted inhibitor arcs, then an optimal supervisor can be derived.

V. REDUCTION OF SUPERVISORY STRUCTURES

This section presents an ILP-based method for the synthesis of an observer place p_o with a weighted inhibitor arc $H(p_o, t)$, which is used to control as many t-dangerous MTSIs as possible for any bounded net model. Therefore, the resulting supervisor can have fewer observer places.

For a transition t and the set of its related dangerous MTSIs Ω_t, let $\mathbb{N}_t = \{k | (M_k, t) \in \Omega_t\}$. A set of binary variables q_k's is used to indicate whether an MTSI (M_k, t) is controlled or not by p_o with $H(p_o, t)$. Then, the following constriant can replace Eq. (15) as a disabling condition.

$$\sum_{i=1}^{n} l_i \cdot M_k(p_i) \geq -Q \cdot (1 - q_k) + \gamma \tag{17}$$

where Q is a big enough positive integer constant. In Eq. (17), $q_k = 1$ implies that (M_k, t) is controlled by p_o with $H(p_o, t)$, and $q_k = 0$ otherwise. Then, by combining Eqs. (16) and (17), we formulate an ILP for the design of p_o with $H(p_o, t)$, which is denoted as the maximal number of t-dangerous MTSIs problem (MNMP(t)):

$$\begin{aligned}
\text{MNMP}(t):& \\
&\max q = \sum_{k \in \mathbb{N}_t} q_k \\
&\text{subject to } \sum_{i=1}^{n} l_i \cdot M_j(p_i) \leq \gamma - 1, \quad \forall M_j \in \mathcal{E}_t \\
&\qquad \sum_{i=1}^{n} l_i \cdot M_k(p_i) \geq -Q \cdot (1 - q_k) + \gamma, \\
&\qquad \forall (M_k, t) \in \Omega_t \\
&\qquad l_i \in \{0, 1, 2, \ldots\}, \quad i \in \{1, 2, \ldots, n\}
\end{aligned} \tag{18}$$

$$\gamma \in \{1, 2, 3, \ldots\}$$
$$q_k \in \{0, 1\}, \quad k \in \mathbb{N}_t. \tag{19}$$

The objective function q is used to maximize the number of MTSIs in Ω_t that can be controlled by an observer place p_o with $H(p_o, t)$. Let q^* denote its optimal value. If $q^* = 0$, it means that no MTSI in Ω_t can be controlled by p_o with any weighted inhibitor arc.

As is well known, an ILP is NP-hard. The complexity of solving an ILP is dependent on the number of constraints and variables in it. On this occasion, the number of constraints and variables of MNMP(t) is presented in Table 1. Compared with the work in [11], in Eqs. (18) and (19), we need to consider all the places, all the t-enabled good markings and all the t-dangerous MTSIs, since the application scope of this work is extended to any bounded net system.

TABLE 1. Number of constraints and variables in MNMP(t).

Eq.	number of constraints	variable	number of variables
(18)	$\|\mathcal{E}_t\|$	l_i	$\|P\|$
(19)	$\|\Omega_t\|$	q_k	$\|\Omega_t\|$
		γ	1
total	$\|\mathcal{E}_t\| + \|\Omega_t\|$	total	$\|P\| + \|\Omega_t\| + 1$

Theorem 1: The observer place p_o with $H(p_o, t)$ derived by MNMP(t) is optimal and can control some MTSIs in Ω_t, if $q^* > 0$.

Proof: By Proposition 1, an observer place cannot forbid any LM. Eq. (18) guarantees that the additional weighted inhibitor arc can enable t at any marking in $\mathcal{E}_t$. Hence, p_o is optimal. Since $q^* > 0$, we can confirm that there exists at least a (M, t) in Ω_t such that $q_k = 1$. Then, Eq. (19) implies that MTSI (M, t) is controlled by p_o with $H(p_o, t)$. The conclusion holds. ■

VI. AN OPTIMAL OBSERVER PLACE WITH A DATA INHIBITOR ARC

In this section, an ILP-based method is proposed for the computation of an observer place with a data inhibitor arc to optimally control an MTSI of a net model.

A data inhibitor arc [7] is an arc from a place p to a transition t with a set of nonnegative integers $\mathbb{A}(p, t) = \{a_1, a_2, \ldots, a_k\}$ labeled on it, which is denoted as Å(p, t) and can be graphically shown in Fig. 4.

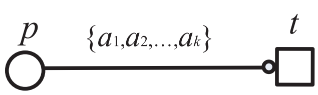

FIGURE 4. Data inhibitor arc.

In a data inhibitor arc, t is disabled by p at a marking M if $M(p) \in \{a_1, a_2, \ldots, a_k\}$. Once t is enabled and fires, it does not change the number of tokens in p. In this paper, a transition t can be associated with both a normal (regular) arc $W(p, t) = w$ or $W(t, p) = w$ and a data inhibitor arc Å(p, t), which can be graphically represented in a compact way as shown in Fig 5.

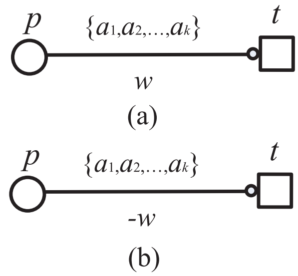

FIGURE 5. (a) $W(p, t) = w$ and $\mathbb{A}(p, t) = \{a_1, a_2, \ldots, a_k\}$, and (b) $W(t, p) = w$ and $\mathbb{A}(p, t) = \{a_1, a_2, \ldots, a_k\}$.

In the following, two transition firing rules are demonstrated for the case in Fig. 5.

1) For the case in Fig. 5(a), t is enabled by p at M if $M(p) \geq w$ and $M(p) \notin \{a_1, a_2, \ldots, a_k\}$, otherwise t is disabled. Once t is enabled and fires, it leads to M' with $M'(p) = M(p) - w$.

2) For the case in Fig. 5(b), t is enabled by p at M if $M(p) \notin \{a_1, a_2, \ldots, a_k\}$, otherwise t is disabled. Once t is enabled and fires, it leads to M' with $M'(p) = M(p) + w$.

In this section, an MTSI (M, t) is said to be controlled by an observer place p_o if its transition is disabled by p_o with Å(p, t).

First, p_o should not prohibit any LM as we have described in Section IV, we can use Eq. (14) as the reachability condition and it does not pose any restriction on the plant. Thus, we do not need to consider the reachability condition for all the LMs as the case in [7], which can greatly reduce the number of constraints in the ILP problem.

At any t-enabled good marking $M_j \in \mathcal{E}_t$, the data inhibitor arc associated with p_o should enable t, i.e., $M_j(p_o) \neq a$. Then, we can modify $M_j(p_o) \neq a$ as $M_j(p_o) \leq a - 1$ or $M_j(p_o) \geq a + 1$. According to Eq. (7), we have $M(p_o) = \sum_{i=1}^{n} l_i \cdot M(p_i)$. In this case, we formulate another enabling condition as follows:

$$\sum_{i=1}^{n} l_i \cdot M_j(p_i) \leq a - 1 \quad \text{or} \quad \sum_{i=1}^{n} l_i \cdot M_j(p_i) \geq a + 1, \quad \forall M_j \in \mathcal{E}_t. \tag{20}$$

It is obvious that Eq. (20) cannot be selected as constraints in ILPs directly. Thus, a set of binary variables r_j's $\in \{0, 1\}$ is introduced. Accordingly, Eq. (20) is modified as

$$\sum_{i=1}^{n} l_i \cdot M_j(p_i) \leq Q \cdot r_j + a - 1, \quad \forall M_j \in \mathcal{E}_t \tag{21}$$

and

$$\sum_{i=1}^{n} l_i \cdot M_j(p_i) \geq -Q \cdot (1 - r_j) + a + 1, \quad \forall M_j \in \mathcal{E}_t \tag{22}$$

where Q is a positive integer constant that is big enough. Eqs. (21) and (22) can guarantee that $\sum_{i=1}^{n} l_i \cdot M_j(p_i) \leq a - 1$

if $r_j = 0$ and $\sum_{i=1}^{n} l_i \cdot M_j(p_i) \geq a+1$ if $r_j = 1$. Hence, they can replace the enabling condition Eq. (20).

For a (M, t), t should be disabled by p_o at M. Thus, we have $M(p_o) < W(p_o, t)$ or $M(p_o) = a$. Next, we show that we need to consider $M(p_o) = a$ only to control (M, t).

Theorem 2: Let p_o be an observer place satisfying Eq. (14) and (M, t) be an MTSI. If $M(p_o) < W(p_o, t)$, then the data inhibitor arc with $\mathbb{A}(p_o, t) = \{a\}$ and $a = \sum_{i=1}^{n} l_i \cdot M(p_i)$ does not disable t at any marking in $\mathcal{E}_t$.

Proof: Let M_j be a marking in $\mathcal{E}_t$. By Definition 2, if t fires at M, it leads to an LM M'. According to Eq. (14), we have $M_j(p_o) \geq W(p_o, t)$. Since $a = \sum_{i=1}^{n} l_i \cdot M(p_i) = M(p_o) < W(p_o, t)$, $M_j(p_o) > a$ is true. Hence, the data inhibitor arc with $\mathbb{A}(p_o, t) = \{a\}$ does not disable t at M_j. The conclusion holds. ■

Theorem 2 implies that once (M, t) is optimally controlled by an observer place with $M(p_o) < W(p_o, t)$, there exists a data inhibitor arc with $\mathbb{A}(p, t) = \{a\}$ and $a = M(p_o)$ such that t is disabled at M and enabled at any t-enabled good marking. On this occasion, we do not need to consider the condition $M(p_o) < W(p_o, t)$ to control (M, t), i.e., we need to consider $M(p_o) = a$ only to control (M, t). Accordingly, the condition $M(p_o) = a$ can be written as

$$\sum_{i=1}^{n} l_i \cdot M(p_i) = a. \tag{23}$$

By combining Eqs. (21), (22), and (23), we formulate an ILP to design an optimal observer place with $\mathring{A}(p_o, t)$ to control a t-dangerous MTSI (M, t), which can be denoted as the controlled t-dangerous MTSI problem (CTMP):

$$\begin{aligned}
\text{CTMP}: \quad & \min a \\
& \text{subject to } \sum_{i=1}^{n} l_i \cdot M(p_i) = a & (24)\\
& \sum_{i=1}^{n} l_i \cdot M_j(p_i) \geq -Q\cdot(1-r_j)+a+1, \\
& \forall (M_j, t) \in \mathcal{E}_t & (25)\\
& \sum_{i=1}^{n} l_i \cdot M_j(p_i) \leq Q\cdot r_j + a - 1, \\
& \forall (M_j, t) \in \mathcal{E}_t \\
& l_i \in \{0, 1, 2, \ldots\}, \quad i \in \{1, 2, \ldots, n\} \\
& a \in \{0, 1, 2, 3, \ldots\} \\
& r_j \in \{0, 1\}, \quad j \in \mathbb{N}_{\mathcal{E}_t}. & (26)
\end{aligned}$$

Actually, it is not necessary to choose an objective function for CTMP since any feasible solution of CTMP can be used to design an optimal observer place with $\mathring{A}(p_o, t)$. In order to restrict the value of a in $\mathbb{A}(p_o, t)$ as small as possible, we select the objective function *min a* in practice.

Theorem 3: CTMP has a solution for any bounded PN.

Proof: Let (N, M_0) be a net model with n places and p_o an observer place satisfying $\sum_{i=1}^{n} l_i \cdot \mu_i - \mu_o = 0$. We prove the theorem by constructing l_i's $(i = \{1, 2, \ldots, n\})$ such that $M_1(p_o) \neq M_2(p_o)$ for any two reachable markings M_1 and M_2 with $M_1 \neq M_2$ and proving that the constructed l_i's $(i = \{1, 2, \ldots, n\})$ are a solution of CTMP.

1) Suppose that $n = 1$, i.e., we need to consider one place p_1 only in a net model. Let $l_1 = 1$, then p_o is computed to satisfy the constraint: $\mu_1 - \mu_o = 0$. In this case, $M_1(p_o) = M_1(p_1)$ and $M_2(p_o) = M_2(p_1)$. Since $M_1 \neq M_2$, we have $M_1(p_o) \neq M_2(p_o)$.

2) Suppose that $n = 2$, i.e., we need to consider two places p_1 and p_2 in a net model. Let $l_1 = 1$ and $l_2 = k_{p_1} + 1$, where k_{p_1} is the bound of p_1. Then, p_o is computed to satisfy the constraint: $\mu_1 + (k_{p_1}+1)\cdot\mu_2 - \mu_o = 0$. In this case, $M_1(p_o) = M_1(p_1) + (k_{p_1}+1)\cdot M_1(p_2)$ and $M_2(p_o) = M_2(p_1) + (k_{p_1}+1)\cdot M_2(p_2)$. By contradiction, suppose that $M_1(p_o) = M_2(p_o)$. Then, we have $M_1(p_1) + (k_{p_1}+1)\cdot M_1(p_2) = M_2(p_1) + (k_{p_1}+1)\cdot M_2(p_2)$, i.e., $M_1(p_1) - M_2(p_1) = (k_{p_1}+1)\cdot(M_2(p_2) - M_1(p_2))$. By considering that $-k_{p_1} \leq M_1(p_1) - M_2(p_1) \leq k_{p_1}$ and $M_2(p_2) - M_1(p_2)$ is an integer, we have $M_1(p_1) = M_2(p_1)$ and $M_2(p_2) = M_1(p_2)$, i.e., $M_1 = M_2$ that contradicts $M_1 \neq M_2$. Consequently, we have $M_1(p_o) \neq M_2(p_o)$.

3) Suppose that $n = 3$, i.e., we need to consider three places p_1, p_2, and p_3 in a net model. Let $l_1 = 1$, $l_2 = k_{p_1} + 1$, and $l_3 = k_2 + 1$, where $k_2 = l_1 \cdot k_{p_1} + l_2 \cdot k_{p_2}$ and k_{p_i} is the bound of p_i $(i = 1, 2)$. Then, p_o is computed to satisfy the constraint: $\mu_1 + (k_{p_1}+1)\cdot\mu_2 + (k_2+1)\cdot\mu_3 - \mu_o = 0$. In this case, $M_1(p_o) = M_1(p_1) + (k_{p_1}+1)\cdot M_1(p_2) + (k_2+1)\cdot M_1(p_3)$ and $M_2(p_o) = M_2(p_1) + (k_{p_1}+1)\cdot M_2(p_2) + (k_2+1)\cdot M_2(p_3)$. By contradiction, suppose that $M_1(p_o) = M_2(p_o)$. Then, we have $M_1(p_1) + (k_{p_1}+1)\cdot M_1(p_2) + (k_2+1)\cdot M_1(p_3) = M_2(p_1) + (k_{p_1}+1)\cdot M_2(p_2) + (k_2+1)\cdot M_2(p_3)$, i.e., $M_1(p_1) + (k_{p_1}+1)\cdot M_1(p_2) - M_2(p_1) - (k_{p_1}+1)\cdot M_2(p_2) = (k_2+1)\cdot(M_2(p_3) - M_1(p_3))$. By considering that $-k_2 \leq M_1(p_1) + (k_{p_1}+1)\cdot M_1(p_2) - M_2(p_1) - (k_{p_1}+1)\cdot M_2(p_2) \leq k_2$ and $M_2(p_3) - M_1(p_3)$ is an integer, we have $M_1(p_1) + (k_{p_1}+1)\cdot M_1(p_2) - M_2(p_1) - (k_{p_1}+1)\cdot M_2(p_2) = 0$ and $M_2(p_3) - M_1(p_3) = 0$. By considering case 2), $M_1(p_1) + (k_{p_1}+1)\cdot M_1(p_2) - M_2(p_1) - (k_{p_1}+1)\cdot M_2(p_2) = 0$ implies that $M_1(p_1) = M_2(p_1)$ and $M_2(p_2) = M_1(p_2)$. Consequently, we have $M_1 = M_2$ that contradicts $M_1 \neq M_2$. Thus, we have $M_1(p_o) \neq M_2(p_o)$.

4) We consider that there are n places p_1, p_2, $\ldots$, and p_n in a net model. Let $l_1 = k_0+1, l_2 = k_1+1, \ldots, l_n = k_{n-1}+1$, where $k_0 = 0$, $k_1 = k_{p_1}+1$, $k_i = k_{i-1} + (k_{i-1}+1)\cdot k_{p_i}$, and k_{p_i} is the bound of p_i $(i = 1, 2, \ldots, n)$. Then, p_o is computed to satisfy the constraint: $\sum_{i=1}^{n}(k_{i-1}+1)\cdot\mu_i + (k_{n-1}+1)\cdot\mu_n - \mu_o = 0$. In this case, $M_1(p_o) = \sum_{i=1}^{n-1}(k_{i-1}+1)\cdot M_1(p_1) + (k_{n-1}+1)\cdot M_1(p_n)$ and $M_2(p_o) = \sum_{i=1}^{n-1}(k_{i-1}+1)\cdot M_2(p_1) + (k_{n-1}+1)\cdot M_2(p_n)$. By contradiction, suppose that $M_1(p_o) = M_2(p_o)$. Then, we have $\sum_{i=1}^{n-1}(k_{i-1}+1)\cdot M_1(p_1) + (k_{n-1}+1)\cdot M_1(p_n) = (k_{i-1}+1)\cdot M_2(p_1) + (k_{n-1}+1)\cdot M_2(p_n)$, i.e., $\sum_{i=1}^{n-1}(k_{i-1}+1)\cdot(M_1(p_i) - M_2(p_i)) = (k_{n-1}+1)\cdot(M_2(p_n) - M_1(p_n))$. Similarly to case 3), we can obtain that $M_1(p_i) = M_2(p_i)$ $(i = 1, 2, \ldots, n)$. Consequently, we have $M_1 = M_2$ that contradicts $M_1 \neq M_2$. Thus, we have $M_1(p_o) \neq M_2(p_o)$.

TABLE 2. Number of constraints and variables in CTMP.

Eq.	number of constraints	variable	number of variables
(24)	1	l_i	$\lvert P\rvert$
(25)	$\lvert\mathcal{E}_t\rvert$	r_j	$\lvert\mathcal{E}_t\rvert$
(26)	$\lvert\mathcal{E}_t\rvert$	a	1
total	$2\lvert\mathcal{E}_t\rvert$+1	total	$\lvert P\rvert+\lvert\mathcal{E}_t\rvert+1$

Since $M_1(p_o) \neq M_2(p_o)$ for any two reachable marking M_1 and M_2 with $M_1 \neq M_2$, Eqs. (24), (25), and (26) are satisfied and a can be computed by Eq. (24).

Finally, we can conclude that the obtained l_i's ($i = 1, 2, \ldots, n$) and a are a feasible solution of CTMP. ■

Theorem 4: The observer place p_o obtained by CTMP is optimal and can control a t-dangerous MTSI in Ω_t.

Proof: By Proposition 1, an observer place does not forbid any LM. Eqs. (25) and (26) can gurarantee that the additional data inhibitor arc does not disable a transition $t \in T_D$ at any marking in $\mathcal{E}_t$. Thus, p_o is optimal. Eq. (24) indicates that a t-dangerous MTSI is controlled by p_o. The conclusion holds. ■

Remark 1: From the proof of Theorem 3, it can be seen that an observer place with Å(p_o, t) can be derived by constructing the coefficients l_i's ($i = (\{1, 2, \ldots, n\})$ and a without solving CTMP. In this case, it will yield too many regular (normal) arcs connected with an observer place (since all l_i's ($i = (\{1, 2, \ldots, n\})$ are greater than zero), and a big value of a. Thus, we need to solve CTMP in practice and the experimental results show that it can derive an observer place with less regular (normal) arcs and a small a in $\mathbb{A}(p_o, t)$. □

Table 2 shows the number of constraints and variables in CTMP, which can sharply affect the computational cost of solving an ILP.

Definition 8: Let Ω_t be a set of t-dangerous MTSIs. If an optimal observer place with Å(p_o, t) satisfies constraints $\sum_{i=1}^{n} l_i \cdot \mu_i - a = 0$, then the set of t-dangerous MTSIs that can be controlled by the observer place with Å(p_o, t) is denoted as $\Omega^d_{p_o} = \{(M, t) \in \Omega_t \mid \sum_{i=1}^{n} l_i \cdot M(p_i) = a\}$.

Definition 8 implies that an optimal observer place with a data inhibitor arc may control more than one t-dangerous MTSI. Thus, the structure of supervisor can be simpler and we can solve fewer times of ILPs.

VII. DEADLOCK PREVENTION STRATEGY

In this section, we present a two-stage deadlock prevention strategy according to the contents in the above sections, which can achieve the optimal control purpose for any bounded PN model. Let p_o be an observer place with $H(p_o, t)$ and the set of MTSIs that are controlled by p_o with $H(p_o, t)$ be $\Omega^w_{p_o}$. Now, Algorithm 1 shows the proposed control strategy.

The computational cost of Algorithm 1 is discussed below. First, we should compute the RG of a net, which is subject to the state explosion problem since the number of reachable markings increases exponentially wrt the size of a net.

Algorithm 1 Design of Optimal Observer Places With Weighted and Data Inhibitor Arcs via MNMP(t) and CTMP

Input: a PN model (N, M_0) suffering from deadlocks.
Output: an optimally controlled PN model (N_c, M_c).
1. Generate the RG of the given PN (N, M_0).
2. Compute the set T_D.
3. $O := \emptyset$. /*O represents the set of observer places with weighted and data inhibitor arcs to be designed.*/
4. **foreach** $t \in T_D$ **do**
  Derive the set $\mathcal{G}_t$ and the set $\mathcal{D}_t$ from the set $\mathcal{M}_L$.
  Derive the set $\mathcal{E}_t$ from $\mathcal{G}_t$.
  Compute the set Ω_t according to the set $\mathcal{D}_t$.
 4.1.**while** $\{\Omega_t \neq \emptyset\}$ **do**
   Solve MNMP(t).
   If there is no optimal solution with $q^* > 0$, **then** go to Step 4.2.
    else Obtain the solution l_i ($i \in \{1, 2, \ldots, n\}$) and γ.
   endif
   Design observer place p_o with a weighted inhibitor arc $H(p_o, t)$ using the approach in Section IV.
   $O := O \cup \{p_o\} \cup \{H(p_o, t)\}$ and $\Omega_t = \Omega_t \backslash \Omega^w_{p_o}$.
  endwhile
 4.2 **while** $\{\Omega_t \neq \emptyset\}$ **do**
   foreach $(M, t) \in \Omega_t$
    Solve CTMP.
    Obtain the solution l_i ($i \in \{1, 2, \ldots, n\}$) and a.
    Design observer place p_o with a data inhibitor arc Å(p_o, t) using the approach in Section VI.
    $O := O \cup \{p_o\} \cup \{$Å$(p_o, t)\}$ and $\Omega_t = \Omega_t \backslash \Omega^d_{p_o}$.
  endwhile
5. Add all observer places with weighted and data inhibitor arcs to (N, M_0) and denote the resulting system as (N_c, M_c).
6. Output (N_c, M_c).
7. End.

Second, we need to solve ILPs, which are NP-hard. When we solve an ILP, the computational time is mainly dependent on the number of constraints and variables. Therefore, a two-stage control method is proposed to decrease the computational burden of solving ILPs compared with the work in [7].

Theorem 5: Algorithm 1 can find an optimal PN supervisor for any bounded PN model.

Proof: If MNMP(t) has a solution with $q^* > 0$, according to Theorem 1, the obtained observer place is optimal and can control some MTSIs. Then, we remove all the MTSIs controlled by the derived observer place with weighted inhibitor arc. The process in this stage cannot terminate until all t-dangerous MTSIs are controlled or the ILP MNMP(t) has no solution with $q^* > 0$. If MNMP(t) has no solution with $q^* > 0$ and there still exist some t-dangerous MTSIs in Ω_t, in the second control stage, at each iteration, for a t-dangerous MTSI that cannot be controlled at the first control stage, we design an optimal observer place with a data inhibitor arc. By Theorem 3, we know that CTMP has a solution for any bounded net model. Then, according to Theorem 3 and Definition 8, we can definitely obtain an optimal observer place with a data inhibitor arc to control one or more t-dangerous MTSIs for any bounded PN. This stage terminates after all t-dangerous MTSIs are controlled. Finally, for each $t \in T_D$, its dangerous MTSIs are controlled. Thus, all MTSIs are controlled. The resulting controlled net is live and all LMs are kept. ■

Remark 2: Now, we discuss the details of Algorithm 1. Step 1 computes the RG of the given PN (N, M_0) and Step 2 derives the set of t-dangerous transitions T_D from the RG. Step 3 initializes the set of observer places with weighted and data inhibitor arcs as O. In Step 4, for each t-dangerous transition $t \in T_D$, the algorithm first obtains the set of t-enabled good markings $\mathcal{E}_t$ and the set of t-dangerous MTSIs Ω_t. Then, in Step 4.1, at each iteration, an optimal observer place with a weighted inhibitor arc is designed by solving an ILP named MNMP(t) to control as many t-dangerous MTSIs as possible and the controlled MTSIs are removed from the set Ω_t. This process terminates after all t-dangerous MTSIs are controlled or MNMP(t) has no optimal solution with a positive objective value q^*. If MNMP(t) has no optimal solution with $q^* > 0$ and there still exist some uncontrolled t-dangerous MTSIs, then Step 4.2 designs an optimal observer place with a data inhibitor arc by solving an ILP named CTMP to control an MTSI at each iteration. And the controlled MTSIs are removed from the set Ω_t. The second control stage terminates after all MTSIs are controlled. Finally, Step 5 adds all observer places with weighted and data inhibitor arcs to the plant and the resulting net system is live with all LMs kept according to Theorem 5. Moreover, since the proposed algorithm introduces weighted and data inhibitor arcs to the plant, the resulting net is no longer a PN and thus the conventional net analysis cannot be done on it.

In Algorithm 1, an observer place with a weighted inhibitor arc can optimally control the MTSIs for one dangerous transition. Thus, the controlled results are independent of the selected order of dangerous transitions in the first stage and they do not impact on CTMP in the second stage. □

Remark 3: According to Theorem 5, given a net (N, M_0), for each $t \in T_D$, if all the t-dangerous MTSIs are controlled by MNMP(t), then we can use the observer places with weighted inhibitor arcs to optimally control this net only. □

VIII. EXPERIMENTAL RESULTS

In this section, we show the performance of the proposed control strategy for the example in Fig. 6 from [52] in detail. In practice, the proposed method and the methods in the related literature are implemented in a notebook computer under the Windows 7 operating system with an Intel Core 2.6 GHz CPU and 8 GB memory. Lingo [26] is selected as an ILP solver to obtain optimal solution for each ILP in this section.

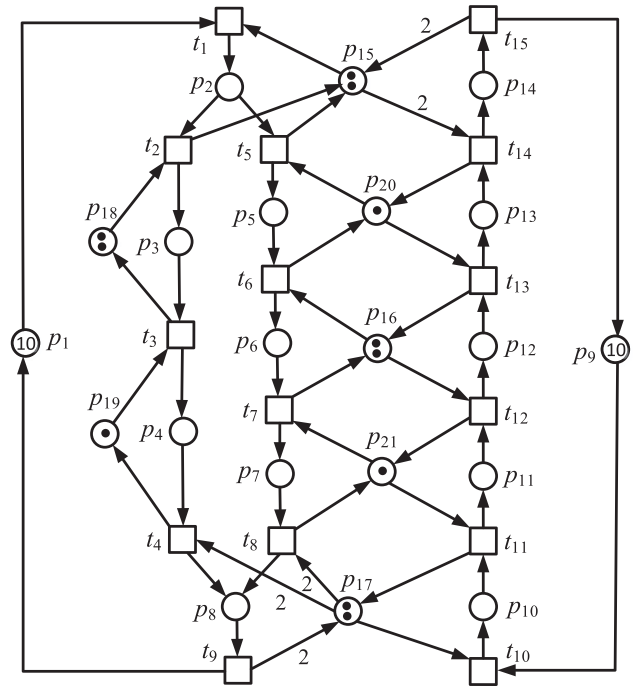

FIGURE 6. A PN model in [52].

The net has 3982 markings in its RG and 2888 LMs. It has six dangerous transitions t_1, t_5, t_6, t_7, t_{10}, and t_{12}, i.e., $T_D = \{t_1, t_5, t_6, t_7, t_{10}, t_{12}\}$. For these dangerous transitions, we obtain $|\Omega_{t_1}| = 33$, $|\Omega_{t_5}| = 197$, $|\Omega_{t_6}| = 54$, $|\Omega_{t_7}| = 206$, $|\Omega_{t_{10}}| = 390$, and $|\Omega_{t_{12}}| = 68$; $|\mathcal{E}_{t_1}| = 1552$, $|\mathcal{E}_{t_5}| = 419$, $|\mathcal{E}_{t_6}| = 428$, $|\mathcal{E}_{t_7}| = 336$, $|\mathcal{E}_{t_{10}}| = 1078$, and $|\mathcal{E}_{t_{12}}| = 676$.

For dangerous transition t_1, we first use MNMP(t_1) to design observer places with weighted inhibitor arcs to optimally control as many MTSIs in Ω_{t_1} as possible. The results are shown in Table 3.

TABLE 3. The results of MNMP(t_1) for Fig. 6.

i	N_{con}	N_{var}	${}^\bullet p_{o_i}$	$p_{o_i}^\bullet$	$H(p_{o_i}, t_1)$	$M_0(p_{o_i})$	time
1	1585	55	$t_1\ t_5\ 2t_{10}$	$t_4\ t_7\ 2t_{13}$	13	0	6 s
2	1570	40	$2t_2\ t_6\ t_{10}$	$2t_4\ t_7\ t_{14}$	11	0	4 s
3	1564	34	$2t_1\ t_6\ t_{10}$	$2t_3\ 2t_5\ t_7\ t_{14}$	11	0	1 s
4	1558	28	$t_1\ 2t_6\ 2t_{10}\ 2t_{12}$	$t_4\ t_5\ 2t_7\ 2t_{11}\ 2t_{14}$	13	0	< 1 s
5	1556	26	$t_1\ 2t_6\ 2t_{10}\ 2t_{13}$	$t_4\ t_5\ 2t_7\ 2t_{12}\ 2t_{14}$	13	0	< 1 s

In the first control stage, we can optimally control 30 MTSIs in Ω_{t_1}. For the rest of three MTSIs (M_1, t_1), (M_2, t_1) and (M_3, t_1) in Ω_{t_1}, where $M_1 = 6p_1 + p_2 + p_3 + p_4 + p_6 + 6p_9 + p_{10} + p_{11} + p_{12} + p_{13} + p_{15} + p_{17} + p_{18}$, $M_2 = 7p_1+p_2+p_3+p_4+5p_9+2p_{10}+p_{11}+p_{12}+p_{13}+p_{15}+p_{16}+p_{18}$, and $M_3 = 7p_1 + p_2 + p_3 + p_4 + 5_9 + p_{10} + p_{11} + 2p_{12} + p_{13} + p_{15} + p_{17} + p_{18}$, we solve CTMP to design an observer place with a data inhibitor arc to optimally control each of them, respectively. Actually, by Algorithm 1, we need to solve two ILPs only to optimally control the three MTSIs. The results of the second stage to control MTSIs in Ω_{t_1} are shown in Table 4.

For dangerous transition t_{10}, we first use MNMP(t_{10}) to design observer places with weighted inhibitor arcs to

TABLE 4. The results of CTMP to control MTSIs in Ω_{t_1} for Fig. 6.

i	N_{con}	N_{var}	${}^\bullet p_{o_i}$	$p_{o_i}^\bullet$	$\mathbb{A}(p_{o_i}, t_1)$	$M_0(p_{o_i})$	time
1	3105	1574	$2t_4$ $3t_8$ $4t_{15}$	$2t_1$ t_5 t_{10} $2t_{11}$ t_{14}	$\{5\}$	24	5 h 33 min 52 s
2	3105	1574	$2t_6$ t_8 $2t_9$ $2t_{12}$ $4t_{15}$	$2t_1$ t_5 $2t_7$ $3t_{10}$ $2t_{13}$ t_{14}	$\{5\}$	24	4 h 34 min 20 s

optimally control as many MTSIs in $\Omega_{t_{10}}$ as possible. The results are shown in Table 5.

In the first control stage, we can optimally control 388 MTSIs in $\Omega_{t_{10}}$. For the rest of two MTSIs (M_4, t_{10}) and (M_5, t_{10}), where $M_4 = 6p_1+p_2+2p_3+p_4+6p_9+p_{10}+p_{11}+p_{12}+p_{13}+p_{15}+p_{16}+p_{17}$ and $M_5 = 6p_1+2p_2+p_3+p_4+6p_9+p_{10}+p_{11}+p_{12}+p_{13}+p_{16}+p_{17}+p_{18}$, we solve CTMP to design an observer place with a data inhibitor arc to optimally control each of them, respectively. The results of the second stage to control MTSIs in $\Omega_{t_{10}}$ are shown in Table 6.

TABLE 5. The results of MNMP(t_{10}) for Fig. 6.

i	N_{con}	N_{var}	${}^\bullet p_{o_i}$	$p_{o_i}^\bullet$	$H(p_{o_i}, t_{10})$	$M_0(p_{o_i})$	time
1	1468	412	t_1 $15t_5$ $6t_6$ $22t_7$ $6t_{10}$	t_4 $44t_8$ $6t_{13}$	44	0	3 min 38 s
2	1110	54	$2t_1$ t_{10}	$2t_3$ t_5 t_7 t_{14}	12	0	4 s
3	1094	38	t_1 $2t_6$ $2t_{11}$	t_4 t_5 $2t_7$ $2t_{14}$	12	0	1 s
4	1090	34	t_1 t_5 $2t_{10}$	t_4 $2t_6$ $2t_{13}$	12	0	1 s
5	1086	30	t_1 $2t_6$ $2t_{10}$ $2t_{12}$	t_4 t_5 $2t_7$ $2t_{11}$ $2t_{14}$	12	0	1 s
6	1082	26	t_1 $2t_6$ $2t_{10}$ $2t_{13}$	t_4 t_5 $2t_7$ $2t_{12}$ $2t_{14}$	12	0	< 1 s

TABLE 6. The results of CTMP to control MTSIs in $\Omega_{t_{10}}$ for Fig. 6.

i	N_{con}	N_{var}	${}^\bullet p_{o_i}$	$p_{o_i}^\bullet$	$\mathbb{A}(p_{o_i}, t_{10})$	$M_0(p_{o_i})$	time
1	2157	1100	t_1 $4t_9$ $2t_{14}$	$5t_2$ $3t_5$ $2t_8$ $2t_{10}$	$\{5\}$	24	1 h 31 min 11 s
2	2157	1100	$2t_2$ t_5 $4t_9$ $2t_{15}$	$3t_1$ $2t_3$ t_4 $2t_8$ $2t_{10}$	$\{5\}$	23	3 h 02 min 3 s

TABLE 7. The results of MNMP(t_5) for Fig. 6.

i	N_{con}	N_{var}	${}^\bullet p_{o_i}$	$p_{o_i}^\bullet$	$H(p_{o_i}, t_5)$	$M_0(p_{o_i})$	time
1	616	219	t_1 $14t_6$ $5t_{10}$ $14t_{12}$	t_4 t_5 $14t_7$ $19t_{13}$	39	0	49 s
2	455	58	$3t_6$ $2t_{10}$	$3t_7$ $2t_{12}$	7	0	< 1 s

For dangerous transitions t_5, t_6, t_7, and t_{12}, after the implementation of the first control stage, all the MTSIs in Ω_{t_5}, Ω_{t_6}, Ω_{t_7}, and $\Omega_{t_{12}}$ are optimally controlled, respectively. The experimental results are shown in Tables 7, 8, 9, and 10. Specifically, for dangerous transition t_7, since ${}^\bullet p_o = \{t_{10}\}$ and $p_o^\bullet = \{t_{11}\}$, we can use p_{10} in the plant as the observer place, i.e., no additional observer place is needed.

Finally, the optimal net supervisor is obtained with all 2888 LMs reachable. The final net supervisor consists of 15 observer places with weighted inhibitor arcs and four observer places with data inhibitor arcs, i.e., 19 observer places in total. Table 11 shows the results.

TABLE 8. The results of MNMP(t_6) for Fig. 6.

i	N_{con}	N_{var}	${}^\bullet p_{o_i}$	$p_{o_i}^\bullet$	$H(p_{o_i}, t_6)$	$M_0(p_i)$	time
1	482	76	$3t_6$ t_{10}	$3t_7$ t_{12}	4	0	< 1 s

TABLE 9. The results of MNMP(t_7) for Fig. 6.

i	N_{con}	N_{var}	${}^\bullet p_{o_i}$	$p_{o_i}^\bullet$	$H(p_{o_i}, t_7)$	$M_0(p_{o_i})$	time
1	542	228	t_{10}	t_{11}	1	0	< 1 s

TABLE 10. The results of MNMP(t_{12}) for Fig. 6.

i	N_{con}	N_{var}	${}^\bullet p_{o_i}$	$p_{o_i}^\bullet$	$H(p_{o_i}, t_{12})$	$M_0(p_{o_i})$	time
1	744	90	t_5 t_{12}	t_6 t_{13}	2	0	< 1 s

TABLE 11. The performance of Algorithm 1 for Fig. 6.

	t_1	t_5	t_6	t_7	t_{10}	t_{12}	total
No. observers[1] in the 1st stage	5	2	1	0	6	1	15
No. observers in the 2nd stage	2	0	0	0	2	0	4

[1] observers: additional observer places for the plant.

It can be seen that after the first stage control (by using observer places with weighted inhibitor arcs), the number of t-dangerous MTSIs is sharply reduced, which can reduce the computational cost at the second stage (by using observer places with data inhibitor arcs).

In fact, due to the non-convexity of its LM space, there exists no pure net supervisor to optimally control the system, e.g., the work in [1], [2], [19]–[21], and [41]–[44] cannot yield an optimal supervisor for this example. Then, we show that nonpure net supervisors obtained by the self-loop-based methods in [3] and [6] cannot optimally control the net for this example either. Let $\mathcal{E}_t$ and $\mathcal{D}_t$ be the sets of t-enabled good and t-dangerous markings of the net, respectively. We have three markings (only the tokens in operation places[1] are considered in [3] and [6]): $M_1 = p_2+2p_3+p_4+p_6+p_8+p_{11}+p_{12}+p_{13}$, $M_2 = p_2+p_4+p_6+2p_{10}+p_{11}+p_{12}+p_{13}$, and $M_3 = p_2+p_3+p_4+p_6+p_{10}+p_{11}+p_{12}+p_{13}$, where M_1 and $M_2 \in \mathcal{E}_{t_1}$, and $M_3 \in \mathcal{D}_{t_1}$. Also, we have $\sum_{i \in \mathbb{N}_A} l_i \cdot [N](p_i, t_1) = l_2$.[2] In [3], t_1 should be enabled at every marking in $\mathcal{E}_{t_1}$ while disabled at the markings in $\mathcal{D}_{t_1}$. Thus, the ILP MMP(t_1) in [3] has the following three constraints:

$$2l_2 + 2l_3 + l_4 + l_6 + l_8 + l_{11} + l_{12} + l_{13} \le \beta - \omega \tag{27}$$

$$2l_2 + l_4 + l_6 + 2l_{10} + l_{11} + l_{12} + l_{13} \le \beta - \omega \tag{28}$$

$$\begin{aligned} 2l_2 + l_3 + l_4 + l_6 + l_{10} + l_{11} + l_{12} + l_{13} &\ge -Q \cdot (1 - f_3) \\ &\quad + \beta - \omega + 1 \\ l_i \in \{0, 1, 2, \ldots\} \quad &\forall i \in \mathbb{N}_A \\ \beta, \omega \in \{1, 2, 3, \ldots\} & \\ f_3 \in \{0, 1\} & \end{aligned} \tag{29}$$

where Q is a big enough positive integer.

[1] An operation place represents an operation to be processed for a part in a production sequence, whose set is denoted as P_A.

[2] $\mathbb{N}_A$ denotes $\{i | p_i \in P_A\}$.

Then, we prove that $f_3 = 1$ is not a feasible solution for the aforementioned constraints. By contradiction, suppose that $f_3 = 1$. Then, Eq. (29) is modified as

$$2l_2 + l_3 + l_4 + l_6 + l_{10} + l_{11} + l_{12} + l_{13} \geq \beta - \omega + 1. \tag{30}$$

Further, it can be rewritten as

$$-4l_2 - 2l_3 - 2l_4 - 2l_6 - 2l_{10} - 2l_{11} - 2l_{12} - 2l_{13} \leq -2\beta + 2\omega - 2. \tag{31}$$

By adding Eqs. (27) and (28), we have the following constraint:

$$4l_2 + 2l_3 + 2l_4 + 2l_6 + l_8 + 2l_{10} + 2l_{11} + 2l_{12} + 2l_{13} \leq 2\beta - 2\omega. \tag{32}$$

By adding Eqs. (31) and (32), we have $l_8 \leq -2$, which contradicts $l_8 \in \{0, 1, 2, \ldots\}$. Thus, we can conclude that $f_3 = 0$. In this case, we cannot obtain an optimal monitor with a self-loop for the MTSI (M_3, t_1), i.e., the method in [3] cannot lead to an optimal supervisor for this net. Similarly, the method in [6] cannot obtain an optimal supervisor for the net either, since the two proposed ILPs, namely IMNM and MNCP have no solution with a positive objective value, respectively.

TABLE 12. The comparison of the experimental results for the net shown in Fig. 6 by MNTMP(t) [4], PTMP(t) [7], and MNMP(t).

t_i	N_{con} in [4]	N_{var} in [4]	time	N_{con} in [7]	N_{var} in [7]	time	N_{con} in MNMP(t)	N_{var} in MNMP(t)	time
t_1	3288	1666	57 min 6 s	105353	51271	o.t.	1585	55	6 s
t_5	1678	1025	24 min 7 s	48194	82762	o.t.	616	219	49 s
t_6	1124	605	43 s	49166	23188	o.t.	482	76	< 1 s
t_7	1548	969	13 s	141526	69444	o.t.	542	228	< 1 s
t_{10}	3768	2263	o.t.	844118	420832	o.t.	1468	412	3 min 38 s
t_{12}	1676	895	5 s	94892	46058	o.t.	744	90	< 1 s

Another approach for nonpure net supervisor is proposed in [4]. However, there is no formal proof given to demonstrate that the strategy definitely yields an optimal supervisor for any bounded net model. Moreover, when we apply MNTMP(t) in [4] to this net, the number of variables and constraints, and the computation time are shown in Table 12. From Table 12, we can see that MNTMP(t_{10}) cannot be solved within 72 hours and "o.t." denotes that the computation is out of time. In this case, the method in [4] cannot lead to an optimal supervisor for the net.

In [7], the proposed technique can definitely yield an optimal supervisor for any bounded PN in theory. However, it is inapplicable for complex system caused by the heavy computational cost in solving the proposed ILPs.

When the ILP PTMP(t) in [7] is applied to this net, the number of constraints and variables, and the computation time are shown in Table 12. From Table 12, we can see that the set of ILPs cannot be solved in a reasonable time. Moreover, another ILP PAMT [7] for this net has 1295438 constraints and 649185 variables, which cannot be solved in a reasonable time either. Therefore, in this case, we cannot derive an optimal supervisor by using the method in [7].

IX. CONCLUSIONS

This work presents a two-stage method to obtain an optimal liveness-enforcing supervisor for any bounded PN. Compared with the work in [1], [2], [19], [41], the proposed method can optimally control a net system that cannot be optimally controlled by any pure net supervisor. Compared with the work in [3], [4], [6], [11], the proposed method can definitely lead to an optimal net supervisor for any bounded PN on the premise that such a supervisor exists. Moreover, compared with the work in [7], the work can significantly reduce the computational cost in total, i.e., the proposed method is much more efficient than that in [7].

However, the developed approach is still subject to the computational complexity problem caused by the computation of the full RG and the process for solving ILPs. Another problem of the developed approach is that it cannot lead to a supervisor with the minimal number of observer places. Future work is needed to avoid generating the full RG and make the structure of the supervisor more simpler. To reduce computational cost, we will explore the learning-based method [62] to construct a liveness-enforcing supervisor when a plant model is not fully known.

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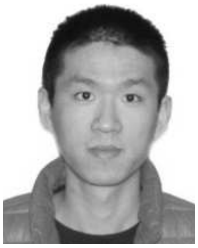

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