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Adaptive Model-Free Control Based on an Ultra-Local Model With Model-Free Parameter Estimations for a Generic SISO System

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ABSTRACT In this paper, a new structure to design model-free control (MFC) based on the ultra-local model is presented for an unknown nonlinear single-input single-output dynamic system. The proposed structure includes two adaptive laws corresponding to the unknown linear and nonlinear terms. Utilizing the adaptive law for linear term, the controller gain is going to be updated online using a differential Riccati equation. Subsequently, the control policy which includes an optimal term as well as a term for compensating the system unknown dynamics is generated. Here, the two proposed adaptive laws are model-free estimation algorithms, in which the need for any regressor parameter and also the persistent excitation condition is eliminated. Finally, two simulation studies are presented to show that the proposed adaptive MFC (AMFC) policy outperforms the two well-known controllers. Moreover, the AMFC is applied on a Duffing-Holmes chaotic oscillator plant and the convincing performance of the algorithm is observed through the simulation results.

INDEX TERMS Model-free control, optimal control, adaptive control, nonlinear system, ultra-local model, model-free estimation, chaotic systems, SISO system.

I. INTRODUCTION

The design procedure of controller for nonlinear dynamic systems can be categorized as *model-based* and *model-free* control policies. For a model-based controller to have acceptable performance, an accurate model of the dynamic system should be defined *a priori* in an off-line manner. Due to the uncertainties in a dynamic system and environmental unknown disturbances, the process for extracting the exact model is too expensive and time-consuming. Hence, the model-based controllers alone are not practical in their original format, since they need an extra process for generating the dynamic model of the plant. There are lots of adaptive algorithms proposed to deal with this problem. In the adaptive control algorithms, a structure of the dynamic system is known and only the unknown parameters are estimated online to provide the controller with the plant dynamic model. Although the adaptive control algorithms have solved the issue of unavailability of an accurate model for the plant, the assumption of known structure for the dynamic system is still a constraint in the design procedure. Therefore, a model-free controller of which does not require knowledge of the structure is proposed to remove this constraint [1]. The model-free controllers consider a general structure for an unknown dynamic system and use the measured input-output data to estimate the unknown dynamic system in an online manner and generate the control policy. Nowadays, model-free control approaches play an important role in control [2], some new model-free control algorithms are designed recently [3]–[6], and several applications can be found for these algorithms [7]–[12].

The MFC technique which is first proposed by Fliess for SISO systems, uses an ultra-local model to approximate the whole nonlinear dynamic system [13]. The ultra-local model which is an *affine* dynamic model with regards to the control input variable, includes a lumped unknown nonlinear function and *a priori*-known constant input gain. Estimation of the unknown nonlinear term is performed by a simple algebraic equation utilizing the past input-output data. A shortcoming of the initial MFC algorithm is that it has not been formulated with a set of analyses that guarantees stability [18]. The stability analysis for the MFC technique is provided in [14] and an application of the algorithm is presented for a twin-rotor system. In [15], first the ultra-local model is transformed to a linear time-invariant state-space system. Then an adaptive observer is designed to estimate both system states and unknown nonlinear dynamics of the system. The work by Thabet *et al.* [15] reveals the model-based adaptive observer to estimate the unknown input gains and unknown nonlinear terms. Moreover, since the estimation is performed utilizing a model-based algorithm, there is a requirement of PE condition for a regressor parameter. Similarly, a parametric model for estimating both unknown nonlinear term and input gain is presented in [16] with PE condition observed. In [17], the MFC is proposed for a multi-input multi-output (MIMO) dynamic system. In that paper, the MFC is formulated in a linear time-invariant system and an optimal MFC is presented. The control policy includes an optimal term which is derived from the solution of a linear quadratic regulator (LQR) problem. Since the optimal control problem is solved off-line, the controller gains should be defined before the system starts to operate. In [18], two MFC sliding mode algorithms are proposed and compared with a simple MFC according to the experimental results. Moreover, a fuzzy MFC is proposed in [19], in which a fuzzy inference system (FIS) is used as an online estimator for estimating the unknown lumped nonlinear term. A robust adaptive iterative learning control for unknown nonlinear systems is proposed in [20]. In that work, the unknown nonlinear terms are presented by multiplication of a vector of unknown parameters and a vector of known nonlinear functions (or regressors). It is to note that the main controller gains should be determined off-line (which is a barrier against going forward with fully autonomous dynamic systems) and the adaptive laws are model-based algorithms which require PE condition for regressors. Recently, Wang *et al.* [21], [22] and Zhao *et al.* [23] have developed adaptive controllers which use FIS and also Artificial Neural Network (ANN) to deal with the unmodeled dynamics and unknown uncertainties of a nonlinear system. In those algorithms, the estimations are regressor-based adaptive laws which require the definition of regressor variables and the PE condition for the regressors is a must.

Due to the PE condition, convergence of the adaptive law to the solution will be achieved if and only if the input signal is sufficiently rich (SR) [24]. The PE condition, as which is applicable for all model-based estimation, is alleviated in our regressor-free-based adaptive laws. Moreover, the need for defining regressor variables are removed in the proposed algorithm, which makes the implementation more convenient.

In this paper, a new structure for MFC based on the ultra-local model is presented. The structure is new, since the unknown lumped nonlinear term is considered to include an unknown linear term plus an unknown nonlinear function. According to the new structure, two adaptive laws are proposed for estimating both linear and nonlinear terms. Here, the adaptive laws are going to be generated in order to drive the tracking error to zero (*direct adaptation*). When the tracking objective is reached, the adaptive laws are turned off thereby, halting further adaptation. This does not affect the tracking objective. Utilizing this technique, we can use adaptive controllers for any time-varying reference signal which can be non-SR. Having updated value for the unknown linear term, the main controller gain is computed online using a scalar DRE. Such new MFC with optimal term incorporated yields a new structure of controller we called Adaptive Model-Free Control or AMFC. Stability and optimality analyses are provided the proposed AMFC. Our contributions can be listed as follows

- a new structure for MFC is proposed, which segments unknown lumped nonlinear term into an unknown linear-in-state part and an unknown nonlinear part;
- online tuning for the controller gain is provided;
- the proposed MFC includes an optimal control policy;
- the proposed adaptive laws for estimating unknown dynamics of the system are model-free estimation algorithms which lead to no requirement for PE condition and also no need to define regressor variables.

In the following sections, first a new structure for the ultra-local model is proposed. Then, the design procedure for the AMFC including the stability and optimality analyses is presented in Section III. Finally, the simulation results are presented in Section IV to compare the performance of the proposed AMFC with two well-known controllers over a nonlinear system and a delay system. The simulation results for application of the AMFC on an oscillatory system is presented in Section V.

II. THE NEW STRUCTURE FOR ULTRA-LOCAL MODEL

Definition 1: The unknown nonlinear dynamics of a SISO nonlinear system can be represented using the following *ultra-local model* [13]

$$\begin{aligned} \dot{x} &= f(x) + bu \\ y &= x \end{aligned} \tag{1}$$

where, $x \in \mathcal{R}$ is the system state, $f : \mathcal{R} \to \mathcal{R}$ is the unknown Lipschitz bounded nonlinear function depending only on x (f also can include bounded disturbances), parameter $b \in \mathcal{R}$ is a non-zero non-physical priori-known constant input gain which is chosen by the practitioner such that bu and $\dot{x}$ are of the same order [13] (the value of b mostly can be considered as 1), u is the control input and y is the system output.

Here, we express f as follows

$$f(x) = ax + g, \tag{2}$$

where $a = a(t) \in \mathcal{R}$ is an unknown time-varying state gain which is bounded and Lipschitz; and $g = g(x)$ is another unknown Lipschitz bounded nonlinear function with the Lebesgue measurable property. According to (2), we have divided f into one linear-in-state part and one nonlinear part. Hence, the system proposed in (1) can be presented as follows

$$\begin{aligned} \dot{x} &= ax + bu + g \\ y &= x. \end{aligned} \tag{3}$$

III. DESIGN PROCEDURE FOR ADAPTIVE MODEL-FREE CONTROL

Definition 2: Considering a time-varying reference signal y_d, the tracking error can be proposed as [33]

$$e = y_d - y = y_d - x. \tag{4}$$

The tracking objective is defined as converging e to zero, when time goes to infinity. Moreover, we define a joint cost function

$$\sigma = e + \zeta \tag{5}$$

where, $\zeta = \int edt$. By accompanying the tracking error with its time integral, the steady-state error can be eliminated [31].

A. AMFC POLICY DESIGN PROCEDURE

Lemma 1 [30]: Based on the Separation Principle, the combination of a stable controller and a stable observer leads to a stable dynamic system. For further readings, please refer to [30].

Lemma 2 [26]: Recalling the sliding-mode differentiator [26], the derivative of a reference signal y_d can be estimated as

$$\dot{y}_d = \nu, \tag{6}$$

where

$$\begin{aligned} \dot{z} &= \nu \\ \nu &= -k_1|z - y_d|^{1/2} sgn(z - y_d) + \nu_1 \\ \dot{\nu}_1 &= -k_2 sgn(z - y_d) \end{aligned} \tag{7}$$

with $k_1 > 0$ and $k_2 > 0$ are two constant parameters and $sgn(.)$ is the signum function. The differentiator presented in (6) and (7), is going to be used in the design procedure for AMFC, wherever $\dot{y}_d$ is required. For further readings, please refer to [26].

Theorem 1: For the SISO dynamic system proposed in (3), if we construct the control input $u = u_1 + u_2$ as follows

$$\begin{aligned} u_1 &= \frac{1}{2} rbP\sigma \\ u_2 &= \frac{1}{b}[\dot{y}_d - \hat{a}(x - \sigma) - \hat{g} - \zeta + [1 + \frac{2q}{P}]\sigma] - \frac{3}{4} rbP\sigma \end{aligned} \tag{8}$$

where $r > 0$, $q > 0$ are two constant values and $P > 0$ is updated online using the following scalar DRE

$$\dot{P} = 2\hat{a}P - rb^2P^2 + 2q, \quad P(0) > 0 \tag{9}$$

with the following adaptive laws

$$\begin{aligned} \dot{\hat{g}} &= -\gamma_1 P\sigma - \rho_1\gamma_1\hat{g} \\ \dot{\hat{a}} &= -\gamma_0 P\sigma(x - \sigma) - \rho_0\gamma_0\hat{a} \end{aligned} \tag{10}$$

where γ_0, γ_1, ρ_0 and ρ_1 are constant positive gains; then the tracking objective presented in *Definition 2* will be achieved.

Proof: By considering the estimation errors $\tilde{g} = g - \hat{g}$ and $\tilde{a} = a - \hat{a}$, we define the following Lyapunov function

$$V = \frac{1}{2}P\sigma^2 + \frac{1}{2\gamma_1}\tilde{g}^2 + \frac{1}{2\gamma_0}\tilde{a}^2. \tag{11}$$

The time derivative of V is

$$\dot{V} = P\sigma\dot{\sigma} + \frac{1}{2}\sigma^2\dot{P} + \frac{1}{\gamma_1}\dot{\tilde{g}}\tilde{g} + \frac{1}{\gamma_0}\dot{\tilde{a}}\tilde{a}. \tag{12}$$

Then, we have

$$\dot{V} = P\sigma(\dot{y}_d - ax - bu - g + e) + \frac{1}{2}\sigma^2\dot{P} + \frac{1}{\gamma_1}\dot{\tilde{g}}\tilde{g} + \frac{1}{\gamma_0}\dot{\tilde{a}}\tilde{a}. \tag{13}$$

We add and subtract $Pa\sigma^2$ to the right-hand side of (13). Thus, it will be presented as

$$\dot{V} = P\sigma(\dot{y}_d - a(x - \sigma) - bu - g + e - a\sigma) + \frac{1}{2}\sigma^2\dot{P} + \frac{1}{\gamma_1}\dot{\tilde{g}}\tilde{g} + \frac{1}{\gamma_0}\dot{\tilde{a}}\tilde{a}. \tag{14}$$

Then, by adding and subtracting $P\sigma(\hat{g}+\hat{a}(x-\sigma))$ to the right-hand side of (14), we have

$$\dot{V} = P\sigma[\dot{y}_d - \hat{a}(x - \sigma) - bu - \hat{g} + e - a\sigma] + \frac{1}{2}\sigma^2\dot{P} + [\frac{1}{\gamma_1}\dot{\tilde{g}}\tilde{g} - P\sigma\tilde{g}] + [\frac{1}{\gamma_0}\dot{\tilde{a}}\tilde{a} - P\sigma(x - \sigma)\tilde{a}]. \tag{15}$$

Hence,

$$\begin{aligned} \dot{V} = &P\sigma[\dot{y}_d - \hat{a}(x - \sigma) - bu - \hat{g} + e - a\sigma] \\ &+ \frac{1}{2}\sigma^2\dot{P} + [\frac{1}{\gamma_1}\dot{g}\tilde{g} - \frac{1}{\gamma_1}\dot{\hat{g}}\tilde{g} - P\sigma\tilde{g}] \\ &+ [\frac{1}{\gamma_0}\dot{a}\tilde{a} - \frac{1}{\gamma_0}\dot{\hat{a}}\tilde{a} - P\sigma(x - \sigma)\tilde{a}]. \end{aligned} \tag{16}$$

Referring to *Definition 1*, $a = a(t)$ and $g = g(x(t))$ are time-varying but bounded, i.e. $\dot{g} \neq 0$ and $\dot{a} \neq 0$. By adding and subtracting

$$s_1 = [\frac{1}{4\rho_1}(\frac{1}{\gamma_1}\dot{g} + \rho_1 g)^2 + \rho_1\tilde{g}^2 + \rho_1\tilde{g}\hat{g}] \tag{17}$$

and

$$s_2 = [\frac{1}{4\rho_0}(\frac{1}{\gamma_0}\dot{a} + \rho_0 a)^2 + \rho_0\tilde{a}^2 + \rho_0\tilde{a}\hat{a}] \tag{18}$$

and also the term $P\sigma\zeta$, we lead to

$$\begin{aligned} \dot{V} = &P\sigma[\dot{y}_d - \hat{a}(x - \sigma) - bu - \hat{g} - \zeta + (1 - a)\sigma] + \frac{1}{2}\sigma^2\dot{P} \\ &- [(\rho_1\hat{g} + \frac{1}{\gamma_1}\dot{\hat{g}} + P\sigma)\tilde{g}] - [\frac{1}{4\rho_1}(\frac{1}{\gamma_1}\dot{g} + \rho_1 g)^2 + (\sqrt{\rho_1}\tilde{g})^2 \\ &-2(\sqrt{\rho_1}\tilde{g})(\frac{1}{2\sqrt{\rho_1}})(\rho_1\tilde{g} + \rho_1\hat{g} + \frac{1}{\gamma_1}\dot{g})] + \frac{1}{4\rho_1}(\frac{1}{\gamma_1}\dot{g} + \rho_1 g)^2 \\ &- [(\rho_0\hat{a} + \frac{1}{\gamma_0}\dot{\hat{a}} + P\sigma(x - \sigma))\tilde{a}] - [\frac{1}{4\rho_0}(\frac{1}{\gamma_0}\dot{a} + \rho_0 a)^2 \\ &+ (\sqrt{\rho_0}\tilde{a})^2 - 2(\sqrt{\rho_0}\tilde{a})(\frac{1}{2\sqrt{\rho_0}})(\rho_0\tilde{a} + \rho_0\hat{a} + \frac{1}{\gamma_0}\dot{a})] \\ &+ \frac{1}{4\rho_0}(\frac{1}{\gamma_0}\dot{a} + \rho_0 a)^2. \end{aligned} \tag{19}$$

Using the adaptive laws proposed in (10), the third and sixth terms in (19) are zero. Hence, we reach to

$$\dot{V} = P\sigma[\dot{y}_d - \hat{a}(x-\sigma) - bu - \hat{g} - \zeta + (1-a)\sigma] + \frac{1}{2}\sigma^2\dot{P} + \frac{1}{4\rho_1}(\frac{1}{\gamma_1}\dot{g} + \rho_1 g)^2 - [(\sqrt{\rho_1}\tilde{g}) - \frac{1}{2\sqrt{\rho_1}}(\frac{1}{\gamma_1}\dot{g}+\rho_1 g)]^2 + \frac{1}{4\rho_0}(\frac{1}{\gamma_0}\dot{a} + \rho_0 a)^2 - [(\sqrt{\rho_0}\tilde{a}) - \frac{1}{2\sqrt{\rho_0}}(\frac{1}{\gamma_0}\dot{a}+\rho_0 a)]^2. \quad (20)$$

Since f is bounded and Lipshitz, g and a are two bounded Lipschitz functions (refer to *Definition 1*). Consequently, we have $|g| \leq M_g$, $|\dot{g}| \leq M_{\dot{g}}$, $|a| \leq M_a$, $|\dot{a}| \leq M_{\dot{a}}$, where M_g, $M_{\dot{g}}$ M_a, $M_{\dot{a}}$ are four positive constants. Thus, we lead to the following inequality

$$\dot{V} \leq P\sigma[\dot{y}_d - \hat{a}(x-\sigma) - bu - \hat{g} - \zeta + (1-a)\sigma] + \frac{1}{2}\sigma^2\dot{P} - H_1 + \delta, \quad (21)$$

where

$$H_1 = [(\sqrt{\rho_1}\tilde{g}) - \frac{1}{2\sqrt{\rho_1}}(\frac{1}{\gamma_1}\dot{g} + \rho_1 g)]^2 + [(\sqrt{\rho_0}\tilde{a}) - \frac{1}{2\sqrt{\rho_0}}(\frac{1}{\gamma_0}\dot{a} + \rho_0 a)]^2 \quad (22)$$

and

$$\delta = \frac{1}{4\rho_1}(\frac{1}{\gamma_1}M_{\dot{g}} + \rho_1 M_g)^2 + \frac{1}{4\rho_0}(\frac{1}{\gamma_0}M_{\dot{a}} + \rho_0 M_a)^2. \quad (23)$$

Note that δ is a positive constant. Then by replacing $u = u_1 + u_2$ as proposed in (8), we have

$$\dot{V} \leq P\sigma[(-\frac{2q}{P}\sigma) + (-a\sigma) + (-\frac{1}{2}rb^2P\sigma) + (+\frac{3}{4}rb^2P\sigma)] + \frac{1}{2}\sigma^2\dot{P} - H_1 + \delta. \quad (24)$$

The equation in (24) can be written as

$$\dot{V} \leq \sigma^2[-2q - aP + \frac{1}{4}rb^2P^2 + \frac{1}{2}\dot{P}] - H_1 + \delta. \quad (25)$$

Then, by utilizing

$$\dot{P} = 2aP - rb^2P^2 + 2q, \quad (26)$$

we reach to

$$\dot{V} \leq -[q + \frac{1}{4}rb^2P^2]\sigma^2 - H_1 + \delta. \quad (27)$$

Here, u_1 can be considered as the *tracking part* of the control signal, while u_2 is the *compensating part*. Finally, by defining $H_2 = \{[q + \frac{1}{4}rb^2P^2]\sigma^2 + H_1\} > 0$, we have

$$\dot{V} \leq -H_2 + \delta. \quad (28)$$

Referring to the LaSalle-Yoshizawa theorem [25], V is uniformly ultimately bounded (UUB). Since V includes the tracking error and the estimation errors, we can deduce that σ, $\tilde{g}$ and $\tilde{a}$ converge to a small radially bounded space around origin. Finally, since $\tilde{a}$ is converging to zero, we can use $\hat{a}$ instead of a in (26) by recalling *Lemma 1*. Then, we reach to (9). This completes the proof.

Remark 1: The parameter δ defined in (23), can be expanded as

$$\delta = [(\frac{1}{4\rho_1\gamma_1^2}M_{\dot{g}}^2) + (\frac{1}{4}\rho_1 M_g^2) + (\frac{1}{2\gamma_1}M_{\dot{g}}M_g)] + [(\frac{1}{4\rho_0\gamma_0^2}M_{\dot{a}}^2) + (\frac{1}{4}\rho_0 M_a^2) + (\frac{1}{2\gamma_0}M_{\dot{a}}M_a)]. \quad (29)$$

By choosing γ_1 and γ_0 large enough and also ρ_1 and ρ_0 adequately small, the value of δ will decrease and can reach to zero. Therefore, the convergence of σ, $\tilde{a}$ and $\tilde{g}$ can be satisfied faster.

Remark 2: The value of P as the main gain for proposed controller and adaptive laws in *Theorem 1*, is updated online utilizing (9). Referring to this property, there is not too much effort to tune other controller gains (i.e. r, q) off-line. Moreover, since the value of P are computed using the DRE in (26), it is confirmed that $P > 0$ which is a requirement for the stability analysis proposed in *Theorem 1*. For further discussions, refer [27] and [28].

Definition 3: For the dynamic system proposed in (3) with the tracking objective defined in *Definition 2*, we define a *cost-to-go* function for time interval $[t, \infty)$ as

$$J = \int_t^\infty L(\sigma, u)d\tau, \quad (30)$$

where

$$L = q\sigma^2 + \frac{1}{r}u_1^2 \quad (31)$$

is the *utility* function which presents the cost at each time step of system operation.

Lemma 3: The cost-to-go function J can be represented by

$$J = \frac{1}{2}P\sigma^2 \quad (32)$$

as an energy function at the current time step t.

Proof: Consider

$$\mathcal{S} = \int_t^\infty \frac{d}{d\tau}[\frac{1}{2}P\sigma^2]d\tau. \quad (33)$$

Then, we have

$$\mathcal{S} = V_1(\infty) - V_1(t) \quad (34)$$

where

$$V_1(t) = \frac{1}{2}P\sigma^2(t). \quad (35)$$

Recalling *Theorem 1*, we know that V_1 converges to zero when time reaches to infinity (i.e. $V_1(\infty) = 0$). Hence,

$$\mathcal{S} = -V_1(t). \quad (36)$$

By adding and subtracting S (defined in (33) and (36)) from the right-hand side of (30), we have [29]

$$J = V_1(t) + \int_t^\infty [\frac{d}{d\tau}V_1 + q\sigma^2 + \frac{1}{r}u_1^2]d\tau. \quad (37)$$

Besides, referring to *Lemma 1* and utilizing *Theorem 1*, we can have the system dynamics as follows

$$\dot{x} = \hat{a}x + bu + \hat{g}. \tag{38}$$

Hence, by replacing V_1 from (35) in (37) and also utilizing (38), we have

$$J = V_1(t) + \int_t^{\infty} [P\sigma(\dot{y}_d - \hat{a}x - bu - \hat{g} + e) + \frac{1}{2}\sigma^2\dot{P} + q\sigma^2 + \frac{1}{r}u_1^2]d\tau. \tag{39}$$

Then, by utilizing $u = u_1 + u_2$ defined in (8), we reach to

$$J = V_1(t) + \int_t^{\infty} [-2q - \hat{a}P - \frac{1}{2}rb^2P^2 + \frac{3}{4}rb^2P^2 + \frac{1}{2}\dot{P} + q + \frac{1}{4}rb^2P^2]\sigma^2 d\tau, \tag{40}$$

where the term inside the bracket is zero using the scalar DRE defined in (9). Finally, it follows to (32) and the proof is completed.

Proposition 1 [29]: For the dynamic system defined in (3) and the cost-to-go and utility functions defined in *Definition 3*, the optimal control can be achieved by utilizing the Hamilton-Jacobi-Bellman (HJB) equation as follows [29]

$$0 = \min_{u=u_{op}} \{L(\sigma, u) + \frac{dJ(\sigma)}{dt}\}. \tag{41}$$

Theorem 2: For the dynamic system proposed in (3) with the cost-to-go and utility functions defined in (32) and (31), the designed control input suggested in (8) includes an optimal control policy.

Proof: By replacing J and L from (31) and (32) in (41), we have

$$0 = \min_{u=u_{op}} [q\sigma^2 + \frac{1}{r}u_1^2 + P\sigma\dot{\sigma} + \frac{1}{2}\sigma^2\dot{P}]. \tag{42}$$

By computing $\dot{\sigma}$ according to (4) and (5) and using (38), we lead to

$$0 = \min_{u=u_{op}} [q\sigma^2 + \frac{1}{r}u_1^2 + P\sigma(\dot{y}_d - \hat{a}x - bu - \hat{g} + e) + \frac{1}{2}\sigma^2\dot{P}]. \tag{43}$$

Then by utilizing u proposed in (8), we reach to

$$0 = \min_{u=u_{op}} [q\sigma^2 + \frac{1}{4}rb^2P^2\sigma^2 + (-2q - \hat{a}P - \frac{1}{2}rb^2P^2 + \frac{3}{4}rb^2P^2 + \frac{1}{2}\dot{P})\sigma^2], \tag{44}$$

where it is zero utilizing the DRE defined in (9). Moreover, by computing the derivative of (43) with respect to u_1, we have

$$0 = +\frac{2}{r}u_1 - bP\sigma, \tag{45}$$

which can be satisfied by replacing u_1 from (8). It means that $u = u_{op} + u_2$, where $u_{op} = u_1$ satisfies the HJB equation presented in (41). Then, the proof is completed.

IV. COMPARATIVE RESULTS

A. CASE-1: UNSTABLE NONLINEAR SYSTEM

In this section, an unstable nonlinear system is considered for evaluating the performance of the proposed AMFC. Dynamic system for the unstable plant is considered as follows [13]

$$\dot{x} = x + u^3$$
$$y = x. \tag{46}$$

Here, we have compared the performance of AMFC with a well-known sliding-mode controller (SMC) defined as [26]

$$u = -\frac{\dot{y}_d - e + \rho_s sat(\sigma)}{b}$$
$$sat(\sigma) = \frac{\sigma}{|\sigma| + \epsilon} \tag{47}$$

and an MFC policy proposed in [13] as

$$u = -\frac{\hat{f} - \dot{y}_d - k_p e - k_i \zeta}{b}$$
$$\hat{f} = \frac{1}{l_0}\int_{t-l_0}^{t} [\dot{y}_d - bu + k_p e + k_i \zeta]d\tau. \tag{48}$$

The MFC presented in (48) is known as intelligent-PI (iPI) in the literature. Besides, we have considered the cost function

$$\mathcal{C} = \int_0^{t_f} [e^2 + u^2]d\tau, \tag{49}$$

TABLE 1. Properties of the controllers in Case-1.

Parameter	SMC	iPI	AMFC
Tuning parameters	$\rho_s = 10$ $\epsilon = 0.1$	$k_p = 100$ $k_i = 100$ $l_0 = 10$	$\gamma_0 = 1e3$ $\gamma_1 = 1e5$ $\rho_0 = 1e-3$ $\rho_1 = 1e-2$
Value of $\mathcal{C}$	23.26	23.26	23.54
Dominant freq. (Hz)	40, 89 118, 160	23, 63	-

where t_f is the terminal time for the simulation study, as a measure to compare the controllers regarding their performance. It should be noted that $\mathcal{C}$ includes the tracking error and the control effort, both. Hence the controller with less tracking error but too much control effort is also penalized. Tuning parameters for the controllers are defined such that we can have almost equal values of $\mathcal{C}$ among them, since the controllers are strong enough to have the same tracking performance (with minor differences). In this way, we can compare the controllers regarding the fluctuations in the control signals. The properties of three mentioned controllers for simulation of the dynamic system in (46) are presented in Table 1. Here, $r = q = 1$. The tuning parameters for AMFC are chosen to have fast convergence in the adaptive laws, as suggested in *Remark 1*. As mentioned before, the values of $\mathcal{C}$ corresponding to each controller are kept almost the same by choosing the appropriate tuning parameters in SMC and iPI.

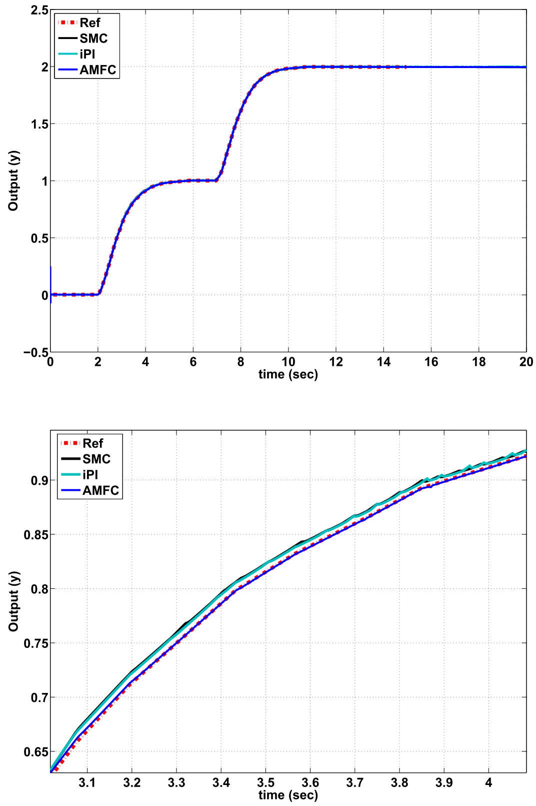

FIGURE 1. Case-1: Tracking performance; the whole (top) and in detail (bottom).

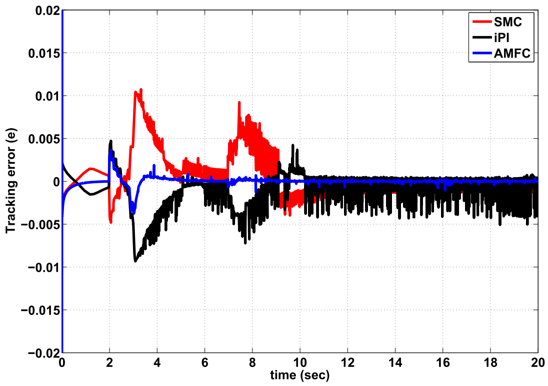

FIGURE 2. Case-1: Tracking error.

The simulation results for comparison of the controllers' performance are depicted in Fig. (1) to Fig. (5). As can be seen, the tracking performance of AMFC is superior to those of iPI and SMC. Moreover, the convergence of unknown linear and nonlinear parameters and consequently the controller gain to finite values are observed using AMFC. The control signals produced by SMC and iPI exhibit fluctuations and aggressive perturbations. On contrary, the signal for AMFC is suggesting a more *smooth* control effort. This is shown in Fig. (6), where the fast Fourier transform (FFT) of the control input signals produced by SMC, iPI and AMFC are compared. The control signals generated by SMC and iPI have several dominant frequencies less than 150 Hz as presented in Table 1. The control signal produced by AMFC does not have any

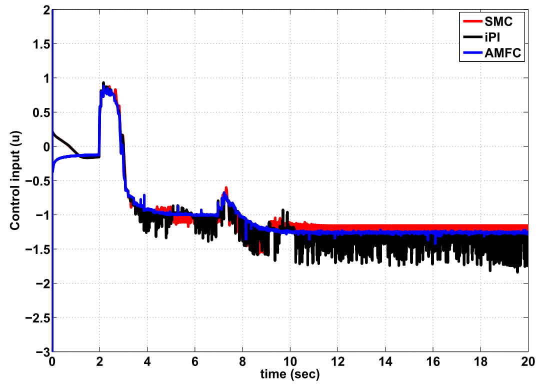

FIGURE 3. Case-1: Control signal.

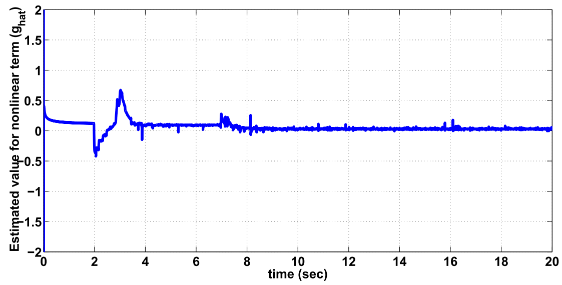

FIGURE 4. Case-1: Estimated value for nonlinear term in AMFC.

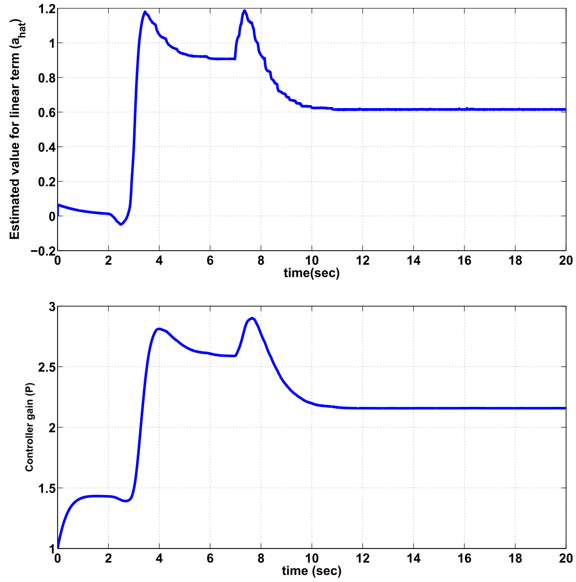

FIGURE 5. Case-1: Top: Estimated value for linear term; and bottom: Controller gain in AMFC.

dominant frequencies. The dominant frequencies in a control signal can excite the natural and structural frequencies of any dynamical system and consequently can lead to severe problems in the system. Hence, more smooth control signal is preferred [32].

B. CASE-2: DELAY SYSTEM

In this section, a delay system with dynamics as follows [13]

$$\dot{y}(t) = y(t) + 5y(t - \tau) + u(t) \tag{50}$$

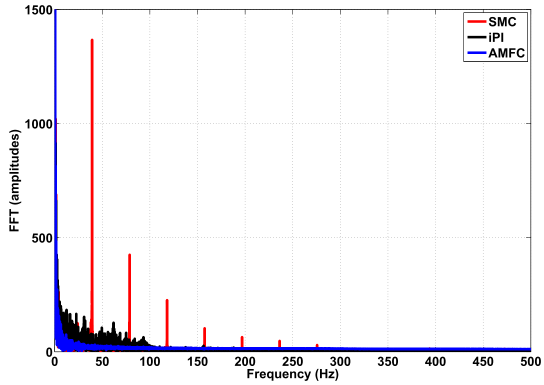

FIGURE 6. Case-1: FFT for control inputs.

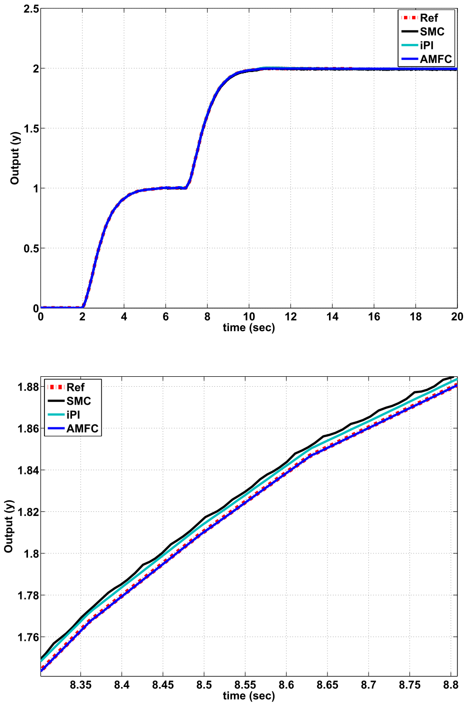

FIGURE 7. Case-2: Tracking performance; the whole (top) and in detail (bottom).

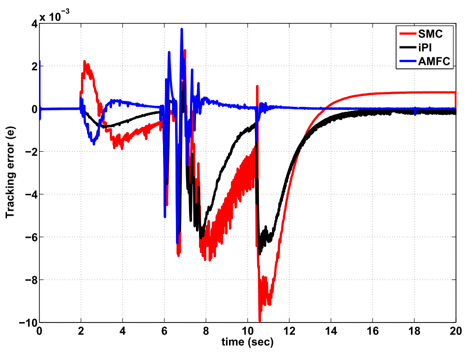

FIGURE 8. Case-2: Tracking error.

where τ is a time-varying delay function as

$$\tau(t) = \tau(t - T_0) + 10T_0 sgn(N(t)), \tau(0) = 2.5s, \quad (51)$$

is considered for simulation study. Here, T_0 is a constant value and $N(t)$ is a zero-mean Gaussian distribution with standard

TABLE 2. Properties of the controllers in Case-2.

Parameter	SMC	iPI	AMFC
Tuning parameters	$\rho_s = 50$ $\epsilon = 0.1$	$k_p = 500$ $k_i = 500$ $l_0 = 10$	$\gamma_0 = 1e3$ $\gamma_1 = 1e5$ $\rho_0 = 1e-3$ $\rho_1 = 1e-2$
Value of $\mathcal{C}$	1475	1486	1484
Dominant freq. (Hz)	28	28	-

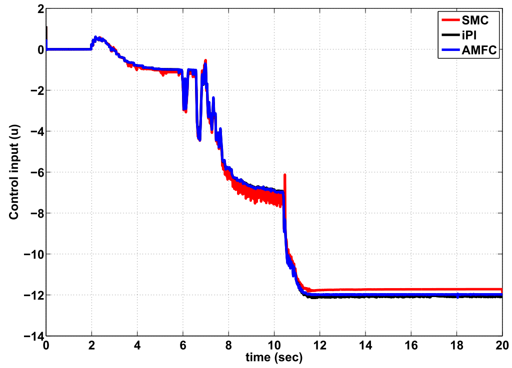

FIGURE 9. Case-2: Control signal.

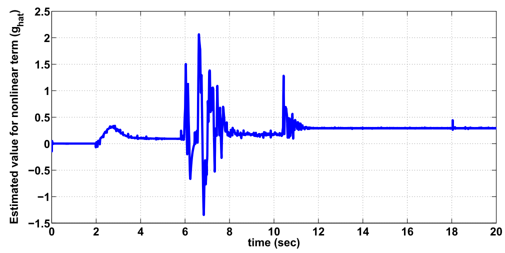

FIGURE 10. Case-2: Estimated nonlinear term in AMFC.

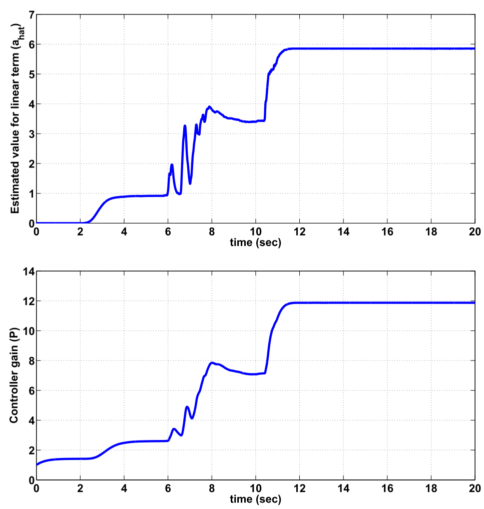

FIGURE 11. Case-2: Top: Estimated value for linear term; and bottom: Controller gain in AMFC.

deviation equal to 1. The tuning parameters and the properties of the controllers for simulation of the dynamic system in (50) is presented in Table 2. The AMFC is tuned same as in Case-1

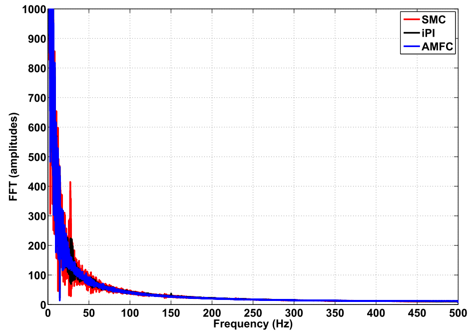

FIGURE 12. Case-2: FFT of control inputs.

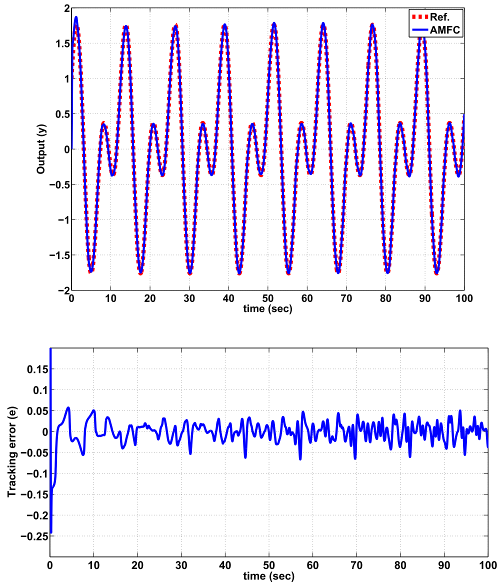

FIGURE 13. Application: Tracking performance.

without any change in tuning parameters. Again, the values of $\mathcal{C}$ are almost equal for the three controllers. The simulation results for the delay system are presented in Fig. (7) to Fig. (11), proving that the AMFC's performance supersedes the rest. In addition, the asymptotic convergence of $\hat{g}$, $\hat{a}$ and consequently P are achieved in AMFC. Online adaptation of P, reduces the efforts needed for off-line tuning of the other control parameters in AMFC. In this case, there is a dominant frequency about 28 Hertz in the control signals of SMC and iPI, while the AMFC control signal is more smooth (Fig. (12)).

V. APPLICATION

In this section, the proposed AMFC is applied to a Duffing-Holmes chaotic system which is a well-known dynamic oscillator. The dynamic system is [33]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -p_1 x_1 - p x_2 - x_1^3 + q \cos \omega t + h(x,u) + d(t) \\ y &= x_1, \end{aligned} \tag{52}$$

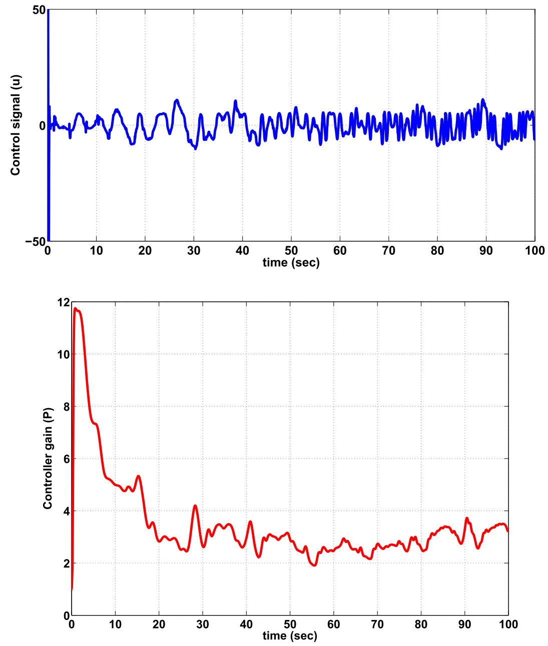

FIGURE 14. Application: top: control signal; and bottom: controller gain.

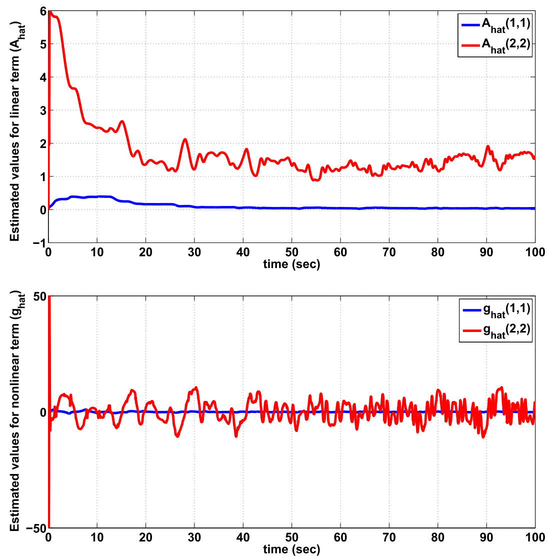

FIGURE 15. Application: top: estimated values for linear term; and bottom: estimated values for nonlinear term.

where $p_1 = 0.3 + 0.2 \sin 10\, t$, $q = 5 + 0.1 \cos t$, $p = 0.2 + 0.2 \cos 5t$, $\omega = 0.5 + 0.1 \sin t$, $h(x, u) = u + 0.5 \cos u$ and the external disturbance is $d = 0.4 \sin 0.2\pi t + 0.3 \sin x_1\, x_2$. The desired trajectory for this system is considered as $y_d = \sin t + \cos 0.5\, t$. The tuning parameters for AMFC algorithm in this case is presented in Table 3. Also, the simulation results are depicted in Fig. 13 to Fig. 15. The tracking objective is satisfied. Since the plant in (52) is a double-integrator dynamic system, there are two system states and one control inputs. In this regards, there are two unknown linear terms and two unknown nonlinear terms which are estimated online (as shown in Fig. 15).

TABLE 3. Properties of the AMFC in application case.

Parameter	AMFC
Tuning parameters	$\gamma_0 = 1$ $\gamma_1 = 1e3$ $\rho_0 = 1e-2$ $\rho_1 = 1e-1$

VI. CONCLUSION

In this paper, the design procedure for an adaptive model-free control policy based on the ultra-local model is presented for a SISO system. Here, a technique is proposed for determining the main controller gain in an online manner. Such efficacy brings convenience in terms of controller tuning as there is a very minimal adjustment needed on the available tuning *knobs*. Utilizing model-free estimation algorithms for estimating both unknown nonlinear and linear terms, the need for PE condition is removed. According to the simulation results, the tracking performance of AMFC is more accurate than those of SMC and iPI controllers. In addition, the control policy generated by AMFC is smoother than the ones generated by SMC and iPI, which have some dominant frequencies that can lead to a resonance in the dynamic system. The proposed online adaptive model-free control policy has a smooth control effort that makes the algorithm applicable directly on dynamic systems, as it is shown in application for an oscillator plant. Additionally, it can be implemented on any SISO and MIMO single agent system and be further extended to accommodate for a multi-agent network problem.

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