

# Efficient Leakage-Resilient Authenticated Key Agreement Protocol in the Continual Leakage eCK Model

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**ABSTRACT** Based on users' permanent private keys and ephemeral secret keys (randomness secret values), authenticated key agreement (AKA) protocols are used to construct a common session key between two session parties while authenticating each other. Recently, the design of leakage-resilient AKA (LR-AKA) resisting side-channel attacks has received significant attention from researchers. By side-channel attacks, an adversary is allowed to obtain fractional leakage information of private (secret) keys during the computation rounds of LR-AKA protocols. However, most LR-AKA protocols have a restriction, namely, the overall fractional leakage information must be bounded. In this paper, we propose an efficient LR-AKA protocol with overall unbounded leakage property in the continual leakage extended Canetti-Krawczyk model. Security analysis is given to demonstrate that our LR-AKA protocol is provably secure in the generic bilinear group model. By comparisons, our protocol is better than the previously proposed LR-AKA protocols in terms of computation cost, security model, and leakage properties.

**INDEX TERMS** Cryptography, key agreement, authentication, leakage-resilience, generic bilinear group.

## I. INTRODUCTION

Recently, side-channel attacks have received significant attention from researchers because most of the existing cryptographic schemes/protocols did not resist this kind of attacks. Side-channel attacks mean that, when users execute these cryptographic schemes/protocols, adversaries could obtain fractional leakage information of the permanent/ephemeral private (secret) keys of users by employing several particular properties, such as differential power consumption [1], fault/bug attacks [2], [3] and timing attacks [4], [5]. Indeed, most adversary models or security notions did not concern with side-channel attacks, where they have assumed that fractional leakage information of the private (secret) keys of users could not be leaked to adversaries. Therefore, these cryptographic schemes/protocols could be broken in an environment with side-channel attacks.

Leakage-resilient cryptography is an emerging approach of resisting side-channel attacks. Indeed, the cryptographic schemes/protocols based on leakage-resilient cryptography must tolerate the fractional leakage of private (secret) keys while retaining security. Recently, numerous leakage-resilient signature schemes [6]–[10], and leakage-resilient

encryption schemes [11]–[15] have been proposed. Since authenticated key agreement (AKA) is an important cryptographic primitive, it is critical to study leakage-resilient AKA (LR-AKA) protocols.

Based on users' permanent private keys and ephemeral secret keys (randomness secret values), AKA protocols are used to construct a common session key between two communication parties while authenticating each other. For the security model, Bellare and Rogaway [16] presented the first adversary model of AKA protocols. Afterward, Canetti and Krawczyk [17] extended Bellare and Rogaway's model to present a new adversary model, called CK model. Since the CK model did not address several possible attacks, LaMacchia *et al.* [18] further presented the extended Canetti–Krawczyk (eCK) model by considering stronger adversaries, who can compromise either permanent private keys or ephemeral secret keys. The eCK model has widely been used to show the security of AKA protocols [19]–[22]. However, the aforementioned AKA models and protocols did not take into account side-channel attacks with fractional leakage of permanent/ephemeral private (secret) keys. In this article, we will aim at the design

of an efficient LR-AKA protocol with strong security properties, especially overall unbounded leakage property and capturing general leakage attacks.

### A. RELATED WORK

Indeed, the construction of LR-AKA protocols can be constructed straightforwardly by employing leakage-resilient encryption or signature schemes. Several LR-AKA protocols have been proposed. In 2009, Alwen *et al.* [23] employed a secure signature scheme to propose a leakage-resilient AKA protocol. Afterward, Dodis *et al.* [24] proposed two LR-AKA protocols by, respectively, employing leakage-resilient encryption and signature schemes. Meanwhile, they also proved that both protocols are secure in the leakage-resilient CK model of LR-AKA protocols. Moreover, by combing a secure AKA protocol under CK model and a random message unforgeable signature scheme, Yang *et al.* [25] proposed a secure LR-AKA protocol in the leakage-resilient CK model and the auxiliary input model. It is obvious that the leakage-resilient CK model did not address the compromise of ephemeral secret keys.

Moreover, Moriyama and Okamoto [26] introduced a leakage-resilient eCK model of LR-AKA protocols while proposing a concrete LR-AKA protocol to concern with the compromise of ephemeral secret keys. However, their LR-AKA protocol concerned with only the fractional leakage of the permanent private key, but not the ephemeral secret key. Very recently, to address the security incompleteness of the aforementioned LR-AKA protocols, Chen *et al.* [27] proposed a new adversary model, termed leakage-resilient eCK model with auxiliary inputs. Their model allows the fractional leakage of both the permanent private key and ephemeral secret key while enabling an adversary to issue leakage queries during the challenge session of LR-AKA protocols. Nevertheless, the LR-AKA protocols mentioned above have the restriction that the total amount of fractional leakage information must be bounded, called the bounded leakage model.

Alawatugoda *et al.* [28] and [29] presented a generic leakage-resilient eCK model of LR-AKA protocols, called after-the-fact leakage eCK model. In this model, adversaries are also allowed to obtain fractional leakage information of the permanent/ephemeral private (secret) keys even after the session key is established during the test/challenge session. In addition, the after-the-fact leakage eCK model has two variants, namely, bounded leakage and continual leakage. The former bounds the total amount of fractional leakage information of each user's permanent/ephemeral private (secret) keys for the entire protocol execution, whereas the continual leakage variant allows adversaries to reveal a fixed amount of leakage for each protocol session while possessing overall unbounded leakage property during protocol execution. Alawatugoda *et al.* [29] also proposed a concrete construction of LR-AKA protocol. However, the proposed LR-AKA protocol is only secure in the

bounded leakage variant of after-the-fact leakage eCK model. Afterwards, Alawatugoda *et al.* [30] proposed the first concrete and secure LR-AKA protocol in the continual leakage variant of after-the-fact leakage eCK model (abbreviated "continual leakage eCK model"). They employed Dziembowski and Faust's key refreshing technique [31] to update the permanent private keys after each protocol session. However, the key refreshing procedure adopts the inner-product extractor method to compute the next permanent private key so that the required computational cost is heavy for achieving secure key refreshing procedure. The secure key refreshing procedure is a protocol which requires  $O(n^2)$  computation operation, where  $n$  is a security parameter and depends on the leakage amount in each round of the secure key refreshing procedure. Indeed, Alawatugoda *et al.* [30] pointed out that it is worthwhile to investigate other techniques to realize continual after-the-fact leakage resilience LR-AKA protocol without inner-product extractor method.

### B. CONTRIBUTION AND ORGANIZATION

In this article, we propose a novel and efficient LR-AKA protocol with overall unbounded leakage property in the continual leakage eCK model. In the continual leakage eCK model, our LR-AKA protocol allows adversaries to obtain fractional leakage information of both user's permanent private keys and ephemeral secret keys involved in the session key after the test/challenge session. The point is that the security model possesses the overall unbounded leakage property [13]. As the splitting storage idea [31], we also partition a permanent private key into two components, and refresh these two components by employing the multiplicative blinding technique [13], [15], [32] instead of the time-consuming inner-product extractor method. In this case, two new components of the permanent private key can be re-used safely since the leakage would be restricted to two "current" components and therefore no adversary can learn the useful information about a permanent private key.

In the generic bilinear group model [33], we demonstrate that our LR-AKA protocol is provably secure in the continual leakage eCK model. Table 1 lists the property comparisons among the aforementioned protocols [23]–[27], [29], [30] and our LR-AKA protocol in terms of AKA model, admitted leakage of randomness, overall leakage amount and computational cost. Obviously, our protocol is better than the others. Finally, performance analysis is made to demonstrate that the proposed LR-AKA protocol is suitable for both PC and mobile devices.

The rest of this paper is structured as follows. Section II gives preliminaries that include the generic bilinear group model and the associated assumptions. In Section III, the framework and security notions of LR-AKA protocols in the continual leakage eCK model are presented. A novel and efficient LR-AKA protocol with overall unbounded leakage property is proposed in Section IV. Security analysis of our LR-AKA protocol is presented in Section V. Section VI

**TABLE 1. Property comparisons among our protocol and the previously proposed protocols.**

Protocols	AKA model	Admitted Leakage of Randomness	Overall leakage	Computational cost
Alwen <i>et al.</i> 's protocol [23]	CK	No	Bounded	$O(1)$
Dodis <i>et al.</i> 's protocol [24]	CK	No	Bounded	$O(1)$
Yang <i>et al.</i> 's protocol [25]	CK	No	Bounded (Auxiliary input)	$O(1)$
Moriyama and Okamoto's protocol [26]	eCK	No	Bounded	$O(1)$
Alawatugoda <i>et al.</i> 's protocol [29]	eCK	No	Bounded	$O(1)$
Chen <i>et al.</i> 's protocol [27]	eCK	Yes	Bounded (Auxiliary input)	$O(1)$
Alawatugoda <i>et al.</i> 's protocol [30]	eCK	Yes	Unbounded	$O(n^2)$
Our protocol	eCK	Yes	Unbounded	$O(1)$

demonstrates performance analysis. Finally, conclusions are given in Section VII.

## II. PRELIMINARIES

Here, we compendiously present properties of bilinear groups [34], [35] and entropy, concepts of the generic bilinear group model [9], [32], [33], and several associated hard assumptions.

### A. BILINEAR GROUPS

Let  $G$  be a multiplicative cyclic group generated by  $g$  and let its order be a large prime  $p$ . Let  $G_T$  be also a multiplicative cyclic group of the same order  $p$ . An admissible bilinear pairing is a map  $\hat{e}: G \times G \rightarrow G_T$  which possesses the following properties:

1. *Bilinearity*: for all  $a, b \in \mathbb{Z}_p^*$ ,  $\hat{e}(g^a, g^b) = \hat{e}(g, g)^{ab} = \hat{e}(g^b, g^a)$ .
2. *Non – degeneracy*:  $\hat{e}(g, g) \neq 1$ .
3. *Computability*: for all  $g_1, g_2 \in G$ , the operation  $\hat{e}(g_1, g_2)$  can be computed efficiently.

For the admissible map  $\hat{e}$ ,  $G$  and  $G_T$  are called a bilinear group and the target group of the map  $\hat{e}$ , respectively. In addition,  $\hat{e}(g, g)$  may be viewed as a generator of  $G_T$ , denoted by  $g_T$ . Such groups appear in hyper-elliptic curves or supersingular elliptic curves over finite field. We refer the reader to [34] and [35] for further details.

### B. GENERIC BILINEAR GROUP MODEL

The generic group model is an adversary model for cryptographic schemes and protocols, which was first proposed by Shoup [36]. Based on the generic group model, Boneh *et al.* [33] further present the generic bilinear group model by adding bilinear pairing operation. The generic bilinear group model involves three group operations, namely, a multiplication in a group  $G$ , a multiplication in a group  $G_T$  and a bilinear pairing  $\hat{e}$  between  $G$  and  $G_T$ . When an adversary would like to perform a group operation, it just

issues a group query (oracle) to a challenger. Upon receiving this group query, the challenger uses the two elements to generate third element in  $G$ , records it in a list and sends it to the adversary. Namely, the adversary may have access to a randomly chosen element (encoding) of a group, which is maintained and controlled by the challenger.

In the generic bilinear group model, the elements of a group are encoded with bit strings. In such a case, the elements of  $G$  and  $G_T$  are, respectively, encoded to bit strings by using two random injective maps  $\psi_G: \mathbb{Z}_p \rightarrow \Omega$  and  $\psi_T: \mathbb{Z}_p \rightarrow \Omega_T$ , where both  $\Omega$  and  $\Omega_T$  are sets of bit strings while satisfying  $\Omega \cap \Omega_T = \phi$  and  $|\Omega| = |\Omega_T| = p$ . Also, let  $Q_G$ ,  $Q_T$  and  $Q_p$  denote, respectively, group queries (oracles) on the multiplication operation in  $G$ , the multiplication operation in  $G_T$  and the bilinear pairing operation  $\hat{e}$ . For any  $a, b \in \mathbb{Z}_p^*$ , the following properties hold.

- $Q_G(\psi_G(a), \psi_G(b)) \rightarrow \psi_G(a + b \text{ mod } p)$ .
- $Q_T(\psi_T(a), \psi_T(b)) \rightarrow \psi_T(a + b \text{ mod } p)$ .
- $Q_p(\psi_G(a), \psi_G(b)) \rightarrow \psi_T(ab \text{ mod } p)$ .

It is worth mentioning that  $\psi_G(1) = g$  and  $\psi_T(1) = \hat{e}(g, g) = g_T$  are generators of  $G$  and  $G_T$ , respectively. One main employment of the generic bilinear group model is to analyze computational hardness assumptions. In the generic bilinear group model, if an adversary with non-negligible probability can find a collision element for a group operation, we say that it solves the computational hardness assumption, i.e. solving the discrete logarithm problem in the multiplicative group  $G$  or  $G_T$ .

*Definition 1:* Discrete logarithm (DL) problem and its associated assumption: Given two group elements  $P, Q \in G$ , where  $G$  is a multiplicative cyclic group with order to be a large prime  $p$ . The DL problem in  $G$  is to compute an integer  $c \in \mathbb{Z}_p^*$  such that  $Q = P^c$ , where  $P$  is a generator of  $G$ . The advantage of any probabilistic polynomial time (PPT) algorithm  $A$  in solving the DL problem in  $G$  is defined by  $Adv_A^{DL} = \text{pr}[A(P, Q \in G) = c | c \in \mathbb{Z}_p^*]$ . The DL assumption is that the advantage  $Adv_A^{DL}$  is negligibly small for any probabilistic polynomial time algorithm  $A$  [33].

In addition, we also need another computational hardness assumption in our protocol.

*Definition 2:* Computational Diffie-Hellman (CDH) problem: Given  $(g, g^a, g^b) \in G^3$  for unknown  $a, b \in \mathbb{Z}_p^*$ , where  $G$  is a multiplicative cyclic group with order to be a large prime  $p$  and with a generator  $g$ . The CDH problem in  $G$  is to compute  $g^{ab}$ . The advantage of any PPT algorithm  $A$  in solving the CDH problem in  $G$  is defined by  $Adv_A^{CDH} = \text{pr}[A(g, g^a, g^b) = g^{ab} | a, b \in \mathbb{Z}_p^*]$ . The CDH assumption is that the advantage  $Adv_A^{CDH}$  is negligibly small [37] for any PPT algorithm  $A$ .

### C. ENTROPY

Entropy is used to measure the amount of all possible statuses (states) in thermodynamic equilibrium systems. Indeed, the statistical exposition of entropy is able to evaluate the probability estimation of uncertainty. Let  $X$  be a discrete random variable which takes on a finite set of values

$x_1, x_2, \dots, x_n$  with probability  $Pr[x_1], Pr[x_2], \dots, Pr[x_n]$  such that  $\sum_{x \in X} Pr[x] = 1$ . Moreover, the min-entropy of a random variable denotes the measure of the largest probability (worst-case predictability). The average conditional min-entropy of a random variable denotes the measure of the worst-case predictability under a correlated discrete random variable with some events. We formally define the two kinds of min-entropies as below.

1.  $H_\infty(X) = -\log_2(\max_x Pr[X = x])$ : the min-entropy of the discrete random variable  $X$ .
2.  $\tilde{H}_\infty(X|Y) = -\log_2(E_{y \leftarrow Y}[\max_x Pr[X = x|Y = y]])$ : the average conditional min-entropy of the discrete random variable  $X$  under the correlated discrete random variable  $Y$  with an event  $Y = y$ .

In the leakage circumstance, to measure the average conditional min-entropy of a discrete random variable (i.e. a private/secret key), Dodis et al. [38] presented the following consequence.

*Lemma 1:* Given a discrete random variable  $X$ , let  $f(X)$  denote the fractional leakage information on  $X$ , where  $f : x \rightarrow \{0, 1\}^\lambda$  is a leakage function on  $X$  and the output bit-length of  $f$  is limited to  $\lambda$  bits. The average conditional min-entropy of the discrete random variable  $X$  under the fractional leakage information  $f(X)$  satisfies the inequality  $\tilde{H}_\infty(X|f(X)) \geq H_\infty(X) - \lambda$ .

Based on the Schwartz-Zippel lemma [39], [40], Galindo and Vivek [9] demonstrated the property of probability distributions of a non-zero polynomial under a leakage function with the maximal output bit-length  $\lambda$  as below.

*Lemma 2:* Let  $G \in Z_p[W_1, W_2, \dots, W_n]$  denote a non-zero polynomial of total degree  $d$  with the probability distributions  $P_i$  (for  $i = 1, 2, \dots, n$ ) on  $Z_p$  such that  $H_\infty(P_i) \geq \log p - \lambda$  and  $0 \leq \lambda \leq \log p$ . If  $w_i \leftarrow_{P_i} Z_p$  (for  $i = 1, 2, \dots, n$ ) are mutually independent, we have the consequence  $Pr[G(W_1 = w_1, W_2 = w_2, \dots, W_n = w_n) = 0] \leq \frac{d}{p} 2^\lambda$ . Meanwhile, if  $\lambda < \log(p) - \omega(\log \log p)$ , the probability  $Pr[W_1 = w_1, W_2 = w_2, \dots, W_n = w_n] = 0$  is negligible.

### III. ADVERSARY MODEL AND SECURITY NOTIONS

In the section, we introduce the security notions of the continual leakage variant of after-the-fact leakage eCK model [30], i.e. the continual leakage eCK model. We first present properties of the continual leakage model.

#### A. CONTINUAL LEAKAGE MODEL

The continual leakage model is used to model the leakage abilities of an adversary, which allows an adversary to continually reveal a fixed amount of leakage for each protocol session while possessing overall unbounded leakage property during the whole system lifecycle [9], [13], [32]. To achieve the overall unbounded leakage property, private (secret) keys must provide the *stateful* property. To do so, each private (secret) key is partitioned into two components and stored in different places of the memory. Generally, a cryptographic protocol/scheme comprises several

computation rounds. After (or before) executing a computation round in the cryptographic protocol/scheme, the system refreshes the involved private (secret) key while the associated public key remains unchanged. In the following, we summarize four properties of the continual leakage model.

- *Only computation leakage:* Only the fractional leakage information of permanent/temporary private (secret) keys involved or accessed in the current computation round could be revealed to a side-channel adversary.
- *Bounded leakage of single computation round:* The amount of fractional leakage information in a single computation round is limited to some  $\lambda$  bits. Namely, the leakage information of each computation round is bounded to a fraction of private (secret) keys.
- *Independent leakage between computation rounds:* The leakage information of the computation rounds is independent with each other.
- *Overall unbounded leakage:* The total amount of leakage information is unbounded, namely, it possesses overall unbounded leakage property during the whole system lifecycle. Thus, after (or before) executing a computation round, the system refreshes the involved or accessed private (secret) keys.

#### B. THE ADVERSARIAL MODEL OF THE LR-AKA PROTOCOL

As mentioned earlier, the eCK model of AKA protocols introduced by LaMacchia et al. [18] is an accredited adversary model and has been widely used to show the security of AKA protocols. Based on the eCK model, several leakage-resilient eCK models [26]–[30] have been presented. Alawatugoda et al.'s continual leakage eCK model [29], [30] is the most accredited model, which allows an adversary to ask all kinds of queries as the abilities of the adversary in the eCK model. In addition, the adversary may also issue leakage queries to obtain the fractional leakage information of permanent/ephemeral private (secret) keys. Typically, a AKA protocol consists of two phases, namely, initial setup phase and session key construction phase. In the continual leakage eCK model, the session key construction phase of leakage-resilient (LR)-AKA protocols is further divided to two sub-phases, namely, key refreshing and key agreement. To represent the fractional leakage information of LR-AKA protocol obtained by the adversary, we choose two leakage functions  $f_{i,s}$  and  $h_{i,s}$ , respectively, to model the adversary's abilities in the key refreshing and key agreement sub-phases, where  $i$  and  $s$  denote the  $s$ -th session of the user with identity  $ID_i$ . The output length of  $f_{i,s}$  and  $h_{i,s}$  are bounded by  $\lambda$ , where  $\lambda$  is the leakage parameter. That is,  $|f_{i,s}| \leq \lambda$  and  $|h_{i,s}| \leq \lambda$ , where  $|f|$  denotes the output length of leakage function  $f$ . Moreover, two leakage functions  $f_{i,s}$  and  $h_{i,s}$  can be efficiently computed. We define the outputs of two leakage functions as follows.

- $\Delta f_{i,s} = f_{i,s}$  (*private keys*).
- $\Delta h_{i,s} = h_{i,s}$  (*private keys, ephemeral secret keys*).

Here, *private keys* denote the private keys involved in the computations of the key refreshing and key agreement sub-phases while *ephemeral secret keys* denote the ephemeral secret keys involved in the computation of the key agreement sub-phase.

Based on the continual leakage eCK model, we present the associated security game  $G_{CL-eCK}$  that is played by an adversary  $A$  and a challenger  $B$ . Let the adversary  $A$  be a  $q$ -query probabilistic polynomial-time (PPT) algorithm that can issue queries to the challenger  $B$  at most  $q$  times. Let the oracle  $\Pi_i^s$  denote the  $s$ -th session of the user with identity  $ID_i$ .

#### GAME $G_{CL-eCK}$

In the game  $G_{CL-eCK}$ , there are two phases, namely, *Initial Setup* and *Query*. The adversary  $A$  may issue six kinds of queries in any order for totally at most  $q$  times in the Query phase.  $A$  wins the game if  $A$  can determine whether or not a bit string is the real session key at the end of the game. Two phases are described as below:

- *Initial Setup*: The challenger  $B$  generates the system parameters and then sends the public parameters to the adversary  $A$ .
- *Query*: In this phase,  $A$  can issue the following six queries adaptively for totally at most  $q$  times.
  - $Send(\Pi_i^s, m)$ : Upon receiving this query along with the communication message  $m$  to the oracle  $\Pi_i^s$ ,  $B$  sends the corresponding results to  $A$  by running the protocol  $\Pi_i^s$  according to  $m$ .
  - $Reveal(\Pi_i^s)$ :  $A$  can issue this query to obtain the session key of the oracle  $\Pi_i^s$  if  $\Pi_i^s$  has accepted the session; otherwise, it returns a null value.
  - $Ephemeral-secret-leakage(\Pi_i^s)$ :  $A$  can issue this query to obtain the ephemeral secret keys of  $\Pi_i^s$ .
  - $Corrupt(ID_i)$ :  $A$  can issue this query to obtain the private key of the user with identity  $ID_i$ .
  - $Leak(\Pi_i^s, f_{i,s}, h_{i,s})$ :  $A$  can issue this query along with the target oracle  $\Pi_i^s$  and two leakage function  $f_{i,s}$  and  $h_{i,s}$  to obtain the fractional leakage information of the private keys and ephemeral secret keys in the key refreshing and key agreement sub-phases of  $\Pi_i^s$ .
  - $Test(\Pi_i^s)$ : When the adversary  $A$  issues this query, the challenger  $B$  flips an unbiased coin bit  $cb$ . The challenger  $B$  returns the session key if  $cb = 1$ ; otherwise, it returns a random value. The query phase ends whenever the Test query has been issued by  $A$ . If  $A$  can return a coin bit  $cb' = cb$ , then  $A$  wins the game  $G_{CL-eCK}$ . We emphasize that the adversary  $A$  is allowed to issue the  $Test(\Pi_i^s)$  query only once.

In the following, we describe the relationships between the real world attack scenarios and different queries in the continual leakage eCK model.

- *Passive adversarial capabilities*: The  $Send$  query addresses the power of a passive adversary who may control and obtain message flows between two communication parties.

- *Malware attacks*: The  $Corrupt$  query addresses the malware attack in the situation that the adversary  $A$  may reveal the private keys of users. The  $Ephemeral-secret-leakage$  query addresses the malware attacks in the situation that the adversary  $A$  may reveal the ephemeral secret keys of any connections.
- *Weak random number generators*: Due to the weak random number generators, the adversary can determine the random number correctly. The  $Ephemeral-secret-leakage$  query addresses such kinds of ability of the adversary.
- *Known-session-key security*: The  $Reveal$  query addresses the known-session-key security in the sense that an adversary cannot reveal other session keys when it compromises a session key.
- *Side-channel attacks*: Whenever the leakage happens due to any kinds of side-channel attacks, the adversary may obtain the fractional leakage information of the parameters involved in the computation. The  $Leak$  query addresses the side-channel attacks.

*Definition 3 (Partnership)*: Two oracles  $\Pi_i^s$  and  $\Pi_j^t$  are *partners* if they can authenticate mutually and accept a common session key.

*Definition 4 (Freshness)*: In the security game  $G_{CL-eCK}$ , the partnership session between  $\Pi_i^s$  and  $\Pi_j^t$  is said to be *fresh* if none of the following conditions holds:

- The adversary has issued  $Reveal(\Pi_i^s)$  or  $Reveal(\Pi_j^t)$ .
- The adversary has issued both  $Corrupt(ID_i)$  and  $Ephemeral-secret-leakage(\Pi_i^s)$ .
- The adversary has issued both  $Corrupt(ID_j)$  and  $Ephemeral-secret-leakage(\Pi_j^t)$ .

#### IV. THE PROPOSED LR-AKA PROTOCOL

In this section, we propose a novel and efficient LR-AKA protocol in the continual leakage eCK model. Our LR-AKA protocol consists of two phases, initial setup and session key construction. Moreover, the session key construction phase consists of two sub-phases including the key refreshing and key agreement. In the following, we demonstrate how Alice and Bob construct a common session key by using our LR-AKA protocol.

- *Initial setup phase*: Given the security parameter  $\kappa$ , the system first generates the bilinear groups  $\{p, G, G_T, g, \hat{e}\}$  as defined in Section 2.1. Moreover, an additional generator  $h \in G$  is randomly chosen. Let the public parameters  $PP$  be  $\{G, G_T, \hat{e}, p, g, h\}$ . Without loss of generality, Alice and Bob generate their own private key pairs and public key pair as follows. Also, see Fig. 1.
  - Alice first picks two random values  $a, \alpha_0 \in \mathbb{Z}_p^*$ , and computes two initial private key pairs  $(SA_{0,1}, SA_{0,2}) = (g^{a_0}, g^{a-\alpha_0})$  and  $(XA_{0,1}, XA_{0,2}) = (\hat{e}(SA_{0,1}, h), \hat{e}(SA_{0,2}, h))$ . Obviously, we have  $SA = SA_{0,1} \cdot SA_{0,2} = g^a$  and  $XA = XA_{0,1} \cdot XA_{0,2} = \hat{e}(g^a, h)$ . Alice computes the public key pair  $(TA, PA) = (\hat{e}(g, SA), \hat{e}(g, g)^{XA})$ .

Alice	Bob
Private key pairs: ( $SA_{0,1}, SA_{0,2}$ )= $(g^{\alpha_0}, g^{\alpha_0})$ ( $XA_{0,1}, XA_{0,2}$ )= $(\hat{e}(SA_{0,1}, h), \hat{e}(SA_{0,2}, h))$	Private key pairs: ( $SB_{0,1}, SB_{0,2}$ )= $(g^{\beta_0}, g^{\beta_0})$ ( $XB_{0,1}, XB_{0,2}$ )= $(\hat{e}(SB_{0,1}, h), \hat{e}(SB_{0,2}, h))$
Public key pair: ( $TA, PA$ )= $(\hat{e}(g, SA), \hat{e}(g, g)^{SA})$ , where $SA = g^{\alpha}$ and $XA = \hat{e}(g^{\alpha}, h)$	Public key pair: ( $TB, PB$ )= $(\hat{e}(g, SB), \hat{e}(g, g)^{SB})$ , where $SB = g^{\beta}$ and $XB = \hat{e}(g^{\beta}, h)$

FIGURE 1. The Initial setup phase.

- Similarly, Bob picks two random values  $b, \beta_0 \in Z_p^*$ , and computes two private key pairs and the public key pair as Alice did. Bob's two initial private key pairs are  $(SB_{0,1}, SB_{0,2})$  and  $(XB_{0,1}, XB_{0,2})$  while the public key pair is  $(TB, PB) = (\hat{e}(g, SB), \hat{e}(g, g)^{SB})$ , where  $SB = SB_{0,1} \cdot SB_{0,2} = g^b$  and  $XB = XB_{0,1} \cdot XB_{0,2} = \hat{e}(g^b, h)$ .

- *Session key construction phase:* To construct a session key between Alice's  $i$ -th and Bob's  $j$ -th session, they perform the key refreshing and key agreement sub-phases as follows. Also, see Fig. 2.

- *Key refreshing:* Alice chooses a random number  $\alpha_i \in Z_p^*$  and refreshes her private key pairs by  $(SA_{i,1}, SA_{i,2}) = (SA_{i-1,1} \cdot g^{\alpha_i}, SA_{i-1,2} \cdot g^{-\alpha_i})$  and  $(XA_{i,1}, XA_{i,2}) = (XA_{i-1,1} \cdot \hat{e}(g^{\alpha_i}, h), XA_{i-1,2} \cdot \hat{e}(g^{-\alpha_i}, h))$ . By the same way, Bob also picks a random number  $\beta_j \in Z_p^*$  and refreshes his private key pairs  $(SB_{j,1}, SB_{j,2}) = (SB_{j-1,1} \cdot g^{\beta_j}, SB_{j-1,2} \cdot g^{-\beta_j})$  and  $(XB_{j,1}, XB_{j,2}) = (XB_{j-1,1} \cdot \hat{e}(g^{\beta_j}, h), XB_{j-1,2} \cdot \hat{e}(g^{-\beta_j}, h))$ . This key refreshing procedure adopts the so-called multiplicative blinding technique. Obviously, we have  
 $SA_{i,1} \cdot SA_{i,2} = g^{\alpha} = SA = SA_{0,1} \cdot SA_{0,2}$ .  
 $XA_{i,1} \cdot XA_{i,2} = \hat{e}(g^{\alpha}, h) = XA = XA_{0,1} \cdot XA_{0,2}$ .  
 $SB_{j,1} \cdot SB_{j,2} = g^{\beta} = SB = SB_{0,1} \cdot SB_{0,2}$ .  
 $XB_{j,1} \cdot XB_{j,2} = \hat{e}(g^{\beta}, h) = XB = XB_{0,1} \cdot XB_{0,2}$ .
- *Key agreement:* Alice chooses an ephemeral secret key  $x$ , computes  $X = g^x$  and sends the value  $X$  to Bob. Bob also chooses an ephemeral secret key  $y$ , computes  $Y = g^y$  and sends the value  $Y$  to Alice. By using Bob's public key  $(TB, PB)$  and  $Y$ , and her own current private key pairs  $(SA_{i,1}, SA_{i,2})$  and  $(XA_{i,1}, XA_{i,2})$ , Alice computes a session key  $SK_{A,i}$  as below:

- (1)  $KA_{i,1} = TB^x$ .
- (2)  $KA_{i,2} = \hat{e}(Y, SA_{i,1}) \cdot \hat{e}(Y, SA_{i,2})$ .
- (3)  $KA_{i,3} = Y^x$ .
- (4)  $KA_{i,4} = (PB^{XA_{i,1}})^{XA_{i,2}}$ .
- (5)  $SK_{A,i} = KA_{i,1} \oplus KA_{i,2} \oplus KA_{i,3} \oplus KA_{i,4}$ .

Similarly, Bob uses his current private key pairs  $(SB_{j,1}, SB_{j,2})$  and  $(XB_{j,1}, XB_{j,2})$  to compute a session key  $SK_{B,j}$  as below:

- (1)  $KB_{j,1} = TA^y$ .
- (2)  $KB_{j,2} = \hat{e}(X, SB_{j,1}) \cdot \hat{e}(X, SB_{j,2})$ .
- (3)  $KB_{j,3} = Y^x$ .

Alice	Bob
<i>Key Refreshing:</i> Choose $\alpha_i \in Z_p^*$ ( $SA_{i,1}, SA_{i,2}$ )= $(SA_{i-1,1} \cdot g^{\alpha_i}, SA_{i-1,2} \cdot g^{-\alpha_i})$ ( $XA_{i,1}, XA_{i,2}$ )= $(XA_{i-1,1} \cdot \hat{e}(g^{\alpha_i}, h), XA_{i-1,2} \cdot \hat{e}(g^{-\alpha_i}, h))$	<i>Key Refreshing:</i> Choose $\beta_j \in Z_p^*$ ( $SB_{j,1}, SB_{j,2}$ )= $(SB_{j-1,1} \cdot g^{\beta_j}, SB_{j-1,2} \cdot g^{-\beta_j})$ ( $XB_{j,1}, XB_{j,2}$ )= $(XB_{j-1,1} \cdot \hat{e}(g^{\beta_j}, h), XB_{j-1,2} \cdot \hat{e}(g^{-\beta_j}, h))$
<i>Key agreement:</i> Choose $x \in Z_p^*$ $X = g^x$	<i>Key agreement:</i> Choose $y \in Z_p^*$ $Y = g^y$
$X \xrightarrow{\quad}$	$\xleftarrow{\quad} Y$
$KA_{i,1} = TB^x$ $KA_{i,2} = \hat{e}(Y, SA_{i,1}) \cdot \hat{e}(Y, SA_{i,2})$ $KA_{i,3} = Y^x$ $KA_{i,4} = (PB^{XA_{i,1}})^{XA_{i,2}}$ $SK_{A,i} = KA_{i,1} \oplus KA_{i,2} \oplus KA_{i,3} \oplus KA_{i,4}$	$KB_{j,1} = TA^y$ $KB_{j,2} = \hat{e}(X, SB_{j,1}) \cdot \hat{e}(X, SB_{j,2})$ $KB_{j,3} = Y^x$ $KB_{j,4} = (PA^{XB_{j,1}})^{XB_{j,2}}$ $SK_{B,j} = KB_{j,1} \oplus KB_{j,2} \oplus KB_{j,3} \oplus KB_{j,4}$

FIGURE 2. The session key construction phase.

$$(4) \quad KB_{j,4} = (PA^{XB_{j,1}})^{XB_{j,2}}$$

$$(5) \quad SK_{B,j} = KB_{j,1} \oplus KB_{j,2} \oplus KB_{j,3} \oplus KB_{j,4}$$

In the following, we show the correctness of the section keys  $KA_i$  and  $KB_j$ . Since  $SK_{A,i} = KA_{i,1} \oplus KA_{i,2} \oplus KA_{i,3} \oplus KA_{i,4}$  and  $SK_{B,j} = KB_{j,1} \oplus KB_{j,2} \oplus KB_{j,3} \oplus KB_{j,4}$ , We show the equality  $SK_{A,i} = SK_{B,j}$  by the following equalities:

$$\begin{aligned} KA_{i,1} &= TB^x = \hat{e}(g, g)^{bx} = \hat{e}(g^x, g^b) \\ &= \hat{e}(X, g^b) = \hat{e}(X, SB_{j,1} \cdot SB_{j,2}) \\ &= \hat{e}(X, SB_{j,1}) \cdot \hat{e}(X, SB_{j,2}) = KB_{j,2}. \\ KA_{i,2} &= \hat{e}(Y, SA_{i,1}) \cdot \hat{e}(Y, SA_{i,2}) \\ &= \hat{e}(Y, SA_{i,1} \cdot SA_{i,2}) \\ &= \hat{e}(Y, g^{\alpha}) = \hat{e}(g^y, g^{\alpha}) \\ &= \hat{e}(g, g^{\alpha})^y = TA^y = KB_{j,1}. \\ KA_{i,3} &= Y^x = g^{yx} = g^{xy} = X^y = KB_{j,3}. \\ KA_{i,4} &= (PB^{XA_{i,1}})^{XA_{i,2}} \\ &= PB^{XA_{i,1} \cdot XA_{i,2}} = \hat{e}(g, g)^{XB \cdot XA} \\ &= PA^{XB} = PA^{XB_{j,1} \cdot XB_{j,2}} = KB_{j,4}. \end{aligned}$$

Hence, we have  $SK_{A,i} = SK_{B,j}$ .

## V. SECURITY ANALYSIS

In this section, we present the security analysis of our LR-AKA protocol in the continual leakage eCK model. Based on the generic bilinear group model [13], [32], [33], we demonstrate that our LR-AKA protocol is provably secure in the continual leakage eCK model.

*Theorem 1:* Assume that  $A$  is a  $q$ -query PPT adversary of the proposed LR-AKA protocol in the continual leakage eCK model. Based on the generic bilinear group model and CDH assumption, the proposed LR-AKA protocol is provably secure.

*Proof:* Let  $A$  be an adversary that can adaptively issue the queries at most  $q$  times in the security game  $G_{CL-eCK}$  which is played by a challenger  $B$  and the adversary  $A$ .

The advantage that  $A$  breaks the proposed LR-AKA protocol is bounded by the success probability of  $A$  in the game  $G_{CL-eCK}$ . In the adversary model defined in Section 3.2,  $A$  can issue the *Ephemeral-secret-leakage* and *Corrupt* queries to obtain the ephemeral secret key and the private key pairs of a participant, respectively. Without loss of generality, let  $\Pi_i^s$  and  $\Pi_j^t$  be the two oracles of a partnership session, where  $\Pi_i^s$  denotes the  $s$ -th session of the user with identity  $ID_i$  while  $\Pi_j^t$  represent the  $t$ -th session of the user with identity  $ID_j$ . The session key of a partnership session between  $\Pi_i^s$  and  $\Pi_j^t$  can be obtained by making use of both the private key pairs and the ephemeral secret key of one participant  $\Pi_i^s$  or  $\Pi_j^t$ . Without breaking the freshness of the partnership session between  $\Pi_i^s$  and  $\Pi_j^t$ , we discuss four situations as below:

- Situation 1:  $A$  may obtain the private key pairs of both  $\Pi_i^s$  and  $\Pi_j^t$ , but not the ephemeral secret key of  $\Pi_i^s$  or  $\Pi_j^t$ .
- Situation 2:  $A$  may obtain the ephemeral secret keys of both  $\Pi_i^s$  and  $\Pi_j^t$ , but not the private key pairs of  $\Pi_i^s$  or  $\Pi_j^t$ .
- Situation 3:  $A$  may obtain the ephemeral secret key of  $\Pi_i^s$  and the private key pairs of  $\Pi_j^t$ , but not the ephemeral secret key of  $\Pi_j^t$  or the private key pairs of  $\Pi_i^s$ .
- Situation 4:  $A$  obtains the ephemeral secret key of  $\Pi_j^t$  and the private key pairs of  $\Pi_i^s$ , but not the ephemeral secret key of  $\Pi_i^s$  or the private key pairs of  $\Pi_j^t$ .

We employ three lemmas to establish the security for each situation. The security of Situation 1 is given in Lemma 3. By Lemma 4, we show the security of Situations 3 and 4. Finally, in Lemma 5, we give the security of Situation 2. Hence, by applying Lemmas 3, 4 and 5, our LR-AKA protocol is provable secure in the continual leakage eCK model. Q.E.D.

**Lemma 3:** Assume that  $A$  is a  $q$ -query PPT adversary of the proposed LR-AKA protocol in the continual leakage eCK game  $G_{CL-eCK}$ . Assume that  $A$  is allowed to obtain the private key pairs of both  $\Pi_i^s$  and  $\Pi_j^t$ , but not the ephemeral secret keys of  $\Pi_i^s$  or  $\Pi_j^t$ . Under the CDH assumption, the probability that  $A$  wins the continual leakage eCK game  $G_{CL-eCK}$  is negligibly small.

*Proof:* Note that the adversary  $A$  can compute  $KA_{i,1}$  ( $= KB_{j,2}$ ),  $KA_{i,2}$  ( $= KB_{j,1}$ ) and  $KA_{i,4}$  ( $= KB_{j,4}$ ). Hence, to obtain the session key  $SK_{A,i}$  ( $= KA_{i,1} \oplus KA_{i,2} \oplus KA_{i,3} \oplus KA_{i,4}$ ),  $A$  must be able to compute  $KA_{i,3}$  ( $= Y^x = g^{xy} = X^y = KB_{j,3}$ ) from the instance  $(G, p, g, X = g^x, Y = g^y)$ , where  $x$  and  $y$  are the ephemeral secret key. However, this is the hard CDH problem and so the advantage that  $A$  can obtain  $g^{xy}$  is negligibly small. In the continual leakage eCK game  $G_{CL-eCK}$ ,  $A$  can collect at most  $\lambda$  bits leakage information about  $x$  or  $y$  by issuing the Leak query with two leakage functions  $h_{i,s}$  and  $h_{j,t}$ , respectively. Since the ephemeral secret key  $x$  and  $y$  are randomly selected for each session,  $A$  can obtain at most  $\lambda$  bits leakage information from each ephemeral secret key. Hence it is hard to determine  $KA_{i,3}$  or  $KB_{j,3}$ . Based on the CDH assumption [37], the probability that  $A$  wins the game  $G_{CL-eCK}$  is negligibly small. Q.E.D.

**Lemma 4:** Assume that  $A$  is a  $q$ -query PPT adversary of the proposed LR-AKA protocol in the continual leakage eCK game  $G_{CL-eCK}$ . Assume that  $A$  is allowed to obtain the ephemeral secret key of  $\Pi_i^s$  and the private key pairs of  $\Pi_j^t$ , or the ephemeral secret key of  $\Pi_j^t$  and the private key pairs of  $\Pi_i^s$ . Based on the bilinear generic group model, the advantage that  $A$  wins the game is negligibly small.

*Proof:* By assumption (of Situation 3),  $A$  is allowed to obtain the ephemeral secret key  $x$  of  $\Pi_i^s$  and the private key pairs  $(SB_{j,1}, SB_{j,2})$  and  $(XB_{j,1}, XB_{j,2})$  of  $\Pi_j^t$ , but not the ephemeral secret key  $y$  of  $\Pi_j^t$  or the private key pairs  $(SA_{i,1}, SA_{i,2})$  and  $(XA_{i,1}, XA_{i,2})$  of  $\Pi_i^s$ . In this case, by using  $x$ ,  $(SB_{j,1}, SB_{j,2})$  and  $(XB_{j,1}, XB_{j,2})$ , the adversary  $A$  can compute  $KA_{i,1}$  ( $= KB_{j,2}$ ),  $KA_{i,3}$  ( $= KB_{j,3}$ ) and  $KA_{i,4}$  ( $= KB_{j,4}$ ). Hence, to obtain the session key,  $A$  must be able to compute  $KA_{i,2}$  ( $= \hat{e}(Y, SA_{i,1}) \cdot \hat{e}(Y, SA_{i,2}) = \hat{e}(Y, SA_{i,1} \cdot SA_{i,2}) = \hat{e}(Y, g^a) = \hat{e}(g^y, g^a) = TA^y = KB_{j,1}$ ). However, this is hard. Compute  $KA_{i,2}$  can be viewed as a key encapsulation by employing leakage resilient ElGamal encryption scheme proposed by Kiltz and Pietrzak [13]. Kiltz and Pietrzak have shown the security of the key encapsulation for  $\hat{e}(Y, SA_{i,1}) \cdot \hat{e}(Y, SA_{i,2}) = TA^y$  under the continual leakage model and based on the generic bilinear group model. Thus, the probability that  $A$  can obtain  $KA_{i,2} = \hat{e}(Y, SA_{i,1}) \cdot \hat{e}(Y, SA_{i,2}) = TA^y = KB_{j,1}$  is negligibly small. Similarly, Situation 4 is also a key encapsulation from Alice to Bob, we omit the proof. Q.E.D.

**Lemma 5:** Assume that  $A$  is a  $q$ -query PPT adversary of the proposed LR-AKA protocol in the continual leakage eCK game  $G_{CL-eCK}$ . Assume that  $A$  is allowed to obtain the ephemeral secret keys of both  $\Pi_i^s$  and  $\Pi_j^t$ , but not the private key pairs of both  $\Pi_i^s$  and  $\Pi_j^t$ . Based on the generic bilinear group model, the probability that  $A$  wins the continual leakage eCK game  $G_{CL-eCK}$  is negligibly small.

*Proof:* The proof of the lemma is presented in Appendix.

## VI. PERFORMANCE ANALYSIS

The performance analysis of our LR-AKA protocol is given here. We adopt the following notations to analyze the computational costs.

- $TG_e$ : The computational cost of a bilinear pairing operation  $\hat{e}: G \times G \rightarrow G_T$ .
- $TG_m$ : The computational cost of an exponentiation operation in  $G$  or  $G_T$ .

Note that the computational cost of the multiplication operation in  $G$  or  $G_T$  is trivial and negligible with compared to  $TG_e$  and  $TG_m$  [35]. The simulation results in [41]–[43] on mobile device and PC are treated as the benchmark of evaluating the running time of  $TG_e$  and  $TG_m$ . The mobile device is a Linux personal digital assistant (PDA) with a PXA270 624-MHz processor. The PC is equipped with a Pentium 3 GHz processor under Microsoft window system. In addition, for the equivalent level of 1024-bit RSA security, the employed bilinear pairings (i.e. Tate pairings) are defined over the elliptic curve  $E/F_p: y^2 = x^3 + x$  with embedding

**TABLE 2. Running time (in milliseconds) of various operations on mobile device and PC.**

	$TG_e$	$TG_m$
PC (3 GHz processor)	20.1	6.38
Mobile device (624 MHz processor)	96.2	30.67

**TABLE 3. Computational cost and running time (in milliseconds) of our LR-AKA protocol.**

	Initial Setup	Key refreshing	Key agreement
Computational cost	$3TG_e$ $+3TG_m$	$2TG_e$ $+2TG_m$	$2TG_e$ $+4TG_m$
Running time on PC	79.44	52.96	65.72
Running time on mobile device	380.61	253.74	315.08

degree 2, where  $p$  is a 512-bit prime such that  $p + 1 = 12qr$  while  $q$  is a 160-bit prime. Table 2 lists the running time (in milliseconds) of two operations on mobile device and PC, respectively. In Table 3, we list the computational cost and running time (in milliseconds) of three phases in the proposed LR-AKA protocol. By Table 3, the performance of our protocol is not only suiting for the platforms of PC but also for mobile devices.

## VII. CONCLUSION

In the article, we proposed a novel and efficient LR-AKA protocol by using the multiplicative blinding technique instead of the time-consuming inner-product extractor method to achieve key refreshing. The idea of the multiplicative blinding technique in the continual leakage eCK model is to partition the private key into two components while the leakage of two components is independent each other. After two current components are involved to construct a session key, they must be refreshed to become two new components of the private key for reuse. By the key refreshing method, the proposed LR-AKA protocol possesses overall unbounded leakage property because an adversary can only learn fractional leakage information of two current components. In the generic bilinear group (GBG) model, we demonstrated that our LR-AKA protocol is provably secure in the continual leakage eCK model. In addition, performance analysis demonstrated that the proposed LR-AKA protocol is suitable for both mobile device and PC. A more challenging issue is to propose a LR-AKA protocol without random oracle model while possessing overall unbounded leakage property in the continual leakage eCK model.

## APPENDIX

### PROOF OF LEMMA 5

*Proof:* By assumption,  $A$  may obtain the ephemeral secret keys  $x$  and  $y$  of both  $\Pi_i^s$  and  $\Pi_j^t$  by issuing *Ephemeral-secret-leakage* query, but not the private key pairs  $(SA_{i,1}, SA_{i,2})$  and  $(XA_{i,1}, XA_{i,2})$  of  $\Pi_i^s$ , or the private key pairs  $(SB_{j,1}, SB_{j,2})$  and  $(XB_{j,1}, XB_{j,2})$  of  $\Pi_j^t$ . In this case,

$A$  can compute  $KA_{i,1}(= KB_{j,2})$ ,  $KA_{i,2}(= KB_{j,1})$  and  $KA_{i,3}(= KB_{j,3})$ . Hence, to obtain the session key,  $A$  must be able to compute  $KA_{i,4} = (PB^{XA_{i,1}})^{XA_{i,2}}$  or  $KB_{j,4} = (PA^{XB_{j,1}})^{XB_{j,2}}$  from the instance  $(G, G_T, \hat{e}, p, g, PA = \hat{e}(g, g)^{XA}, PB = \hat{e}(g, g)^{XB})$ . In the following, we prove that the advantage that  $A$  wins the game  $G_{CL-eCK}$  is negligibly small, based on the generic bilinear group model. In the generic bilinear group model, the elements of a group are encoded with bit strings. In order for  $A$  to perform the multiplication operation on  $G$ , the multiplication operation on  $G_T$  and the bilinear pairing operation  $\hat{e}$ , the adversary  $A$  should be able to issue associated group queries (oracles). Hence, in the following, we modify the game  $G_{CL-eCK}$  defined in Section 3.2 by adding these three group queries  $Q_G, Q_T$  and  $Q_P$ .

- *Initial Setup:* In this phase, the challenger  $B$  first builds several lists by performing the following steps:

1. The challenger  $B$  builds two lists  $L_G$  and  $L_T$  to record pairs of elements in the groups  $G$  and  $G_T$ , respectively.

- The list  $L_G$  consists of elements of the form  $(PG_{m,n,r}, \psi G_{m,n,r})$ . Each  $PG_{m,n,r}$  is a multivariate polynomial that consists of a finite numbers of variates in  $G$  with coefficients in  $Z_p$ . For a given  $PG_{m,n,r}$ ,  $B$  uses a bit string  $\psi G_{m,n,r}$  to communicate with  $A$ . Here, the indices  $m, n$  and  $r$  indicate, respectively, the type of query,  $n$ -th query, and the  $r$ -th element in  $G/G_T$ . Two tuples  $(g, \psi G_{I,1,1})$  and  $(h, \psi G_{I,1,2})$  are initially added in the list  $L_G$ .

- The list  $L_T$  consists of elements of the form  $(PT_{m,n,r}, \psi T_{m,n,r})$  and records the elements of  $G_T$ . The indices  $m, n$  and  $r$  have the same meanings as above.  $PT_{m,n,r}$  is a multivariate polynomial with coefficients in  $Z_p$  and variates in  $G$  or  $G_T$ . For a given  $PT_{m,n,r}$ ,  $B$  uses the bit string  $\psi T_{m,n,r}$  to communicate with  $A$ . It is worth mentioning that all the  $PG_{m,n,r}$  in  $L_G$  and  $PT_{m,n,r}$  in  $L_T$  are different multivariate polynomials. In addition, all the bit strings  $\psi G_{m,n,r}$  and  $\psi T_{m,n,r}$  are distinct bit strings. In the *Query* phase described later, the challenger  $B$  adopts two rules to update two lists  $L_G$  and  $L_T$  as below.

- (1) When recording a multivariate polynomial  $PG_{m,n,r}/PT_{m,n,r}$  in  $L_G/L_T$ ,  $B$  first checks whether or not  $PG_{m,n,r}/PT_{m,n,r}$  has been recorded in  $L_G/L_T$ . If so,  $B$  obtains the corresponding bit string  $\psi G_{m,n,r}/\psi T_{m,n,r}$ . Otherwise,  $B$  randomly chooses a new bit string  $\psi G_{m,n,r}/\psi T_{m,n,r}$  in  $L_G$  and  $L_T$ .  $B$  records  $(PG_{m,n,r}, \psi G_{m,n,r})/(PT_{m,n,r}, \psi T_{m,n,r})$  in  $L_G/L_T$ .

- (2) When recording a bit string  $\psi G_{m,n,r}/\psi T_{m,n,r}$  in  $L_G$ ,  $B$  first checks whether or not  $\psi G_{m,n,r}/\psi T_{m,n,r}$  has been recorded before. If so,  $B$  obtains the corresponding multivari-



ate polynomial  $PG_{m,n,r}/PT_{m,n,r}$ . Otherwise,  $B$  chooses a new variate  $VG_{m,n,r}/VT_{m,n,r}$  and records  $(VG_{m,n,r}, \psi G_{m,n,r})/(VT_{m,n,r}, \psi T_{m,n,r})$  in  $L_G/L_T$ .

- The challenger  $B$  constructs a list  $L_K$  to record the user's private key pairs and public keys. More precisely, the elements in  $L_K$  are of the form  $(ID, SA, XA, TA, PA)$ , where  $ID$  is in  $Z_p$  and  $SA, XA, TA, PA$  are multivariate polynomials recorded in  $L_G$  and  $L_T$ . And  $B$  generates and adds  $l$  honest users in  $L_K$  and sends these user's  $(ID, TA, PA)$  to the adversary  $A$ . The key generation of a user is described as below:

- $B$  randomly chooses the user's identity  $ID_i \in Z_p^*$ .
- $B$  picks a new variable  $T_i$ , and computes four multivariate polynomials:
  - $PSA_i = T_i \cdot g$ .
  - $PXA_i = T_i \cdot g \cdot h$ .
  - $PTA_i = T_i \cdot g \cdot g$ .
  - $PPA_i = g \cdot g \cdot T_i \cdot g \cdot h$ .
- $B$  records  $(ID_i, PSA_i, PXA_i, PTA_i, PPA_i)$  in  $L_K$ .
- $B$  records  $T_i$  and  $PSA_i$  in  $L_G$ , and  $PXA_i, PTA_i$  and  $PPA_i$  in  $L_T$ .

- The challenger  $B$  builds a list  $REC$  to record the details of the sessions. For each oracle  $\Pi_i^s$ ,  $B$  records the session in the form  $(\psi PNID_{i,s}, \psi PNPk_{i,s}, ES_{i,s}, SK_{i,s}, \psi SM_{i,s}, PSM_{i,s}, \psi RM_{i,s}, PRM_{i,s})$ , where

- $\psi PNID_{i,s}$ : The partner's identity.
- $ES_{i,s}$ : The ephemeral key of  $\Pi_i^s$ .
- $SK_{i,s}$ : The session key of this session.
- $\psi SM_{i,s}$ : The transcript (bit string) of message sent by  $\Pi_i^s$ .
- $PSM_{i,s}$ : The multivariate polynomial of  $\psi SM_{i,s}$ .
- $\psi RM_{i,s}$ : The transcript (bit string) of message sent by partner.
- $PRM_{i,s}$ : The multivariate polynomial of  $\psi RM_{i,s}$ .

- At the end of this phase, the challenger  $B$  sends the public parameters  $PP$  to  $A$  (using bit strings).

- *Query*: In this phase,  $A$  can issue the following queries adaptively for totally at most  $q$  times.

- Group query*  $Q_G(\psi G_{Q,i,1}, \psi G_{Q,i,2}, \text{operation})$ : When  $A$  issues the  $i$ -th group query  $Q_G$  with two bit strings  $(\psi G_{Q,i,1}, \psi G_{Q,i,2})$  and an operation (multiplication or division), the challenger  $B$  performs the following steps.

- $B$  first records  $\psi G_{Q,i,1}$  and  $\psi G_{Q,i,2}$  in  $L_G$  and obtains the corresponding multivariate polynomials  $PG_{Q,i,1}$  and  $PG_{Q,i,2}$ .
- $B$  computes and sets the polynomial  $PG_{Q,i,3} = PG_{Q,i,1} + PG_{Q,i,2}$  if the operation is multiplication, or  $PG_{Q,i,3} = PG_{Q,i,1} - PG_{Q,i,2}$  if the operation is division.
- $B$  records  $PG_{Q,i,3}$  in  $L_G$  and obtain the corresponding bit string  $\psi G_{Q,i,3}$ . Finally,  $B$  returns  $\psi G_{Q,i,3}$  to  $A$ .

- Group query*  $Q_T(\psi T_{Q,i,1}, \psi T_{Q,i,2}, \text{operation})$ : When  $A$  issues the  $i$ -th group query  $Q_T$  with two bit strings  $(\psi T_{Q,i,1}, \psi T_{Q,i,2})$  and an operation (multiplication or division). Then, with respect to the group  $G_T$  and the list  $L_T$ ,  $B$  performs similar steps in the Group query  $Q_G$  above.  $B$  finally returns  $\psi T_{Q,i,3}$  to  $A$ .

- Pairing query*  $Q_P(\psi G_{P,i,1}, \psi G_{P,i,2})$ : For the  $i$ -th pairing query  $Q_P$ , by taking as input two bit strings  $\psi G_{P,i,1}$  and  $\psi G_{P,i,2}$ ,  $B$  performs the following steps:

- $B$  first records two bit strings  $\psi G_{P,i,1}$  and  $\psi G_{P,i,2}$  in  $L_G$  and obtains the corresponding multivariate polynomials  $PG_{P,i,1}$  and  $PG_{P,i,2}$ .
- $B$  computes the polynomial  $PT_{P,i,1} = PG_{P,i,1} \cdot PG_{P,i,2}$ .
- $B$  records  $PT_{P,i,1}$  in  $L_T$ , obtains the corresponding bit string  $\psi T_{P,i,1}$ , and returns  $\psi T_{P,i,1}$  to  $A$ .

- Send* $(\Pi_i^s, \psi RM_{i,s}, \psi PNID_{i,s}, \psi PNPk_{i,s})$ :  $A$  can issue the *Send* query with an oracle  $\Pi_i^s$ , the partner's information  $(\psi PNID_{i,s}, \psi PNPk_{i,s})$  and the message  $\psi RM_{i,s}$ .  $B$  first checks whether or not  $\Pi_i^s$  is recorded in the list  $REC$ . If so, then  $B$  returns  $\psi SM_{i,s}$  to  $A$  provided that the other inputs are the same with  $\Pi_i^s$  in  $REC$ ; otherwise,  $B$  returns "false" to  $A$ . If  $\Pi_i^s$  is not recorded in the list  $REC$ ,  $B$  performs the following steps:

- $B$  creates a new record  $(\Pi_i^s, \psi RM_{i,s}, \psi PNID_{i,s}, \psi PNPk_{i,s})$  of  $\Pi_i^s$  in  $REC$ .
- $B$  randomly chooses an ephemeral secret key  $ES_{i,s} \in Z_p^*$  and sets  $PSM_{i,s} = ES_{i,s} \cdot g$ .
- $B$  stores  $PSM_{i,s}$  in  $L_G$  and obtains the corresponding bit string  $\psi SM_{i,s}$ .
- $B$  stores  $PSM_{i,s}$  and  $\psi SM_{i,s}$  in the record of  $\Pi_i^s$  in the list  $REC$ .
- $B$  returns  $\psi SM_{i,s}$  to  $A$ .

- Reveal* $(\Pi_i^s)$ :  $A$  can issue this query to obtain the session key  $SK_{i,s}$  of  $\Pi_i^s$ . Upon receiving the *Reveal* query,  $B$  first checks whether or not  $\Pi_i^s$  has been recorded in the list  $REC$ . If  $\Pi_i^s$  is not recorded in  $REC$ ,  $B$  returns "false" to  $A$ . Otherwise,  $B$  runs the following steps:

- $B$  first obtains  $(\psi RM_{i,s}, \psi PNID_{i,s}, \psi PNPk_{i,s}, ES_{i,s}, PSM_{i,s})$  by searching  $\Pi_i^s$  in  $REC$ .
- $B$  searches  $\psi PNID_{i,s}$  in  $L_K$  to obtains two multivariate polynomials  $PPNTA$  and  $PPNPA$ .
- $B$  searches  $\psi RM_{i,s}$  in  $L_G$  and obtains the multivariate polynomial  $PRM_{i,s}$ .
- $B$  obtains the multivariate polynomials of private key pair  $(PSA_i, PXA_i)$  by searching  $ID_i$  in  $L_K$ .
- $B$  sets a new variable  $TES_{i,s}$  to represent  $ES_{i,s}$ .
- $B$  computes four multivariate polynomials:

- $K_1 = TES_{i,s} \cdot PPNTA$ .
- $K_2 = PRM_{i,s} \cdot PSA_i$
- $K_3 = TES_{i,s} \cdot PRM_{i,s}$ .

- $K_4 = PPNPA \cdot PXA_i$ .
- (7)  $B$  records  $K_3$  in  $L_G$ , and records  $K_1, K_2$  and  $K_4$  in  $L_T$ . Also,  $B$  obtains the corresponding bit strings  $\psi K_1, \psi K_2, \psi K_3$  and  $\psi K_4$ .
- (8)  $B$  sets the session key  $SK_{i,s} = \psi K_1 \oplus \psi K_2 \oplus \psi K_3 \oplus \psi K_4$  in the tuple of  $\Pi_i^s$  of the list  $REC$ .
- *Ephemeral-secret-leakage*( $\Pi_i^s$ ): Upon receiving the *Reveal* query along with  $\Pi_i^s$ ,  $B$  checks whether or not  $\Pi_i^s$  has been recorded in  $REC$ . If so,  $B$  returns  $ES_{i,s}$  to  $A$ . Otherwise,  $B$  returns “false” to  $A$ .
- *Corrupt*( $ID_i$ ): Upon receiving the *Corrupt* query along with  $ID_i$ ,  $B$  first checks whether or not  $ID_i$  has been recorded in  $L_K$ . If not,  $B$  returns “false” to  $A$ . Otherwise,  $B$  can obtain  $(ID_i, PSA_i, PXA_i, PTA_i, PPA_i)$  from  $L_K$ , and returns the corresponding bit strings of  $(PSA_i, PXA_i)$  to  $A$ .
- *Leak query*( $f_{i,s}, h_{i,s}, \Pi_i^s$ ): Upon receiving the *Leak* query along with  $f_{i,s}, h_{i,s}$  and  $\Pi_i^s$ ,  $B$  computes and sends the fractional leakage information  $(\Delta f_{i,s}, \Delta h_{i,s})$  to  $A$ , where  $\Delta f_{i,s} = f_{i,s} (SA_{i-1,1}, SA_{i-1,2}, XA_{i-1,1}, XA_{i-1,2}, \alpha_i)$  and  $\Delta h_{i,s} = h_{i,s} (SA_{i,2}, SA_{i,2}, XA_{i,2}, XA_{i,2}, ES_{i,s})$ , where  $ES_{i,s}$  denotes the random value  $x$  or  $y$ . Note that two leakage functions must satisfy  $|f_{i,s}| \leq \lambda$  and  $|h_{i,s}| \leq \lambda$ .
- *Test*( $\Pi_i^s$ ): When  $A$  issues this query, the challenger  $B$  first checks whether or not  $\Pi_i^s$  has been record in  $REC$ . If not,  $B$  returns “false” to  $A$ . Also, if  $SK_{i,s}$  has been set in  $REC$ ,  $B$  also returns “false” to  $A$ . Otherwise,  $B$  performs the following steps:
  - (1)  $B$  issues the *Reveal*( $\Pi_i^s$ ) to obtain the session key  $SK_{i,s}$ .
  - (2)  $B$  flips an unbiased coin bit  $cb \in \{0, 1\}$ . If  $cb = 1$ ,  $B$  returns  $SK_{i,s}$  to  $A$ ; otherwise,  $B$  returns a random bit string to  $A$ .

Now we analyze the advantage of the adversary  $A$  winning the game.

- (1) Let us discuss the numbers of elements added in of  $L_G$  and  $L_T$  after all kinds of queries.
  - For each query of  $Q_G, Q_T$  or  $Q_P$ , at most 3 elements are recorded in  $L_G$  or  $L_T$ .
  - For each *Send* query, at most 1 new element is recorded in  $L_G$  or  $L_T$ .
  - For each *Reveal* query, at most 5 new elements are recorded in  $L_G$  or  $L_T$ .
  - For each *Ephemeral-secret-leakage* or *Corrupt* query, no new element recorded.
  - For each *Test* query, at most 5 new elements are recorded in  $L_G$  or  $L_T$ .

Let  $q_Q$  denote the total number of group queries  $Q_G, Q_T$  and  $Q_P$ , and let  $q_S, q_R$  and  $q_T$ , denote the numbers of *Send*, *Reveal* and *Test* queries, respectively. As before,  $|L_G|$  and  $|L_T|$  denote, respectively, the numbers of elements in  $L_G$  and  $L_T$ . Then, we have  $|L_G| + |L_T| \leq 2 + 3q_Q + q_S + 5q_R + 5 \leq 5q$ .

- (2) Let us discuss the degrees of polynomials in  $L_G$ .
  - Every polynomial of new variates  $VG_{m,n,r}, T_i$  and  $TES_{i,s}$  in  $L_G$  has degree 1.
  - Every polynomial of the private key  $PSA_i$  has degree 2.
  - In the *Reveal* query,  $K_3$  and  $PrM_{i,s}$  have the same degree.
  - In  $Q_G$ , the degree of  $PG_{Q,i,3}$  is equal to the maximal degree of  $PG_{Q,i,1}$  and  $PG_{Q,i,2}$ , since the polynomial  $PG_{Q,i,3} = PG_{Q,i,1} + PG_{Q,i,2}$ .

Hence, every polynomial in  $L_G$  has degree at most 2.

- (3) Let us discuss the degrees of polynomials in  $L_T$ .
  - Every polynomial of new variates  $VT_{m,n,r}$  in  $L_T$  has degree 1.
  - Every polynomial of the private key  $PXA_i = T_i \cdot g \cdot h$  has degree 3.
  - The polynomials of the private key pairs  $PTA_i$  and  $PPA_i$  have degrees 3 and 5, respectively.
  - In  $Q_P$ , each polynomial  $PT_{P,i,k}$  has degree at most 4 since each polynomial in  $L_G$  has degree at most 2.
  - In the *Reveal* query, the degree of  $K_1 = TES_{i,s} \cdot PPNTA$  is 4, the degree of  $K_2 = PrM_{i,s} \cdot PSA_i$  is 4, and the degree of  $K_4 = PPNPA \cdot PXA_i$  is 8.
  - In  $Q_T$ , the degree of  $PT_{Q,i,3}$  is equal to the maximal degree of  $PT_{Q,i,1}$  and  $PT_{Q,i,2}$  since the polynomial  $PT_{Q,i,3} = PT_{Q,i,1} + PT_{Q,i,2}$ .

Hence, every polynomial in  $L_T$  has degree at most 8.

Next, for each variable in  $L_G$  and  $L_T$ , the challenger  $B$  chooses a random value in  $Z_p^*$ , denoted by  $v_1, v_2, \dots, v_n$ . The adversary  $A$  is said to win the game  $G_{CL-eCK}$  if one of the following two cases occurs:

- *Case 1*: The adversary  $A$  finds a collision in  $G$  or  $G_T$ :
  - ◊ There exist two polynomials  $PG_i$  and  $PG_j$  in  $L_G$  such that  $PG_i(v_1, v_2, \dots, v_n) = PG_j(v_1, v_2, \dots, v_n)$ .
  - ◊ There exist two polynomials  $PT_i$  and  $PT_j$  in  $L_T$  such that  $PT_i(v_1, v_2, \dots, v_n) = PT_j(v_1, v_2, \dots, v_n)$ .
- *Case 2*: The adversary  $A$  outputs  $cb' = cb$  after the *Test* query.

In the real adversary game, the advantage of the adversary  $A$  in the simulated game  $G_{CL-eCK}$  is an upper bound of the success probability of  $A$ . For convenience, we first compute  $A$ 's success probability under the situation that  $A$  cannot issue the *Leak* query in the query phase. Afterwards, we discuss the other situation.

- Without the *Leak* query: If  $A$  does not use the *Leak* query, the success probability can be computed by the following two cases:

- *Case 1*: The adversary  $A$  can find a collision in  $G$  or  $G_T$ . In this case,  $A$  can resolve the discrete logarithm problem in  $G$  or  $G_T$ . Assume that  $PG_i$  and  $PG_j$  are two distinct polynomials in  $L_G$ . We compute the probability of the event when the polynomials  $PG_C = PG_i - PG_j$  is a zero polynomial.

By applying Lemma 2 with  $\lambda = 0$  in Section 2, the probability of  $PG_C(v_1, v_2, \dots, v_n) = 0$  in  $Z_p^*$  is at most  $2/p$ . Since there are  $\binom{|L_G|}{2}$  different pairs  $(PG_i, PG_j)$  in  $L_G$ , the probability that Case 1 occurs is at most  $(2/p)\binom{|L_G|}{2}$ . Similarly, since every polynomial in  $L_T$  has degree at most 8, the collision probability in  $L_T$  is at most  $(8/p)\binom{|L_T|}{2}$ .

- *Case 2*: The adversary  $A$  cannot find any collision in  $G$  or  $G_T$ . In this case,  $A$ 's view in the game  $G_{CL-eCK}$  is identical to that in the real game. This means that  $A$  does not obtain useful information for guessing a coin bit  $cb$  in the game  $G_{CL-eCK}$ . Hence, the success probability that  $A$  outputs a correct coin bit  $cb' = cb$  is  $1/2$  on average.

Now, let's evaluate the success probability  $Pr_{A-wol}$  that  $A$  wins the game  $G_{CL-eCK}$  without the *Leak* query. By the discussion above, the probability for Case 1 to occur satisfies the inequality

$$\begin{aligned} Pr[Case1] &\leq [(2/p)\binom{|L_G|}{2} + (8/p)\binom{|L_T|}{2}] \\ &\leq (8/p)(|L_G| + |L_T|)^2 \leq 200q^2/p. \end{aligned}$$

On the other hand, in Case 2,  $A$  has probability  $1/2$  to output a correct coin bit. Therefore, the success probability  $Pr_{A-wol}$  satisfies

$$\begin{aligned} Pr_{A-wol} &\leq Pr[Case1] + Pr[Case2] \\ &\leq 200q^2/p + (1/2). \end{aligned}$$

The advantage that  $A$  wins the game  $G_{CL-eCK}$  without issuing the *Leak* query is

$$Adv_{A-wol} \leq |200q^2/p + (1/2) - (1/2)| = 200q^2/p.$$

Obviously,  $Adv_{A-wol}$  is negligible if  $q = \text{poly}(\log p)$ .

- With the *Leak* query: Note that, under Situation 2 described in the proof of Theorem 1,  $A$  is allowed to issue the *Ephemeral-secret-leakage*( $\Pi_i^s$ ) and *Ephemeral-secret-leakage*( $\Pi_j^s$ ), but not the *Corrupt*( $ID_i$ ) or *Corrupt*( $ID_j$ ) during the query phase. Two fractional leakage information  $\Delta f_{i,s}$  and  $\Delta h_{i,s}$ , respectively, are used to represent the outputs of two leakage functions  $f_{i,s}$  and  $h_{i,s}$ . By  $\Delta f_{i,s}$  and  $\Delta h_{i,s}$ , the leaked information about  $f_{i,s}(SA_{i-1,1}, SA_{i-1,2}, XA_{i-1,1}, XA_{i-1,2}, \alpha_i)$  and  $h_{i,s}(SA_{i,2}, SA_{i,2}, XA_{i,2}, ES_{i,s})$  are discussed below:

- $\alpha_i$ : The random value  $\alpha_i$  is used to generate the next private key pairs. Since the value  $\alpha_i$  is randomly chosen for each key refreshing, the  $\lambda$  bits of  $\alpha_i$  is useless for the next session.
- $ES_{i,s}$ : The ephemeral secret key  $ES_{i,s}$  denotes the random value  $x$  or  $y$  in the oracle  $\Pi_i^s$  and can only be leaked once. Since  $A$  can obtain the ephemeral secret keys completely by issuing *Ephemeral-secret-leakage*( $\Pi_i^s$ ) and *Ephemeral-secret-leakage*( $\Pi_j^s$ ), the leakage information of  $ES_{i,s}$  is useless.
- $(SA_{i-1,1}, SA_{i,1}, SA_{i-1,2}, SA_{i,2})$ : The user's first private key satisfies the equality  $SA = SA_{i-1,1} \cdot$

$SA_{i-1,2} = SA_{i,1} \cdot SA_{i,2}$ . And the leakage information of  $SA_{i,1}$  and  $SA_{i,2}$  is independent of that of  $SA_{i-1,1}$  and  $SA_{i-1,2}$ . Thus,  $A$  can learn at most  $\lambda$  bits of  $SA$ .

- $(XA_{i-1,1}, XA_{i,1}, XA_{i-1,2}, XA_{i,2})$ : The user's second private key satisfies the equality  $XA = XA_{i-1,1} \cdot XA_{i-1,2} = XA_{i,1} \cdot XA_{i,2}$ . And the leakage information of  $XA_{i,1}$  and  $XA_{i,2}$  is independent of that of  $XA_{i-1,1}$  and  $XA_{i-1,2}$ . Thus,  $A$  can learn at most  $\lambda$  bits of  $XA$ .

Now, let us discuss the successful probability  $Pr_{A-wl}$  that  $A$  wins the game  $G_{CL-eCK}$  with the *Leak* query. Indeed, in Situation 2,  $A$  can obtain the ephemeral secret keys. Then, by getting a participant's private keys  $SA$  and  $XA$ , the adversary  $A$  can always output a correct coin bit. To evaluate the successful probability  $Pr_{A-wl}$ , we define two events as follows.

- (1) The event  $EP$  denotes that  $A$  may obtain  $SA$  and  $XA$  completely from the fractional leakage information  $\Delta f_{i,s}$  and  $\Delta h_{i,s}$ . In addition, the events  $\overline{EP}$  denote the complement events of  $EP$ .
- (2) The event  $EC$  denotes that  $A$  may output a correct coin bit  $cb' = cb$ . The successful probability  $Pr_{A-wl}$  that  $A$  wins the game  $G_{CL-eCK}$  with the *Leak* query is bounded as below.

$$\begin{aligned} Pr_{A-wl} &= Pr[EC] \\ &= Pr[EC \cap EP] + Pr[EC \cap \overline{EP}] \\ &= Pr[EC \cap EP] + Pr[EC|\overline{EP}] \cdot Pr[\overline{EP}]. \end{aligned}$$

Since  $Pr[EC \cap EP] \leq Pr[EP]$  and  $Pr[\overline{EP}] = 1 - Pr[EP]$ , we obtain  $Pr_{A-wl} \leq Pr[EP] + Pr[EC|\overline{EP}] \cdot (1 - Pr[EP])$ . Under the condition,  $A$  can't obtain any useful information to output a correct coin bit. Hence,  $Pr[EC|\overline{EP}]$  is  $1/2$  on average. Thus,

$$\begin{aligned} Pr_{A-wl} &\leq Pr[EP] + (1/2) \cdot (1 - Pr[EP]) \\ &= 1/2(1 + Pr[EP]). \end{aligned}$$

Therefore, the advantage that  $A$  wins the game  $G_{CL-eCK}$  with the *Leak* query is  $Adv_{A-wl} \leq |Pr_{A-wl} - 1/2| = (1/2)Pr[EP]$ . Since the advantage  $Adv_{A-wol}$  that  $A$  wins the game  $G_{CL-eCK}$  without the *Leak* query is  $Adv_{A-wol} \leq 100q^2/p = O(q^2/p)$ , the advantage that  $A$  wins the game  $G_{CL-eCK}$  with issuing the *Leak* query  $Adv_{A-wl} \leq O((q^2/p) * 2^\lambda)$  because  $A$  can learn at most  $\lambda$  bits of the private keys  $SA$  and  $XA$ . By Lemma 2, if  $\lambda \leq \log p - \omega(\log \log p)$ , we say that the proposed LR-AKA protocol is secure in the continual leakage model. Q.E.D.

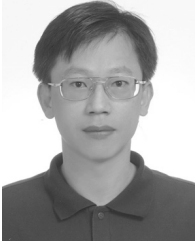
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