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Spectral and Energy Efficiency of Massive MIMO for Hybrid Architectures Based on Phase Shifters

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ABSTRACT This paper investigates the downlink achievable spectral efficiency (SE) and energy efficiency (EE) of massive MIMO for hybrid architectures based on phase shifters, where the base station (BS) has perfect channel state information and baseband processing is done zero-forcing precoding. We derive an approximated upper bound on the achievable SE for hybrid architecture with ideal phase shifters. Based on the derived analytical expression, we find that the total achievable SE increases with the number of BS antennas and users and the signal-to-noise ratio (SNR). In order to acquire the required limited feedback, we propose an algorithm to generate the corresponding quantized matrix and study hybrid architectures with quantized phase shifters. Compared to full-digital architectures, results show that hybrid architectures enjoy a much higher achievable EE in massive MIMO systems, whereas its achievable SE is inferior to full-digital architectures. In addition, results also showcase that the achievable SE of hybrid architectures can be improved by increasing the bits of phase shifters, and there exists an optimal SNR and antenna number to maximize the achievable EE of the system.

INDEX TERMS Energy efficiency, hybrid architectures, massive MIMO, spectral efficiency, zero-forcing.

I. INTRODUCTION

Massive multi-input-multi-output (MIMO), where base station (BS) is equipped with a large number of antennas has emerged as one of the promising technologies for 5th generation (5G) communications since it provides both higher spectral efficiency (SE) and energy efficiency (EE) [1], [2]. In conventional MIMO system, each antenna requires a dedicated radio frequency (RF) chain, which includes low-noise amplifier, digital to analog converter (DAC), high resolution analog to digital converter (ADC) and so on [3]. However, when the number of BS antennas is very large (i.e., massive MIMO system), dedicating a separate RF chain for each antenna not only consumes high levels of energy but also increases the hardware cost of system [4]–[6].

To reduce the hardware cost and power consumption, two types of newborn architectures were proposed for massive MIMO systems. One architectures is a mixed-ADC receiver architecture [7]–[10], in which a fraction of the ADCs has full-resolution to promote system performance and the rest utilize a low resolution ADCs in consideration of the hardware cost and energy consumption. Liang *et al.* [17] reported that a mixed-ADC architecture is able to enhance the system performance and reduces channel estimation overhead, which effectively balance hardware cost and power consumption. In [8], a mixed-ADC receiver architecture was investigated for multiuser massive MIMO systems, in which a family of Bayes detectors were developed by conducting Bayesian estimate of the user signals with generalized approximate message passing algorithm. The work of [9] investigated the uplink achievable SE of massive MIMO systems with a mixed ADC receiver architecture and derived a closed-form approximation of the achievable SE with the maximum-ratio combining detector. In order to reduce power consumption, Zhang *et al.* [11] proposed the power allocation algorithms for mixed-ADC massive MIMO systems, which satisfy the total power constraint and guarantee the minimum transmit power of each user to meet quality of service.

Another architectures is hybrid analog/digital precoding architectures, which uses a small number of RF chains to control the large-scale antenna array. For example, low-complexity hybrid beamforming algorithms were proposed for single-user MIMO-OFDM systems in [12], which aimed to maximize either the power of the received signal or the total achievable SE over different sub-carriers. In order to maximize the sum rate, Bogale *et al.* [13] discussed the tradeoff between the required number of RF chains and phase shifters, which makes hybrid beamforming achieve the same performance of full digital precoding. As a newborn architecture, Tan *et al.* [14] proposed hybrid architectures based on discrete fourier transform processing, where baseband processing is performed by zero-forcing precoding and investigates the achievable SE of systems. Most studies of massive MIMO systems have emphasized the achievable SE of system. Yet, the work of [15] proposed a successive interference cancelation (SIC)-based hybrid precoding. The result showed that SIC-based hybrid precoding is near-optimal and enjoys higher energy efficiency than the spatially sparse precoding and the fully digital precoding. To maximize the total achievable EE and reduce the cost of RF circuits, Zi *et al.* [16] analyzed the achievable SE hybrid precoding algorithm by utilizing user scheduling and resource management schemes. In all the aforementioned studies, there is no whole theoretical analysis on the achievable SE and EE for hybrid massive MIMO systems. Therefore, the exploitation of the achievable SE and EE in hybrid massive MIMO systems is of special interest.

Motivated by the above gaps, we investigate the achievable SE and EE of multiuser massive MIMO for hybrid architectures with phase shifters. Specifically, our main contributions are summarized as follows:

- For hybrid architectures with ideal phase shifters, we derive an upper bound on the achievable SE for finite number of BS antennas. Based on the derived analytical expressions, the effects of the number of BS antennas, the SNR, and the number of users are presented. We also study hybrid architectures with quantized phase shifters, which utilizes a codebook of finite phase angle.
- We consider a realistic power consumption model and use it to evaluate the total achievable EE for hybrid architectures. By applying the analytical result of the achievable SE, closed-form expressions for the optimal SNR and the number of BS antennas by maximizing the achievable EE are derived.
- Compared to the existing full-digital architectures, results show that hybrid architectures offer the higher achievable EE, whereas its achievable SE is inferior to that of full-digital architectures. Furthermore, numerical results also showcase that hybrid architectures based on ideal phase shifters exhibit a considerable achievable EE that of quantized phase shifters with a large scale number of antennas and low SNR regime.

Notation: Vectors and matrix are expressed as lowercase boldface, $|\cdot|$ and $\mathbb{E}\{\cdot\}$ denote absolute value and expectation operation, respectively; $\lfloor\cdot\rfloor$ represents rounding to the nearest integer; $\mathbf{I}_K$ denotes an $K \times K$ identity matrix; $[\mathbf{A}]_{n,m}$ is the element at the n-th row and m-th column of $\mathbf{A}$, $W_0(\cdot)$ denotes the Lambert function[28, eq. (3.1)], and $j = \sqrt{-1}$ denotes the imaginary unit.

II. SYSTEM MODEL

In this section, we describe the massive MIMO system that deploys a hybrid architecture based on phase shifters. Some key assumptions made for the system model are also highlighted.

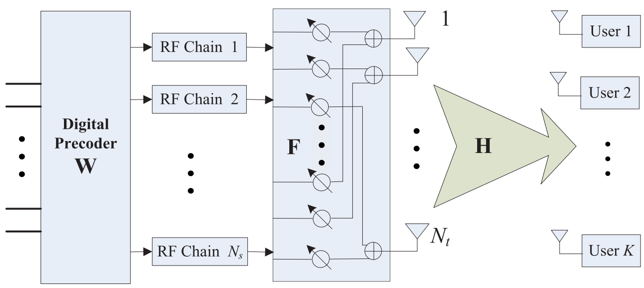

FIGURE 1. Multiuser massive MIMO system with hybrid architecture based on phase shifters.

A. CHANNEL MODEL

Consider a narrowband massive MIMO system of hybrid architecture based on phase shifters as shown in Fig. 1, where the BS is equipped with N_t transmit antennas, which connects to N_s RF chains, and simultaneously serves K single-antenna users. Without loss of generality, we assume that the number of BS antennas is much larger than the number of users, i.e., $N_t \gg K$ and the number of RF chains equals to the number of activated users in hybrid architecture, i.e., $N_s = K$. In this paper, we adopt the equal power allocation scheme for all K activated users and let $\mathbf{x}$ be the $K \times 1$ the signal vector for a total of K users, satisfying $\mathbb{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{I}_K$. If Gaussian inputs are used, the received signal after analog signal processing and digital precoding, can be expressed as

$$\mathbf{y} = \sqrt{\frac{P}{K}}\mathbf{G}\mathbf{F}\mathbf{W}^H\mathbf{x} + \mathbf{n}, \tag{1}$$

where P denotes the total transmitter power of the BS, $\mathbf{W}$ and $\mathbf{F}$ denote the digital precoding matrix of $K \times K$ dimension, and $\mathbf{F}$ represents the analog processing matrix of $N_t \times K$ dimension corresponding to the phase shifters or switches, $\mathbf{n} \in \mathbb{C}^{K\times 1}$ is the additive white Gaussian noise, and $\mathbf{G}$ be the $K \times N_t$ channel matrix from the users to the BS. The channel matrix is modeled as

$$\mathbf{G} = \mathbf{H}\mathbf{D}^{1/2}, \tag{2}$$

where $\mathbf{H} \in \mathbb{C}^{K\times N_t}$ contains the fast-fading coefficients, whose entries are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero-mean and unit-variance denoted $\mathcal{CN}(0, 1)$, and $\mathbf{D}$ is an $K \times K$ diagonal matrix with diagonal elements given by $[\mathbf{D}]_{k,k} = \beta_k$. Herein, the large fading $\beta_k = z_k r_k^{-\gamma}$ models both path loss and shadowing, where r_k is the distance from the kth user to the BS, γ is the decay exponent, and z_k is a log-normal random variable.

For hybrid architecture based on phase shifters, we consider the analog processing matrix $\mathbf{F}$ is implemented by using analog phase shifters and magnitude of its entries should satisfies a constant modulus constraint such that $|[\mathbf{F}]_{n,k}|^2 = 1/N_t$, which efficiently improves hardware cost and power consumption of system. Meanwhile, we assume that the angles of the analog phase shifters can be extracted by taking the conjugate transpose of the channel matrix $[\mathbf{G}]_{n,k}$. In this case, the phases of the conjugate transpose are able to harvest the large array gain in massive MIMO systems, which has been reported by [17]. With these assumptions, the analog processing matrix $\mathbf{F}$ is designed as

$$[\mathbf{F}]_{n,k} = \frac{1}{\sqrt{N_t}} e^{j\varphi_{n,k}}, \tag{3}$$

where $\varphi_{n,k}$ is the phase angle of the (n, k)-th element.

As shown in Fig. 1, the downlink hybrid architecture precoding is divided among analog and digital processing successively, corresponding to matrixes read as $\mathbf{F}$ and $\mathbf{W}$, respectively. Before analog processing and signal transmission, we know that an equivalent downlink channel matrix $\mathbf{G}_{eq}$ becomes a composite matrix that consists practical channel matrix and analog matrix, i.e., $\mathbf{G}_{eq} = \mathbf{GF}$. Therefore, we easily know that the precoding matrices is equivalent to $\mathbf{G}_{eq}$.

III. SPECTRAL EFFICIENCY ANALYSIS

In this section, we first investigate the achievable SE of hybrid MIMO systems and derive an approximated upper bound on the achievable SE for hybrid architecture with ideal phase shifters. Based on the derived analytical expression, the effects of the number of BS antennas, the SNR, and the number of users are discussed. Then, we propose an algorithm to generate the corresponding quantized matrix and study the achievable SE of hybrid architecture with quantized phase shifters.

A. THE ACHIEVABLE ERGODIC SE

Assume that the CSI is known perfectly and instantaneously at the BS. This assumption is widely adopted for the precoding design problem of massive MIMO system [18], such that the CSI at the receiver can be obtained via training and subsequently shared with the transmitter via limited feedback in the frequency division duplex (FDD) mode. For the time division duplex (TDD) mode, the CSI can be obtained through uplink channel estimation then applied to downlink precoding based on the channel reciprocity. Moreover, in multiuser MIMO systems, ZF precoding is able to completely cancel out inter-user interference, which can perform almost as well as dirty paper coding (DPC) [19]. Therefore, we concentrate on adopting the ZF precoding scheme in this paper. The baseband precoding matrix is given by

$$\mathbf{W} = \mathbf{G}_{eq}^H \left(\mathbf{G}_{eq}\mathbf{G}_{eq}^H\right)^{-1} \mathbf{\Lambda}, \tag{4}$$

where $\mathbf{G}_{eq} = \mathbf{GF}$ and $\mathbf{\Lambda}$ denotes a normalized diagonal matrix, i.e. $[\Lambda]_{k,k} = 1/\sqrt{[\mathbf{W}^H\mathbf{W}]_{k,k}}$. For the case under the above consideration, the achievable SE of the k-th user with ZF precoding can be given by

$$R_k = \mathbb{E}\left\{\log_2\left(1 + \frac{P}{K\left[\left(\mathbf{G}_{eq}\mathbf{G}_{eq}^H\right)^{-1}\right]_{k,k}}\right)\right\}. \tag{5}$$

Considering all the users, the total achievable SE of the hybrid architecture MIMO system in bits/s/Hz is calculated as

$$R_{\text{sum}} = \sum\nolimits_{k=1}^{K} R_k. \tag{6}$$

From (5), it is worth noting that the expectation is taken over all channel realizations of $\mathbf{H}_{eq}$ and the equivalent channel is needed to be ergodic. We observe that it is difficult to derive closed-form expressions on the achievable SE for hybrid architecture. Alternatively, we seek to a tractable bound on the total achievable SE of hybrid architecture massive MIMO system, which enable us to draw engineering insights into the performance of system.

B. SPECTRAL EFFICIENCY OF IDEAL PHASE SHIFTERS

In this section, we derive an approximated upper bounds on the achievable SE. Based on the derived theoretical result, the effect of several interesting insights, such as the number of antennas, the SNR, and the number of users, on the achievable SE is revealed.

We now present the following theorem, which shows an approximated upper bound on the achievable SE.

Theorem 1: For the hybrid architecture with ideal phase shifters in Fig. 1, the upper bound on the achievable SE of each user with ZF precoding is approximated as

$$\hat{R}_k^U = \log_2\left(1 + \frac{P\beta_k}{K}\left(\frac{\pi\,(N_t - 1)}{4} + 1\right)\right). \tag{7}$$

Proof: Based on the Jensen's inequality, the upper bound of the achievable SE can be expressed as

$$R_k \leq R_k^U = \log_2\left(1 + \frac{P}{K}\mathbb{E}\left\{\frac{1}{\left[\left(\mathbf{G}_{eq}\mathbf{G}_{eq}^H\right)^{-1}\right]_{k,k}}\right\}\right). \tag{8}$$

According to the channel model in (2), we have

$$R_k^U = \log_2\left(1 + \frac{P\beta_k}{K}\mathbb{E}\left\{\frac{1}{\left[\left(\mathbf{H}_{eq}\mathbf{H}_{eq}^H\right)^{-1}\right]_{k,k}}\right\}\right). \tag{9}$$

The diagonal and off diagonal entries of $\mathbf{H}_{eq}$ are given by

$$\left[\mathbf{H}_{eq}\right]_{k,k} = \mathbf{h}_k^H \mathbf{f}_k = \frac{1}{\sqrt{N_t}} \sum_{n=1}^{N_t} \left|[\mathbf{H}]_{n,k}\right| \tag{10}$$

and

$$\left[\mathbf{H}_{eq}\right]_{k,j} = \mathbf{h}_k^H \mathbf{f}_j = \frac{1}{\sqrt{N_t}} \sum_{n=1}^{N_t} [\mathbf{H}]_{n,j} e^{j\varphi_{n,j}}, \tag{11}$$

where $\mathbf{h}_k$ and $\mathbf{f}_k$ are the k-th column of $\mathbf{H}$ and $\mathbf{F}$, respectively. With the aid of the results in [17], it can be shown that the mean and variance of the diagonal entries of $\mathbf{H}_{eq}$ equal to $\sqrt{\pi N_t}/2$ and $1-\pi/4$, respectively, and the mean and variance of the off-diagonal entries of $\mathbf{H}_{eq}$ equal to 0 and 1. Given a massive MIMO system, we can infer that the mean value of diagonal entries plays an important dominant since the mean value of diagonal entries is much larger than the mean value of off-diagonal entries. Therefore, the upper bound of the achievable SE is approximated as

$$\hat{R}_k^U = \log_2\left(1 + \frac{P\beta_k}{K}\mathbb{E}\left\{\left[\mathbf{H}_{eq}\right]_{k,k}^2\right\}\right). \tag{12}$$

By applying the identify equal $\mathbb{E}\left\{\alpha^2\right\} = (\mathbb{E}\{\alpha\})^2 + \mathrm{var}(\alpha)$, we obtain

$$\mathbb{E}\left\{\left[\mathbf{H}_{eq}\right]_{k,k}^2\right\} = \frac{\pi N_t}{4} - \frac{\pi}{4} + 1. \tag{13}$$

Substituting (13) into (12) and performing some basic algebraic manipulations, we obtain the desired result. ■

From *Theorem* 1, it is worth noting that although $\hat{R}_k^{U,1}$ is approximated upper bound, it shall reveals important trends of the ergodic achievable SE with respect to several physical parameters. Furthermore, it can be seen that the upper bound is very simple and efficiently programmed. In addition, we know that the achievable SE is a function of the SNR, the number of BS antennas and users. The following corollary presents the achievable SE limit as the number of BS antennas and the SNR grows without bound.

Corollary 1: For the special case of $N_t \to \infty$ or $P \to \infty$, the total achievable SE is a logarithmic increasing function with the number of BS antennas and the SNR as follows:

$$\lim_{N_t \text{ or } P \to \infty} \hat{R}_k^U = \hat{R}_k^\infty = \log_2\left(1 + N_t P\beta_k\left(\frac{\pi}{4K}\right)\right). \tag{14}$$

Proof: The result is directly obtained by setting $N_t \to \infty$ or $P \to \infty$ and omitting the constant term. ■

From *Corollary* 1, we observe that for a fixed number of users, the upper bound on the achievable SE in (14) becomes a strictly logarithmic increasing function with the number of BS antennas and the SNR. This implies that the achievable can be boosted by increasing the number of BS antennas or improving the transmitter power. We then investigate the impact of the number of users on the achievable SE in the following corollary.

Corollary 2: For the special case of $K \to \infty$ with $N_t/K = \alpha$, for some fixed α, the total achievable SE is a monotonically increasing function of K.

Proof: According to (7), the upper bound on total achievable SE can be expressed as

$$R_{\text{sum}} = K\log_2\left(1 + \frac{P\beta_k}{K}\left(\frac{\pi\ (N_t - 1)}{4} + 1\right)\right). \tag{15}$$

For the sake of convenience, we denote $y = P\beta_k(\pi\ (N_t - 1)/4 + 1)$. By differentiating the total achievable SE in (15) with respect to the number of users K, the first order partial derivative of $R_{\text{sum}}(K)$ can be written as

$$\frac{\partial R_{\text{sum}}(K)}{\partial K} = \frac{1}{\ln 2}\left(\ln\left(\frac{K+y}{K}\right) + \frac{K}{K+y} - 1\right). \tag{16}$$

From (16), it can be seen that $\partial R_{\text{sum}}(K)/\partial K \geq 0$ for any number of users, which indicates that R_{sum} is always a monotonically increasing function of K. ■

Remark 1: According to (7), we observe that the achievable SE per user decreases with the number of activated users, due to average power designated for each user. Nevertheless in *Corollary* 2, it can be seen that fixed the number of BS antennas and SNR, the total achievable SE of system can be improved by increasing the number of activated users. This indicates that increasing the number of activated users benefits the total achievable SE of hybrid massive MIMO systems.

To this end, we have deduced tractable upper bound on the achievable SE of hybrid architecture with ideal phase shifters and discussed the impact of physical insights (SNR, the number of BS antennas and activated users) on the achievable SE. It should be emphasized that for such hybrid architecture, only all of RF chains are available infinite resolution phase shifters so that RF chains are able to generate the accurate phase angles. However, in practical implementation, such shifters with continuous phase is not feasible due to components require for accurate phase control [20]. Moreover, finite resolution analog phase are more attractive in limited feedback systems [21]. Hence, studying the achievable SE of the achievable SE of hybrid architectures with quantized phase shifters is of special interest.

C. SPECTRAL EFFICIENCY OF QUANTIZED PHASE SHIFTERS

In this subsection, we first discuss how the analog processing matrix for finite resolution phase shifters can be constructed, and then we provide the approach how to generate the corresponding analog processing matrix.

For hybrid MIMO systems based on finite resolution phase shifters, we define $\mathbf{F}$ as the analog processing matrix, which can be obtained via a codebook of finite phase angles. Let $\epsilon_{n,k}$ be the error between $\varphi_{n,k}$ and $\hat{\varphi}_{n,k}$, where $\varphi_{n,k}$ denotes the ideal phase and $\hat{\varphi}_{n,k}$ denotes the quantized phase that is selected from a codebook $\mathcal{S}$

$$\mathcal{S} = \left\{0, \pm\left(\frac{2\pi}{2^b}\right), \pm 2\left(\frac{2\pi}{2^b}\right), \cdots \pm 2^{b-1}\left(\frac{2\pi}{2^b}\right)\right\}, \tag{17}$$

where b denotes the number of quantization bits while the quantized phase $\hat{\varphi}_{n,k}$ is one of the entries of the set $\mathcal{S}$.

Thus, the phase of each entry of $\mathbf{F}$ is chosen from a codebook based on the closest Euclidean distance, which reads as

$$[\mathbf{\Delta}]_{n,k} = \arg\min \left\| \left[\hat{\mathbf{\Psi}}\right]_{n,k} - [\mathbf{\Psi}]_{n,k} \right\|_2^2, \tag{18}$$

where $\mathbf{\Delta}$ is the error matrix of phase shifter, whose entries read as $\epsilon_{n,k}$, $[\Psi]_{n,k}$ is the real phase shifter matrix of the instantaneous channel, whose entries treat as $\varphi_{n,k}$ that can be computed by $\angle[\mathbf{G}]_{n,k}$, and $\left[\hat{\mathbf{\Psi}}\right]_{n,k}$ is the quantized phase matrix, whose entries read as $\hat{\varphi}_{n,k}$. Our target is to find a feasible angle $\left[\hat{\mathbf{\Psi}}\right]_{n,k}$, which is sufficiently close (in terms of Euclidean distance) to the optimal but unpractical angle $[\mathbf{\Psi}]_{n,k}$. Thus, the quantized phase matrix $\mathbf{F}$ can be calculated as

$$[\mathbf{F}]_{n,k} = \frac{1}{\sqrt{N_t}} e^{i\hat{\varphi}_{n,k}}, \tag{19}$$

For hybrid architecture with finite resolution phase shifters, $\mathbf{F}$ denotes finite resolution phase matrix, whose entries can be generated via channel matrix $\mathbf{H}$. When the resolution of the phase shifters is high, finding the optimal phase angle is complicated due to using an exhaustive search method. For our simulation results, we will assume that the RF chains are constructed by using 1-bit resolution phase shifters. Therefore, $\hat{\varphi}_{n,k}$ has only two phase candidates $\{0, \pi\}$. In the following section, we will investigate the achievable EE of massive MIMO for hybrid architectures.

IV. ENERGY EFFICIENCY ANALYSIS

Green communication is treated as key requirement for next generation systems and the total EE should be adopted as the performance criterion, which is defined as the ratio of the total achievable SE to the total power consumption. Therefore, the total achievable EE of system in bit/Joule can be established as

$$\eta_E = \frac{BR_{\text{sum}}}{P_{\text{total}}}, \tag{20}$$

where B is the available bandwidth, R_{sum} was defined in (6), and P_{total} denotes the overall power consumption of system, which will be discussed in the following.

Building on the prior works of [23]–[25], we consider a realistic power consumption model of massive MIMO systems, which can be categorized as follows: Circuit power P_{C}, signal processing power P_{SP}, signal transmission power P_{T}, and fixed system power P_0. Thus, the total power consumption can be computed as

$$P_{\text{total}} = P_{\text{C}} + P_{\text{SP}} + P_{\text{T}} + P_0. \tag{21}$$

In the following, we give a more detailed discussion of each power consumption part.

1) Circuit power: The circuit power of system is mainly caused by circuit dissipation. Herein, we introduce power consumption of hybrid architectures based on phase shifters for massive MIMO system, which can be given by [23, eq. (7)]

$$P_{\text{C}} = N_t (K+1) P_{\text{LAN}} + N_t K P_{\text{PS}} + K(P_{\text{RF}} + P_{\text{ADC}}) + P_{\text{BB}}, \tag{22}$$

where P_{LAN} denotes the power consumption by a single LNA, P_{PS} denotes the power consumption by a phase shifter, P_{BB} denotes the power consumption by a baseband processor, P_{ADC} denotes the power consumption by a single ADC, and P_{RF} is the power consumption by a RF chain block (a mixer, a local oscillator buffer, a low pass filter and a baseband amplifier), respectively.

2) Signal processing power: The total power consumption caused by signal processing can be expressed as [24, eq. (25)]

$$P_{\text{SP}} = KP_{\text{cd}} + \left(\frac{2K^2N_t + 2KN_t}{LT} + \frac{2K^3}{3LT}\right) + \frac{N_t K}{L}, \tag{23}$$

where the first term $P_{\text{cd}} = P_{\text{code}} + P_{\text{dec}}$, in which they account for the power required of the coding and decoding symbols, respectively. The second term comes from the computation of the ZF precoding matrix using the LU-based matrix inversion, U and L denote the number of coherence block per sec and the computational efficiency with $U = 1800$ and $L = 12.8$, respectively, and T represents the coherence time with $T = 32$ ms.[1] The third term is due to the multiplication of the precoding matrix with the vector of information symbols during data transmission.

3) Signal transmission power: The power consumption of signal transmission process is proportional to the average transmit power of the BS, which can be computed as $P_{\text{T}} = P/\eta$, where η denotes the efficiency of average transmit power at the BS.

4) System fixed power: The system structure leads to a stable power consumption.

In order to analyze the total optimal EE with respect to the physical parameters of system, we formulate the following optimization problem. By substituting (21) into (20) and performing some basic algebraic manipulations, the total EE of system can be calculated as

$$\eta_E = \frac{\log_2\left(1 + \frac{P}{K}\left(\frac{\pi(N_t-1)}{4} + 1\right)\right)}{\frac{P}{K\eta} + \frac{3N_tK}{LT} + \frac{2K^2}{3LT} + I_1 + N_t I_2 + \frac{N_t P_{\text{LAN}} + I_3}{LTK}}, \tag{24}$$

where $I_1 = P_{\text{cd}} + P_{\text{RF}} + P_{\text{ADC}}$, $I_2 = P_{\text{PS}} + P_{\text{LAN}} + 1/L + 2/LT$, and $I_3 = P_{\text{BB}} + P_0$. From (24), it is worth pointing out that the total EE is a function of SNR, the number of BS antennas and the number of users, and other constant power consumption parameters. We are now interested in figuring out the optimal SNR and the number of BS antennas to maximum the total

[1]These parameters are obtained from a practical energy consumption model, as was shown in [25].

EE of the massive MIMO system using the upper bound on the achievable SE of hybrid architectures in *Theorem* 1.

Proposition 1: For fixed number of BS antennas and users, the optimal SNR is given as (25) shown at the bottom of this page, where $W(\cdot)$ is the first real branch of the Lambert function [28].

Proof: The result is directly obtained by capitalizing on the results presented in [26, Lemma 1] and setting the other parameters being constant except the SNR. ■

Proposition 1 indicates that there exists an optimal SNR to maximize total EE and shows that the optimal SNR is the function of the number of BS antennas and users. This is because $W(x)$ is a monotonically increasing function with respect to x,

Proposition 2: For fixed number of users and the transmitter power, the optimal number of antennas N_t^{opt} is $\lfloor N \rfloor$ and $\lceil N \rceil$, where N is expressed as (26) shown at the bottom of this page.

Proof: The result is directly obtained by capitalizing on the results presented in [26, Lemma 1] and setting the other parameters being constant except N_t. ■

Proposition 2 provides explicit guidelines on how many antennas can be deployed at the BS in hybrid MIMO system to maximize total EE. In addition, Note that Proposition 1 typically gives a non-integer value on N_t^{opt}, but the quasiconcavity of the problem (26) implies that the optimal number of antennas N_t is attained at one of the two closest integers.

Remark 2: From *Proposition* 1 and *Proposition* 2, we know that there indeed exists an optimal SNR and number of BS antennas to maximize the total EE of system. However, it is challenge to directly find the relationship between the number of BS antennas and the SNR due to the Lambert function. Given a fixed number of BS antennas, there exists a globally optimal transmit power which yields the best EE performance. The following section will show some simulation results of the achievable EE with respect to three main design parameters: The number of of BS antennas (N_t), and the number of activated users (K) and the SNR (P).

V. NUMERICAL RESULTS

In this section, we provide simulation results to validate the derived analytical expressions. We consider a circular cell with a radius (from center to edge) of 500 meters and the users are located in a ring region at a guard zone of 20 meters with a random distribution, the available bandwidth is set to $B = 1$ Hz, the number of users is set to $K = 8$, the decay exponent is $\gamma = 2.1$, and the shadow-fading standard deviation is $\sigma_{\text{shad}} = 4.9$ dB. Note that β_k is fixed once the k-th user is dropped in the cell as in [1] and [27], which agrees with the setting of the analytical derivation that β_k is fixed, and the expectation is taken over the fast-fading coefficients. Hence, the analytical results are in accordance with the numerical ones. The coefficients of the users' large-fading β_k ($k = 1, \ldots, 8$) are randomly generated as follows: $\{0.19,\ 3.35,\ 5.28,\ 0.15,\ 11.84,\ 1.79,\ 3.03,\ 0.08\} \times 10^{-3}$. With the energy consumption models [23], [25] $P_{\text{LAN}} = 0.02$ Watt; $P_{\text{PS}} = 0.03$ Watt; $P_{\text{SW}} = 0.005$ Watt; $P_{\text{FR}} = 0.04$ Watt; $P_{\text{ADC}} = 0.2$ Watt; $P_{\text{BB}} = 0.2$ Watt; $P_{\text{cod}} = 4$ Watt; $P_{\text{dec}} = 0.5$ Watt; $P_0 = 2$ Watt for hybrid architectures, and the efficiency of average transmit power at the BS is set to $\eta = 0.5$. All simulation results were obtained by averaging over 100, 000 independent channel realizations.

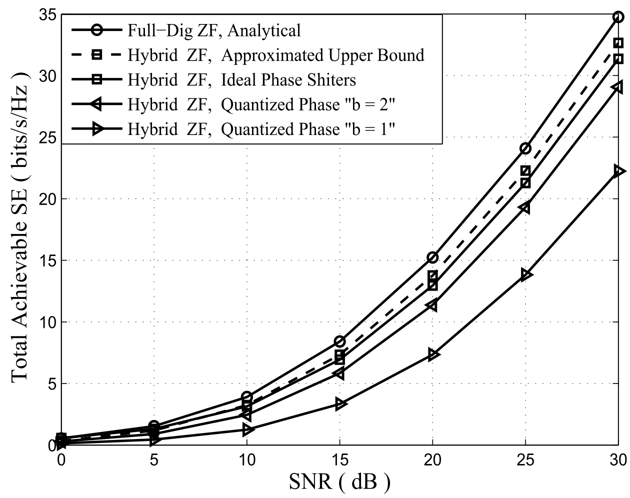

FIGURE 2. Simulated and approximated upper bound on the total achievable SE versus SNR for full digital architectures, ideal phase shifters and quantized phase shifters ($N_t = 128$ and $K = 8$).

In Fig. 2, the simulated total achievable SE along with the derived analytical bounds of *Theorem* 1 are plotted against the average SNR for ideal phase shifters. We can see that the analytical curve always remains very close to the simulated curve in the entire SNR regime and this validates the analytical results in *Theorem* 1. For comparison, the total approximated achievable SE for the full digital architectures is also provided [18, eq. (10)]. We observe that the total achievable SE of hybrid architecture based on ideal phase

$$P^{\text{opt}} = \frac{4\exp\left\{W\left(\eta\left(\frac{\pi(N_t-1)+4}{4e}\right)\left(\frac{9N_tK+2K^2}{3LT}+I_1+N_tI_2+\frac{N_tP_{LAN}+I_3}{LTK}\right)-\frac{1}{e}\right)+1\right\}}{\pi\left(N_t-1\right)+4}, \quad (25)$$

$$N = \frac{4K}{\pi P}\left(\exp\left\{W\left(\frac{\frac{P\pi}{4K}\left(\frac{P}{K\eta}+\frac{2K^3+3I_3}{3LTK}+I_1\right)-\left(1+\frac{P}{K}\left(\frac{\pi}{4}+1\right)\right)\left(\frac{3K^2+P_{LAN}}{LT}+I_2\right)}{\left(\frac{3K^2+P_{\text{LAN}}}{LT}+I_2\right)\left(\frac{P}{K\eta}+\frac{2K^3+3I_3}{3LTK}+I_1\right)}\right)\right\}-\left(1+\frac{P}{K}\left(\frac{\pi}{4}+1\right)\right)\right), \quad (26)$$

shifters achieves almost the near performance as that of the full digital architectures. However, the total achievable SE of the hybrid architectures based on quantized (1- and 2-bit resolution) phase shifters is inferior to that of the full digital architectures. As expected, the performance loss is caused by inaccurate angle phase information. Compared with the infinite resolution case, the achievable SE with 1-bit resolution phase shifters is up to 43% loss at $P = 20$ dB. However, this gap can be reduced by increasing the bits resolution of phase shifter. Clearly, we observe that at $P = 20$ dB regime, the total achievable SE with 2-bit resolution phase shifters has 55% improve compared to the 1-bit resolution phase shifters case, and the degradation is only 12% compared to the infinite resolution case.

In Fig. 3, we depict the total achievable EE against the average SNR for hybrid architectures based on ideal phase shifters and 1- and 2-bit resolution phase shifters. In order to compare to existing the massive MIMO system, we consider a full digital architecture, in which the power consumption is the same as the hybrid architectures except the circuit power part, which can be given by: $P_C = N_t P_{BS} + P_{SYS} + KP_{UE}$, where $P_{BS} = 1$ Watt; $P_{SYS} = 0.3$ Watt; $P_{UE} = 2$ Watt. These parameters are obtained from the results of [25]. We observe that the total achievable EE for hybrid architectures with ideal phase shifters is always superior to full digital architectures, for the reason that hybrid architectures is employed a small number of RF chains. Whereas the architectures with 2-bit resolution phase shifters is close to the full digital architectures and with 1-bit resolution phase shifters yields an inferior performance. This is because the degradation of the achievable SE with quantized phase shifters as compared to the infinite resolution case is significant, especially for low bit case. In addition, it can be seen that there exists an optimal SNR to maximize the total achievable EE, which is in line with *Proposition* 1.

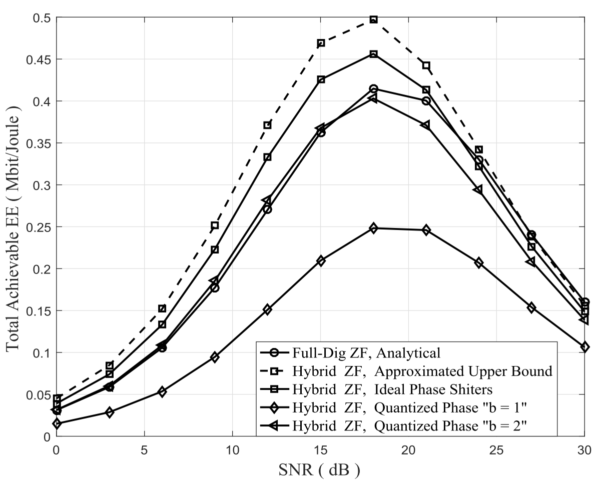

FIGURE 3. **The total achievable EE versus SNR for full digital architectures, ideal phase shifters and quantized phase shifters ($N_t = 128$ and $K = 8$).**

Fig. 4 illustrates the total achievable EE with respect to the number of BS antennas considering different massive MIMO

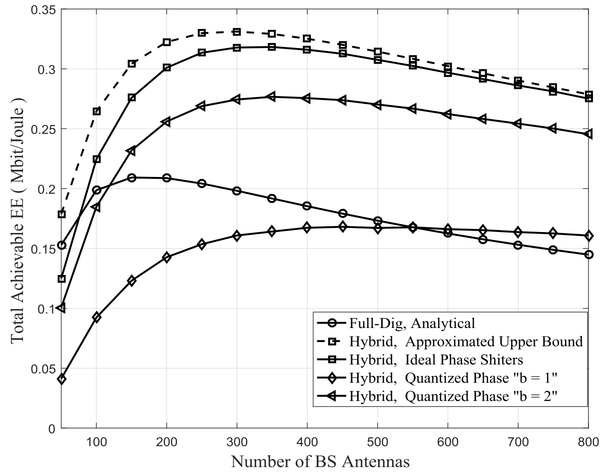

FIGURE 4. **The total achievable EE versus the number of BS antennas for full digital architectures, ideal phase shifters and quantized phase shifters (SNR = 20 dB and $K = 8$).**

systems, including digital and hybrid architectures based on ideal phase shifters and 1- and 2-bit resolution phase shifters. It can be seen that the total EE first increases the numbers of BS antennas and then decreases with increasing numbers of BS antennas. Hence, there exists an optimal antenna number of antennas to maximize the total EE, which efficiency validates the analytical results in *Proposition* 2. Furthermore, as increasing number of BS antennas, the achievable EE of hybrid architectures always outperforms that of the full digital architecture.

VI. CONCLUSION

In this paper, we first investigated the achievable SE of massive MIMO system for hybrid architectures based on phase shifters. We derived a novel upper bound on the achievable SE of the proposed hybrid architecture with ideal phase shifter and studied hybrid architecture with quantized phase shifters. Our results indicated that the achievable SE of hybrid architectures with finite resolution phase shifters has a degradation as compared to the full digital architecture. However, this gap can be reduced by increasing the number of quantization bits.

In addition, we explored the total EE of massive MIMO system by considering a realistic power consumption model. Based on the derived analytical expressions, closed-form expressions for the optimal SNR and the number of BS antennas that maximize the total EE can be derived. Compared to existing full-digital architectures, results showed that although the achievable SE of hybrid architectures are inferior to that of full-digital architectures, the total EE outperforms that of the full-digital architectures.

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