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On the Performance of RF-FSO System Over Rayleigh and Kappa-Mu/Inverse Gaussian Fading Environment

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ABSTRACT In this paper, we investigate a decode-and-forward cooperative communication based radio frequency (RF)-free space optical (FSO) system. An asymmetric environment is considered where source to relay channel is subjected to Rayleigh fading and relay to destination path is assumed to follow κ - μ /Inverse Gaussian (IG) distribution. The analytical framework proposed is based on a versatile κ - μ /IG model which includes one-sided Gaussian/IG, Rice (Nakagami- n)/IG, Nakagami- m /IG and Rayleigh/IG distributions as special cases. The expressions for moment generating function (MGF) and its special case are derived for this mixed RF and FSO network. The MGF is further utilized to derive the expression of end-to-end ergodic channel capacity for the RF-FSO communication system. Thereafter, end-to-end bit error rate is determined for BPSK, QPSK, DPSK and NCFSK modulation schemes. In addition, we have also derived analytical and asymptotic expressions of MGF and outage probability of the proposed RF-FSO system. Furthermore, we have also investigated the diversity gain and coding gain of the considered RF-FSO system.

INDEX TERMS RF-FSO system, κ - μ /Inverse Gaussian distribution, Rayleigh fading, Ergodic capacity, BER, outage probability.

I. INTRODUCTION

Due to high achievable optical bandwidth, greater security against eavesdropping, low power consumption, and communication over long distances (upto several kilometers), the free space optical communication (FSO) system surpasses the RF systems [1]. This promising technology has unique features like high directivity through narrow beams, license-free transmission with high bandwidth and security, ease-of-installation and cost-effectiveness which makes it more fascinating for the emerging demands of the wireless systems [2]. The RF-FSO system provides connectivity of the main backbone (RF) network with last-mile access (FSO) network [3]. The hybrid RF-FSO systems harness the advantages of both RF and FSO links. RF has the advantage of being low cost and non-line-of-sight (nLOS) communication while FSO being optical communication has the advantage of low latency coupled with high data rate. In the proposed system the combined advantages of both RF and FSO system results in cost effective last-mile connection, with increased reliability, connectivity and increased data rate to the end-user and rapid deployment at low cost [4]. The performance

of RF-FSO system [5] has been analysed for Gamma-Gamma (GG) distribution in terms of Meijer's G function and the better performance is observed in FSO relative to RF only mode. In [5]–[7] performance analysis of amplify-and-forward (AF) relaying protocol for a dual-hop system is presented while [3], [8] present the analysis of decode-and-forward (DF) based RF-FSO system. For the implementation of AF protocol, it is assumed that the channel coefficients are known to the destination for the optimal decoding. Also, in AF scheme, sampling, amplifying and retransmitting analog values is non-trivial [9]. It not only amplifies desired signal but also the noise [10]. But, in DF protocol, the relay first decodes and then re-encodes the signal before transmitting it to the destination [9] and hence results in the possibility to vary the communication rate in $S-R$ and $R-D$ link. Recently, it has been found that the performance of DF based relay system can be improved using various coding schemes [11].

Relaying provides capacity enhancement along with wider coverage area. The concept of dual-hop relaying of an asymmetric environment, composed of RF and FSO link, was first proposed in [12]. Asymmetric system provides different

fading models for $S - R$ and $R - D$ link, which are more practical. The received signals are transmitted via different communication systems in real-life environment [13]. References [5] and [13]–[15] considered asymmetric fading environment (also called mixed fading channels). Practically, the networks are asymmetric in nature, where different hops undergo through different channel conditions and the signals commute through physically different paths [15].

Traditionally, lognormal distribution has been used to model large scale fading. Lognormal (LN) distribution is commonly used to model weak turbulence [16] and the turbulence when the receiver's aperture is larger than the corresponding length of the fluctuations [17]. Later on, Gamma distribution emerged to characterize the shadowing effect more accurately. Chatzidiamentis *et al.* [17] demonstrated that Kolmogorov-Smirnov (KS) statistical tests prove that Inverse Gaussian (IG) in place of Gamma distribution is more tractable analytically and hence, is a better substitute of Lognormal (LN) distribution [18]–[20]. For various turbulence conditions in free space optical (FSO) communications, IG distribution efficiently approximates LN distribution. In context to optical systems, it is also employed for modeling the statistical behavior of avalanche photo diode receivers [21]. The cumulant function of IG distribution is inverse of the cumulant function of normal distribution [22].

The most widely known distributions for modeling small scale fading are Rayleigh, Nakagami- m , Nakagami- q (Hoyt), and Weibull for nLOS communications and Nakagami- n (Rice) model for line-of-sight (LOS) communication scenarios [18]. The fading distribution, α - μ [23], [24], also known as generalized Gamma (Stacy) distribution, includes Gamma, Nakagami- m , Exponential, Weibull, One-sided Gaussian, and Rayleigh distributions as special cases. Similar other models, namely, η - μ and α - μ , [25]–[28], are general fading distributions representing small scale variations which are again bundles of many distributions. The η - μ distribution describes the behavior of a signal propagating in a non-homogeneous environment, while the behavior of multipath waves propagating in a homogeneous environment is described by κ - μ fading distribution [25].

It is widely known that multipath and shadowing effects occur simultaneously and thus, their effect should be analyzed together. A new generic propagation model proposed in [29], named, Málaga includes the effect of coupled to LoS scattering component, which was its novelty over other existing models. The composite fading distribution, GG can be derived from Málaga distribution and has gained wide acceptance to model weak-to-strong turbulence [30] in FSO system. In GG distribution, large scale fluctuations are not modeled by lognormal distribution, rather by the Gamma distribution. But Gamma distribution is not a good approximation for LN distribution with large variance [31] and [32]. Moreover the limitation of Málaga distribution is that, the amount of fading parameter β is only defined for natural number [29].

Therefore, this motivated us to implement another generic composite distribution, κ - μ /Inverse Gaussian, which has

recently emerged as a versatile fading distribution to accurately model real wireless channels. It includes distributions like one-sided Gaussian/IG, Rice (Nakagami- n)/IG, Nakagami- m /IG and Rayleigh/IG distributions [18] as special cases. The irradiance PDF [18] of κ - μ /Inverse Gaussian composite distribution is used to model moderate-to-strong atmospheric turbulence in FSO systems. After scanning the open technical literature and to the best of our knowledge, it has been found that the performance of the κ - μ /Inverse Gaussian composite model over decode-and-forward (DF) based RF-FSO system has not been analyzed yet. Since, IG distribution provides better modeling of weak turbulence in comparison to Gamma distribution [19], therefore, we analyze the relay-based RF-FSO communication system considering both small-scale and large-scale fading in κ - μ /Inverse Gaussian distribution. The MGF based performance of RF-FSO system is analysed for mixed wireless communication system with Nakagami- m distribution considered at RF link and Gamma-Gamma distribution considered at FSO link for DF and AF relaying scheme in [3] and [6] respectively.

This paper presents a dual-hop network with Rayleigh distribution in the first hop and κ - μ /Inverse Gaussian distribution in the second hop. To achieve benefits from the user's co-operation as in [33], the DF relaying protocol is used to encode and re-transmit the signal towards the destination after demodulating and decoding the received signal from the source.

A. CONTRIBUTION

The list of present contributions are stated as:

- We analytically evaluate the performance of a RF-FSO system under Rayleigh- κ - μ /Inverse Gaussian fading channels with arbitrary parameters.
- In particular, we determine the expressions of moment generating function (MGF) and its derivative.
- Utilising these results, we evaluate performance metrics such as ergodic channel capacity, bit error rate (BER), and outage probability (OP).
- Also, asymptotic expressions of MGF, BER and OP are derived at high SNR regime to validate the mathematical analysis.
- The coding gain and diversity gain are also identified to employ better insight of the system.
- Capitalizing on the derived expression of MGF and relating the specific case of Bessel's function, a novel identity of MGF is deduced.

B. STRUCTURE

The remainder of this paper is organized as follows: Section II describes the system and channel models. The novel expressions of MGF, Channel Capacity, BER, and OP are derived in Section III. Thereafter, to gain better insight of the system performance, the asymptotic expressions and some specific cases are also evaluated. Numerical results are dis-

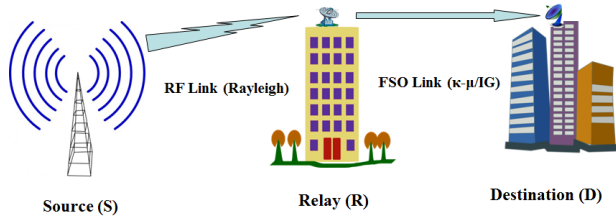


FIGURE 1. Asymmetric RF-FSO system model.

cussed in Section IV and finally the paper is concluded in Section V.

II. SYSTEM AND CHANNEL MODELS

A dual-hop asymmetric DF based wireless relay network is considered. As shown in Fig. 1, we assume that due to long distance, nLOS communication takes place between source and destination. Destination and relay are considered to be placed on the high-rise buildings providing LOS communication between the two ends. In this system, source (S) communicates with destination (D) through relay (R). A scenario of asymmetric wireless channel is generated, where the RF link between S – R is Rayleigh distributed and FSO link between R – D is κ-μ/Inverse Gaussian distributed. The relay receives RF signal from the source and converts it to optical signal with optical conversion ratio ‘η’. The instantaneous signal-to-noise ratio (SNR) of the Rayleigh distributed RF link between S – R is $\gamma_{S,R}$. The received RF signal $y_{S,R}$ at R can be expressed as

$$y_{S,R} = h_{S,R}x + n'_{S,R}, \tag{1}$$

where x is the transmitted signal from source over the RF channel with fading gain, $h_{S,R}$ and $n'_{S,R}$ is the AWGN noise of S – R link. The SNR of the S – R link is given by $\gamma_{S,R} = \Omega_{S,R}|h_{S,R}|^2/N_0 = |h_{S,R}|^2\bar{\gamma}$, where $\bar{\gamma}$ is the average SNR of S – R link, $\Omega_{S,R}$ is the average power of the signal received at relay, and N_0 is the variance of AWGN. Further, the PDF of Rayleigh faded S – R link is given by [35]

$$f_{\gamma_{S,R}}(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right). \tag{2}$$

The second hop between R and D constitutes of FSO link and undergoes through κ-μ/Inverse Gaussian composite distribution with instantaneous SNR, $\gamma_{R,D}$. We assume that the channel state information (CSI) is available at both the relay and destination terminals. The received signal $y_{R,D}$ at destination D is given by

$$y_{R,D} = \eta I \tilde{x} + n'_{R,D}, \tag{3}$$

where \tilde{x} denotes the signal estimated at relay R and $n'_{R,D}$ is the AWGN noise associated with R – D link. The intensity of irradiance is denoted by I . The instantaneous electrical SNR of R – D link is given by $\gamma_{R,D} = (\eta I)^2/N_0$. The average electrical SNR is introduced as $\bar{\gamma} = \frac{(\eta E\{I\})^2}{N_0}$ [36], where

$E[\cdot]$ denotes the expectation. To make the analysis simpler, the average SNR of R – D link, $\bar{\gamma}$, is considered to be same as that of S – R link. In DF based relaying protocol, the relay decodes the received signal and re-encodes before further transmission. γ_{DF} is the SNR of the relaying link S – R – D and is given by $\gamma_{DF} = \min(\gamma_{S,R}, \gamma_{R,D})$. It is the minimum SNR between the SNRs of two hops S – R and R – D. The PDF of equivalent SNR at the destination for the DF based dual-hop system is stated as [37]

$$f_Y(\gamma) = f_{\gamma_{S,R}}(\gamma) + f_{\gamma_{R,D}}(\gamma) - f_{\gamma_{S,R}}(\gamma)F_{\gamma_{R,D}}(\gamma) - f_{\gamma_{R,D}}(\gamma)F_{\gamma_{S,R}}(\gamma), \tag{4}$$

where $F_{\gamma_{S,R}}$ is CDF of S – R link. In (4), $f_Y(\gamma)$ is obtained after determining the minimum SNR of the two hops between S – R and R – D.

A. IRRADIANCE PDF IN FSO COMMUNICATION

We consider an irradiance PDF of the κ-μ/IG composite distribution to deduce strong atmospheric turbulence between R and D which is derived by superimposing multipath and shadowing distribution. Mathematically, the PDF, $f_{\gamma_{R,D}}$ of R – D link [18] is given as

$$f_{\gamma_{R,D}}(\gamma) = \sum_{i=0}^p \frac{\Gamma(p+i)p^{1-2i}\kappa^i\mu^{\mu+2i}(1+\kappa)^{\mu+i}\lambda^{\left(\frac{\mu+i}{2}+\frac{3}{4}\right)}}{i!2^{\left(\frac{\mu+i}{2}-\frac{1}{4}\right)}(p-i)!\Gamma(\mu+i)e^{\left(-\frac{\lambda}{\theta}+\mu\kappa\right)}\sqrt{\pi\theta}} \times \left(\frac{\gamma^{\mu+i-1}K_{\mu+i+\frac{1}{2}}\left[\sqrt{\frac{\mu(1+\kappa)\gamma\lambda}{2\theta\bar{\gamma}}+\frac{\lambda^2}{4\theta^2}}\right]}{\bar{\gamma}^{\mu+i}\left[\frac{\mu(1+\kappa)\theta\gamma}{\bar{\gamma}}+\frac{\lambda}{2}\right]^{\left(\frac{\mu+i}{2}+\frac{1}{4}\right)}} \right), \tag{5}$$

where λ is the shape parameter, θ relates to the mean value of the corresponding fluctuations, $\bar{\gamma}$ is average SNR, $K_\nu(\cdot)$ is the modified Bessel function of second kind of order ν [43], $\Gamma(\cdot)$ is Gamma function [43, eq. 8.310.1], and $p \rightarrow \infty$. The physical parameters κ and μ describe small scale variations in the LOS communication. μ specifies multi-path clustering and κ denotes the ratio between total power of the dominant component and the total power of the scattered waves. The CDF of R-D link from [34, eq. 6], can be expressed as

$$F_{\gamma_{R,D}}(\gamma) = -\sum_{i=0}^p A(i)\Gamma(\mu+i)\sum_{k=1}^{\mu+i} \frac{2^k\gamma^{\mu+i-k}}{(\beta b)^k(\mu+i-k)!} \times \left(\frac{K_{\mu+i-k+1/2}(b\sqrt{\alpha+\beta\gamma})}{(\sqrt{\alpha+\beta\gamma})^{(\mu+i-k+\frac{1}{2})}} \right), \tag{6}$$

where

$$A(i) = \frac{\Gamma(p+i)p^{1-2i}\kappa^i\mu^{\mu+2i}(1+\kappa)^{\mu+i}\lambda^{\left(\frac{\mu+i}{2}+\frac{3}{4}\right)}}{i!2^{\left(\frac{\mu+i}{2}-\frac{1}{4}\right)}(p-i)!\Gamma(\mu+i)e^{-\frac{\lambda}{\theta}+\mu\kappa}\sqrt{\pi\theta}(\bar{\gamma}^{\mu+i})},$$

$$\alpha = \frac{\lambda}{2}, \beta = \frac{\mu(1+\kappa)\theta}{\bar{\gamma}}, \text{ and } b = \frac{1}{\theta}\sqrt{\frac{\lambda}{2}}.$$

Theorem 1: The moment generating function (MGF) for dual-hop DF based RF-FSO scheme is given by

$$\begin{aligned}
 M_{\gamma}(s) &= \left(\frac{1}{1 + \bar{\gamma}s} \right) + \sum_{i=0}^p A(i) \sum_{k=1}^{\mu+i} B(i, k) \sum_{j=0}^{\mu+i-k} \binom{\mu+i-k}{j} (-\alpha)^j \\
 &\times \sum_{t=0}^{\infty} \left[\left\{ r^{(-\mu-i+k+2t-\frac{1}{2})} \left(\frac{s\bar{\gamma}+1}{\bar{\gamma}\beta} \right)^{(j-t-\frac{1}{2})} \frac{\Gamma(-j+t+\frac{1}{2}, \left(\frac{s\bar{\gamma}+1}{\bar{\gamma}\beta} \right) \alpha)}{\Gamma(t+1)\Gamma(-\mu-i+k+t+\frac{1}{2})} \right\} \right. \\
 &\left. - \left\{ r^{(\mu+i-k+2t+\frac{1}{2})} \frac{\Gamma(\mu+i-k-j+t+1, \left(\frac{s\bar{\gamma}+1}{\bar{\gamma}\beta} \right) \alpha)}{\Gamma(t+1)\Gamma(\mu+i-k+t+\frac{3}{2})} \left(\frac{s\bar{\gamma}+1}{\bar{\gamma}\beta} \right)^{-(\mu+i-k-j+t+1)} \right\} \right] \\
 &+ \sum_{i=0}^p A(i) D(i) \sum_{j=0}^{\mu+i-1} \binom{\mu+i-1}{j} (-\alpha)^j \\
 &\times \sum_{t=0}^{\infty} \left[\left\{ r^{(-\mu-i+2t-\frac{1}{2})} \frac{\Gamma(-j-t+\frac{5}{2}, \left(\frac{s\bar{\gamma}+1}{\bar{\gamma}\beta} \right) \alpha)}{\Gamma(t+1)\Gamma(-\mu-i+t+\frac{1}{2})} \left(\frac{s\bar{\gamma}+1}{\bar{\gamma}\beta} \right)^{(j+t-\frac{5}{2})} \right\} \right. \\
 &\left. - \left\{ r^{(\mu+i+2t+\frac{1}{2})} \frac{\Gamma(\mu+i-j+t, \left(\frac{s\bar{\gamma}+1}{\bar{\gamma}\beta} \right) \alpha)}{\Gamma(t+1)\Gamma(\mu+i+t+\frac{3}{2})} \left(\frac{s\bar{\gamma}+1}{\bar{\gamma}\beta} \right)^{-(\mu+j-i-t)} \right\} \right] \tag{7}
 \end{aligned}$$

$$\begin{aligned}
 M_{\gamma}(s) \xrightarrow{\bar{\gamma} \rightarrow \infty} &\left(\frac{1}{1 + \bar{\gamma}s} \right) + \sum_{i=0}^p A(i) \sum_{k=1}^{\mu+i} B(i, k) \sum_{j=0}^{\mu+i-k} \binom{\mu+i-k}{j} (-\alpha)^j \sum_{t=0}^{\infty} \frac{\exp\left(\frac{-s\alpha}{\beta}\right)}{\Gamma(t+1)} \\
 &\times \left[\left\{ \frac{r^{-(\mu+i-k-2t+\frac{1}{2})}}{\Gamma(-\mu-i+k+t+\frac{1}{2})} \sum_{m=0}^{M-1} \frac{(-1)^m \Gamma(j-t+m+\frac{1}{2})}{\left(\frac{s}{\beta}\right)^{m+1} \alpha^{(m+j-t+\frac{1}{2})} \Gamma(j-t+\frac{1}{2})} \right\} \right. \\
 &\left. - \left\{ \frac{r^{(\mu+i-k+2t+\frac{1}{2})}}{\Gamma(\mu+i-k+t+\frac{3}{2})} \sum_{m=0}^{M-1} \frac{(-1)^m \Gamma(-\mu-i+k+j-t+m)}{\left(\frac{s}{\beta}\right)^{m+1} \alpha^{(m-\mu-i+k+j-t)} \Gamma(-\mu-i+k+j-t)} \right\} \right] \\
 &+ \sum_{i=0}^p A(i) D(i) \sum_{j=0}^{\mu+i-1} \binom{\mu+i-1}{j} (-\alpha)^j \sum_{t=0}^{\infty} \frac{\exp\left(\frac{-s\alpha}{\beta}\right)}{\Gamma(t+1)} \\
 &\times \left[\left\{ \frac{r^{(-\mu-i+2t-\frac{1}{2})}}{\Gamma(-\mu-i+t+\frac{1}{2})} \sum_{m=0}^{M-1} \frac{(-1)^m \Gamma(j+t+m-\frac{3}{2})}{\left(\frac{s}{\beta}\right)^{m+1} \alpha^{(m+j+t-\frac{3}{2})} \Gamma(j+t-\frac{3}{2})} \right\} \right. \\
 &\left. - \left\{ \frac{r^{(\mu+i+2t+\frac{1}{2})}}{\Gamma(\mu+i+t+\frac{3}{2})} \sum_{m=0}^{M-1} \frac{(-1)^m \Gamma(1-\mu-i+j-t+m)}{\left(\frac{s}{\beta}\right)^{m+1} \alpha^{(m-\mu-i+j-t+1)} \Gamma(1-\mu-i+j-t)} \right\} \right] \tag{8}
 \end{aligned}$$

III. CHANNEL STATISTICS

A. MOMENT GENERATING FUNCTION (MGF)

The moment generating function of the proposed RF-FSO system is stated in Theorem 1, where $r = \frac{\beta}{2}$,

$$\begin{aligned}
 B(i, k) &= \frac{2^k \Gamma(\mu+i) \exp\left(\left(s + \frac{1}{\bar{\gamma}}\right) \frac{\alpha}{\beta}\right)}{(\beta b)^k \bar{\gamma}(\mu+i-k)!} \\
 &\times \frac{\pi \left(\frac{1}{\beta}\right)^{(\mu+i-k+1)}}{2 \sin\left(\left(\mu+i-k+\frac{1}{2}\right) \pi\right)},
 \end{aligned}$$

and

$$D(i) = \frac{1}{\beta^{\mu+i}} \frac{\pi}{2 \sin\left(\left(\mu+i+\frac{1}{2}\right) \pi\right)} \exp\left(\left(s + \frac{1}{\bar{\gamma}}\right) \frac{1}{\beta}\right).$$

Proof: The detailed proof is given in Appendix A.

B. HIGH SNR ANALYSIS OF MGF

To provide the further insight of attained expression of MGF, an asymptotic expression for high SNR regime is derived. By setting $\bar{\gamma} \rightarrow \infty$, neglecting higher order terms and using

the fact [43, eq. 8.357],

$$\Gamma(\alpha, x) \approx x^{\alpha-1} \exp(-x) \left[\sum_{m=0}^{M-1} \frac{(-1)^m \Gamma(1-\alpha+m)}{x^m \Gamma(1-\alpha)} \right]$$

the asymptotic expression of $M_\gamma(s)$, where $M = 1, 2, 3, \dots$, can be expressed as in (8), as shown at the top of the previous page.

C. SPECIFIC CASE OF MGF, $M_\gamma^*(s)$

In particular, novel identity for a specific case of modified Bessel function of the second kind, $K_\nu(x)$, is stated in Theorem 2.

D. HIGH SNR ANALYSIS OF $M_\gamma^*(s)$

At high SNR, the expression of $M_\gamma^*(s)$ in (9), as shown at the top of the next page can be approximated in (10), as shown at the top of the next page.

Proof: The detailed proof is provided in Appendix C.

IV. PERFORMANCE ANALYSIS

A. ERGODIC CHANNEL CAPACITY

Due to the time varying characteristic of wireless channel, we consider the average/ergodic capacity of RF-FSO system. We present ergodic channel capacity based on unified MGF [39] as

$$C_{avg} \approx \frac{B}{\log(2)} \sum_{n=1}^N v_n Ei(-s_n) \left[\frac{d}{ds} M_\gamma(s) \right]_{s \rightarrow s_n}, \quad (11)$$

where N is any positive integer, B is the channel bandwidth and $Ei(\cdot)$ is the exponential integral function [40, eq. 6.15.2]. The coefficients v_n and s_n are given [39] as

$$v_n = \frac{\pi^2 \sin(((2n-1)/2N)\pi)}{4N \cos^2((\pi/4) \cos(((2N-1)/2N)\pi) + (\pi/4))}$$

and

$$s_n = \tan\left(\frac{\pi}{4} \cos\left(\left(\frac{2n-1}{2N}\right)\pi\right) + \frac{\pi}{4}\right).$$

The derivative of MGF is stated in (12), shown on an upcoming page, as $M'_\gamma(s)$, where $\Theta = \mu + i - k$. After substituting (12) to (11), we obtain the ergodic channel capacity of the proposed mixed RF-FSO model.

B. BIT ERROR RATE ANALYSIS

The average bit error rate (BER) is one of the most important criteria that reveals the performance behavior of any system. From [38], the average BER, P_e , can be evaluated using MGF based method for BPSK/BFSK modulation scheme, expressed as

$$P_e = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} M_\gamma\left(\frac{g}{\sin^2\theta}\right) d\theta. \quad (13)$$

For BPSK, value of $g = 1$ and for coherent detection of BFSK, $g = 1/2$. As, no closed form solution of BER is available for this expression therefore, the approximated MGF based Symbol Error Rate (SER) for L -PSK is used for

numerical analysis [41]. The expression of approximate SER is given by

$$P_e \approx \left(\frac{\Omega}{2\pi} - \frac{1}{6}\right) M_\gamma\left(\sin^2\left(\frac{\pi}{L}\right)\right) + \frac{1}{4} M_\gamma\left(\frac{4}{3} \sin^2\left(\frac{\pi}{L}\right)\right) + \left(\frac{\Omega}{2\pi} - \frac{1}{4}\right) M_\gamma\left(\frac{4}{3} \left(\frac{\sin^2\left(\frac{\pi}{L}\right)}{\sin^2(\Omega)}\right)\right), \quad (14)$$

where $\Omega = \pi(L-1)/L$ and $M_\gamma(\cdot)$ denotes the MGF derived in (7), as shown at the top of the previous page. For BPSK, value of L is 2 and for QPSK modulation L is 4. Also, the average BER, P_e , for differential coherent detection of the phase-shift-keying (DPSK) and non-coherent detection of orthogonal frequency-shift-keying (NCFSK) [35]–[38] is given by

$$P_e = \frac{1}{2} M_\gamma(a). \quad (15)$$

For DPSK, $a = 1$ and for NCFSK, $a = 1/2$. Substituting the value of $M_\gamma(s)$ from (7) to (15), we obtain average BER for differential DPSK and NCFSK modulation scheme.

C. OUTAGE PROBABILITY

An important performance evaluation metric, OP, measures the probability of failing to achieve an output SNR threshold required for a desired service. For the dual-hop RF-FSO communication system, OP of end-to-end relay system [42] is given by

$$P_{out}(\gamma_{th}) = Pr(\min(\gamma_{S,R}, \gamma_{R,D}) < \gamma_{th}). \quad (16)$$

Further, (16) can be re-written as

$$P_{out}(\gamma_{th}) = 1 - Pr(\gamma_{S,R} > \gamma_{th}) Pr(\gamma_{R,D} > \gamma_{th}). \quad (17)$$

The expression of OP for the considered system can be stated as

$$P_{out}(\gamma_{th}) = 1 - \left[\exp\left(-\frac{\gamma_{th}}{\bar{\gamma}}\right) \sum_{i=0}^p \frac{2A(i)}{b^2} \left(\frac{1}{\beta}\right)^{(\mu+i)\mu+i-1} \sum_{j=0}^{(\mu+i-\frac{5}{2}-2j)} \left(\frac{2}{b}\right)^{(\mu+i-\frac{5}{2}-2j)} \times (-\alpha)^j C_j^{\mu+i-1} G_{1,3}^{3,0}\left(\frac{b^2}{4}(\alpha+\beta\gamma_{th}) \middle| \begin{matrix} -\frac{\mu}{2}, -\frac{i}{2}, j+\frac{9}{4} \\ \frac{\nu}{2}, \frac{\nu+2}{2}, -\frac{\mu}{2}-\frac{i}{2}, j+\frac{5}{4} \end{matrix} \right) \right], \quad (18)$$

where $G(\cdot)$ is Meijer’s G function [43, eq. 9.301].

Proof: The detailed proof is provided in Appendix D.

D. ASYMPTOTIC ANALYSIS OF OUTAGE PROBABILITY

The CDF of the equivalent SNR of the considered RF-FSO system is given by [3]

$$F_\gamma(\gamma) = F_{\gamma_{S,R}}(\gamma) + F_{\gamma_{R,D}}(\gamma) - F_{\gamma_{S,R}}(\gamma) F_{\gamma_{R,D}}(\gamma),$$

which further can be reduced to an approximated expression at $\bar{\gamma} \rightarrow \infty$ by neglecting the higher order terms of average SNR. The asymptotic outage probability expression is obtained as

$$P_{out}(\gamma_{th}) \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} F_{\gamma_{S,R}}(\gamma_{th}) + F_{\gamma_{R,D}}(\gamma_{th}). \quad (19)$$

Theorem 2: For $\nu = m - \frac{1}{2}$; $m \in \mathbb{Z}^+$ of modified Bessel function of the second kind, $K_\nu(x)$, the MGF $M_\gamma^*(s)$, can be expressed as follows:

$$\begin{aligned}
 M_\gamma^*(s) &= \frac{1}{1 + \bar{\gamma}s} + \sum_{i=0}^p \sum_{k=1}^{\mu+i} \left(\frac{A(i) 2^{k+1} \Gamma(\mu+i)}{(b)^k (\mu+i-k)!} \right) \left(\frac{1}{\beta^{\mu+i\bar{\gamma}}} \right) \sqrt{\frac{\pi}{2b}} \sum_{l=0}^{\mu+i-k} \sum_{m=0}^{\mu+i-k} \binom{\mu+i-k}{m} \left(\frac{(-\alpha)^m (\mu+i-k+l)!}{(2b)^l (\mu+i-k-l)!} \right) \\
 &\times \sum_{q=0}^{\mu+i-k-2m-l} \binom{\mu+i-k-2m-l}{q} (\alpha)^{\mu+i-k-2m-l-q} \Gamma(q+1) \left(2\beta \left(s + \frac{1}{\bar{\gamma}} \right) \right)^{-\left(\frac{q+1}{2}\right)} \beta^q \\
 &\times \exp\left(\frac{\left(2\alpha \left(s + \frac{1}{\bar{\gamma}} \right) - b\beta \right)^2}{8\beta \left(s + \frac{1}{\bar{\gamma}} \right)} \right) D_{-(q+1)} \left(\frac{2\alpha \left(s + \frac{1}{\bar{\gamma}} \right) - b\beta}{\sqrt{2 \left(\beta \left(s + \frac{1}{\bar{\gamma}} \right) \right)}} \right) \exp\left(\left(\frac{-\alpha^2}{\beta} + \frac{\alpha}{\beta} \right) \left(s + \frac{1}{\bar{\gamma}} \right) - b\alpha \right) \\
 &+ \sum_{i=0}^p \sum_{m=0}^{\mu+i-1} \binom{\mu+i-1}{m} (-\alpha)^m \sqrt{\frac{2\pi}{b}} \exp\left(-\left(s + \frac{1}{\bar{\gamma}} \right) \left(\frac{\alpha^2 - \alpha}{\beta} \right) - b\alpha \right) \exp\left(\frac{\left(2\alpha \left(s + \frac{1}{\bar{\gamma}} \right) + b\beta \right)^2}{8\beta \left(s + \frac{1}{\bar{\gamma}} \right)} \right) \\
 &\times \sum_{l=0}^{\mu+i} \sum_{q=0}^{\mu+i-2m-2-l} \binom{\mu+i-k-2m-l}{q} \frac{(\mu+i+l)!}{l! (\mu+i-l)!} \frac{1}{(2b)^l} \alpha^{(\mu+i-2m-2-l-q)} \\
 &\times \beta^{(q-\mu-i+1)} \Gamma(q+1) \left(2\beta \left(s + \frac{1}{\bar{\gamma}} \right) \right)^{-\left(\frac{q+1}{2}\right)} D_{-(q+1)} \left(\frac{2\alpha \left(s + \frac{1}{\bar{\gamma}} \right) + b\beta}{\sqrt{2 \left(\beta \left(s + \frac{1}{\bar{\gamma}} \right) \right)}} \right) \tag{9}
 \end{aligned}$$

where $D_n(z)$ is Parabolic Cylinder function [44].

Proof: The detailed proof is provided in Appendix B.

$$\begin{aligned}
 M_\gamma^*(s) \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} &\left(\frac{1}{1 + \bar{\gamma}s} \right) + \sum_{i=0}^p \left(\frac{\Gamma(p+i) p^{1-2i} \kappa^i \mu^{\mu+2i} (1+\kappa)^{\mu+i} \lambda^{\left(\frac{\mu+i}{2} + \frac{3}{4}\right)} \Gamma(\mu+i)}{i! 2^{\left(\frac{\mu+i}{2} - \frac{1}{4}\right)} (p-i)! e^{\left(-\frac{i}{\theta} + \mu\kappa\right)} \sqrt{\pi\theta}} \right) \left\{ \sqrt{\frac{\pi}{2b\sqrt{\alpha}}} \exp(-b\sqrt{\alpha}) \right. \\
 &\times \sum_{k=1}^{\mu+i} \left(\frac{2^k}{(\mu(1+\kappa)\theta b)^k (\mu+i-k)!} \sum_{w=0}^{\mu+i-k} \frac{(\mu+i-k+w)! \Gamma(\mu+i-k+1)}{w! (\mu+i-k-w)! (2b\sqrt{\alpha})^w} \frac{1}{s^{(\mu+i-k+1)} \bar{\gamma}^{(\mu+i-k+1)}} \right) \\
 &\left. + \left(\sqrt{\frac{\pi\theta}{\lambda}} \right) \exp\left(\frac{-\lambda}{2\theta} \right) \sum_{w=0}^{\mu+i} \left(\frac{(\mu+i+k)! \left(\frac{\theta}{\lambda}\right)}{k! (\mu+i-w)! s^{(\mu+i)} \bar{\gamma}^{(\mu+i)}} \right) \right\} \tag{10}
 \end{aligned}$$

The CDF of S-R link, $\left(1 - \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \right) \rightarrow 0$ when $\bar{\gamma} \rightarrow \infty$ and hence, asymptotic expression of outage probability reduces to $F_{\gamma_{R,D}}(\gamma_{th})$ given by

$$\begin{aligned}
 P_{out}(\gamma_{th}) \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} &\sum_{i=0}^p A(i) \Gamma(\mu+i) \sum_{k=1}^{\mu+i} \frac{2^k \gamma_{th}^{(\mu+i-k)}}{(\mu(1+\kappa)\theta b)^k} \\
 &\times \frac{1}{\bar{\gamma}^{(\mu+i-k)} (\mu+i-k)!} \frac{K_{\mu+i+\frac{1}{2}}(b\sqrt{\alpha})}{(\sqrt{\alpha})^{(\mu+i-k+\frac{1}{2})}}. \tag{20}
 \end{aligned}$$

The outage probability at high SNR regime is characterized by two parameters, viz., diversity gain G_d and coding gain

G_c and can be given as $P_{out}^\infty \simeq G_c (\bar{\gamma})^{-G_d}$. In this context and based on (20), it can be inferred that

$$\begin{aligned}
 G_c &= \left(\frac{\Gamma(p) \mu^\mu (1+\kappa)^{\mu-1} \lambda^{\left(\frac{\mu}{2} + \frac{3}{4}\right)}}{2^{\left(\frac{\mu}{2} - \frac{5}{4}\right)} (p-1)! e^{\left(-\frac{i}{\theta} + \mu\kappa\right)} \sqrt{\pi\theta}} \right) \\
 &\times \left(\frac{\gamma_{th}^{(\mu-1)} K_{\mu+\frac{1}{2}}(b\sqrt{\alpha})}{\mu! (\theta b) (\sqrt{\alpha})^{(\mu-\frac{1}{2})}} \right)
 \end{aligned}$$

and $G_d = (2\mu - 1)$. The diversity gain depends on fading severity, modeled by small scale variations in the LOS communication component, μ .

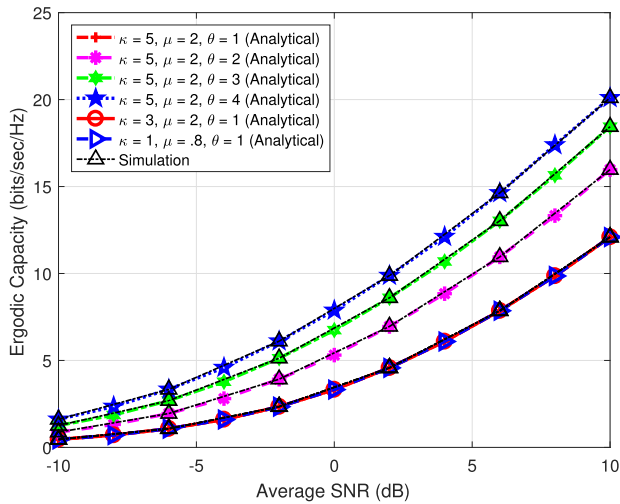


FIGURE 2. Capacity (bits/sec/Hz) vs average SNR.

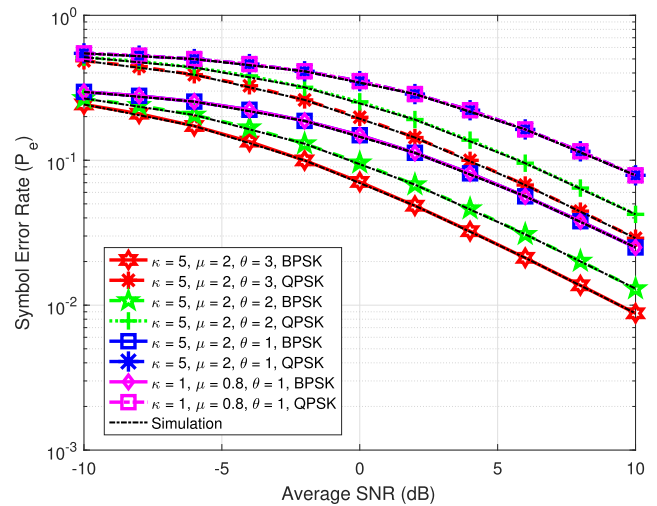


FIGURE 4. Symbol error rate vs average SNR.

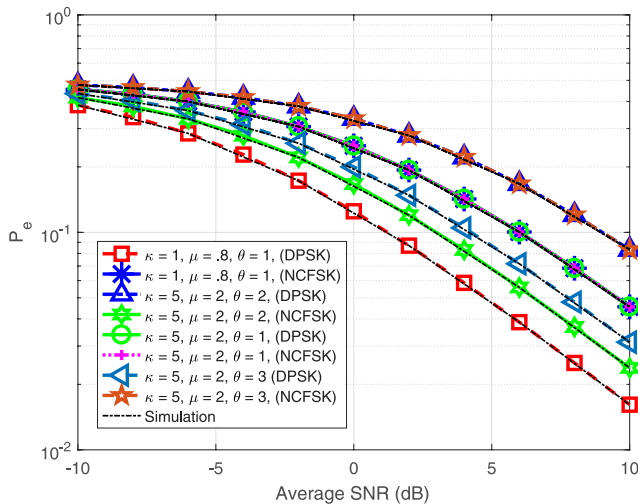


FIGURE 3. BER vs average SNR.

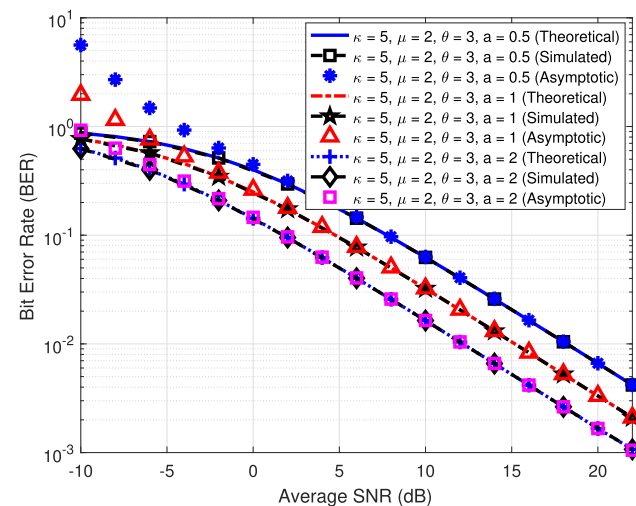


FIGURE 5. BER at high SNR vs average SNR.

in Fig. (3) reduces significantly. This behavior is noticed, as the variance is also dependent on the mean ‘ θ ’ and is given by θ^3/λ [22].

Fig. 4 shows the variation of Symbol Error Rate (P_e) with average SNR for BPSK and QPSK modulation schemes as represented in (14). It is clearly shown that for the fixed value of mean fluctuations, $\theta = 1$ and for the different sets of multipath clustering κ, μ namely, (5, 2) and (1, 0.8), the plots overlap with each other. It indicates that there is no significant variation in the value of P_e with the change in the values of multipath clustering and ratio of the total power of the dominant component and the total power of the scattered waves. For each BPSK and QPSK modulation scheme, the increase in θ reflects reduction in Symbol Error Rate. The behavior of the asymptotic BER is illustrated in Fig. 5. It can be clearly noticed that the asymptotic results starts showing excellent agreement with analytical results at high SNRs.

In addition to it, the outage probability presented in Fig. 6 analytically describes the effect of average SNR and λ to the expression derived in (18). As the value of κ

decreases for a fixed value of $\gamma_{th} = 5$ dB, OP increases. To plot OP, we substitute $\mu = .1$ and $\theta = 1$. We can also deduce that for a fixed value of κ , improved plots of OP are obtained for $\lambda = 3$ as compared to $\lambda = 2.5$. Since, κ is the ratio of total power of dominant components to the total power of scattered waves [18], therefore, Fig. 6 proves that as the power of dominant components, κ increases, outage probability also improves. Also, the effect of increase in value of μ for different values of λ has been illustrated in Fig. 7. We considered $\theta = 1$ and $\kappa = 2$ to plot the theoretical results of OP in Fig. 7. It can be observed that outage probability improves with the increase in value of μ (from 0.5 to 0.7) for two different sets of λ (3.0 and 3.1).

Further, in Fig. 8, it can be observed that asymptotic results of OP at high SNR in (20) are in perfect agreement with the actual results of outage probability for different threshold SNRs and hence validate the correctness of the proposed analysis. To plot Fig. 8, we consider $\kappa = 8, \mu = 2$, and $\theta = 0.85$.

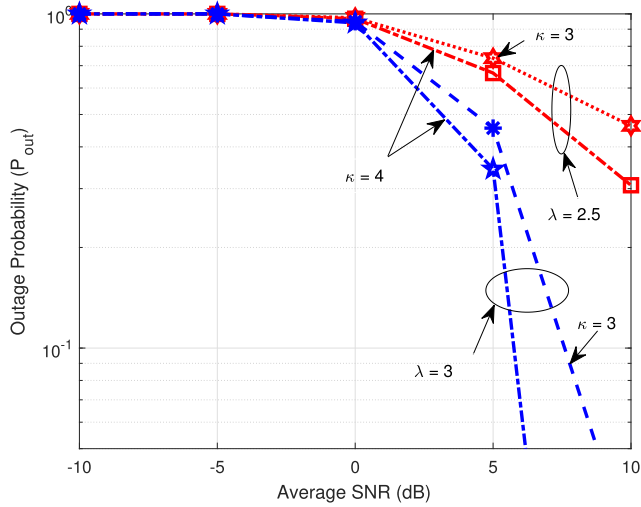


FIGURE 6. Outage probability vs average SNR.

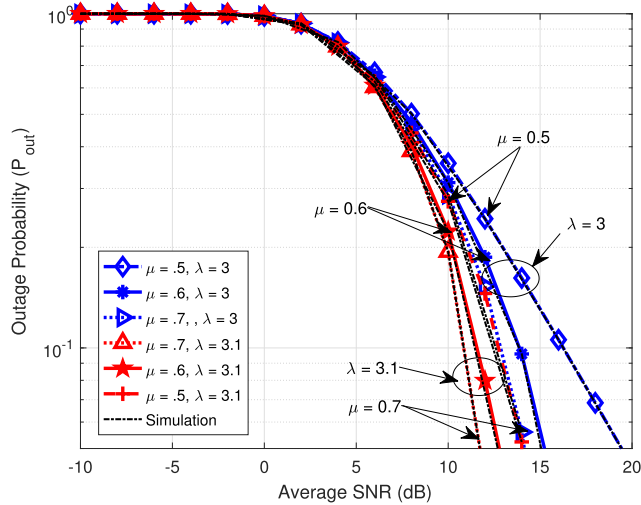


FIGURE 7. Outage probability vs average SNR.

VI. CONCLUSION

In the present work, the mathematical expressions of MGF with its asymptotic analysis is derived for a dual-hop; DF based RF-FSO system. Further, we derived an expression of a specific case of MGF with its approximation at high SNR. In addition, the MGF based analytical expressions for the ergodic channel capacity and BER are obtained. The numerical values demonstrated the impact of θ and average signal to noise ratio on the system performance of a dual-hop asymmetric channels. Moreover, both analytical and asymptotic expressions of OP for the system under consideration are derived. The main contribution of the paper is analysis of the behavior of FSO link in a dual-hop RF-FSO system configured by a composite versatile model of κ - μ /Inverse Gaussian distribution.

APPENDIX A
PROOF OF THEOREM 1

The MGF of end-to-end SNR, γ , is given by [38]

$$M_\gamma(s) = \int_0^\infty f_\gamma(\gamma) \exp(-s\gamma) d\gamma. \quad (21)$$

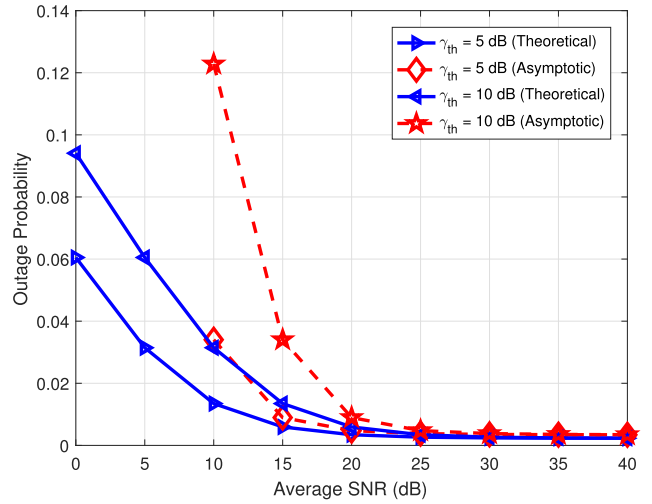


FIGURE 8. Outage probability at high SNR regime vs average SNR.

Substituting the value of $f_\gamma(\gamma)$ from (4), we obtain the expression of MGF as

$$M_\gamma(s) = \int_0^\infty \left(f_{\gamma_{S,R}}(\gamma) + f_{\gamma_{R,D}}(\gamma) - f_{\gamma_{S,R}}(\gamma)F_{\gamma_{R,D}}(\gamma) - f_{\gamma_{R,D}}(\gamma)F_{\gamma_{S,R}}(\gamma) \right) \exp(-s\gamma) d\gamma. \quad (22)$$

To make the analysis simpler, we can write

$$M_\gamma(s) = I_1 + I_2 - I_3 - I_4, \quad (23)$$

where

$$I_1 = \int_0^\infty f_{\gamma_{S,R}}(\gamma) \exp(-s\gamma) d\gamma, \quad (24)$$

$$I_2 = \int_0^\infty f_{\gamma_{R,D}}(\gamma) \exp(-s\gamma) d\gamma, \quad (25)$$

$$I_3 = \int_0^\infty f_{\gamma_{S,R}}(\gamma)F_{\gamma_{R,D}}(\gamma) \exp(-s\gamma) d\gamma, \quad (26)$$

and

$$I_4 = \int_0^\infty f_{\gamma_{R,D}}(\gamma)F_{\gamma_{S,R}}(\gamma) \exp(-s\gamma) d\gamma. \quad (27)$$

The value of MGF in I_1 is $\left(\frac{1}{1+s\gamma}\right)$. The CDF, $F_{\gamma_{S,R}}$, of Rayleigh distributed $S - R$ link is expressed as $1 - \exp\left(-\frac{\gamma}{\gamma}\right)$. After substituting $F_{\gamma_{S,R}}$ in I_4 , the expression of I_4 becomes

$$I_4 = \int_0^\infty f_{\gamma_{R,D}} \exp(-s\gamma) d\gamma - \int_0^\infty f_{\gamma_{R,D}} \exp\left(-\left(s + \frac{1}{\gamma}\right)\gamma\right) d\gamma, \quad (28)$$

Substituting the values of $I_1, I_2,$ and I_3 from (24), (25), and (26) respectively and I_4 from (28) to $M_\gamma(s)$ in (23), it is observed that the first term of I_4 gets canceled

with I_2 and hence, $M_\gamma(s)$ reduces to:

$$M_\gamma(s) = \frac{1}{1 + \bar{\gamma}s} - \int_0^\infty (f_{\gamma_{S,R}}(\gamma)F_{\gamma_{R,D}}(\gamma) \exp(-s\gamma) d\gamma - f_{\gamma_{R,D}} \exp\left(-\left(s + \frac{1}{\bar{\gamma}}\right)\gamma\right)) d\gamma. \quad (29)$$

Now, to solve $M_\gamma(s)$ in (29), change of variable method is used. The value of $f_{\gamma_{R,D}}$ and $F_{\gamma_{R,D}}$ from (5) and (6) respectively, are substituted in (29). Thereafter, the Bessel functions, $K_{\mu+i+\frac{1}{2}}\left[\sqrt{\frac{\mu(1+\kappa)\gamma}{2\theta\bar{\gamma}} + \frac{\lambda^2}{4\theta^2}}\right]$ and $K_{\mu+i-k+\frac{1}{2}}(b\sqrt{\alpha + \beta\gamma})$ in PDF and CDF of $R - D$ link respectively are expanded by using the relation [43, eq. 8.485] given by

$$K_\nu(z) = \frac{\pi}{2} \left(\frac{I_{-\nu}(z) - I_\nu(z)}{\sin(\nu\pi)} \right), \quad (30)$$

where

$$I_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{k=0}^\infty \frac{1}{\Gamma(k+1)\Gamma(\nu+k+1)} \left(\frac{x}{2}\right)^{2k}. \quad (31)$$

After applying Binomial expansion to the obtained expression, we solve the integral using the relation [43, eq. 3.381.3] given by

$$\int_u^\infty x^{\nu-1} \exp(-\mu x) dx = \mu^{-\nu} \Gamma(\nu, \mu u). \quad (32)$$

This completes the proof of MGF.

**APPENDIX B
PROOF OF SPECIAL CASE OF MGF**

To determine special case of MGF at $\nu = m - \frac{1}{2}$, we write (29) as

$$M_\gamma^*(s) = \frac{1}{1 + \bar{\gamma}s} - (M1 - M2), \quad (33)$$

where, we substitute

$$M1 = \int_0^\infty f_{\gamma_{S,R}}(\gamma) F_{\gamma_{R,D}}(\gamma) \exp(-s\gamma) d\gamma \quad (34)$$

and

$$M2 = \int_0^\infty f_{\gamma_{R,D}}(\gamma) \exp\left(-\left(s + \frac{1}{\bar{\gamma}}\right)\gamma\right) d\gamma. \quad (35)$$

Substituting $f_{\gamma_{S,R}}(\gamma)$ and $F_{\gamma_{R,D}}(\gamma)$ from (2) and (6) respectively in $M1$. Thereafter, $M1$ becomes

$$M1 = - \int_0^\infty \frac{1}{\bar{\gamma}} \exp\left(\frac{-\gamma}{\bar{\gamma}}\right) \sum_{i=0}^p A(i) \Gamma(\mu + i) \times \sum_{k=1}^{\mu+i} \frac{2^k \gamma^{\mu+i-k}}{(\beta b)^k (\mu + i - k)!} \times \left(\frac{K_{\mu+i-k+\frac{1}{2}}(b\sqrt{\alpha + \beta\gamma})}{(\sqrt{\alpha + \beta\gamma})^{(\mu+i-k+\frac{1}{2})}} \right) \exp(-s\gamma) d\gamma. \quad (36)$$

After substituting $\sqrt{\alpha + \beta\gamma} = \alpha + \beta t$, $M1$ can be written as

$$M1 = - \sum_{i=0}^p \sum_{k=1}^{\mu+i} \left(\frac{A(i) 2^{k+1} \Gamma(\mu + i)}{(b)^k (\mu + i - k)!} \right) \times \int_0^\infty \left(\frac{((\alpha + \beta t)^2 - \alpha)^{\mu+i-k}}{\beta^{\mu+i} \bar{\gamma}} \right) \times \left(\frac{K_{\mu+i-k+\frac{1}{2}}(b(\alpha + \beta t))}{(\alpha + \beta t)^{(\mu+i-k+\frac{1}{2})}} \right) \times \exp\left(-\left(\frac{((\alpha + \beta t)^2 - \alpha)}{\beta}\right) \left(\frac{s\bar{\gamma} + 1}{\bar{\gamma}}\right)\right) dt. \quad (37)$$

We expand $((\alpha + \beta t)^2 - \alpha)^{\mu+i-k}$ term by using binomial expansion given by

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k. \quad (38)$$

Importantly, the Bessel term of integral in (37) is expanded by using the relation [43, eq. 8.468]

$$K_{n+\frac{1}{2}}(z) = \sqrt{\frac{\pi}{2z}} \exp(-z) \sum_{k=0}^n \frac{(n+k)!}{k!(n-k)!(2z)^k}. \quad (39)$$

Thereafter, substituting (39) in (37) we obtain (40) as,

$$M1 = - \sum_{i=0}^p \sum_{k=1}^{\mu+i} \left(\frac{A(i) 2^{k+1} \Gamma(\mu + i)}{(b)^k (\mu + i - k)!} \right) \sqrt{\frac{\pi}{2b}} \left(\frac{1}{\beta^{\mu+i} \bar{\gamma}} \right) \times \sum_{l=0}^{\mu+i-k} \sum_{m=0}^{\mu+i-k} \left(\frac{(-\alpha)^m (\mu + i - k + l)!}{(2b)^l (\mu + i - k - l)! l!} \right) \times \binom{\mu + i - k}{m} \int_0^\infty (\alpha + \beta t)^{\mu+i-k-2m-l} \times \exp\left\{-\left(\frac{\alpha^2 + \beta^2 t^2 + 2\alpha\beta t - \alpha}{\beta}\right) \left(s + \frac{1}{\bar{\gamma}}\right) - b(\alpha + \beta t)\right\} dt. \quad (40)$$

We substitute integral term in (40) as I given by

$$I = \int_0^\infty (\alpha + \beta t)^{\mu+i-k-2m-l} \exp\left(-\left(\frac{\alpha^2 + \beta^2 t^2 + 2\alpha\beta t - \alpha}{\beta}\right) \times \left(s + \frac{1}{\bar{\gamma}}\right) - b(\alpha + \beta t)\right) dt. \quad (41)$$

$$\begin{aligned}
 M1 = & - \sum_{i=0}^p \sum_{k=1}^{\mu+i} \left(\frac{A(i) 2^{k+1} \Gamma(\mu+i)}{(b)^k (\mu+i-k)!} \right) \left(\frac{1}{\beta^{\mu+i\bar{\gamma}}} \right) \sqrt{\frac{\pi}{2b}} \sum_{l=0}^{\mu+i-k} \sum_{m=0}^{\mu+i-k} \binom{\mu+i-k}{m} \left(\frac{(-\alpha)^m (\mu+i-k+l)!}{(2b)^l (\mu+i-k-l)! l!} \right) \\
 & \times \sum_{q=0}^{\mu+i-k-2m-l} \binom{\mu+i-k-2m-l}{q} (\alpha)^{\mu+i-k-2m-l-q} \Gamma(q+1) \left(2\beta \left(s + \frac{1}{\bar{\gamma}} \right) \right)^{-\left(\frac{q+1}{2}\right)} \beta^q \\
 & \times D_{-(q+1)} \left(\frac{2\alpha \left(s + \frac{1}{\bar{\gamma}} \right) - b\beta}{\sqrt{2 \left(\beta \left(s + \frac{1}{\bar{\gamma}} \right) \right)}} \right) \exp \left(\left(\frac{-\alpha^2}{\beta} + \frac{\alpha}{\beta} \right) \left(s + \frac{1}{\bar{\gamma}} \right) - b\alpha \right) \exp \left(\frac{\left(2\alpha \left(s + \frac{1}{\bar{\gamma}} \right) - b\beta \right)^2}{8\beta \left(s + \frac{1}{\bar{\gamma}} \right)} \right) \quad (44)
 \end{aligned}$$

$$\begin{aligned}
 M2 = & \sum_{i=0}^p A(i) \sum_{m=0}^{\mu+i-1} \binom{\mu+i-1}{m} (-\alpha)^m \sqrt{\frac{2\pi}{b}} \exp \left(- \left(s + \frac{1}{\bar{\gamma}} \right) \left(\frac{\alpha^2 - \alpha}{\beta} \right) - b\alpha \right) \\
 & \times \sum_{l=0}^{\mu+i} \sum_{q=0}^{\mu+i-2m-2-l} \binom{\mu+i-2m-l-2}{q} \frac{(\mu+i+l)!}{l! (\mu+i-l)!} \frac{1}{(2b)^l} \alpha^{\mu+i-2m-2-l-q} \beta^{(q-\mu-i+1)} \\
 & \times \Gamma(q+1) \left(2\beta \left(s + \frac{1}{\bar{\gamma}} \right) \right)^{-\left(\frac{q+1}{2}\right)} D_{-(q+1)} \left(\frac{2\alpha \left(s + \frac{1}{\bar{\gamma}} \right) + b\beta}{\sqrt{2 \left(\beta \left(s + \frac{1}{\bar{\gamma}} \right) \right)}} \right) \exp \left(\frac{\left(2\alpha \left(s + \frac{1}{\bar{\gamma}} \right) + b\beta \right)^2}{8\beta \left(s + \frac{1}{\bar{\gamma}} \right)} \right) \quad (45)
 \end{aligned}$$

Applying binomial expansion again to $(\alpha + \beta t)^{\mu+i-k-2m-l}$ in (41) we get

$$\begin{aligned}
 I = & \sum_{q=0}^{\mu+i-k-2m-l} \binom{\mu+i-k-2m-l}{q} (\alpha)^{\mu+i-k-2m-l-q} \beta^q \\
 & \times \int_0^\infty t^q \exp \left(-t^2 \beta \left(s + \frac{1}{\bar{\gamma}} \right) - t \left(2\alpha \left(s + \frac{1}{\bar{\gamma}} \right) - b\beta \right) \right. \\
 & \left. + \left(\frac{-\alpha^2 + \alpha}{\beta} \right) \left(s + \frac{1}{\bar{\gamma}} \right) - b\alpha \right) dt. \quad (42)
 \end{aligned}$$

To solve I , we apply [44, eq. 2.3.15.3] given by

$$\int_0^\infty x^{\alpha-1} e^{-\rho x^2 - qx} dx = \Gamma(\alpha) \exp \left(\frac{q^2}{8\rho} \right) D_{-\alpha} \left(q/\sqrt{2\rho} \right), \quad (43)$$

where $D_n(z)$ is Parabolic Cylinder function. After solving (42), the deduced expression of I is then substituted to (40) to obtain $M1$ in (44), as shown at the top of this page. Similarly, $M2$ is obtained in (45), as shown at the top of this page. The $M_\gamma^*(s)$ of DF relay based RF-FSO system is obtained by substituting $M1$ and $M2$ from (44) and (45) respectively to (33), which completes the proof.

APPENDIX C
PROOF OF ASYMPTOTIC EXPRESSION OF $M_\gamma^*(s)$

We utilize the fact [43, eq. 8.468]

$$\begin{aligned}
 K_{\mu+i-k+\frac{1}{2}} \left(b\sqrt{\alpha + \beta\gamma} \right) \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} \sqrt{\frac{\pi}{2b\sqrt{\alpha}}} \exp(-b\sqrt{\alpha}) \\
 \times \sum_{w=0}^{\mu+i-k} \frac{(\mu+i-k+w)!}{w! (\mu+i-k-w)! (2b\sqrt{\alpha})^w}, \quad (46)
 \end{aligned}$$

to substitute the Bessel terms of MGF obtained by solving (29). Similarly, we substitute

$$\begin{aligned}
 K_{\mu+i+\frac{1}{2}} \left(\sqrt{\frac{\mu(1+\kappa)\gamma\lambda}{2\theta\bar{\gamma}} + \frac{\lambda^2}{4\theta^2}} \right) \\
 \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} \sqrt{\frac{\pi\theta}{\lambda}} \exp \left(-\frac{\lambda}{2\theta} \right) \sum_{w=0}^{\mu+i} \frac{(\mu+i+w)!}{w! (\mu+i-w)! \left(\frac{\lambda}{\theta} \right)^w}. \quad (47)
 \end{aligned}$$

Further, using the relation [43, eq. 3.381.4] given by

$$\int_0^\infty x^{\nu-1} \exp(-\mu x) dx = \frac{1}{\mu^\nu} \Gamma(\nu), \quad (48)$$

we evaluate the obtained expression, which yields the asymptotic expression of $M_\gamma^*(s)$, in (10).

APPENDIX D
PROOF OF OUTAGE PROBABILITY

The probability of $\gamma_{S,R} > \gamma_{th}$ at relay, R is obtained by the CDF of the rayleigh distribution at the threshold value γ_{th} .

$$Pr(\gamma_{S,R} > \gamma_{th}) = \exp \left(\frac{-\gamma_{th}}{\bar{\gamma}} \right). \quad (49)$$

whereas, $Pr(\gamma_{R,D} > \gamma_{th})$ is given by

$$\begin{aligned}
 Pr(\gamma_{R,D} > \gamma_{th}) \\
 = \int_{\gamma_{th}}^\infty f_{\gamma_{R,D}}(\gamma) d\gamma \\
 = \int_{\gamma_{th}}^\infty \sum_{i=0}^p \frac{A(i) \gamma^{\mu+i-1} K_{\mu+i+\frac{1}{2}}(b\sqrt{\alpha + \beta\gamma})}{(\sqrt{\alpha + \beta\gamma})^{\mu+i+\frac{1}{2}}} d\gamma, \quad (50)
 \end{aligned}$$

After substituting $\frac{b^2}{4}(\alpha + \beta\gamma) = x$ in (52), the expression reduces to

$$Pr(\gamma_{R,D} > \gamma_{th}) = \int_{\frac{b^2}{4}(\alpha + \beta\gamma_{th})}^{\infty} \sum_{i=0}^p \frac{\left(\frac{4x}{b^2\beta} - \frac{\alpha}{\beta}\right)^{\mu+i-1} 4A(i) K_\nu(2\sqrt{x})}{b^2\beta \left(\frac{2\sqrt{x}}{b}\right)^{\mu+i+\frac{1}{2}}} dx, \quad (51)$$

where $\nu = \mu + i + \frac{1}{2}$. Thereafter, applying binomial expansion

$$\left(\frac{4x}{b^2\beta} - \frac{\alpha}{\beta}\right)^{\mu+i-1} = \left(\frac{1}{\beta}\right)^{\mu+i-1} \sum_{j=0}^{\mu+i-1} C_j^{\mu+i-1} \left(\frac{4x}{b^2}\right)^{\mu+i-1-j} (-\alpha)^j \quad (52)$$

and substituting [45, eq. 8.4.23.1]

$$K_\nu(2\sqrt{x}) = \frac{1}{2} G_{0,2}^{2,0} \left(x \mid \frac{\nu}{2}, -\frac{\nu}{2}\right). \quad (53)$$

to (51), we obtain the CDF of κ - μ /IG distributed FSO path as,

$$Pr(\gamma_{R,D} > \gamma_{th}) = \int_{\frac{b^2}{4}(\alpha + \beta\gamma_{th})}^{\infty} \sum_{i=0}^p \frac{2A(i)}{b^2} \left(\frac{1}{\beta}\right)^{\mu+i} \sum_{j=0}^{\mu+i-1} C_j^{\mu+i-1} \times (-\alpha)^j \left(\frac{2}{b}\right)^{\mu+i-\frac{5}{2}-2j} x^{\left(\frac{\mu}{2} + \frac{i}{2} - j - \frac{5}{4}\right)} G_{0,2}^{2,0} \left(x \mid \frac{\nu}{2}, -\frac{\nu}{2}\right) dx. \quad (54)$$

Thereafter, substituting $\frac{x}{\frac{b^2}{4}(\alpha + \beta\gamma_{th})} = t$, (54) reduces to (55)

$$Pr(\gamma_{R,D} > \gamma_{th}) = \int_1^{\infty} \sum_{i=0}^p \frac{2A(i)}{b^2} \left(\frac{1}{\beta}\right)^{\mu+i} \sum_{j=0}^{\mu+i-1} C_j^{\mu+i-1} \left(\frac{2}{b}\right)^{\mu+i-\frac{5}{2}-2j} \times (-\alpha)^j \left(\frac{b^2}{4}(\alpha + \beta\gamma_{th})\right)^{\left(\frac{\mu}{2} + \frac{i}{2} - j - \frac{1}{4}\right)} t^{\left(\frac{\mu}{2} + \frac{i}{2} - j - \frac{1}{4}\right)} \times G_{0,2}^{2,0} \left(\frac{b^2}{4}(\alpha + \beta\gamma_{th}) t \mid \frac{\nu}{2}, -\frac{\nu}{2}\right) dt, \quad (55)$$

Using relation [45, eq. 8.4.2.2] in (55), we get

$$Pr(\gamma_{R,D} > \gamma_{th}) = \sum_{i=0}^p \frac{2A(i)}{b^2} \left(\frac{1}{\beta}\right)^{\mu+i} \sum_{j=0}^{\mu+i-1} C_j^{\mu+i-1} \left(\frac{2}{b}\right)^{\mu+i-\frac{5}{2}-2j} (-\alpha)^j \times \int_0^{\infty} \left\{ \left(\frac{b^2}{4}(\alpha + \beta\gamma_{th})\right)^{\left(\frac{\mu}{2} + \frac{i}{2} - j - \frac{1}{4}\right)} G_{1,1}^{0,1} \left(t \mid \frac{\mu}{2} + \frac{i}{2} - j - \frac{1}{4}\right) \times G_{0,2}^{2,0} \left(\frac{b^2}{4}(\alpha + \beta\gamma_{th}) t \mid \frac{\nu}{2}, -\frac{\nu}{2}\right) \right\} dt. \quad (56)$$

Now, solving (56) using the relation [45, eq. 2.24.1]

$$Pr(\gamma_{R,D} > \gamma_{th}) = \sum_{i=0}^p \frac{2A(i)}{b^2} \left(\frac{1}{\beta}\right)^{\mu+i} \sum_{j=0}^{\mu+i-1} C_j^{\mu+i-1} \left(\frac{2}{b}\right)^{\mu+i-\frac{5}{2}-2j} \times (-\alpha)^j G_{1,3}^{3,0} \left(\frac{b^2}{4}(\alpha + \beta\gamma_{th}) \mid \frac{\nu}{2}, \frac{\nu+2}{2}, -\frac{\mu}{2} - \frac{i}{2} + j + \frac{9}{4}\right). \quad (57)$$

Now, substituting $Pr(\gamma_{S,R} > \gamma_{th})$ and $Pr(\gamma_{R,D} > \gamma_{th})$ from (49) and (57) respectively in (17), we obtain the expression of OP.

This completes the proof of outage probability.

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