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An Improved Gaussian Filter for Dynamic Positioning Ships With Colored Noises and Random Measurements Loss

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ABSTRACT An improved Gaussian Filter (GF) is designed for nonlinear Dynamic Positioning (DP) ships with cross-correlated colored noises and random measurements loss. For the actual nonlinear Dynamic Position System (DPS), the state noises and measurement noises do not satisfy the assumption of Gaussian white noises and the loss of measurements may occur randomly. Therefore, the following circumstances are considered: the state noises and measurement noises are cross-correlated colored noises at the same and adjacent sampling moments; the measurement loss occurs randomly for the data transmission between the sensor units and the estimator units. In order to get the estimator for nonlinear DP ships with cross-correlated colored noises and random measurements loss, a GF framework based on Bayesian theory is proposed, and then the Cubature Mix Kalman Filter based on spherical-radial method is obtained. In the end, the simulation results show that the proposed algorithm has better estimation performance than Unscented Kalman Filter with Measurements Loss and standard Cubature Kalman Filter.

INDEX TERMS Colored noise, filtering algorithms, nonlinear systems, noise measurement, loss measurement.

NOMENCLATURE

The abbreviations used in this article are as follows:

DP	Dynamic Positioning
DPS	Dynamic Position System
DGPS	Differential Global Positioning System
TWS	Taut Wire System
IMU	Inertial Measurement Unit
NED	North-East-Down
3-DOF	3-Degrees of Freedom
GF	Gaussian Filter
CMKF	Cubature Mix Kalman Filter
CKF	Cubature Kalman Filter
KF	Kalman Filter
EKF	Extended Kalman Filter
UT	Unscented Transform
UKF	Unscented Kalman Filter
UKF-PL	Unscented Kalman Filter with Measurements Loss

I. INTRODUCTION

Recently, Dynamic Positioning (DP) ships have been widely used in offshore oil and gas exploration, submarine pipe-laying, ships platform operation and other fields [1], [2]. Dynamic Positioning System (DPS) are typically equipped with redundant position reference system, such as Differential Global Positioning System(DGPS), Inertial Measurement Unit (IMU), Hydro-acoustic reference system, Taut Wire System (TWS), Gyrocompass, Microwave and Laser position reference system [3]. The accurate system state estimation affects the control performance of DP ships directly. However, due to the impact of the environmental interference, the state feedback measurement signals to controller are imprecise. Therefore, an estimator should be adopted to obtain accurate state information.

For DP ships, the system noises generally do not meet the assumption of Gaussian white noises. And because of the uncertain interference and other factors, the received measured data may also have random measurements loss. Thus, the following circumstances are considered: 1) The state noises and measurement noises are colored noises and there are auto-correlation at the adjacent sampling moments. 2) The measurement noises and the state noises are cross-correlated at the same and adjacent sampling moments. 3) The measurements loss occur randomly during the communication from sensor units to estimator units. Therefore, it has great practical application values for studying of nonlinear estimator for DP ships with cross-correlated colored noises and random measurements loss.

The Bayesian theory has been widely used for general nonlinear systems[4]. The Bayesian estimator is obtained based on posteriori probability density function and the current measurements, which provides a basic state prediction and state update framework. A variety of different filters have been developed based on Bayesian theory. Such as, the linear optimal Kalman Filter (KF) for linear system is deduced directly[5], the Extended Kalman Filter (EKF) for nonlinear system is obtained by first-order Taylor expansion[6]. EKF has a similar filtering structure with KF in essentially, but EKF needs to calculate the Jacobi matrix of nonlinear system. So the error of EKF is greater than KF, and even diverges for strong nonlinear system. In order to improve the estimation accuracy of nonlinear system, the Unscented Kalman Filter (UKF) is proposed which approximate posterior probability density function by Unscented Transform (UT)[7]. The UKF does not need to calculate system Jacobi matrix which reduces the linearization error. And the UKF has third-order Taylor accuracy in theory. But, the UT on which UKF relies has numerical instability and is prone to dimensional curse for high-dimensional nonlinear system. Therefore, the Cubature Kalman Filter (CKF) was proposed which approximates the posterior probability density function by spherical-radial method [8]. The CKF has better numerical stability and higher estimation accuracy.

However, the proposed filtering algorithms above are valid only for ideal model. Namely, the state noises and the measurement noises should satisfy the assumption of Gaussian white noises, and all measurements should arrive at estimator timely and accurately. In practice, these assumptions are generally difficult to satisfy. Therefore, some improved filters have been proposed for the non-standard Gaussian models. Such as, several Kalman filters and fusion algorithms were designed for non-standard models with random parameter matrix [9], correlated noises [10], [11], measurements delay[12], [13] or measurements loss[14], [15]. But these improved algorithms are all based on standard KF frame, and also required the system to be linear in generally. Therefore, it does not be applied to nonlinear DP ships directly. On the other hand, several improved nonlinear state estimators have been studied for nonlinear system with measurements delay [16], measurements loss [17]–[19] or correlated noises[20]–[23]. But there are still some shortcomings for these algorithms, e.g. it is only assumed that there is a synchronous correlation between the state noises and measurement noises, and without considering the cross-correlate at the adjacent sampling moments. In addition, no consideration is given to the fact that the state noise and the measurement noise are colored noise. Therefore, the noises studied in this paper which mainly refers to that the state noises and measurement noises are auto-correlated at the adjacent sampling moments. And there are cross-correlations at the same and adjacent sampling moments.

In this paper, a recursive filter is proposed for nonlinear system with cross-correlation colored noises and random measurements loss. First of all, the following cases are considered: The state noises and measurement noises are colored noises and also have cross-correlation at the same and adjacent sampling moments; the measurements loss may occur randomly during the communication between sensor units and estimator units. Secondly, a recursive Gaussian Filter (GF) is designed based on Bayesian theory, and the Cubature Mix Kalman Filter (CMKF) is proposed based on spherical-radial method.

The main contributions are: 1) the nonlinear kinematic model and measurement model are built for ships with cross-correlated colored noises and random measurements loss. 2) A recursive GF framework based on Bayesian theory is proposed for nonlinear system with cross-correlated colored noises and random measurements loss. 3) A CMKF algorithm is obtained by spherical-radial method and the simulation results show the effectiveness of the proposed algorithm.

The remainder of this paper is structured as following: The second part is model and problem formation. The third part is the proposed estimator. The simulations results are shown in fourth section and the conclusions are given in the end.

II. MODEL AND PROBLEM FORMATION

The ships state is generally described by kinematic model [1]. So, the 3-Degrees of Freedom (3-DOF) nonlinear kinematic model and measurement model for ships is established.

A. THE 3-DOF KINEMATIC MODEL

In order to describe the motion of DP ships, the North-East-Down (NED) coordinate system $n = (x_n, y_n, z_n)$ with origin is defined [24], where,the x_n axis points towards the North, the y_n axis points towards the East, and the z_n axis points downwards the centre of the Earth. Newton's law still holds for local area motion in NED coordinate system as the $\{n\}$ coordinate system is inertial coordinate system [1]. At the same time, the body fixed coordinate system $\{b\} = (x_b, y_b, z_b)$ is defined; the coordinate origin o_b is located at the centre of mass or centroid of ships and move with the ships. The coordinate is shown as Fig. 1.

The surge, sway and yaw for ships under low sea conditions is considered, and 3-DOF kinematic model is built as:

$$\dot{\eta} = J(\psi)\upsilon. \tag{1}$$

Where, the vector $\eta = [x_N, y_E, \psi]^T$ represents the north, east and heading, the $[\bullet]^T$ represents the transpose of $[\bullet]$. The symbol $\dot{\eta}$ is the first derivative of vector η. The vector $\upsilon = [u, v, r]^T$ represents the velocity of surge, sway and yaw. The transition matrix $J(\psi)$ is expressed as :

$$J(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

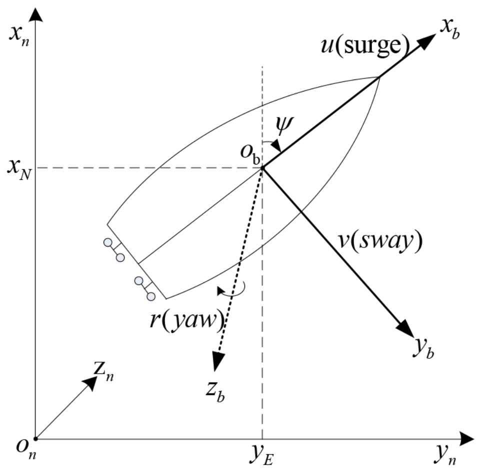

FIGURE 1. Coordinate system description.

The surface ships have the characteristics of low speed and weak manoeuvrability [3]. So a continuous noises acceleration model as (2) is established:

$$\ddot{\eta} = \xi. \tag{2}$$

Where, ξ expresses zero-mean Gaussian noises. The symbol $\ddot{\eta}(t)$ is the second derivative of η. So the kinematic model for ships is rewrite as:

$$\begin{bmatrix} \dot{\eta} \\ \ddot{\eta} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta \\ \dot{\eta} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \xi. \tag{3}$$

Next, by Euler method, the discrete form is obtained as (4):

$$x_k = f_{k-1}(x_{k-1}) + \omega_{k-1}. \tag{4}$$

$x_k = [x_{N,k}, y_{E,k}, \psi_k, u_k, v_k, r_k]^T$ represents position and velocity, ω_k denotes state noises. Due to the recursive relationship between states, there is a correlation between the discrete state noise at time k and the noise at time $k-1$, so the following colored noise model is built to describe the discretization state noise ω_k:

$$\omega_k = \Gamma_{k-1}\omega_{k-1} + \zeta_{k-1}. \tag{5}$$

Γ_{k-1} is state matrix of state colored noise. The symbol ζ_{k-1} is zero-mean Gaussian noise. The symbol T is sampling time. Then the discrete state space function is shown as:

$$f_{k-1}(x_{k-1}) = \begin{bmatrix} x_{N,k-1} & 0 & 0 & Tu_{k-1}\cos\psi_{k-1} & -Tv_{k-1}\sin\psi_{k-1} & 0 \\ 0 & y_{E,k-1} & 0 & Tu_{k-1}\sin\psi_{k-1} & Tv_{k-1}\cos\psi_{k-1} & 0 \\ 0 & 0 & \psi_{k-1} & 0 & 0 & Tr_{k-1} \\ 0 & 0 & 0 & u_{k-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{k-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{k-1} \end{bmatrix}$$

B. MEASUREMENT MODEL

Let $x_{1,k} = [x_{N,k}, y_{E,k}, u_k, v_k]^T$ represents the actual north, east, surge, and sway, the $z_{1,\mathrm{k}}$ represents the corresponding measurements by DGPS. Let $x_{2,k} = [\psi_k, r_k]^T$ represents the actual heading and yaw, the $z_{2,\mathrm{k}}$ represents the corresponding measurements by Gyrocompass. Thus, the discrete measurement equations are established as equations (6) and (7), $e_{1,k}$ and $e_{2,k}$ represent measurements noises:

$$z_{1,\mathrm{k}} = h_{1,k}(x_{1,k}) + e_{1,k}. \tag{6}$$

$$z_{2,\mathrm{k}} = h_{2,k}(x_{2,k}) + e_{2,k}. \tag{7}$$

For conveniently, the discrete model is expressed as:

$$z'_k = h_k(x_k) + e_k. \tag{8}$$

where, $z'_k = [z_{1,\mathrm{k}}, z_{2,\mathrm{k}}]^T$, $e_k = [e_{1,k}, e_{2,k}]^T$, $h_k(x_k) = [h_{1,k}(x_{1,k}), h_{2,k}(x_{2,k})]^T$. The measurements are obtained by states, so the measurement noise e_k can be modelled as the following colored noises:

$$e_k = \Phi_{k-1}e_{k-1} + \varepsilon_{k-1}. \tag{9}$$

the symbol Φ_{k-1} is state matrix of measurement colored noise, and ε_{k-1} denotes zero-mean Gaussian noises.

The state noises ω_k and the measurement noises e_k are cross-correlated at the same sampling moment as the ships and measurement units working in the same environment [11], i.e. $E[\omega_k e_{\mathrm{k}}^T] \neq 0$. At the same time, considering of (4) and (8) [11], there is correlation between the measured noise e_k and state noise ω_{k-1}, i.e. $E[\omega_{k-1} e_{\mathrm{k}}^T] \neq 0$.

On the other hand, due to the impact of unstable interference, there exists random measurements loss when the measurements are transmitted from sensor units to estimator units. And the Bernoulli distribution is adopted to describe the phenomenon of random measurements loss [15]. It is assumed that the sensor measurements loss parameter is γ_k, and z_k is received measurements, then the measurement model is rewritten as (10):

$$z_{\mathrm{k}} = \gamma_k h_k(x_k) + e_k. \tag{10}$$

If $\gamma_k \equiv I$, it means all measurements arrive at estimator, namely there is no measurements loss in transmit process. And the measurements equation (10) is simplified to (8). If $\gamma_k \equiv 0$, it means that all measurements loss occurs, namely the information received by estimator contains only noises term, and the measurement model is rewrite as:

$$z_k = e_k. \tag{11}$$

C. PROBLEM FORMATION

The main objective is to design the corresponding recursive nonlinear filter for nonlinear system (4) and (10) with cross-correlated colored noises and random measurements loss, then the performance of proposed algorithm is verified through several simulation experiments. For the convenience of problem description and algorithm derivation, the following Assumptions are given:

Assumption 1: the state noises and measurement noises are zero-mean cross-correlation colored noises at the same and adjacent sampling moments. I.e. the following statistical characteristics are satisfied:

$$\begin{aligned} E[\omega_k \omega_m^T] &= Q_{k,m}\delta_{k-m} + Q_{k,m-1}\delta_{k-m+1} \\ E[e_k e_m^T] &= R_{k,m}\delta_{k-m} + R_{k,m-1}\delta_{k-m+1} \\ E[\omega_k e_m^T] &= S_{k,m}\delta_{k-m} + S_{k,m-1}\delta_{k-m+1} \\ E[\omega_k] &= 0, \quad E[e_k] = 0. \end{aligned} \tag{12}$$

where, the symbol E represents expectation, the symbol δ_{k-m} represents Dirac function, i.e.$\delta_{k-m} = 1$, when $m = k$; $\delta_{k-m} = 0$ when $m \neq k$. The state noises covariance and measurement noises covariance are denoted as $Q_{k,k}$ and $R_{k,k}$, respectively. The auto-correlation covariance of state noises and auto-correlation covariance of measurement noises are $Q_{k,k-1}$ and $R_{k,k-1}$. The cross-covariance of state noises and measurement noises at the same and adjacent sampling moments are $S_{k,k}$ and $S_{k,k-1}$, respectively.

Assumption 2: The Bernoulli distribution γ_k as following is used to describe the loss of measurements:

$$\gamma_k = \begin{cases} 1, & \bar{\gamma}_k \\ 0, & 1 - \bar{\gamma}_k. \end{cases}$$

The $\bar{\gamma}_k$ represents probability of parameter $\gamma_k = 1$. The $1 - \bar{\gamma}_k$ represents the probability of parameter $\gamma_k = 0$. The measurement signal arrives at estimator timely when $\gamma_k = 1$; while $\gamma_k = 0$ means measurements loss occurs. The parameter γ_k is independent with states and noises, and the following statistical characteristics are obtained:

$$\begin{aligned} E[\gamma_k] = \bar{\gamma}_k, \quad & E[\gamma_k \gamma_k^T] = \bar{\gamma}_k(1 - \bar{\gamma}_k), \\ E[\gamma_k x_m^T] = 0, \quad & E[\gamma_k \omega_m^T] = 0, \; E[\gamma_k e_m^T] = 0. \end{aligned} \tag{13}$$

By Assumption 2, the expectation of measurements is:

$$\begin{aligned} E[z_k] &= \bar{\gamma}_k z_{k,(\gamma_k=1)} + (1 - \bar{\gamma}_k) z_{k,(\gamma_k=0)} \\ &= \bar{\gamma}_k(h_k(x_k) + e_k) + (1 - \bar{\gamma}_k) e_k \\ &= \bar{\gamma}_k h_k(x_k) + e_k. \end{aligned} \tag{14}$$

The initial state x_0 is independent with other signals, and satisfies Gaussian distribution with initial values $\bar{x}_{0|0}$ and initial error covariance $P_{0|0} = E[(x_0 - \bar{x}_{0|0})(x_0 - \bar{x}_{0|0})^T]$. The joint distributions of system noises and measurements are $p(\omega_k, z_k)$ and $p(e_k, z_k)$ which obey Gaussian distribution, and conditional probability densities $p(\omega_k|z_k)$ and $p(e_k|z_k)$ obey Gaussian distribution.

The correlation terms caused by noises are $E[h_{k-1}(x_{k-1}) e_{k-1}^T|Z_{1:k-1}]$ and $E[f_{k-1}(x_{k-1})\omega_{k-1}^T|Z_{1:k-1}]$. The symbol $Z_{1:k-1}$ represents measurements from z_1 to z_2, namely $Z_{1:k-1} = \{z_1, z_2 \ldots z_{k-1}\}$. As the noises interference is generally smaller than state, the correlation noises terms abovementioned are calculated by local linearization:

$$f_{k-1}(x_{k-1}) = F_{k-1}x_{k-1}, h_{k-1}(x_{k-1}) = H_{k-1}x_{k-1}. \tag{15}$$

where, F_{k-1} and H_{k-1} represent Jacobi matrices of nonlinear system which are calculated as:

$$\begin{aligned} f_{k-1}(x_{k-1}) &= F_{k-1}x_{k-1} \\ h_{k-1}(x_{k-1}) &= H_{k-1}x_{k-1}. \end{aligned} \tag{16}$$

The state prediction value $\hat{x}_{k|k-1}$ is calculated as:

$$\hat{x}_{k|k-1} = E[x_k|Z_{1:k-1}] = \int_{R^{n_x}} x_k p(x_k|Z_{1:k-1}) dx_k. \tag{17}$$

Define the state prediction error $\tilde{x}_{k|k-1} = x_k - \hat{x}_{k|k-1}$, the innovation $\tilde{z}_{k|k-1} = z_k - \hat{z}_{k|k-1}$, the measurement residual $\tilde{z}_{k-1|k-1} = z_{k-1} - \hat{z}_{k-1|k-1}$. And the state prediction error covariance $P_{k|k-1}^{xx}$ is calculated as:

$$P_{k|k-1}^{xx} = E[\tilde{x}_{k|k-1}\tilde{x}_{k|k-1}^T|Z_{1:k-1}]. \tag{18}$$

so, the measurement predictive value $\hat{z}_{k|k-1}$, predictive error cross-covariance $P_{k|k-1}^{xz}$ and measurement predictive error covariance $P_{k|k-1}^{zz}$ are calculated as following, respectively:

$$\hat{z}_{k|k-1} = E[z_k|x_k, Z_{1:k-1}]. \tag{19}$$

$$P_{k|k-1}^{xz} = E[\tilde{x}_{k|k-1}\tilde{z}_{k|k-1}^T|Z_{1:k-1}]. \tag{20}$$

$$P_{k|k-1}^{zz} = E[\tilde{z}_{k|k-1}\tilde{z}_{k|k-1}^T|x_k, Z_{1:k-1}]. \tag{21}$$

The cross-covariance of state noises and measurements is defined as $P_{k-1|k-1}^{\omega z}$, and the measurement estimation error cross-covariance is defined as $P_{k-1|k-1}^{zz}$, which are calculated as following:

$$P_{k-1|k-1}^{\omega z} = E[\omega_{k-1} z_{k-1}^T|Z_{1:k-1}]. \tag{22}$$

$$P_{k-1|k-1}^{zz} = E[\tilde{z}_{k-1|k-1}\tilde{z}_{k-1|k-1}^T|Z_{1:k-1}]. \tag{23}$$

Then the recursive GF will be given in the next section.

III. DESIGN OF THE GAUSSIAN FILTER

The nonlinear system (4) and (10) with cross-correlated colored noises and random measurements loss are considered here. Then a recursive GF framework is designed and a CMKF algorithm is obtained by spherical-radial method.

A. RECURSIVE GAUSSIAN FILTERING FRAMEWORK

The designed GF is shown as Theorem 1, which consists of state prediction and state update.

Theorem 1: The GF framework is designed as following 1) and 2) for the nonlinear system described by (4) and (10).

1) State prediction:

$$\begin{aligned} \hat{x}_{k|k-1} = & \int f_{k-1}(x_{k-1})p(x_{k-1}|Z_{1:k-1})dx_{k-1} \\ & + (S_{k-1} + \bar{\gamma}_{k-1}Q_{k-1,k-2}H_{k-1})(P_{k-1|k-1}^{zz})^{-1} \\ & (z_{k-1} - \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})). \end{aligned} \tag{24}$$

$$\begin{aligned} P_{k|k-1}^{xx} = & \int f_{k-1}(x_{k-1})f_{k-1}^T(x_{k-1})p(x_{k-1}|Z_{1:k-1})dx_{k-1} \\ & - \hat{x}_{k|k-1}\hat{x}_{k|k-1}^T + Q_{k-1} + F_{k-1}Q_{k-2,k-1} \\ & + Q_{k-2,k-1}^T F_{k-1}^T - (S_{k-1} + Q_{k-1,k-2}^T H_{k-1}^T \bar{\gamma}_{k-1}^T) \\ & (P_{k-1|k-1}^{zz})^{-1}(S_{k-1} + Q_{k-1,k-2}^T H_{k-1}^T \bar{\gamma}_{k-1}^T)^T. \end{aligned} \tag{25}$$

where,

$$P^{zz}_{k-1|k-1} = \int \gamma_{k-1}h_{k-1}(x_{k-1})h^T_{k-1}(x_{k-1})\gamma^T_{k-1} p(x_{k-1}|Z_{1:k-1})dx_{k-1} + R_{k-1} - \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})h^T_{k-1}(\hat{x}_{k-1|k-1})\bar{\gamma}^T_{k-1} + \bar{\gamma}_{k-1}H_{k-1}S_{k-2,k-1} + S^T_{k-2,k-1}H^T_{k-1}\bar{\gamma}^T_{k-1}. \tag{26}$$

2) State update:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - \hat{z}_{k|k-1}). \tag{27}$$

$$P^{xx}_{k|k} = P^{xx}_{k|k-1} - P^{xz}_{k|k-1}(P^{zz}_{k|k-1})^{-1}(P^{xz}_{k|k-1})^T. \tag{28}$$

$$K_k = P^{xz}_{k|k-1}(P^{zz}_{k|k-1})^{-1}. \tag{29}$$

where,

$$\hat{z}_{k|k-1} = \bar{\gamma}_k \int_{R^{n_x}} h_k(x_k)p(x_k|Z_{1:k-1})dx_k + R_{k,k-1} (P^{zz}_{k-1|k-1})^{-1}(z_{k-1} - \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})). \tag{30}$$

$$P^{xz}_{k|k-1} = \int x_k h^T_k(x_k)\gamma^T_k p(x_k|Z_{1:k-1})dx_k - \hat{x}_{k|k-1}\hat{z}^T_{k|k-1} + S_{k,k-1}. \tag{31}$$

$$P^{zz}_{k|k-1} = \int_{R^{n_x}} \gamma_k h_k(x_k)h^T_k(x_k)\gamma^T_k p(x_k|Z_{1:k-1})dx_k - \hat{z}_{k|k-1}\hat{z}^T_{k|k-1} + R_k + \bar{\gamma}_k H_k S_{k,k-1} + S^T_{k,k-1}H^T_k\bar{\gamma}^T_k - R_{k,k-1}(P^{zz}_{k-1|k-1})^{-1}R^T_{k,k-1}. \tag{32}$$

Proof: the state noises and measurement noises are cross-correlated colored noises and the following formulas are established by Assumption 1:

$$E[\omega_{k-2}e^T_{k-1}] = S_{k-2,k-1}, \quad E[\omega_{k-1}e^T_{k-1}] = S_{k-1}$$
$$E[\omega_{k-2}\omega^T_{k-1}] = Q_{k-2,k-1}, \quad E[\omega_{k-1}\omega^T_{k-1}] = Q_{k-1}$$
$$E[e_{k-2}e^T_{k-1}] = R_{k-2,k-1}, \quad E[e_{k-1}e^T_{k-1}] = R_{k-1}. \tag{33}$$

Then, (34) is obtained by (22):

$$P^{\omega z}_{k-1|k-1} = E[\omega_{k-1}(\gamma_{k-1}h_{k-1}(x_{k-1}) + e_{k-1})^T|Z_{1:k-1}] = S_{k-1} + Q^T_{k-1,k-2}H^T_{k-1}\bar{\gamma}^T_{k-1}. \tag{34}$$

And, the (12), (15), and (16) are rewrite as:

$$E[h_{k-1}(x_{k-1})e^T_{k-1}|Z_{1:k-1}] = H_{k-1}S_{k-1,k-2}. \tag{35}$$

$$E[f_{k-1}(x_{k-1})\omega^T_{k-1}|Z_{1:k-1}] = F_{k-1}Q_{k-2,k-1}. \tag{36}$$

$$E[h_k(x_k)\omega^T_{k-1}|Z_{1:k-1}] = H_kF_{k-1}Q_{k-2,k-1} + H_kQ_{k-1}. \tag{37}$$

The measurement cross-covariance $P^{zz}_{k-1|k-1}$ is obtained as (38) by (10), and (23).

$$\begin{aligned} P^{zz}_{k-1|k-1} &= E[(z_{k-1} - \hat{z}_{k-1|k-1})(z_{k-1} - \hat{z}_{k-1|k-1})^T|Z_{1:k-1}] \\ &= E[(\gamma_{k-1}h_{k-1}(x_{k-1})h^T_{k-1}(x_{k-1})\gamma^T_{k-1} + \gamma_{k-1}h_{k-1}(x_{k-1})e^T_{k-1} - (\gamma_{k-1}h_{k-1}(x_{k-1}) \\ &\quad + e_{k-1})h^T_{k-1}(\hat{x}_{k-1|k-1})\bar{\gamma}^T_{k-1} + e_{k-1}h^T_{k-1}(x_{k-1})\gamma^T_{k-1} \\ &\quad + e_{k-1}e^T_{k-1} - \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})(\gamma_{k-1}h_{k-1}(x_{k-1}) \\ &\quad + e_{k-1})^T + \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1}) h^T_{k-1}(\hat{x}_{k-1|k-1})\bar{\gamma}^T_{k-1}|Z_{1:k-1}. \end{aligned} \tag{38}$$

where,

$$\begin{aligned} &E[(\gamma_{k-1}h_{k-1}(x_{k-1}) + e_{k-1}) \times h^T_{k-1}(\hat{x}_{k-1|k-1})\bar{\gamma}^T_{k-1}|Z_{1:k-1}] \\ &= E[\bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})h^T_{k-1}(\hat{x}_{k-1|k-1})\bar{\gamma}^T_{k-1}]. \end{aligned} \tag{39}$$

Considering that $E[\gamma_{k-1}h_{k-1}(x_{k-1})e^T_{k-1}|Z_{1:k-1}]=E[e_{k-1} h^T_{k-1}(x_{k-1})\gamma^T_{k-1}|Z_{1:k-1}]$ and substituting (35), and (39) into (38), then (26) is proved.

As the joint distribution of state noises and measurements obeys Gaussian distribution, namely:

$$p(\omega_{k-1}, z_{k-1}) = \mathrm{N}\left[\begin{pmatrix}\omega_{k-1}\\ z_{k-1}\end{pmatrix}; \begin{pmatrix}0\\ \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})\end{pmatrix}, \begin{pmatrix}Q_{k-1} & P^{\omega z}_{k-1|k-1}\\ {P^{\omega z}_{k-1|k-1}}^T & P^{zz}_{k-1|k-1}\end{pmatrix}\right]. \tag{40}$$

As the state noises ω_{k-1} and measurements z_{k-2} are uncorrelated, so $p(\omega_{k-1}, Z_{1:k-1}) = p(\omega_{k-1}, z_{k-1})$, and the conditional probability density satisfies following Gaussian distribution by joint distribution [25]:

$$\begin{aligned} &p(\omega_{k-1}|Z_{1:k-1}) \\ &= \mathrm{N}(P^{\omega z}_{k-1|k-1}(P^{zz}_{k-1|k-1})^{-1}(z_{k-1} - \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})), \\ &\quad Q_{k-1} - P^{\omega z}_{k-1|k-1}(P^{zz}_{k-1|k-1})^{-1}(P^{\omega z}_{k-1|k-1})^T). \end{aligned} \tag{41}$$

similarly, the joint distribution of measurement noises e_k and measurements z_{k-1} also satisfies Gaussian distribution, i.e.

$$p(e_k, z_{k-1}) = \mathrm{N}\left[\begin{pmatrix}e_k\\ z_{k-1}\end{pmatrix}; \begin{pmatrix}0\\ \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})\end{pmatrix}, \begin{pmatrix}R_k & R_{k,k-1}\\ R^T_{k,k-1} & P^{zz}_{k-1|k-1}\end{pmatrix}\right]. \tag{42}$$

Considering that measurement noises e_k and measurements z_{k-2} are uncorrelated. So, the probability densities $p(e_k, Z_{1:k-1}) = p(e_k, z_{k-1})$, and the conditional probability density of e_k satisfies following Gaussian distribution:

$$\begin{aligned} &p(e_k|Z_{1:k-1}) \\ &= \mathrm{N}(R_{k,k-1}(P^{zz}_{k-1|k-1})^{-1}(z_{k-1} - \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})), \\ &\quad R_k - R_{k,k-1}(P^{zz}_{k-1|k-1})^{-1}R^T_{k,k-1}). \end{aligned} \tag{43}$$

And state prediction $\hat{x}_{k|k-1}$ can be expressed as (44) :

$$\hat{x}_{k|k-1} = E[x_k|Z_{1:k-1}] = E[f_{k-1}(x_{k-1})|Z_{1:k-1}] + E[\omega_{k-1}|Z_{1:k-1}]. \tag{44}$$

Then, substituting (34), and (41) into (44), then (24) is proved.

And the state prediction error covariance (18) can be expressed as (45):

$$\begin{aligned} P^{xx}_{k|k-1} &= E[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})^T|Z_{1:k-1}] \\ &= E\left[\begin{array}{l}(f_{k-1}(x_{k-1}) + \omega_{k-1} - \hat{x}_{k|k-1})(f_{k-1}(x_{k-1})+ \\ \omega_{k-1} - \hat{x}_{k|k-1})^T|Z_{1:k-1}\end{array}\right] \\ &= \mathrm{E}\left[\begin{array}{l} f_{k-1}(x_{k-1})f^T_{k-1}(x_{k-1}) + f_{k-1}(x_{k-1})\omega^T_{k-1} - \\ (f_{k-1}(x_{k-1}) + \omega_{k-1})\hat{x}^T_{k|k-1} + \omega_{k-1}\omega^T_{k-1} + \\ \omega_{k-1}f^T_{k-1}(x_{k-1}) - \hat{x}_{k|k-1}(f_{k-1}(x_{k-1})+ \\ \omega_{k-1})^T + \hat{x}_{k|k-1}\hat{x}^T_{k|k-1}|Z_{1:k-1}\end{array}\right]. \end{aligned} \tag{45}$$

Substituting (34), (36), and (41) into (45), then the prediction error cross-covariance (25) is proved.

As the joint distribution of states and measurements satisfies following Gaussian distribution:

$$\begin{aligned} &p(x_k, z_k|Z_{1:k-1}) \\ &= \mathrm{N}\left[\begin{pmatrix} x_k \\ z_k \end{pmatrix}; \begin{pmatrix} \hat{x}_{k|k-1} \\ \hat{z}_{k|k-1} \end{pmatrix}, \begin{pmatrix} P^{xx}_{k|k-1} & P^{xz}_{k|k-1} \\ P^{xz\,T}_{k|k-1} & P^{zz}_{k|k-1} \end{pmatrix}\right]. \end{aligned} \tag{46}$$

with

$$p(z_k|Z_{1:k-1}) = p(z_k|z_{k-1}) = N(z_k; \hat{z}_{k|k-1}, P^{zz}_{k|k-1}). \tag{47}$$

And, the posterior probability density of states is calculated as (48) by Bayesian theory [4]:

$$p(x_k|Z_{1:k}) = \frac{p(x_k, z_k|Z_{1:k-1})}{p(z_k|Z_{1:k-1})} = N(x_k; \hat{x}_{k|k}, P^{xx}_{k|k}). \tag{48}$$

Then, (27)-(29) are established.

The measurement prediction value $\hat{z}_{k|k-1}$ defined by (19) is calculated as:

$$\begin{aligned} \hat{z}_{k|k-1} &= E[z_k|Z_{1:k-1}] = E[\gamma_k h_k(x_k) + e_k|Z_{1:k-1}] \\ &= E[\gamma_k h_k(x_k)|Z_{1:k-1}] + E[e_k|Z_{1:k-1}]. \end{aligned} \tag{49}$$

Substituting (33), and (43) into (49), then (30) is proved. The prediction error cross-covariance is calculated as (50) by the definition (20):

$$\begin{aligned} P^{xz}_{k|k-1} &= E[(x_k - \hat{x}_{k|k-1})(z_k - \hat{z}_{k|k-1})^T|Z_{1:k-1}] \\ &= E[(x_k - \hat{x}_{k|k-1})(\gamma_k h_k(x_k) + e_k - \hat{z}_{k|k-1})^T|Z_{1:k-1}] \\ &= \mathrm{E}\left[\begin{array}{l} x_k h^T_k(x_k)\gamma^T_k + x_k e^T_k - x_k\hat{z}^T_{k|k-1} - \\ \hat{x}_{k|k-1}(\gamma_k h_k(x_k) + e_k)^T + \hat{x}_{k|k-1}\hat{z}^T_{k|k-1}|Z_{1:k-1}\end{array}\right]. \end{aligned} \tag{50}$$

Then, (31) is proved by (33) and (50). And similar to the derivation of state prediction error covariance $P^{xx}_{k|k-1}$, the measurement prediction error covariance (32) is also set up.

Thus, the proposed GF framework for nonlinear system with cross-correlated colored noises and random measurements loss is obtained as Theorem 1.

B. CUBATURE MIXING KALMAN FILTER (CMKF)

The implementation of the above recursive GF is finally transformed into the realization of nonlinear integral. There are two common approaches: one method is to linearize the nonlinear system, for example, EKF algorithm. The other method is sigma point approaches, such as, UKF by UT and CKF by spherical-radial method.

UKF does not need to calculate Jacobin matrix of nonlinear system compared with EKF, and estimation accuracy is higher than EKF. However, in the development of integration rule used for UKF, it required the solution of $2n_x + 1$ simultaneous equations [7]. In the spherical-radial method of CKF, the number of simultaneous equations is reduced to two by taking advantage of fully symmetric nature of the Gaussian-weighted integrals [8]. And CKF is a simplified form of UKF after removing the sigma point at the origin. Therefore, the spherical-radial method is adopted in this paper for the implementation of recursive GF algorithm. The standard Gaussian weighted integral is calculated as following by spherical-radial cubature rules:

$$\begin{aligned} I_{\mathrm{N}}(f) &= \int f(x)\mathrm{N}(x; 0, I)\,dx \approx \sum_{i=1}^{M} w_i f(\xi_i). \\ \xi_i &= \sqrt{M/2}[1]_i, w_i = 1/M, \quad i = 1, 2 \ldots M = 2n. \end{aligned}$$

where, w_i is weight corresponding to spatial cubature point. $[1]_i$ is vector point generator for initial cubature point.

The probability density functions satisfy the Gaussian distribution, namely:

$$\begin{aligned} p(x_k|Z_{1:k}) &= N(x_k; \hat{x}_{k|\mathrm{k}}, P^{xx}_{k|k}). \\ p(x_k|Z_{1:k-1}) &= N(x_k; \hat{x}_{k|\mathrm{k}-1}, P^{xx}_{k|k-1}). \end{aligned}$$

Then a CMKF with cross-correlated colored noises and measurements loss is given as Step (a)-Step (b) by Gaussian weighted integral and spherical-radial method.

Step (a): state prediction

1) Calculate the Cholesky factorization $l_{k-1|k-1}$ of state estimation error covariance $P^{xx}_{k-1|k-1}$:

$$l_{k-1|k-1} = chol(P^{xx}_{k-1|k-1}). \tag{51}$$

2) The initial cubature points :

$$\begin{aligned} \chi_{i,k-1|k-1} &= l_{k-1|k-1}\xi_i + \hat{x}_{k-1|k-1} \\ \xi_i &= [1]_i\sqrt{M/2} \quad i = 1, 2 \ldots, M = 2n. \end{aligned} \tag{52}$$

3) The transmit cubature points:

$$\chi^{*}_{i,k|k-1} = f_k(\chi_{i,k-1|k-1}). \tag{53}$$

$$\chi^{**}_{i,k-1|k-1} = \bar{\gamma}_{k-1}h_{k-1}(\chi_{i,k-1|k-1}). \tag{54}$$

4) The measurement estimated error covariance $P^{zz}_{k-1|k-1}$:

$$\begin{aligned} &P^{zz}_{k-1|k-1} \\ &= \frac{1}{M}\sum_{i=1}^{M} \chi^{**}_{i,k-1|k-1}\chi^{**}_{i,k-1|k-1}{}^T \\ &\quad - \bar{\gamma}_{k-1}h_{\mathrm{k-1}}(\hat{x}_{k-1|k-1})h^T_{k-1}(\hat{x}_{k-1|k-1})\bar{\gamma}^T_{k-1} \\ &\quad + R_{k-1} + \bar{\gamma}_{k-1}H_{k-1}S_{k-2,k-1} + S^T_{k-2,k-1}H^T_{k-1}\bar{\gamma}^T_{k-1}. \end{aligned} \tag{55}$$

5) The state prediction :

$$\hat{x}_{k|k-1} = \frac{1}{M}\sum_{i=1}^{M}\chi^{*}_{i,k|k-1} + (S_{k-1} + \bar{\gamma}_{k-1}Q_{k-1,k-2}H_{k-1})(P^{zz}_{k-1|k-1})^{-1}(z_{k-1} - \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})). \quad (56)$$

6) The state prediction error covariance $P^{xx}_{k|k-1}$:

$$P^{xx}_{k|k-1} = \frac{1}{M}\sum_{i=1}^{M}\chi^{*}_{i,k|k-1}\chi^{*T}_{i,k|k-1} - \hat{x}_{k|k-1}\hat{x}^{T}_{k|k-1} + Q_{k-1} + F_{k-1}Q_{k-2,k-1} + Q^{T}_{k-2,k-1}F^{T}_{k-1} - (S_{k-1} + Q^{T}_{k-1,k-2}H^{T}_{k-1}\bar{\gamma}^{T}_{k-1})(P^{zz}_{k-1|k-1})^{-1}(S_{k-1} + Q^{T}_{k-1,k-2}H^{T}_{k-1}\bar{\gamma}^{T}_{k-1})^{T}. \quad (57)$$

Step (b): state update

1) Calculate the Cholesky factorization $l_{k|k-1}$ of state prediction error covariance $P^{xx}_{k|k-1}$:

$$l_{k|k-1} = chol(P^{xx}_{k|k-1}). \quad (58)$$

2) The prediction cubature points $\chi'_{i,k|k-1}$:

$$\chi'_{i,k|k-1} = l_{k|k-1}\xi_i + \hat{x}_{k|k-1}. \quad (59)$$

3) The transmit cubature points $Z_{i,k|k-1}$:

$$Z_{i,k|k-1} = \bar{\gamma}_k h_k(\chi'_{i,k|k-1}). \quad (60)$$

4) The measurement prediction $\hat{z}_{k|k-1}$:

$$\hat{z}_{k|k-1} = \frac{1}{M}\sum_{i=1}^{M}Z_{i,k|k-1} + R_{k,k-1}(P^{zz}_{k-1|k-1})^{-1}(z_{k-1} - \bar{\gamma}_{k-1}h_{k-1}(\hat{x}_{k-1|k-1})). \quad (61)$$

5) The measurement prediction error covariance $P^{zz}_{k|k-1}$:

$$P^{zz}_{k|k-1} = \frac{1}{M}\sum_{i=1}^{M}Z_{i,k|k-1}Z^{T}_{i,k|k-1} - \hat{z}_{k|k-1}\hat{z}^{T}_{k|k-1} + R_k + \bar{\gamma}_k H_k S_{k,k-1} + S^{T}_{k,k-1}H^{T}_{k}\bar{\gamma}^{T}_{k} - R_{k,k-1}(P^{zz}_{k-1|k-1})^{-1}R^{T}_{k,k-1}. \quad (62)$$

6) The prediction error cross-covariance $P^{xz}_{k|k-1}$:

$$P^{xz}_{k|k-1} = \frac{1}{M}\sum_{i=1}^{M}\chi'_{i,k|k-1}Z^{T}_{i,k|k-1} - \hat{x}_{k|k-1}\hat{z}^{T}_{k|k-1} + S_{k,k-1}. \quad (63)$$

7) The state and covariance update by (27)-(29).

In order to facilitate description, a brief flow for proposed CMKF algorithm is given: Firstly, calculate state prediction $\hat{x}_{k|k-1}$, state prediction error covariance $P^{xx}_{k|k-1}$, measurement prediction error covariance $P^{zz}_{k|k-1}$ and prediction error cross-covariance $P^{xz}_{k|k-1}$ by (56), (57), and (62)-(63), respectively. Then, calculate measurement estimated error covariance $P^{zz}_{k-1|k-1}$, state update $\hat{x}_{k|k}$, estimation error covariance $P^{xx}_{k|k}$ and state gain K_k by (55), and (27)-(29).

Remark 1: If all measurements arrive at estimator, the proposed CMKF algorithm is simplified to cubature Kalman filter with cross-correlated colored noises (CKF-CN).

Remark 2: If the system noises are independent Gaussian noises and all measurements arrive at estimator timely, namely $S_{kk-1} = S_{k-1k-1} = 0$, $Q_{k-1,k-2} = R_{k-1,k-2} = 0$ and parameter $\gamma_k \equiv 1$. By substituting these parameters into Step (a) and Step (b) in Section III.B, then proposed CMKF is simplified to standard CKF as reference [8].

IV. SIMULATION EXPERIMENTS

The proposed algorithm is implemented by nonlinear system (4) and (10). In order to verify the effectiveness of CMKF: (1) The comparative simulations of CMKF, and CKF in [8] are performed in Scenario 1 under condition of cross-correlated colored noises; (2) The comparative simulations of CMKF, CKF, and UKF-PL algorithm in [19] are carried out in Scenario 2 under the condition of cross-correlate colored noises and random measurements loss.

The initial state $\hat{x}_{0|0} = [10, 20, 10, 1, 1.5, 0.1]^T$, the initial error covariance $p^{xx}_{0|0} = diag[1, 1, 1, 1.5, 1.5, 0.5]$, the measure matrix $h_k(\bullet) = diag[1, 1, 1, 1, 1, 1]$, the state noises covariance $Q_{k,k} = diag[20, 20, 20, 20, 20, 20]$, the correlation covariance $Q_{k,k-1} = diag[0.3, 0.3, 0.3, 0.3, 0.3, 0.3]$, the measurement noises covariance $R_{k,k} = diag[1, 1, 1, 1, 1, 1]$, the measurement noises correlation covariance $R_{k,k-1} = 0.02\ diag[1, 1, 1, 1, 1, 1]$, the cross-covariance between state noises and measurement noises at same and adjacent sampling moments are $S_{k,k} = diag[5, 5, 5, 5, 5, 5]$ and $S_{k,k-1} = 0.07diag[1, 1, 1, 1, 1, 1]$, respectively.

The Root-mean-square error (RMSE) is chosen as performance index for algorithms, where, $\tilde{x}^{i}_{k|k}(j) = x^{i}_{k}(j) - \hat{x}^{i}_{k|k}(j)$ denotes the estimation error of $i-$th parameters in j-Monte Carlo simulation experiments. Here, 100-Monte Carlo simulation experiments were made, namely $N = 100$.

$$RMSE : \tilde{x}^{i}_{k|k} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}[\tilde{x}^{i}_{k|k}(j)]^2}.$$

A. SCENARIO 1

In this case, it is assumed that all measurements reach estimator, i.e. $\bar{\gamma}_k \equiv 1$. The RMSEs of north, east, heading, surge, sway, and yaw under 100-Monte Carlo simulation experiments by CKF and CMKF are shown as Fig. 2 and Fig. 3, respectively. The RMSE of CKF is represented by black line and the RMSE of CMKF is represented by red line.

It is shown that the RMSE of CMKF is smaller than CKF. So, CMKF is effectively for DP ships with cross-correlated colored noises, and the estimation performance is better than CKF. On the other hand, the numerical results are presented in Table 1 for the average RMSE, which

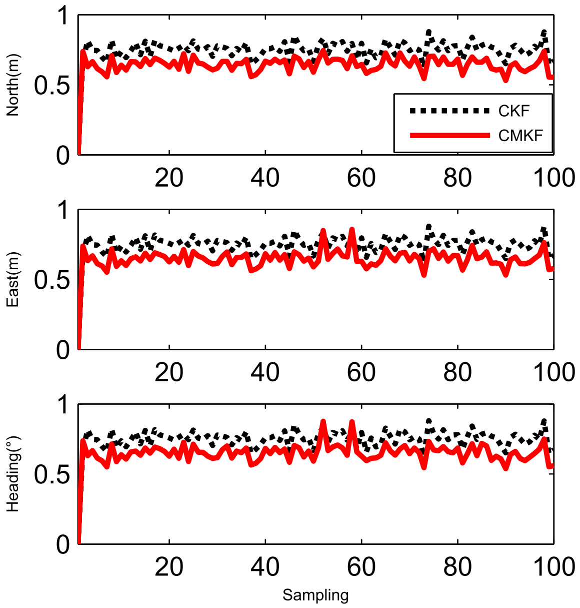

FIGURE 2. The RMSEs of North-East-heading.

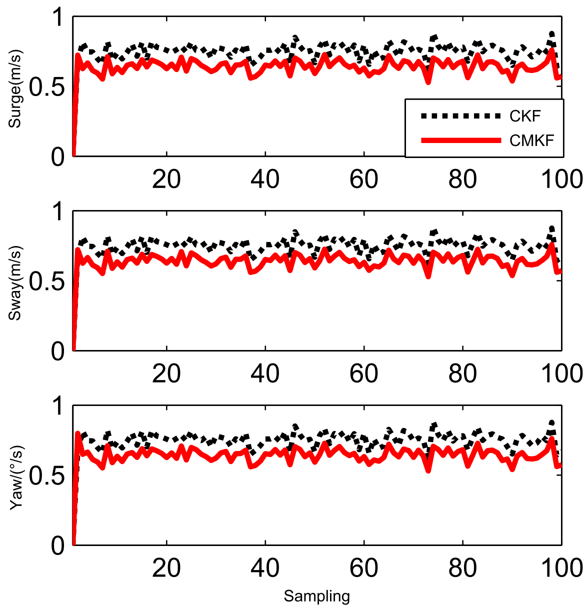

FIGURE 3. The RMSEs of Surge-Sway-Yaw.

shows that the accuracy of CMKF is higher than CKF further.

TABLE 1. Average RMSEs of states by different algorithms.

	x_N	y_E	ψ	u	v	r
CKF	0.75	0.75	0.75	0.75	0.75	0.75
CMKF	0.65	0.65	0.67	0.64	0.64	0.64

B. SCENARIO 2

In this section, the parameter $\bar{\gamma}_k = 0.8$, the other parameters are the same as Scenario 1, and the effectiveness of CMKF is further illustrated by comparison experiments with CKF, and UKF-PL in [19]. The RMSEs of CKF, UKF-PL, and CMKF are represented by Fig. 4 - Fig. 5, respectively.

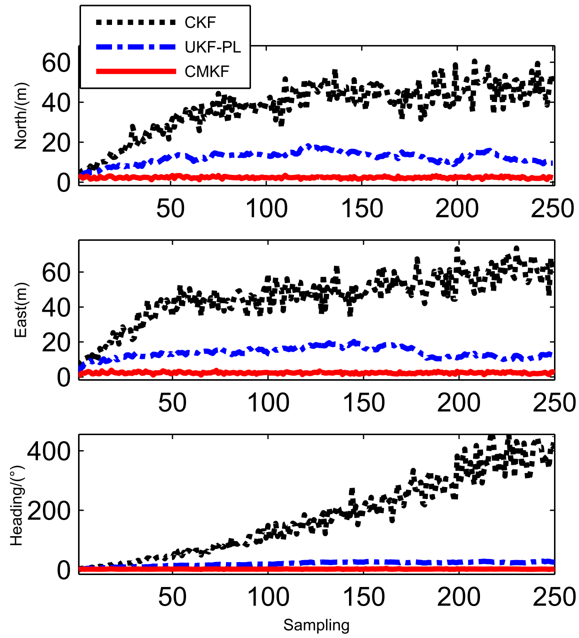

FIGURE 4. The RMSE of North-East-heading.

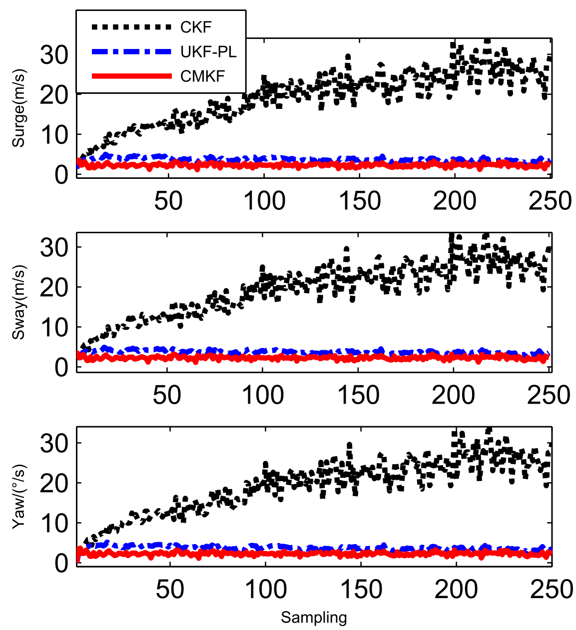

FIGURE 5. The RMSEs of Surge-Sway-Yaw.

It shows that the traditional CKF is no longer applicable after several steps. At the same time, the accuracy of CMKF is higher than UKF-PL because the cross-correlated colored noises and measurements loss are taken into account in CMKF. And the average RMSEs of UKF-PL and CMKF in different measurements loss probability are given as Table 2 and Table 3, respectively.

TABLE 2. Average RMSEs of UKF-PL for different $\bar{\gamma}_k$.

$\bar{\gamma}_k$	x_N	y_E	ψ	u	v	r
1.0	0.75	0.75	0.75	0.75	0.75	0.75
0.9	6.43	6.79	21.83	2.32	2.32	2.33
0.8	10.81	11.19	40.09	3.87	3.86	3.88
0.7	12.46	15.74	44.67	4.51	4.51	4.53
0.6	20.58	25.54	87.60	7.07	7.07	7.09
0.5	48.39	60.56	343.7	17.80	17.81	17.82

TABLE 3. Average RMSEs of CMKF for different $\bar{\gamma}_k$.

$\bar{\gamma}_k$	x_N	y_E	ψ	u	v	r
1.0	0.65	0.65	0.67	0.64	0.64	0.64
0.9	1.69	1.70	1.67	1.67	1.67	1.67
0.8	2.37	2.39	2.58	2.31	2.31	2.31
0.7	2.86	2.88	3.14	2.77	2.77	2.77
0.6	3.38	3.43	4.15	3.22	3.22	3.22
0.5	3.80	3.89	5.17	3.55	3.55	3.55

For $\bar{\gamma}_k = 1$, the CMKF in this section is simplified to CMKF in Scenario 1, so it has equal average RMSEs as Table 1 and Table 3. At the same time, the UKF-PL is simplified to UKF in Table 2. It shows that UKF and CKF have the similar estimation accuracy, and are lower than CMKF because the influence of cross-correlated colored noises is not taken into account for UKF and CKF.

And the average RMSEs of UKF-PL and CMKF are decreasing along with the increasing of $\bar{\gamma}_k$; the average RMSEs of CMKF is smaller than UKF-PL for the same $\bar{\gamma}_k$. The estimation accuracy of CMKF changes little with the increasing of measured data loss, and the average RMSE of u, v and r is almost the same(the values are equal when two bits of valid numbers are retained here).

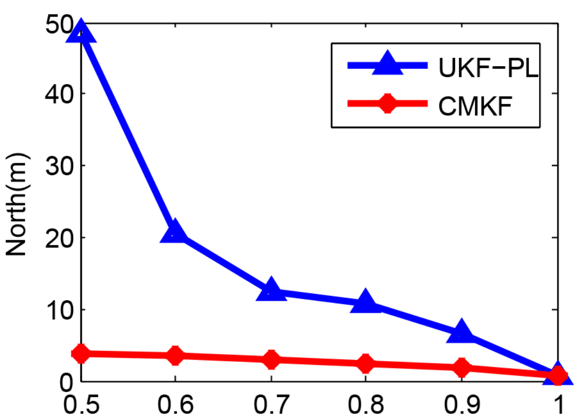

FIGURE 6. Average RMSEs along with the increasing of $\bar{\gamma}_k$.

The Fig. 6 shows the North average RMSEs as the increasing of $\bar{\gamma}_k$ which further demonstrates the CMKF outperforms UKF-PL. Meanwhile, the CMKF in this paper considers the cross-correlated colored noises and random measurements loss simultaneously, so the computational cost of the CMKF is larger than that of the CKF and UKF-PL algorithms.

V. CONCLUSIONS

An alternative GF framework for nonlinear Dynamic Positioning ships with cross-correlated colored noises and randomly measurements loss is proposed based on Bayesian theory. And then a CMKF algorithm is obtained by spherical-radial method for DP ships. In the end, the simulation results show that the proposed CMKF is effectiveness for DP ships with cross-correlated colored noises and measurements loss simultaneously, and CMKF has better estimation accuracy than UKF-PL and CKF. So, the application range of nonlinear GF is extended, and the proposed CMKF could be used as an alternative state estimation method for DP ships.

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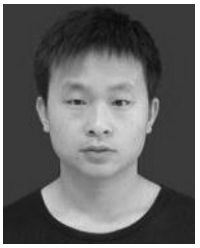

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