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Robust Adaptive Variable Structure Tracking Control for Spacecraft Chaotic Attitude Motion

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ABSTRACT A continuous globally stable control algorithm is presented to track angular velocity for spacecraft chaotic attitude motion affected by external disturbances using adaptive variable structure controller. Affected by some external disturbances, the spacecraft attitude dynamics system can generate many types of chaotic motion. Once it is required that a spacecraft with chaotic attitude motion should track the other spacecraft chaotic attitude plant to achieve angular velocity synchronization, the design of a robust tracking controller becomes necessary. The controller design is based on adaptive control theory and variable structure control theory, and adopts integral sliding surface and a single vector adjusted dynamically. Numerical simulations are performed to demonstrate the effectiveness and feasibility of the proposed adaptive variable structure controller.

INDEX TERMS Spacecraft chaotic motion, attitude system, tracking control, adaptive control, variable structure control.

I. INTRODUCTION

Spacecraft have been widely used in widespread communications, remote sensing and related scientific research in space. The attitude control system has great influence on spacecraft pointing accuracy and stabilization precision. Generally, any spacecraft in orbit is influenced by several kinds of external disturbance torques, such as the aerodynamic drag torque, the gravity gradient torque, the solar radiation pressure and the magnetic torque caused by the Earth's magnetic field. Although the external disturbances are small compared to the weight of the spacecraft, it often consists of periodic and secular terms, and the long-time disturbances on spacecraft may have significant influence on its actual attitude motion [1]. When certain conditions between the moment of inertia and the external disturbances of the spacecraft are met, it can lead to chaotic motion in the spacecraft [2]. A robust tracking controller is needed to track a chaotic attitude plant to achieve angular velocity synchronization.

The chaos phenomenon has been extensively studied by many researchers due to its unstable and complex behavior and wide range of applications in many industrial systems and different sciences [3], [4]. A few typical chaotic systems that result from external disturbances on spacecraft are Newton-Leipnik system [5], Lorenz system [6], Chen system [7], Lu system [8], Genesio-Tesi system [9], Rucklidge system [10], Liu system [11] and Rossler system [12]. The research on synchronization control in chaotic system has recently become a subject of great interest, and considerable efforts have been made to study the control and synchronization problems of different chaotic systems, such as adaptive control [13], sliding mode control [14], linear matrix inequality techniques [15], fuzzy logic control [16], state observer control [17], active control [18] and passive control [19].

Many flight experiences during the aerospace history have witnessed unexpected behaviors in spacecraft attitude motion, which results from the external disturbances that had not been taken into consideration in spacecraft design. Numerous theoretical studies have pointed out that chaotic spacecraft motion exists under the action of different kinds of external disturbances. Tong and Rimrott [20] have studied the planar vibration of an asymmetric satellite in elliptical orbit under the action of gravity gradient torque.

Meehan and Asokanthan [21], [22] conducted research on the chaotic motion of a spinning spacecraft which results from circumferential nutation damper or unbalanced rotor or vibrations in appendages. Salarieh and Alasty [23] investigated the problem of synchronization between two chaotic gyros using a modified sliding mode control method. Aghababa [24] adopted adaptive finite-time controller to achieve the synchronization of two chaotic flywheel governor systems and verified its robustness. Beletsky *et al.* [25] have conducted research on the numerical realization method to analyze the chaos in spacecraft attitude motion in circular polar orbit only influenced by the geomagnetic field. However, to the best of the authors' knowledge, the approach in the case that a spacecraft with chaotic attitude motion should track another spacecraft chaotic attitude plant to achieve angular velocity synchronization has not been investigated, and the main contribution of this paper is to conduct theoretical research on the design of adaptive variable structure tracking controller for spacecraft chaotic motion to meet the requirement of angular velocity synchronization.

The remainder of this paper is organized as follows. Section II introduces the spacecraft attitude dynamics equation and expands it in its component form. Besides, this section describes several types of chaotic phenomena in spacecraft attitude motion and describes the purpose of this work. Section III presents the robust adaptive variable structure controller based on adaptive control theory and variable structure control theory. Numerical simulations are given to illustrate the performance of the proposed technique in section IV. Finally, some conclusions of this work are addressed.

II. PROBLEM STATEMENT

The attitude dynamics equation of rigid spacecraft is

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I}\boldsymbol{\omega}) = \boldsymbol{T}_c + \boldsymbol{T}_d \tag{1}$$

where, $\boldsymbol{\omega} = \begin{bmatrix}\omega_1 & \omega_2 & \omega_3\end{bmatrix}^T$ is the angular velocity of the spacecraft body reference frame with respect to the earth-centered inertial reference frame in the spacecraft body reference frame, $\boldsymbol{I}$ denotes the spacecraft inertia matrix, $\boldsymbol{T}_c$ denotes the control input torque; $\boldsymbol{T}_d$ is the external disturbance torque, which can be generally expressed in the following nonlinear form:

$$\boldsymbol{T}_d = \boldsymbol{D}\boldsymbol{\omega} + \boldsymbol{M} \tag{2}$$

where, $\boldsymbol{D} = \left[d_{ij}\right]_{3\times 3} \in \boldsymbol{R}^{3\times 3}(i,j = 1,2,3)$, which can be a constant matrix or matrix varying with angular velocity;

$\boldsymbol{M} = [m_i]_{3\times 1} \in \boldsymbol{R}^{3\times 1}(i = 1,2,3)$, which can be a constant matrix or matrix varying with angular velocity, or even periodic matrix or long-term matrix.

Consider the three axes of the spacecraft body coordinate system for the inertial principal axes, then $\boldsymbol{I} = \mathrm{diag}(I_1, I_2, I_3)$. In addition, use Levi Civita symbol in three dimensions to express vector products, denoted as ε_{kij}, and the corresponding definition is given as follows:

$$\varepsilon_{kij} = \begin{cases} +1, & if\ (k,i,j)\ is\ (1,2,3),(2,3,1)\ or\ (3,1,2) \\ -1, & if\ (k,i,j)\ is\ (3,2,1),(1,3,2)\ or\ (2,1,3) \\ 0, & if\ i=j\ or\ j=k\ or\ k=i \end{cases} \tag{3}$$

For any two vectors $\boldsymbol{p} = [p_i]_{3\times 1}(i = 1,2,3)$ and $\boldsymbol{q} = [q_j]_{3\times 1}(j = 1,2,3)$, we have that $\sum_{i,j} \varepsilon_{kij}p_iq_j = (\boldsymbol{p}\times\boldsymbol{q})_k$, where $()_k$ represents the k-th component of the vector product.

Substitute Eq.(2) into Eq.(1) and expand it in its component form, we have

$$\begin{cases} I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 = T_{c1} + d_{11}\omega_1 + d_{12}\omega_2 \\ \qquad + d_{13}\omega_3 + m_1 \\ I_2\dot{\omega}_2 - (I_3 - I_1)\omega_1\omega_3 = T_{c2} + d_{21}\omega_1 + d_{22}\omega_2 \\ \qquad + d_{23}\omega_3 + m_2 \\ I_3\dot{\omega}_3 - (I_1 - I_2)\omega_1\omega_2 = T_{c3} + d_{31}\omega_1 + d_{32}\omega_2 \\ \qquad + d_{33}\omega_3 + m_3 \end{cases} \tag{4}$$

where, I_1, I_2 and I_3 denote the three components of inertia matrix; ω_1, ω_2 and ω_3 denote the three components of angular velocity; T_{c1}, T_{c2} and T_{c3} denote the three components of control input torque.

Then, we have

$$\begin{cases} \dot{\omega}_1 = I_1^{-1}(I_2 - I_3)\omega_2\omega_3 + I_1^{-1}d_{11}\omega_1 + I_1^{-1}d_{12}\omega_2 \\ \qquad + I_1^{-1}d_{13}\omega_3 + I_1^{-1}T_{c1} + I_1^{-1}m_1 \\ \dot{\omega}_2 = I_2^{-1}(I_3 - I_1)\omega_1\omega_3 + I_2^{-1}d_{21}\omega_1 + I_2^{-1}d_{22}\omega_2 \\ \qquad + I_2^{-1}d_{23}\omega_3 + I_2^{-1}T_{c2} + I_2^{-1}m_2 \\ \dot{\omega}_3 = I_3^{-1}(I_1 - I_2)\omega_1\omega_2 + I_3^{-1}d_{31}\omega_1 + I_3^{-1}d_{32}\omega_2 \\ \qquad + I_3^{-1}d_{33}\omega_3 + I_3^{-1}T_{c3} + I_3^{-1}m_3 \end{cases} \tag{5}$$

Define the relative inertia ratios as $a_1 = I_1^{-1}(I_2 - I_3)$, $a_2 = I_2^{-1}(I_3 - I_1)$ and $a_3 = I_3^{-1}(I_1 - I_2)$, and $[u_i]_{3\times 1} = \left[I_i^{-1}T_{ci}\right]_{3\times 1}$ $(i = 1,2,3)$ is the angular acceleration generated by the control input torque, and $\left[b_{ij}\right]_{3\times 3} = \left[I_i^{-1}d_{ij}\right]_{3\times 3}$, $[c_i]_{3\times 1} = \left[I_i^{-1}m_i\right]_{3\times 1}$ $(i,j = 1,2,3)$.

Then, Eq.(5) can be transformed into

$$\begin{cases} \dot{\omega}_1 = a_1\omega_2\omega_3 + b_{11}\omega_1 + b_{12}\omega_2 + b_{13}\omega_3 + u_1 + c_1 \\ \dot{\omega}_2 = a_2\omega_1\omega_3 + b_{21}\omega_1 + b_{22}\omega_2 + b_{23}\omega_3 + u_2 + c_2 \\ \dot{\omega}_3 = a_3\omega_1\omega_2 + b_{31}\omega_1 + b_{32}\omega_2 + b_{33}\omega_3 + u_3 + c_3 \end{cases} \tag{6}$$

The matrix form of Eq.(6) is

$$\dot{\boldsymbol{\omega}} = \boldsymbol{B}\boldsymbol{\omega} + f(\boldsymbol{\omega}) + \boldsymbol{u} \tag{7}$$

where,

$$\boldsymbol{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix},$$

$$f(\boldsymbol{\omega}) = \begin{bmatrix} f_1(\boldsymbol{\omega}) \\ f_2(\boldsymbol{\omega}) \\ f_3(\boldsymbol{\omega}) \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \boldsymbol{A}\begin{bmatrix} \omega_2\omega_3 \\ \omega_1\omega_3 \\ \omega_1\omega_2 \end{bmatrix} + \boldsymbol{C},$$

$$\boldsymbol{A} = \mathrm{diag}(a_1, a_2, a_3), \boldsymbol{C} = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}^T,$$
$$\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$$

For uncontrolled spacecraft attitude system, namely, the control input torque satisfies $\boldsymbol{u} = \boldsymbol{0}$, different external disturbance torque may lead to different kinds of chaos in spacecraft attitude motion, such as Newton-Leipnik system, Lorenz system, Chen system, Lu system, Genesio-Tesi system, Rucklidge system, Liu system and Rossler system and so on. The analysis of these chaotic systems, their characteristics and chaotic attractors, and corresponding necessary condition for spacecraft properties can be seen in Table 1.

The divergence of $\dot{\boldsymbol{\omega}}$ is

$$\nabla \cdot \dot{\boldsymbol{\omega}} = \frac{\partial \dot{\omega}_1}{\partial \omega_1} + \frac{\partial \dot{\omega}_2}{\partial \omega_2} + \frac{\partial \dot{\omega}_3}{\partial \omega_3} \tag{8}$$

For one of the above eight typical chaotic systems, when certain conditions between the moment of inertia of spacecraft and corresponding external disturbance torques are met, we can calculate $\nabla \cdot \dot{\boldsymbol{\omega}} < 0$, which indicates that the spacecraft attitude dynamics system is a dissipative system and its solution is bounded with increasing time.

The desired reference angular velocity is defined in the following chaotic attitude equation

$$\dot{\hat{\boldsymbol{\omega}}} = \hat{B}\hat{\omega} + g(\hat{\boldsymbol{\omega}}) \tag{9}$$

where,

$$\hat{\boldsymbol{B}} = \begin{bmatrix} \hat{b}_{11} & \hat{b}_{12} & \hat{b}_{13} \\ \hat{b}_{21} & \hat{b}_{22} & \hat{b}_{23} \\ \hat{b}_{31} & \hat{b}_{32} & \hat{b}_{33} \end{bmatrix}, \quad g(\hat{\boldsymbol{\omega}}) = \begin{bmatrix} g_1(\hat{\boldsymbol{\omega}}) \\ g_2(\hat{\boldsymbol{\omega}}) \\ g_3(\hat{\boldsymbol{\omega}}) \end{bmatrix} + \begin{bmatrix} \hat{c}_1 \\ \hat{c}_2 \\ \hat{c}_3 \end{bmatrix}$$
$$= \hat{\boldsymbol{A}} \begin{bmatrix} \hat{\omega}_2\hat{\omega}_3 \\ \hat{\omega}_1\hat{\omega}_3 \\ \hat{\omega}_1\hat{\omega}_2 \end{bmatrix} + \hat{\boldsymbol{C}},$$
$$\hat{\boldsymbol{A}} = \mathrm{diag}(\hat{a}_1, \hat{a}_2, \hat{a}_3), \quad \hat{\boldsymbol{C}} = \begin{bmatrix} \hat{c}_1 & \hat{c}_2 & \hat{c}_3 \end{bmatrix}^T$$

The tracking error is defined as $\boldsymbol{e} = \boldsymbol{\omega} - \hat{\boldsymbol{\omega}}$, with its components as $e_i (i = 1, 2, 3)$, and we can get the error system as

$$\begin{aligned} \dot{\boldsymbol{e}} = \dot{\boldsymbol{\omega}} - \dot{\hat{\omega}} &= \begin{bmatrix} \dot{e}_1 & \dot{e}_2 & \dot{e}_3 \end{bmatrix}^T \\ &= [\lambda_1(e_1, e_2, e_3) + u_1 \quad \lambda_2(e_1, e_2, e_3) + u_2 \\ &\qquad \lambda_3(e_1, e_2, e_3) + u_3]^T \end{aligned} \tag{10}$$

Assumption 1: There exists a constant $l > 0$ such that $\lambda_i(e_1, e_2, e_3) \le l \min_i |e_i| \, (i = 1, 2, 3)$.

This assumption will be used in the stability analysis, and it is easy to check such an assumption in practice.

The tracking problem is solved if $\lim_{t\to\infty} \boldsymbol{e} = 0$, which is equivalent to the stabilization of $\boldsymbol{e}$, and the equation that governs the spacecraft's motion is given by

$$\dot{\boldsymbol{e}} = \boldsymbol{B\omega} - \hat{\boldsymbol{B}}\hat{\boldsymbol{\omega}} + f(\boldsymbol{\omega}) - g(\hat{\boldsymbol{\omega}}) + \boldsymbol{u} \tag{11}$$

The control objective of this paper is to design an adaptive variable structure controller $\boldsymbol{u}(t)$ for the plant (7), whose tracking error dynamics is given by Eq.(11), such that, for all physically realizable initial conditions, the following is achieved: $\lim_{t\to\infty} \boldsymbol{e} = 0$.

III. ADAPTIVE VARIABLE STRUCTURE TRACKING CONTROLLER

The controller design is based on the following sliding surface:

$$s_i = e_i + \int_0^t re_i(\tau)d\tau \;\; (i = 1, 2, 3; \, r > 0) \tag{12}$$

When the system reaches the sliding surface and moves on it, the following conditions should be satisfied:

$$s_i = e_i + \int_0^t re_i(\tau)d\tau = 0 \tag{13}$$
$$\dot{s}_i = \dot{e}_i + re_i = 0 \tag{14}$$

From Eq.(14), we have

$$\dot{e}_i = -re_i \tag{15}$$

Therefore, Eq.(15) is asymptotically stable, namely, $\lim_{t\to\infty} e_i(t) = 0 (i = 1, 2, 3)$.

In this case, the controller is proposed in the form

$$u_i = -\sigma k_i \, |e_i| \, \mathrm{sgn}(s_i) \;\; (i = 1, 2, 3, \sigma > 0) \tag{16}$$

It can be seen that the term on right-hand side contains a variable k_i that will be adjusted dynamically to guarantee asymptotic disturbance rejection. If $k(0) > 0$ is given, $k(t) > 0$ can be ensured for all time, which will be discussed in the following part.

Choose the Lyapunov candidate function as

$$V = \frac{1}{2}\sum_{i=1}^{3} s_i^2 + \frac{1}{2\varsigma}\sum_{i=1}^{3} \sigma(k_i - k^*)^2 \tag{17}$$

where, $k^* > (l + r)/\sigma$. Taking the first derivative of V along the motion of the error system yields

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{3} s_i\dot{s}_i + \frac{\sigma}{\varsigma}\sum_{i=1}^{3} (k_i - k^*)\dot{k}_i \\ &= \sum_{i=1}^{3} s_i(\dot{e}_i + re_i) + \frac{\sigma}{\varsigma}\sum_{i=1}^{3} (k_i - k^*)\dot{k}_i \\ &= \sum_{i=1}^{3} s_i(\lambda_i + u_i + re_i) + \frac{\sigma}{\varsigma}\sum_{i=1}^{3} (k_i - k^*)\dot{k}_i \\ &\le l\sum_{i=1}^{3} |s_i| \, |e_i| + r\sum_{i=1}^{3} s_ie_i + \frac{\sigma}{\varsigma}\sum_{i=1}^{3} (k_i - k^*)\dot{k}_i \\ &\quad - \sum_{i=1}^{3} s_i(\sigma k_i \, |e_i| \, sgn(s_i)) \end{aligned}$$

TABLE 1. Chaotic system characteristics and chaotic attractor.

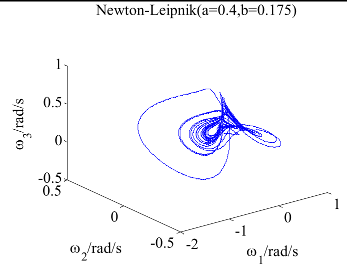
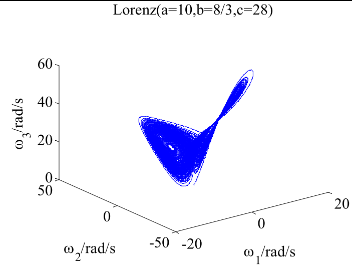
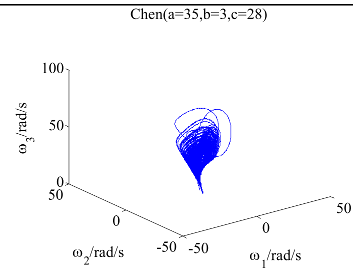
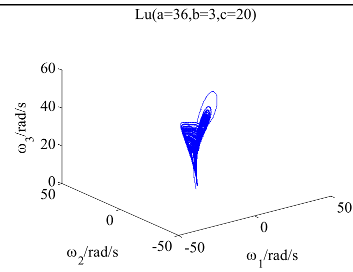
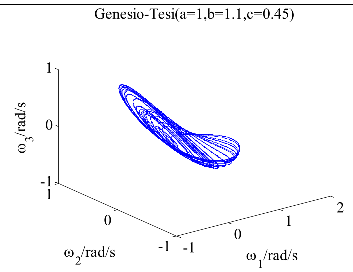
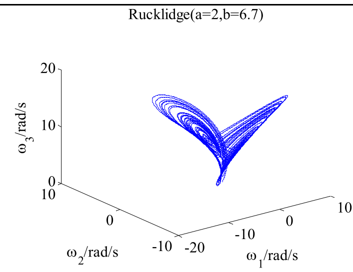
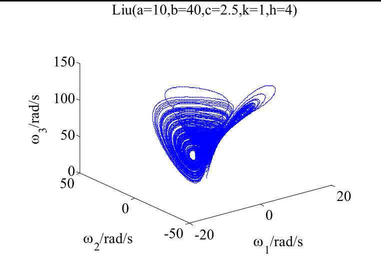
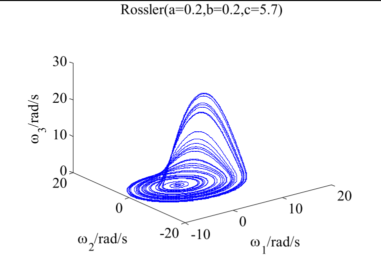
Chaotic system	Characteristics	Spacecraft	Chaotic attractor
Newton-Leipnik system	$\boldsymbol{A}=\mathrm{diag}(0,0,0),\boldsymbol{C}=\mathbf{0}$ $\boldsymbol{B}=\begin{bmatrix}-a & 1+10\omega_3 & 0\\ -1+5\omega_3 & -0.4 & 0\\ -5\omega_2 & 0 & b\end{bmatrix}$	$I_1=I_2=I_3$ $\boldsymbol{T}_d=\begin{bmatrix}I_1(-a\omega_1+\omega_2+10\omega_2\omega_3)\\ I_2(-\omega_1-0.4\omega_2+5\omega_1\omega_3)\\ I_3(b\omega_3-5\omega_1\omega_2)\end{bmatrix}$	
Lorenz system	$\boldsymbol{A}=\mathrm{diag}(0,-1,1),\boldsymbol{C}=\mathbf{0}$ $\boldsymbol{B}=\begin{bmatrix}-a & a & 0\\ c & -1 & 0\\ 0 & 0 & -b\end{bmatrix}$	$I_1=2I_2=2I_3$ $\boldsymbol{T}_d=\begin{bmatrix}I_1a(\omega_2-\omega_1)\\ I_2(c\omega_1-\omega_2)\\ -I_3b\omega_3\end{bmatrix}$	
Chen system	$\boldsymbol{A}=\mathrm{diag}(0,-1,1),\boldsymbol{C}=\mathbf{0}$ $\boldsymbol{B}=\begin{bmatrix}-a & a & 0\\ c-a & c & 0\\ 0 & 0 & -b\end{bmatrix}$	$I_1=2I_2=2I_3$ $\boldsymbol{T}_d=\begin{bmatrix}I_1a(\omega_2-\omega_1)\\ I_2(c\omega_1-a\omega_1+c\omega_2)\\ -I_3b\omega_3\end{bmatrix}$	
Lu system	$\boldsymbol{A}=\mathrm{diag}(0,-1,1),\boldsymbol{C}=\mathbf{0}$ $\boldsymbol{B}=\begin{bmatrix}-a & a & 0\\ 0 & c & 0\\ 0 & 0 & -b\end{bmatrix}$	$I_1=2I_2=2I_3$ $\boldsymbol{T}_d=\begin{bmatrix}I_1a(\omega_2-\omega_1)\\ I_2c\omega_2\\ -I_3b\omega_3\end{bmatrix}$	
Genesio-Tesi system	$\boldsymbol{A}=\mathrm{diag}(0,0,0),\boldsymbol{C}=\mathbf{0}$ $\boldsymbol{B}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -a+\omega_1 & -b & -c\end{bmatrix}$	$I_1=I_2=I_3$ $\boldsymbol{T}_d=\begin{bmatrix}I_1\omega_2\\ I_2\omega_3\\ -I_3(a\omega_1-\omega_1^2+b\omega_2+c\omega_3)\end{bmatrix}$	
Rucklidge system	$\boldsymbol{A}=\mathrm{diag}(0,0,0),\boldsymbol{C}=\mathbf{0}$ $\boldsymbol{B}=\begin{bmatrix}-a & b & -\omega_2\\ 1 & 0 & 0\\ 0 & \omega_2 & -1\end{bmatrix}$	$I_1=I_2=I_3$ $\boldsymbol{T}_d=\begin{bmatrix}I_1(-a\omega_1+b\omega_2-\omega_2\omega_3)\\ I_2\omega_1\\ I_3(\omega_2^2-\omega_3)\end{bmatrix}$	

TABLE 1. *(Continued.)* Chaotic system characteristics and chaotic attractor.

Liu system	$A=\mathrm{diag}(0,0,0), C=\mathbf{0}$ $B=\begin{bmatrix}-a & a & 0\\ b & 0 & -k\omega_1\\ h\omega_1 & 0 & -c\end{bmatrix}$	$I_1=I_2=I_3$ $T_d=\begin{bmatrix}I_1a(\omega_2-\omega_1)\\ I_2(b\omega_1-k\omega_1\omega_3)\\ I_3(h\omega_1^2-c\omega_3)\end{bmatrix}$	
Rossler system	$A=\mathrm{diag}(0,0,0)$ $C=[0\quad 0\quad b]^{\mathrm{T}}$ $B=\begin{bmatrix}0 & -1 & -1\\ 1 & a & 0\\ \omega_3 & 0 & -c\end{bmatrix}$	$I_1=I_2=I_3$ $T_d=\begin{bmatrix}-I_1(\omega_1+\omega_2)\\ I_2(\omega_1+a\omega_2)\\ I_3(\omega_1\omega_3-c\omega_3)+I_3b\end{bmatrix}$	

$$= l\sum_{i=1}^{3}|s_i||e_i| + r\sum_{i=1}^{3}s_ie_i + \frac{\sigma}{\varsigma}\sum_{i=1}^{3}(k_i-k^*)\dot{k}_i - \sigma\sum_{i=1}^{3}k_i|e_i||s_i|$$

$$= l\sum_{i=1}^{3}|s_i||e_i| + r\sum_{i=1}^{3}s_ie_i - \frac{\sigma}{\varsigma}\sum_{i=1}^{3}k^*\dot{k}_i + \frac{\sigma}{\varsigma}\sum_{i=1}^{3}k_i(\dot{k}_i-\varsigma|e_i||s_i|) \tag{18}$$

The adjustment law for $k_i(t)$ is now chosen as

$$\dot{k}_i = \varsigma|e_i||s_i| \tag{19}$$

This yields

$$\dot{V} \le l\sum_{i=1}^{3}|s_i||e_i| + r\sum_{i=1}^{3}s_ie_i - \sigma k^*\sum_{i=1}^{3}|e_i||s_i|$$
$$\le l\sum_{i=1}^{3}|s_i||e_i| + r\sum_{i=1}^{3}|s_i||e_i| - \sigma k^*\sum_{i=1}^{3}|e_i||s_i|$$
$$= (l+r-\sigma k^*)\sum_{i=1}^{3}|s_i||e_i| < 0 \tag{20}$$

According to Lyapunov stability theory, the tracking error system is asymptotically stable.

IV. NUMERICAL SIMULATIONS

To validate the effectiveness and feasibility of the proposed adaptive variable structure tracking controller in this paper, we take the Newton-Leipnik system for example, which is shown below.

Choose the moment of inertia and external disturbance torque acting on the spacecraft as

$$I_1 = I_2 = I_3 = 1\mathrm{kg}\cdot m^2,$$

$$\boldsymbol{T}_d = \begin{bmatrix} I_1(-a\omega_1+\omega_2+10\omega_2\omega_3)\\ I_2(-\omega_1-0.4\omega_2+5\omega_1\omega_3)\\ I_3(b\omega_3-5\omega_1\omega_2)\end{bmatrix}$$

Then, the target system is

$$\dot{\hat{\omega}} = \begin{bmatrix}-a & 1 & 0\\ -1 & -0.4 & 0\\ 0 & 0 & b\end{bmatrix}\begin{bmatrix}\hat{\omega}_1\\ \hat{\omega}_2\\ \hat{\omega}_3\end{bmatrix} + \begin{bmatrix}10\hat{\omega}_2\hat{\omega}_3\\ 5\hat{\omega}_1\hat{\omega}_3\\ -5\hat{\omega}_1\hat{\omega}_2\end{bmatrix} \tag{21}$$

The tracking system is

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix}-a & 1 & 0\\ -1 & -0.4 & 0\\ 0 & 0 & b\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} + \begin{bmatrix}10\omega_2\omega_3\\ 5\omega_1\omega_3\\ -5\omega_1\omega_2\end{bmatrix} + \boldsymbol{u} \tag{22}$$

Combining Eqs.(21) and (22), the error system can be obtained as

$$\dot{\boldsymbol{e}} = \begin{bmatrix}-a & 1 & 0\\ -1 & -0.4 & 0\\ 0 & 0 & b\end{bmatrix}\begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix} + \begin{bmatrix}10\omega_2\omega_3-10\hat{\omega}_2\hat{\omega}_3\\ 5\omega_1\omega_3-5\hat{\omega}_1\hat{\omega}_3\\ 5\hat{\omega}_1\hat{\omega}_2-5\omega_1\omega_2\end{bmatrix} + \boldsymbol{u} \tag{23}$$

The sliding mode surface is chosen as $s_i = e_i + \int_0^t re_i(\tau)d\tau (i = 1, 2, 3; r > 0)$, the adjustment law for $k_i(t)$ is

$$\begin{cases}\dot{k}_1 = \varsigma|e_1||s_1|\\ \dot{k}_2 = \varsigma|e_2||s_2|\\ \dot{k}_3 = \varsigma|e_3||s_3|\end{cases} \tag{24}$$

The adaptive variable structure tracking controller is

$$\begin{cases}u_1 = -\sigma k_1|e_1|\mathrm{sgn}(s_1)\\ u_2 = -\sigma k_2|e_2|\mathrm{sgn}(s_2)\\ u_3 = -\sigma k_3|e_3|\mathrm{sgn}(s_3)\end{cases} \tag{25}$$

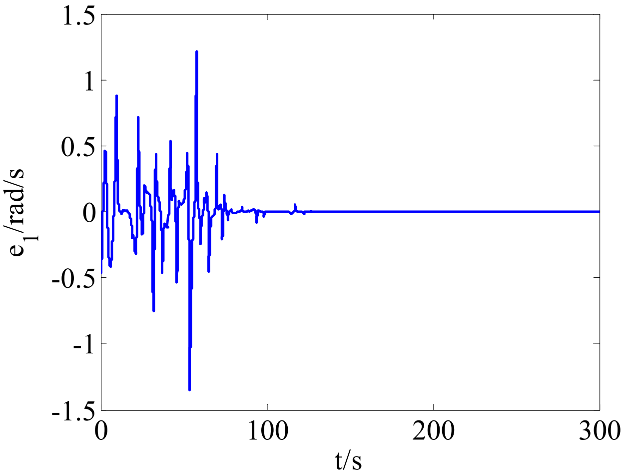


FIGURE 1. Response of the error e_1.

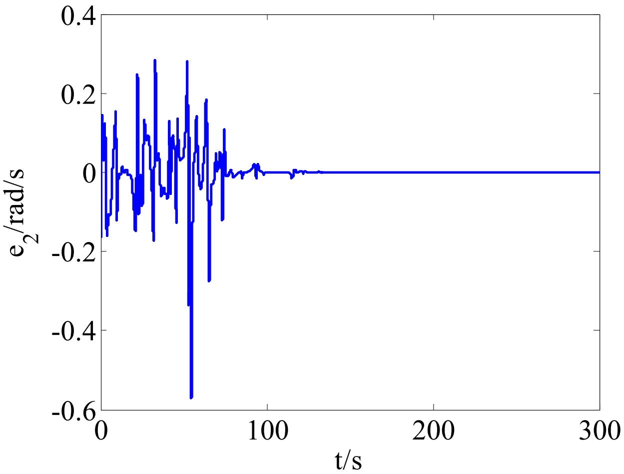


FIGURE 2. Response of the error e_2.

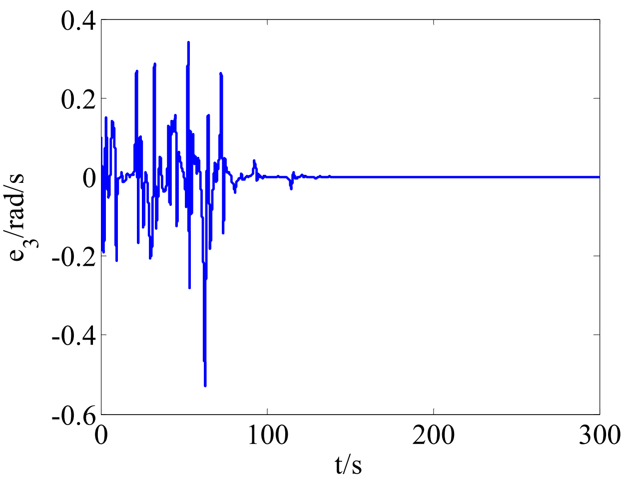


FIGURE 3. Response of the error e_3.

The corresponding known parameters are chosen as $a = 0.4$, $b = 0.175$, $r = 4$, $\sigma = 0.052$, $\varsigma = 5$. The initial states are $k_0 = \left[0.4\ 0.2\ 0.2\right]^T$, $\omega_0 = [\,0.3\ 0.1\ 0.1\,]^T$,

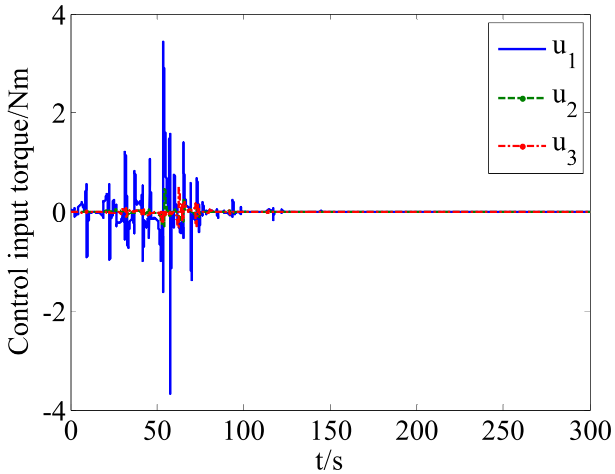


FIGURE 4. Response of the control input torque.

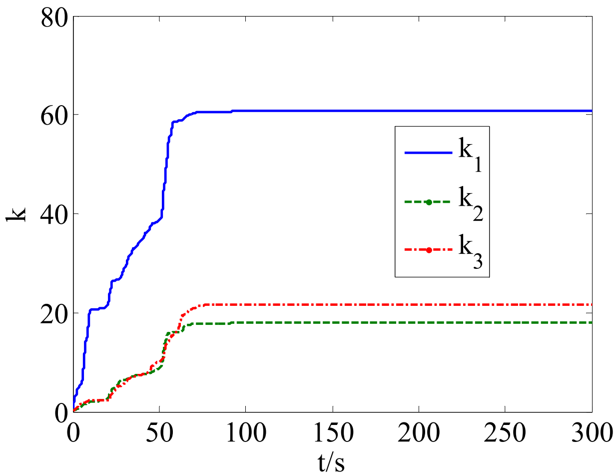


FIGURE 5. Response of $k(t)$.

and $\hat{\omega}_0 = [\,0.7\ 0.25\ 0.25\,]^T$. The unit of angular velocity is rad/s. The simulation step is chosen as 0.001s, and the corresponding simulation time is chosen as 300s.

The response of the closed-loop system can be seen in Figs.1-5, from which we can see that the control objective is achieved despite the presence of the external disturbances. The error $\boldsymbol{e}$ will converge to zero as time increases, and the control input torque will also converge to zero, which indicates that the tracking system will achieve synchronization with the target system. Generally, the variable structure control method has control chattering problem, but it isn't obvious due to the small tracking errors in the stable phase. It is also seen in Fig.5 that k_i(t) increases during the transient phase, but then converges to a constant value when the error system is stable.

V. CONCLUSIONS

For the tracking control problem of spacecraft chaotic attitude motion affected by external disturbances, this paper presents a continuous globally stable tracking control algorithm which is based on adaptive control theory and variable structure control theory. In the proposed controller, the integral sliding surface is adopted, and a single vector is adjusted dynamically in such a fashion that the angular velocity error will tend to zero asymptotically. The stability proof is conducted via a Lyapunov analysis of the spacecraft error dynamics. Numerical simulations also illustrate the tracking performance of spacecraft chaotic motion using the adaptive variable structure controller proposed in this paper.

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