Received October 22, 2017, accepted December 11, 2017, date of publication December 27, 2017,
date of current version February 14, 2018.

Digital Object Identifier 10.1109/ACCESS.2017.2786216

Suboptimal Learning Control for Nonlinearly Parametric Time-Delay Systems Under Alignment Condition

YOUFANG YU[1], JUN WAN[2], AND HONGBO BI[3]
[1]Applied Engineering College, Zhejiang Business College, Hangzhou 310053, China
[2]School of Electrical Engineering, Zhejiang University of Water Resources and Electric Power, Hangzhou 310018, China
[3]College of Electrical Engineering and Information Engineering, Quzhou University, Quzhou 324000, China

Corresponding author: Youfang Yu (youfang_yu@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61573322 and Grant 61673050 and in part by the Science and Technology Project of Zhejiang Province under Grant 2015C31061.

ABSTRACT In order to achieve perfect trajectory-tracking performance over the entire time interval at a higher convergence speed, this paper proposes a suboptimal learning control scheme for a class of nonlinearly parametric time-delay systems under alignment condition. The controller is designed by integrating the robust learning control with the suboptimal control, in which, the control Lyapunov function and Sontag formula are employed in generating a suboptimal controller for the nominal system, while robust learning control mechanism is applied to deal with nonlinearly parametric time-delay uncertainties. As the iteration number increases, the system state can follow its reference signal over the full time interval. The proposed method extends some existing results. Numerical simulations demonstrate that our suboptimal iterative learning control scheme improves the convergence performance in comparison with traditional solutions.

INDEX TERMS Suboptimal control, iterative learning control, time delay, nonlinearly parametric systems, alignment condition.

I. INTRODUCTION

Nowadays people's living standard is continuously rising, which intensifies the need of high-precision control methodology for improving the performance of the automation systems. In the presence of complicated uncertainties and inaccuracy in modeling, those control systems adopting traditional design techniques, such as PID control, optimal control, etc., can hardly achieve satisfactory tracking performance. As an effective control technique to overcome the limitation of traditional control designs, iterative learning control (ILC), which is good at tackling repetitive control tasks over a finite time interval, has been proposed and well developed for more than three decades [1]–[4]. ILC has attracted increasing attention for its perfect trajectory-tracking performance over the full time interval. So far, it has been widely applied in many industries [5]–[9], including robotic manipulators, hard disk drives, servo control system, and so on.

In most existing studies of ILC, it is commonly assumed that the initial system state in each iteration should be reset to the exact beginning of the desired trajectory; that is to say, the initial system error is necessarily equal to zero [10]. Otherwise, a slight initial system error will lead to divergence of the tracking error. However, this identical initial condition is very hard to meet in practical industries. For this reason researchers have come up with various solutions, such as initial impulsive compensation [11], [12], time-varying boundary layer [1], initial rectifying action [13]–[15], average operator [16], and error-tracking method [17]–[19], to mitigate the tough initial condition. Among them, the alignment condition is applicable to the controlled systems whose reference trajectories are spatially closed. While ILC systems operate under alignment condition, the final state of the previous iteration is directly employed as the initial state at the current iteration, instead of initial resetting at each iteration in traditional ILC algorithm. Some promising results have been proposed in [2], [20], and [21] respectively for uncertain robot manipulators [2], multiple-output nonparametric systems [20], and state-constrained systems [21].

Time delay is often encountered in many applications inherently, such as batch processes, turbojet engine, electrical networks. The time delays in controlled systems may degrade system performance, and even lead to system divergences in serious cases. The early results on ILC for time-delay system mainly belong to contraction mapping ILC area, see [22]–[25]. Recently, time delay is still a hot research topic in the study of ILC. As reported in [26], an iterative learning controller was developed on the basis of 2-D system theory for linear continuous multi-variable systems with time delay. A robust ILC design for uncertain time-delay systems was proposed in [27], by using LMI approach. Some promising ILC results have been derived for nonlinearly parametric systems with unknown time-varying delays, including adaptive ILC [28], [29] and adaptive repetitive learning control [30].

So far, most ILC algorithms aim at achieving precise tracking performance after a large number of iterations. It is noteworthy that the convergence speed of the algorithms is an important factor in practical industries [31]. In order to improve the convergence speed of ILC systems, researchers have developed optimal ILC methods since the early 1990s, with many meaningful results reported in [32]–[37]. However, in the controller design for nonlinear systems, few ILC schemes have been proposed by integrating adaptive learning control with standard optimal control. A main reason behind this is that, as an indispensable step in standard optimal control algorithms, solving Hamilton-Jacobi-Bellman (HJB) equations is of great difficulty, especially when the controlled systems are general nonlinear systems. In view of this, suboptimal ILC solutions have been presented as an alternative. A suboptimal ILC control scheme was firstly proposed for a class of MIMO nonlinear parametric system with time-varying uncertainties in [38]. In this scheme, the feedback term corresponding to nominal system is given by Sontag formula, while time-varying uncertainties are compensated by using the learning method. Inspired by [38], later on [39] designed a suboptimal iterative learning controller and a suboptimal repetitive learning controller for a class of SISO non-parametric systems, with robust control and learning control synthetically applied to cope with nonparametric uncertainties.

We remark that the above-mentioned ILC results can only either apply to the systems with time-delays, or design controllers under alignment condition, or investigate suboptimal ILC algorithms. Though improving the convergence speed of time-delay systems under alignment condition is useful in modern industrial processes, to the best of our knowledge, there has been little literature focusing on suboptimal ILC under alignment condition. This motivates us to undertake the current research.

In this paper, we present a suboptimal ILC scheme for a class of nonlinearly parametric time-delay systems under alignment condition, so as to meet the practical needs. The contribution of this paper is threefold: (1) A suboptimal adaptive iterative learning controller, consisting of a suboptimal feedback term, a parameters updating law and a robust feedback term, is developed to achieve perfect tracking performance; (2) A novel control Lyapunov functional based on filtering errors, state and parameter estimation information is constructed during the controller design and convergence analysis; (3) The proposed suboptimal learning control approach can tackle nonlinearly parametric time-delay systems under alignment condition, without requiring the initial condition, and improve the convergence speed in comparison with traditional ILC algorithms. Theoretical analysis and numerical simulations synthetically show the effectiveness of the proposed scheme.

The remainder of this paper is organized as follows. The problem formulation is presented in Section II. A suboptimal controller is designed in Section III, with detailed convergence analysis being given in Section IV. Simulation results are shown in Section V. Finally, Section VI concludes the work.

II. PROBLEM FORMULATION

Consider a class of nonlinearly parametric systems with time-delays,

$$\begin{cases} \dot{x}_{i,k} = x_{i+1,k}, \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_{n,k} = f(\boldsymbol{x}_k) + \eta(\boldsymbol{x}_k(t-\tau(t)), \theta(t), t) \\ \qquad\quad + g(\boldsymbol{x}_k)\big(u_k + \boldsymbol{\vartheta}^T(t)\boldsymbol{\psi}(\boldsymbol{x}_k, t)\big), \\ \boldsymbol{x}_0(t) = \boldsymbol{\varpi}(t), \quad \forall t \in [-\tau_{\max}, 0], \end{cases} \tag{1}$$

in which, $k = 0, 1, 2, \ldots$ is the iteration index, $t \in [0, T]$. $\boldsymbol{x}_k \triangleq [x_{1,k}, x_{2,k}, \ldots, x_{n,k}]^T \in \boldsymbol{R}^n$ is the system state, $\boldsymbol{\vartheta}(t) \in \boldsymbol{R}^m$ is an unknown time-varying part that is iteration-independent, $\boldsymbol{\psi}(\boldsymbol{x}_k, t) \in \boldsymbol{R}^m$ is a known state dependent function, $u_k \in \boldsymbol{R}$ is the system input, and $f(\boldsymbol{x}_k(t-\tau(t)), \theta(t), t) \in R$ is an unknown smooth nonlinear function with $\tau(t) \in [-\tau_{\max}, 0]$ being the unknown time-delay of the system state $\boldsymbol{x}_k$. The nominal system corresponding to (1) is

$$\begin{cases} \dot{x}_{i,k} = x_{i+1,k}, \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_{n,k} = f(\boldsymbol{x}_k) + g(\boldsymbol{x}_k)u_k, \end{cases} \tag{2}$$

where, $g(\boldsymbol{x}_k) \geq \underline{g}$ holds with $\underline{g}$ an unknown positive constant. Without loss of generality, we make the following assumptions:

Assumption 1: $\forall \boldsymbol{\xi}_1, \boldsymbol{\xi}_2 \in \boldsymbol{R}^n$, the inequality

$$|\eta(\boldsymbol{\xi}_1, \theta(t), t) - \eta(\boldsymbol{\xi}_2, \theta(t), t)| \leq \|\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2\| h(\theta, t), \tag{3}$$

holds, where $h(\theta, t) > 0$ is an unknown smooth positive function.

Assumption 2: The time delay $\tau(t)$ satisfies $\dot{\tau}(t) \leq \phi < 1$, i.e.,

$$-\frac{1-\dot{\tau}(t)}{1-\phi} \leq -1.$$

The nominal system corresponding to (1) is

$$\begin{cases} \dot{x}_{i,k} = x_{i+1,k}, \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_{n,k} = f(\boldsymbol{x}_k) + g(\boldsymbol{x}_k)u_k. \end{cases} \tag{4}$$

For the given reference signal $\boldsymbol{x}_d(t) = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]^T$, $\boldsymbol{x}_d(t) \in C^n[0, T]$, our control objective is to let the system state $\boldsymbol{x}_k(t)$ track its reference signal $\boldsymbol{x}_d$ under alignment condition, i.e. $\boldsymbol{x}_k(0) = \boldsymbol{x}_{k-1}(T)$ and $\boldsymbol{x}_d(T) = \boldsymbol{x}_d(0)$.

Throughout the paper, for brevity we abbreviate $f(\boldsymbol{x}_k(t-\tau(t)), \theta(t), t)$ and $g(\boldsymbol{x}_k)$ to $f_k(t-\tau(t))$ and g_k, respectively, and we often omit arguments as long as no confusion arises.

III. CONTROL SYSTEM DESIGN

Let us define $\boldsymbol{e}_k(t) = [e_{1,k}, \ldots, e_{n,k}]^T = \boldsymbol{x}_k(t) - \boldsymbol{x}_d(t)$ and

$$s_k = (\frac{d}{dt} + \lambda)^{n-1} e_{1,k}, \tag{5}$$

where λ is a positive constant. Expanding the right side of (5) yields

$$s_k = c_1 e_{1,k} + \ldots + c_{n-1} e_{n-1,k-1} + e_{n,k}, \tag{6}$$

where $c_j = \frac{(n-1)!}{(j-1)!(n-j)!}\lambda^{n-j}$, $j = 1, 2, \ldots, n-1$.

From (1), we can easily obtain

$$\begin{cases} \dot{e}_{i,k} = e_{i+1,k}, \quad i = 1, 2, \ldots, n-1, \\ \dot{e}_{n,k} = f_k + \eta_k(t-\tau(t)) + g_k u_k - x_d^{(n)}. \end{cases} \tag{7}$$

Then we choose a candidate control Lyapunov function at the kth iteration as

$$V_k = \frac{1}{2}s_k^2 + \underline{\boldsymbol{e}}_k^T P \underline{\boldsymbol{e}}_k, \tag{8}$$

with $\underline{\boldsymbol{e}}_k(t) = [e_{1,k}, \ldots, e_{n-1,k}]^T$. V_k is a control Lyapunov function if and only if it complies with the condition that while $\boldsymbol{e}_k \neq 0$,

$$b_k = 0 \Rightarrow a_k < 0, \tag{9}$$

with

$$b_k = \frac{\partial V_k}{\partial \boldsymbol{e}_k^T}\boldsymbol{\beta}_g^T, \quad a_k = \frac{\partial V_k}{\partial \boldsymbol{e}_k^T}\boldsymbol{\beta}_e^T, \tag{10}$$

$\boldsymbol{\beta}_g = [0, \ldots, 0, g_k]^T$, $\boldsymbol{\beta}_e = [e_{2,k}, \ldots, e_{n,k}, f_k - f(\boldsymbol{x}_d)]^T$. Taking the time derivative of V_k yields

$$\begin{aligned} \dot{V}_k &= \frac{\partial V_k}{\partial \boldsymbol{e}_k^T}\boldsymbol{\beta}_e^T d\tau(t) + \frac{\partial V_k}{\partial \boldsymbol{e}_k^T}\boldsymbol{\beta}_g^T(u_k + \eta_k(t-\tau(t)) + \boldsymbol{\vartheta}^T\boldsymbol{\psi}_k) \\ &= a_k + b_k\big(u_k + g_k^{-1}(\eta_k(t-\tau(t)) \\ &\qquad + f(\boldsymbol{x}_d) - x_d^{(n)}) + \boldsymbol{\vartheta}^T\boldsymbol{\psi}_k\big). \end{aligned} \tag{11}$$

By Assumption 1, we have

$$\begin{aligned} & g_k^{-1}\eta_k(t-\tau(t)) \\ &\quad = g_k^{-1}(\eta_k(t-\tau(t)) - \eta_d(t-\tau(t)) + \eta_d(t-\tau(t))) \\ &\quad \leq |g_k^{-1}|\|\boldsymbol{e}_k(t-\tau(t))\|h(\theta,t) + g_k^{-1}\eta_d(t-\tau(t)) \\ &\quad \leq \frac{1}{2}g_k^{-2}h^2(\theta,t) + \frac{1}{2}\boldsymbol{e}_k^T(t-\tau(t))\boldsymbol{e}_k(t-\tau(t)) \\ &\qquad + g_k^{-1}\eta_d(t-\tau(t)). \end{aligned} \tag{12}$$

Combining (11) with (12) gives

$$\dot{V}_k \leq a_k + b_k(u_k + \boldsymbol{p}^T\boldsymbol{\varphi}_k) + \frac{1}{2}\boldsymbol{e}_k^T(t-\tau(t))\boldsymbol{e}_k(t-\tau(t)), \tag{13}$$

with $\boldsymbol{p} = \big(\frac{1}{2}h^2(\theta,t), \eta_d(t-\tau(t)) + f(\boldsymbol{x}_d) - x_d^{(n)}, \boldsymbol{\vartheta}^T\big)^T$ and $\boldsymbol{\varphi}_k = (g_k^{-2}, g_k^{-1}, \boldsymbol{\psi}_k^T)^T$. On the basis of (13) and by Sontag formula, we propose the following suboptimal ILC law for system (1) as

$$u_k = u_{ok} - \boldsymbol{p}_k^T\boldsymbol{\varphi}_k - \frac{s_k\boldsymbol{e}_k^T(t)\boldsymbol{e}(t)}{g_k(1-\phi)(s_k^2+\epsilon_2^2)}, \tag{14}$$

in which,

$$u_{ok} = -(\mu + \frac{a_k + \sqrt{a_k^2 + b_k^4}}{b_k^2 + \epsilon_1\beta(s_{ak})})b_k \tag{15}$$

and

$$\boldsymbol{p}_k = sat(\boldsymbol{p}_{k-1}) + \gamma_1 b_k\boldsymbol{\varphi}_k, \boldsymbol{p}_{-1} = 0. \tag{16}$$

Here, $\epsilon_1 = \epsilon_3 + \epsilon_4 e^{-\gamma_2 k}$, $s_{ak} \triangleq |s_k|$, $\beta(s_{ak}) = \big(\frac{10(\epsilon_2 - s_{ak})^3}{\epsilon_2^3} - \frac{15(\epsilon_2 - s_{ak})^4}{\epsilon_2^4} + \frac{6(\epsilon_2 - s_{ak})^5}{\epsilon_2^5}\big)\vartheta$,

$$\vartheta = \begin{cases} 0, & s_{ak} > \epsilon_2, \\ 1, & otherwise. \end{cases} \tag{17}$$

$sat(\boldsymbol{p}_{k-1}) \triangleq \big(sat(p_{1,k-1}), sat(p_{2,k-1})\big)^T$, and

$$sat(p_{i,k-1}) \triangleq \begin{cases} \bar{p}sgn(p_{i,k-1}), & |p_{i,k-1}| > \bar{p}, \\ p_{i,k-1}, & otherwise. \end{cases} \quad (i = 1, 2)$$

with $\bar{p}$ the bound of elements in vector $\boldsymbol{p}$, $\mu > 0, \epsilon_2 > 0, \epsilon_3 > 0, \epsilon_4 > 0, \gamma_1 > 0, \gamma_2 > 0$.

Remark 1: In order to suppress the possible flutter, (15) is a substitute for (18), which is designed according to Sontag formula.

$$u_{ok} = \begin{cases} -(\mu + \frac{a_k + \sqrt{a_k^2 + b_k^4}}{b_k^2})b_k & b_k \neq 0 \\ -\mu b_k & b_k = 0 \end{cases} \tag{18}$$

IV. CONVERGENCE ANALYSIS

In this subsection, we will analyze stability and error convergence of the closed-loop system. Here we present the main result in the following.

Theorem 1: Given the dynamic system (1) satisfying Assumptions 1 and 2, the proposed suboptimal learning controller given in (14) - (16), ensures

$$|s_k(t)| \leq \epsilon_2, \quad \forall t \in [0, T] \tag{19}$$

and

$$|e_{1,k}^{(i)}(t)| \leq (2\lambda)^i\frac{\epsilon_2}{\lambda^{n-1}}, \quad \forall t \in [0, T],\ i = 0, 1, \ldots, n-1$$

hold as the iteration number increases, and all system variables in the closed-loop system are guaranteed to be bounded.

Proof: part i: Test of control Lyapunov function

Through direct calculation, we have

$$b_k = \frac{\partial(\frac{1}{2}s_k^2 + \underline{\boldsymbol{e}}_k^T P\underline{\boldsymbol{e}}_k)}{\partial\boldsymbol{e}_k^T}\boldsymbol{\beta}_g^T = g_k s_k. \tag{20}$$

Since $g_k > 0$, when $b_k = 0$, from(20) we conclude that $s_k = 0$, and hence that

$$\dot{\underline{\boldsymbol{e}}}_k = [e_{2,k}, \ldots, e_{n-1,k}, -\sum_{i=1}^{n-1} c_i e_{i,k}]^T = A\underline{\boldsymbol{e}}_k. \tag{21}$$

From (21), we obtain

$$\begin{aligned} a_k &= \frac{\partial(\underline{\boldsymbol{e}}_k^T P \underline{\boldsymbol{e}}_k)}{\partial \underline{\boldsymbol{e}}_k^T}\boldsymbol{\beta}_e = \frac{\partial(\underline{\boldsymbol{e}}_k^T P \underline{\boldsymbol{e}}_k)}{\partial \underline{\boldsymbol{e}}_k^T}\dot{\underline{\boldsymbol{e}}}_k = \frac{d(\underline{\boldsymbol{e}}_k^T P \underline{\boldsymbol{e}}_k)}{dt} \\ &= \underline{\boldsymbol{e}}_k^T (A^T P + PA)\underline{\boldsymbol{e}}_k = -\underline{\boldsymbol{e}}_k^T Q \underline{\boldsymbol{e}}_k. \end{aligned}$$

Therefore, from $\boldsymbol{e}_k \neq 0$ and $b_k = 0$ we get $a_k < 0$, i.e., V_k is a control Lyapunov function.

part ii: Analysis of error convergence

Denoting $V_{2,k} = V_k + \frac{1}{2(1-\phi)} \int_{t-\tau(t)}^{t} \boldsymbol{e}_k^T \boldsymbol{e}_k d\sigma$, we choose a Lyapunov functional at the kth iteration as

$$L_k = V_{2,k} + \frac{1}{2\gamma_1} \int_0^t \tilde{\boldsymbol{p}}_k^T \tilde{\boldsymbol{p}}_k d\sigma, \tag{22}$$

with $\tilde{\boldsymbol{p}}_k = \boldsymbol{p} - \boldsymbol{p}_k$.

Firstly, let us examine the finiteness of $L_0(t)$. Taking the derivative of $V_{2,k}$, from (13) we obtain

$$\dot{V}_{2,k} \leq a_k + b_k(u_k + \boldsymbol{p}^T \boldsymbol{\varphi}_k) + \frac{1}{2(1-\phi)} \boldsymbol{e}_k^T(t)\boldsymbol{e}_k(t). \tag{23}$$

When $|s_k| > \epsilon_2$, $\epsilon_1 \beta(s_{ak}) = 0$ holds, so we can further assert that

$$a_k - \frac{a_k + \sqrt{a_k^2 + b_k^4}}{b_k^2 + \epsilon_1 \beta(s_{ak})})b_k^2 \leq -\sqrt{a_k^2 + b_k^4} \tag{24}$$

and

$$\frac{1}{2(1-\phi)} \boldsymbol{e}_k^T(t)\boldsymbol{e}_k(t)(1 - \frac{2 s_k b_k}{g_k(s_k^2 + \epsilon^2)}) \leq 0 \tag{25}$$

hold. On account of (24) and (25), substituting (14) to (23) yields

$$\dot{V}_{2,k} \leq -\mu b_k^2 + b_k \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k, \tag{26}$$

which implies that

$$\dot{V}_{2,0} \leq -\mu b_0^2 + b_0 \tilde{\boldsymbol{p}}_0^T \boldsymbol{\varphi}_0. \tag{27}$$

Now we take the time derivative of $\frac{1}{2\gamma_1} \int_0^t \tilde{\boldsymbol{p}}_k^T \tilde{\boldsymbol{p}}_k d\sigma$, the 2rd term on the left side of the equation (22), we obtain

$$\begin{aligned} &(\frac{1}{2\gamma_1} \int_0^t \tilde{\boldsymbol{p}}_k^T \tilde{\boldsymbol{p}}_k d\sigma)' \\ &= \frac{1}{2\gamma_1} \tilde{\boldsymbol{p}}_k^T (\boldsymbol{p} - \boldsymbol{p}_k) \\ &= -\frac{1}{2\gamma_1} \tilde{\boldsymbol{p}}_k^T \boldsymbol{p}_k + \frac{1}{2\gamma_1} (\boldsymbol{p} - \boldsymbol{p}_k)^T \boldsymbol{p} \\ &= -\frac{1}{2\gamma_1} \tilde{\boldsymbol{p}}_k^T \boldsymbol{p}_k + \frac{1}{2\gamma_1} \boldsymbol{p}^T \boldsymbol{p} - \frac{1}{2\gamma_1} \boldsymbol{p}_k^T (\boldsymbol{p}_k + \tilde{\boldsymbol{p}}_k) \\ &= -\frac{1}{\gamma_1} \tilde{\boldsymbol{p}}_k^T \boldsymbol{p}_k + \frac{1}{2\gamma_1} \boldsymbol{p}^T \boldsymbol{p} - \frac{1}{2\gamma_1} \boldsymbol{p}_k^T \boldsymbol{p}_k. \end{aligned} \tag{28}$$

When $k = 0$, by (16), we know $\boldsymbol{p}_0 = \gamma_1 b_0 \boldsymbol{\psi}_0$. From this and (28), we have

$$(\frac{1}{2\gamma_1} \int_0^t \tilde{\boldsymbol{p}}_0^T \tilde{\boldsymbol{p}}_0 d\sigma)' = -\tilde{\boldsymbol{p}}_0^T b_0 \boldsymbol{\psi}_0 + \frac{1}{2\gamma_1} \boldsymbol{p}^T \boldsymbol{p} - \frac{1}{2\gamma_1} \boldsymbol{p}_0^T \boldsymbol{p}_0. \tag{29}$$

On account of (26) and (29), when $|s_k| > \epsilon_2$, we have

$$\dot{L}_0(t) \leq -\mu b_0^2 + \frac{1}{2\gamma_1} \boldsymbol{p}^T \boldsymbol{p} - \frac{1}{2\gamma_1} \boldsymbol{p}_0^T \boldsymbol{p}_0 \leq \frac{1}{2\gamma_1} \boldsymbol{p}^T \boldsymbol{p}. \tag{30}$$

Then by (30), we can easily draw a conclusion that

$$0 \leq L_0(T) < +\infty. \tag{31}$$

Next, let us consider the difference of $L_k(t)$ between two adjacent iterations. While $k > 0$,

$$\begin{aligned} &L_k - L_{k-1} \\ &\leq V_{2,k}(0) - \int_0^t \left(\mu b_k^2 + \sqrt{a_k^2 + b_k^4}\right) d\sigma + \int_0^t b_k \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k d\sigma \\ &\quad - V_{2,k-1} + \frac{1}{2\gamma_1} \int_0^t (\tilde{\boldsymbol{p}}_k^T \tilde{\boldsymbol{p}}_k - \tilde{\boldsymbol{p}}_{k-1}^T \tilde{\boldsymbol{p}}_{k-1}) d\tau(t). \end{aligned} \tag{32}$$

Applying (16) and the property $(\boldsymbol{p} - (\boldsymbol{p}_{k-1}))^T (\boldsymbol{p} - (\boldsymbol{p}_{k-1}) \geq (\boldsymbol{p} - sat(\boldsymbol{p}_{k-1}))^T (\boldsymbol{p} - sat(\boldsymbol{p}_{k-1})$, we have

$$\begin{aligned} &\frac{1}{2\gamma_1} (\tilde{\boldsymbol{p}}_k^T \tilde{\boldsymbol{p}}_k - \tilde{\boldsymbol{p}}_{k-1}^T \tilde{\boldsymbol{p}}_{k-1}) + b_k \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k \\ &\leq b_k \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k + \frac{1}{2\gamma_1} \big((\boldsymbol{p} - \boldsymbol{p}_k)^T (\boldsymbol{p} - \boldsymbol{p}_k) \\ &\quad - (\boldsymbol{p} - sat(\boldsymbol{p}_{k-1}))^T (\boldsymbol{p} - sat(\boldsymbol{p}_{k-1})\big) \\ &= \frac{1}{2\gamma_1} (2\boldsymbol{p} - \boldsymbol{p}_k - sat(\boldsymbol{p}_k))^T (sat(\boldsymbol{p}_k) - \boldsymbol{p}_k\big) + b_k \tilde{\boldsymbol{p}}_k^T \boldsymbol{\varphi}_k \\ &\leq \frac{1}{\gamma_1} (\boldsymbol{p} - \boldsymbol{p}_k)^T (sat(\boldsymbol{p}_k) - \boldsymbol{p}_k + \gamma_1 b_k \boldsymbol{\varphi}_k\big) \\ &= 0. \end{aligned} \tag{33}$$

Combing (33) with (32) yields

$$\begin{aligned} &L_k - L_{k-1} \\ &\leq V_{2,k}(0) - \int_0^t \left(\mu b_k^2 + \sqrt{a_k^2 + b_k^4}\right) d\sigma - V_{2,k-1}. \end{aligned} \tag{34}$$

It follows that

$$\begin{aligned} L_k(t) &\leq V_{2,k}(0) - \int_0^t \left(\mu b_k^2 + \sqrt{a_k^2 + b_k^4}\right) d\sigma \\ &\quad + \frac{1}{2\gamma_1} \int_0^t \tilde{\boldsymbol{p}}_{k-1}^T \tilde{\boldsymbol{p}}_{k-1} d\sigma \\ &\leq L_{k-1}(T) - \int_0^t \left(\mu b_k^2 + \sqrt{a_k^2 + b_k^4}\right) d\sigma \end{aligned} \tag{35}$$

Under the alignment condition, we have $\boldsymbol{x}_{k-1}(T) = \boldsymbol{x}_k(0)$ and $\boldsymbol{x}_d(T) = \boldsymbol{x}_d(0)$. Hence, we can obviously obtain $\boldsymbol{e}_k(0) = \boldsymbol{e}_{k-1}(T)$. Applying this conclusion, from (34) we also can get

$$L_k(T) - L_{k-1}(T) \leq -\int_0^T \left(\mu b_k^2 + \sqrt{a_k^2 + b_k^4}\right) d\sigma, \tag{36}$$

which further yields

$$L_k(T) \leq L_{k-1}(T) \leq \ldots \leq L_0(T). \tag{37}$$

Combining (35) with (37) gives

$$L_k(t) \leq L_0(T) - \int_0^t \left(\mu b_k^2 + \sqrt{a_k^2 + b_k^4}\right) d\sigma \tag{38}$$

Owing to the property (31), it follows from (38) that

$$0 \leq L_k(t) < +\infty. \tag{39}$$

Therefore, by the definition of L_k, we can draw a conclusion that s_k and $\underline{\boldsymbol{e}}_k$ are bounded, which furthermore leads to the boundedness of $\boldsymbol{e}_k$, $\|\dot{\boldsymbol{e}}_k\|$. Therefore, we assert that $|\dot{s}_k| < +\infty$ while $|s_k| > \epsilon_2$, which implies that s_k is continuous while $|s_k| > \epsilon_2$. According to the above-mentioned facts, all other system signals can easily be proved to be bounded. From (36) and (37), we obtain

$$L_k(T) \leq L_0(T) - \underline{g}^2 \sum_{i=1}^{k} \int_0^T s_i^2(\sigma) d\sigma. \tag{40}$$

Suppose that after k iterations, there still exists $|s_k(t_\epsilon)| > \epsilon_2$ at any time point $t_\epsilon \in (0, T)$. By the continuity of s_k, there must exist a little positive constant t_δ, satisfying that $|s_k(t)| > \epsilon_2$ holds for $t \in [t_\epsilon - \frac{t_\delta}{2}, t_\epsilon + \frac{t_\delta}{2}]$. From this and (40), we obtain

$$L_k(T) < L_0(T) - \mu k \epsilon_2^2 t_\delta \underline{g}^2. \tag{41}$$

From this inequality we can conclude that

$$L_k(T) < 0 \tag{42}$$

holds while $k > \frac{L_0(T)}{\mu \epsilon_2^2 t_\delta \underline{g}^2}$. It is easily seen that (42) is contrary to the positiveness of L_k. Hence, while $k > \frac{L_0(T)}{\mu \epsilon_2^2 t_\delta \underline{g}^2}$,

$$|s_k(t)| \leq \epsilon_2, t \in (0, T). \tag{43}$$

Similarly, we can prove $|s_k(t)| \leq \epsilon_2$ holds for $t = 0$ and $t = T$. Hence, $|s_k(t)| \leq \epsilon_2, \forall t \in [0, T]$ holds as the iteration number increases, which further implies

$$|e_{1,k}^{(i)}(t)| \leq (2\lambda)^i \frac{\epsilon_2}{\lambda^{n-1}}, \quad i = 0, 1, \ldots, n-1 \tag{44}$$

as the iteration number increases [40]. ■

In view of (44), through choosing an appropriate small positive number ϵ_2, we can get the pre-specified control precision.

V. ILLUSTRATIVE EXAMPLE

Consider the following system:

$$\begin{cases} \dot{x}_{1,k} = x_{2,k}, \\ \dot{x}_{2,k} = f(\boldsymbol{x}_k) + e^{-\theta(x_{1,k}^2(t-\tau(t)) + x_{2,k}^2(t-\tau(t)))} \\ \qquad\qquad + g(\boldsymbol{x}_k)\big(u_k + \vartheta(t)\psi(\boldsymbol{x}_k, t)\big), \\ [x_{1,0}(t), x_{2,0}(t)]^T = [1.5, 0.2], \quad t \in [-\tau_{\max}, 0], \end{cases} \tag{45}$$

where $\theta = |\cos(2t)|$, $\tau(t) = 1 - 0.5\sin^2 t$, $f(\boldsymbol{x}_k) = -0.1x_{2,k} - 0.1x_{1,k}^3$, $g(\boldsymbol{x}_k) = 1 + 0.1x_{1,k}^2$, $\vartheta = 12$, $\psi(\boldsymbol{x}_k, t) = \frac{\cos t}{1+0.1x_{1,k}^2}$, $\tau_{\max} = 1$ and $\dot{\tau}(t) \leq 0.5$. We can easily verify that system (45) satisfies Assumptions 1 and 2. The control objective is to make the system state $[x_{1,k}, x_{2,k}]^T$ track its reference trajectory $[x_{1,d}, x_{2,d}]^T = [\cos(\pi t), -\pi \sin(\pi t)]^T$ over $[0, T]$.

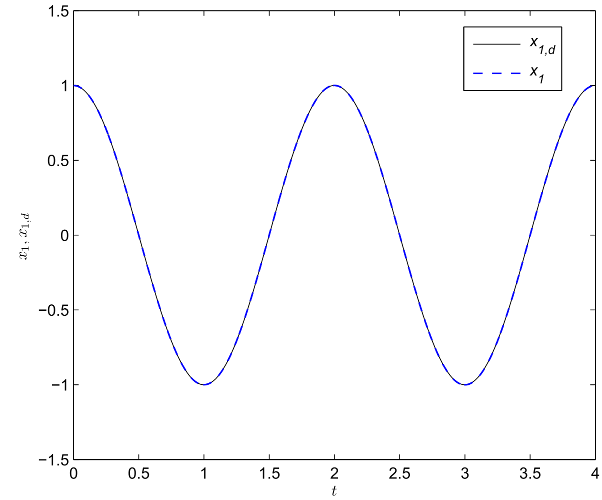

FIGURE 1. System state x_1 and its reference signal $x_{1,d}$.

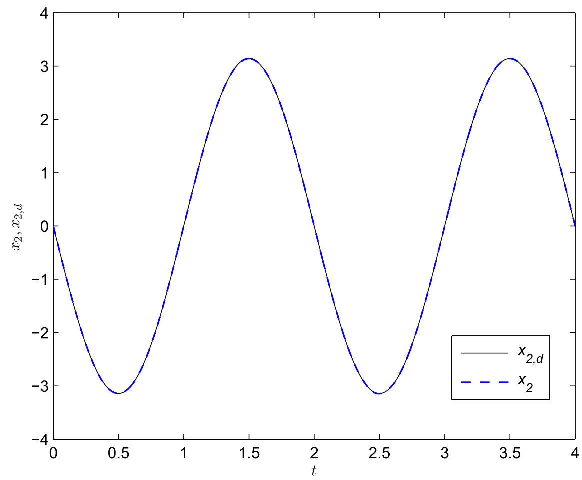

FIGURE 2. System state x_2 and its reference signal $x_{2,d}$.

Choosing $V_k = \frac{1}{2}s_k^2 + e_{1,k}^2$ with $s_k = 2e_{1,k} + e_{2,k}$, the learning control law (14) is implemented with $T = 4$, $\mu = 5, \gamma_1 = 2, \epsilon_1 = \epsilon_2 = 0.001 + 1.5e^{-0.2k}, c_1 = 2, \bar{p} = 30$. After 35 cycles, the simulation results are shown in Figs. 1–7. As shown in Figs. 1–2, x_1 and x_2 respectively follow their reference trajectories over $[0, T]$ at the 35th iteration. Figs. 3–4 show the state tracking error profiles over $[0, T]$ at the 35th iteration. According to Figs. 1–4, we conclude that $\boldsymbol{x}_k(t)$ can precisely track $\boldsymbol{x}_d(t)$ over [0, T] as the iteration number increases. The control input signal at the 35th iteration is shown in Fig. 5. In Figs. 6–7, profile ilc2 represents the error convergence of suboptimal ILC algorithm proposed in this paper, where $|e1|_{sup} \triangleq \max_{t\in[0,T]} |e_{1k}(t)|$, $|e2|_{sup} \triangleq \max_{t\in[0,T]} |e_{2k}(t)|$.

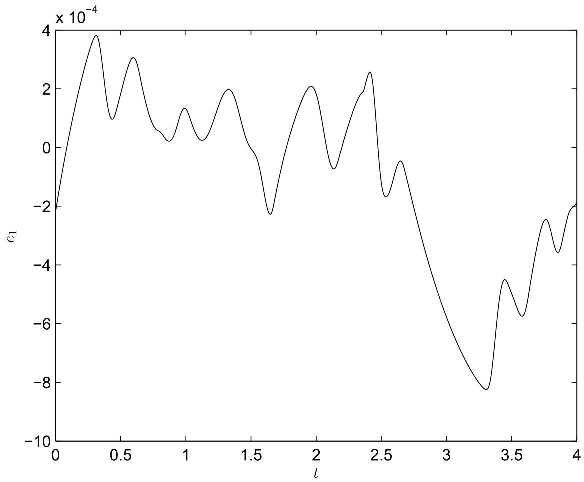

FIGURE 3. State error e_1.

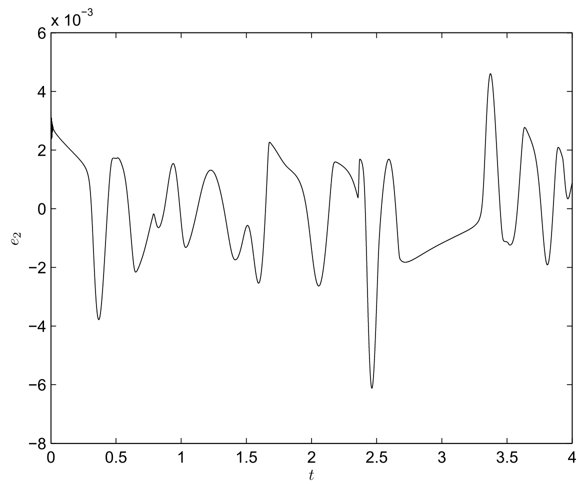

FIGURE 4. State error e_2.

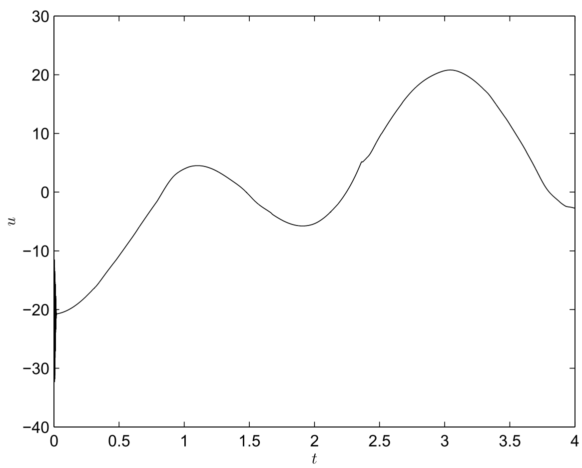

FIGURE 5. Control input.

For comparison, a non-optimal ILC law is given as follows:

$$u_k = -\gamma_3 \boldsymbol{e}_k^T P\boldsymbol{b} - \frac{1}{g_k}(f(\boldsymbol{x}_k) + \alpha_1 e_{1,k} + \alpha_2 e_{2,k}) - \boldsymbol{p}_{n,k}^T \boldsymbol{\varphi}_{n,k}, \tag{46}$$

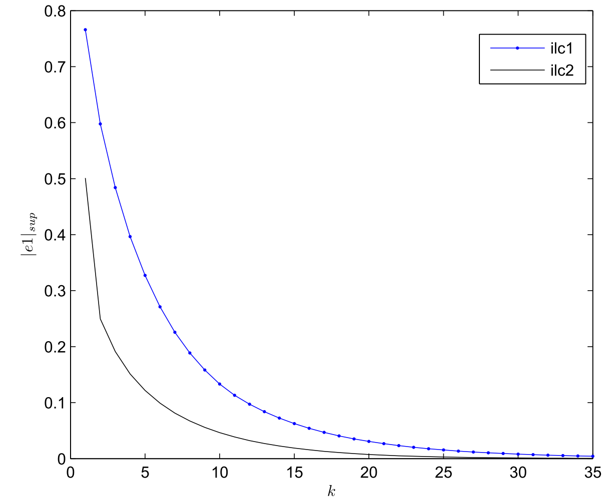

FIGURE 6. Profile of e_{1k} convergence.

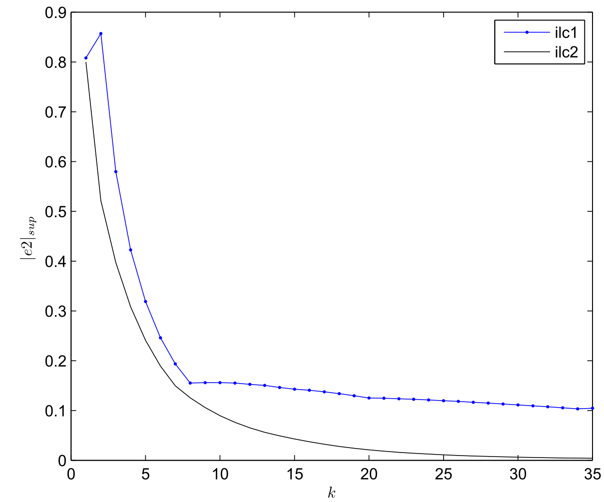

FIGURE 7. Profile of e_{2k} convergence.

$$\boldsymbol{p}_{n,k} = sat(\boldsymbol{p}_{n,k-1}) + \gamma_4 \boldsymbol{e}_k^T P\boldsymbol{b}\boldsymbol{\varphi}_{n,k}, \boldsymbol{p}_{n,-1} = 0, \tag{47}$$

in which $\boldsymbol{\varphi}_k = (\boldsymbol{e}_k^T P\boldsymbol{b}/g_k, 1/g_k, \boldsymbol{\psi}_k)^T, \alpha_1 = 2, \alpha_2 = 1, \gamma_3 = 5, \gamma_4 = 2$ and

$$P = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}. \tag{48}$$

For

$$A = \begin{pmatrix} 0 & 1 \\ -\alpha_1 & -\alpha_2 \end{pmatrix}, \quad Q = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \tag{49}$$

$A^T P + PA = -Q$ holds. In Figs. 6–7, profile ilc1 represents the error convergence of normal ILC algorithm given in (46). We see that both ilc1 and ilc2 approach zero as the iteration number increases, and the latter converges to zero faster than the former. This means that the suboptimal ILC algorithm proposed in this paper has a higher converge speed. The simulation results effectively verify the theoretical analysis in this paper.

VI. CONCLUSION

This paper has proposed a suboptimal iterative learning control scheme for nonlinearly parametric time-delay systems under alignment condition. The controller is designed by integrating adaptive learning control with suboptimal control. While the closed-loop system operates as the iteration number increases, the filtering error may converge to a pre-specified neighborhood of the origin at a higher convergence speed, and the system state can precisely track its reference signal over the full time interval. Simulation results show that, compared with the traditional non-optimal adaptive learning control, our algorithm can improve the convergence speed.

REFERENCES

[1] C.-J. Chien, C.-T. Hsu, and C.-Y. Yao, "Fuzzy system-based adaptive iterative learning control for nonlinear plants with initial state errors," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 5, pp. 724–732, Oct. 2004.

[2] A. Tayebi, "Adaptive iterative learning control for robot manipulators," *Automatica*, vol. 40, no. 7, pp. 1195–1203, Jul. 2004.

[3] L. Zhang and S. Liu, "Basis function based adaptive iterative learning control for non-minimum phase systems," *Acta Automat. Sinica*, vol. 40, no. 12, pp. 2716–2725, Dec. 2014.

[4] H. B. Bi, M. X. Sun, and J. Q. Chen, "Monotonic convergence of feedback-aided iterative learning control algorithms in the sense of Lebesgue-p norm," *Control Theory Appl.*, vol. 33, no. 7, pp. 965–972, Jul. 2016.

[5] X.-H. Bu and Z.-S. Hou, "Stability of iterative learning control with data dropouts via asynchronous dynamical system," *Int. J. Autom. Comput.*, vol. 8, no. 1, pp. 29–36, Feb. 2011.

[6] T. J. Hu, K. H. Low, L. Shen, and X. Xu, "Effective phase tracking for bioinspired undulations of robotic fish models: A learning control approach," *IEEE/ASME Trans. Mechatronics*, vol. 19, no. 1, pp. 191–200, Feb. 2014.

[7] Y. Wang, E. Dassau, and F. J. Doyle, III, "Closed-loop control of artificial pancreatic β-cell in type 1 diabetes mellitus using model predictive iterative learning control," *IEEE Trans. Biomed. Eng.*, vol. 57, no. 2, pp. 211–219, Feb. 2010.

[8] X. Dai, S. Tian, Y. Peng, and W. Luo, "Closed-loop P-type iterative learning control of uncertain linear distributed parameter systems," *IEEE/CAA J. Automatica Sinica*, vol. 1, no. 3, pp. 267–273, Jul. 2014.

[9] D. Q. Huang and J.-X. Xu, "Steady-state iterative learning control for a class of nonlinear PDE processes," *J. Process Control*, vol. 21, no. 8, pp. 1155–1163, Sep. 2011.

[10] J.-X. Xu and R. Yan, "On initial conditions in iterative learning control," *IEEE Trans. Autom. Control*, vol. 50, no. 9, pp. 1349–1354, Sep. 2005.

[11] B. Porter and S. S. Mohamed, "Iterative learning control of partially irregular multivariable plants with initial impulsive action," *Int. J. Syst. Sci.*, vol. 22, no. 3, pp. 447–454, 1991.

[12] X. Ruan and Z. Z. Bien, "Pulse compensation for PD-type iterative learning control against initial state shift," *Int. J. Syst. Sci.*, vol. 43, no. 11, pp. 2172–2184, Nov. 2012.

[13] M. Sun and D. Wang, "Iterative learning control with initial rectifying action," *Automatica*, vol. 38, no. 7, pp. 1177–1182, 2002.

[14] X.-D. Li, M.-M. Lv, and J. K. L. Ho, "Adaptive ILC algorithms of nonlinear continuous systems with non-parametric uncertainties for non-repetitive trajectory tracking," *Int. J. Syst. Sci.*, vol. 47, no. 10, pp. 2279–2289, 2016.

[15] Q. Z. Yan, M. X. Sun, and H. Li, "Iterative learning control for nonlinear uncertain systems with arbitrary initial state," *Acta Automat. Sinica*, vol. 42, no. 4, pp. 545–555, Apr. 2016.

[16] K.-H. Park, "An average operator-based PD-type iterative learning control for variable initial state error," *IEEE Trans. Autom. Control*, vol. 50, no. 6, pp. 865–869, Jun. 2005.

[17] M.-X. Sun and Q.-Z. Yan, "Error tracking of iterative learning control systems," *Acta Automat. Sinica*, vol. 39, no. 3, pp. 251–262, Mar. 2013.

[18] Q.-Z. Yan and M.-X. Sun, "Error trajectory tracking by robust learning control for nonlinear systems," *Control Theory Appl.*, vol. 30, no. 1, pp. 23–30, Jan. 2013.

[19] Q. Z. Yan, M. X. Sun, and H. Li, "Consensus-error-tracking learning control for nonparametric uncertain multi-agent systems," *Control Theory Appl.*, vol. 33, no. 6, pp. 793–799, Jun. 2016.

[20] J.-X. Xu, X. Jin, and D. Huang, "Composite energy function-based iterative learning control for systems with nonparametric uncertainties," *Int. J. Adapt. Control Signal Process.*, vol. 28, no. 1, pp. 1–13, Jan. 2014.

[21] X. Jin and J.-X. Xu, "Iterative learning control for output-constrained systems with both parametric and nonparametric uncertainties," *Automatica*, vol. 49, no. 8, pp. 2508–2516, Aug. 2013.

[22] K. S. Lee, S. H. Bang, and K. S. Chang, "Feedback-assisted iterative learning control based on an inverse process model," *J. Process Control*, vol. 4, no. 2, pp. 77–89, May 1994.

[23] Y. Chen, Z. Gong, and C. Wen, "Analysis of a high-order iterative learning control algorithm for uncertain nonlinear systems with state delays," *Automatica*, vol. 34, no. 3, pp. 345–353, Mar. 1998.

[24] K. H. Park, Z. Bien, and D. H. Hwang, "Design of an iterative learning controller for a class of linear dynamic systems with time delay," *IEE Proc.-Control Theory Appl.*, vol. 145, no. 6, pp. 507–512, Nov. 1998.

[25] M. Sun and D. Wang, "Initial condition issues on iterative learning control for non-linear systems with time delay," *Int. J. Syst. Sci.*, vol. 32, no. 11, pp. 1365–1375, Jul. 2001.

[26] X.-D. Li, T. W. S. Chow, and J. K. L. Ho, "2-D system theory based iterative learning control for linear continuous systems with time delays," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 52, no. 7, pp. 1421–1430, Jul. 2005.

[27] D. Meng, Y. Jia, J. Du, and F. Yu, "Robust iterative learning control design for uncertain time-delay systems based on a performance index," *IET Control Theory Appl.*, vol. 4, no. 5, pp. 759–772, May 2010.

[28] J.-M. Li, Y.-L. Wang, and X.-M. Li, "Adaptive iterative learning control for nonlinear parameterized-systems with unknown time-varying delays," *Control Theory Appl.*, vol. 28, no. 6, pp. 861–868, Jun. 2011.

[29] R. Zhang, Z. Hou, R. Chi, and H. Ji, "Adaptive iterative learning control for nonlinearly parameterised systems with unknown time-varying delays and input saturations," *Int. J. Control*, vol. 88, no. 6, pp. 1133–1141, 2015.

[30] W. Chen and L. Zhang, "Adaptive iterative learning control for nonlinearly parameterized systems with unknown time-varying delays," *Int. J. Control, Autom., Syst.*, vol. 8, no. 2, pp. 177–186, 2010.

[31] Q. Wei, R. Song, B. Li, and X. Lin, "A novel policy iteration-based deterministic Q-learning for discrete-time nonlinear systems," *Sci. China*, vol. 58, no. 12, pp. 1–15, Dec. 2015.

[32] N. Amann, D. H. Owens, and E. Rogers, "Iterative learning control using optimal feedback and feedforward actions," *Int. J. Control*, vol. 65, no. 2, pp. 277–293, 1996.

[33] J.-X. Xu and Y. Tan, "Robust optimal design and convergence properties analysis of iterative learning control approaches," *Automatica*, vol. 38, no. 11, pp. 1867–1880, Nov. 2002.

[34] S. Gunnarsson and M. NorrlöK, "On the design of ILC algorithms using optimization," *Automatica*, vol. 37, no. 12, pp. 2011–2016, Dec. 2001.

[35] R.-H. Chi and Z.-S. Hou, "Dual-stage optimal iterative learning control for nonlinear non-affine discrete-time systems," *Acta Automat. Sinica*, vol. 33, no. 10, pp. 1061–1065, Oct. 2007.

[36] S. Yang, Z. Qu, X. Fan, and X. Nian, "Novel iterative learning controls for linear discrete-time systems based on a performance index over iterations," *Automatica*, vol. 44, no. 5, pp. 1366–1372, May 2008.

[37] P. Janssens, G. Pipeleers, and J. Swevers, "A data-driven constrained norm-optimal iterative learning control framework for LTI systems," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 2, pp. 546–551, Mar. 2013.

[38] J.-X. Xu and Y. Tan, "A suboptimal learning control scheme for non-linear systems with time-varying parametric uncertainties," *Optim. Control Appl. Methods*, vol. 22, no. 3, pp. 111–126, May 2001.

[39] Q. Yan and M. Sun, "Suboptimal learning control for nonlinear systems with both parametric and nonparametric ncertainties," *Acta Automat. Sinica*, vol. 41, no. 9, pp. 1659–1668, Sep. 2015.

[40] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1991.

YOUFANG YU received the B.S. degree in electronic engineering and the M.S. degree in agricultural electrification and automation from Zhejiang University, Hangzhou, in 1999 and 2006, respectively. From 1999 to 2003, she was a Lecturer with the Anhui Vocational and Technical College, Hefei. From 2006 to 2008, she became an Electronic Engineer with the Research and Development Department, Zhejiang Jincheng Technology Development Co. Ltd., Hangzhou. Since 2008, she has been a Lecturer with the Applied Engineering College, Zhejiang Business College, Hangzhou. Her current research interests include automatic control, industrial automation, electronic technology, and biomass technology.

JUN WAN is currently an Associate Professor with the Zhejiang University of Water Resources and Electric Power. His main research interests include electrical equipment and its control and analysis.

HONGBO BI received the B.S. degree in information and computation science from the North China University of Water Resources and Electric Power, Zhengzhou, in 2007, and the Ph.D. degree in control theory and control engineering from the Zhejiang University of Technology, Hangzhou, in 2015. Since 2015, he has been with the College of Electrical and Information Engineering, Quzhou University, Quzhou, where he holds a lecturer position. His current research interests include iterative learning control and iterative learning identification.