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Dual Hesitant Bipolar Fuzzy Hamacher Prioritized Aggregation Operators in Multiple Attribute Decision Making

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ABSTRACT In this paper, we investigate the dual hesitant bipolar fuzzy multiple attribute decision making problems in which there exists a prioritization relationship over attributes. Then, motivated by the idea of Hamacher operations and prioritized aggregation operators, we have developed some Hamacher prioritized aggregation operators for aggregating dual hesitant bipolar fuzzy information: dual hesitant bipolar fuzzy Hamacher prioritized average operator, dual hesitant bipolar fuzzy Hamacher prioritized geometric operator, dual hesitant bipolar fuzzy Hamacher prioritized weighted average operator, dual hesitant bipolar fuzzy Hamacher prioritized weighted geometric operator. Then, we have utilized these operators to develop some approaches to solve the dual hesitant bipolar fuzzy multiple attribute decision making problems. Finally, a real-world example is then analyzed to illustrate the relevance and effectiveness of the proposed methodology.

INDEX TERMS Multiple attribute decision making (MADM), Bipolar fuzzy set, Dual hesitant bipolar fuzzy set, Dual hesitant bipolar fuzzy Hamacher prioritized weighted average (DHBFHPWA) operator, dual hesitant bipolar fuzzy Hamacher prioritized weighted geometric (DHBFHPWG) operator.

I. INTRODUCTION

Atanassov [1], [2] introduced the concept of intuitionistic fuzzy set(IFS) characterized by a membership function and a non-membership function, which is a generalization of the concept of fuzzy set [3] whose basic component is only a membership function. Xu [4] developed the intuitionistic fuzzy arithmetic aggregation operators. Xu [5] developed some intuitionistic fuzzy geometric aggregation operators. The intuitionistic fuzzy set has received more and more attention since its appearance [6]–[25]. More recently, the bipolar fuzzy set (BFS) [26], [27] has emerged lately as an alternative tool to depict uncertainty in MADM problems. A pair of numbers, namely, the positive membership degree and the negative membership degree, is employed to define an object in a BFS. But different from the IFS, the range of membership degree of the bipolar fuzzy set is $[-1, 1]$. BFSs have been applied in many research areas including but not limited to bipolar logical reasoning and set theory [28], [29], traditional Chinese medicine theory [30], [31], bipolar cognitive mapping [32], [33], computational psychiatry [34], [35], decision analysis and organizational modeling [36], [37], photonics [38], quantum computing [39], [40], biosystem regulation [30], [41], [42], quantum cellular combinatorics [39], physics and philosophy [43] and graph theory [44]–[48]. Recently, Gul [49] defined some bipolar fuzzy aggregations operators, such as, bipolar fuzzy averaging weighted aggregation operators and bipolar fuzzy geometric aggregations operators. Wei *et al.* [50] proposed some hesitant bipolar fuzzy aggregation operators in multiple attribute decision making. Lu *et al.* [51] proposed some bipolar 2-tuple linguistic aggregation operators in multiple attribute decision making.

Xu & Wei [52] defined the dual hesitant bipolar fuzzy sets(DHBFSs) and developed some dual hesitant bipolar fuzzy aggregation operators for multiple attribute decision making. We note that almost all the dual hesitant bipolar fuzzy aggregation operators [52] used in the literature employed the algebraic product or sum of DHBFSs. Constructed on the basis of general t-norm and t-conorm, Hamacher product and Hamacher sum [53] could be applied, respectively, to surrogate the algebraic product and algebraic sum. For studies on Hamacher aggregation operators and their applications, the reader is referred to [54]–[56]. In this study, we consider how to extend Hamacher operators and prioritized aggregation operators to aggregate the dual hesitant bipolar fuzzy information. In order to do so, the remainder of this paper is organized as follows. In the next section, we briefly review the basic concepts of the DHBFSs and the fundamental operational laws of DHBFNs. In Section 3, we develop dual hesitant bipolar fuzzy Hamacher prioritized aggregation operators. In Section 4, models are developed that apply the proposed aggregation operators to solve MADM problems. An illustrative example is analyzed in Section 5. Some remarks are given in Section 6 to conclude the paper.

II. PRELIMINARIES

A. THE BIPOLAR FUZZY SET

In this section, we present a short overview of BFSs [26], [27].

Definition 1 [26], [27]: Let X be a fix set. A BFS is an object having the form

$$B = \left\{ \left\langle x, \left(\mu_B^+ (x), \nu_B^- (x) \right) \right\rangle | x \in X \right\} \tag{1}$$

where the positive membership degree function $\mu_B^+ (x)$: $X \rightarrow [0, 1]$ denotes the satisfaction degree of an element x to the property corresponding to a BFS B and the negative membership degree function $\nu_B^- (x)$: $X \rightarrow [-1, 0]$ denotes satisfaction degree of an element x to some implicit counter property corresponding to a BFS B, respectively, and, for every $x \in X$.

Definition 2 [49]: Some basic operations on BFNs are expressed as follows:

(1) $\tilde{b}_1 \oplus \tilde{b}_2 = \left(\mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+, -\left| \nu_1^- \right| \left| \nu_2^- \right| \right)$;
(2) $\tilde{b}_1 \otimes \tilde{b}_2 = \left(\mu_1^+ \mu_2^+, \nu_1^- + \nu_2^- - \nu_1^- \nu_2^- \right)$;
(3) $\lambda \tilde{b} = \left(1 - \left(1 - \mu^+ \right)^\lambda, -\left| \nu^- \right|^\lambda \right), \lambda > 0$;
(4) $\left(\tilde{b} \right)^\lambda = \left(\left(\mu^+ \right)^\lambda, -1 + \left| 1 + \nu^- \right|^\lambda \right), \lambda > 0$;
(5) $\tilde{b}^c = \left(1 - \mu^+, \left| \nu^- \right| - 1 \right)$;
(6) $\tilde{b}_1 \subseteq \tilde{b}_2$, if and only if $\mu_1^+ \leq \mu_2^+$ and $\nu_1^- \geq \nu_2^-$;
(7) $\tilde{b}_1 \cup \tilde{b}_2 = \left(\max \left\{ \mu_1^+, \mu_2^+ \right\}, \min \left\{ \nu_1^-, \nu_2^- \right\} \right)$;
(8) $\tilde{b}_1 \cap \tilde{b}_2 = \left(\min \left\{ \mu_1^+, \mu_2^+ \right\}, \max \left\{ \nu_1^-, \nu_2^- \right\} \right)$.

Based on the Definition 2, we can introduce the **Theorem 1** easily.

Theorem 1 [49]: Let $\tilde{b}_1 = \left(\mu_1^+, \nu_1^- \right)$ and $\tilde{b}_2 = \left(\mu_2^+, \nu_2^- \right)$ be two BFNs, $\lambda, \lambda_1, \lambda_2 > 0$, then

(1) $\tilde{b}_1 \oplus \tilde{b}_2 = \tilde{b}_2 \oplus \tilde{b}_1$;
(2) $\tilde{b}_1 \otimes \tilde{b}_2 = \tilde{b}_2 \otimes \tilde{b}_1$;
(3) $\lambda \left(\tilde{b}_1 \oplus \tilde{b}_2 \right) = \lambda \tilde{b}_1 \oplus \lambda \tilde{b}_2$;
(4) $\left(\tilde{b}_1 \otimes \tilde{b}_2 \right)^\lambda = \left(\tilde{b}_1 \right)^\lambda \otimes \left(\tilde{b}_2 \right)^\lambda$;
(5) $\lambda_1 \tilde{b}_1 \oplus \lambda_2 \tilde{b}_1 = \left(\lambda_1 + \lambda_2 \right) \tilde{b}_1$;
(6) $\left(\tilde{b}_1 \right)^{\lambda_1} \otimes \left(\tilde{b}_1 \right)^{\lambda_2} = \left(\tilde{b}_1 \right)^{(\lambda_1 + \lambda_2)}$;
(7) $\left(\left(\tilde{b}_1 \right)^{\lambda_1} \right)^{\lambda_2} = \left(\tilde{b}_1 \right)^{\lambda_1 \lambda_2}$.

B. DUAL HESITANT BIPOLAR FUZZY SET (DHBFS)

In the following, motivated by the bipolar fuzzy set (BFS) [26], [27] and dual hesitant fuzzy set (DHFS) [57], [58], Xu & Wei [52] proposed the dual hesitant bipolar fuzzy sets (DHBFSs).

Definition 3 [52]: Let X be a fixed set, then a dual hesitant bipolar fuzzy set (DHBFS) on X is described as:

$$D = \left(\left\langle x, \mu^+ (x), \nu^- (x) \right\rangle | x \in X \right) \tag{2}$$

where the positive membership degree function $\mu_B^+ (x)$: $X \rightarrow [0, 1]$ denotes some possible satisfaction degree of an element x to the property corresponding to a DHBFS D and the negative membership degree function $\nu_B^- (x)$: $X \rightarrow [-1, 0]$ denotes some possible satisfaction degree of an element x to some implicit counter property corresponding to a DHBFS D, respectively, and, for every $x \in X$, with the conditions:

$$0 \leq \gamma^+ \leq 1, \quad -1 \leq \eta^- \leq 0$$

where $\gamma^+ \in \mu^+ (x)$, $\eta^- \in \nu^- (x)$, $\gamma^{\max} \in \mu^+ (x) = \cup_{\gamma^+ \in \mu^+ (x)} \max \left\{ \gamma^+ \right\}$, $\eta^{\min} \in \nu^- (x) = \cup_{\eta^- \in \nu^- (x)} \min \left\{ \eta^- \right\}$ for all $x \in X$. For convenience, the pair $d (x) = \left(\mu^+ (x), \nu^- (x) \right)$ is called a dual hesitant bipolar fuzzy number (DHBFN) denoted by $d = \left(\mu^+, \nu^- \right)$, with the conditions: $\gamma^+ \in \mu^+ (x)$, $\eta^- \in \nu^- (x)$, $\gamma^{\max} \in \mu^+ (x) = \cup_{\gamma^+ \in \mu^+ (x)} \max \left\{ \gamma^+ \right\}$, $\eta^{\max} \in \nu^- (x) = \cup_{\eta^- \in \nu^- (x)} \max \left\{ \eta^- \right\}$, $0 \leq \gamma^+ \leq 1$, $-1 \leq \eta^- \leq 0$, $0 \leq \gamma^{\max} \leq 1$, $-1 \leq \eta^{\min} \leq 0$.

To compare the DHBFN, Xu & Wei [52] gave the following comparison laws:

Definition 4 [52]: Let $d_i = \left(\mu_i^+, \nu_i^- \right)$ $(i = 1, 2)$ be any two DHBFNs,

$$s (d) = \frac{1}{2} \left(1 + \frac{1}{\# \mu^+} \sum\nolimits_{\gamma^+ \in \mu^+} \gamma^+ + \frac{1}{\# \nu^-} \sum\nolimits_{\eta^- \in \nu^-} \eta^- \right)$$

the score function of $d = \left(\mu^+, \nu^- \right)$, and

$$a (d) = \frac{1}{2} \left(\frac{1}{\# \mu^+} \sum\nolimits_{\gamma^+ \in \mu^+} \gamma^+ - \frac{1}{\# \nu^-} \sum\nolimits_{\eta^- \in \nu^-} \eta^- \right)$$

the accuracy function of $d = \left(\mu^+, \nu^- \right)$, where $\# \mu^+$ and $\# \nu^-$ are the numbers of the elements in μ^+ and ν^- respectively, then

- If $s (d_1) > s (d_2)$, then d_1 is superior to d_2, denoted by $d_1 \succ d_2$;
- If $s (d_1) = s (d_2)$, then
  (1) If $a (d_1) = a (d_2)$, then d_1 is equivalent to d_2, denoted by $d_1 \sim d_2$;
  (2) If $a (d_1) > a (d_2)$, then d_1 is superior to d_2, denoted by $d_1 \succ d_2$.

Then, Xu & Wei [52] defined some new operations on the DHBFN d, d_1 and d_2:

(1) $d^{\lambda} = \cup_{\eta^{+}\in\mu^{+},\eta^{-}\in\nu^{-}}\{\{(\eta^{+})^{\lambda}\},\{-1+|1+\eta^{-}|^{\lambda}\}\}$, $\lambda > 0$;

(2) $\lambda d = \cup_{\eta^{+}\in\mu^{+},\eta^{-}\in\nu^{-}}\{\{1-(1-\mu^{+})^{\lambda}\},\{-|\eta^{-}|^{\lambda}\}\}$, $\lambda > 0$;

(3) $d_1 \oplus d_2 = \cup_{\gamma_1^{+}\in\mu_1^{+},\gamma_2^{+}\in\mu_2^{+},\eta_1^{-}\in\nu_1^{-},\eta_2^{-}\in\nu_2^{-}}\{\{\gamma_1^{+}+\gamma_2^{+}-\gamma_1^{+}\gamma_2^{+}\},\{-|\eta_1^{-}||\eta_2^{-}|\}\}$;

(4) $d_1 \otimes d_2 = \cup_{\gamma_1^{+}\in\mu_1^{+},\gamma_2^{+}\in\mu_2^{+},\eta_1^{-}\in\nu_1^{-},\eta_2^{-}\in\nu_2^{-}}\{\{\gamma_1^{+}\gamma_2^{+}\},\{\eta_1^{-}+\eta_2^{-}-\eta_1^{-}\eta_2^{-}\}\}$.

C. HAMACHER OPERATIONS OF DUAL HESITANT BIPOLAR FUZZY SETS

Let $d_i = (\mu_i^{+},\nu_i^{-})\ (i=1,2)$ be any two DHBFNs, and $d = (\mu^{+},\nu^{-})$ denote DHBFN. On the basis of general t-norm and t-conorm, Hamacher product and Hamacher sum [53], we define the following basic Hamacher operators of DHBFNs with $\gamma > 0$.

(1) $\lambda d = \cup_{\gamma^{+}\in\mu^{+},\eta^{-}\in\nu^{-}}$

$$\times\left\{\begin{array}{l}\left\{\dfrac{(1+(\gamma-1)\gamma^{+})^{\lambda}-(1-\gamma^{+})^{\lambda}}{(1+(\gamma-1)\gamma^{+})^{\lambda}+(\gamma-1)(1-\gamma^{+})^{\lambda}}\right\},\\ \left\{\dfrac{-\gamma|\eta^{-}|^{\lambda}}{(1+(\gamma-1)(1+\eta^{-}))^{\lambda}+(\gamma-1)|\eta^{-}|^{\lambda}}\right\}\end{array}\right\},\quad \lambda>0;$$

(2) $d^{\lambda} = \cup_{\gamma^{+}\in\mu^{+},\eta^{-}\in\nu^{-}}$

$$\times\left\{\begin{array}{l}\left\{\dfrac{\gamma(\gamma^{+})^{\lambda}}{(1+(\gamma-1)(1-\gamma^{+}))^{\lambda}+(\gamma-1)(\gamma^{+})^{\lambda}}\right\},\\ \left\{-\dfrac{(1+(\gamma-1)|\eta^{-}|)^{\lambda}-(1+\eta^{-})^{\lambda}}{(1+(\gamma-1)|\eta^{-}|)^{\lambda}+(\gamma-1)(1+\eta^{-})^{\lambda}}\right\}\end{array}\right\},\quad \lambda>0;$$

(3) $d_1 \oplus d_2 = \cup_{\gamma_1^{+}\in\mu_1^{+},\gamma_2^{+}\in\mu_2^{+},\eta_1^{-}\in\nu_1^{-},\eta_2^{-}\in\nu_2^{-}}$

$$\times\left\{\begin{array}{l}\left\{\dfrac{\gamma_1^{+}+\gamma_2^{+}-\gamma_1^{+}\gamma_2^{+}-(1-\gamma)\gamma_1^{+}\gamma_2^{+}}{1-(1-\gamma)\gamma_1^{+}\gamma_2^{+}}\right\},\\ \left\{\dfrac{-\eta_1^{-}\eta_2^{-}}{\gamma+(1-\gamma)(\eta_1^{-}+\eta_2^{-}-\eta_1^{-}\eta_2^{-})}\right\}\end{array}\right\};$$

(4) $d_1 \otimes d_2 = \cup_{\gamma_1^{+}\in\mu_1^{+},\gamma_2^{+}\in\mu_2^{+},\eta_1^{-}\in\nu_1^{-},\eta_2^{-}\in\nu_2^{-}}$

$$\times\left\{\begin{array}{l}\left\{\dfrac{\gamma_1^{-}\gamma_2^{-}}{\gamma+(1-\gamma)(\gamma_1^{-}+\gamma_2^{-}-\gamma_1^{-}\gamma_2^{-})}\right\},\\ \left\{\dfrac{\eta_1^{-}+\eta_2^{-}-\eta_1^{-}\eta_2^{-}-(1-\gamma)\eta_1^{-}\eta_2^{-}}{1-(1-\gamma)\eta_1^{-}\eta_2^{-}}\right\}\end{array}\right\}.$$

III. DUAL HESITANT BIPOLAR FUZZY HAMACHER PRIORITIZED AGGREGATION OPERATORS

A. DUAL HESITANT BIPOLAR FUZZY HAMACHER PRIORITIZED ARITHMETIC AGGREGATION OPERATORS

The prioritized average (PA) operator was originally introduced by Yager [59], which was defined as follows:

Definition 5 [59]: Let $G = \{G_1, G_2, \cdots, G_n\}$ be a collection of attribute and that there is a prioritization between the attribute expressed by the linear ordering $G_1 \succ G_2 \succ G_3 \cdots \succ G_n$, indicate attribute G_j has a higher priority than G_k, if $j < k$. The value $G_j(x)$ is the performance of any alternative x under attribute G_j, and satisfies $G_j(x) \in [0,1]$. If

$$PA(G_j(x)) = \sum_{j=1}^{n} w_j G_j(x) \tag{3}$$

where

$$w_j = \frac{T_j}{\sum_{j=1}^{n} T_j},\quad T_j = \prod_{k=1}^{j-1} G_k(x)\quad (j=2,\cdots,n),\ T_1 = 1.$$

Then PA is called the prioritized average (PA) operator.

The prioritized average [59] operators, however, have usually been used in situations where the input arguments are the exact values. We shall extend the PA operators to accommodate the situations where the input arguments are DHBFNs. In this Section, we shall investigate the PA operator under dual hesitant bipolar fuzzy environments. Based on Definition 5, we give the definition of the dual hesitant bipolar fuzzy Hamacher prioritized average (DHBFHPA) operator as follows:

Let $\tilde{d}_j = (\mu_j^{+},\nu_j^{-})\ (j=1,2,\cdots,n)$ be a collection of DHBFNs. We next establish dual hesitant bipolar fuzzy Hamacher prioritized arithmetic aggregation operators.

Definition 6: The dual hesitant bipolar fuzzy Hamacher prioritized average (DHBFHPA) operator is

$$\begin{aligned}&\mathrm{DHBFHPA}(\tilde{d}_1,\tilde{d}_2,\cdots,\tilde{d}_n)\\ &= \frac{T_1}{\sum_{j=1}^{n}T_j}\tilde{d}_1 \oplus \frac{T_2}{\sum_{j=1}^{n}T_j}\tilde{d}_2 \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^{n}T_j}\tilde{d}_n\\ &= \mathop{\oplus}_{j=1}^{n}\left(\frac{T_j\tilde{d}_j}{\sum_{j=1}^{n}T_j}\right)\end{aligned} \tag{4}$$

where $T_j = \prod_{k=1}^{j-1} s(\tilde{d}_j)\ (j=2,\cdots,n)$, $T_1 = 1$ and $s(\tilde{d}_j)$ is the score values of $\tilde{d}_j\ (j=1,2,\cdots,n)$.

Theorem 2 can be shown by its definition and mathematical induction.

Theorem 2: The DHBFHPA operator returns a DHBFN with (5), as shown at the top of the next page.

We subsequently discuss two special cases of the DHBFHPA operator.

- If $\gamma = 1$, the DHBFHPA operator is equivalent to the dual hesitant bipolar fuzzy prioritized

$$\begin{aligned}
&\text{DHBFHPA}\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= \frac{T_1}{\sum_{j=1}^{n} T_j}\tilde{d}_1 \oplus \frac{T_2}{\sum_{j=1}^{n} T_j}\tilde{d}_2 \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^{n} T_j}\tilde{d}_n \\
&= \mathop{\oplus}_{j=1}^{n}\left(\frac{T_j\tilde{d}_j}{\sum_{j=1}^{n} T_j}\right) = \cup_{\gamma_j^+\in\mu_j^+,\eta_j^-\in\nu_j^-}\left\{\begin{array}{l}\left\{\dfrac{\prod_{j=1}^{n}\left(1+(\gamma-1)\gamma_j^+\right)^{T_j\Big/\sum_{j=1}^{n}T_j}-\prod_{j=1}^{n}\left(1-\gamma_j^+\right)^{T_j\Big/\sum_{j=1}^{n}T_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\gamma_j^+\right)^{T_j\Big/\sum_{j=1}^{n}T_j}+(\gamma-1)\prod_{j=1}^{n}\left(1-\gamma_j^+\right)^{T_j\Big/\sum_{j=1}^{n}T_j}}\right\}, \\ \left\{\dfrac{-\gamma\prod_{j=1}^{n}\left|\eta_j^-\right|^{T_j\Big/\sum_{j=1}^{n}T_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1+\eta_j^-\right)\right)^{T_j\Big/\sum_{j=1}^{n}T_j}+(\gamma-1)\prod_{j=1}^{n}\left|\eta_j^-\right|^{T_j\Big/\sum_{j=1}^{n}T_j}}\right\}\end{array}\right\}
\end{aligned} \tag{5}$$

average (DHBFPA) operator:

$$\begin{aligned}
&\text{DHBFPA}\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= \frac{T_1}{\sum_{j=1}^{n} T_j}\tilde{d}_1 \oplus \frac{T_2}{\sum_{j=1}^{n} T_j}\tilde{d}_2 \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^{n} T_j}\tilde{d}_n \\
&= \mathop{\oplus}_{j=1}^{n}\left(\frac{T_j\tilde{d}_j}{\sum_{j=1}^{n} T_j}\right) \\
&= \cup_{\gamma_j^+\in\mu_j^+,\eta_j^-\in\nu_j^-}\left\{\begin{array}{l}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j^+\right)^{T_j\Big/\sum_{j=1}^{n}T_j}\right\}, \\ \left\{-\prod_{j=1}^{n}\left|\eta_j^-\right|^{T_j\Big/\sum_{j=1}^{n}T_j}\right\}\end{array}\right\}
\end{aligned} \tag{6}$$

- If $\gamma = 2$, the DHBFHPA operator coincides with the dual hesitant bipolar fuzzy Einstein prioritized average (DHBFEPA) operator (7), as shown at the top of the next page.

If we consider the weights of $\tilde{d}_j\,(j = 1, 2, \cdots, n)$, where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of $\tilde{d}_j\,(j = 1, 2, \cdots, n)$ with $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$. Then, based on Definition 5, we give the definition of the dual hesitant bipolar fuzzy Hamacher prioritized weighted average (DHBFHPWA) operator as follows:

Definition 7: The dual hesitant bipolar fuzzy Hamacher prioritized weighted average (DHBFHPWA) operator is

$$\begin{aligned}
&\text{DHBFHPWA}_{\omega}\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= \frac{\omega_1 T_1}{\sum_{j=1}^{n} \omega_j T_j}\tilde{d}_1 \oplus \frac{\omega_2 T_2}{\sum_{j=1}^{n} \omega_j T_j}\tilde{d}_2 \oplus \cdots \oplus \frac{\omega_n T_n}{\sum_{j=1}^{n} \omega_j T_j}\tilde{d}_n \\
&= \mathop{\oplus}_{j=1}^{n}\left(\frac{\omega_j T_j\tilde{d}_j}{\sum_{j=1}^{n} \omega_j T_j}\right)
\end{aligned} \tag{8}$$

where $T_j = \prod_{k=1}^{j-1} s\left(\tilde{d}_j\right)(j = 2, \cdots, n)$, $T_1 = 1$ and $s\left(\tilde{d}_j\right)$ is the score values of $\tilde{d}_j\,(j = 1, 2, \cdots, n)$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of $\tilde{d}_j\,(j = 1, 2, \cdots, n)$ with $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$.

Theorem 3 can be shown by its definition and mathematical induction.

Theorem 3: The DHBFHPWA operator returns a DHBFN with (9), as shown at the top of the next page.

We subsequently discuss two special cases of the DHBFHPA operator.

- If $\gamma = 1$, the DHBFHPWA operator is equivalent to the dual hesitant bipolar fuzzy prioritized weighted average (DHBFPWA) operator (10), as shown at the top of the next page.
- If $\gamma = 2$, the DHBFHPWA operator coincides with the dual hesitant bipolar fuzzy Einstein prioritized weighted average (DHBFEPWA) operator (11), as shown in Section III-B.

$$
\begin{aligned}
&\mathrm{DHBFEPA}\left(\tilde{d}_1,\tilde{d}_2,\cdots,\tilde{d}_n\right)\\
&=\frac{T_1}{\sum_{j=1}^{n}T_j}\tilde{d}_1\oplus\frac{T_2}{\sum_{j=1}^{n}T_j}\tilde{d}_2\oplus\cdots\oplus\frac{T_n}{\sum_{j=1}^{n}T_j}\tilde{d}_n\\
&=\bigoplus_{j=1}^{n}\left(\frac{T_j\tilde{d}_j}{\sum_{j=1}^{n}T_j}\right)=\cup_{\gamma_j^+\in\mu_j^+,\eta_j^-\in\nu_j^-}\left\{\begin{array}{l}\left\{\dfrac{\prod_{j=1}^{n}\left(1+\gamma_j^+\right)^{T_j/\sum_{j=1}^{n}T_j}-\prod_{j=1}^{n}\left(1-\gamma_j^+\right)^{T_j/\sum_{j=1}^{n}T_j}}{\prod_{j=1}^{n}\left(1+\gamma_j^+\right)^{T_j/\sum_{j=1}^{n}T_j}+\prod_{j=1}^{n}\left(1-\gamma_j^+\right)^{T_j/\sum_{j=1}^{n}T_j}}\right\},\\ \left\{\dfrac{-2\prod_{j=1}^{n}\left|\eta_j^-\right|^{T_j/\sum_{j=1}^{n}T_j}}{\prod_{j=1}^{n}\left(2+\eta_j^-\right)^{T_j/\sum_{j=1}^{n}T_j}+\prod_{j=1}^{n}\left|\eta_j^-\right|^{T_j/\sum_{j=1}^{n}T_j}}\right\}\end{array}\right\}
\end{aligned}\tag{7}
$$

$$
\begin{aligned}
&\mathrm{DHBFHPWA}_{\omega}\left(\tilde{d}_1,\tilde{d}_2,\cdots,\tilde{d}_n\right)\\
&=\frac{\omega_1T_1}{\sum_{j=1}^{n}\omega_jT_j}\tilde{d}_1\oplus\frac{\omega_2T_2}{\sum_{j=1}^{n}\omega_jT_j}\tilde{d}_2\oplus\cdots\oplus\frac{\omega_nT_n}{\sum_{j=1}^{n}\omega_jT_j}\tilde{d}_n\\
&=\bigoplus_{j=1}^{n}\left(\frac{\omega_jT_j\tilde{d}_j}{\sum_{j=1}^{n}\omega_jT_j}\right)=\cup_{\gamma_j^+\in\mu_j^+,\eta_j^-\in\nu_j^-}\left\{\begin{array}{l}\left\{\dfrac{\prod_{j=1}^{n}\left(1+(\gamma-1)\gamma_j^+\right)^{\omega_jT_j/\sum_{j=1}^{n}\omega_jT_j}-\prod_{j=1}^{n}\left(1-\gamma_j^+\right)^{\omega_jT_j/\sum_{j=1}^{n}\omega_jT_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\gamma_j^+\right)^{\omega_jT_j/\sum_{j=1}^{n}\omega_jT_j}+(\gamma-1)\prod_{j=1}^{n}\left(1-\gamma_j^+\right)^{\omega_jT_j/\sum_{j=1}^{n}\omega_jT_j}}\right\},\\ \left\{\dfrac{-\gamma\prod_{j=1}^{n}\left|\eta_j^-\right|^{\omega_jT_j/\sum_{j=1}^{n}\omega_jT_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1+\eta_j^-\right)\right)^{\omega_jT_j/\sum_{j=1}^{n}\omega_jT_j}+(\gamma-1)\prod_{j=1}^{n}\left|\eta_j^-\right|^{\omega_jT_j/\sum_{j=1}^{n}\omega_jT_j}}\right\}\end{array}\right\}
\end{aligned}\tag{9}
$$

$$
\begin{aligned}
&\mathrm{DHBFPWA}_{\omega}\left(\tilde{d}_1,\tilde{d}_2,\cdots,\tilde{d}_n\right)\\
&=\frac{\omega_1T_1}{\sum_{j=1}^{n}\omega_jT_j}\tilde{d}_1\oplus\frac{\omega_2T_2}{\sum_{j=1}^{n}\omega_jT_j}\tilde{d}_2\oplus\cdots\oplus\frac{\omega_nT_n}{\sum_{j=1}^{n}\omega_jT_j}\tilde{d}_n\\
&=\bigoplus_{j=1}^{n}\left(\frac{\omega_jT_j\tilde{d}_j}{\sum_{j=1}^{n}\omega_jT_j}\right)=\cup_{\gamma_j^+\in\mu_j^+,\eta_j^-\in\nu_j^-}\times\left\{\begin{array}{l}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j^+\right)^{\omega_jT_j/\sum_{j=1}^{n}\omega_jT_j}\right\},\\ \left\{-\prod_{j=1}^{n}\left|\eta_j^-\right|^{\omega_jT_j/\sum_{j=1}^{n}\omega_jT_j}\right\}\end{array}\right\}
\end{aligned}\tag{10}
$$

B. DUAL HESITANT BIPOLAR FUZZY HAMACHER PRIORITIZED GEOMETRIC AGGREGATION OPERATORS

Applying the dual hesitant bipolar fuzzy Hamacher prioritized arithmetic aggregation operators and the concept of geometric mean [60]–[66], we can define dual hesitant bipolar fuzzy Hamacher prioritized geometric aggregation operators.

Definition 8: The dual hesitant bipolar fuzzy Hamacher prioritized geometric (DHBFHPG) operator

$$
\begin{aligned}
&\mathrm{DHBFEPWA}_{\omega}\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= \frac{\omega_1 T_1}{\sum_{j=1}^{n} \omega_j T_j} \tilde{d}_1 \oplus \frac{\omega_2 T_2}{\sum_{j=1}^{n} \omega_j T_j} \tilde{d}_2 \oplus \cdots \oplus \frac{\omega_n T_n}{\sum_{j=1}^{n} \omega_j T_j} \tilde{d}_n \\
&= \bigoplus_{j=1}^{n} \left(\frac{\omega_j T_j \tilde{d}_j}{\sum_{j=1}^{n} \omega_j T_j} \right) = \cup_{\gamma_j^+ \in \mu_j^+, \eta_j^- \in \nu_j^-} \left\{ \begin{array}{l} \left\{ \dfrac{\prod_{j=1}^{n} \left(1+\gamma_j^+\right)^{\omega_j T_j / \sum_{j=1}^{n} \omega_j T_j} - \prod_{j=1}^{n} \left(1-\gamma_j^+\right)^{\omega_j T_j / \sum_{j=1}^{n} \omega_j T_j}}{\prod_{j=1}^{n} \left(1+\gamma_j^+\right)^{\omega_j T_j / \sum_{j=1}^{n} \omega_j T_j} + \prod_{j=1}^{n} \left(1-\gamma_j^+\right)^{\omega_j T_j / \sum_{j=1}^{n} \omega_j T_j}} \right\}, \\ \left\{ \dfrac{-2 \prod_{j=1}^{n} \left|\eta_j^-\right|^{\omega_j T_j / \sum_{j=1}^{n} \omega_j T_j}}{\prod_{j=1}^{n} \left(2+\eta_j^-\right)^{\omega_j T_j / \sum_{j=1}^{n} \omega_j T_j} + \prod_{j=1}^{n} \left|\eta_j^-\right|^{\omega_j T_j / \sum_{j=1}^{n} \omega_j T_j}} \right\} \end{array} \right\}
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&\mathrm{DHBFHPG}\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= \left(\tilde{d}_1\right)^{T_1 / \sum_{j=1}^{n} T_j} \otimes \left(\tilde{d}_2\right)^{T_2 / \sum_{j=1}^{n} T_j} \otimes \cdots \otimes \left(\tilde{d}_n\right)^{T_n / \sum_{j=1}^{n} T_j} \\
&= \bigotimes_{j=1}^{n} \left(\tilde{d}_j\right)^{T_j / \sum_{j=1}^{n} T_j} = \cup_{\gamma_j^+ \in \mu_j^+, \eta_j^- \in \nu_j^-} \left\{ \begin{array}{l} \left\{ \dfrac{\gamma \prod_{j=1}^{n} \left(\gamma_j^+\right)^{T_j / \sum_{j=1}^{n} T_j}}{\prod_{j=1}^{n} \left(1+(\gamma-1)\left(1-\gamma_j^+\right)\right)^{T_j / \sum_{j=1}^{n} T_j} + (\gamma-1) \prod_{j=1}^{n} \left(\gamma_j^+\right)^{T_j / \sum_{j=1}^{n} T_j}} \right\}, \\ \left\{ -\dfrac{\prod_{j=1}^{n} \left(1+(\gamma-1)\left|\eta_j^-\right|\right)^{T_j / \sum_{j=1}^{n} T_j} - \prod_{j=1}^{n} \left(1+\eta_j^-\right)^{T_j / \sum_{j=1}^{n} T_j}}{\prod_{j=1}^{n} \left(1+(\gamma-1)\left|\eta_j^-\right|\right)^{T_j / \sum_{j=1}^{n} T_j} + (\gamma-1) \prod_{j=1}^{n} \left(1+\eta_j^-\right)^{T_j / \sum_{j=1}^{n} T_j}} \right\} \end{array} \right\}
\end{aligned}
\tag{13}
$$

is defined as

$$
\begin{aligned}
&\mathrm{DHBFHPG}\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= \tilde{d}_1^{\,T_1 / \sum_{j=1}^{n} T_j} \otimes \tilde{d}_2^{\,T_2 / \sum_{j=1}^{n} T_j} \otimes \cdots \otimes \tilde{d}_n^{\,T_n / \sum_{j=1}^{n} T_j} \\
&= \bigotimes_{j=1}^{n} \tilde{d}_j^{\,T_j / \sum_{j=1}^{n} T_j}
\end{aligned}
\tag{12}
$$

where $T_j = \prod_{k=1}^{j-1} s\left(\tilde{d}_j\right) (j = 2, \cdots, n)$, $T_1 = 1$ and $s\left(\tilde{d}_j\right)$ is the score values of $\tilde{d}_j \, (j = 1, 2, \cdots, n)$.

By definition and mathematical induction, we can prove the following theorem.

Theorem 4: The DHBFHPG operator returns a DHBFN, and (13), as shown at the top of this page. In (13) $T_j = \prod_{k=1}^{j-1} s\left(\tilde{d}_j\right) (j = 2, \cdots, n)$, $T_1 = 1$ and $s\left(\tilde{d}_j\right)$ is the score values of $\tilde{d}_j \, (j = 1, 2, \cdots, n)$.

Next we present two special cases of the DHBFHPG operator.

- If $\gamma = 1$, DHBFHPG operator reduces to the dual hesitant bipolar fuzzy prioritized geometric (DHBFPG) operator (14), as shown at the top of the next page.
- If $\gamma = 2$, DHBFHPG operator reduces to the dual hesitant bipolar fuzzy Einstein prioritized geometric (DHBFEPG) operator (15), as shown at the top of the next page.

If we consider the weights of $\tilde{d}_j \, (j = 1, 2, \cdots, n)$, where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of

$$\begin{aligned}
&\mathrm{DHBFPG}\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= (\tilde{d}_1)^{T_1 \Big/ \sum_{j=1}^n T_j} \otimes (\tilde{d}_2)^{T_2 \Big/ \sum_{j=1}^n T_j} \otimes \cdots \otimes (\tilde{d}_n)^{T_n \Big/ \sum_{j=1}^n T_j} \\
&= \otimes_{j=1}^n \left(\tilde{d}_j\right)^{T_j \Big/ \sum_{j=1}^n T_j} = \cup_{\gamma_j^+ \in \mu_j^+, \eta_j^- \in \nu_j^-} \left\{ \begin{array}{l} \left\{ \prod_{j=1}^n \left(\gamma_j^+\right)^{T_j \Big/ \sum_{j=1}^n T_j} \right\}, \\ \left\{ -1 + \prod_{j=1}^n \left(1 + \eta_j^-\right)^{T_j \Big/ \sum_{j=1}^n T_j} \right\} \end{array} \right\}
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\mathrm{DHBFEPG}\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= (\tilde{d}_1)^{T_1 \Big/ \sum_{j=1}^n T_j} \otimes (\tilde{d}_2)^{T_2 \Big/ \sum_{j=1}^n T_j} \otimes \cdots \otimes (\tilde{d}_n)^{T_n \Big/ \sum_{j=1}^n T_j} \\
&= \otimes_{j=1}^n \left(\tilde{d}_j\right)^{T_j \Big/ \sum_{j=1}^n T_j} = \cup_{\gamma_j^+ \in \mu_j^+, \eta_j^- \in \nu_j^-} \left\{ \begin{array}{l} \left\{ \dfrac{2 \prod_{j=1}^n \left(\gamma_j^+\right)^{T_j \Big/ \sum_{j=1}^n T_j}}{\prod_{j=1}^n \left(2 - \gamma_j^+\right)^{T_j \Big/ \sum_{j=1}^n T_j} + \prod_{j=1}^n \left(\gamma_j^+\right)^{T_j \Big/ \sum_{j=1}^n T_j}} \right\}, \\ \left\{ -\dfrac{\prod_{j=1}^n \left(1 + \left|\eta_j^-\right|\right)^{T_j \Big/ \sum_{j=1}^n T_j} - \prod_{j=1}^n \left(1 + \eta_j^-\right)^{T_j \Big/ \sum_{j=1}^n T_j}}{\prod_{j=1}^n \left(1 + \left|\eta_j^-\right|\right)^{T_j \Big/ \sum_{j=1}^n T_j} + \prod_{j=1}^n \left(1 + \eta_j^-\right)^{T_j \Big/ \sum_{j=1}^n T_j}} \right\} \end{array} \right\}
\end{aligned} \tag{15}$$

$\tilde{d}_j\,(j = 1, 2, \cdots, n)$ with $\omega_j > 0$, $\sum_{j=1}^n \omega_j = 1$. Then, based on Definition 5, we give the definition of the dual hesitant bipolar fuzzy Hamacher prioritized weighted geometric (DHBFHPWG) operator as follows:

Definition 9: The dual hesitant bipolar fuzzy Hamacher prioritized weighted geometric (DHBFHPWG) operator is

$$\begin{aligned}
&\mathrm{DHBFHPWG}_\omega\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= \left(\tilde{d}_1\right)^{\omega_1 T_1 \Big/ \sum_{j=1}^n \omega_j T_j} \otimes \left(\tilde{d}_2\right)^{\omega_2 T_2 \Big/ \sum_{j=1}^n \omega_j T_j} \otimes \cdots \\
&\otimes \left(\tilde{d}_n\right)^{\omega_n T_n \Big/ \sum_{j=1}^n \omega_j T_j} = \otimes_{j=1}^n \left(\tilde{d}_j\right)^{\omega_j T_j \Big/ \sum_{j=1}^n \omega_j T_j}
\end{aligned} \tag{16}$$

where $T_j = \prod_{k=1}^{j-1} s\left(\tilde{d}_j\right)\,(j = 2, \cdots, n)$, $T_1 = 1$ and $s\left(\tilde{d}_j\right)$ is the score values of $\tilde{d}_j\,(j = 1, 2, \cdots, n)$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of $\tilde{d}_j\,(j = 1, 2, \cdots, n)$ with $\omega_j > 0$, $\sum_{j=1}^n \omega_j = 1$.

Theorem 5 can be shown by its definition and mathematical induction. *Theorem 5:* The DHBFHPWG operator returns a DHBFN with (17), as shown at the top of the next page.

We subsequently discuss two special cases of the DHBFHPWG operator.

- If $\gamma = 1$, the DHBFHPWG operator is equivalent to the dual hesitant bipolar fuzzy prioritized weighted geometric (DHBFPWG) operator :

$$\begin{aligned}
&\mathrm{DHBFHPWG}_\omega\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= \left(\tilde{d}_1\right)^{\omega_1 T_1 \Big/ \sum_{j=1}^n \omega_j T_j} \otimes \left(\tilde{d}_2\right)^{\omega_2 T_2 \Big/ \sum_{j=1}^n \omega_j T_j} \otimes \cdots \\
&\otimes \left(\tilde{d}_n\right)^{\omega_n T_n \Big/ \sum_{j=1}^n \omega_j T_j} = \otimes_{j=1}^n \left(\tilde{d}_j\right)^{\omega_j T_j \Big/ \sum_{j=1}^n \omega_j T_j} \\
&= \cup_{\gamma_j^+ \in \mu_j^+, \eta_j^- \in \nu_j^-} \\
&\times \left\{ \begin{array}{l} \left\{ \prod_{j=1}^n \left(\gamma_j^+\right)^{\omega_j T_j \Big/ \sum_{j=1}^n \omega_j T_j} \right\}, \\ \left\{ -1 + \prod_{j=1}^n \left(1 + \eta_j^-\right)^{\omega_j T_j \Big/ \sum_{j=1}^n \omega_j T_j} \right\} \end{array} \right\}
\end{aligned} \tag{18}$$

$$
\begin{aligned}
&\mathrm{DHBFHPWG}_{\omega}\left(\tilde{d}_1,\tilde{d}_2,\cdots,\tilde{d}_n\right)\\
&=\left(\tilde{d}_1\right)^{\omega_1T_1\Big/\sum_{j=1}^{n}\omega_jT_j}\otimes\left(\tilde{d}_2\right)^{\omega_2T_2\Big/\sum_{j=1}^{n}\omega_jT_j}\otimes\cdots\otimes\left(\tilde{d}_n\right)^{\omega_nT_n\Big/\sum_{j=1}^{n}\omega_jT_j}\\
&=\mathop{\otimes}_{j=1}^{n}\left(\tilde{d}_j\right)^{\omega_jT_j\Big/\sum_{j=1}^{n}\omega_jT_j}=\cup_{\gamma_j^+\in\mu_j^+,\eta_j^-\in\nu_j^-}\left\{\begin{array}{l}\left\{\dfrac{\gamma\prod_{j=1}^{n}\left(\gamma_j^+\right)^{\omega_jT_j\Big/\sum_{j=1}^{n}\omega_jT_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-\gamma_j^+\right)\right)^{\omega_jT_j\Big/\sum_{j=1}^{n}\omega_jT_j}+(\gamma-1)\prod_{j=1}^{n}\left(\gamma_j^+\right)^{\omega_jT_j\Big/\sum_{j=1}^{n}\omega_jT_j}}\right\},\\
\left\{-\dfrac{\prod_{j=1}^{n}\left(1+(\gamma-1)\left|\eta_j^-\right|\right)^{\omega_jT_j\Big/\sum_{j=1}^{n}\omega_jT_j}-\prod_{j=1}^{n}\left(1+\eta_j^-\right)^{\omega_jT_j\Big/\sum_{j=1}^{n}\omega_jT_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left|\eta_j^-\right|\right)^{\omega_jT_j\Big/\sum_{j=1}^{n}\omega_jT_j}+(\gamma-1)\prod_{j=1}^{n}\left(1+\eta_j^-\right)^{\omega_jT_j\Big/\sum_{j=1}^{n}\omega_jT_j}}\right\}\end{array}\right\}
\end{aligned}
\tag{17}
$$

- If $\gamma = 2$, the DHBFHPWG operator coincides with the dual hesitant bipolar fuzzy Einstein prioritized weighted geometric (DHBFEPWG) operator (19), as shown at the bottom of the next page.

IV. MODELS FOR MULTIPLE ATTRIBUTE DECISION MAKING WITH DUAL HESITANT BIPOLAR FUZZY INFORMATION

We next apply the dual hesitant bipolar aggregation operators developed in the previous section to solve MADM problems with dual hesitant bipolar fuzzy information. The following assumptions or notations are used to represent the MADM problem for potential evaluation of emerging technology commercialization with dual hesitant bipolar fuzzy information. Denote a discrete set of alternatives by $A = \{A_1, A_2, \cdots, A_m\}$ and $G = \{G_1, G_2, \cdots, G_n\}$ be a collection of attribute and that there is a prioritization between the attribute expressed by the linear ordering $G_1 \succ G_2 \succ G_3 \cdots \succ G_n$, indicate attribute G_j has a higher priority than G_s, if $j < s$. If the decision makers provide several values for the alternative A_i under the attribute G_j with anonymity, these values can be considered as a DHBFNs $\tilde{d}_{ij}$. In the case where two decision makers provide the same value, then the value emerges only once in $\tilde{d}_{ij}$. Let $w = (w_1, w_2, \cdots, w_n)$ be the weight vector of attributes, where $w_j \geq 0$, $j = 1, 2, \cdots, n$, $\sum_{j=1}^{n} w_j = 1$. Suppose that $\tilde{D} = \left\{\tilde{d}_{ij}\right\}_{m\times n} = \left\{\mu_{ij}^+, \nu_{ij}^-\right\}_{m\times n}$ is the dual hesitant bipolar fuzzy decision matrix, where μ_{ij}^+ and ν_{ij}^- indicate, respectively, the positive degree and negative degree assessed by the decision maker that the alternative A_i satisfies the attribute G_j, $\mu_{ij}^+ \in [0, 1]$, $\nu_{ij}^- \in [-1, 0]$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$.

The process of utilizing the DHBFHPWA (or DHBFHPWG) operator to solve a MADM problem is presented below.

Step 1: Calculate the values of $T_{ij}(i = 1, 2, \cdots, m, j = 2, \cdots, n)$ as follows:

$$T_{ij} = \prod_{\lambda=1}^{j-1} s\left(\tilde{d}_{i\lambda}\right) \quad (i = 1, 2, \cdots, m, j = 2, \cdots, n) \tag{20}$$

$$T_{i1} = 1, \quad i = 1, 2, \cdots, m \tag{21}$$

Step 1: Applying the DHBFHPWA operator to process the information in matrix $\tilde{D}$, derive the overall values $\tilde{d}_i\,(i = 1, 2, \cdots, m)$ of the alternative A_i. The equation (22), as shown at the bottom of the next page.

If the DHBFHPWG operator is chosen instead, we have (23), as shown in Section V.

Step 2: Calculate the scores $S\left(\tilde{d}_i\right)$ $(i = 1, 2, \cdots, m)$.

Step 3: Rank all the alternatives A_i $(i = 1, 2, \cdots, m)$ in terms of $s(\tilde{d}_i)$ $(i = 1, 2, \cdots, m)$. If there is no difference between two scores $s(\tilde{d}_i)$ and $s(\tilde{d}_j)$, then calculate the accuracy degrees $a(\tilde{d}_i)$ and $a\left(\tilde{d}_j\right)$ to rank the alternatives A_i and A_j.

Step 4: Select the best alternative(s).

V. NUMERICAL EXAMPLE

In order to strengthen academic education, promote the building of teaching body, the school of management in a Chinese university wants to introduce oversea outstanding teachers (adapted from [67]). This introduction has been raised great attention from the school, university president, dean of management school and human resource officer sets up the panel of decision makers which will take the whole responsibility for this introduction. They made strict evaluation for 5 candidates A_i $(i = 1, 2, 3, 4, 5)$ according to the following four attributes: ① G_1 is the morality; ② G_2 is the research capability; ③ G_3 is the teaching skill; ④ G_4 is the education background. University president have the absolute priority for decision making, dean of the management school comes next. Besides, this introduction will be in strict accordance with the principle of combine ability with political integrity.

TABLE 1. Dual hesitant bipolar fuzzy decision matrix.

	G_1	G_2	G_3	G_4
A_1	{{0.3,0.4},{-0.6}}	{{0.4,0.5},{-0.3,-0.4)}	{{0.2,0.3},{-0.7)}	{{0.4,0.5},{-0.5}}
A_2	{{0.6},{-0.4}}	{{0.2,0.4,0.5},{-0.4}}	{{0.2},{-0.6,-0.7,-0.8}}	{{0.5),{-0.4,-0.5)}
A_3	{{0.5,0.7},{-0.2}}	{{0.2},{-0.7,-0.8}}	{{0.2,0.3,0.4},{-0.6}}	{{0.5,0.6,0.7},{-0.3}}
A_4	{{0.7},{-0.3})}	{{0.6,0.7,0.8},{-0.2}}	{{0.1,0.2},{-0.3}}	{{0.1},{-0.6,-0.7,-0.8}}
A_5	{{0.6,0.7},{-0.2}}	{{0.2,0.3,0.4},{-0.5}}	{{0.4,0.5},{-0.2}}	{{0.2,0.3,0.4},{-0.5}}

The prioritization relationship for the criteria is as below, $G_1 \succ G_2 \succ G_3 \succ G_4$. The five possible candidates A_i $(i = 1, 2, 3, 4, 5)$ are to be evaluated using the DHBFNs by the three decision makers under the above four attributes, and construct, respectively, the dual hesitant bipolar fuzzy decision matrix are shown in Table 1.

$$
\begin{aligned}
&\mathrm{DHBFEPWG}_{\omega}\left(\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_n\right) \\
&= \left(\tilde{d}_1\right)^{\omega_1 T_1 \Big/ \sum_{j=1}^{n} \omega_j T_j} \otimes \left(\tilde{d}_2\right)^{\omega_2 T_2 \Big/ \sum_{j=1}^{n} \omega_j T_j} \otimes \cdots \otimes \left(\tilde{d}_n\right)^{\omega_n T_n \Big/ \sum_{j=1}^{n} \omega_j T_j} \\
&= \bigotimes_{j=1}^{n} \left(\tilde{d}_j\right)^{\omega_j T_j \Big/ \sum_{j=1}^{n} \omega_j T_j} = \cup_{\gamma_j^+ \in \mu_j^+, \eta_j^- \in \nu_j^-} \left\{ \begin{array}{l} \left\{ \dfrac{2 \prod_{j=1}^{n} \left(\gamma_j^+\right)^{\omega_j T_j \Big/ \sum_{j=1}^{n} \omega_j T_j}}{\prod_{j=1}^{n} \left(2 - \gamma_j^+\right)^{\omega_j T_j \Big/ \sum_{j=1}^{n} \omega_j T_j} + \prod_{j=1}^{n} \left(\gamma_j^+\right)^{\omega_j T_j \Big/ \sum_{j=1}^{n} \omega_j T_j}} \right\}, \\ \left\{ -\dfrac{\prod_{j=1}^{n} \left(1 + \left|\eta_j^-\right|\right)^{\omega_j T_j \Big/ \sum_{j=1}^{n} \omega_j T_j} - \prod_{j=1}^{n} \left(1 + \eta_j^-\right)^{\omega_j T_j \Big/ \sum_{j=1}^{n} \omega_j T_j}}{\prod_{j=1}^{n} \left(1 + \left|\eta_j^-\right|\right)^{\omega_j T_j \Big/ \sum_{j=1}^{n} \omega_j T_j} + \prod_{j=1}^{n} \left(1 + \eta_j^-\right)^{\omega_j T_j \Big/ \sum_{j=1}^{n} \omega_j T_j}} \right\} \end{array} \right\}
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\tilde{d}_i &= \left(\mu_i^+, \nu_i^-\right) = \mathrm{DHBFHPWA}_{\omega}\left(\tilde{d}_{i1}, \tilde{d}_{i2}, \cdots, \tilde{d}_{in}\right) \\
&= \frac{\omega_1 T_{i1}}{\sum_{j=1}^{n} \omega_j T_{ij}} \tilde{d}_{i1} \oplus \frac{\omega_2 T_{i2}}{\sum_{j=1}^{n} \omega_j T_{ij}} \tilde{d}_{i2} \oplus \cdots \oplus \frac{\omega_n T_{in}}{\sum_{j=1}^{n} \omega_j T_{ij}} \tilde{d}_{in} \\
&= \bigoplus_{j=1}^{n} \left(\frac{\omega_j T_{ij} \tilde{d}_{ij}}{\sum_{j=1}^{n} \omega_j T_{ij}} \right) = \cup_{\gamma_{ij}^+ \in \mu_{ij}^+, \eta_{ij}^- \in \nu_{ij}^-} \left\{ \begin{array}{l} \left\{ \dfrac{\prod_{j=1}^{n} \left(1 + (\gamma - 1)\gamma_{ij}^+\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^{n} \omega_j T_{ij}} - \prod_{j=1}^{n} \left(1 - \gamma_{ij}^+\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^{n} \omega_j T_{ij}}}{\prod_{j=1}^{n} \left(1 + (\gamma - 1)\gamma_{ij}^+\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^{n} \omega_j T_{ij}} + (\gamma - 1) \prod_{j=1}^{n} \left(1 - \gamma_{ij}^+\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^{n} \omega_j T_{ij}}} \right\}, \\ \left\{ \dfrac{-\gamma \prod_{j=1}^{n} \left|\eta_{ij}^-\right|^{\omega_j T_{ij} \Big/ \sum_{j=1}^{n} \omega_j T_{ij}}}{\prod_{j=1}^{n} \left(1 + (\gamma - 1)\left(1 + \eta_{ij}^-\right)\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^{n} \omega_j T_{ij}} + (\gamma - 1) \prod_{j=1}^{n} \left|\eta_{ij}^-\right|^{\omega_j T_{ij} \Big/ \sum_{j=1}^{n} \omega_j T_{ij}}} \right\} \end{array} \right\}
\end{aligned} \tag{22}
$$

The information about the attribute weights is known as follows: $\omega = (0.20, 0.10, 0.30, 0.40)$.

In the following, we utilize the approach developed to select the desirable candidate with dual hesitant bipolar fuzzy information.

Step 1: Utilize (19)-(20) to calculate the values of T_{ij} $(i = 1, 2, \cdots, m, j = 2, \cdots, n)$ as follows:

$$T_{ij} = \begin{bmatrix} 1.000 & 0.350 & 0.193 & 0.053 \\ 1.000 & 0.600 & 0.290 & 0.072 \\ 1.000 & 0.700 & 0.158 & 0.055 \\ 1.000 & 0.700 & 0.525 & 0.223 \\ 1.000 & 0.725 & 0.290 & 0.181 \end{bmatrix}$$

Step 2: We utilize the decision information given in matrix $\tilde{D}$, and the DHBFHPWA operator to obtain the overall preference values $\tilde{d}_i$ of the candidate A_i $(i = 1, 2, 3, 4, 5)$. Let $\gamma = 3$, we have $\tilde{d}_1$–$\tilde{d}_5$, as shown at the top of the next page.

Step 2: Calculate the scores $s\left(\tilde{d}_i\right)$ $(i = 1, 2, 3, 4, 5)$ of the overall DHBFNs $\tilde{d}_i$ $(i = 1, 2, 3, 4, 5)$:

$$s\left(\tilde{d}_1\right) = 0.5127, \quad s\left(\tilde{d}_2\right) = 0.5946, \quad s\left(\tilde{d}_3\right) = 0.6369$$
$$s\left(\tilde{d}_4\right) = 0.5434, \quad s\left(\tilde{d}_5\right) = 0.6540$$

Step 3: Rank all the candidates A_i $(i = 1, 2, 3, 4, 5)$ in accordance with the scores $s\left(\tilde{d}_i\right)$ $(i = 1, 2, 3, 4, 5)$ of the overall DHBFNs: $A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$, and thus the most desirable candidate is A_5.

Based on the DHBFHPWG operator, then, in order to select the most desirable candidates, we can develop another approach developed to select the best candidates with dual hesitant bipolar fuzzy information, which can be described as following:

Step 1′: See Step 1.

Step 2′: Aggregate all the dual hesitant bipolar fuzzy numbers in the Table 1 by using the DHBFHPWG operator to derive the overall DHBFNs $\tilde{d}_i$ $(i = 1, 2, \cdots, 5)$ of the candidate A_i. Take candidate A_1 for an example (Let $\gamma = 3$), we have $\tilde{d}_1$–$\tilde{d}_5$, as shown at the top of the Conclusion Section.

Step 2′: Calculate the scores $s\left(\tilde{d}_i\right)$ $(i = 1, 2, 3, 4, 5)$ of the overall DHBFNs $\tilde{d}_i$ $(i = 1, 2, 3, 4, 5)$ of the candidate A_i:

$$s\left(\tilde{d}_1\right) = 0.3777, \quad s\left(\tilde{d}_2\right) = 0.4753, \quad s\left(\tilde{d}_3\right) = 0.5246$$
$$s\left(\tilde{d}_4\right) = 0.5200, \quad s\left(\tilde{d}_5\right) = 0.5851$$

Step 3′: Rank all the candidates A_i $(i = 1, 2, 3, 4, 5)$ in accordance with the scores $s\left(\tilde{d}_i\right)$ $(i = 1, 2, 3, 4, 5)$ of the overall DHBFNs $\tilde{d}_i$ $(i = 1, 2, \cdots, 5)$: $A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$ and thus the most desirable candidate is A_5.

TABLE 2. Order of different methods.

	Ordering
DHBFWA[52]	$A_5 > A_3 > A_2 > A_4 > A_1$
DHBFWG[52]	$A_5 > A_3 > A_4 > A_2 > A_1$
DHBFHPWA	$A_5 > A_3 > A_2 > A_4 > A_1$
DHBFHPWG	$A_5 > A_3 > A_4 > A_2 > A_1$

From the above analysis, it is easily seen that although the overall rating values of the alternatives are slightly different by using two operators respectively. However, the most desirable candidate is A_5.

In what follows, we compare with the dual hesitant bipolar fuzzy weighted average (DHBFWA) operator

$$\begin{aligned}
\tilde{d}_i &= \left(\mu_i^+, \nu_i^-\right) = \mathrm{DHBFHWG}_\omega\left(\tilde{d}_{i1}, \tilde{d}_{i2}, \cdots, \tilde{d}_{in}\right) \\
&= \left(\tilde{d}_{i1}\right)^{\omega_1 T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}} \otimes \left(\tilde{d}_{i2}\right)^{\omega_2 T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}} \otimes \cdots \otimes \left(\tilde{d}_{in}\right)^{\omega_n T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}} \\
&= \mathop{\otimes}_{j=1}^{n} \left(\tilde{d}_{ij}\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}} \\
&= \cup_{\gamma_{ij}^+ \in \mu_{ij}^+, \eta_{ij}^- \in \nu_{ij}^-} \left\{ \begin{array}{l} \left\{ \dfrac{\gamma \prod_{j=1}^n \left(\gamma_{ij}^+\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}}}{\prod_{j=1}^n \left(1 + (\gamma - 1)\left(1 - \gamma_{ij}^+\right)\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}} + (\gamma - 1) \prod_{j=1}^n \left(\gamma_{ij}^+\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}}} \right\}, \\ \left\{ -\dfrac{\prod_{j=1}^n \left(1 + (\gamma - 1)\left|\eta_{ij}^-\right|\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}} - \prod_{j=1}^n \left(1 + \eta_{ij}^-\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}}}{\prod_{j=1}^n \left(1 + (\gamma - 1)\left|\eta_{ij}^-\right|\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}} + (\gamma - 1) \prod_{j=1}^n \left(1 + \eta_{ij}^-\right)^{\omega_j T_{ij} \Big/ \sum_{j=1}^n \omega_j T_{ij}}} \right\} \end{array} \right\}
\end{aligned} \tag{23}$$

$$
\begin{aligned}
\tilde{d}_1 &= \text{DHBFHPWA}_\omega\left(\tilde{d}_{11}, \tilde{d}_{12}, \tilde{d}_{13}, \tilde{d}_{14}\right) \\
&= \frac{\omega_1 T_{11}}{\sum\limits_{j=1}^{n}\omega_j T_{1j}}\tilde{d}_{11} \oplus \frac{\omega_2 T_{12}}{\sum\limits_{j=1}^{n}\omega_j T_{1j}}\tilde{d}_{12} \oplus \frac{\omega_3 T_{13}}{\sum\limits_{j=1}^{n}\omega_j T_{1j}}\tilde{d}_{13} \oplus \frac{\omega_4 T_{14}}{\sum\limits_{j=1}^{n}\omega_j T_{1j}}\tilde{d}_{14} \\
&= \cup_{\gamma_{1j}^{+}\in\mu_{1j}^{+},\eta_{1j}^{-}\in\nu_{1j}^{-}} \left\{ \begin{array}{l} \left\{ \dfrac{\prod\limits_{j=1}^{4}\left(1+(\gamma-1)\gamma_{1j}^{+}\right)^{\omega_1 T_{1j}\Big/\sum\limits_{j=1}^{n}\omega_j T_{1j}} - \prod\limits_{j=1}^{4}\left(1-\gamma_{1j}^{+}\right)^{\omega_1 T_{1j}\Big/\sum\limits_{j=1}^{n}\omega_j T_{1j}}}{\prod\limits_{j=1}^{4}\left(1+(\gamma-1)\gamma_{1j}^{+}\right)^{\omega_1 T_{1j}\Big/\sum\limits_{j=1}^{n}\omega_j T_{1j}} + (\gamma-1)\prod\limits_{j=1}^{4}\left(1-\gamma_{1j}^{+}\right)^{\omega_1 T_{1j}\Big/\sum\limits_{j=1}^{n}\omega_j T_{1j}}} \right\}, \\ \left\{ \dfrac{-\gamma\prod\limits_{j=1}^{4}\left|\eta_{1j}^{-}\right|^{\omega_1 T_{1j}\Big/\sum\limits_{j=1}^{n}\omega_j T_{1j}}}{\prod\limits_{j=1}^{4}\left(1+(\gamma-1)\left(1+\eta_{1j}^{-}\right)\right)^{\omega_1 T_{1j}\Big/\sum\limits_{j=1}^{n}\omega_j T_{1j}} + (\gamma-1)\prod\limits_{j=1}^{4}\left|\eta_{1j}^{-}\right|^{\omega_1 T_{1j}\Big/\sum\limits_{j=1}^{n}\omega_j T_{1j}}} \right\} \end{array} \right\} \\
&= \text{DHBFHPWA}_\omega\left\{ \begin{array}{l} \{\{0.3, 0.4\}, \{-0.5\}\}, \{\{0.4, 0.5\}\}, \{-0.3, -0.4\}\}, \\ \{\{0.2, 0.3\}, \{-0.5)\}, \{\{0.4, 0.5\}, \{-0.5\}\} \end{array} \right\} \\
&= \{\{0.2994, 0.3066, 0.3177, 0.3249, 0.3113, 0.3185, 0.3296, 0.3367, 0.3636, 0.3706, \\
&\quad 0.3815, 0.3885, 0.3752, 0.3823, 0.3931, 0.4000\}, \{-0.3213, -0.3279\}\} \\
\tilde{d}_2 &= \text{DHBFHPWA}_\omega\left(\tilde{d}_{21}, \tilde{d}_{22}, \tilde{d}_{23}, \tilde{d}_{24}\right) \\
&= \text{DHBFHPWA}_\omega\left\{ \begin{array}{l} \{\{0.6\}, \{-0.4\}\}, \{\{0.2, 0.4, 0.5\}, \{-0.4\}\}, \\ \{\{0.2\}, \{-0.6, -0.7, -0.8\}\}, \{\{0.5\}, \{-0.4, -0.5\}\} \end{array} \right\} \\
&= \left\{ \begin{array}{l} \{0.4475, 0.4772, 0.4927\}, \\ \{-0.2764, -0.2794, -0.2821, -0.2851, -0.2868, -0.2899\} \end{array} \right\} \\
\tilde{d}_3 &= \text{DHBFHPWA}_\omega\left(\tilde{d}_{31}, \tilde{d}_{32}, \tilde{d}_{33}, \tilde{d}_{34}\right) \\
&= \text{DHBFHPWA}_\omega\left\{ \begin{array}{l} \{\{0.5, 0.7\}, \{-0.2\}\}, \{\{0.2\}\}, \{-0.7, -0.8\}\}, \\ \{\{0.2, 0.3, 0.4\}, \{-0.6)\}, \{\{0.5, 0.6, 0.7\}, \{-0.3\}\} \end{array} \right\} \\
&= \{\{0.4007, 0.4082, 0.4169, 0.4141, 0.4214, 0.4301, 0.4275, 0.4348, 0.4434, \\
&\quad 0.5401, 0.5467, 0.5543, 0.5519, 0.5583, 0.5659, 0.5637, 0.5701, 0.5775\}, \\
&\quad \{-0.2149, -0.2183\}\} \\
\tilde{d}_4 &= \text{DHBFHPWA}_\omega\left(\tilde{d}_{41}, \tilde{d}_{42}, \tilde{d}_{43}, \tilde{d}_{44}\right) \\
&= \text{DHBFHPWA}_\omega\left\{ \begin{array}{l} \{\{0.7\}, \{-0.3\}\}, \{\{0.6, 0.7, 0.8\}, \{-0.2\}\}, \\ \{\{0.1, 0.2\}, \{-0.3\}\}, \{\{0.1\}, \{0.6, -0.7, -0.8\}\} \end{array} \right\} \\
&= \left\{ \begin{array}{l} \{0.4266, 0.4557, 0.4445, 0.4731, 0.4673, 0.4954\}, \\ \{-0.2229, -0.2264, -0.2294\} \end{array} \right\} \\
\tilde{d}_5 &= \text{DHBFHPWA}_\omega\left(\tilde{d}_{51}, \tilde{d}_{52}, \tilde{d}_{53}, \tilde{d}_{54}\right) \\
&= \text{DHBFHPWA}_\omega\left\{ \begin{array}{l} \{\{0.6, 0.7\}, \{-0.2\}\}, \{\{0.2, 0.3, 0.4\}\}, \{-0.5\}, \\ \{\{0.4, 0.5\}, \{-0.2)\}, \{\{0.2, 0.3, 0.4\}, \{-0.5\}\} \end{array} \right\} \\
&= \{\{0.4355, 0.4512, 0.4670, 0.4558, 0.4712, 0.4867, 0.4512, 0.4667, 0.4823, \\
&\quad 0.4712, 0.4864, 0.5017, 0.4670, 0.4823, 0.4976, 0.4867, 0.5017, 0.5168, \\
&\quad 0.4952, 0.5101, 0.5250, 0.5144, 0.5290, 0.5436, 0.5101, 0.5247, 0.5394, \\
&\quad 0.5290, 0.5433, 0.5577, 0.5250, 0.5394, 0.5539, 0.5436, 0.5577, 0.5718\}, \{-0.1974\}\}
\end{aligned}
$$

and dual hesitant bipolar fuzzy weighted geometric (DHBFWG) operator [52]. The result is shown in Table 2.

From the above analysis, it can be seen that four operators have the same best emerging technology enterprise A_5 and two methods' ranking results are slightly different. But, the our proposed operators consider the prioritization relationship over attributes, but DHBFWA and DHBFWG operator [52] fail to do so. This verifies the method we proposed is reasonable and effective.

$$
\begin{aligned}
\tilde{d}_1 &= \mathrm{DHBFHPWG}_\omega\left(\tilde{d}_{11}, \tilde{d}_{12}, \tilde{d}_{13}, \tilde{d}_{14}\right)\\
&= \left(\tilde{d}_{11}\right)^{\omega_1 T_{11} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}} \otimes \left(\tilde{d}_{12}\right)^{\omega_2 T_{12} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}} \otimes \left(\tilde{d}_{13}\right)^{\omega_1 T_{13} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}} \otimes \left(\tilde{d}_{14}\right)^{\omega_1 T_{14} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}}\\
&= \cup_{\gamma_{1j}^{+}\in\mu_{1j}^{+},\eta_{1j}^{-}\in\nu_{1j}^{-}}\left\{\left\{\frac{\gamma\prod_{j=1}^{4}\left(\gamma_{1j}^{+}\right)^{\omega_1 T_{1j} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}}}{\prod_{j=1}^{4}\left(1+(\gamma-1)\left(1-\gamma_{1j}^{+}\right)\right)^{\omega_1 T_{1j} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}}+(\gamma-1)\prod_{j=1}^{4}\left(\gamma_{1j}^{+}\right)^{\omega_1 T_{1j} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}}}\right\},\right.\\
&\qquad\left.\left\{-\frac{\prod_{j=1}^{4}\left(1+(\gamma-1)\left|\eta_{1j}^{-}\right|\right)^{\omega_1 T_{1j} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}}-\prod_{j=1}^{4}\left(1+\eta_{1j}^{-}\right)^{\omega_1 T_{1j} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}}}{\prod_{j=1}^{4}\left(1+(\gamma-1)\left|\eta_{1j}^{-}\right|\right)^{\omega_1 T_{1j} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}}+(\gamma-1)\prod_{j=1}^{4}\left(1+\eta_{1j}^{-}\right)^{\omega_1 T_{1j} \Big/ \sum_{j=1}^{n}\omega_j T_{1j}}}\right\}\right\}\\
&= \mathrm{DHBFHPWG}_\omega\left\{\begin{array}{l}\{\{0.3, 0.4\}, \{-0.5\}\}, \{\{0.4, 0.5\}\}, \{-0.3, -0.4\}\},\\ \{\{0.2, 0.3\}, \{-0.5)\}, \{\{0.4, 0.5\}, \{-0.5\}\}\end{array}\right\}\\
&= \{\{0.2951, 0.3002, 0.3168, 0.3222, 0.3036, 0.3088, 0.3258, 0.3312, 0.3558, 0.3616,\\
&\qquad 0.3806, 0.3867, 0.3655, 0.3714, 0.3907, 0.3969\}, \{-0.5847, -0.5937\}\}\\
\tilde{d}_2 &= \mathrm{DHBFHPWG}_\omega\left(\tilde{d}_{21}, \tilde{d}_{22}, \tilde{d}_{23}, \tilde{d}_{24}\right)\\
&= \mathrm{DHBFHPWG}_\omega\left\{\begin{array}{l}\{\{0.6\}, \{-0.4\}\}, \{\{0.2, 0.4, 0.5\}, \{-0.4\}\},\\ \{\{0.2\}, \{-0.6, -0.7, -0.8\}\}, \{\{0.5\}, \{-0.4, -0.5\}\}\end{array}\right\}\\
&= \left\{\begin{array}{l}\{0.4032, 0.4449, 0.4610\},\\ \{-0.4497, -0.4574, -0.4795, -0.4870, -0.5170, -0.5242\}\end{array}\right\}\\
\tilde{d}_3 &= \mathrm{DHBFHPWG}_\omega\left(\tilde{d}_{31}, \tilde{d}_{32}, \tilde{d}_{33}, \tilde{d}_{34}\right)\\
&= \mathrm{DHBFHPWG}_\omega\left\{\begin{array}{l}\{\{0.5, 0.7\}, \{-0.2\}\}, \{\{0.2\}\}, \{-0.7, -0.8\}\},\\ \{\{0.2, 0.3, 0.4\}, \{-0.6)\}, \{\{0.5, 0.6, 0.7\}, \{-0.3\}\}\end{array}\right\}\\
&= \{\{0.3745, 0.3798, 0.3848, 0.3938, 0.3993, 0.4045, 0.4091, 0.4147, 0.4201,\\
&\qquad \left.\begin{array}{l}0.4745, 0.4806, 0.4864, 0.4967, 0.5030, 0.5089, 0.5142, 0.5206, 0.5266\},\\ \{-0.3824, -0.4185\}\end{array}\right\}\\
\tilde{d}_4 &= \mathrm{DHBFHPWG}_\omega\left(\tilde{d}_{41}, \tilde{d}_{42}, \tilde{d}_{43}, \tilde{d}_{44}\right)\\
&= \mathrm{DHBFHPWG}_\omega\left\{\begin{array}{l}\{\{0.7\}, \{-0.3\}\}, \{\{0.6, 0.7, 0.8\}, \{-0.2\}\},\\ \{\{0.1, 0.2\}, \{-0.3\}\}, \{\{0.1\}, \{0.6, -0.7, -0.8\}\}\end{array}\right\}\\
&= \left\{\begin{array}{l}\{0.3047, 0.3654, 0.3138, 0.3757, 0.3228, 0.3860\},\\ \{-0.3429, -0.3667, -0.3972\}\end{array}\right\}\\
\tilde{d}_5 &= \mathrm{DHBFHPWG}_\omega\left(\tilde{d}_{51}, \tilde{d}_{52}, \tilde{d}_{53}, \tilde{d}_{54}\right)\\
&= \mathrm{DHBFHPWG}_\omega\left\{\begin{array}{l}\{\{0.6, 0.7\}, \{-0.2\}\}, \{\{0.2, 0.3, 0.4\}\}, \{-0.5\},\\ \{\{0.4, 0.5\}, \{-0.2)\}, \{\{0.2, 0.3, 0.4\}, \{-0.5\}\}\end{array}\right\}\\
&= \{\{0.3976, 0.4219, 0.4412, 0.4166, 0.4416, 0.4614, 0.4219, 0.4470, 0.4670,\\
&\qquad 0.4416, 0.4674, 0.4879, 0.4412, 0.4670, 0.4875, 0.4614, 0.4879, 0.5088,\\
&\qquad 0.4355, 0.4611, 0.4814, 0.4556, 0.4818, 0.5026, 0.4611, 0.4875, 0.5084,\\
&\qquad 0.4818, 0.5088, 0.5302, 0.4814, 0.5084, 0.5298, 0.5026, 0.5302, 0.5519\}, \{-0.3039\}\}
\end{aligned}
$$

VI. CONCLUSION

In this paper, we investigate the dual hesitant bipolar fuzzy multiple attribute decision making problems with in which there exists a prioritization relationship over attributes. Then, motivated by the idea of Hamacher operations and prioritized aggregation operators, we have developed some

Hamacher prioritized aggregation operators for aggregating dual hesitant bipolar fuzzy information: dual hesitant bipolar fuzzy Hamacher prioritized average (DHBFHPA) operator, dual hesitant bipolar fuzzy Hamacher prioritized geometric (DHBFHPG) operator, dual hesitant bipolar fuzzy Hamacher prioritized weighted average (DHBFHPWA) operator, dual hesitant bipolar fuzzy Hamacher prioritized weighted geometric (DHBFHPWG) operator. Then, we have utilized these operators to develop some approaches to solve the dual hesitant bipolar fuzzy multiple attribute decision making problems. Finally, a practical example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Several directions for future research may be promising. First, the aggregation operator proposed in this paper can be introduced into other fuzzy and uncertain environments [68]–[76]. Second, applications of the proposed MADM method can be explored to tackle practical problems in other areas, such as selecting information systems, evaluating the financial risks or software quality [77]–[92]. The common feature of these practical problems is that multiple attributes involved are interdependent and have different priority levels. Third, the complexity of the proposed method can be improved with the help of computer technology. In the future, we will devote ourselves to reducing the complexity of the method as well as increasing accuracy.

REFERENCES

[1] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, pp. 87–96, Aug. 1986.

[2] K. Atanassov, "More on intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 33, pp. 37–46, Oct. 1989.

[3] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 2, pp. 338–356, 1965.

[4] Z. Xu, "Intuitionistic fuzzy aggregation operators," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 6, pp. 1179–1187, Dec. 2007.

[5] Z. Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets," *Int. J. Gen. Syst.*, vol. 35, no. 4, pp. 417–433, 2006.

[6] Z. Xu and R. R. Yager, "Dynamic intuitionistic fuzzy multi-attribute decision making," *Int. J. Approx. Reasoning*, vol. 48, no. 1, pp. 246–262, 2008.

[7] Z. Xu, "Approaches to multiple attribute group decision making based on intuitionistic fuzzy power aggregation operators," *Knowl.-Based Syst.*, vol. 24, no. 6, pp. 749–760, 2011.

[8] Z. Xu and Q. Chen, "A multi-criteria decision making procedure based on interval-valued intuitionistic fuzzy bonferroni means," *J. Syst. Sci. Syst. Eng.*, vol. 20, no. 2, pp. 217–228, 2011.

[9] Z. Xu and M. M. Xia, "Induced generalized intuitionistic fuzzy operators," *Knowl.-Based Syst.*, vol. 24, no. 2, pp. 197–209, 2011.

[10] G. Wei, "Approaches to interval intuitionistic trapezoidal fuzzy multiple attribute decision making with incomplete weight information," *Int. J. Fuzzy Syst.*, vol. 17, no. 3, pp. 484–489, 2015.

[11] G. Wei, "Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making," *Appl. Soft Comput.*, vol. 10, no. 2, pp. 423–431, 2010.

[12] G. Wei, H.-J. Wang, and R. Lin, "Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information," *Knowl. Inf. Syst.*, vol. 26, no. 2, pp. 337–349, 2011.

[13] G. Wei, "Gray relational analysis method for intuitionistic fuzzy multiple attribute decision making," *Expert Syst. Appl.*, vol. 38, no. 9, pp. 11671–11677, 2011.

[14] F. Jin, L. Pei, H. Chen, and L. Zhou, "Interval-valued intuitionistic fuzzy continuous weighted entropy and its application to multi-criteria fuzzy group decision making," *Knowl.-Based Syst.*, vol. 59, pp. 132–141, Mar. 2014.

[15] R. Verma and B. D. Sharma, "A new measure of inaccuracy with its application to multi-criteria decision making under intuitionistic fuzzy environment," *J. Intell. Fuzzy Syst.*, vol. 27, no. 4, pp. 1811–1824, 2014.

[16] T.-Y. Chen, "The inclusion-based TOPSIS method with interval-valued intuitionistic fuzzy sets for multiple criteria group decision making," *Appl. Soft Comput.*, vol. 26, pp. 57–73, Jan. 2015.

[17] G. Wei, "GRA method for multiple attribute decision making with incomplete weight information in intuitionistic fuzzy setting," *Knowl.-Based Syst.*, vol. 23, no. 3, pp. 243–247, 2010.

[18] X. Qi, C. Liang, and J. Zhang, "Generalized cross-entropy based group decision making with unknown expert and attribute weights under interval-valued intuitionistic fuzzy environment," *Comput. Ind. Eng.*, vol. 79, pp. 52–64, Jan. 2015.

[19] G. Wei and X. Zhao, "Some induced correlated aggregating operators with intuitionistic fuzzy information and their application to multiple attribute group decision making," *Expert Syst. Appl.*, vol. 39, no. 2, pp. 2026–2034, 2012.

[20] G. Wei, "Maximizing deviation method for multiple attribute decision making in intuitionistic fuzzy setting," *Knowl.-Based Syst.*, vol. 21, no. 8, pp. 833–836, 2008.

[21] G. Wei, X. Zhao, and R. Lin, "Some induced aggregating operators with fuzzy number intuitionistic fuzzy information and their applications to group decision making," *Int. J. Comput. Intell. Syst.*, vol. 3, no. 1, pp. 84–95, 2010.

[22] G. Wei and J. M. Merigó, "ethods for strategic decision-making problems with immediate probabilities in intuitionistic fuzzy setting," *Sci. Iranica*, vol. 19, no. 6, pp. 1936–1946, 2012.

[23] X. Zhao and G. Wei, "Some intuitionistic fuzzy Einstein hybrid aggregation operators and their application to multiple attribute decision making," *Knowl.-Based Syst.*, vol. 37, pp. 472–479, Jan. 2013.

[24] G. Wei, H. J. Wang, R. Lin, and X. Zhao, "Grey relational analysis method For intuitionistic fuzzy multiple attribute decision making with preference information on alternatives," *Int. J. Comput. Intell. Syst.*, vol. 4, no. 2, pp. 164–173, 2011.

[25] G. Wei and X. Zhao, "Minimum deviation models for multiple attribute decision making in intuitionistic fuzzy setting," *Int. J. Comput. Intell. Syst.*, vol. 4, no. 2, pp. 174–183, 2011.

[26] W.-R. Zhang, "Bipolar fuzzy sets and relations: A computational framework for cognitive modeling and multiagent decision analysis," in *Proc. IEEE Conf.*, Dec. 1994, pp. 305–309.

[27] W.-R. Zhang, "(Yin) (Yang) bipolar fuzzy sets," in *Proc. FUZZY-IEEE*, May 1998, pp. 835–840.

[28] W.-R. Zhang and L. Zhang, "Bipolar logic and bipolar fuzzy logic," *Inf. Sci.*, vol. 165, nos. 3–4, pp. 265–287, 2004.

[29] Y. Han, P. Shi, and S. Chen, "Bipolar-valued rough fuzzy set and its applications to decision information system," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 6, pp. 2358–2370, Jun. 2015.

[30] W.-R. Zhang, H. J. Zhang, Y. Shi, and S. S. Chen, "Bipolar linear algebra and YinYang-N-element cellular networks for equilibrium-based biosystem simulation and regulation," *J. Biol. Syst.*, vol. 17, no. 4, pp. 547–576, 2009.

[31] M. Lu and J. R. Busemeyer, "Do traditional Chinese theories of Yi Jing ('Yin-Yang' and Chinese medicine) go beyond Western concepts of mind and matter," *Mind Matter*, vol. 12, no. 1, pp. 37–59, 2014.

[32] W.-R. Zhang, "Equilibrium relations and bipolar cognitive mapping for online analytical processing with applications in international relations and strategic decision support," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 33, no. 2, pp. 295–307, Apr. 2003.

[33] W.-R. Zhang, "Equilibrium energy and stability measures for bipolar decision and global regulation," *Int. J. Fuzzy Syst.*, vol. 5, no. 2, pp. 114–122, 2003.

[34] W.-R. Zhang, A. K. Pandurangi, and K. E. Peace, "YinYang dynamic neurobiological modeling and diagnostic analysis of major depressive and bipolar disorders," *IEEE Trans. Biomed. Eng.*, vol. 54, no. 10, pp. 1729–1739, Oct. 2007.

[35] W.-R. Zhang, A. K. Pandurangi, Y. Q. Zhang, and Z. Zhao, "Mental-Squares: A generic bipolar Support Vector Machine for psychiatric disorder classification, diagnostic analysis and neurobiological data mining," *Int. J. Data Mining Bioinform.*, vol. 5, no. 5, pp. 532–572, 2011.

[36] G. Fink and M. Yolles, "Collective emotion regulation in an organization—A plural agency with cognition and affect," *J. Org. Change Manage.*, vol. 28, no. 5, pp. 832–871, 2015.

[37] P. P. Li, "The global implications of the indigenous epistemological system from the east: How to apply yin-yang balancing to paradox management," *Cross Cultural Strategic Manage.*, vol. 23, no. 1, pp. 42–47, 2016.

[38] W.-R. Zhang and F. Marchetti, "A logical exposition of dirac 3-polarizer experiment and its potential impact on computational biology," in *Proc. ACM Conf. Bioinform., Comput. Biol., Health Informat. (ACM BCB)*, 2015, pp. 517–518.

[39] W.-R. Zhang, "Bipolar quantum logic gates and quantum cellular combinatorics—A logical extension to quantum entanglement," *J. Quantum Inf. Sci.*, vol. 3, no. 2, pp. 93–105, 2013.

[40] W.-R. Zhang and K. E. Peace, "Causality is logically definable—Toward an equilibrium-based computing paradigm of quantum agent and quantum intelligence," *J. Quantum Inf. Sci.*, vol. 4, pp. 227–268, Dec. 2014.

[41] W.-R. Zhang and S.-S. Chen, "Equilibrium and non-equilibrium modeling of YinYang WuXing for diagnostic decision analysis in traditional Chinese medicine," *Int. J. Inf. Technol. Decision Making*, vol. 8, no. 3, pp. 529–548, 2009.

[42] W.-R. Zhang, *YinYang Bipolar Relativity: A Unifying Theory of Nature, Agents and Causality with Applications in Quantum Computing, Cognitive Informatics and Life Sciences*. New York, NY, USA: IGI Global, 2011.

[43] W.-R. Zhang, "G-CPT symmetry of quantum emergence and submergence-an information conservational multiagent cellular automata unification of CPT symmetry and cp violation for equilibrium-based many world causal analysis of quantum coherence and decoherence," *J. Quantum Inf. Sci.*, vol. 6, no. 2, pp. 62–97, 2016.

[44] M. Akram, "Bipolar fuzzy graphs," *Inf. Sci.*, vol. 181, no. 24, pp. 5548–5564, 2011.

[45] H.-L. Yang, S.-G. Li, W.-H. Yang, and Y. Lu, "Notes on 'Bipolar fuzzy graphs,'" *Inf. Sci.*, vol. 242, pp. 113–121, Sep. 2013.

[46] S. Samanta and M. Pal, "Bipolar fuzzy hypergraphs," *Int. J. Fuzzy Logic Syst.*, vol. 2, no. 1, pp. 17–28, 2012.

[47] S. Samanta and M. Pal, "Irregular bipolar fuzzy graphs," *Int. J. Appl. Fuzzy Sets*, vol. 2, pp. 91–102, Jan. 2012.

[48] S. Samanta and M. Pal, "Some more results on bipolar fuzzy sets and bipolar fuzzy intersection graphs," *J. Fuzzy Math.*, vol. 22, no. 2, pp. 253–262, 2014.

[49] Z. Gul, "Some bipolar fuzzy aggregations operators and their applications in multicriteria group decision making," M.S. thesis, Hazara Univ., Masnehra, Pakistan, 2015.

[50] G. Wei, F. E. Alsaadi, T. Hayat, and A. Alsaedi, "Hesitant bipolar fuzzy aggregation operators in multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 33, no. 2, pp. 1119–1128, 2017.

[51] M. Lu, G. Wei, F. E. Alsaadi, T. Hayat, and A. Alsaedi, "Bipolar 2-tuple linguistic aggregation operators in multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 33, no. 2, pp. 1197–1207, 2017.

[52] X. R. Xu and G. Wei, "Dual hesitant bipolar fuzzy aggregation operators in multiple attribute decision making," *Int. J. Knowl.-Based Intell. Eng. Syst.*, vol. 21, no. 3, pp. 155–164, 2017.

[53] H. Hamachar, "Uber logische verknunpfungenn unssharfer Aussagen und deren Zugenhorige Bewertungsfunktione Trappl, Klir, Riccardi (Eds.)," *Progr. Cybern. Syst. Res.*, vol. 3, pp. 276–288, 1978.

[54] L. Y. Zhou, X. Zhao, and G. Wei, "Hesitant fuzzy hamacher aggregation operators and their application to multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 26, no. 6, pp. 2689–2699, 2014.

[55] P. Liu, "Some hamacher aggregation operators based on the interval-valued intuitionistic fuzzy numbers and their application to group decision making," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 83–97, Feb. 2014.

[56] C. Tan, W. Yi, and X. Chen, "Hesitant fuzzy Hamacher aggregation operators for multicriteria decision making," *Appl. Soft Comput.*, vol. 26, pp. 325–349, Jan. 2015.

[57] B. Zhu, Z. Xu, and M. Xia, "Dual hesitant fuzzy sets," *J. Appl. Math.*, vol. 2012, Feb. 2012, Art. no. 879629. [Online]. Available: http://www.hindawi.com/journals/jam/2012/879629/

[58] H. J. Wang, X. Zhao, and G. Wei, "Dual hesitant fuzzy aggregation operators in multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 26, no. 5, pp. 2281–2290, 2014.

[59] R. R. Yager, "Prioritized aggregation operators," *Int. J. Approx. Reasoning*, vol. 48, pp. 263–274, Apr. 2008.

[60] F. Chiclana, F. Herrera, and E. Herrera-Viedma, "The ordered weighted geometric operator: Properties and application in MCDM problems," in *Proc. 8th Int. Conf Inf. Process. Manage. Uncertainty Knowl.-Based Syst.*, 2000, pp. 985–991.

[61] Z. Xu and Q. L. Da, "An overview of operators for aggregating information," *Int. J. Intell. Syst.*, vol. 18, no. 9, pp. 953–969, 2003.

[62] G. Wei, F. E. Alsaadi, T. Hayat, and A. Alsaedi, "Hesitant fuzzy linguistic arithmetic aggregation operators in multiple attribute decision making," *Iranian J. Fuzzy Syst.*, vol. 13, no. 4, pp. 1–16, 2016.

[63] R. Lin, X. Zhao, H. Wang, and G. Wei, "Hesitant fuzzy linguistic aggregation operators and their application to multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 27, no. 1, pp. 49–63, 2014.

[64] R. Lin, X. Zhao, and G. Wei, "Models for Selecting an ERP System with Hesitant Fuzzy Linguistic Information," *J. Intell. Fuzzy Syst.*, vol. 26, no. 5, pp. 2155–2165, 2014.

[65] X. Zhao, R. Lin, and G. Wei, "Hesitant triangular fuzzy information aggregation based on einstein operations and their application to multiple attribute decision making," *Expert Syst. Appl.*, vol. 41, no. 4, pp. 1086–1094, 2014.

[66] G. Wei, "Picture fuzzy aggregation operators and their application to multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 33, no. 2, pp. 713–724, 2017.

[67] G. Wei and M. Lu, "Dual hesitant Pythagorean fuzzy Hamacher aggregation operators in multiple attribute decision making," *Arch. Control Sci.*, vol. 27, no. 3, pp. 365–395, 2017.

[68] G. Wei, "Picture 2-tuple linguistic Bonferroni mean operators and their application to multiple attribute decision making," *Int. J. Fuzzy Syst.*, vol. 19, no. 4, pp. 997–1010, 2017.

[69] S.-P. Wan and J.-Y. Dong, "Power geometric operators of trapezoidal intuitionistic fuzzy numbers and application to multi-attribute group decision making," *Appl. Soft Comput.*, vol. 29, pp. 153–168, Apr. 2015.

[70] H. Garg, "Generalized intuitionistic fuzzy multiplicative interactive geometric operators and their application to multiple criteria decision making," *Int. J. Mach. Learn.*, vol. 7, no. 6, pp. 1075–1092, 2016.

[71] G. Wei, M. Lu, F. E. Alsaadi, T. Hayat, and A. Alsaedi, "Pythagorean 2-tuple linguistic aggregation operators in multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 33, no. 2, pp. 1129–1142, 2017.

[72] M. Lu, G. Wei, F. E. Alsaadi, T. Hayat, and A. Alsaedi, "Hesitant pythagorean fuzzy Hamacher aggregation operators and their application to multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 33, no. 2, pp. 1105–1117, 2017.

[73] G. Wei, F. E. Alsaadi, T. Hayat, and A. Alsaedi, "A linear assignment method for multiple criteria decision analysis with hesitant fuzzy sets based on fuzzy measure," *Int. J. Fuzzy Syst.*, vol. 19, no. 3, pp. 607–614, 2007.

[74] G. Wei and M. Lu, "Pythagorean fuzzy maclaurin symmetric mean operators in multiple attribute decision making," *Int. J. Intell. Syst.*, to be published, doi: 10.1002/int.21911.

[75] G. Wei, "Pythagorean fuzzy interaction aggregation operators and their application to multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 33, no. 4, pp. 2119–2132, 2017.

[76] G. Wei and J. Wang, "A comparative study of robust efficiency analysis and data envelopment analysis with imprecise data," *Expert Syst. Appl.*, vol. 81, pp. 28–38, Sep. 2017.

[77] G. Wei, "Interval valued hesitant fuzzy uncertain linguistic aggregation operators in multiple attribute decision making," *Int. J. Mach. Learn. Cybern.*, vol. 7, no. 6, pp. 1093–1114, 2016.

[78] G. Wei, "Picture fuzzy cross-entropy for multiple attribute decision making problems," *J. Bus. Econ. Manage.*, vol. 17, no. 4, pp. 491–502, 2016.

[79] G. Wei, "Interval-valued dual hesitant fuzzy uncertain linguistic aggregation operators in multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 33, no. 3, pp. 1881–1893, 2017.

[80] G. Wei, "Some cosine similarity measures for picture fuzzy sets and their applications to strategic decision making," *Informatics*, vol. 28, no. 3, pp. 547–564, 2017.

[81] J. M. Merigo and M. Casanovas, "Induced aggregation operators in decision making with the Dempster-Shafer belief structure," *Int. J. Intell. Syst.*, vol. 24, pp. 934–954, Aug. 2009.

[82] G. W. Wei, "Picture fuzzy Hamacher aggregation operators and their application to multiple attribute decision making," *Fund. Informat.*, vol. 157, no. 3, pp. 271–320, 2018, doi: 10.3233/FI-2018-1628.

[83] Z. Zhang, C. Wang, D. Tian, and K. Li, "Induced generalized hesitant fuzzy operators and their application to multiple attribute group decision making," *Comput. Ind. Eng.*, vol. 67, pp. 116–138, Jan. 2014.
[84] Z. Zhang and C. Wu, "A decision support model for group decision making with hesitant multiplicative preference relations," *Inf. Sci.*, vol. 282, pp. 136–166, Oct. 2014.
[85] Z. Zhang, "Deriving the priority weights from incomplete hesitant fuzzy preference relations based on multiplicative consistency," *Appl. Soft Comput.*, vol. 46, pp. 37–59, Sep. 2016.
[86] G. W. Wei, F. E. Alsaadi, and T. Hayat, "Ahmed alsaedi, picture 2-tuple linguistic aggregation operators in multiple attribute decision making," *Soft Comput.*, vol. 22, no. 3, pp. 989–1002, 2018.
[87] S.-M. Yu, H.-Y. Zhang, and J.-Q. Wang, "Hesitant fuzzy Linguistic Maclaurin symmetric mean operators and their applications to multi-criteria decision-making problem," *Int. J. Intell. Syst.*, to be published, doi: 10.1002/int.21907.
[88] L. Wang, H.-Y. Zhang, and J.-Q. Wang, "Frank Choquet Bonferroni mean operators of bipolar neutrosophic sets and their application to multi-criteria decision-making problems," *J. Intell. Fuzzy Syst.*, to be published, doi: 10.1007/s40815-017-0373-3.
[89] J.-Q. Wang, Y. Yang, and L. Li, "Multi-criteria decision-making method based on single-valued neutrosophic Linguistic Maclaurin symmetric mean operators," *Neural Comput. Appl.*, to be published, doi: 10.1007/s00521-016-2747-0.
[90] G. W. Wei and Y. Wei, "Similarity measures of pythagorean fuzzy sets based on cosine function and their applications," *Int. J. Intell. Syst.*, vol. 33, no. 3, pp. 634–652, 2018.
[91] G. Wei and M. Lu, "Pythagorean fuzzy power aggregation operators in multiple attribute decision making," *Int. J. Intell. Syst.*, vol. 33, no. 1, pp. 169–186, 2018.
[92] G. Wei, "Picture uncertain Linguistic Bonferroni mean operators and their application to multiple attribute decision making," *Kybernetes*, vol. 46, no. 10, pp. 1777–1800, 2017.

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