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Linear Space-Time Interference Alignment for *K*-User MIMO Interference Channels

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ABSTRACT Interference is believed to be the most significant bottleneck for the next-generation wireless networks to achieve high throughout. Interference alignment (IA), as a novel interference management scheme to break through the traditional interference cancelation, not only makes the complete mitigation of interference possible but also achieves a theoretical breakthrough in promoting the wireless network capacity region. In this paper, by combining the space and time, we proposed a linear space-time (LST) IA algorithm based on the extension of the channel in time dimension for K-user multi-input multi-output interference channel. The proposed LST-IA scheme effectively reduces the number of antennas required for eliminating interference completely in systems, and the closed-form solution of precoding matrices and detector matrices is obtained as well. Compared with the classical IA algorithms, the simulation results demonstrate that the proposed scheme shows distinguished advantages in terms of sum-rate and bit error rate in the strong interference communication scenarios.

INDEX TERMS Multi-input multi-output (MIMO), linear interference alignment, interference channel (IC), channel extension.

I. INTRODUCTION

Interference is one of the basic characteristics in wireless communication systems. Its bottleneck effect on the developments of the next-generation 5G wireless networks is increasingly evident with the continued growth of the mobile Internet and smart devices [1]. The pressing need to improve the efficiency of wireless networks has led to intensive studies of interference and its effect on communication systems [2]. With the developments and evolutions of the wireless technology, on one hand, the communication data traffic and rate surge, on the other hand, the available spectrum resources of the communication systems are increasingly depleted. However, the conventional strategies to deal with interference are orthogonalization; that is, treating other transmitters' signals as noise, or decoding interference, which are powerless to eliminate interference in this dilemma due to their inherent limitations [3].

In order to resolve this dilemma, the effective interference management techniques are gaining attractions, especially in the design of the next-generation 5G wireless communication systems. Interference alignment (IA), as a novel interference management scheme [3], confines the interferences at each receiver into a reduced-dimension subspace by designing the precoding matrices at each transmitter, thus leaving some receiver dimensions free of interference. The research results of the IA show that it is a promising candidate for achieving high throughput in wireless interference networks. Until now, the studies on IA are divided into two categories. One category mainly focused on the theoretical respect, including the achievable degrees of freedom (DoF) and the feasibility conditions of the IA, while the second concentrated on the design of the IA schemes.

In the aspect of the theoretical research IA, the maximum DoF characterizes the number of data streams that can be transmitted free of interferences in the systems, which is the first-order approximation of sum rate capacity at high signal-to-noise ratio (SNR) regime. Cadambe and Jafar [4] show that the DoF is $K/2$ for the K user single antenna interfering channel (IC). In [5], the spatially-normalized information-theoretic DoF through linear beamforming schemes was investigated for a symmetric 3-user Gaussian MIMO IC. In [6], an IA scheme was presented, where $2(M-1)$ DoF was supported when M antennas are used at both the Base Station (BS) and user terminal. The same number of DoF was obtained with $M-1$ antenna elements at user terminal and M antennas at BS by the scheme proposed in [7]. Combining algebraic tools with an induction analysis that indirectly considered the inequalities, Sun and Luo [8] theoretically investigated the maximum DoF achievable via vector space IA for MIMO IC with an arbitrary number of channel extensions and arbitrary diversity order. In [9], the sum-DoF was studied in the two-cell MIMO interfering multiple access channel (IMAC) for a general topology, in which each transmitter was provided with the past state information of the channel from the respective receiver. For the feasibility study of the IA, it was pointed out that in the K-user MIMO-IC, the problem of the feasibility of IA was regarded as the solvability of its corresponding multivariate polynomial system [10]. By the equivalently solvable problem of the polynomial equation to the independent problem of the equations, the sufficient conditions for the feasibility of IA were deduced in [11]. The theoretical research on IA and its related conclusions were very instructive to the designs of the specific IA schemes.

Besides the theoretical investigations of IA, the second research direction is to design the specific IA algorithms. In terms of the existence of the closed-form precoder matrices and the detector matrices, the IA algorithms among these works were divided into the iterative IA approaches and the linear IA approaches. The iterative IA algorithms iteratively optimized the transceiver design to align the interferences. Generally speaking, two types of iterative algorithms had been developed.

The first type of iterative algorithms, including the algorithms in [12]–[14], performed the optimization on the cost functions over both the precoder matrices and detector matrices. Utilizing the reciprocity of wireless channel in time division duplex (TDD) systems, Gomadam *et al.* [12] proposed two representative iterative algorithms, namely minimizing the interference leakage (Min-IL) algorithm and maximizing the signal to noise plus interference ratio (Max-SINR) algorithm. The Min-IL and Max-SINR algorithms respectively utilized the leakage interference power and SINR as the cost function, and updated the precoder matrices and detector matrices by switching the uplink and downlink to optimize cost function. The Min-IL and Max-SINR algorithms represent the general design ideas of the iterative IA algorithms. By alternating the minimization over the precoder matrices at the transmitters and the interference subspaces at the receivers, Peter and Heath [13] proposed an algorithm for IA in the MIMO IC with an arbitrary number of users, antennas, or spatial streams. In [14], an iterative IA algorithm was proposed based on the Gauss-Newton method, which was well-known for its quadratic convergence rate. The most obvious drawback of this type of iterative algorithms was its high design complexity due to the involvement of both the transmitters and receivers.

The second type of the iterative algorithms was called as one-sided algorithms, which conducted the optimization for IA at either the transmitter or the receiver side. Aiming to minimize the power of the interference leaked in the desired signal subspace, Ghauch and Papadias [15] presented a typical one-sided algorithm, i.e., minimum interference strength (Min-IS) algorithm, to design the transmit precoders. Later on, Wang *et al.* [16] achieved the one-sided algorithm for IA solution at the transmitters by minimizing the spatial distance among different interference subspaces. Chen *et al.* [17] introduced the optimization on matrix manifolds into the precoder design for the IA, and limited the optimization only at the transmitters. In [18], a precoder optimization algorithm to maximize the sum rate was proposed for MIMO interference networks in an iterative manner. With more geometrical insights to the IA problem, Bazzi *et al.* [19] achieved the IA with the precoder matrices design only and the detector matrices were designed independently of the IA conditions. Compared with the first type of iterative IA algorithms, the complexity of the second one was reduced dramatically. However, the iterative IA algorithms had common problems of that the convergence speed is relatively slow and the convergence performance is sometimes difficult to guarantee.

Unlike the iterative IA algorithms mentioned above, the linear IA algorithms seek a closed-form solution to completely eliminate the interference. For K-user IC with time-varying fading, a closed-form IA algorithm was developed in [20]. Sung *et al.* [21] designed the linear precoder and detector matrices over MIMO IC, in which the precoder basis vectors were designed to determine the signal subspaces such that the maximum DoF and chordal distance were achieved and the detector matrices were optimized based on the block interference suppression to maximize the individual rate. In [22], a subspace IA was proposed to align interferences into a multi-dimensional subspace (instead of one dimension) for simultaneous alignments at multiple non-intended BSs for IMAC. Due to the fact that inter-user interference (IUI) and inter-cell interference (ICI) coexisted in the multi-cell interfering broadcast channels (IBC), Park and Lee [23] proposed a zero-forcing (ZF) scheme with the aim of maximizing the sum rate performance in a multiple-input single-output (MISO) scenario. Furthermore, in [24], the ZF IA scheme for the MIMO-IBC was extended to the case of multiple receiver antennas. For the symmetric MIMO X channel with constant channel coefficients, the layered IA method was proposed in [25], where both vector and real IA techniques were exploited together with joint processing at receiver sides. Focusing on a two-cell MIMO-IBC, Shin *et al.* [26] proposed a novel IA method jointly designing the precoder matrices and detector matrices in a closed-form expression without iterative computations, which achieved the optimal DoF both analytically and numerically. By extending the work in [26], a new method using the principle of multiple access channel (MAC)-broadcast channel (BC) duality to perform IA while maximizing the capacity of users in each cell was developed [27], which outperformed the IA method in [26] in terms of capacity and complexity. However, the linear IA algorithms not only had strict requirements on the configuration of antennas, but also usually required the global channel knowledge.

With the recent developments on IA techniques, the specific design of the IA algorithms should not limit only over space dimension. In fact, either spatial dimensions or time/frequencey dimensions can be exploited for IA. For two-user X channel, Li *et al.* [28] incorporated Alamouti designs before using beamformers to align symbols at unintended receivers, and each receiver removed aligned interference followed by symbol decoupling via interference cancellation. Zaki *et al.* [29] proposed a scheme that combined the interference-cancellation capability of Alamouti codes with the subspace-overlapping property of IA for the 3-user MIMO IC, which is limited and hard to extend to more users scenarios. In [30], the implications of joint space-frequency precoding for IA were investigated, in which both a sum DoF gain as well as a substantial power gain were achieved by employing space-frequency precoding instead of space-only precoding over the subcarriers. More practically, a space-time IA (ST-IA) approach proposed by Lee and Heath [31] proved that in the underdetermined MISO BC with N_t transmit antennas and $K = N_t + 1$ users N_t sum DoF were achievable for the enough small feedback delay. However, the feedback delay needed to remain less or equal to $\frac{T_c}{N_t+1}$, where T_c is the coherence time. By extending the works to MIMO BC in [32], the full sum DoF was achieved with bigger feedback delay than the results in [31], and the result was also extended to the MIMO IC. Recently, a new IA scheme in [33] was introduced that jointly performs the alignment over space, time and frequency, and a closed-form necessary condition on the feasibility of asymmetric achievable DoF in the corresponding scheme was derived.

The extended IA works mentioned in [28]–[33] perform the IA by jointly designing the transmit scheme within space, time and/or frequency dimensions, and by suppressing the aligned interference at the receiver. However, these works generally constrain in some specific scenarios. Hence, we propose a more general IA scheme, named the linear space-time IA (LST-IA) in this paper, by utilizing the extension of channel over the time dimension for K-user MIMO IC. The LST-IA scheme obtains the closed-form precoder matrices and the detector matrices, which not only eliminates the interference completely, but also simplifies the implementation of IA. Additionally, the introduction of the time dimension significantly reduces the number of antennas required for effective interference suppression, especially the number of receiving antennas.

The organization of the paper is as follows. System model is presented in Section II. In Section III, the LST-IA scheme is presented and its computational complexity is analyzed. Section IV shows several simulation results where the LST-IA scheme is compared to some IA algorithms. Finally, Section V provides concluding remarks.

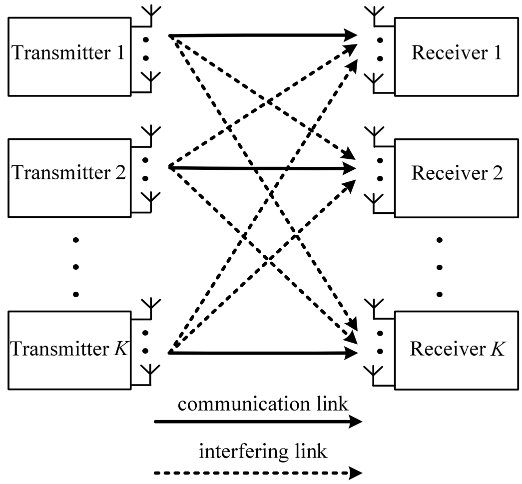

FIGURE 1. MIMO-IC network comprised of K transmitter-receiver pairs.

Throughout the paper, transpose, conjugate transpose, inverse, and trace of a matrix $\mathbf{X}$ are represented by $\mathbf{X}^T$, $\mathbf{X}^H$, $\mathbf{X}^{-1}$, and $\mathrm{tr}(\mathbf{X})$, respectively. $\mathrm{Null}(\mathbf{X})$ denotes an orthonormal basis for the null space of the matrix $\mathbf{X}$. $\complement_A S$ denotes the complement of set S on set A. $A \backslash a$ denotes the set composed of the elements of set A except element $a \in A$.

II. SYSTEM MODEL

Consider a MIMO-IC as shown in Fig. 1, which is comprised of K transmitter-receiver pairs. The transmitter i and receiver k for $i, k \in \Psi \triangleq \{1, 2, \cdots, K\}$ are respectively equipped with N_t and $N_r \leq N_t$ antennas. The transmitter i is assumed to send $d < N_r$ data streams to its corresponding receiver i. Therefore, for receiver $j \neq i$, the transmitter i is the interfering source (IFS). For illustration purposes, we denote this MIMO-IC as an $[K, N_t \times N_r, d]$ system.

In the system, the receiver k will not only receive the signals from the its corresponding transmitter k, but also the interfering signals from IFS $i \in \Psi\backslash k$. Therefore, at the time slot t, the received signal $\mathbf{y}_k(t) \in \mathbb{C}^{N_r \times 1}$ at the receiver k can be modeled as

$$\mathbf{y}_k(t) = \mathbf{H}_{kk}(t)\mathbf{V}_k(t)\mathbf{s}_k + \sum_{i=1, i \neq k}^{K} \mathbf{H}_{ki}(t)\mathbf{V}_i(t)\mathbf{s}_i + \mathbf{z}_k(t), \quad (1)$$

where $\mathbf{H}_{ki}(t) \in \mathbb{C}^{N_r \times N_t}$, $\mathbf{V}_i(t) \in \mathbb{C}^{N_t \times d}$, $\mathbf{s}_i \in \mathbb{C}^{d \times 1}$, and $\mathbf{z}_k(t) \in \mathbb{C}^{N_r \times 1}$ denote the flat fading channel from transmitter i to the receiver k, the precoder matrix associated with the transmitter i, the expected signals at the receiver i with $\mathrm{E}\{||\mathbf{s}_i||^2\} = \mathbf{I}_d$, and the additive Gaussian noise vector with zero mean and variance σ^2 per entry at the time slot t, respectively. After precoding, the transmitted signal is assumed to be satisfied with the power constraint $\mathrm{E}\{||\mathbf{V}_i(t)\mathbf{s}_i||^2\} \leq P_i/d$, where P_i denotes the maximum transmitted power of transmitter i.

III. LINEAR SPACE-TIME INTERFERENCE ALIGNMENT FOR K-USER MIMO IC

A. PREPROCESSING THE RECEIVED SIGNALS

The total number of the transmission time slots required for the proposed LST-IA scheme is set as T. After receiving the corresponding transmitted signal over the T consecutive time slots, a simple and linear preprocessing is performed over them, and the resultant signal is represented by $\mathbf{y}_k \in \mathbb{C}^{N_r \times 1}$ for $k = 1, 2, \ldots, K$.

$$\begin{aligned}\mathbf{y}_k &= \sum_{t=1}^{T} \mathbf{y}_k(t) \\ &= \sum_{t=1}^{T} \mathbf{H}_{kk}(t)\mathbf{V}_k(t)\mathbf{s}_k + \sum_{i=1, i\neq k}^{K} \sum_{t=1}^{T} \mathbf{H}_{ki}(t)\mathbf{V}_i(t)\mathbf{s}_i \\ &\quad + \sum_{t=1}^{T} \mathbf{z}_k(t).\end{aligned} \tag{2}$$

After this, $\mathbf{y}_k$ is processed using the detector matrix $\mathbf{U}_k \in \mathbb{C}^{N_r \times d}$ to form an estimate of its desired transmit signal, which is defined as

$$\begin{aligned}\bar{\mathbf{y}}_k &= \mathbf{U}_k^H \mathbf{y}_k \\ &= \mathbf{U}_k^H \sum_{t=1}^{T} \mathbf{H}_{kk}(t)\mathbf{V}_k(t)\mathbf{s}_k + \mathbf{U}_k^H \sum_{i=1, i\neq k}^{K} \sum_{t=1}^{T} \mathbf{H}_{ki}(t)\mathbf{V}_i(t)\mathbf{s}_i \\ &\quad + \mathbf{U}_k^H \sum_{t=1}^{T} \mathbf{z}_k(t).\end{aligned} \tag{3}$$

On the right-hand side of (3), the first term $\mathbf{U}_k^H \sum_{t=1}^{T} \mathbf{H}_{kk}(t) \mathbf{V}_k(t)\mathbf{s}_k$ is the desired signal vector sent by the transmitter k, while the second term $\mathbf{U}_k^H \sum_{i=1, i\neq k}^{K} \sum_{t=1}^{T} \mathbf{H}_{ki}(t)\mathbf{V}_i(t)\mathbf{s}_i$ represents the interference signals from other transmitters.

Assuming the LST-IA scheme is feasible for the above MIMO IC, the alignment is achieved when the precoder matrices and detector matrices $\{\mathbf{U}_k, \mathbf{V}_i\}_{k,i=1}^{K}$ satisfy the following IA conditions, and they are

$$\mathrm{Rank}(\mathbf{U}_k^H \sum_{t=1}^{T} \mathbf{H}_{kk}(t)\mathbf{V}_k(t)) = d, \tag{4}$$

$$\mathbf{U}_k^H \sum_{i=1, i\neq k}^{K} \sum_{t=1}^{T} \mathbf{H}_{ki}(t)\mathbf{V}_i(t) = 0. \tag{5}$$

The polynomial (4) is a rank constraint to extract the desired signals for each receiver, and (5) ensures the elimination of interferences from the unwanted transmitters, i.e., IFSs. Once the conditions above are satisfied, the desired signal s_k in (3) is theoretically solvable.

B. DESIGNING THE PRECODER MATRICES AND DETECTOR MATRICES

Firstly, $\bar{\mathbf{y}}_k$ can be rewritten as

$$\bar{\mathbf{y}}_k = \mathbf{U}_k^H \mathbf{H}_{kk}\mathbf{s}_k + \mathbf{U}_k^H \sum_{i=1, i\neq k}^{K} \mathbf{H}_{ki}^{IF} s_i + \mathbf{U}_k^H \mathbf{z}_k, \tag{6}$$

where $\mathbf{H}_{kk} = \sum_{t=1}^{T} \mathbf{H}_{kk}(t)\mathbf{V}_k(t) \in \mathbb{C}^{N_r \times d}$ is the equivalent channel matrix associated with the desired sources, $\mathbf{H}_{ki}^{IF} = \sum_{t=1}^{T} \mathbf{H}_{ki}(t)\mathbf{V}_i(t) \in \mathbb{C}^{N_r \times d}$ is the equivalent IC associated with IFS i, $\mathbf{z}_k = \sum_{t=1}^{T} \mathbf{z}_k(t)$, and $i \in \{1, 2, \ldots, k-1, k+1, \ldots K\}$.

Let $\mathbf{H}_k^{IF} \in \mathbb{C}^{N_r \times (K-1)d}$ denote the total equivalent IC matrix related to the receiver k, which can be written as

$$\mathbf{H}_k^{IF} = [\mathbf{H}_{k1}^{IF}, \mathbf{H}_{k2}^{IF}, \cdots, \mathbf{H}_{k(k-1)}^{IF}, \mathbf{H}_{k(k+1)}^{IF}, \ldots, \mathbf{H}_{kK}^{IF}]. \tag{7}$$

In order to align interferences and guarantee the existence of the detector matrix $\mathbf{U}_k$, the following condition must hold by reasonably designing the precoder matrices

$$\mathrm{rank}(\mathbf{H}_k^{IF}) \leq N_r - d. \tag{8}$$

From formula (8), the number of linearly independent columns in $\mathbf{H}_k^{IF}$ is not more than $N_r - d$, which means the dimension of the interference subspace is less than or equal to $N_r - d$. To further analyze the matrix $\mathbf{H}_k^{IF}$, $\mathbf{H}_{ki}^{IF}$ is

$$\mathbf{H}_{ki}^{IF} = [\mathbf{H}_{ki1}^{IF}, \mathbf{H}_{ki2}^{IF}, \ldots, \mathbf{H}_{kid}^{IF}], \tag{9}$$

where $\mathbf{H}_{kin}^{IF} \in \mathbb{C}^{N_r \times 1}$ denotes the n^{th} column of $\mathbf{H}_{ki}^{IF}$ for $n \in \{1, 2, \ldots, d\}$. And we have

$$\mathbf{H}_{ki}^{IF} = \sum_{t=1}^{T} \mathbf{H}_{ki}(t)\mathbf{V}_i(t). \tag{10}$$

Utilizing (10), the n^{th} column of $\mathbf{H}_{ki}^{IF}$ is then represented by

$$\mathbf{H}_{kin}^{IF} = \sum_{t=1}^{T} \mathbf{H}_{ki}(t)\mathbf{V}_{in}(t), \tag{11}$$

where $\mathbf{V}_{in}(t)$ is the n^{th} column of $\mathbf{V}_i(t)$.

For the receiver K, there exist $K - 1$ IFSs. However, the rank of $\mathbf{H}_k^{IF}$ is no more than $N_r - d$. Thereby, if $N_r - d \geq K - 1$, the independent interference subspace can be allocated for each IFS. The dimensions of the interference subspaces for all the IFSs vary with the relationship between $N_r - d$ and $K - 1$, but always are greater than or equal to 1. In this case, IA is conducted only in intra-IFS. If $N_r - d < K - 1$, IA must be conducted in intra-IFS and in inter-IFS. This is, for some IFSs, the independent interference subspaces are allocated and the IA is performed only in intra-IFS. For the rest of the IFSs, the IA must be carried out in inter-IFS and intra-IFS successively. Since the design of precoder matrices cannot be unified, the following two cases are described.

1) $N_r - d \geq K - 1$

For receiver k, the D_{ki}-dimension independent subspace is assumed to be allocated to IFS i, and D_{ki} is satisfied with

$$\begin{cases} \sum_{i=1, i\neq k}^{K} D_{ki} = N_r - d, \\ D_{ki} \geq 1, \\ |D_{ki} - D_{km}| \leq 1, \end{cases} \quad (12)$$

where $|\cdot|$ denotes the absolute value operator, and $i, m \in \{1, 2, \ldots, k-1, k+1, \ldots, K\}$. In (12), the first expression ensures the dimensions of interferences of each receiver are set as $N_r - d$, the second expression requires the interference dimension allocated to IFS i is more than 1, while the third expression is a constraint condition to guarantee the difference of interference dimensions for different IFSs not more than 1. The matrix $\mathbf{H}_{ki}^{IF}$ has D_{ki} independent columns, which means that the first D_i columns of $\mathbf{V}_i(t)$ can be arbitrarily initialized as long as the power constraint condition is met, where D_i is chosen as $\min(D_{1i}, \ldots, D_{ji}, \ldots, D_{Ki})$ with $j \neq i$ in order to make the LST-IA theoretically realizable. And the D_{ki}-dimension interference subspace composed of $\mathbf{H}_{ki1}^{IF}, \mathbf{H}_{ki2}^{IF}, \ldots, \mathbf{H}_{kiD_{ki}}^{IF}$ can be regarded as the alignment baseline of the IFS i for the receiver k, which is denoted by $\mathbf{R}_{ki} = [\mathbf{H}_{ki1}^{IF}\ \mathbf{H}_{ki2}^{IF} \cdots \mathbf{H}_{kiD_{ki}}^{IF}]$. From the point of preserving the power of the desired signals, $\mathbf{V}_{i1}(t), \mathbf{V}_{i2}(t), \ldots, \mathbf{V}_{iD_i}(t)$ can be designed as the D_i eigenvectors corresponding to the first D_i maximum eigenvalues of $\mathbf{H}_{kk}^H \mathbf{H}_{kk}$. The design of the other columns of $\mathbf{V}_i(t)$ can be described as follows.

According to the analysis above, for receiver k, the precoder matrix $\mathbf{V}_i(t)$ is designed to satisfy the rank constraint condition associated with $\mathbf{V}_i(t)$:

$$\text{rank}(\mathbf{H}_{ki}^{IF}) = D_{ki}, \quad (13)$$

where $i = 1, 2, \ldots, K$ with $i \neq k$. For simplicity, the first D_{ki} columns of $\mathbf{H}_{ki}^{IF}$ are assumed to be linearly independent. Then according to the formula (13), we have

$$\mathbf{H}_{kin}^{IF} = \alpha_{n,1}\mathbf{H}_{ki1}^{IF} + \alpha_{n,2}\mathbf{H}_{ki2}^{IF} + \cdots + \alpha_{n,D_{ki}}\mathbf{H}_{kiD_{ki}}^{IF}, \quad (14)$$

where $n \in \{D_{ki}+1, D_{ki}+2, \ldots, d\}$, and $\alpha_{n,1}, \alpha_{n,2}, \ldots, \alpha_{n,D_{ki}}$ respectively denote combination coefficients. Namely, $\mathbf{H}_{kin}^{IF}$ can be represented by the linear combination of $\mathbf{H}_{ki1}^{IF}, \mathbf{H}_{ki2}^{IF}, \ldots, \mathbf{H}_{kiD_{ki}}^{IF}$, and the corresponding interferences are aligned into the space spanned by $\mathbf{R}_{ki}$. And if $D_{ki} > D_i$, $V_{i(D_i+1)}(t), V_{i(D_i+2)}(t), \ldots, V_{iD_{ki}}(t)$ are unknown, and then the same to $\mathbf{H}_{ki(D_i+1)}^{IF}, \mathbf{H}_{ki(D_i+2)}^{IF}, \ldots, \mathbf{H}_{kiD_{ki}}^{IF}$ from formula (11). Hence, the corresponding linear combination coefficients $\alpha_{n,D_i+1}, \alpha_{n,D_i+2}, \ldots, \alpha_{n,D_{ki}}$ only need be set to zeros for simplifying the subsequent derivations and this implication does not violate the concept of the IA. Combining formulas (11) and (14), a set of equations related to $\mathbf{H}_{ki}(t)$ and $\mathbf{V}_i(t)$ can be obtained for $t = 1, 2, \ldots, T$ and expressed in matrix form as (15), shown at the bottom of this page.

In (15), $\tilde{\mathbf{H}}_{ki}^{IF} \in \mathbb{C}^{N_r \times TN_t}$, $\bar{\mathbf{H}}_{ki}^{IF} \in \mathbb{C}^{N_r \times (d-D_{ki})}$, $\bar{\mathbf{V}}_i \in \mathbb{C}^{TN_t \times (d-D_{ki})}$ is the partitioned matrix composed of the precoder matrix $\mathbf{V}_{in}(t)$ related with IFS i for $D_{ki}+1 \leq n \leq d$ and $t = 1, 2, \ldots, T$, and $\mathbf{H}_{ki(D_{ki}+1)}^{IF}, \mathbf{H}_{ki(D_{ki}+2)}^{IF}, \ldots, \mathbf{H}_{kid}^{IF}$ respectively denote the linear combination of the $\mathbf{H}_{ki1}^{IF}, \mathbf{H}_{ki2}^{IF}, \ldots, \mathbf{H}_{kiD_i}^{IF}$ from (14). By designing precoding matrix $\bar{\mathbf{V}}_i$ in (15), the IFS i for the receiver k is aligned on the baseline $\mathbf{R}_{ki}$.

Due to the broadcast features of the wireless signals, the transmitter i is the IFS for all the receivers except the receiver i, which means each interfered receiver should be taken into consideration in the design of $\mathbf{V}_i(t)$. Therefore, totally $K-1$ sets of equations are obtained as (15). By using the simultaneous equations, the total system of equations is shown as (16), at the bottom of this page.

In (16), $k \in \{1, 2, \ldots, K\}, k \neq i$, $\tilde{\mathbf{H}}_i^{IF} \in \mathbb{C}^{(K-1)N_r \times TN_t}$, $\tilde{\mathbf{V}}_i \in \mathbb{C}^{TN_t \times (d-D_i)}$, $\hat{\mathbf{H}}_{ki}^{IF} \in \mathbb{C}^{N_r \times (d-D_i)}$, and $\hat{\mathbf{H}}_i^{IF} \in$

$$\underbrace{\begin{bmatrix} (\mathbf{H}_{ki}(1))^T \\ (\mathbf{H}_{ki}(2))^T \\ \vdots \\ (\mathbf{H}_{ki}(T))^T \end{bmatrix}^T}_{\tilde{\mathbf{H}}_{ki}^{IF}} \underbrace{\begin{bmatrix} \mathbf{V}_{i(D_{ki}+1)}(1) & \mathbf{V}_{i(D_{ki}+2)}(1) & \cdots & \mathbf{V}_{i(d)}(1) \\ \mathbf{V}_{i(D_{ki}+1)}(2) & \mathbf{V}_{i(D_{ki}+2)}(2) & \cdots & \mathbf{V}_{i(d)}(2) \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{V}_{i(D_{ki}+1)}(T) & \mathbf{V}_{i(D_{ki}+2)}(T) & \cdots & \mathbf{V}_{i(d)}(T) \end{bmatrix}}_{\bar{\mathbf{V}}_i} = \underbrace{\begin{bmatrix} (\mathbf{H}_{ki(D_{ki}+1)}^{IF})^T \\ (\mathbf{H}_{ki(D_{ki}+2)}^{IF})^T \\ \vdots \\ (\mathbf{H}_{kid}^{IF})^T \end{bmatrix}^T}_{\bar{\mathbf{H}}_{ki}^{IF}}. \quad (15)$$

$$\underbrace{\begin{bmatrix} \tilde{\mathbf{H}}_{1i}^{IF} \\ \vdots \\ \tilde{\mathbf{H}}_{ki}^{IF} \\ \vdots \\ \tilde{\mathbf{H}}_{Ki}^{IF} \end{bmatrix}}_{\tilde{\mathbf{H}}_i^{IF}} \underbrace{\begin{bmatrix} \mathbf{V}_{i(D_i+1)}(1) & \mathbf{V}_{i(D_i+2)}(1) & \cdots & \mathbf{V}_{i(d)}(1) \\ \mathbf{V}_{i(D_i+1)}(2) & \mathbf{V}_{i(D_i+2)}(2) & \cdots & \mathbf{V}_{i(d)}(2) \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{V}_{i(D_i+1)}(T) & \mathbf{V}_{i(D_i+2)}(T) & \cdots & \mathbf{V}_{i(d)}(T) \end{bmatrix}}_{\tilde{\mathbf{V}}_i} = \underbrace{\begin{bmatrix} \hat{\mathbf{H}}_{1i}^{IF} \\ \vdots \\ \hat{\mathbf{H}}_{ki}^{IF} \\ \vdots \\ \hat{\mathbf{H}}_{Ki}^{IF} \end{bmatrix}}_{\hat{\mathbf{H}}_i^{IF}}. \quad (16)$$

$\mathbb{C}^{(K-1)N_r\times(d-D_i)}$, respectively. When $D_{ki}=D_i$, $\hat{\mathbf{H}}_{ki}^{IF}=\bar{\mathbf{H}}_{ki}^{IF}$. When $D_{ki}>D_i$, $\hat{\mathbf{H}}_{ki}^{IF}=[\bar{\mathbf{H}}_{ki}^{IF}\ \Upsilon]$, where $\Upsilon\in\mathbb{C}^{N_r\times(D_{ki}-D_i)}$ is a matrix formed by arbitrary complex numbers. Since the channel coefficients are independent and identical distributed, $\tilde{\mathbf{H}}_i^{IF}$ is full-rank in terms of row or column with the probability of 1. In order to ensure that $\tilde{\mathbf{V}}_i$ has at least one solution with $d-D_i$ dimensions for all the cases of $\hat{\mathbf{H}}_i^{IF}$, the following conditions must be satisfied.

$$\begin{cases} TN_t-(K-1)N_r>d-D_i, \\ \text{rank}(\tilde{\mathbf{H}}_i^{IF})=(K-1)N_r. \end{cases} \tag{17}$$

Finally, with the power constraint, the resultant precoder matrices for $n=D_i+1, D_i+2,\ldots,d$ can be represented as

$$\mathbf{V}_{in}(t)=\sqrt{1/\rho}\mathbf{V}_{in}(t), \tag{18}$$

where ρ denotes the maximum value of $\text{tr}\left(\mathbf{V}_{in}^H(t)\mathbf{V}_{in}(t)\right)$ for $t=1,2,\ldots,T$.

2) $1\le N_r-d<K-1$

In the case of $1\le N_r-d<K-1$, each IFS cannot be allocated an independent interference subspace. Thereby the interference subspace of some IFSs must be overlapped. For this case, the design of the precoder matrices can be summarized as follows: the IA in inter-IFS is first performed to obtain the alignment baseline of each IFS, and then the IA is performed in intra-IFS to align the interference streams to their corresponding alignment baselines.

Taking the receiver 1 as the interfered target to analyze. The N_r-d dimensions of $\mathbf{H}_1^{IF}$ are assumed to respectively assign to the transmitter $2,3,\ldots,N_r-d+1$, and let the set $\Xi_1^{(1)}=\{2,3,\ldots,N_r-d+1\}$ denote the indices of these N_r-d transmitter interfered with transmitter 1 (More schemes to assign the dimensions of $\mathbf{H}_1^{IF}$ can be founded and the same effects can be obtained from these schemes). Then we have

$$\begin{cases} \text{rank}(\mathbf{H}_{12}^{IF})=\text{rank}(\mathbf{H}_{13}^{IF})=\cdots=\text{rank}(\mathbf{H}_{1(N_r-d+1)}^{IF})=1, \\ \text{rank}([\,\mathbf{H}_{12}^{IF}\ \mathbf{H}_{13}^{IF}\ \cdots\ \mathbf{H}_{1(N_r-d+1)}^{IF}\,])=N_r-d. \end{cases} \tag{19}$$

Let

$$\mathbf{R}_{12}=[\mathbf{H}_{121}^{IF}], \tag{20}$$
$$\mathbf{R}_{13}=[\mathbf{H}_{131}^{IF}], \tag{21}$$
$$\vdots$$
$$\mathbf{R}_{1(N_r-d+1)}=[\mathbf{H}_{1(N_r-d+1)1}^{IF}]. \tag{22}$$

For IFSs $N_r-d+2, N_r-d+3,\ldots,K$, the dimension of the interference subspace for each of them is set as 1 and the interference subspace of them must be overlapped with the subspace spanned by one of $\mathbf{R}_{1i}$ for $i\in\Xi_1^{(1)}$. Therefore, one obtains

$$\mathbf{R}_{1(N_r-d+2)}=\beta_1\mathbf{R}_{1i_{11}}=[\mathbf{H}_{1(N_r-d+2)1}^{IF}], \tag{23}$$
$$\mathbf{R}_{1(N_r-d+3)}=\beta_2\mathbf{R}_{1i_{12}}=[\mathbf{H}_{1(N_r-d+3)1}^{IF}], \tag{24}$$
$$\vdots$$
$$\mathbf{R}_{1K}=\beta_x\mathbf{R}_{1i_{1x}}=[\mathbf{H}_{1K1}^{IF}], \tag{25}$$

where $x=K-1-(N_r-d)$, $i_{11},i_{12},\ldots,i_{1x}\in\Xi_1^{(1)}$, and $\beta_1,\beta_2,\ldots,\beta_x$ represent any real numbers, respectively. In fact, the numbers $i_{11},i_{12},\ldots,i_{1x}$ may be equal, and thus set $\Xi_1^{(2)}$ is consisted of the different numbers in $\{i_{11},i_{12},\ldots,i_{1x}\}$. For the purpose of convenient analysis, we here still use $\Xi_1^{(2)}=\{i_{11},i_{12},\ldots,i_{1x}\}$. Combining formula (11), a set of equations can be obtained according to formulas from (24) to (25),

$$\beta_1\tilde{\mathbf{H}}_{1i_{11}}^{IF}V_{i_{11}1}=\tilde{\mathbf{H}}_{1(N_r-d+2)}^{IF}\mathbf{V}_{(N_r-d+2)1}, \tag{26}$$
$$\beta_2\tilde{\mathbf{H}}_{1i_{12}}^{IF}V_{i_{12}1}=\tilde{\mathbf{H}}_{1(N_r-d+3)}^{IF}\mathbf{V}_{(N_r-d+3)1}, \tag{27}$$
$$\vdots$$
$$\beta_x\tilde{\mathbf{H}}_{1i_{1x}}^{IF}\mathbf{V}_{i_{1x}1}=\tilde{\mathbf{H}}_{1K}^{IF}\mathbf{V}_{K1}, \tag{28}$$

where $\mathbf{V}_{i1}=[\,\mathbf{V}_{i1}^T(1)\ \mathbf{V}_{i1}^T(2)\cdots\mathbf{V}_{i1}^T(T)\,]^T\in\mathbb{C}^{TN_t\times 1}$ and $i=1,2,\ldots,K$. Let $\mathrm{C}=\{1,2,3,\ldots,K\}$ and $\mathrm{B}_1=\mathrm{C}\backslash 1$ as a universal set of $\Xi_1^{(1)}$ (For receiver k, $\mathrm{B}_k=\mathrm{C}\backslash k$ denotes the set consisted of the elements of the set C except element k.). An investigation of (26)-(28) reveals that the elements of the sets $\Xi_1^{(2)}$ and $\complement_{\mathrm{B}_1}\Xi_1^{(1)}$ have the respective correspondences. If $\Xi_1^{(2)}$ and $\complement_{\mathrm{B}_1}\Xi_1^{(1)}$ are determined, the forms of the formulas (26)-(28) are specific. From the analysis above, there are multiple alternative options for the elements contained in the sets $\Xi_1^{(1)}$ and $\Xi_1^{(2)}$. For simplicity, any proper subset of B_1 with cardinality N_r-d can be taken as $\Xi_1^{(1)}$, while one of the subsets of $\Xi_1^{(1)}$ with cardinality $K-1-(N_r-d)$ can be as $\Xi_1^{(2)}$.

In the same way, the similar formulas as (26)-(28) can be obtained after B_k, $\Xi_k^{(1)}$ and $\Xi_k^{(2)}$ are determined for the rest of receivers $k\neq 1$. And the total number of $N=KN_r((K-1)-(N_r-d))$ formulas are obtained with $M\le KTN_t$ variables in order to design V_{i1} for $i=1,2,\ldots,K$. The specific value of M is related to $\Xi_k^{(2)}$ and $\complement_{\mathrm{B}_k}\Xi_1^{(1)}$ for $k=1,2,\ldots,K$. Hence the indexes of all the transmitters should be contained in $\Xi_1^{(2)},\Xi_2^{(2)},\ldots,\Xi_K^{(2)}$ and $\complement_{\mathrm{B}_1}\Xi_1^{(1)},\complement_{\mathrm{B}_2}\Xi_1^{(1)},\ldots,\complement_{\mathrm{B}_1}\Xi_K^{(1)}$ in order to guarantee these formula solvable, which means that $M=KTN_t$ holds. Even it is the case, the sets $\Xi_k^{(1)}$ and $\Xi_k^{(2)}$ for $k=1,2,\ldots,K$ have a variety of options. In the paper, a simple scheme is presented to construct $\Xi_k^{(1)}$ and $\Xi_k^{(2)}$ for $k=1,2,\ldots,K$, which is given in Table 1 in the pseudo-code form.

Note: In Table 1, $\mathrm{B}_k(i)$ denotes the i^{th} element of B_k in the ascending order. Utilizing the method in Table 1, the sets $\Xi_1^{(2)},\Xi_2^{(2)},\ldots,\Xi_K^{(2)}$ and $\complement_{\mathrm{B}_1}\Xi_1^{(1)},\complement_{\mathrm{B}_2}\Xi_2^{(1)},\ldots,\complement_{\mathrm{B}_K}\Xi_K^{(1)}$ are guaranteed to include all the indexes of IFSs.

From analysis above, K groups equations as (26)-(28) related with $\mathbf{V}_{i1}$ are obtained for $i=1,2,\ldots,K$. Hence, there exist $N=KN_r((K-1)-(N_r-d))$ equations that contain $M=KTN_t$ variables. According to the Bezout theorem that a multivariate polynomial equation is solvable

TABLE 1. The method to construct the sets $\Xi_k^{(1)}$ and $\Xi_k^{(2)}$ for $k = 1, 2, \ldots, K$.

for $k = 1, 2, ..., K$;
  $C = \{1, 2, ..., K\}$, $B_k = C \backslash k$
  for $i = 1, 2, ..., N_r - d$
   $\Xi_k^{(1)}(i) = B_k(i)$
  end for
  if $k \leq N_r - d$
   for $j = 1, 2, ..., (K - 1 - (N_r - d))$
    $\Xi_k^{(2)}(j) = k + 1$
   end for
  else
   for $j = 1, 2, ..., (K - 1 - (N_r - d))$;
    $\Xi_k^{(2)}(j) = 1$
   end for
  end if
end for

if and only if the number of equations does not exceed the number of variables. Therefore, the conditions to guarantee the existence of nonzero solution of $\mathbf{V}_{i1}$ for $i = 1, 2, \ldots, K$ can be expressed as

$$TKN_t > KN_r\left((K-1) - (N_r - d)\right). \tag{29}$$

It is noted that there are multiple options to align $K - 1$ IFSs into $N_r - d$-dimension interference subspace. In the paper, a simple solution is presented to assign $N_r - d$ interference dimensions to $N_r - d$ IFSs one by one. The scheme not only ensures the existence of nonzero solution of $\mathbf{V}_{11}, \mathbf{V}_{12}, \ldots, \mathbf{V}_{K1}$ and solves its all at one-time, but also guarantees the subsequent conduction of IA in inter-IFS and intra-IFS and the complete alignment of the residual interferences. Regardless of the assignment schemes, the proposed IA method in the paper comes down to design $\Xi_k^{(1)}$ and $\Xi_k^{(2)}$ for $k = 1, 2, \ldots, K$. The only difference is that the elements of the $\Xi_k^{(1)}$ and $\Xi_k^{(2)}$ for $k = 1, 2, \ldots, K$ are different for different assignment schemes. When $\Xi_1^{(2)}, \Xi_2^{(2)}, \ldots, \Xi_K^{(2)}$ and $\complement_{B_1}\Xi_1^{(1)}, \complement_{B_2}\Xi_1^{(1)}, \ldots, \complement_{B_1}\Xi_K^{(1)}$ do not contain all transmitter indexes, the first column of precoding matrix related with the uncontained transmitters can be initialized as any value that satisfies the power constraint.

Similar to the case of $N_r - d \geq K - 1$, imposing the power constraint on $\mathbf{V}_{11}(t), \mathbf{V}_{21}(t), \ldots, \mathbf{V}_{K1}(t)$ yields

$$\mathbf{V}_{i1}(t) = \sqrt{1/\sigma}\,\mathbf{V}_{i1}(t), \tag{30}$$

where σ is the maximum value of $\mathrm{tr}(\mathbf{V}_{i1}^H(t)\mathbf{V}_{i1}(t))$ for $i = 1, 2, \ldots, K$ and $t = 1, 2, \ldots, T$.

After obtaining $\mathbf{V}_{11}(t), \mathbf{V}_{21}(t), \ldots, \mathbf{V}_{K1}(t)$, the alignment baseline of each IFS is determined, and then IA in intra-IFS is conducted. It is of interest to note that the design of the other columns of $\mathbf{V}_i(t)$ for $i = 1, 2, \ldots, K$ is similar as the case of $N_r - d \geq K - 1$ to design the $D_i + 1^{th}$ to d^{th} columns in $\mathbf{V}_i(t)$. Due to the fact that $D_i = 1$, the constraint condition in formula (17) can be transformed as follows.

$$\begin{cases} TN_t - (K-1)N_r > d - 1, \\ \mathrm{rank}(\tilde{H}_i^{IF}) = (K-1)N_r. \end{cases} \tag{31}$$

Obviously, the constraint condition in (31) includes the condition in (29).

Thus far, the precoder matrices design has been completed and the inequality (8) is satisfied. Correspondingly, the receiver detector matrix for the receiver k is given by

$$\mathbf{U}_k = \mathrm{null}(\mathbf{H}_k^{IF}), \tag{32}$$

It is noted that for the case $N_r - d \geqslant K - 1$, the local CSI is required since IA is conducted only in intra-IFS, while for the case $1 \leqslant N_r - d < K - 1$, the global CSI is required since the IA is carried out in inter-IFS and intra-IFS successively.

C. COMPUTATIONAL COMPLEXITY ANALYSIS

In the respect of the algorithm complexity analysis, the complex multiplication (CM) as a indicator is widely used. In this section, CM is also employed to analyze the computational complexity of the proposed LST-IA scheme, and comparison is also conducted with that of the classic IA algorithms.

The number of CM required by the main steps of the LST-IA scheme are given as follows.

1) DETERMINING THE ALIGNMENT BASELINE

For $N_r - d \geq K - 1$, this step does not cause any complexity overhead. For $N_r - d < K - 1$, IA is conducted in inter-IFSs, the key of which is to solve $K - 1$ groups of equations as (26)-(28). According to the conclusions in [27], the number of the required CMs to solve all these equations is approximately

$$\mathcal{O}\{\min((KTN_t)^2 KN_r(K - 1 + d - N_r), \quad KTN_t\,(KN_r(K - 1 + d - N_r))^2)\}. \tag{33}$$

2) PERFORMING IA IN INTRA-IFSS

This step is mainly related to solve the equation (16), and the number of the required CMs is approximately $\mathcal{O}\{\min((TN_t)^2(K-1)N_r,\ TN_t((K-1)N_r)^2)\}$.

3) COMPUTING RECEIVER BEAMFORMING MATRIX

As shown in (32), the number of CMs to compute the null space of $\mathbf{H}_k^{IF} \in \mathbb{C}^{N_r \times (K-1)N_t}$ is approximately $\mathcal{O}\left\{\min\left({N_r}^2(K-1)N_t, N_r((K-1)N_t)^2\right)\right\}$.

Taking into consideration the fact that N_t is far greater than the N_r in downlink, and the number and the growth rate of N_t must far exceed K and d in order to eliminate the interferences completely, the total number of the required CMs for the LST-IA scheme now is:

◇ when $1 \leq K - 1 < N_r - d$

$$\mathcal{O}\left\{KTN_t(KN_r(K - 1 + d - N_r))^2 + TN_t((K-1)N_r)^2 + {N_r}^2(K-1)N_t\right\}. \tag{34}$$

◇ when $N_r - d \geq K - 1$

$$\mathcal{O}\left\{TN_t((K-1)N_r)^2 + {N_r}^2(K-1)N_t\right\}. \tag{35}$$

In [34], the computational complexity of the Min-IL algorithm is given by

$$\mathcal{O}\Big\{LK((K-1)(N_rN_t+N_r{}^2)d+(N_r{}^2+2N_r)) + LK((K-1)(N_rN_t+N_t{}^2)d+(N_t{}^2+2N_t))\Big\}, \quad (36)$$

where L denotes the iteration times. The experiments show that hundreds of iterations are generally required by the Min-IL algorithm in order to effectively suppress the interference, and it increases with the number of IFSs.

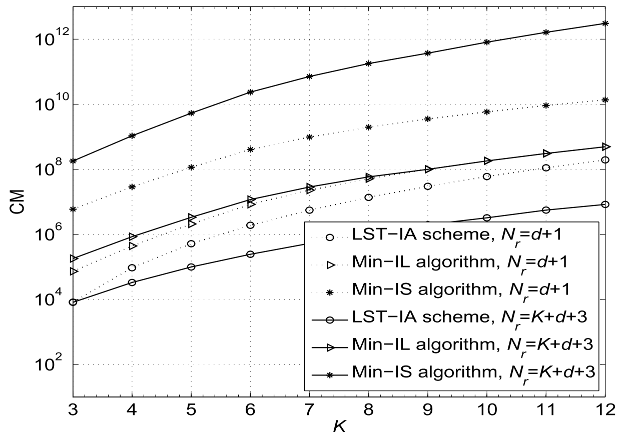

FIGURE 2. **CM comparison between the LST-IA scheme and Min-IL algorithm with *K*.**

The number of the CMs required by the Min-IS algorithm is approximately [15]

$$\mathcal{O}\Big\{K^2L((K-1)(N_rN_td+N_r{}^2d)+N_r{}^3+3dN_r{}^2N_t) + KLd(K(N_r{}^2+N_r)+4KN_r{}^4N_t+N_td)\Big\}. \quad (37)$$

In what follows, to clearly observe the complexity comparisons, two experiments are conducted for proposed LST-IA scheme, the Min-IL and Min-IS algorithms.

For an $[K, N_t \times N_r, 3]$ system, Fig.2 depicts the complexity comparison versus the number of users K for the two cases of $1 \le N_r - d < K - 1$ and $N_r - d \ge K - 1$, respectively. For Min-IL and Min-IS algorithms, the iteration times of $L = \{25, 50, 100, 200, 300, 400, 500, 600, 700, 800\}$ are correspondingly set for $K = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. For the two cases, the number of the receiving antennas are set as $d + 1$ and $K + d + 3$, respectively. The number of the transmitting antennas is set as the minimum values, which satisfy the relationship $N_t + N_r \ge (K + 1)d$ and $N_t \ge N_r$. It is seen that, with the increase of K, the computational complexities of the LST-IA scheme and other two iterative algorithms increase for the two cases, but our proposed LST-IA scheme is obviously lower than that of the Min-IL and Min-IS algorithms. Additionally, for the cases of $1 \le N_r - d < K - 1$ and $N_r - d \ge K - 1$, the Min-IL algorithm almost has the same complexity when the K is relatively large because the key factor that affected the complexity is just the K from formula (36).

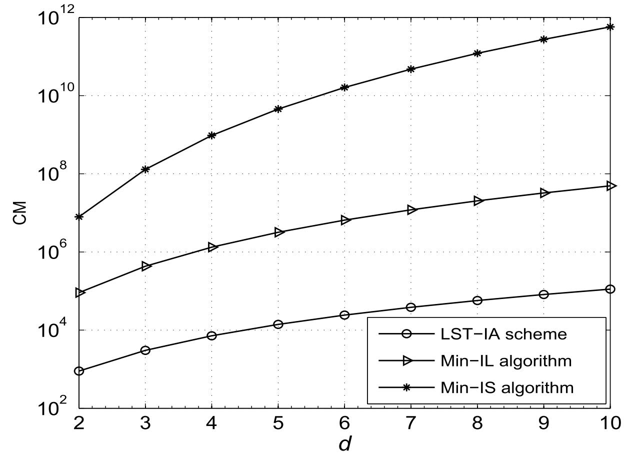

FIGURE 3. **CM comparison with *d*.**

For an $[3, N_t \times N_r, d]$ system, Fig.3 illustrates the complexity versus the parameter d for the LST-IA scheme, the Min-IL algorithm and the Min-IL algorihtm for $N_t = N_r$. For the Min-IL and Min-IS algorithms, the iterative time L is set as 15 for $d = 2$ and L increases with increment of 10 when d increases 1. As shown in Fig. 3, although the complexities of the the LST-IA scheme, the Min-IL algorithm and the Min-IS algorithm increase with the parameter d, the complexity of LST-IA scheme is much smaller than other two IA schemes, which indicates that our proposed LST-IA schemes is more feasible in multi-stream communication systems.

IV. COMPUTER SIMULATIONS

In this section, the performance of the LST-IA scheme is evaluated via Monte Carlo simulations in $[5, N_t \times N_r, 3]$ system, and is also compared with other typical IA algorithms of Min-IL and Min-IS.

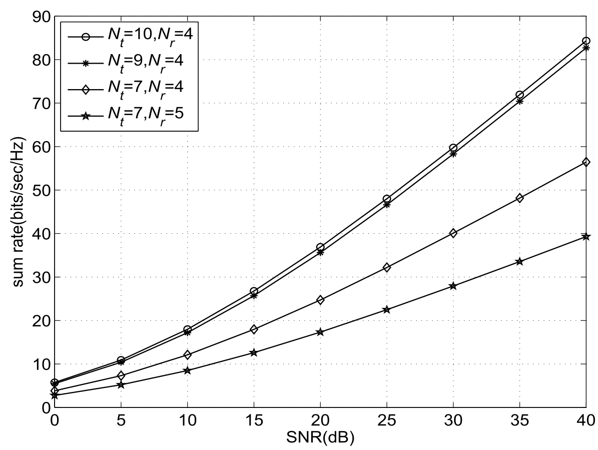

FIGURE 4. **Sum-rate comparison against different antennas configurations for LST-IA scheme.**

First, Fig. 4 illustrates the sum-rate of LST-IA scheme versus signal-noise-ratio (SNR) for several configurations of the transmitting and receiving antennas. From Fig.4, the sum-rate of the LST-IA scheme for the $[5, 10 \times 4, 3]$ system is almost the same as that for $[5, 9 \times 4, 3]$ system. This is because that the complete interference mitigation has more strong effect on the sum-rate performance than the increased transmit antennas, although for the $[5, 9 \times 4, 3]$ and $[5, 10 \times 4, 3]$ systems, the interferences can be completely mitigated with the required minimum time slot $T = 2$. Additionally, for the $[5, 7 \times 5, 3]$ system, the obvious degradation of the sum-rate of the LST-IA scheme can be observed in contrast to the $[5, 7 \times 4, 3]$ system. According to the analysis, the required minimum time slot for the $[5, 7 \times 5, 3]$ system is 5, while it is 4 for the $[5, 7 \times 4, 3]$ system, which leads to the different DoF between these two systems and then the different sum-rates.

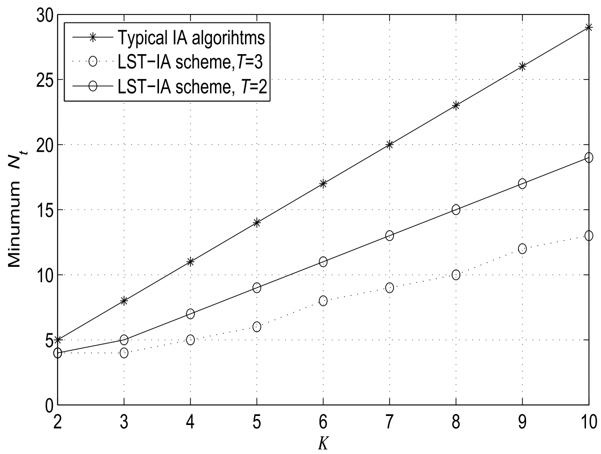

FIGURE 5. **Minimum number of transmitting antennas N_t versus K.**

Second, the simulations are conducted to verify that the LST-IA scheme can significantly reduce number of antennas required to eliminate all the interferences. In the MIMO-IC, the relationship $\min\{N_t, N_r\} > d$ is generally satisfied. In the downlink, the receiving antennas are generally limited, and here we set $N_r = d + 1$. According to the feasible conditions in [10], two typical IA algorithms, including Min-IL algorithm and Min-IS algorithm (Min-IL algorithm belongs to two-side approach and Min-IS algorithm belongs to one-side approach), required that the number of the transmitting and receiving antennas satisfied $N_t + N_r \geq (K + 1)d$ in multi-user MIMO-IC in order to efficiently mitigate the interferences. But in the LST-IA scheme, the number of the transmitting and receiving antennas is constrained by the time slot T in (17). Fig. 5 provides the simulation results of the minimum number of the transmitting antennas required versus K for the two typical IA algorithms and our proposed LST-IA scheme. In addition, Fig.6 depicts the comparison of the minimum number of the transmitting antennas required versus d for the two typical IA algorithms and our proposed LST-IA scheme. In the simulation, the time slot T for our scheme is set as 2 and 3, respectively. From these two figures, it is demonstrated that the total number of transceiver antennas required by the LST-IA scheme is significantly reduced compared to the classical IA algorithms. The superiority of

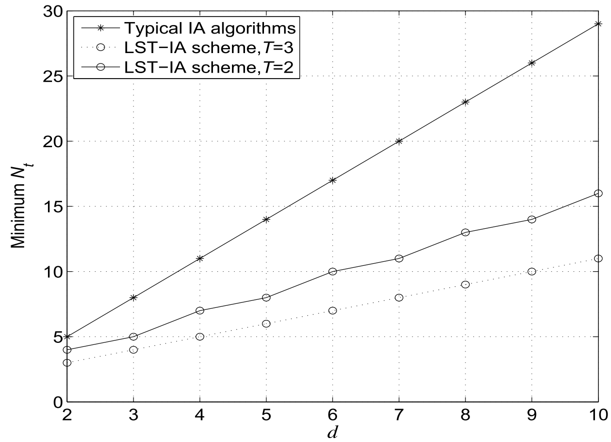

FIGURE 6. **Minimum number of transmitting antennas N_t versus d.**

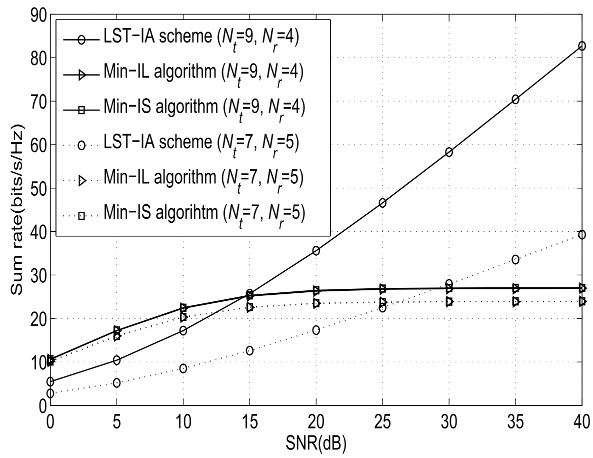

FIGURE 7. **Sum-rate versus SNR.**

the LST-IA scheme over two typical IA algorithms benefits from the introduction of the time dimension in designing the IA scheme. Namely, it is that the reduction of transceiver antennas is exchanged from sacrificing more time dimension resource.

Finally, the sum-rate and BER performance of our proposed LST-IA scheme for each channel use are compared with the Min-IL and Min-IS algorithms for $[5, 9 \times 4, 3]$ and $[5, 7 \times 5, 3]$ systems, which are depicted in Fig. 7 and Fig. 8, respectively. According to expression (17) and (31), the minimum time slot T set as 2 and 4 for $[5, 9 \times 4, 3]$ system and $[5, 7 \times 5, 3]$ system, respectively. In the comparison, the iterative times of the Min-IL and Min-IS algorithms are set as 100. In order to keep the same data rate for our proposed LST-IA scheme and other two counterpart algorithms for each channel use, BPSK modulation is adopted for these two counterpart algorithms, while QPSK and 16QAM modulation are respectively used in $[5, 9 \times 4, 3]$ and $[5, 7 \times 5, 3]$ systems for the LST-IA scheme. In MIMO-IC, each transmitter is the IFS of other transmitters. With the increase of the SNR, the power of IFS are strengthened. From the Fig.7 and Fig. 8, the LST-IA scheme demonstrates superior performance over

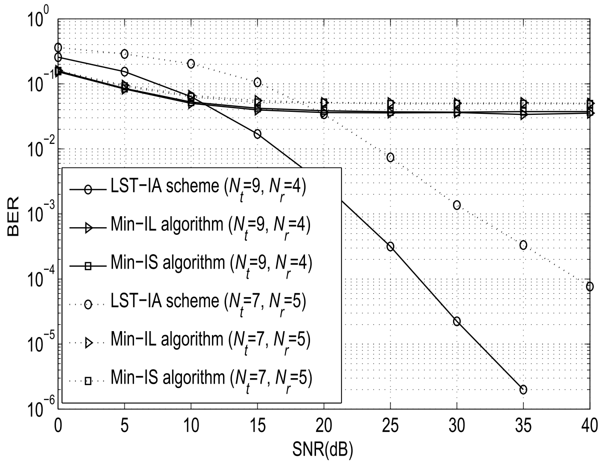

FIGURE 8. **BER versus SNR.**

other two algorithms in terms of the sum-rate and BER in stronger interference scenarios. This is because that the LST-IA scheme can completely eliminate all the interference signals in both systems. For the Min-IL and Min-IS algorithms, more transceiver antennas are required in order to gain the ability to suppress the interferences, which indicates that the LST-IA scheme effectively reduces the required number of transceiver antennas under the premise of mitigating all interferences, and hence it is more practical. In the relatively low SNR region, noise power instead of the power of the interfering signals became the main factor to degrade the system performance, and the preprocessing of the received signals in the LST-IA scheme adopts the simple superposition of all receivers. Hence, the performance of the proposed LST-IA scheme is worse than these two competing algorithms, but the performance gap is not large. In addition, it should be noted that Min-IL algorithm has almost the same sum rate as the Min-IS algorithm in [15].

V. CONCLUSIONS

To efficiently suppress the interference in K-user MIMO system, this paper presents the LST-IA scheme by combining the space-time resources, which not only achieves the aim of simplifying the realization of IA, but also reduces the total number of antennas required for removing all the interferences. The simulation results show that the LST-IA scheme is superior to classical IA algorithms requiring relatively smaller number of the transceiving antennas, and has the potential to achieve more better tradeoff between the number of transceiver antennas and sum rate. It is of interest to point out that the proposed scheme presents modest noise supersession ability at relatively low SNR region, and enhancing the system performance in this case will be our next focus.

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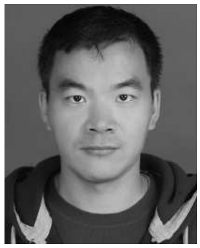

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