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Classification of Methods in the SINS/CNS Integration Navigation System

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ABSTRACT The known methods used for strapdown inertial navigation system (SINS)/celestial navigation system (CNS) integration are classified based on two categories of measurement in this paper. One category is called the "attitude observation method," in which the measurement is derived by the difference between the optimal attitude information of the star sensor and the SINS. The other category is called the "star vector observation method," in which the measurement is derived by the difference between the original star vector information of the star sensor. The attitude angle observation equation of the first category is generally obtained by using the relationship between the attitude angle errors and the Phi-angle (or tilt errors), and the attitude matrix observation equation is obtained by using the relationship between the attitude matrix and the Psi-angle (or platform errors). However, the interrelationship between these two observation equations has not been developed in previous studies. A simpler attitude angle observation method based on the Psi-angle instead of the Phi-angle is proposed to reveal the interrelationship between these two methods. This proposed method is basically the principle behind the SINS/CNS integration and depicts the physical meaning clearly. In addition, the internal relationships of the second category and the interrelationship of these two categories are also analyzed to show their equivalence to each other. Numerical simulations verify the correctness of the analysis. Experimental studies indicate that the integration accuracy of the two categories is also exactly equivalent.

INDEX TERMS Strap-down inertial navigation system (SINS), celestial navigation system (CNS), attitude observation, star vector observation.

I. INTRODUCTION

The initial misalignment errors and the inertial instrument errors of SINS further accumulated because of the existence of double integrators that make the SINS results unsuitable for high-precision navigation in ballistic and spacecraft applications [1]. To enhance the performance and reliability of the navigation system, GPS (global positioning system) is popularly used along with SINS [2]. However, the anti-jamming problems and degraded accuracy in hostile environments makes GPS a poor choice for autonomous and precise navigation. In comparison to GPS, CNS has better and wider utilization of a star sensor on ballistics and satellite. SINS/CNS, manufactured by NASA, has been widely used in the trident ballistic missions and the Mars missions [3]. SINS/CNS is characterized by being accurate, autonomous, reliable, inexpensive, and practically independent of all external inputs but only for low dynamic applications [4], [5].

The advancement in optoelectronics and image processing techniques has made possible the development of the LFOV (Large Field of View) CCD (Charged Coupled Device) star sensor, which can provide arc-second level accuracy of attitude in the inertial frame. Consequently, SINS/CNS has been established as a very high-accuracy integration navigation system. To date, several comparatively mature schemes of SINS/CNS integrated mode have been investigated. The theoretical relevance of these schemes has not been sufficiently discussed in the past studies. Revealing the relationship between the several known algorithms is a significant research effort in the field of navigation. There are two major categories of SINS/CNS integration methods. One category is the combination based on the optimal attitude information, such as the attitude matrix, the attitude angles and the quaternion determined by the original star vector information of the star sensor. An innovative SINS/CNS deep integrated scheme based on an attitude matrix was introduced by He *et al.* [6]. They calculated the Psi-angle using the star sensor and SINS attitude matrix in inertial reference. As reported in [7] and [8], the star sensor attitude angles from the body frame to the navigation frame can be obtained by the body attitude with respect to the inertial coordinate and the SINS position; subsequently, the Phi-angle can be estimated based on the difference between the star sensor attitude angles and the SINS attitude angles. Hao *et al.* [9] and Hong *et al.* [10] used the attitude information provided by CNS in the quaternion observation equation; this approach can not only meet the requirements of integration accuracy but also reduce the amount of calculation, thereby improving the accuracy of remote-range ballistic missile stability. All methods mentioned previously use the attitude information integration method. The attitude angle observation equation is deduced by using the inertial error dynamic equations in the Phi-angle formulation, which can estimate the value of the tilt errors. The other attitude observation equation is deduced by using the inertial error dynamic equations in the Psi-angle formulation, which can estimate the value of platform error angles. Although the Phi-angle formulation and the Psi-angle formulation are equivalent in terms of inertial error [11], the relationship in the SINS/CNS integrated method still must be further studied.

The other integration methods are based on a combination of star sensor original information, such as the star vectors and the celestial angles; the principle of these methods has been discussed adequately in previous studies [12], [13]. The star sensor original information should be projected in the computer frame provided by SINS. Consequently, the observation equation is deduced in the Psi-angle formulation and is able to estimate the values of the platform error angles.

A variety of combination methods have been studied previously because of their unclear interrelationship. Bar-Itzhack [14] showed the connection between the derivative approach and the estimation approach for angular velocity estimation of a gyro-less spacecraft. Different velocity errors in either a computer frame or a true frame are naturally equivalent in the Doppler-Inertial error analysis [15]. There must be a certain interrelation between the two different categories of combination methods in the SINS/CNS Integration Navigation System; it is the purpose of this paper to elucidate their common denominator. The mathematical equivalence is proven theoretically in the attitude information and the star vector observation method for SINS/CNS integration system. The difference caused by the stochastic character of different integration methods is not within the scope of this work and is a topic of further study. In addition, numerical simulations and experimental studies were conducted to verify the analysis conclusion. In this paper, the sole purpose of the deduction and verifications that are presented is to show the interrelationship between the methods.

This paper is organized as follows. Section 1 presents the introduction and discusses the background and significance of the study. Section 2 builds the integration methods, such as attitude angle observation, attitude matrix observation, celestial angled observation and star vector observation in the navigation frame. Section 3 provides the internal relationship and equivalence deduction in each category method and the interrelationship between the two categories. Section 4 presents simulations and experimental studies and demonstrates the efficiency of the deduction. The final section provides the conclusion and presents some novel points regarding classification of algorithms in the SINS/CNS integration system.

II. PRINCIPLES OF THE INTEGRATION METHODS

A. DEFINITION OF THE REFERENCE FRAMES

The star sensor can report the body attitude with respect to the inertial coordinate system continuously and help provide navigation information through coordinate transformation. The coordinate frames used in this paper are defined as follows.

The geocentric inertial frame (i-frame) is a coordinate frame that has its origin at the center of the earth and is non-rotating with respect to the fixed stars.

The earth frame (e-frame) has its origin at the earth's center of mass and is fixed in the earth.

The computer frame (n(c)-frame) is the local-level coordinate frame located at the computed position.

The platform frame (n(s)-frame) is the navigation frame built by the computed attitude of the SINS.

The navigation frame (n-frame) is the Wander Azimuth (WA) frame, which is rotated with respect to the local geographic east-north-upward (E-N-U) navigation frame z-axis by the WA angles a.

The body frame (b-frame) is the frame in which the accelerations and angular rates generated by SINS are resolved. The star sensor and SINS undergo strap down installation, and the star senor frame (s-frame) coincides with the b-frame.

B. PRINCIPLE OF SINS/CNS BASED ON ATTITUDE ANGLE OBSERVATION

The star sensor provides the body attitude with respect to the inertial coordinate system continuously by observing the relative position of a star in space. The star sensor attitude matrix is calculated by the following mathematical relationship.

$$\mathbf{C}_b^{n(c)} = \mathbf{C}_e^{n(c)}\mathbf{C}_i^e\mathbf{C}_b^i \tag{1}$$

where $\mathbf{C}_b^{n(c)}$ is the star sensor attitude matrix from the body frame to the calculating navigation frame obtained by SINS position. $\mathbf{C}_b^i$ is the transformation matrix from the body frame to the geocentric inertial frame of the star sensor. $\mathbf{C}_e^{n(c)}$ is the SINS position transformation matrix. $\mathbf{C}_i^e$ is the transformation matrix from the geocentric inertial frame to the earth frame related to the earth's rotation and the navigation time. The star sensor attitude angle based on the SINS position can be calculated using the above formula, and its error can be defined as

$$\begin{cases} \delta\theta_{n(c)} = \hat{\theta}_{n(c)} - \theta \\ \delta\gamma_{n(c)} = \hat{\gamma}_{n(c)} - \gamma \\ \delta\varphi_{n(c)} = \hat{\varphi}_{n(c)} - \varphi \end{cases} \tag{2}$$

where $\delta\theta_{n(c)}$, $\delta\gamma_{n(c)}$, and $\delta\varphi_{n(c)}$ are the star sensor pitch error, roll error and yaw error, respectively. $\hat{\theta}_{n(c)}$, $\hat{\gamma}_{n(c)}$, and $\hat{\varphi}_{n(c)}$ are the star sensor pitch, roll and yaw, respectively. $\theta\gamma$ and φ are the corresponding true values of attitude. The definition of attitude referring to [16] is adopted. The transformation from the navigation frame to the body axes proceeds through yaw, pitch, and roll rotations.

Similarly, the attitude angle error of SINS can be defined in a manner similar to the star sensor attitude error.

$$\begin{cases} \delta\theta_{n(c)} = \hat{\theta}_{n(c)} - \theta \\ \delta\gamma_{n(c)} = \hat{\gamma}_{n(c)} - \gamma \\ \delta\varphi_{n(c)} = \hat{\varphi}_{n(c)} - \varphi \end{cases} \tag{3}$$

where $\delta\theta_{n(c)}$, $\delta\gamma_{n(c)}$, and $\delta\varphi_{n(c)}$ are the SINS pitch error, roll error and yaw error, respectively. $\hat{\theta}_{n(c)}$, $\hat{\gamma}_{n(c)}$, and $\hat{\varphi}_{n(c)}$ are the SINS pitch attitude, roll attitude and yaw attitude, respectively. The observation equation based on the difference in attitude angles between the star sensor and the SINS can be written as below, and its details can be found in [7].

$$\begin{aligned} \mathbf{Z} &= \begin{bmatrix} \hat{\theta}_{n(s)} - \hat{\theta}_{n(c)} \\ \hat{\gamma}_{n(s)} - \hat{\gamma}_{n(c)} \\ \hat{\varphi}_{n(s)} - \hat{\varphi}_{n(c)} \end{bmatrix} = \begin{bmatrix} \delta\theta_{n(s)} - \delta\theta_{n(c)} \\ \delta\gamma_{n(s)} - \delta\gamma_{n(c)} \\ \delta\varphi_{n(s)} - \delta\varphi_{n(c)} \end{bmatrix} \\ &= \mathbf{H}_{\varphi}^{1} \begin{bmatrix} \varphi_{n(s)x} \\ \varphi_{n(s)y} \\ \varphi_{n(s)z} \end{bmatrix} - \mathbf{H}_{\delta\theta}^{1} \begin{bmatrix} \delta\theta_{n(c)x} \\ \delta\theta_{n(c)y} \\ \delta\theta_{n(c)z} \end{bmatrix} \end{aligned} \tag{4}$$

where

$$\mathbf{H}_{\varphi}^{1} = \begin{bmatrix} -\cos\hat{\varphi}_{n(s)} & -\sin\hat{\varphi}_{n(s)} & 0 \\ \sin\hat{\varphi}_{n(s)}\sec\hat{\theta}_{n(s)} & -\cos\hat{\varphi}_{n(s)}\sec\hat{\theta}_{n(s)} & 0 \\ -\tan\hat{\theta}_{n(s)}\sin\hat{\varphi}_{n(s)} & \tan\hat{\theta}_{n(s)}\cos\hat{\varphi}_{n(s)} & -1 \end{bmatrix}$$

$$\mathbf{H}_{\delta\theta}^{1} = \begin{bmatrix} -\cos\hat{\varphi}_{n(c)} & -\sin\hat{\varphi}_{n(c)} & 0 \\ \sin\hat{\varphi}_{n(c)}\sec\hat{\theta}_{n(c)} & -\cos\hat{\varphi}_{n(c)}\sec\hat{\theta}_{n(c)} & 0 \\ -\tan\hat{\theta}_{n(c)}\sin\hat{\varphi}_{n(c)} & \tan\hat{\theta}_{n(c)}\cos\hat{\varphi}_{n(c)} & -1 \end{bmatrix}$$

Z is the observation vector, and $\varphi_{n(s)x}$, $\varphi_{n(s)y}$ and $\varphi_{n(s)z}$ can be called tilt errors [11]. $\delta\theta_{n(c)x}$, $\delta\theta_{n(c)y}$ and $\delta\theta_{n(c)z}$ are the position errors. This observation equation is suitable for the Phi-angle formulation. Another useful form is the Psi-angle formulation, which yields simpler attitude error dynamic equations and offers the potential for reduced computational demands of the on-board computer for the star sensor.

C. PRINCIPLE OF SINS/CNS BASED ON ATTITUDE MATRIX/QUATERNION OBSERVATION

The attitude matrix is a type of superficial characteristic for attitude that is the same as that of quaternion. In other words, they can be equivalent to each other. Thus, it is appropriate to select one of the methods for analysis. The star sensor provides the body attitude with respect to the inertial coordinate system, and it can also provide the body attitude with respect to the computer frame n(c)-frame, with the help of the position and the predictable motion of the earth in the SINS/CNS integration navigation system. The SINS provides the body attitude with respect to the platform frame (n(s)-frame). The platform error as a measurement can be written as

$$\begin{aligned} \mathbf{C}_b^{n(s)} &= [\mathbf{I} + (\boldsymbol{\psi}_{n(s)}\times)](\mathbf{C}_b^{n(c)}) \\ &\Rightarrow [\mathbf{I} + (\boldsymbol{\psi}_{n(s)}\times)] = \mathbf{C}_b^{n(s)}(\mathbf{C}_e^{n(c)}\mathbf{C}_i^{e}\mathbf{C}_b^{i})^T \end{aligned} \tag{5}$$

where $\mathbf{C}_b^{n(s)}$ is the attitude matrix from the body frame to the navigation frame obtained by SINS. $\boldsymbol{\psi}_{n(s)}$ is the platform misalignments of SINS in the n(s) frame. $\boldsymbol{\psi}_{n(s)}\times$ is the askew matrix of $\boldsymbol{\psi}_{n(s)}$. Using the small-angle approximation and neglecting the higher-order errors, the skew symmetric matrix of platform error vector can be simplified as

$$\boldsymbol{\psi}_{n(s)}\times = \begin{bmatrix} 0 & -\psi_{n(s)z} & \psi_{n(s)y} \\ \psi_{n(s)z} & 0 & -\psi_{n(s)x} \\ -\psi_{n(s)y} & \psi_{n(s)x} & 0 \end{bmatrix} \tag{6}$$

where $\psi_{n(s)x}$ $\psi_{n(s)y}$ $\psi_{n(s)z}$ are the platform errors projected in the three directions. The observation equation based on attitude information is defined as follows (7), as shown at the bottom of the next page:

where $\mathbf{C}_b^{n(s)}(i,j)$ is the i_{th} column and the j_{th} row element of the matrix $\mathbf{C}_b^{n(s)}$, and $\mathbf{C}_b^{n(c)}(i,j)$ is the i_{th} column and the j_{th} row element of the matrix $\mathbf{C}_b^{n(c)}$. It is easy to see that the observation matrix $\mathbf{H}_{\psi}^{2} = \mathbf{I}_{3\times 3}$

D. PRINCIPLE OF SINS/CNS BASED ON CELESTIAL ANGLES OBSERVATION

A unit vector pointing to a star in the reference frame is given by [17].

$$\boldsymbol{l}_n = \begin{bmatrix} \sin AZ_n \cos EL_n & \cos AZ_n \cos EL_n & \sin EL_n \end{bmatrix}^T \tag{8}$$

where $\boldsymbol{l}$ is the unit star vector, AZ is the celestial azimuth angle and EL is the celestial elevation angle. The nearly fixed positions of star in space and the predictable motion of the earth define a known star vector in the i-frame. The star vector is then transformed from the i-frame into the computer frame n(c)-frame through the transformation matrices computed from the position of SINS and the stars' catalogs. Alternatively, the star vector can be obtained from the b-frame into the platform-frame n(s)-frame through the transformation matrices computed from the attitude of the SINS. The relationship of the star vector projected in different frames can be expressed as [13]

$$\boldsymbol{l}_{n(s)} = [\mathbf{I} - (\boldsymbol{\psi}_{n(s)}\times)]\boldsymbol{l}_{n(c)} \tag{9}$$

The corresponding measurement information of the celestial angles ΔEL and ΔAZ are the main objectives of this integration algorithm, which can be written as

$$\Delta EL = EL_{n(s)} - EL_{n(c)}, \quad \Delta AZ = AZ_{n(s)} - AZ_{n(c)} \tag{10}$$

where EL_{ref} and AZ_{ref} are called the celestial angles in the reference frame, including the computer-frame n(c)-frame and platform-frame n(s)-frame in [5]. Substituting (8) and (10) into equation (9), using the small-angle approximation for both ΔEL and ΔAZ and neglecting the higher-order errors, results in its simplified form as follows in [13],

$$\mathbf{Z} = \begin{bmatrix} \Delta EL \\ \Delta AZ \end{bmatrix} = \mathbf{H}_{\psi}^{3} \cdot \begin{bmatrix} \psi_{n(s)x} \\ \psi_{n(s)y} \\ \psi_{n(s)z} \end{bmatrix} \tag{11}$$

where $\mathbf{Z}$ is the measurement vector of the celestial angle observation method. Next, the observation matrix can be written as

$$\mathbf{H}_{\psi}^{3} = \begin{bmatrix} -\cos AZ_{n(c)} & \sin AZ_{n(c)} & 0 \\ -\tan EL_{n(c)} \sin AZ_{n(c)} & -\tan EL_{n(c)} \cos AZ_{n(c)} & 1 \end{bmatrix}$$

E. PRINCIPLE OF SINS/CNS BASED ON STAR VECTOR OBSERVATION

Generally, the vector of star $\boldsymbol{l}_{n(c)}$ in the n(c)-frame can be obtained autonomously by the star sensor, and the position can be obtained by SINS. Thus, the vector of star $\boldsymbol{l}_{n(s)}$ in the p-frame can be described as follows [13]:

$$\boldsymbol{l}_{n(s)} = [I - (\boldsymbol{\psi}_{n(s)} \times)]\boldsymbol{l}_{n(c)} \tag{12}$$

where $\mathbf{C}_b^{n(s)}$ is the attitude matrix determined by SINS. $\boldsymbol{l}_b$ is the star vector in the b-frame.

The corresponding measurement information of the star vector can be written as

$$\begin{aligned} \mathbf{Z} = \boldsymbol{l}_{n(s)} - \boldsymbol{l}_{n(c)} &= -(\boldsymbol{\psi}_{n(s)} \times)(\hat{\boldsymbol{C}}_{\mathrm{e}}^{n(c)} \mathbf{C}_i^e \boldsymbol{l}_i) \\ &= [(\hat{\boldsymbol{C}}_{\mathrm{e}}^{n(c)} \mathbf{C}_i^e \boldsymbol{l}_i)\times]\boldsymbol{\psi}_{n(s)} \end{aligned} \tag{13}$$

where $(\hat{\boldsymbol{C}}_{\mathrm{e}}^{n(c)} \mathbf{C}_i^e \boldsymbol{l}_i \times)$ is the skew symmetric matrix of the star vector projected in the computer-frame n-frame. The general observation matrix of this method is listed as follows (14), as shown at the bottom of the next page:

The majority of the methods of data fusion related to SINS and star sensor integration are shown in Table 1 and are classified into two categories.

III. DETAILED ANALYSIS OF THE INTERRELATIONSHIP

A. CLASSIFICATION OF THE FOUR METHODS

The four methods are based on one key equation that relates the platform error or the tilt error, expressed by the derivative of the attitude or the star vector information from the star sensor and SINS. The differences are the measurement information and the attitude dynamic error model in the 'φ' and 'ψ' formulations. Different measurement information leads to different measurement noise characteristics and different degrees of observability. However, different attitude dynamic error formulations reveal the same attitude error propagation characteristic. This relationship is basically the principle behind the four methods. The major integration methods mentioned above can be classified into two categories according to the output information of the star sensor. The first method is called the "attitude observation method" applied to the large field of view (LFOV) star sensor; its measurement is derived between the indirectly optimal attitude information of star sensor and attitude of SINS. The second method is called the "star vector observation method", in which the measurement is obtained between the directly original star vector information of the star sensor and the attitude of SINS and is applied to the narrow field of view (NFOV) star sensor.

B. EQUIVALENCE BETWEEN THE TWO METHODS IN THE FIRST CATEGORY

The attitude angle observation equation of the first category is generally derived by using the relationship between the attitude angle errors and the tilt errors. The other observation equation is derived by using the relationship between the star vector and the platform error angles. A superficial inspection of the methods may lead to the conclusion that the attitude angles observation equations are not related to one another. However, they can be unified through the same dynamic error 'ψ' formulation. The equivalence between the attitude angles and the attitude matrix observation is deduced as follows.

1) ATTITUDE ANGLE OBSERVATION EQUATION BASED ON THE 'ψ' FORMULATION

In this part, a new and simpler attitude angle observation equation based on the platform error angles is proposed. To demonstrate the correspondence between the two approaches, the inter-relation between the attitude angle errors and platform error angles, which is crucial for a navigation system and not developed in previous studies, should be deduced. Consider the 'φ' formulation that defines the attitude error as [11]:

$$\boldsymbol{\varphi} \equiv \boldsymbol{\psi} + \delta\boldsymbol{\theta} \tag{15}$$

where $\boldsymbol{\varphi}$ is the tilt error vector, and $\boldsymbol{\psi}$ is the platform error angle vector defined in platform frame. This angle $\delta\boldsymbol{\theta}$ is the position error vector.

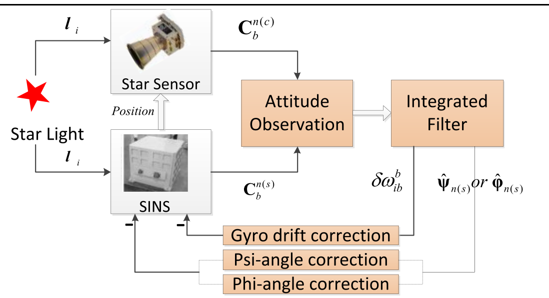
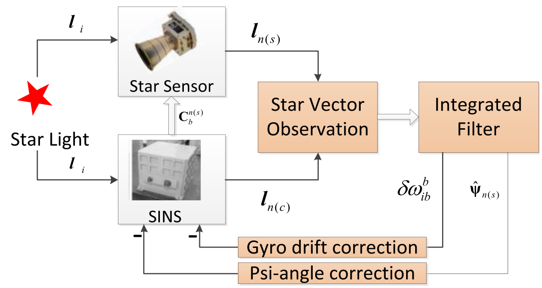
Substituting Eq. (15) into Eq. (4), the observation equation based on the difference of attitude angles is modified again as

$$\mathbf{Z} = \mathbf{H}_{\varphi}^{1} \begin{bmatrix} \psi_{n(s)x} \\ \psi_{n(s)y} \\ \psi_{n(s)z} \end{bmatrix} + (\mathbf{H}_{\varphi}^{1} - \mathbf{H}_{\delta\theta}^{1}) \begin{bmatrix} \delta\theta_{n(c)x} \\ \delta\theta_{n(c)y} \\ \delta\theta_{n(c)z} \end{bmatrix} \tag{16}$$

Let $\delta\theta$, $\delta\gamma$ and $\delta\varphi$ represent the differences between the SINS attitude and the star sensor attitude; it can be concluded

$$\mathbf{Z} = \begin{bmatrix} \mathbf{C}_b^{n(s)}(3,1)\mathbf{C}_{n(c)}^{b}(1,2) + \mathbf{C}_b^{n(s)}(3,2)\mathbf{C}_{n(c)}^{b}(2,2) + \mathbf{C}_b^{n(s)}(3,3)\mathbf{C}_{n(c)}^{b}(3,2) \\ \mathbf{C}_b^{n(s)}(1,1)\mathbf{C}_{n(c)}^{b}(1,3) + \mathbf{C}_b^{n(s)}(1,2)\mathbf{C}_{n(c)}^{b}(2,3) + \mathbf{C}_b^{n(s)}(1,3)\mathbf{C}_{n(c)}^{b}(3,3) \\ \mathbf{C}_b^{n(s)}(2,1)\mathbf{C}_{n(c)}^{b}(1,1) + \mathbf{C}_b^{n(s)}(2,2)\mathbf{C}_{n(c)}^{b}(2,1) + \mathbf{C}_b^{n(s)}(2,3)\mathbf{C}_{n(c)}^{b}(3,1) \end{bmatrix} = \mathbf{I}_{3\times3} \begin{bmatrix} \psi_{n(s)x} \\ \psi_{n(s)y} \\ \psi_{n(s)z} \end{bmatrix} \tag{7}$$

TABLE 1. Integration methods for SINS/CNS.

	Attitude integration		Star vector integration	
Observation	Attitude angle	Attitude matrix/Quaternion	Celestial angle	Star vector
Measurement	$\mathbf{Z}=\begin{bmatrix}\hat{\theta}_{n(s)}-\hat{\theta}_{n(c)}\\ \hat{\gamma}_{n(s)}-\hat{\gamma}_{n(c)}\\ \hat{\varphi}_{n(s)}-\hat{\varphi}_{n(c)}\end{bmatrix}$	$\mathbf{Z}=(\mathbf{C}_e^{n(c)}\mathbf{C}_i^e\mathbf{C}_b^i)\mathbf{C}_b^{n(s)}$ $\mathbf{Z}=(\mathbf{Q}_e^{n(c)}*\mathbf{Q}_i^e*\mathbf{Q}_b^i)*\mathbf{Q}_b^{n(s)}$	$\mathbf{Z}=\begin{bmatrix}EL_{n(s)}-EL_{n(c)}\\ AZ_{n(s)}-AZ_{n(c)}\end{bmatrix}$	$\mathbf{Z}=\mathbf{C}_b^{n(s)}\mathbf{l}_b-\mathbf{C}_e^{n(c)}\mathbf{C}_i^e\mathbf{l}_i$
Observation in the 'ψ' formulation	$\mathbf{Z}=\mathbf{H}_\psi^1\boldsymbol{\psi}$	$\mathbf{Z}=\mathbf{H}_\psi^2\boldsymbol{\psi}$	$\mathbf{Z}=\mathbf{H}_\psi^3\boldsymbol{\psi}$	$\mathbf{Z}=\mathbf{H}_\psi^4\boldsymbol{\psi}$
Observation in the 'φ' formulation	$\mathbf{Z}=\mathbf{H}_\varphi^1\boldsymbol{\varphi}+\mathbf{H}_{\delta\theta}^1\delta\boldsymbol{\theta}$	—	—	—
'φ' error model	$\dot{\mathbf{\Phi}}^n=-(\boldsymbol{\omega}_{in}^n\times)\mathbf{\Phi}^n+\delta\boldsymbol{\omega}_{in}^n+\boldsymbol{\varepsilon}^n$			
'ψ' error model	$\dot{\mathbf{\Psi}}^{n(c)}=-(\boldsymbol{\omega}_{in(c)}^{n(c)}\times)\mathbf{\Psi}^{n(c)}+\boldsymbol{\varepsilon}^{n(c)}$			
Integration schemes				
Star sensor output	Attitude information determined by star vector information		Original star vector information	

that

$$\begin{cases}\delta\theta=\hat{\theta}_{n(s)}-\hat{\theta}_{n(c)}\\ \delta\gamma=\hat{\gamma}_{n(s)}-\hat{\gamma}_{n(c)}\\ \delta\varphi=\hat{\varphi}_{n(s)}-\hat{\varphi}_{n(c)}\end{cases} \tag{17}$$

where $\hat{\theta}_{n(s)}$, $\hat{\gamma}_{n(s)}$, and $\hat{\varphi}_{n(s)}$ are the SINS pitch, roll and yaw attitude angles, respectively. $\hat{\theta}_{n(c)}$, $\hat{\gamma}_{n(c)}$, and $\hat{\varphi}_{n(c)}$ are the star sensor pitch, roll and yaw attitude angles, respectively.

Substituting Eq. (17) into Eq. (16) and using the first-order Taylor expansion while neglecting the high-order items, the observation matrix can be simplified to the $\boldsymbol{\psi}$ dynamic error model.

$$\mathbf{Z}=\mathbf{H}_\varphi^1\begin{bmatrix}\psi_{n(s)x}\\ \psi_{n(s)y}\\ \psi_{n(s)z}\end{bmatrix}+\mathbf{V}\approx\mathbf{H}_\varphi^1\begin{bmatrix}\psi_{n(s)x}\\ \psi_{n(s)y}\\ \psi_{n(s)z}\end{bmatrix} \tag{18}$$

where

$$\mathbf{V}=\begin{bmatrix}V_{11}\\ V_{21}\\ V_{31}\end{bmatrix}=(\mathbf{H}_\varphi^1-\mathbf{H}_{\delta\theta}^1)\begin{bmatrix}\delta\theta_{n(c)x}\\ \delta\theta_{n(c)y}\\ \delta\theta_{n(c)z}\end{bmatrix}$$

The vector $\mathbf{V}$ is a second-order small quantity, and it must be omitted in the linearization observation equation.

2) EQUIVALENCE DEDUCTION

The equivalence of the attitude angles and the attitude matrix observation for the SINS/CNS integration system is deduced below. The 'φ' and 'ψ' error dynamic equations are adopted in the two integration methods. Consider again the observation equation based on the attitude matrix in the SINS/CNS integration system

$$[\mathbf{I}+(\boldsymbol{\psi}_{n(s)}\times)]=\mathbf{C}_b^{n(s)}(\mathbf{C}_b^{n(c)})^T \tag{19}$$

where $\mathbf{C}_b^{n(s)}$ and $\mathbf{C}_b^{n(c)}$ are the orthogonally attitude matrix, which can be expressed as (20), as shown at the bottom of the next page

$$\mathbf{H}_\psi^4=[(\hat{\mathbf{C}}_{\mathrm{e}}^{n(c)}\mathbf{C}_i^e\boldsymbol{l}_i)\times]=(\boldsymbol{l}_{n(c)}\times)=\begin{bmatrix}0 & -\sin EL_{n(c)} & \cos AZ_{n(c)}\cos EL_{n(c)}\\ \sin EL_{n(c)} & 0 & -\sin AZ_{n(c)}\cos EL_{n(c)}\\ -\cos AZ_{n(c)}\cos EL_{n(c)} & \sin AZ_{n(c)}\cos EL_{n(c)} & 0\end{bmatrix} \tag{14}$$

Substituting Eq. (20) into Eq. (19) yields

$$\begin{cases}\psi_{n(s)x} = -(\cos\hat{\varphi}_{n(s)})\delta\theta + (\sin\hat{\varphi}_{n(s)}\cos\hat{\theta}_{n(s)})\delta\gamma \\ \psi_{n(s)y} = -(\sin\hat{\varphi}_{n(s)})\delta\theta - (\cos\hat{\varphi}_{n(s)}\cos\hat{\theta}_{n(s)})\delta\gamma \\ \psi_{n(s)z} = \delta\varphi + \sin\hat{\theta}_{n(s)}\delta\gamma\end{cases} \tag{21}$$

The attitude angle observation vector is obtained from the above formula.

$$\begin{aligned}\mathbf{Z} &= \begin{bmatrix}\delta\theta \\ \delta\gamma \\ \delta\varphi\end{bmatrix} \\ &= \begin{bmatrix}-\cos\hat{\varphi}_{n(s)} & -\sin\hat{\varphi}_{n(s)} & 0 \\ \sin\hat{\varphi}_{n(s)}\sec\hat{\theta}_{n(s)} & -\cos\hat{\varphi}_{n(s)}\cos\hat{\theta}_{n(s)} & 0 \\ -\tan\hat{\theta}_{n(s)}\sin\hat{\varphi}_{n(s)} & \tan\hat{\theta}_{n(s)}\cos\hat{\varphi}_{n(s)} & -1\end{bmatrix} \\ &\quad\times\begin{bmatrix}\psi_{n(s)x} \\ \psi_{n(s)y} \\ \psi_{n(s)z}\end{bmatrix} = \mathbf{H}_{\psi}^{1}\begin{bmatrix}\psi_{n(s)x} \\ \psi_{n(s)y} \\ \psi_{n(s)z}\end{bmatrix}\end{aligned} \tag{22}$$

It is shown in the above formula that the attitude angle observation equation is obtained throughout the deduction based on the attitude matrix observation equation. Thus, the theoretical relevance is sufficiently discussed in the text, and the two integration methods are exactly equivalent. Obviously, the attitude angles have a variety of definitions, which can lead to diverse observation matrices for the attitude observation. The second technique is characterized by a clear physical meaning and is a simplified form relative to the first one.

3) EQUIVALENCE BETWEEN THE TWO METHODS IN THE SECOND CATEGORY

Both the celestial angle method and the star vector method are based on the original star sensor information. From observation equations (11) and (13), the two methods have different observation dimensions, leading to the different constraints to the solution of unknown tilt errors. In fact, they generally have the same rank of observation matrix equal to 2. Apparently, the observation matrix of the celestial angle method is a full row rank matrix. To demonstrate the observation matrix of the star vector method is not a full rank matrix, the determinant of the skew symmetric matrix can be written as

$$\begin{aligned}\det(-(\boldsymbol{l}_{n(c)}\times)) &= \det((\boldsymbol{l}_{n(c)}\times)^T) = (-1)^3\det((\boldsymbol{l}_{n(c)}\times)) \\ &\Rightarrow \det((\boldsymbol{l}_{n(c)}\times)^T) = -\det((\boldsymbol{l}_{n(c)}\times))\end{aligned} \tag{23}$$

Considering the nonnegative characteristic of the determinant,

$$\det((\boldsymbol{l}_{n(c)}\times)) = 0 \Rightarrow rank((\boldsymbol{l}_{n(c)}\times)) < 3 \tag{24}$$

Obviously, the rank of the skew symmetric matrix is equal to 2 in the condition of either celestial angle not being zero. In conclusion, both methods of the second category have the same number of effective observation equations.

In addition, this category can also be unified by the general relationship between the celestial angle and the star vector, as given in Eq. (8). The deduction from the star vector to the celestial angle is performed as follows:

$$\begin{aligned}\mathbf{Z} &= \boldsymbol{l}_{n(s)} - \mathbf{l}_{n(c)} \\ &= \begin{bmatrix}\sin AZ_{n(s)}\cos EL_{n(s)} \\ \cos AZ_{n(s)}\cos EL_{n(s)} \\ \sin EL_{n(s)}\end{bmatrix} - \begin{bmatrix}\sin AZ_{n(c)}\cos EL_{n(c)} \\ \cos AZ_{n(c)}\cos EL_{n(c)} \\ \sin EL_{n(c)}\end{bmatrix}\end{aligned} \tag{25}$$

Substituting Eq. (10) into Eq. (25), the following is obtained (26), as shown at the bottom of the next page:

Using Eq. (14), the general star vector observation equation of Eq. (26) is expressed in the celestial angle formulation. The third row of Eq. (26) can be written as

$$\begin{aligned}\Delta EL &= \cos AZ_{n(c)}\psi_{n(s)x} \\ &\quad - \sin AZ_{n(c)}\psi_{n(s)y}, \quad (EL_{n(c)} \neq \pi/2)\end{aligned} \tag{27}$$

The second row of Eq. (26) can be written as

$$\begin{aligned}-\sin AZ_{n(c)}\cos EL_{n(c)}\Delta AZ &= \sin EL_{n(c)}\cos AZ_{n(c)}\Delta \mathrm{EL} \\ &\quad - \sin EL_{n(c)}\psi_{n(s)x} \\ &\quad + \sin AZ_{n(c)}\cos EL_{n(c)}\psi_{n(s)z}\end{aligned} \tag{28}$$

Substituting Eq. (27) into Eq. (28) and combining similar terms for the tilt errors, the following is obtained:

$$\begin{aligned}\Delta AZ &= \tan EL_{n(c)}\sin AZ_{n(c)}\psi_{n(s)x} \\ &\quad + \tan EL_{n(c)}\cos AZ_{n(c)}\psi_{n(s)y} \\ &\quad - \psi_{n(s)z}, (EL_{n(c)} \neq \pi/2, AZ_{n(c)} \neq 0)\end{aligned} \tag{29}$$

When the celestial angle meets the conditions $EL_{n(c)} \neq \pi/2, AZ_{n(c)} \neq 0$, Eq. (29) can also be obtained by the first row of Eq. (26). Consequently, the two methods in the second category are equivalent.

4) INTERRELATIONSHIP OF THE TWO CATEGORIES

Taking the attitude matrix observation method as an example of the "attitude observation method" and the star vector observation method as an example of the "star vector observation method", the interrelationship can be described as follows. At least two star vectors can determine a coordinate system, and the double star vector observation matrix is a full row rank matrix. Apparently, the tilt errors can be uniquely determined by the double star vector observation or the attitude observation. Thus, the two categories are closely related to each other.

$$\mathbf{C}_b^n = \begin{bmatrix}\cos\gamma_n\cos\varphi_n - \sin\gamma_n\sin\theta_n\sin\varphi_n & -\cos\theta_n\sin\varphi_n & \sin\gamma_n\cos\varphi_n + \cos\gamma_n\sin\theta_n\sin\varphi_n \\ \cos\gamma_n\sin\varphi_n + \sin\gamma_n\sin\theta_n\cos\varphi_n & \cos\theta_n\cos\varphi_n & \sin\gamma_n\sin\varphi_n - \cos\gamma_n\sin\theta_n\cos\varphi_n \\ -\sin\gamma_n\cos\theta_n & \sin\theta_n & \cos\gamma_n\cos\theta_n\end{bmatrix} \tag{20}$$

IV. SIMULATION AND EXPERIMENT

A. NUMERICAL SIMULATIONS

Based on the above deduction, this section presents simulations verifying the interrelationship between the attitude observation method and the star vector observation method. A trajectory generator is developed to provide a typical ballistic path. The constant drift of the gyro is 0.03° /h. The scale factor error of the gyro is $2\times10-5$. The measurement noise of the gyro is Gaussian white noise with a power spectral density of 0.006° /$\sqrt{}$h. The accelerometer bias is $2 \times 10 - 5$ g. The scale factor error of the accelerometer is $5 \times 10 - 5$. The accelerometer measurement noise is Gaussian white noise with an amplitude of $1.5 \times 10 - 5$ g.

The star sensor measurement noise of the celestial angle is Gaussian white noise with an amplitude of 5 arc-seconds. The field of star sensor view is approximately 20°. The error of the star catalog is 1 arc-second. The star sensor measurement is provided at a frequency of 10 Hz. The initial position error is set as 100 m. The initial attitude error is 10 arc-minutes.

According to a typical ballistic maneuver, ground initial alignment and accelerating to near-earth space, the vehicle moved with a horizontal velocity changed with small discontinuous accelerating and decelerating vertical velocity, with conduct any two times of pose adjustment. The star sensor was measured from 389 s to 483 s. The measurement contains static and low dynamic measurements, and the low dynamic measurement is under 0.1°/s.

The platform angular errors are calculated in two cases: attitude angle observation and double star vector observation. In addition, comparative analysis is performed with the true value as reference in Figure 1. The platform angular errors are obtained directly by the observation equation, and no feedback is used to correct the navigation system. To verify the exact equivalence, the random errors are not considered in the simulation. From Figure 1, three curves are basically coincident, and their D-values are far less than 0.1 arc-seconds. The platform error angles increase linearly with a fixed slope because of the initial attitude errors and inertial device constant bias. As the carrier begins to have posture adjustments, the platform error angles change with a higher slope because of scalar factors and misalignments.

To evaluate the integration accuracy of the SINS/CNS system in two different methods, another simulation was conducted with the Kalman Filter (KF). The state vector includes all the gyro errors and the star sensor installation

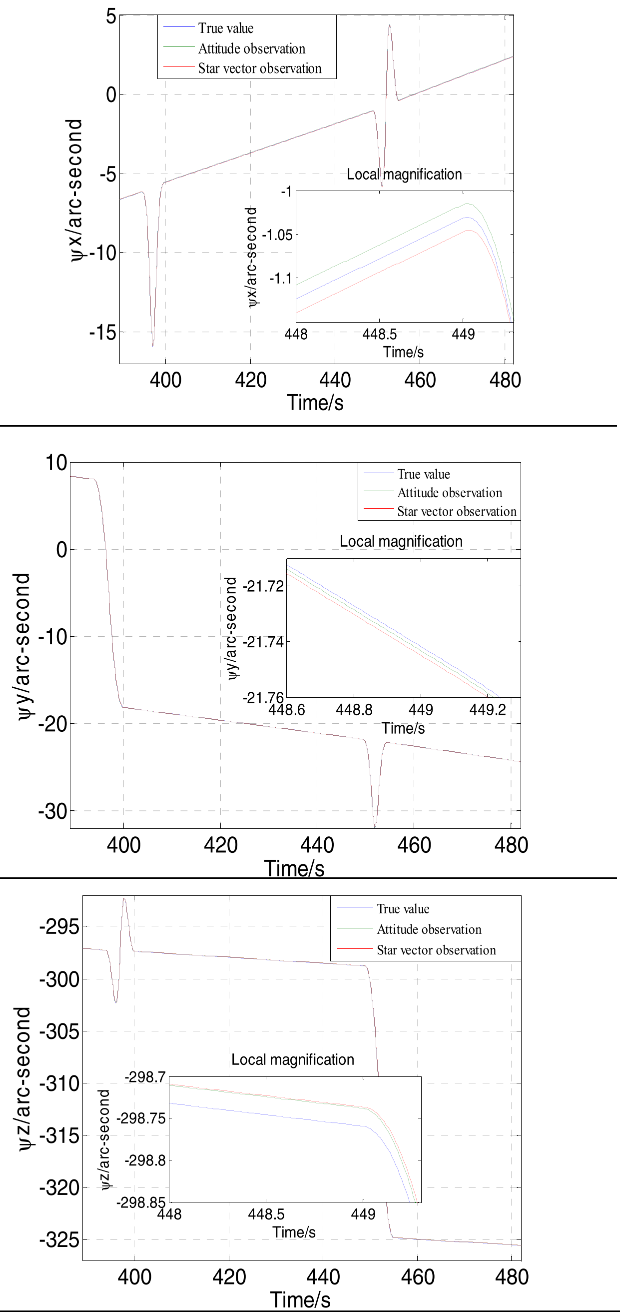


FIGURE 1. The curves of the estimated platform error angles.

errors. The KF's measurement update interval was 0.1 s, and the prediction interval was 0.01 s. To keep the paper

$$\mathbf{Z} = \boldsymbol{l}_{n(s)} - \mathbf{l}_{n(c)} = \begin{bmatrix} \sin(AZ_{n(c)} + \Delta AZ)\cos(EL_{n(c)} + \Delta EL) \\ \cos(AZ_{n(c)} + \Delta AZ)\cos(EL_{n(c)} + \Delta EL) \\ \sin(EL_{n(c)} + \Delta EL) \end{bmatrix} - \begin{bmatrix} \sin AZ_{n(c)} \cos EL_{n(c)} \\ \cos AZ_{n(c)} \cos EL_{n(c)} \\ \sin EL_{n(c)} \end{bmatrix}$$
$$= \begin{bmatrix} \cos EL_{n(c)} \cos AZ_{n(c)} \Delta AZ - \sin EL_{n(c)} \sin AZ_{n(c)} \Delta EL \\ -\sin AZ_{n(c)} \cos EL_{n(c)} \Delta AZ - \sin EL_{n(c)} \cos AZ_{n(c)} \Delta EL \\ \Delta EL \cos EL_{n(c)} \end{bmatrix} = \mathbf{H}_{\psi}^{4} \cdot \begin{bmatrix} \psi_{n(s)x} \\ \psi_{n(s)y} \\ \psi_{n(s)z} \end{bmatrix} \quad (26)$$

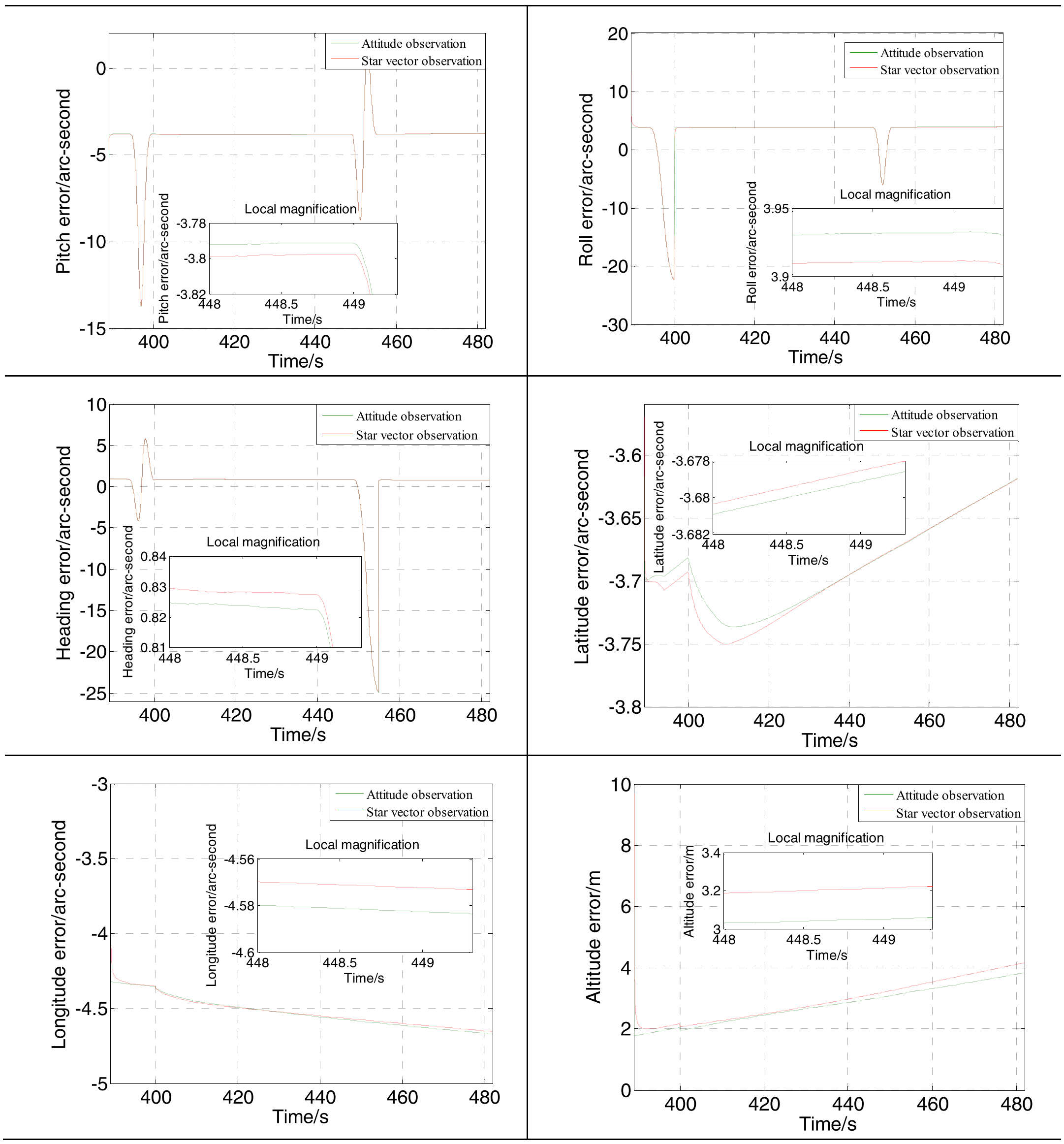


FIGURE 2. **The navigation errors of the two integration methods.**

reasonably concise, the equations of Kalman Filtering are presented in [13]. The platform angular error estimated values are used as feedback to correct the navigation system. The star sensor was not measured in the procedure of pose adjustment. The simulation conditions and the maneuvering were the same as those indicated above. In addition, the star sensor measurement noise is considered in the simulation. As shown in Figure 2, the platform error angles almost do not change when the KF is a closed loop. As the carrier begins to undergo posture adjustments, the platform error angles change with a higher slope because of the unobservable star sensor in the large angular dynamic measurement. All of platform angular errors are close to zero in the observations. The position errors which is irrelevant with star sensor observation, cannot be reduced.

B. EXPERIMENTAL VERIFICATION

To validate the integrated navigation method of SINS/CNS and the equivalence between the attitude observation method and the star vector observation method, a hardware in the loop test was conducted by fixing the SINS and star sensor. The SINS/CNS integration system is mounted on the three-axis table that can accurately reproduce the angular motion. A fiber optic gyroscope (FOG) is chosen as a test gyro, which has constant and random drifts of $0.03°/h$ and $0.006°/\sqrt{h}$, respectively. The scale factor error is 20 ppm, and the installation error is 5 arc-seconds. The constant and random biases of accelerometers are 50 μg and 15 μg, respectively. The scale factor error is 50 ppm, and the installation error is 5 arc-seconds. The SINS measurements are generated with a sample rate of 100 Hz. The precision of the star sensor is

TABLE 2. The attitude angles in three repeated tests.

Method	Test-1			Test-2			Test-3		
	Pitch/°	Roll/°	Head/°	Pitch/°	Roll/°	Head/°	Pitch/°	Roll/°	Head/°
Attitude observation	0.03675	0.00868	20.90233	0.03651	0.00819	20.90144	0.03748	0.00792	20.90274
Star sensor observation	0.03664	0.00846	20.90242	0.03635	0.00838	20.90165	0.03759	0.00793	20.90293
D-value (arc-second)	0.39”	0.32”	-0.32”	0.216”	-0.32”	-0.32”	-0.14”	-0.21”	-0.18”

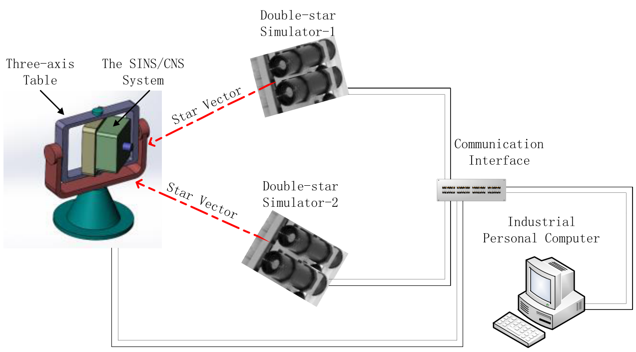


FIGURE 3. Schematic diagram of hardware in the loop test.

5 arc-seconds. The star sensor field of view is approximately 20°. The star sensor measurement update is provided at a frequency of 10 Hz. The fixation installation differences between SINS and the star sensor is on-field calibration and compensated to eliminate the navigation error caused by the installation errors. The position of SINS/CNS is known precisely. The outputs from the SINS/CNS system under test are connected through a suitable interface to a computer simulating the motion. Two groups of static double-star simulators are fixed horizontally at a certain angle, and the relative position is shown in Figure 3.

According to the simulation path, the SINS/CNS integration system conducts ground initial alignment and poses adjustment and measures the star vector three times. The output attitudes of the SINS/CNS integration system were calculated in two different combination categories offline; the attitudes at the end of the test in three repeated experiments are illustrated in Table 2. The consistency of multiple repeated experiments was applied to verify the equivalence between the attitude observation and the star vector observation. The attitude repeatability is approximately 4.9 arc-second, verifying the correctness of the two different integration methods. The maxim attitude D-value of two integration methods is less than 0.39 arc-seconds, in good agreement with the simulation result. Consequently, the results are indicative of the effective estimation regarding the platform angular errors and verify the correctness of the numerical simulation. Moreover, the equivalence of the integration accuracy between the two categories is verified.

V. CONCLUSION

Several comparatively mature schemes of the SINS/CNS integrated mode were classified into two categories. A deeper analysis revealed that these combination methods are closely related. Their theoretical relevance was sufficiently discussed and revealed in this paper. Considering the relationship between the Phi-angle and Psi-angle error dynamic equations, the attitude angle observation formulation was rebuilt in the Psi-angle error dynamic equations. Next, the equivalence of the attitude angles and the attitude matrix observation was deduced.

The main conclusion of the classification and comparative analysis are listed below.

1) In comparison to the observation equations in the Phi-angle formulation associated with tilt errors, the Psi-angle observation formulation is characterized by a clear physical meaning.

2) The SINS integrated with a star sensor can improve the accuracy of the platform angle but not the attitude angle.

3) The equivalence of the celestial angle observation method and the star vector observation method can be derived by the relationship between the celestial angles and the star vector. They have the same effective equation for one time measurement. The two categories are closely related to each other, and the platform errors can be uniquely determined by the double-star vector observation or the attitude observation.

With verified experimental results using hardware in the loop tests, the equivalence and correctness of the analysis was validated. In addition, the purpose of this work was to provide a broad overview of the two categories and show the interrelations and equivalence between them. Normally, the narrow field of view star sensor can provide higher precision star vector than the large field of view star sensor, and the large field of view star sensor can also provide the attitude information. For engineering application, the precision of SINS/CNS integration system depend on the precision of observation.

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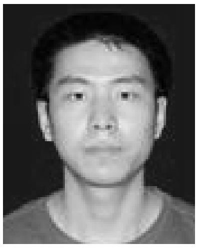

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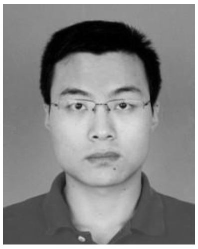

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