IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received October 26, 2017, accepted December 18, 2017, date of publication December 25, 2017,
date of current version February 14, 2018.

Digital Object Identifier 10.1109/ACCESS.2017.2787184

Fatigue Damage Mechanism-Based Dependent Modeling With Stochastic Degradation and Random Shocks

DAN XU, JIAOLAN HE, SHAOBO SUI , SHAN JIANG, AND WEI ZHANG
Science and Technology on Reliability and Environmental Engineering Laboratory, School of Reliability and Systems Engineering, Beihang University, Beijing 100191, China

Corresponding author: Wei Zhang (zhangwei.dse@buaa.edu.cn)

This work was supported by the National Science Foundation of China under Grant 61403010 and Grant 51405009.

ABSTRACT This paper proposes a dependent modeling method for reliability estimation of metal structures under constant amplitude loading and random shocks considering the nonlinear damage accumulation mechanism. It is well known that a large monotonic plastic zone ahead of a crack tip caused by a spike overload will retard subsequent fatigue crack growth. However, most existing degradation-and-shock dependent models cannot account for this retardation phenomenon. Moreover, the increase in damage caused by shock is usually assumed to be independent of fatigue degradation. In this investigation, both fatigue degradation and applied random shock damage are considered to have a coupled effect on the crack propagation process. The nonlinear damage superposition approach is utilized herein to model the interaction between fatigue loading and random shocks. Fatigue degradation is considered as a stochastic process influenced by the uncertainties in material properties, and the applied shocks are regarded as random incidents. Next, the piecewise deterministic Markov process is employed to describe the coupling relationship between this stochastic degradation and the random shock process. In the proposed algorithm, Paris' equation is utilized to describe fatigue degradation, and the Willenborg model is employed to describe retardation caused by random shock loads. A simulation is performed to validate the proposed method, and the proposed method is compared with the traditional method.

INDEX TERMS Dependent modeling, degradation-and-shock, retardation, Paris model, Willenborg model, PDMP.

I. INTRODUCTION

THE airframe, one of the most important components of an airplane, is commonly designed using the damage tolerance concept. During service, the airframe is usually subjected to fatigue loading caused by air flow-induced vibration and occasional shocks caused by turbulence. This loading condition can be idealized as a combination of a fatigue loading sequence and spike overloads. Researchers usually construct a degradation model with shock damage to simulate the actual physical process.

Many studies have modeled fatigue degradation and shock damage. Li and Pham [1] developed a generalized multi-state degraded reliability system subject to multiple independent failures, including two degradation processes and random shocks. Lehmann [2] considered a class of degradation–threshold–shock (DTS) models that provided a rich conceptual framework for the study of degradation and shock. In addition, Castro *et al.* [3] analyzed a condition-based maintenance approach to the DTS model with multiple independent degradation processes and external shocks. Ye *et al.* [4] considered two degradation-and-shock failure models related to the failure time and the failure mode. In these models, the degradation process and shock are assumed to be independent of each other; that is, no interaction effect is considered, which is not consistent with the actual situation.

Other studies have improved the method to a certain extent by considering the correlation between the degradation and the shock process. Those studies have divided random shocks into two types: fatal shocks (which cause the system to fail immediately) and nonfatal shocks (which do not cause catastrophic failure) [5], [6]. In the nonfatal shock situation, there are two effects on the degradation process: the degradation rate increases sharply after a shock [7], [8], and the degradation damage increases upon the occurrence of shocks.

Most studies have considered shock damage increments superimposed on cumulative damage [5], [10]. Wang and Pham [5] developed a time-varying copula method to describe the relationship between degradation processes and random shocks. Shu *et al.* [9] used a single stochastic process to model cumulative degradation with random jump increments caused by random shocks. In a departure from previous research, Song *et al.* [10] extended the cumulative degradation to complex systems with multiple components. When considering the increase in damage, the classification of shocks has also been taken into account. Not every shock affects the degradation process. For example, Jiang *et al.* [11] addressed the safety zone by dividing shocks into three zones based on their magnitudes. According to the interval and the number of shock occurrences, Jiang *et al.* [12] proposed a decrease in the hard failure threshold value to a lower level corresponding to one of three types of shock models: the generalized extreme shock model, the generalized δ-shock model or the generalized m-shock model. For a multi-component system, Song *et al.* [13] considered that each component has its own shock set; as a result, random shocks were classified into different sets, resulting in each component having its own hard failure threshold value . Based on physics-of-failure mechanisms, Keedy and Feng [14] and Feng *et al.* [15] integrated two stochastic-based dependent processes to model the crack growth of stents according to the activity level of patients . These studies assumed only that the shock performance always accelerates, rather than slows, the degradation process. In addition, studies have not considered the coupling relationship between degradation and shock. Chen *et al.* [16], [17] filled this gap based on the local stress-strain approach, and they calculated the effects of shocks using the Miner linear cumulative law . However, analyzing coupling damage by stress-strain alone is not adequate under actual circumstances. Moreover, the linear cumulative law is no longer applicable to the case in which the crack growth rate is reduced because of overload [18], [19].

In our study, the fatigue degradation process is characterized by the crack propagation process. This overall process is described by a specific physical model that incorporates the retardation effect. Moreover, the damage caused by overload has a temporal effect determined by the current cumulative damage; that is, the time at which spike overload occurs will affect the size of the retardation zone. Under overloads, the validated model indicates that the Willenborg model can provide good prediction results [20], [21]. This paper proposes a nonlinear damage superposition approach, which is utilized herein to consider the interaction between fatigue loading and random shocks.

In the proposed algorithm, the degradation process is a deterministic process, and the applied shocks are regarded as random incidents. The state transition between fatigue loading and random shocks is taken into consideration. This consideration requires a state model to describe the corresponding interaction. Piecewise deterministic Markov processes (PDMPs) [22] are processes related to state prediction and are non-diffusion stochastic models that feature a mixture of deterministic and jump motions. The advantage of PDMPs is that they solve the problem of accounting for the interaction between the deterministic dynamics and random shocks of a system. Recently, PDMPs have also been utilized to model corrosion [23]. Nguyen *et al.* [24] adopted a PDMP to address the challenge of estimating the remaining useful life of a closed-loop feedback system. Therefore, this paper implements a piecewise deterministic Markov process (PDMP) to describe the deterministic fatigue degradation process and the random shock process in building a coupling model. In the coupling model, Paris' equation is utilized to describe fatigue degradation, and the Willenborg model is used to describe the retardation caused by random shock loads. The model mainly concentrates on two factors: 1) the greater the cumulative damage before the occurrence of a shock is, the greater the damage caused by random shocks will be, and 2) the superposition of shock damage is nonlinear.

The rest of the paper is organized as follows. In Section II, the framework of the methodology is described. Next, a case study introduces our proposed method and the traditional method in Section III. Section IV concludes the paper.

II. FRAMEWORK OF METHODOLOGY

The framework of the methodology consists of three parts. Part one introduces the physically based fatigue degradation model under constant amplitude loading and the retardation effect caused by overload. Part two describes the shock load model. Part three presents the dependent model under constant amplitude loading and random shocks. One model is the crack growth model that considers the retardation effect based on a PDMP, and the other model is the traditional method, which shows that the effects of shocks produce a direct, random increase in degradation after the occurrence of a shock. Compared with the traditional model, the proposed PDMP-based dependent model is demonstrated to be more effective.

A. DEGRADATION MODEL

1) FATIGUE DEGRADATION MODEL UNDER CONSTANT AMPLITUDE LOADING

In the fatigue damage stage, the Paris model is adopted to describe crack expansion under constant amplitude loading. The Paris model, which is widely used in current crack research, is expressed as follows [25]–[27]:

$$\frac{da}{dn} = C\left(\Delta K\right)^m \tag{1}$$

where C, m are the calibration factors, $C \sim N(\mu_c, \sigma_c^2)$, a is the crack length, and n refers to the number of load cycles suffered by the specimen. ΔK is the variation in the intensity factor over a single load cycle, which is expressed by

$$\Delta K = Y\Delta\sigma\sqrt{\pi a\left(n\right)} \tag{2}$$

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} \tag{3}$$

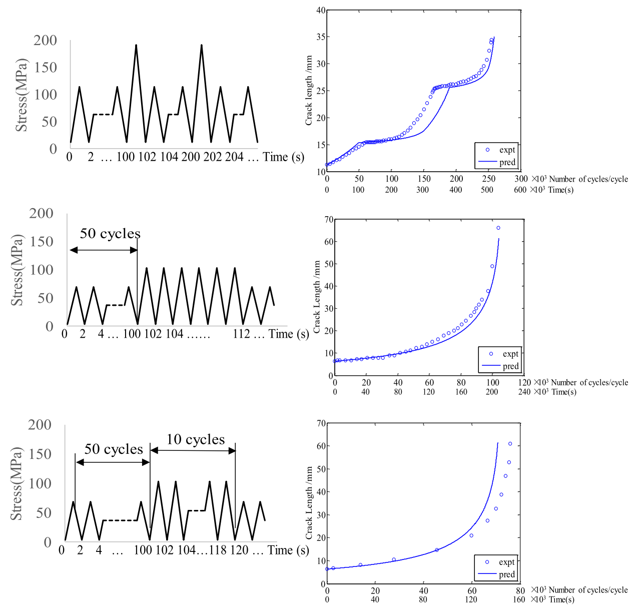


FIGURE 1. Validation of the Willenborg model.

where $\Delta\sigma$ represents the variation of stress within one load cycle, a is the crack length, $\sigma_{\max}$ is the peak load stress within one load cycle, $\sigma_{\min}$ is the corresponding bottom load stress. Y is a geometric parameter, which can be calculated by [30]

$$\begin{cases} Y = \dfrac{1}{\sqrt{\pi a(n)}}\left(\dfrac{2+\theta}{(1-\theta)^{3/2}}\right) \\ \quad (0.886 + 4.64\theta - 13.32\theta^2 + 14.72\theta^3 - 5.6\theta^4) \\ \theta = \dfrac{a(n)}{w} \end{cases} \tag{4}$$

where w is the width of the specimen. The geometry factor can also be simplified in a special situation. For a center crack in a plate, the geometry factor is equal to 1.12, and for a surface crack in a plate, the factor is approximated as $\frac{2.24}{\pi}$.

2) THE RETARDATION MODEL UNDER OVERLOADS

In practice, vibration loads do not have an ideal, constant amplitude but random environmental shocks. For some materials, such as aluminum and iron, these overloads delay surface crack expansion rather than accelerate its growth. This delay represents the overload retardation caused by shocks. Much research has been conducted on this phenomenon. However, these studies have mainly been concerned with the theories used in an attempt to determine the most appropriate mathematic model, e.g., the Wheeler and Willenborg models. The issues of reliability and life prediction under these effects have been less frequently considered.

Some studies have provided fatigue testing data under shock loads [20], [21]. A comparison between Willenborg model predictions and experimental data is shown in Fig. 1; good agreement is observed. Note that the Willenborg model can well account for the retardation effects. In this paper, we study the reliability under overload using the most commonly used retardation model: Willenborg model.

The model proposed by Willenborg is based on plasticity. When a shock load is applied, a large monotonic plastic zone appears near the crack tip, and the crack growth rate reaches the minimum value. In the following cycles, the retardation effect gradually decreases and then vanishes completely when the current plastic zone exceeds the upper boundary value. In the zone of retardation, fatigue crack growth can be expressed by [29]

$$\begin{cases} \dfrac{da(n)}{dn} = C\left(\Delta K_{eff}\right)^m \\ \Delta K_{eff} = Y\Delta\sigma_{eff}\sqrt{\pi a(n)} \\ \Delta\sigma_{eff} = \sigma_{\max eff} - \sigma_{\min eff} \end{cases} \tag{5}$$

$$\sigma_{\max eff} = \begin{cases} \sigma_{\max} - \sigma_{comp}(\sigma_{\max} > \sigma_{comp}) \\ 0(\sigma_{\max} \le \sigma_{comp}) \end{cases}$$

$$\sigma_{\min eff} = \begin{cases} \sigma_{\min} - \sigma_{comp}(\sigma_{\min} > \sigma_{comp}) \\ 0(\sigma_{\min} \le \sigma_{comp}) \end{cases} \tag{6}$$

$$\sigma_{comp} = \sigma_{req} - \sigma_{\max} \tag{7}$$

where ΔK_{eff} refers to the variation of these effective stress intensity factors;$\sigma_{\max eff}$, $\sigma_{\min eff}$ are the effective load stresses corresponding to $\sigma_{\max}, \sigma_{\min}$; σ_{comp} is the equivalent residual stress after the overload; and σ_{req} is the stress at which there is no retardation. The magnitude of σ_{req} is derived by the Irwin function [29], [30] and the geometric principle

$$\rho_{req} = \rho_{res} \tag{8}$$

$$\rho_{res} = \rho_{ol} - (a(n) - a_{ol}) \tag{9}$$

$$\rho_{ol} = \frac{Y^2 a_{ol}}{\alpha}\left(\frac{\sigma_{ol}}{\sigma_{ys}}\right)^2 \tag{10}$$

$$\rho_{req} = \frac{Y^2 a(n)}{\alpha}\left(\frac{\sigma_{req}}{\sigma_{ys}}\right)^2 \tag{11}$$

where a_{ol} a is the crack length at the shock arrival time, $a_{ol} = a(T_i)$. ρ_{ol} is the maximum plastic zone size caused by the shock. a is the current crack length. ρ_{res} is the maximum plastic zone size caused by the residual overload. ρ_{req} indicates the plastic zone size produced by σ_{req}. σ_{req} refers to the shock load stress. σ_{ys} is the yield stress of the material. α is the constant coefficient in the Irwin function. In this paper, the study object is a thin plate of uniform thickness. Crack growth on the plate can be cast as a plane stress problem, where α is equal to 1 [29].

Based on equations (8)-(11), σ_{req} is written as

$$\sigma_{req} = \frac{\sigma_{ys}}{Y}\sqrt{\frac{a_{ol} + \rho_{ol} - a(n)}{a(n)}} \tag{12}$$

The duration of retardation depends on the corresponding size of the plastic zone. Based on equations (21) and (24), the duration can be determined when the stress satisfies the relation

$$\sigma_{\min} > \sigma_{req} - \sigma_{\max} \tag{13}$$

The effective stress variation $\Delta\sigma_{eff}$ in the Willenborg model is equal to the variation $\Delta\sigma$ in the Paris model. In other words,

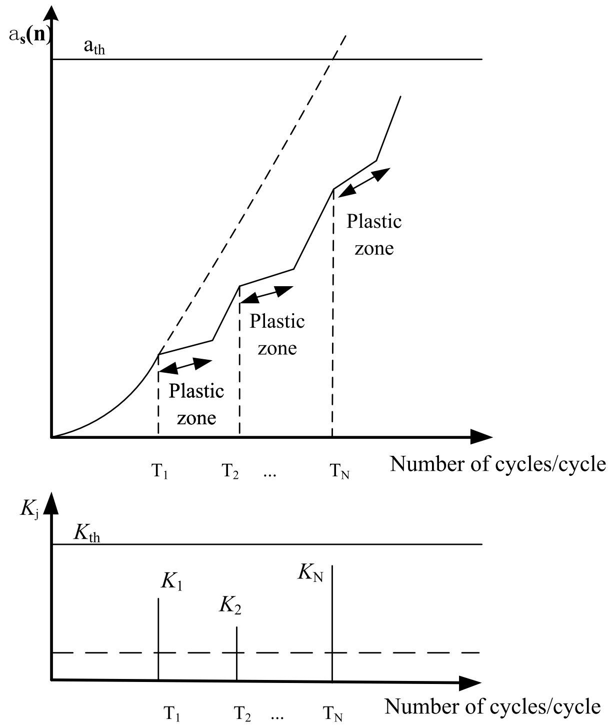


FIGURE 2. The overall process of crack growth.

the overload retardation ends when equation (13) is met. The stress determined in this manner can be substituted into the expression for determining crack length based on equations (12) and (13)

$$a(n) > \frac{1}{1+[Y(1+r)\frac{\sigma_{\max}}{\sigma_{ys}}]^2}(a_{ol}+\rho_{ol}) \tag{14}$$

where min max $r = \sigma_{\min}/\sigma_{\max}$. When the current crack length satisfies equation (14), the retardation disappears, and thus, the rule governing crack growth once again follows the Paris model.

From formulas (1), (2) and (5) of the fatigue models, it is well known that the future crack growth rate depends on the current crack length. Therefore, the traditional linear cumulative law is no longer applicable for calculating the crack length of the segment. For this case, a nonlinear damage accumulation mechanism is proposed in this paper.

B. SHOCK LOAD MODEL

In practice, constant amplitude loading is accompanied by random shocks. The overall process of crack growth is shown in Fig. 2. In Fig. 2, T_N is the time at which the Nth shock occurs, and K_N is the shock's intensity. In this paper, we assume that the shocks are large enough and that all shocks used to calculate the effect of retardation are called overloads. It is assumed that the number of shocks follows a Poisson distribution with intensity λ: $\{N(n), n \geq 0\} \sim Poisson(\lambda)$, and the magnitude of shock σ_{ol} follows a normal distribution:

$$\sigma_{ol} \sim N\left(\mu_{ol}, \delta_{ol}^2\right) \tag{15}$$

Before the first shock arrives, the crack propagation is determined solely by constant amplitude loading. When the Nth shock occurs, the crack will be in the plastic zone caused by overload, and its growth rate will remain low until the plastic zone disappears. The crack growth rate will then revert to the normal degradation rate before the $(N+1)$th shock appears. The shock process is equivalent to a jump process. The degradation process under constant amplitude loading is a deterministic process that is constantly updated over time. $X(n)$ is assumed to be the state of crack growth under degradation and the shock process. $X(n)$ is expressed by

$$X(n) = X_n, n \in [T_N, T_{N+1}) \tag{16}$$

C. DEGRADATION-AND-SHOCK DEPENDENT MODEL

We apply two different methods to build the dependent model coupling with constant amplitude loading and random shocks. In the first model, the effect of overloading on crack growth is considered retardation, as verified in [31] and [32]. The piecewise deterministic Markov process (PDMP) method is adopted to build the crack growth model with retardation, where the retardation effect leads to a lower crack growth rate than before. In the other model, the effect of random shocks on crack growth is considered to be an increase in damage. A model based on overloading as the cause of an increase in damage is utilized to build the crack growth model, which is compared with the proposed PDMP-based modeling method.

1) THE PDMP-BASED DEPENDENT MODEL WITH THE RETARDATION EFFECT

In this section, the crack growth model with retardation caused by overloading is built. In the retardation process, the path of crack growth is affected by many random factors caused by the shock process, such as the shock arrival time, the size of the shock, the corresponding crack length, and the properties of the material. Here, the Willenborg model is used to describe this relationship between the random shock load and the crack growth rate. The Paris model is then utilized to describe crack growth under constant amplitude loading, when there is no retardation. Considering the above-mentioned random factors, the PDMP process is adopted to establish the intrinsic relationship between the random shock process and the deterministic crack growth model. Generally, a PDMP is suitable for describing deterministic processes coupled with random gaps. In this paper, the deterministic process refers to fatigue degradation, whereas random incidents refer to random shocks. Thus, a PDMP is used to describe the coupling relationship between fatigue degradation and random shock. According to [33] and [34], the piecewise deterministic Markov process is determined by the following three characteristics:

① Path φ;

② Jump process $(X_N, T_N, N), N \geq 0$, where X_N is the crack growth state at time T_N;

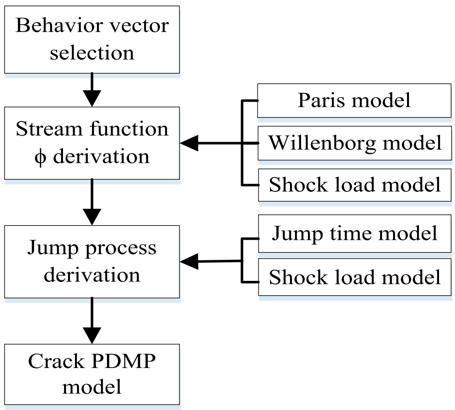


FIGURE 3. Modeling process of the PDMP method.

③ A state transition. This paper describes the state transition between fatigue degradation and random shocks.

This piecewise deterministic Markov process can be summarized as a three-step strategy, as shown in Fig. 3.

Step 1: Select the behavior vector.

In this step, a multi-dimensional behavior vector $Z(n)$ with environmental and stress variation is constructed to describe the crack state of a specimen:

$$Z(n) = \begin{pmatrix} a(n) \\ \frac{da(n)}{dn} \\ K_{\max}(n) \\ X(n) \end{pmatrix} \tag{17}$$

where n is the number of loading cycles; $a(n)$ is the current crack length; $\frac{da(n)}{dn}$ represents the crack growth rate; $K_{\max}$ is the maximum stress intensity factor loaded by environmental stress; and $X(n)$ refers to the crack propagation state between two adjacent shocks.

Step 2: Derive stream function φ.

The aim of this step is to find the deterministic path between two random shocks. Assuming the times at which two random shocks occur are T_N and T_{N+1}, the stream function $\varphi(Z, n)$ is

$$\varphi(Z,n) = \begin{cases} a(n) = X(n) + a(T_N) \\ \dfrac{da(n)}{dn} = \begin{cases} C\left(\Delta K_{eff}\right)^m, a(n) \le \eta_{T_N} \\ C(\Delta K)^m, a(n) > \eta_{T_N} \end{cases} \\ K_{\max}(n) = Y\sqrt{\pi a(n)} * Max(\sigma_{\max}, \sigma_{ol}) \\ X(n) = \int_{T_N}^{n} \dfrac{da(n)}{dn} dn \end{cases} \tag{18}$$

where $\eta(T_N)$ expresses the crack length at the moment at which the Nth shock occurs. Moreover, the boundary condition, regardless of whether the retardation effect occurs, is

$$\eta(T_N) = \frac{1}{1 + \left[Y(1+r)\frac{\sigma_{\max}}{\sigma_{ys}}\right]^2}\left(a(T_N) + \rho_{ol}\right) \tag{19}$$

Step 3: Derive the jump process.

This step involves the derivation of the jump time T_n and the corresponding size of the loading shock. The size of the shock follows a normal distribution $\sigma_{ol} \sim N\left(\mu_{ol}, \delta_{ol}^2\right)$, where μ_{ol} and δ_{ol} are the shock's mean and standard variance. The time interval between two adjacent random shocks follows an exponential distribution with intensity λ. In the process of simulation, this formula can be expressed in order to generate random numbers of exponential distribution,

$$T_{N+1} - T_N = \text{exprnd}(1/\lambda) \tag{20}$$

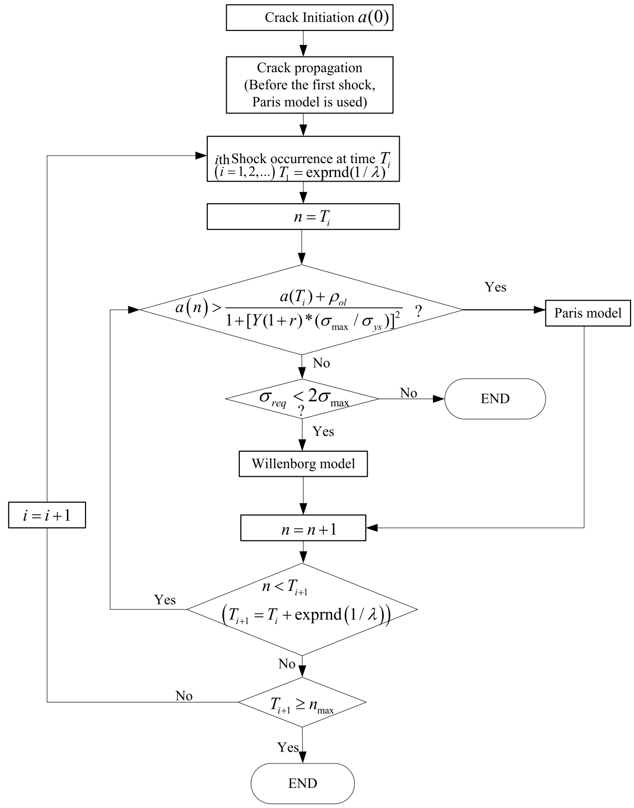


FIGURE 4. Algorithm flowchart of crack growth.

The PDMP algorithm flow is presented as follows in Fig. 4. In Fig. 4, $n_{\max}$ is the given maximum number of loading cycles.

The reliability conditions of the fatigue crack are that the maximum stress intensity factor $K_{\max}$ cannot exceed the threshold $K_{threshold}$ and that the crack length cannot exceed the threshold $a_{threshold}$. The reliability can be written as

$$R(n) = P\left\{K_{\max}(n) \le K_{threshold} \cap a(n) \le a_{threshold}\right\} \tag{21}$$

2) THE TRADITIONAL METHOD-BASED DEPENDENT MODEL

The fatigue degradation and random shock coupling model is used to describe two competing failure processes, continuous degradation and random shocks. In the model, the instantaneous damage from random shocks is assumed to be an increment added to the damage amplitude [11].

In general, fatigue crack propagation is described in terms of a Paris power law formulation, as demonstrated in equation (1). When the degradation or crack size is greater than threshold $a_{threshold}$, continuous failure occurs.

TABLE 1. Parameters of models.

Parameters	Values	Parameters	Values
μ_c	1.92125*E*-10	σ_{ys}	509*Mpa*
σ_c	5*E*-11	$K_{threshold}$	50$MPa.m^{0.5}$
m	3.5990	λ	4*E-4*
$\sigma_{\max}$	68.95*Mpa*	$\sigma_{ol} \sim (\mu_{ol}, \delta_{ol}^2)$	(99*Mpa*,(5*Mpa*)^2)
$\sigma_{\min}$	6.895*MPa*	Y	1.12
$a(0)$	4*mm*	b	0.005$mm/(MPa.m^{0.5})$
$a_{threshold}$	20*mm*	W	1.33

After integrating formula (1), the crack length can be expressed by [25]

$$a(n) = \frac{a(0)}{\left(1 - \frac{1}{2}C'(m-2)a(0)^{(m-2)/2}n\right)^{2/(m-2)}} \tag{22}$$

where $C' = C(Y\Delta\sigma\sqrt{\pi})^m$ and $a(0)$ is the initial crack length.

Instantaneous damage is caused by overloads. Instantaneous failure occurs when the fracture toughness K_j of the jth shock exceeds the threshold $K_{threshold}$. We consider that overloads occur according to a Poisson process, and the number of shocks as a function of the number of load cycles n is $\{Nn, n \geq 0\} \sim Poisson(\lambda)$. K_j is expressed by

$$K_j = Y\sigma_{ol}\sqrt{\pi a(n)} \tag{23}$$

The cumulative damage size depends on the magnitude of the overload. When the magnitude of the overload falls within the interval $(W, K_{threshold})$, the effect of random shocks on a crack is considered to be an increase in damage on the crack [11]. The damage increment can be expressed by

$$H_j = \begin{cases} b(K_j - W) & if\ K_j > W \\ 0 & if\ K_j \leq W \end{cases} \tag{24}$$

where b is a predetermined constant. After the jth shock occurs, the crack length is expressed as $a_s(n) = a(n) + H_j$.

Failure will not occur when the crack length is less than $a_{threshold}$ and the magnitude of the overload is smaller than $K_{threshold}$. The reliability function is the same as formula (21).

III. CASE STUDY AND RESULTS

To illustrate our proposed PDMP-based method, we consider the effects of retardation on fatigue crack growth. Moreover, the larger the crack length at the time at which a shock occurs is, the greater the damage caused by the shock will be. In this section, we first present our PDMP-based method for addressing this problem. Next, the traditional method is presented. A Monte Carlo-based simulation is performed to validate the proposed method via comparison with the traditional method. The model parameters are presented in Table 1. The loading cyclic stress is shown in Fig. 5.

Algorithm 1 Steps for Monte Carlo simulation

Step 1: Define the sample size M and the maximum number of loading cycles $n_{\max}$

Step 2: Set $i = 1$. Sample the calibration factor C; the value can be selected randomly from a normal distribution $C \sim N(\mu_c, \sigma_c^2)$.

Step 3: Set $N = 1$. Sample the time at which the Nth shock occurs and the shock load stress σ_{ol}. The time interval between two consecutive shocks is extracted randomly using an exponential distribution with intensity λ. The values can be selected randomly from a normal distribution $\sigma_{ol} \sim N\left(\mu_{ol}, \delta_{ol}^2\right)$. Assume that $T_0 = 0$ and that the time at which the Nth shock occurs is $T_N = T_{N-1} + \exp rnd(1/\lambda)$.

Step 4: Set $N = N + 1$. If T_N is less than the given maximum number of loading cycles $n_{\max}$, go back to step 3. If T_N is greater than $n_{\max}$, go to step 5.

Step 5: Set $i = i+1$. If $i = M$, the process is over. If $i < M$, go back to step 2

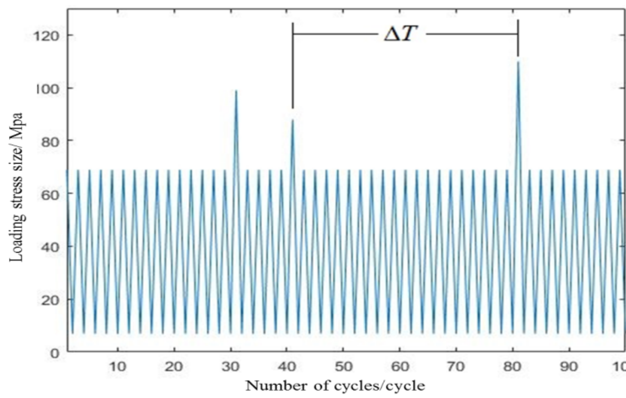


FIGURE 5. Cyclic stress level with shock.

In Monte Carlo simulation, the steps of the random sampling are presented as follows.

In this study, because of the limited computational power of the computer used, $M = 1500$ samples were simulated in MATLAB R2014a and the number of loading cycles was $n_{\max} = 4 \times 10^4$.

In Fig. 5, ΔT refers to the time interval between two random shocks that follow an exponential distribution with intensity λ. The corresponding probability density function and cumulative distribution function are shown in Fig. 6(a) and (b) respectively.

A. DEGRADATION-AND-SHOCK DEPENDENT MODELING METHOD BASED ON PDMP

This section focuses on how to formulate our proposed PDMP-based method. Overloads will result in a retardation effect. The crack growth process under this effect is described in detail.

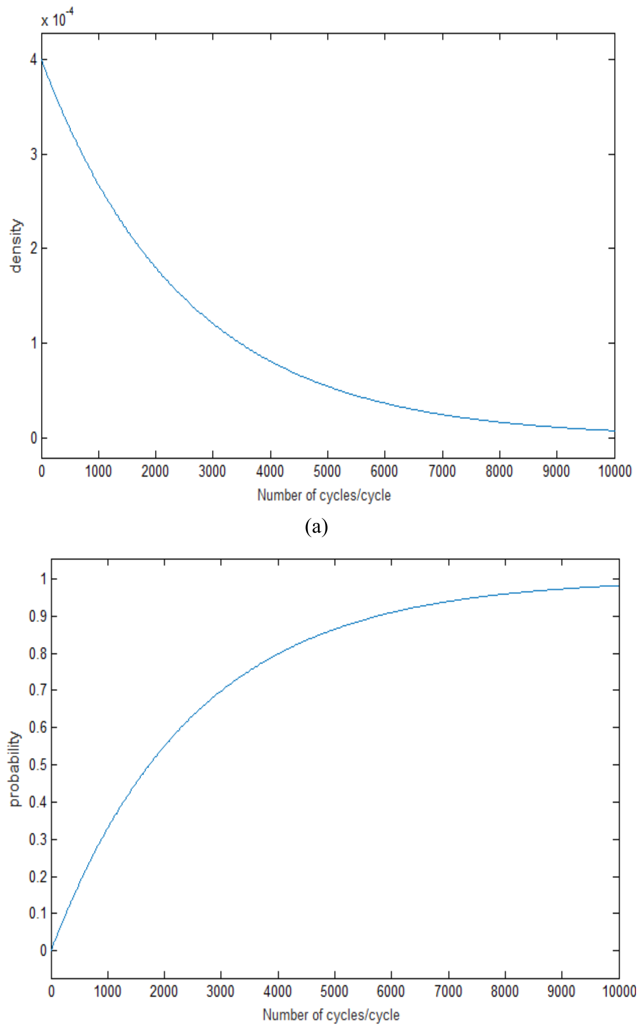


FIGURE 6. Distribution curves. (a) The probability distribution of time interval between two random shocks. (b) The cumulative distribution of time interval between two random shocks.

According to the Willenborg model, retardation occurs if the current crack length satisfies the following condition when overloading occurs,

$$\begin{cases} a\,(n) < \dfrac{1}{1+\left[Y\,(1+r)\,\frac{\sigma_{\max}}{\sigma_{ys}}\right]^2}(a_{ol}+\rho_{ol}) \\ \sigma_{req} < 2\sigma_{\max} \end{cases} \tag{25}$$

Specifically, $T_i < n \leq T_{en}$, where T_i refers to the arrival time of the ith shock, and crack growth is retarded within this interval time. T_{en} is the end time of overload retardation. It's worth noting that when equation (25) is not satisfied, the value of T_{en} is determined and $T_{\text{en}} = n$ at that moment. In the calculation process, it is difficult to obtain the mathematical analytical form of the Willenborg model, so we utilize the forward Euler method with $\Delta n = 1$ to simulate crack growth,

$$\begin{cases} \eta = \dfrac{1}{1+\left[Y\,(1+r)\,\frac{\sigma_{\max}}{\sigma_{ys}}\right]^2}\,(a\,(T_i)+\rho_{ol}) \\ a(n) = a(n-\Delta n) + da \\ da = C\left(\Delta k_{eff}\right)^m \\ \Delta K_{eff} = Y\Delta\sigma_{eff}\sqrt{\pi a(n-\Delta n)} \\ \Delta\sigma_{eff} = 2\sigma_{\max} - \sigma_{req} \\ \sigma_{req} = \frac{\sigma_{ys}}{Y}\sqrt{\frac{a_{ol}+\rho_{ol}-a(n)}{a(n)}} \\ N\,(n) \sim Poisson\,(\lambda) \\ K_{\max}\,(n) = Y\sqrt{\pi a(n)} * Max\,(\sigma_{\max}, \sigma_{ol}) \end{cases} \tag{26}$$

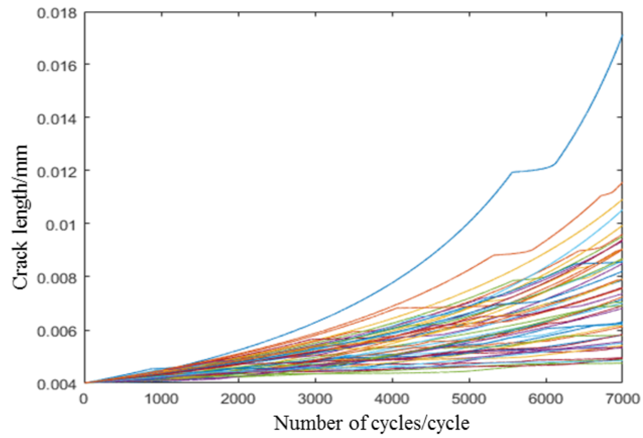


FIGURE 7. Part of the simulated crack path.

Crack growth can be described by a multi-dimensional random variable $Z\,(n)$ consisting of the variation in environmental stress and variation in state, as indicated in equation (17). Considering that materials properties vary within the same batch, the geometric factor is randomized, $C \sim N\,(\mu_c, \delta_c)$.

As the crack grows in the plastic zone, the crack becomes longer. The retardation effect ends when the current crack length $a(n)$ satisfies the condition given by equation (14).

The crack propagation law then follows the Paris model, $T_{en} < n \leq T_{i+1}$, and is written as

$$a(n) = \frac{a(T_{en})}{\left(1 - \frac{1}{2}C'(m-2)a(T_{en})^{(m-2)/2}(n-T_{en})\right)^{2/(m-2)}} T_{en} < n < T_{i+1} \tag{27}$$

where T_{i+1} refers to the arrival time of the $(i+1)$th shock.

Next, the Monte Carlo method is adopted to simulate crack growth according to a PDMP. Part of the simulated crack path results are shown in Fig. 7. It is observed that random factors caused by the shock process affect the crack growth rate, including the shock arrival time corresponding to the retardation start time, the size of the shock, the corresponding crack length, and the properties of the material. This process diversifies the crack path.

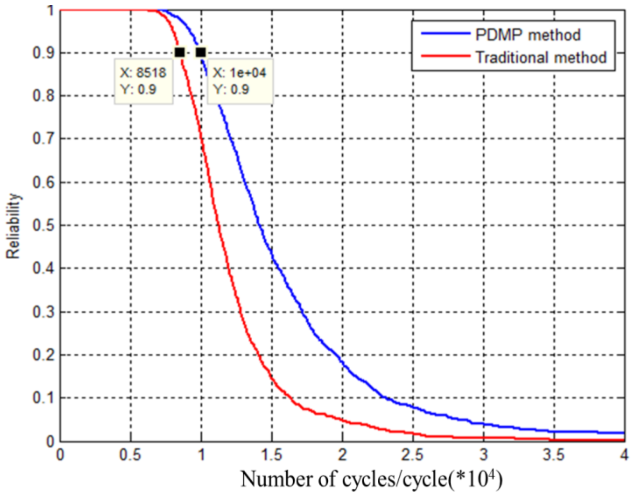


FIGURE 8. Comparison of reliability of specimen.

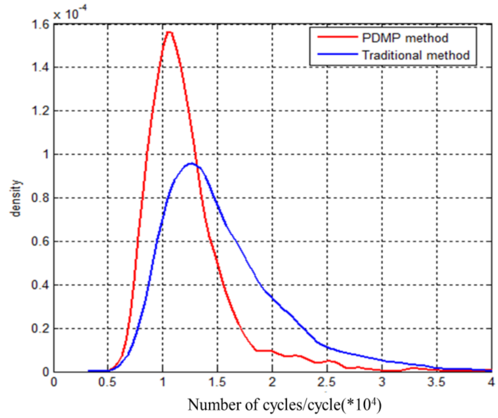


FIGURE 9. Comparison of probability density functions of the first passage time.

B. DEGRADATION-AND-SHOCK DEPENDENT MODEL BASED ON TRADITIONAL METHOD

The retardation effect caused by overloads is not considered in this section, whereas the damage increment caused by overloads is considered. The traditional method based on fatigue damage and shock damage is used to calculate crack propagation considering continuous and instantaneous damage. The main goal is to compare the results of this method with those of the PDMP method; we then explain how the retardation effect substantially delays crack propagation.

Before a shock occurs, crack propagation follows the Paris model, as indicated in equation (22). Once the jth shock occurs and its magnitude exceeds the value W, the crack length will increase because of an increase in damage, as expressed by

$$a_s(n) = a(n) + b(K_j - W) \tag{28}$$

Crack growth will then follow the Paris model again. However, the initial crack length is updated as

$$a(0) = a_s(n) \tag{29}$$

At that moment, the equation describing crack growth when the jth shock occurs at T_j becomes

$$a(n) = \frac{a_s(n)}{\left(1 - \frac{1}{2}C'(m-2)a_s(n)^{(m-2)/2}(n - T_j)\right)^{2/(m-2)}} \tag{30}$$

C. ANALYSIS AND COMPARISON OF THE PDMP METHOD AND THE TRADITIONAL METHOD DEGRADATION

Using both the PDMP-based method and the traditional method, the reliability curves of crack growth and the corresponding probability density functions of the first passage time are shown in Fig. 8 and Fig. 9, respectively.

In Figs. 8 and 9, the curves of the PDMP-based model with retardation are shown in blue and the curves of the traditional model are shown in red, respectively. The curves move to the right with an increase in the number of loading cycles. The estimated accuracy is improved with the progression of the retardation effect. For example, the number of loading cycles at a reliability value of 0.9 is 10,000 cycles based on our proposed method with retardation, and the number of loading cycles is 8518 based on the traditional model, as shown in Fig. 8. The result shows that the crack propagation rate is influenced by overloads. Moreover, this influence delays, rather than accelerates, crack growth. Therefore, it is important to consider the effects of the time at which shocks occur and retardation when evaluating crack growth.

IV. CONCLUSION

This article proposed a novel dependent modeling method that incorporates constant amplitude loading cycles and random shocks based on failure damage mechanisms. This dependent model considers the coupling relationship between degradation and shock damage. Unlike most existing methods (which treat soft shock damage as a mechanism that accelerates crack growth and rarely consider the effects of the time at which shocks occur), the proposed model considers that overloads retard crack propagation. Therefore, in the proposed model, based on nonlinear damage caused by shocks, a Markov process is used to describe the state of crack propagation in the presence of shocks. In this article, the degradation process was considered a deterministic process, and the shock process was assumed to be random; thus, it was natural to model the overall crack growth process as a piecewise deterministic Markov process (PDMP). In addition, retardation was considered via the Willenborg model, and the Paris model was applied to the non-retardation zone. Finally, the Monte Carlo method was adopted to simulate the crack propagation process via our PDMP-based method and the traditional method. The comparison results verified the effects of nonlinear cumulative damage and retardation on the degradation behavior and demonstrated the effectiveness of the proposed method.

One limitation of the current study is that it focused only on uniaxial loading without considering crack arrest. In the future, more complex loading conditions will be investigated, such as adjacent spike loads and large spike loads.

REFERENCES

[1] W. Li and H. Pham, "Reliability modeling of multi-state degraded systems with multi-competing failures and random shocks," *IEEE Trans. Rel.*, vol. 54, no. 2, pp. 297–303, Jun. 2005.

[2] A. Lehmann, "Joint modeling of degradation and failure time data," *J. Stat. Planning Inference*, vol. 139, no. 5, pp. 1693–1706, 2009.

[3] I. T. Castro, N. C. Caballé, and C. J. Pérez, "A condition-based maintenance for a system subject to multiple degradation processes and external shocks," *Int. J. Syst. Sci.*, vol. 46, no. 9, pp. 1692–1704, 2015.

[4] Z. S. Ye, L. C. Tang, and H. Y. Xu, "A distribution-based systems reliability model under extreme shocks and natural degradation," *IEEE Trans. Rel.*, vol. 60, no. 1, pp. 246–256, Mar. 2011.

[5] Y. Wang and H. Pham, "Modeling the dependent competing risks with multiple degradation processes and random shock using time-varying copulas," *IEEE Trans. Rel.*, vol. 61, no. 1, pp. 13–22, Mar. 2012.

[6] K. Rafiee, Q. Feng, and D. W. Coit, "Condition-based maintenance for repairable deteriorating systems subject to a generalized mixed shock model," *IEEE Trans. Rel.*, vol. 64, no. 4, pp. 1–11, Dec. 2015.

[7] Z. Wang, H.-Z. Huang, Y. Li, and N.-C. Xiao, "An approach to reliability assessment under degradation and shock process," *IEEE Trans. Rel.*, vol. 60, no. 4, pp. 852–863, Dec. 2011.

[8] K. Rafiee, Q. Feng, and D. W. Coit, "Reliability modeling for dependent competing failure processes with changing degradation rate," *IIE Trans.*, vol. 46, no. 5, pp. 483–496, 2014.

[9] Y. Shu, Q. Feng, and D. W. Coit, "Life distribution analysis based on Lévy subordinators for degradation with random jumps," *Naval Res. Logistics*, vol. 62, no. 6, pp. 483–492, 2015.

[10] S. Song, D. W. Coit, Q. Feng, and H. Peng, "Reliability analysis for multi-component systems subject to multiple dependent competing failure processes," *IEEE Trans. Rel.*, vol. 63, no. 1, pp. 331–345, Mar. 2014.

[11] L. Jiang, Q. Feng, and D. W. Coit, "Modeling zoned shock effects on stochastic degradation in dependent failure processes," *IIE Trans.*, vol. 47, no. 5, pp. 460–470, 2015.

[12] L. Jiang, Q. Feng, and D. W. Coit, "Reliability and maintenance modeling for dependent competing failure processes with shifting failure thresholds," *IEEE Trans. Rel.*, vol. 61, no. 4, pp. 932–948, Dec. 2012.

[13] S. Song, D. W. Coit, and Q. Feng, "Reliability for systems of degrading components with distinct component shock sets," *Rel. Eng., Syst. Safety*, vol. 132, no. 132, pp. 115–124, 2014.

[14] E. Keedy and Q. Feng, "Reliability analysis and customized preventive maintenance policies for Stents with stochastic dependent scompeting risk processes," *IEEE Trans. Rel.*, vol. 62, no. 4, pp. 887–897, Dec. 2013.

[15] Q. Feng, K. Rafiee, and E. Keedy, "Reliability and condition-based maintenance for multi-stent systems with stochastic-dependent competing risk processes," *Int. J. Adv. Manuf. Technol.*, vol. 80, no. 9, pp. 2027–2040, 2015.

[16] H. Chen, Y. Chen, and Z. Yang, "Coupling damage and reliability model of low-cycle fatigue and high energy impact based on the local stress–strain approach," *Chin. J. Aeronautics*, vol. 27, no. 4, pp. 846–855, 2014.

[17] H. Chen, Y. Chen, and Y. Yang, "A fatigue and low-energy shock-based approach to predict fatigue life," *Chin. J. Aeronautics*, vol. 28, no. 10, pp. 3977–3984, 2014.

[18] M. Lang and G. Marci, "The influence of single and multiple overloads on fatigue crack propagation," *Fatigue Fracture Eng. Mater. Struct.*, vol. 22, no. 4, pp. 257–271, 1999.

[19] P. J. Bernard, T. C. Lindley, and C. E. Richards, "Mechanisms of overload retardation during fatigue crack propagation," Fatigue Crack Growth Under Spectrum Loads, ASTM STP595, 1976, pp. 78–97.

[20] F. Taheri, D. Trask, and N. Pegg, "Experimental and analytical investigation of fatigue characteristics of 350 WT steel under constant and variable amplitude loadings," *Marine Struct.*, vol. 16, no. 1, pp. 69–91, 2003.

[21] T. R. Porter, "Method of analysis and prediction for variable amplitude fatigue crack growth," *Eng. Fracture Mech.*, vol. 4, no. 4, pp. 717–736, 1972.

[22] J. Chiquet, N. Limnios, and M. Eid, "Piecewise deterministic Markov processes applied to fatigue crack growth modelling," *J. Stat. Planning Inference*, vol. 139, no. 5, pp. 1657–1667, 2009.

[23] A. Brandejsky, B. De Saporta, F. Dufour, and C. Elegbede, "Numerical method for the distribution of a service time," in *Proc. Eur. Safety Rel. Conf. Adv. Safety, Rel. Risk Manage. (ESREL)*, Sep. 2011.

[24] D. N. Nguyen, L. Dieulle, and A. Grall, "Remaining useful lifetime prognosis of controlled systems: A case of stochastically deteriorating actuator," *Math. Problems Eng.*, vol. 2015, no. 4, Mar. 2015, Art. no. 356916.

[25] P. C. Paris, M. P. Gomez, and W. E. Anderson, "A rational analytic theory of fatigue," *Trend Eng.*, vol. 13, no. 1, pp. 9–14, 1961.

[26] R. V. Marrey, R. Burgermeister, R. B. Grishaber, and R. O. Ritchie, "Fatigue and life prediction for cobalt-chromium stents: A fracture mechanics analysis," *Biomaterial*, vol. 27, no. 9, pp. 1988–2000, 2006.

[27] E. Keedy and Q. Feng, "A physics-of-failure based reliability and maintenance modeling framework for stent deployment and operation," *Rel. Eng. Syst. Safety*, vol. 103, pp. 94–101, Jul. 2012.

[28] S. Jiang, W. Zhang, and Z. Wang, "A vibration fatigue analysis model considering interaction effects," *J. Vibroeng.*, vol. 17, no. 7, pp. 3650–3661, 2015.

[29] Y. Sha, "Compressive loading effect and overload effect of aluminum alloy fatigue crack propagation," (in Chinese), Ph.D. dissertation, Dept. Mech. Manuf. Autom., Harbin Univ. Sci. Technol., Heilongjiang Sheng, China, 2011.

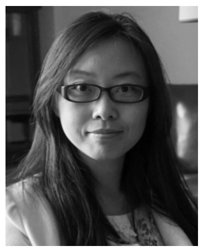
[30] G. R. Irwin, "Plastic zone near a crack tip and fracture toughness," in *Proc. 7th Sagamore Ordnance Mater. Conf.*, vol. 4. 1960, pp. 63–78.

[31] Z. Lu and Y. Liu, "Experimental investigation of random loading sequence effect on fatigue crack growth," *Mater. Des.*, vol. 32, no. 10, pp. 4773–4785, 2011.

[32] S. Jiang, W. Zhang, J. He, and Z. Wang, "Comparative study between crack closure model and Willenborg model for fatigue prediction under overload effects," *Chin. J. Aeronautics*, vol. 29, no. 6, pp. 1618–1625, 2016.

[33] A. Lorton, M. Fouladirad, and A. Grall, "A methodology for probabilistic model-based prognosis," *Eur. J. Oper. Res.*, vol. 225, no. 3, pp. 443–454, 2013.

[34] C. Cocozza-Thivent, R. Eymard, and S. Mercier, "A finite-volume scheme for dynamic reliability models," *IMA J. Numer. Anal.*, vol. 26, no. 3, pp. 446–471, 2006.

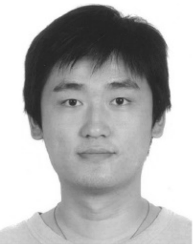


DAN XU received the B.S. and Ph.D. degrees from the School of Mechanical Engineering and Automation, Beihang University, Beijing, China, in 2003 and 2009, respectively. She is currently an Assistant Professor with the School of Reliability and Systems Engineering, BUAA. Her primary research interests include reliability design, assessment based on physics of failure, and degradation modeling.

JIAOLAN HE received the bachelor's degree from the School of Information Engineering, Minzu University of China, Beijing, China, in 2016. She is currently a Graduate Student with the School of Reliability and Systems Engineering, Beihang University. Her primary research interests include degradation modeling and data analysis.

SHAOBO SUI received the master's degree from the School of Reliability and Systems Engineering, Beihang University, Beijing, China, in 2017. He is currently with HUAWEI, Shenzhen, China. His primary research interests include accelerated tests and reliability assessment.

SHAN JIANG received the B.S. degree from the School of Economics and Management, Beihang University, Beijing, China, in 2012, where she is currently pursuing the Ph.D. degree with the School of Reliability and Systems Engineering. Her research areas include engineering and material fatigue and fracture.



WEI ZHANG received the engineering Doctoral degree from Clarkson University in 2012. He was an Assistant Research Scientist with the School for Engineering of Matter, Transport & Energy, Arizona State University. He is currently an Associate Professor of reliability and systems engineering with Beihang University. His research areas include the fatigue and fracture of engineering materials and structures, multiphysics damage modeling and structural durability, and multiscale experimental mechanics.