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Graphical Solution for Arterial Road Traffic Flow Model Considering Spillover

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ABSTRACT This research proposes a graphical solution to arterial road dynamics simulation. An arterial road consists of channelized and upstream sections. Traffic flow is mixed in the upstream section (U-section) and operates without lateral interaction in the channelized section (C-section). Under oversaturated conditions, C-section spillover occurrence governs the traffic flow dynamics of the entire road. In order to capture the arterial road traffic flow dynamics, the proposed method takes advantage of the following components: 1) a decomposition mechanism that uniforms the analysis of each lane; 2) a red–green pair-based vehicle queue tracking component that describes traffic dynamics both temporally and spatially; and 3) a well-defined converted cumulative curve that is capable of generating a queue solution directly. The graphical method permits arbitrary traffic flow and signal settings input and can easily be automated, as demonstrated in the example.

INDEX TERMS Traffic flow, arterial, graphical solution, simulation.

I. INTRODUCTION

The arterial road is one of the most important elements of the road network. It consists of numerous lanes, and can be divided into the channelized section (C-section) and upstream section (U-section), as illustrated in Figure 1. Traffic flow is mixed in the U-section, and enters target lanes in the C-section, where the turning lanes are controlled by signals. When vehicle queues are short, no interaction takes place between the U-section and C-section. When vehicle queues of certain lanes spill back into the U-section, the interface is blocked; therefore, all of the inflow through the interface is delayed. Furthermore, the inflow of other directions will change due to spillover, resulting in the queue behavior being altered.

Various methods have been proposed to study traffic flow dynamics on typical urban roads, most of which are lane-oriented. The seminar work of Lighthill *et al.* [3], [4] marked the start of long-lasting research on road traffic flow dynamics. The model was known as the LWR model for short, and models road traffic flow dynamics in a manner mimicking that of fluid. The model was later discretized, resulting in the famous cell transmission model (CTM) [2], for which the fundamental diagram was trapezoidal. The CTM made use of a set of recursive equations to track the flow rate and density of the cells. Isaak [10] proposed the link transmission model (LTM), and although time is discretized in this approach, space is not; therefore, efficiency is improved. Raadsen *et al.* [8] simplified the model by linearizing the input flow, thereby obtaining an event-based network traffic flow model, known as the event-based link transmission model (eLTM).

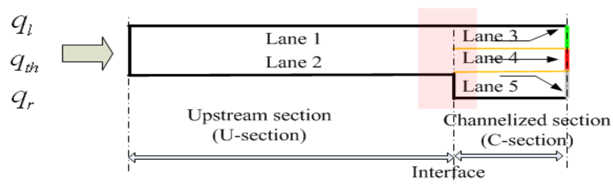

FIGURE 1. Arterial road.

Vehicle queue models can be further classified into point and physical queues [5], and the two classes are also known as the vertical queue and horizontal queue. The former treats the queue as the vehicle number at the stop-line; therefore, it bears no physical size. This simplifies analysis but leads to errors when queues spill back. Another type of link model is the store-and-forward model [1], [9], which is used extensively in the optimal control field. The advantage of this method is that the system can be described by recursive equations; however, traffic flow dynamics are given less consideration.

All of the models described treat roads as simple lanes or an aggregation of lanes. Such simplification may result in significant bias, particularly under spillover conditions. Spillover is an inevitable phenomenon when the vehicle queue is longer than the link length, and creates discontinuity in traffic flow dynamics. Spillover modeling encounters great difficulties for this reason. We propose a converted cumulative curve (CCC) to calculate vehicle queue dynamics. It is demonstrated that, using a carefully designed grid plot, traffic flow dynamics can be solved graphically. The resulting dynamics not only depict the temporal evolution of the queue, but also output the spillover behavior. The remainder of this paper is arranged as follows. Firstly, the basic traffic flow model is presented; then, the graphical solution is developed. Following this, the automation and an example are provided. The paper ends with certain conclusions and remarks.

II. BASIC TRAFFIC FLOW MODELS

Qi *et al.* [6], [7] proposed a simple traffic flow model for arterial roads, which tracks vehicle queue dynamics cycle by cycle. Here, a cycle refers to the duration consisting of a red and green signal. The red signal always precedes the green one, and this is known as a red-green pair (RGP). The model is briefly described in this section.

A. CONCEPTUALIZATION OF ARTERIAL ROAD

The road is decomposed into a series of inter-connected lanes. For a common arterial road, as illustrated in Figure 1, there are two sections: the U-section) and C-section. The input demand includes three directional flows, namely the left-turn flow, q_l, through flow, q_{th}, and right-turn flow, q_r. These are mixed uniformly at the entrance and obey the first-in-first-out (FIFO) principle in the U-section.

The lanes in the C-section operate separately, while the U-section is modeled as a single lane (that is, the demand is evenly distributed across lanes), and all lanes are treated as being signal controlled. In the U-section, when no spillover occurs, the 'signal' will always display green. When there is left-turn flow spillover, through flow spillover, or both, a virtual red signal will come into effect. Therefore, the focus is placed on the signal-controlled lane model. The following two sub-sections outline the lane queuing dynamics and spillover component of the model.

B. LANE QUEUING MODEL

The signal is arranged as consecutive RGPs. During each red signal, the vehicle queue is expected to increase; during each green signal, the vehicle queue is shortened. Thus, the traffic flow dynamics can be tracked cyclically along the time horizon.

The relationship between flow rate and density, namely the fundamental diagram (FD) is described by a triangle, as illustrated in Figure 2 (b). It is effectively determined by the following parameters: v_f, the free flow speed, w, the backward

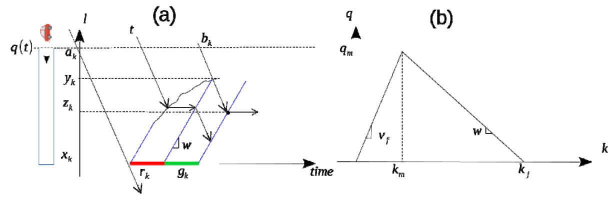

FIGURE 2. Lane queue dynamics and its FD.

wave, q_m, the maximal flow rate that the lane can achieve, and k_m, the density corresponding to q_m. Figure 2 (a) depicts the lane queuing dynamics, where the input demand is indicated by $q(t)$, the red duration at the k-th RGP is r_k, and the green duration is g_k. Suppose that, at the beginning of cycle k, the residual queue length x_k is zero, the last vehicle of the last green signal enters the lane at moment a_k, and it passes the stop-line at moment $\sum_{i=1}^{k-1} C_i$, where $C_i = r_i + g_i$. Furthermore, we have $a_k = \sum_{i=1}^{k-1} C_i - \frac{L}{v_f}$, where L is the lane length. The queue grows as a result of the red signal, and the rate is dependent on the inflow rate $q(t)$, $t \geq a_k$. The signal changes to green at moment $\sum_{i=1}^{k-1} C_i + r_k$. According to the FD, a wave with speed w is formed and propagates upstream. This starting wave intersects with the stopping wave, leading to the queue tail point (y_k, t_{y_k}), which is the maximal queue length measured from the stop-line. The final vehicle in the queue enters the lane at moment b_k. This vehicle trajectory proceeds with a slope v_f in the space-time diagram and intersects with the next RGP starting wave, resulting in a new residual queue length, z_k, or equivalently, x_{k+1}.

Therefore, the lane queue dynamics can be described cycle by cycle. The traffic flow dynamics output during each RGP include the residual queue length z_k and entering moment b_k. These are directly mirrored to x_{k+1} and a_{k+1}, which mark the beginning of the traffic flow dynamics derivation of the next RGP. The three points x_k, y_k, and z_k are known as characteristic queue tails, while a_k, b_k are known as characteristic arrival moments.

Suppose that a vehicle enters the lane at moment t, $t \in [a_k, b_k]$, and the cumulative vehicle number counted from moment a_k is $F_{a_k}(t) = \int_{a_k}^{t} q(t)dt$. Therefore, when this vehicle join the queue, the queue length is $x_k + \frac{F_{a_k}(t)}{k_j}$. The duration for which this vehicle travels can be computed as $\frac{L-(x_k+\frac{F_{a_k}(t)}{k_{jam}})}{v_f}$. Hence, the spatial-temporal coordinates of the queue tail are $\left(x_k + \frac{F_{a_k}(t)}{k_{jam}}, a_k + \frac{L-(x_k+\frac{F_{a_k}(t)}{k_{jam}})}{v_f}\right)$. The characteristic queue tail point $\left(y_k, t_{y_k}\right)$ also satisfies the above relationship. Moreover, this characteristic queue tail point is located at the starting wave trajectory of the RGP cycle k, and therefore also satisfies $t_{y_k} = \sum_{i=1}^{k-1} C_i + r_k + \frac{y_k}{w}$. Combining the above two relationships, (y_k, t_{y_k}) can be solved numerically.

According to shock wave theory, after the meeting of the stopping and starting waves, a new wave forms and its speed is v_f, which coincides with the vehicle trajectory. If the wave passes the stop-line before termination of the green signal, all of the vehicles can be cleared within this RGP; otherwise, a residual queue exists. Under this condition, the trajectory will intersect with the stopping wave of the next RGP. The location and moment of the intersection point are z_k and t_{z_k}. Because both lines are linear, z_k and t_{z_k} can be determined easily:

$$z_k = \frac{wv_f(t_{y_k} + \frac{y_k}{v_f} + \sum_{i=1}^{k} C_i)}{w + v_f}; \quad t_{z_k} = t_{y_k} + \frac{y_k - z_k}{v_f} \quad (1)$$

The coordinates (t_{z_k}, z_k) are the same as $(t_{x_{k+1}}, x_{k+1})$. Therefore, given the basic supply parameters:

- signal: r_k, g_k;
- FD: v_f, k_j, v_m, k_m

and the demand parameters:

- demand: $q_l(t)$, $q_{th}(t)$, $q_r(t)$,

the lane queuing system is effectively described by the following parameter group:

- characteristic arrival moments a_k and b_k;
- queue profile $l(t)$ and its byproducts, namely three characteristic points: x_k, y_k, z_k.

C. SPILLOVER COMPONENT

If the characteristic point y_k is beyond the lane length L at an RGP, a C-section spillover occurs.

1) VIRTUAL RED SIGNAL CONCEPT

A C-section spillover will block the interface and thus prevent vehicles from entering this section. The effect is realized by means of a virtual signal at the interface. It should be noted that FIFO should be maintained, as this is the reason for directional blockage, which creates green signal loss.

2) SPILLOVER DURATION DETERMINATION

The following figure presents the scheme for obtaining the virtual red signal duration. Suppose that the characteristic point Y_k is beyond the lane length L at the k-th RGP. The queue tail trajectory $l(t)$ can be derived in the same manner as when there no spillover exists. Once $l(t)$ is available, the spillover onset moment can be interpolated by $l(t) = L$. The spillover termination moment is obtained in a similar manner. The onset and termination moments are denoted by y_k^u and y_k^v, respectively; thus, $y_k^u = l^{-1}(L)$ and $y_k^v = t_{y_k} - (t_{y_k} - t_{x_k} - r_k)\frac{y_k - L}{y_k - x_k}$, according to the geometrical relationship, and l^{-1} (.) is the inverse curve of $l(t)$.

Thereafter, the inflow rate profile in the C-section is changed and the influenced inflow rate profile should be reconstructed. The inflow rate during $\left[y_k^u, y_k^v\right]$ is zero, and during $[y_k^v, y_k^v + g_k']$ it is q_m. Following this, the inflow rate returns to its original profile.

III. GRAPHICAL SOLUTION DEVELOPMENT

A. SINGLE LANE WITH MIXED INFLOW

Suppose that a lane exists with length L. The inflow is mixed by directional movements, denoted by the cumulative curve

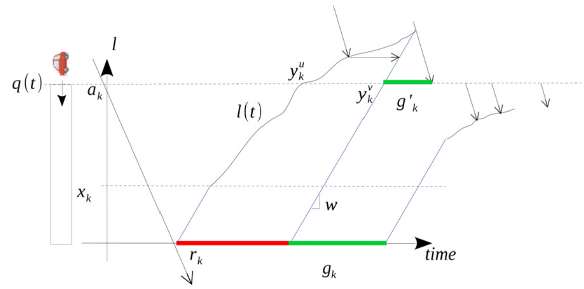

FIGURE 3. Spillover component.

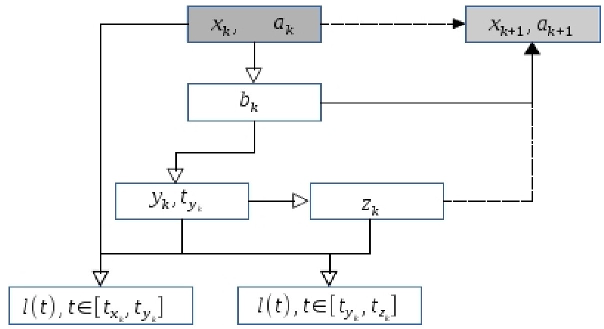

FIGURE 4. Parameters process.

$N(t) = N_l(t)+N_{th}(t)+N_r(t)$, and obeys FIFO. The graphical solution operates in an iterative manner. The basic inputs for each iteration k are the characteristic arrival moment a_k and characteristic queue tail x_k. The moment at which the characteristic queue tail x_k forms is denoted by t_{x_k}, and can be computed as $t_{x_k} = a_k + \frac{L-x_k}{v_f}$. Following one iteration, a_{k+1} and x_{k+1} of the next RGP are generated.

1) OVERALL PARAMETERS PROCESS

a: CASE WITH NO SPILLOVER

The algorithm is implemented in the following order:

- The characteristic arrival moment b_k is obtained first.
- The characteristic queue tail y_k is calculated according to the intersection of the vehicle and starting wave trajectories.
- The characteristic queue tail z_k, which simultaneously represents x_{k+1}, is easily derived by the intersection of the vehicle trajectory and stopping wave of the next RGP, both of which are linear.

The above process is illustrated in the following figure:

b: CASE WHEN SPILLOVER EXISTS

However, when spillover occurs, the above process is interrupted. The Extra stages considering spillover include the following:

- Determine the spillover event duration, $[y_k^u, y_k^v]$.
- Determine the critical moment when the spillover cumulative vehicle number curve coincides with the original one, which is denoted by g_k^v.
- Reconstruct the cumulative vehicle curve, and redefine the characteristic moments and queue tails.

Therefore, the flow chart for spillover in a single lane is illustrated in Figure 5.

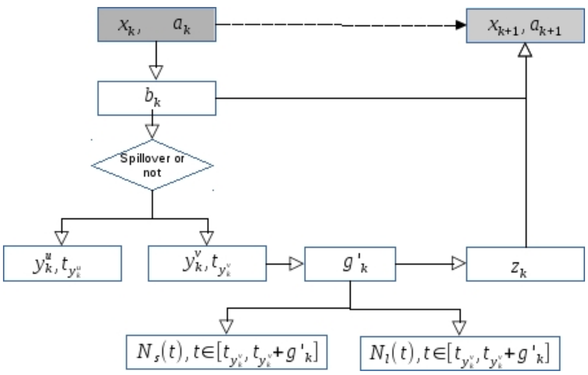

FIGURE 5. **Parameters process with spillover.**

2) ESTABLISHMENT OF CONVERTED CUMULATIVE CURVE: ONE-CYCLE SOLUTION WITHOUT SPILLOVER

a: DEFINITION OF CCC CURVE

Suppose that a vehicle enters the link at time t, $t > \alpha_k$. The cumulative vehicle number between α_k and t is $F_{\alpha_k}(t) = N(t) - N(\alpha_k)$. Therefore, if this vehicle enters the queue tail, namely point A in Figure 6, its spatial location is $x_k + \frac{N(t)-N(\alpha_k)}{k_j}$. The coordinates of point A, namely the queue tail, are $(t + \frac{L-x_k-\frac{N(t)-N(\alpha_k)}{k_j}}{v_f}, x_k + \frac{N(t)-N(\alpha_k)}{k_j})$.

If follows that $t + \frac{L-x_k-\frac{N(t)-N(\alpha_k)}{k_j}}{v_f} - \frac{x_k+\frac{N(t)-N(\alpha_k)}{k_j}}{w} = t'$. Here, t' is temporal moment at the stop-line, as illustrated in Figure 6. The above formula establishes a unique relationship between t and t'. We denote $l(t) = \frac{N(t)}{k_j}$; then, $t + \frac{L-x_k-l(t)+l(a_k)}{v_f} - \frac{x_k+l(t)-l(a_k)}{w} = t'$. The above formula leads to $t' = t - l(t)\frac{w+v_f}{wv_f} + t_f - x_k\frac{w+v_f}{wv_f}$, where $t_f = \frac{L}{v_f}$. Let $\eta = \frac{w+v_f}{wv_f}$; then, we have $t' = t + t_f - \eta x_k - \eta(l(t) - l(a_k)) = t - \eta l(t) + t_f - \eta x_k + \eta l(a_k)$.

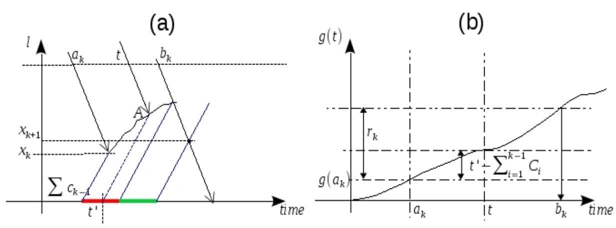

FIGURE 6. **Converted cumulative curve.**

Let $g(t) = t - \eta l(t) = t - l(t)\frac{w+v_f}{wv_f} = t - \frac{N(t)}{q_m}$; then, $t' = t - \eta l(t) - (a_k - \eta l(a_k)) + a_k + t_f - \eta x_k = g(t) - g(a_k) + a_k + t_f - \eta x_k$. Simultaneously, $a_k + t_f - \eta x_k = \sum_{i=1}^{k-1} C_i$, in which C_i is the time duration of RGP cycle i and $\sum_{i=1}^{k-1} C_i$ is the termination moment of RGP cycle $k-1$. Finally, $t' - \sum_{i=1}^{k-1} C_i = g(t) - g(a_k)$.

We therefore obtain two corresponding temporal increments: one from $\sum_{i=1}^{k-1} C_i$ to t', and the other from a_k to t. With t, we can calculate t'; inversely, t can be computed from t'. When we set t' to r_k, we obtain the characteristic arrival moment b_k.

The function $g(t)$ is monotonically increasing with t; that is, $\frac{dg(t)}{dt} \geq 0$. This is known as the **converted cumulative curve (CCC).** Once the CCC is available, the arrival characteristic moments can easily be determined by a simple graphical method, as shown in Figure 6 (b). Suppose the characteristic arrival moment is a_k in sub-figure (b). From $g(a_k)$, we map the red duration r_k to the vertical axis, and obtain $g(a_k) + r_k$. A line is drawn horizontally from this y-axis point and the intersection of the line with CCC directly gives b_k.

Once b_k is available, x_{k+1} is easily solved graphically, as shown in Figure 6 (a) by intersecting the vehicle trajectory with the stopping wave of the next RGP cycle.

Whether b_k can be set to a_{k+1} depends on the green duration g_{k+1}. When the k-th RGP is under-saturated, $a_{k+1} = \sum_{i=1}^{k-1} C_i - \frac{L}{v_f}$.

b: EXAMPLE

The above figure illustrates an example in which the CCC is applied to solve the queue dynamics in a single lane without spillover. It makes use of the CCC curve and spatial-temporal plot, and the temporal axes of the two plots are aligned. The spatial-temporal plot has two parallel time axes, namely an upper and lower axis. The initial conditions are a_k and x_k, and $x_k = 0$ because the cycle $k-1$ is under-saturated, without a residual queue. Therefore, $a_k = \sum_{i=1}^{k-1} C_i - \frac{L}{v_f}$ in the figure. At cycle k, the red signal duration is projected onto the y-axis in sub-figure (a). The baseline of this r_k is $g(a_k)$. The red signal is projected to the right and intersects with the $g(t)$ curve at point A. The x-coordinate of point A is then b_k, which can be projected downwards; thus, we have the characteristic arrival moment b_k at sub-plot (b). At this arrival moment, a vehicle enters the lane with a free-flow speed. The trajectory is linear, and intersects with the starting wave of cycle k at the characteristic queue tail y_k. The queue tail still moves forward, and intersects with the stopping wave of the next cycle $k+1$ at the characteristic queue tail z_k; therefore, we have $x_{k+1} = z_k$.

For the queue tail at any moment between t_{x_k} and t_{y_k} (here $t_{x_k} = \sum_{i=1}^{k-1} C_i$), the solution is as follows. From t_{x_k}, suppose a time increment of Δr_k is provided, where $0 \leq \Delta r_k \leq r_k$. For any such increment, a unique moment t exists, which is indicated on the up-time axis in sub-figure (b). The moment represents a "characteristic" arrival moment if the red signal duration is Δr_k. Therefore, we apply the projection onto the y-axis on the $g(t)$ curve and obtain moment t. The trajectory line begins from moment t and intersects with the virtual starting wave, which leads to the queue tail. The queue tail at other moments can be obtained in a similar manner.

The characteristic arrival moment b_{k+1} is derived according to the same method. However, the trajectory of the vehicle entering the lane at b_{k+1} intersects with the green duration; therefore, cycle $k+1$ is not saturated. Hence, the outflow of this lane in $[b_{k+1} + \frac{L}{v_f}, \sum_{i=1}^{k+1} C_i]$ is the same as the inflow rate, with a $\frac{L}{v_f}$ lag.

Since cycle $k+1$ is not saturated, the characteristic arrival moment a_{k+2} is not b_{k+1}. Moment a_{k+2} is directly plotted in sub-figure (b) as $a_{k+2} = \sum_{i=1}^{k+1} C_i - \frac{L}{v_f}$.

Finally, the traffic flow dynamics for this lane are solved graphically.

3) CCC INFLUENCED BY SPILLOVER: ONE RGP SOLUTION WITH SPILLOVER

When spillover occurs, the cumulative vehicle curve is affected by the inflow restriction; therefore, the CCC should be updated. The cumulative curve is required in order to obtain the spillover onset moment.

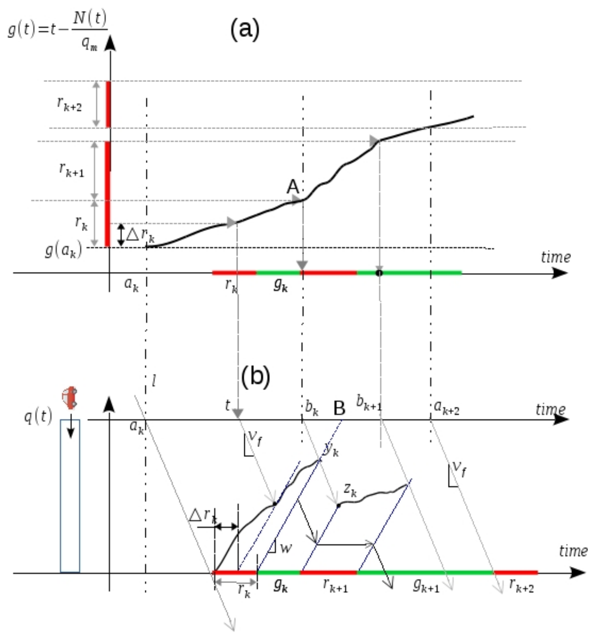

FIGURE 7. Single-lane example.

a: SPILLOVER DURATION

Whether or not spillover exists in the current RGP cycle can easily be judged from the figures. In Figure 7, the resulting moment b_k precedes point B in sub-figure (b), which is the intersection point of the starting wave extension in the current cycle and the temporal axis at the lane entrance. Therefore, spillover does not exist. If b_k lies to the right of point B, spillover exists. In Figure 8, when solving the characteristic arrival moment b_{k+1}, the resulting moment first lies in point A in sub-figure (a), and this moment lags behind moment B in sub-figure (c); hence, spillover exists. Termination of the spillover interval is obtained simply by the intersection of the starting wave line and temporal axis at the lane entrance, namely, y_{k+1}^v, which coincides with point B. However, as the converted curve $g(t)$ contains no lane length information, the solution requires a cumulative curve. The three curves keep the temporal axis aligned, as shown in Figure 8.

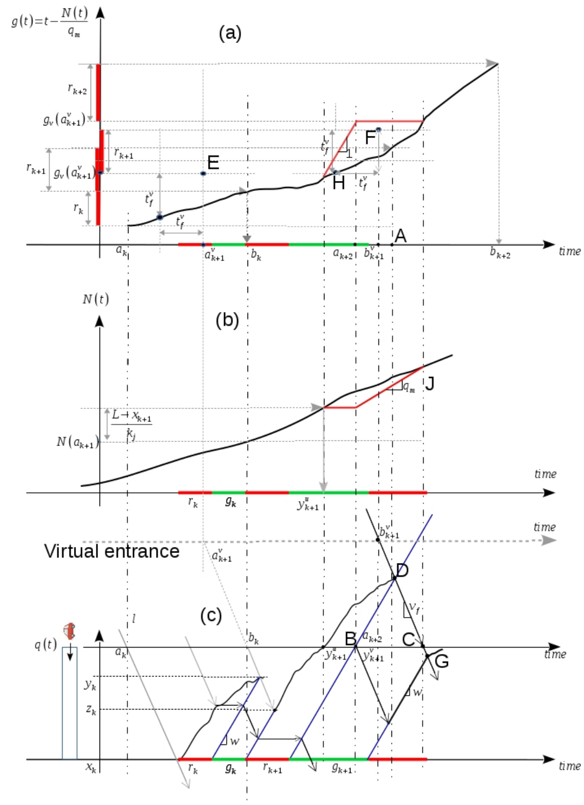

FIGURE 8. CCC influenced by spillover.

Suppose the spillover onset moment is y_{k+1}^u; Then, the cumulative vehicle number between a_{k+1} and y_{k+1}^u should be $\frac{L-x_{k+1}}{k_j}$. In Figure 8 (b), from the baseline of $N(a_{k+1})$, drawing a parallel line with a distance $\frac{L-x_{k+1}}{k_j}$ and projecting it onto the temporal axis, we obtain y_{k+1}^u.

b: CCC RECONSTRUCTION

The queue tail between moments $t_{x_{k+1}}$ and y_{k+1}^u has already been solved. Following spillover, at first the inflow is q_m. However, it remains unknown when the saturation flow rate will end. Suppose that a virtual lane extension exists upstream with sufficient physical length. The queue at this virtual lane extension can be described in the same manner, but with a different lane entrance. Suppose the virtual lane entrance is as shown in Figure 8 (c), where the spillover cycle is $k+1$. We determine the characteristic arrival moment a_{k+1}^v by the line start from b_k in the actual entrance with slope v_f. In principle, the characteristic arrival moment b_{k+1}^v can be solved by the CCC; however, the known CCC is at the actual lane entrance, as opposed to the virtual one. Suppose the CCC at the virtual entrance is $g_v(t)$. We have $g_v(t) = t - \frac{N_v(t)}{q_m}$, where

$N_v(t)$ is the cumulative curve at the virtual lane entrance. Furthermore, suppose that the free-flow travel time between the actual and virtual entrances is t_f^v; then, the cumulative vehicle curve at the virtual entrance is $N_v(t) = N(t - t_f^v)$. Hence, the CCC at the virtual entrance is $g_v(t) = t - \frac{N_v(t)}{q_m} = t - \frac{N(t-t_f^v)}{q_m} = (t - t_f^v) - \frac{N(t-t_f^v)}{q_m} + t_f^v = g(t - t_f^v) + t_f^v$. This implies that by moving curve $g(t)$ at the actual entrance to the right by t_f^v and further moving it upwards by t_f^v, we obtain $g_v(t)$.

Because $g_v(t) = g(t - t_f^v) + t_f^v$, $g_v(a_{k+1}^v)$ is obtained by the following method. The target point $g_v(a_{k+1}^v)$ has its counterpart in $g(t)$. Based on the relationship between $g(t)$ and $g_v(t)$, the counterpart point should be $g(a_{k+1}^v - t_f^v)$. Therefore, by moving $g(a_{k+1}^v - t_f^v)$ upwards by t_f^v, and projecting it onto the vertical line at $t = a_{k+1}^v$, we obtain the resulting point E. Therefore, we also have $g_v(a_{k+1}^v)$, as shown in Figure 8 (a). From the baseline of $g_v(a_{k+1}^v)$, we project the red signal duration of cycle $k + 1$ to the right in order to obtain the characteristic arrival moment b_{k+1}^v. However, again, because we do not have $g_v(t)$, b_{k+1}^v cannot be solved directly. In a similar manner, we move point $g_v\left(a_{k+1}^v + r_{k+1}\right)$ downwards by t_f^v, obtain the counterpart point H in Figure 8 (a), and move it to the right by t_f^v, so the resulting point F is expected, which provides b_{k+1}^v. From moment b_{k+1}^v at the virtual lane entrance in Figure 8 (c), a vehicle trajectory is constructed with free-flow speed, and intersects with the starting wave, which leads to the queue tail in the virtual upstream lane. The trajectory also provides the moment C at which all the residual vehicles are cleared. The inflow rate is regained following moment C. During the interval between moments B and C in Figure 8 (c), the inflow rate at the actual lane entrance is q_m. Therefore, the resulting queue tail increases at a maximal rate, namely w.

Given the queue clearance moment C, and spillover duration $[y_{k+1}^u, y_{k+1}^v]$, reconstruction of the cumulative curve and CCC is a simple process, as shown in sub-figures (a) and (b) in Figure 8. All of the curves are linear during the intervals $[y_{k+1}^u, y_{k+1}^v]$ and $[y_{k+1}^v, C]$. The slope of $g'(t)$ can be determined by its derivative with respect to t. During $[y_{k+1}^u, y_{k+1}^v]$, the derivative is 1, and during $[y_{k+1}^v, C]$, it is 0; hence, we obtain the changed CCC.

Because the CCC is reconstructed, when solving the queue dynamics in cycle $k + 2$, the baseline of $g(a_{k+2})$ is also changed, as shown on the y-axis of Figure 8 (a).

Finally, the queue dynamics due to spillover for the single lane are solved graphically.

c: A SIMPLER SOLUTION

If we are not interested in the vehicle queue in the upstream virtual extension lane, a significantly simpler solution is available. When the spillover duration $[y_{k+1}^u, y_{k+1}^v]$ is available, the unknown parameter is the saturation inflow duration as a result of spillover, g_k^v, which is the temporal difference between point C and y_{k+1}^v.

The cumulative vehicle curve slope within $[y_{k+1}^u, y_{k+1}^v]$ is zero, and q_m within $[y_{k+1}^v, C]$, following which $N(t)$ merges to the original path. Therefore, in Figure 8 (b), temporal point J is readily obtained. Then, point C in sub-figure (c) is simply the mirror point of J on the temporal axis. Moreover, the CCC can easily be constructed, given that the slope within $[y_{k+1}^u, y_{k+1}^v]$ is 1 and that within $[y_{k+1}^v, C]$ is 0.

B. ARTERIAL ROAD SOLUTION

1) CCC CONSTRUCTION

In the single-lane solution, the arrival flow is counted at the lane entrance. On an arterial road, the flow is mixed and enters the C-section based on the expected turning direction. Therefore, in order to apply the single-lane graphical solution to an arterial road, the input cumulative curve, which is calculated at the road entrance, is moved to the C-section entrance. The procedure is as follows:

- Construct the cumulative vehicle number $N(t) = N_l(t) + N_s(t) + N_r(t)$ at the road entrance, where the subscript l means left-turn, s means through flow, and r means right-turn.
- Obtain the cumulative vehicle number at the C-section entrance for each turning direction: $N_l^c(t) = N_l(t - \frac{l_u}{v_f})$, where l_u is the physical length of the U-section; similarly, we have $N_s^c(t)$ and $N_r^c(t)$.

Therefore, the graphical solution can be applied to the C-section lanes. However, when spillover occurs, the U-section exit is blocked and $N_l^c(t), N_s^c(t), N_r^c(t)$, which are counted at the C-section entrance, are subsequently changed. Because the FIFO principle holds in the U-section, the left-turn spillover will block the interface and through flow is also blocked. When the queue is cleared, the inflow in the C-section is compressed compared to the raw inflow. Therefore, in the spillover scenario, the cumulative curves for different directions would be synchronized as a result of FIFO.

2) SYNCHRONIZATION OF CUMULATIVE VEHICLE CURVE UNDER SPILLOVER

The key to the synchronization is to obtain the directional cumulative vehicle curve and CCC. As a result of FIFO, the cumulative vehicle number ratio between different directional flows in any location within the U-section remains unchanged. For example, if the cumulative numbers of the three turning directions are n_l, n_s *and* n_r respectively, at the arterial road entrance, at any location within the U-section, when the left-turn cumulative curve is n_l, the couterpart for the through flow and right-turn flow must be n_s and n_r, respectively. Therefore, once it is determined at the arterial road entrance, the cumulative curve ratio is fixed.

Suppose the left-turn flow experiences spillover during cycle k; then, the resulting spillover duration is $[y_k^u, y_k^v]$. Furthermore, following duration g_k^v, the residual vehicles are cleared. The outflow rate at the U-section exit is q_m. The solution for the directional flow rate is illustrated in Figure 9, which contains four sub-figures. Without loss of generality, sub-figure (a) is the cumulative vehicle number for the

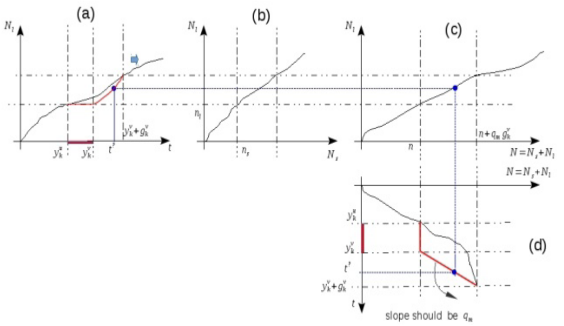

FIGURE 9. Reconstruction of converted cumulative vehicle numbers.

left-turn flow, while (b) shows the proportion line for the left-turn and through flows. This line remains unchanged regardless of where the cumulative vehicle curve is counted. Sub-figure (c) displays the relationship between the left-turn flow and total cumulative curve, which also does not change with time. Sub-figure (d) represents the total cumulative curve with time.

Suppose that, at the spillover onset moment, the through flow cumulative vehicle number is n_s and that of the left-turn flow is n_l; the total vehicle number is n. Therefore, in Figure 9 (d), at moment $y_k^v + g_k^v$, the total cumulative vehicle number is $n+q_m g_k^v$ (which can be projected onto sub-figure (c)). Because the flow rate at the C-section entrance is zero during spillover, the total cumulative vehicle number remains unchanged in sub-figure (d). Thereafter, the cumulative vehicle curve is linear, until moment $y_k^v + g_k^v$, when the curve merges to its original counterpart.

However, the cumulative curve for the left-turn and through flow remains unknown. The solution for the left-turn flow $N_l(t^?)$ at moment $t^?$ is indicated in the figure with the blue dotted line. The steps are as follows:

- For any moment $t^?$, locate it on the time-axis in sub-figures (d) and (a).
- From $t^?$ in sub-figure (d), obtain the total vehicle number by drawing a line horizontally.
- From the intersection point, draw a vertical line and obtain the intersection point with the curve in sub-figure (c).
- From the intersection point in sub-figure (c), draw a horizontal line all the way to sub-figure (a).
- The intersection point of the above horizontal line with a vertical line from $t^?$ at the x-axis in sub-figure (a) provides the solution, $N_l(t^?)$.

The curve can be plotted point by point. The red line in sub-figures (a) and (d) provides the solved cumulative curve for the left-turn flow. The cumulative curves for other directions can be obtained in a similar manner.

3) SYNCHRONIZATION OF CCC

When the cumulative curve is reformulated, the CCC should retain consistency. However, $g(t)$ is not a linear function of t, and does not change linearly with $N(t)$. The reformulation is

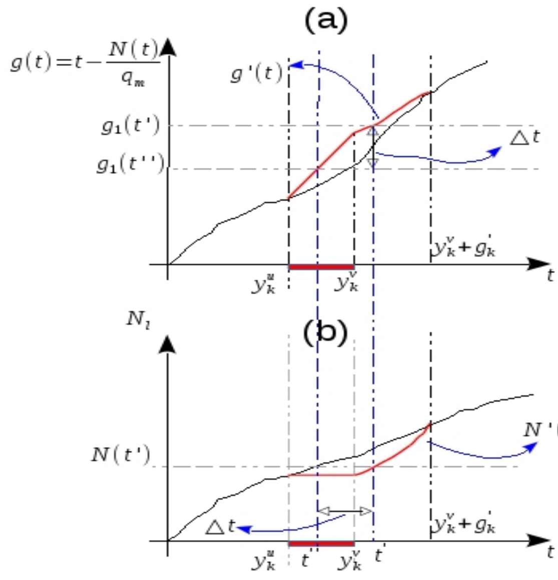

FIGURE 10. Reformulation of CCC.

point-wise interpolated, the method for which is illustrated in Figure 10. Within the spillover duration, the CCC is linear with slope 1. Suppose that the reformulated CCC is $g'(t)$, where $t \in [y_k^v, y_k^v + g_k']$, and g_k' is the duration required to clear the queue in the U-section. Furthermore, suppose we wish to compute $g'(t')$; then, $g'(t') = t' - \frac{N'(t')}{q_m}$. We also have $N'(t') = N(t'')$, as shown in Figure 10(b). Therefore, $g'(t') = t' - \frac{N'(t')}{q_m} = t' - \frac{N(t'')}{q_m} = t'' + t' - \frac{N(t'')}{q_m} - t''$. Because $t'' - \frac{N(t''))}{q_m} = g(t'')$, $g'(t') = t' - t'' + g(t'')$. Finally, we obtain $g'(t')$.

If we are not concerned with the detailed queue evolution, point-wise interpolation is not necessary, and only $g'(a_k)$ will be evaluated.

4) ORDER OF SPILLOVER OCCURRENCE

In an arterial road, spillovers are possible from any turning direction, and obey the following principles:

- When two spillovers occur simultaneously, the ultimate spillover duration is in accordance with the longer one.
- Two spillovers do not overlap; the gap between two spillovers must be positive. This is because the preceding spillover will block the entrance, thereby cutting off the flow.

The above two principles can be generalized to multiple turning direction spillovers.

The second principle means that if one spillover precedes another, we can neglect the second and reconstruct the cumulative curves and CCC. The behavior of the second spillover is consequently changed due to the preceding spillover.

5) SOLUTION SCHEME AND GRID OF PLOTS DESIGN

All elements of the solution have been presented. In order to solve the queue dynamics simultaneously, a grid of plots is designed, as illustrated in Figure 11. Furthermore, the solution requires a table to record the characteristic arrival

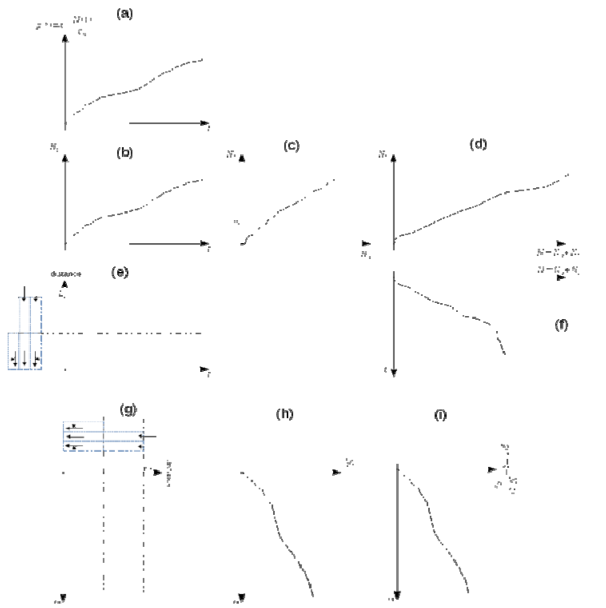

FIGURE 11. Grid of plots.

moments and queue tails, as shown in Table 1. It should be noted that column y_k may contain a tuple of two values if spillover exists; otherwise, it simply contains one value, namely y_k. Moreover, note that the final two columns, namely t_{x_k} and t_{y_k}, can be computed from other columns; thus, they are not mandatory. Column z_k also coincides with column x_k, but with an RGP cycle lag.

TABLE 1. Characteristic points table.

RGP ID	x_k	y_k	z_k	a_k	b_k	t_{x_k}	t_{y_k}
1							
2							

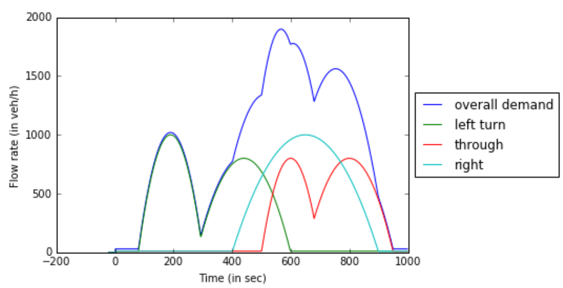

FIGURE 12. Demand settings.

In the plots grid, sub-plots (a), (b), and (e) are for the left-turn lane, while sub-plots (g), (h), and (i) are for the through lane. Sub-plot (a) is a CCC, (b) is a cumulative curve, and (e) is a spatial-temporal plot, which will provide the queue dynamics and vehicle trajectories directly. Similarly, sub-plot (i), (h), and (g) are a CCC, cumulative curve, and spatial-temporal diagram, respectively. Sub-plot (c) is the proportion line between the left-turn flow and through flow cumulative number; (d) is the cumulative curve relationship between the left-turn and overall flow; and (f) is the overall cumulative curve with time. The role of sub-plots (c), (d), and (f) is to ensure the FIFO principle.

The temporal axes of (a), (b), and (e) are aligned; similarly, sub-plots (g), (h) and (i), as well as (c) and (h), have aligned axes, while further aligned axes can be observed in (c), (d), and (f).

Using the plots grid, the overall procedure of the solution is as follows:

- Simultaneously solve the vehicle dynamics at different lanes in the C-section; if there no spillover exists for all directions, the process can iterate, without considering the interaction between the U-section and C-section.
- When one or more spillover occurrences exist, determine whether the spillovers start at the same moment; if so, adopt the longest spillover, and reconstruct the cumulative vehicle curves and CCC; if the onset moments are not the same, adopt the earliest one, and reconstruct the cumulative vehicle curves and CCC.
- Restart the process from the cycle when spillover occurs until the next spillover occurrence.

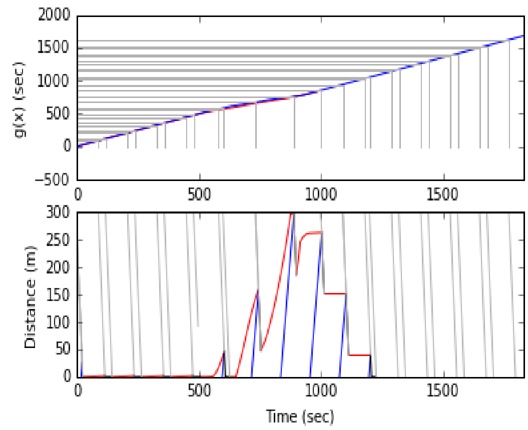

FIGURE 13. Vehicle queue for through flow.

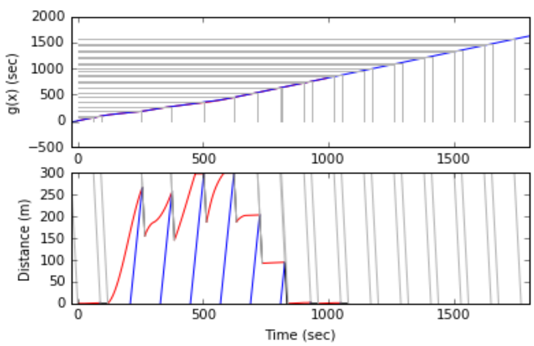

FIGURE 14. Vehicle queue for left-turn flow.

IV. AUTOMATION AND EXAMPLE

A. AUTOMATION

Due to its simplicity, this method is easily automated by means of a program. The pseudo-code is as follows:

Algorithm 1

Input: Cumulative curve at arterial road entrance, $N(t) = \sum_i N_i(t)$; signal settings, $r_{1i}, g_{1i}, r_{2i}, g_{2i}, \ldots r_{ki}, g_{ki}, \ldots$ for each direction i; lengths of the U-section and C-section, lu and ld, respectively.

Output: Vehicle queue dynamics for all directions.

Initialization:

SET the fundamental diagram parameters, v_f, q_m, and k_j.

SET the shifted cumulative curve and CCC at the C-section entrance, $N_i'(t) = N_i(t - \frac{l_u}{v_f})$ and $g_i'(t)$, where $g_i'(t) = t - \frac{N_i'(t)}{q_m}$.

SET current RGP cycle c to 1 for each turning direction; c will increase by 1 when queue dynamics of current RGP cycle are solved.

While not terminating **

RUN solving vehicle queue dynamics from current RGP cycle c until RGP cycle k, so that spillover occurs in cycle k.

Spillover onset moment determination: If any spillover exists, obtain the spillover duration that with minimal onset moment, $t_{spillover} = \{y_{ku}, y_{kv}$.

Demand reformulation: Change demands $N_i'(t)$ and $g_i'(t)$ for all turning directions.

RESET current RGP c for each direction; the reset is determined based on y_k^u.

END WHILE

Note ** termination means that all the demands are cleared, which can be determined from the cumulative curve.

In the automation process, the demand is represented as a piece-wise function. The process does need to divide the space and time. Therefor the computation efficiency can be guaranteed. Also note that, when the whole road experience spillover, the input demand would be limited either. The effect can also be obtained, from the CCC curve at the interface.

B. EXAMPLE

This section presents a simple example of the arterial road dynamics, based on the automated process. In order to create spillovers, the demand is illustrated in the following figure.

The demand of each turning direction is set as the combination of several parabolic curves. Each parabolic curve governs part of the time horizon. The total demand is the sum over all turning demands. Peak durations are observed for both the left-turn and through flows, and these partly overlap. The lengths of l_u and l_d are 100 m and 300 m, respectively. The green and red signals of the left-turn flow are 30 s and 90 s, as are those of the through flow. The through signal lags behind the left-turn signal by 25 s. Jam density k_j is 135 veh/km; free-flow speed is set to 40km/h, and the backward wave speed is −20km/h. The saturation flow rate then can be calculated, which is 1800 veh/h. The resulting queue dynamics are illustrated as follows:

The red lines in the above figures represent the queue tails, while the blue lines indicate the starting waves. The gray lines in the $g(t)$ curve provide solutions for the characteristic arrival moments.

Three spillover events occur. Among these, two spillover intervals are caused by the left-turn flow, at the third and fourth RGPs, with durations of [470.35, 504.0] and [584.11, 624.0], respectively. One spillover interval is caused by the through flow, at the seventh RGP, with a duration of [876.47, 889.0].

V. CONCLUSION AND REMARKS

The arterial road is the basic element of the urban road network, and its traffic flow dynamics provide a basis for understanding the traffic flow dynamics of the entire network. When the vehicle queue is short and within the C-section, the traffic flow dynamics can be described effectively using traditional methods. However, when C-section spillover occurs, the directional traffic flows interact with one another. The current method can simply describe this using numerical or software simulation.

This research proposes a graphical method for solving arterial road traffic flow dynamics. The method defines two characteristic arrival moments and three characteristic queue tail points for each RGP. These parameters can easily be obtained using a graphical method, which is developed based on the defined CCC. When the queue exceeds the C-section, the spillover duration can also be solved. When multiple spillovers occur, the first spillover will come into effect. The proposed model can be viewed as a variant of shock wave theory. When the parameters of the shockwave model are calibrated, the calibration result can be directly embedded in the proposed method.

The method can easily be integrated into the network traffic flow model. However, certain limitations still exist, which will be the focus of our next work:

- The C-section entrance capacity should not be identical to the saturation flow rate at the stop-line. This difference is caused by the mutual interaction among directional flows, such as lane changing. This capacity degradation should furthermore depend on the ratios of different turning flows; hence, the CCC curve requires improvement.
- The proposed method can be extended easily to shared lanes, with exogenous variable. Given the total demand, the exogenous variable which is the lane sharing ratio, will assign the total demand to each lane, either exclusive or shared. Therefore, the proposed method can be applied directly.
- When more than one upstream lanes exist, following spillover (if there is any), the ultimate inflow into the C-section will depend on demand and supply. Demand is the flow rate at the U-section exit, while supply is how

many vehicles the C-section can accommodate per unit time while obeying FIFO. Such a mechanism is effectively solved by the supply-demand scheme in models such as CTM. However, embedding it into the graphical process needs to be considered.

REFERENCES

[1] K. Aboudolas, M. Papageorgiou, and E. Kosmatopoulos, "Store-and-forward based methods for the signal control problem in large-scale congested urban road networks," *Transp. Res. C, Emerg. Technol.*, vol. 17, no. 2, pp. 163–174, 2009.

[2] C. F. Daganzo, "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory," *Transp. Res. B, Methodol.*, vol. 28, no. 4, pp. 269–287, 1994.

[3] M. J. Lighthill and G. B. Whitham, "On kinematic waves I. Flood movement in long rivers," *Proc. R. Soc. Lond. A, Math. Phys. Sci.*, vol. 229, no. 1178, pp. 281–316, 1955.

[4] M. J. Lighthill and G. B. Whitham, "On kinematic waves. II. A theory of traffic flow on long crowded roads," *Proc. R. Soc. Lond. A, Math. Phys. Sci.*, vol. 229, no. 1178, pp. 317–345, 1955.

[5] W.-L. Jin, "Point queue models: A unified approach," *Transp. Res. B, Methodol.*, vol. 77, pp. 1–16, Jul. 2015.

[6] H. Qi, D. Wang, P. Chen, and Y. Bie, "A link traffic model by incorporating both merits of vertical and horizontal queue concept," *KSCE J. Civil Eng.*, vol. 17, no. 5, pp. 1117–1129, 2013.

[7] H.-S. Qi, D.-H. Wang, and P. Chen, "Formation and propagation of local traffic jam," *Discrete Dyn. Nature Soc.*, vol. 2013, 2013, Art. no. 748529.

[8] M. P. H. Raadsen, M. C. J. Bliemer, and M. G. H. Bell, "An efficient and exact event-based algorithm for solving simplified first order dynamic network loading problems in continuous time," *Transp. Res. B, Methodol.*, vol. 92, pp. 191–210, Oct. 2015.

[9] B.-L. Ye, W. Wu, L. Li, and W. Mao, "A hierarchical model predictive control approach for signal splits optimization in large-scale urban road networks," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 8, pp. 2182–2192, Aug. 2016.

[10] I. Yperman, "The link transmission model for dynamic network loading," KU Leuven Energy Inst., Leuven, Belgium, Tech. Rep., 2016.

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