IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received November 21, 2017, accepted December 16, 2017, date of publication December 20, 2017,
date of current version February 14, 2018.

Digital Object Identifier 10.1109/ACCESS.2017.2785347

Downlink Multi-User MIMO Precoding Design Via Signal-Over-Leakage Capacity

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This work was supported by the National Natural Science Foundation of China under Grant 61631017.

ABSTRACT We study the linear precoder design problem in downlink multiuser MIMO (MU-MIMO). We first analyze the leakage-based precoder design under a full multiplexing constraint, which maximizes the signal-to-leakage-and-noise ratio (SLNR) for each individual user. We prove that the SLNR-optimal multistream precoder always maximizes the SLNR via essentially concentrating all the available transmit power on a single data stream, regardless of the rank constraint. We then propose a novel design criterion, called signal-over-leakage capacity (SLC), which corresponds to the achievable rate difference between a virtual signal-only link and a virtual interference-only link, for each individual user. We completely solve the SLC maximization problem and provide a closed-form optimal solution, which distributes the transmit power among multiple data streams, and thus better utilizes the available spatial degrees of freedom in an MU-MIMO system. Numerical experimental results are provided to corroborate the analysis.

INDEX TERMS MIMO, radio transmitters.

I. INTRODUCTION

Linear precoding is a widely used transmitter design for multi-user MIMO (MU-MIMO) communications, in 4G [1] and recently 5G cellular systems [2], [3]. Despite its incapability[1] of achieving the MIMO broadcast channel capacity region [5]–[8], linear precoding is attractive in practice due to its simplicity. For each user, the base station (BS) uses a precoding matrix to linearly transform the data symbols into a signal vector, and the signal vectors for all the users are linearly superposed to form the signal transmitted from the BS to all the users. Each user, upon receiving its signal, demodulates and decodes its corresponding data symbols, treating the signals for the other users as interference.

A number of previous works aim at maximizing the output signal-to-interference-plus-noise ratio (SINR) (or the SINR margin) [9], [10], or minimizing the transmit power while satisfying the SINR targets for all users (or the worst case user) [10], [11]. The solutions to those problems are typically *iterative*, due to the coupled nature of signal and interference. The iterative algorithms for solving the SINR maximization problems are often of considerable complexity, and have no analytical closed-form solutions. In contrast, low-complexity design algorithms, that perfectly cancel the interference for each user via zero-forcing (ZF), have been proposed and extensively analyzed; see, [12]–[14] for a sampling of the vast existing literature. Due to its simplicity, ZF is also a viable option for multi-cell MIMO cooperative networks; see [15] and references therein. Generally speaking, a basic restriction of the ZF solutions is that the number of transmit antennas at the BS needs to be at least no smaller than the total number of receive antennas at all the users. This constraint, however, significantly limits the scalability of MIMO in practice. Other interesting and useful linear precoder designs also exist; see, [16], [17].

Alternatively, the concept of signal *leakage* has been utilized to design the MU-MIMO precoder [18], [19]. For a particular user, its signal leakage is the interference caused by the signal, intended for that user, upon all the remaining users. Consequently, in [18] the signal-to-leakage-and-noise ratio (SLNR) is introduced as a performance metric, which, roughly speaking, is the ratio between the strength of the signal received by the intended user and by all the other users plus noise. The BS thus can design linear precoders that maximize the SLNR for each user, and conveniently, the SLNR maximization problems for all the users are completely decoupled, yielding closed-form solutions. This leads to a low-complexity and autonomous implementation, which is attractive for practical applications. Unlike ZF solutions, the SLNR-based solutions exist for general system

[1]The suboptimality of linear processing is evident in practical systems with a fixed number of antennas/users, although may be asymptotically negligible in the large system limit such as massive MIMO; see, [4].

dimensions, without requiring an excessively large number of transmit antennas at the BS. Furthermore, the SLNR-based precoders outperform the ZF-based ones in the noise-limited regime.

In this paper, we focus on the leakage-based linear precoder design, and propose a novel design criterion called *signal-over-leakage capacity* (SLC). The SLC follows a similar philosophy of SLNR, but is fundamentally different in that it quantifies signal over leakage not using the difference between their *power strengths* as SLNR, but using the difference between the *achievable rates*, one for a virtual link with signal for the intended user only, and the other for a virtual link with signal for all the remaining interfered users. As revealed later in this paper, the SLC-based precoder design is capable of overcoming the drawback of the SLNR-based design, especially in that SLC tends to exploit the available spatial degrees of freedom more effectively in a MU-MIMO system.

The main results in this paper are summarized as follows. First, we perform a thorough analysis of the SLNR precoder design for a multi-stream MU-MIMO system in which each user has a vector of independent symbols. Our analysis reveals that the SLNR-maximizing precoder, even with an explicit rank constraint to satisfy the multi-stream requirement, essentially concentrates all the available power on a single stream corresponding to the largest generalized eigenvalue of a certain matrix pair. This singular behavior stems from the fact that the SLNR quantifies the extent of signal over leakage only through contrasting their *power strengths*, which cannot promote the exploitation of the available multiple streams in the presence of abundant *spatial degrees of freedom*.

Motivated by the aforementioned analysis, we introduce the SLC-based precoder design problem, and provide a *closed-form solution*. The SLC-maximizing solution is related to the generalized eigenvalues of a certain matrix pair, and distributes the available transmit power among multiple spatial dimensions. As a result, it is fundamentally different from the SLNR-maximizing solution. In obtaining the SLC-maximizing solution, we first provide an upper bound of the SLC, and then construct a precoding method that exactly matches the upper bound. By doing so, we prove the optimality of the precoding design. Furthermore, we utilize a *perturbation* argument to establish that the solution is also applicable for rank-deficient scenarios. The SLC-based precoder design aims at striking a balance between the signal strength in individual data streams and the spatial degrees of freedom of the overall received signal. Note that unlike ZF-based precoders, SLC-based precoders do not belong to the class of generalized inverses [20]. Indeed, our simulation experiments show that the SLC-maximizing precoders usually significantly outperform the ZF-based and SLNR-based solutions, under various scenarios.

The remaining part of this paper is organized as follows. Section II describes the system model, and reviews the SLNR precoder design. Section III establishes the SLNR-maximizing precoder design, showing its near-singular structure, either without or with a rank constraint. Section IV introduces the concept of SLC, and solves the SLC-maximizing precoder design problem. Section V presents numerical experimental results. Finally Section VI concludes this paper.

Throughout this paper, matrices are represented in bold upper cases, and vectors in bold lower cases. $\cdot^T$ denotes transpose, $\cdot^*$ denotes conjugate transpose, and $\cdot^\dagger$ denotes pseudo-inverse. $\mathbf{A} \preceq \mathbf{B}$ means that $\mathbf{B}-\mathbf{A}$ is positive semidefinite, and $\mathbf{A} \prec \mathbf{B}$ means that $\mathbf{B}-\mathbf{A}$ is positive definite. $\mathbb{E}$ denotes expectation, Tr denotes trace, $||\cdot||$ denotes the norm of a vector, and $|\cdot|$ denotes the determinant of its operand matrix.

II. SYSTEM MODEL AND PRELIMINARIES

A. SYSTEM MODEL

A downlink MU-MIMO communication system is considered. It is assumed that the transmitter is equipped with N antennas, and K users, each equipped with M antennas,[2] are simultaneously served. Denote $\mathbf{H_k} \in \mathbb{C}^{M\times N}$ as the narrowband channel matrix between the transmitter and user k. The entries of $\mathbf{H_k}$ are modeled as independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. It is assumed that the transmitter has perfect knowledge of $\{\mathbf{H}_k\}_{k=1}^K$. The signal vector intended for user k is denoted as $\mathbf{X_k} \in \mathbb{C}^{N\times 1}$ and can be written as

$$\mathbf{X}_k = \mathbf{W}_k\mathbf{s}_k = \sum_{i=1}^{L} s_{ki}\mathbf{w}_{ki}, \tag{1}$$

where $\mathbf{s}_k = [s_{k1}, \cdots, s_{kL}]^T$ denotes the L independently modulated data symbols for user k, $L \le \min(M, N)$, and $\mathbf{W}_k = [\mathbf{w}_{k1}, \cdots, \mathbf{w}_{kL}] \in \mathbb{C}^{N\times L}$ denotes the precoding matrix for these L data symbols, where $\mathbf{w}_{ki}$ applies on data symbol s_{ki}. The overall $N \times 1$ signal vector is given by the linear superposition of the individual transmitted signal vectors for all the K users,

$$\mathbf{X} = \sum_{k=1}^{K}\mathbf{X}_k = \sum_{k=1}^{K}\mathbf{W}_k\mathbf{s}_k. \tag{2}$$

The data symbol $\mathbf{s}_k$ is normalized so that $\mathbb{E}\left[\mathbf{s}_k\mathbf{s}_k^*\right] = \mathbf{I}_L$, for each $k = 1, \ldots, K$. Two different types of power constraint are considered in this paper. The first one is a *total power constraint*, which requires that the total transmit power across all antennas for any user k is limited to P_k.[3] This suggests

$$\mathrm{Tr}\left(\mathbf{W}_k\mathbf{W}_k^*\right) = \sum_{i=1}^{L}\mathbf{w}_{ki}^*\mathbf{w}_{ki} = P_k, \quad \forall k = 1, \cdots, K. \tag{3}$$

The second power constraint is a *matrix power constraint*, which can be written as

$$\mathbf{W}_k\mathbf{W}_k^* \preceq \mathbf{P}_k, \quad \forall k = 1, \cdots, K, \tag{4}$$

[2]For simplicity we let all the users be equipped with the same number of receive antennas.

[3]Power allocation among active users is not studied. In fact, P_k may be viewed as an outcome of certain power allocation scheme among users.

where $\mathbf{P}_k$ is a positive semidefinite matrix. Note that (4) is a general power constraint, including several other types of power constraint as special cases, e.g., per-stream maximum power constraint. Both (3) and (4) are frequently used in the literature and practical system designs.

The received signal at user k is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{X} + \mathbf{z}_k = \mathbf{H}_k \mathbf{W}_k \mathbf{s}_k + \sum_{j \neq k} \mathbf{H}_k \mathbf{W}_j \mathbf{s}_j + \mathbf{z}_k, \tag{5}$$

where the first term $\mathbf{H}_k \mathbf{W}_k \mathbf{s}_k$ is the desired signal, the second term $\sum_{j\neq k} \mathbf{H}_k \mathbf{W}_j \mathbf{s}_j$ is the interference from other users' signals to user k, and $\mathbf{z}_k$ is the additive Gaussian noise vector whose elements are assumed to be independent and identically distributed with zero mean and unity variance,

$$\mathbb{E}\left[\mathbf{z}_k \mathbf{z}_k^*\right] = \mathbf{I}_M. \tag{6}$$

Figure 1 illustrates a block diagram of the system under consideration.

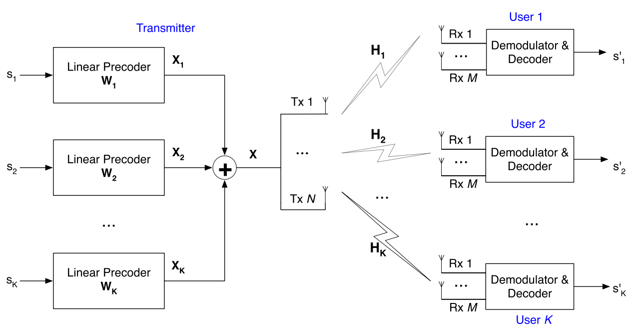


FIGURE 1. Block diagram of a downlink MU-MIMO communication system with linear precoding.

B. SLNR AND PREVIOUS RESULTS

For the system model considered in Section II-A, the SLNR for user k is defined as [18, eqs. (42)–(44)]:

$$\mathsf{SLNR}_k = \frac{\mathbb{E}\left[\mathbf{s}_k^* \mathbf{W}_k^* \mathbf{H}_k^* \mathbf{H}_k \mathbf{W}_k \mathbf{s}_k\right]}{M + \mathbb{E}\left[\sum_{i\neq k}\sum_{j\neq k} \mathbf{s}_k^* \mathbf{W}_k^* \mathbf{H}_i^* \mathbf{H}_j \mathbf{W}_k \mathbf{s}_k\right]}. \tag{7}$$

With the total power constraint (3), the expression of (7) can be rewritten as

$$\mathsf{SLNR}_k = \frac{\mathrm{Tr}\left(\mathbf{W}_k^* \mathbf{H}_k^* \mathbf{H}_k \mathbf{W}_k\right)}{\mathrm{Tr}\left[\mathbf{W}_k^* \left(\frac{M}{P_k}\mathbf{I} + \tilde{\mathbf{H}}_k^* \tilde{\mathbf{H}}_k\right) \mathbf{W}_k\right]}, \tag{8}$$

where we define $\tilde{\mathbf{H}}_k = [\mathbf{H}_1, \cdots, \mathbf{H}_{k-1}, \mathbf{H}_{k+1}, \cdots, \mathbf{H}_K]^T$.

As suggested in [18], with a total power constraint (3) and a rank constraint rank $(\mathbf{W}_k) = L$, an SLNR-based precoder for user k is given by [18, eq. (47)]:

$$\mathbf{W}_{\mathsf{SLNR,k}} = \rho_k \mathbf{T_k} \begin{bmatrix} \mathbf{I_L} \\ \mathbf{0} \end{bmatrix}, \tag{9}$$

i.e., the leading L columns of the eigenspace $\mathbf{T_k}$, where $\mathbf{T_k}$ is an invertible matrix that simultaneously diagonalizes $\mathbf{H}_k^*\mathbf{H}_k$ and $\mathbf{I} + \tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k$. Here the scaling factor ρ_k is chosen such that $\mathrm{Tr}\left(\mathbf{W}_k^*\mathbf{W}_k\right) = P_k$. The resulting SLNR is $\sum_{i=1}^{L} \lambda_i / L$, where $\{\lambda_i\}_{i=1}^N$ are the generalized eigenvalues of $\left(\mathbf{H}_k^*\mathbf{H}_k, \frac{M}{P_k}\mathbf{I} + \tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k\right)$, arranged in decreasing order.

III. SLNR OPTIMIZATION WITH TOTAL POWER CONSTRAINT

A. WITHOUT RANK CONSTRAINT

We first study the SLNR optimization problem with only a total power constraint, without any constraint imposed on the number of independent streams that the transmitter has to transmit to user k simultaneously. This case is of significance because the precoder design now has the freedom to choose any number, up to $\min(M, N)$, of independent data streams for user k in order to maximize its SLNR. In addition, this case also provides the basis for studying the case with a rank constraint in the next subsection. The optimization problem is formally stated as follows.

Problem 1: Select an $N \times L$ precoder matrix $\mathbf{W}_k$ such that the SLNR of user k is maximized with a total power constraint, i.e., solve the following problem:

$$\underset{\mathbf{W}_k}{\text{maximize}} \quad \frac{Tr\left(\mathbf{W}_k^* \mathbf{H}_k^* \mathbf{H}_k \mathbf{W}_k\right)}{Tr\left[\mathbf{W}_k^* \left(\frac{M}{P_k}\mathbf{I} + \tilde{\mathbf{H}}_k^* \tilde{\mathbf{H}}_k\right) \mathbf{W}_k\right]}$$
$$\text{subject to } Tr\left(\mathbf{W}_k^*\mathbf{W}_k\right) = P_k. \tag{10}$$

Problem 1 considers the multi-stream SLNR optimization in a general setting. Despite that the maximum possible rank of $\mathbf{W}_k$ is $\min(M, N)$, the SLNR-maximizing solution to Problem 1 may have any rank ranging from 1 to $\min(M, N)$. This is different from the corresponding problem considered in [18] for multiple streams where a fixed rank is assumed.

The solution to Problem 1 is given by the following theorem.

Theorem 1: The $N \times L$ precoder that maximizes the SLNR in Problem 1 is given by

$$\mathbf{W}^o_{\mathsf{SLNR},P1,k} = \sqrt{P_k}\left[\frac{\mathbf{T}_k^{(1)}}{\|\mathbf{T}_k^{(1)}\|} \quad \mathbf{0} \quad \cdots \quad \mathbf{0}\right], \tag{11}$$

where $\mathbf{T}_k^{(1)}$ is the first column of $\mathbf{T}_k$ corresponding to the largest generalized eigenvalue of

$$\left(\mathbf{H}_k^*\mathbf{H}_k, \frac{M}{P_k}\mathbf{I} + \tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k\right), \tag{12}$$

and $\mathbf{T}_k$ is the nonsingular matrix that simultaneously diagonalizes $\mathbf{H}_k^*\mathbf{H}_k$ and $\frac{M}{P_k}\mathbf{I} + \tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k$, such that all the generalized eigenvalues are arranged in decreasing order. The maximum SLNR is

$$\mathsf{SLNR}^o_{P1,k} = \lambda_{\max}, \tag{13}$$

where $\lambda_{\max}$ is the largest generalized eigenvalue of $\left(\mathbf{H}_k^*\mathbf{H}_k, \frac{M}{P_k}\mathbf{I} + \tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k\right)$.

Proof: See Appendix C. □

Theorem 1 states that, even with the freedom to perform spatial multiplexing with up to $\min(M, N)$ independent streams, the SLNR-maximizing solution under a total power constraint is always to concentrate all the available transmit power on one stream, with the precoder obtained by simultaneously diagonalizing $\mathbf{H}_k^*\mathbf{H}_k$ and $\frac{M}{P_k}\mathbf{I}+\tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k$ and choosing the column of $\mathbf{T}_k$ that corresponds to the largest generalized eigenvalue of that pair of matrices.

One additional remark is that in the original SLNR optimization problem [18, eq. (44)], there is an additional diagonal constraint on $\mathbf{W}_k^*\mathbf{H}_k^*\mathbf{H}_k\mathbf{W}_k$. This constraint is not explicitly specified in Problem 1 (and Problem 2 in the next subsection), but we can readily verify that the solutions to those problems satisfy such a diagonal constraint, by choosing $\mathbf{V}_k = \mathbf{I}$ in the singular value decomposition (SVD) of $\mathbf{D}_k$. The discussion following (49) in Appendix A provides more details to this remark.

B. WITH RANK CONSTRAINT

Let us now consider the SLNR optimization problem with both a total power constraint and a minimum rank constraint. The problem can be formulated as

Problem 2: Select an $N \times L$ precoder matrix $\mathbf{W}_k$ such that the SLNR of user k is maximized, for every $k = 1, \ldots, K$:

$$\begin{aligned} &\underset{\mathbf{W}_k}{\mathit{maximize}} \quad \frac{Tr\left(\mathbf{W}_k^*\mathbf{H}_k^*\mathbf{H}_k\mathbf{W}_k\right)}{Tr\left[\mathbf{W}_k^*\left(\frac{M}{P_k}\mathbf{I}+\tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k\right)\mathbf{W}_k\right]} \\ &\mathit{subject\ to}\ Tr\left(\mathbf{W}_k^*\mathbf{W}_k\right) = P_k, \\ &\qquad rank\left(\mathbf{W}_k\right) \geq L. \end{aligned} \tag{14}$$

The following corollary of Theorem 1 states that, even with the rank constraint, the maximum SLNR is essentially (i.e., arbitrarily close to) $\lambda_{\max}$, which is the solution of Problem 1 without rank constraint.

Corollary 2: For Problem 2, $\forall\epsilon > 0$, $\exists\delta_1, \cdots, \delta_L > 0$ and an $N \times L$ precoder $\mathbf{W}_k$ that satisfies

$$|\mathsf{SLNR}_k\left(\mathbf{W}_k\right) - \lambda_{\max}| \leq \epsilon, \tag{15}$$

where $\lambda_{\max}$ is the largest generalized eigenvalue of $\left(\mathbf{H}_k^*\mathbf{H}_k, \frac{M}{P_k}\mathbf{I}+\tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k\right)$. Moreover, the optimal precoder $\mathbf{W}^o_{\mathsf{SLNR},P2,k}$ has the following form

$$\begin{aligned} &\mathbf{W}^o_{\mathsf{SLNR},P2,k} \\ &= \frac{\sqrt{P_k}}{\sqrt{(1-\delta_1)||\mathbf{T}_k^{(1)}||^2 + \sum_{i=2}^{L}\delta_i||\mathbf{T}_k^{(i)}||^2}} \\ &\quad \times \left[\sqrt{1-\delta_1}\mathbf{T}_k^{(1)} \quad \sqrt{\delta_2}\mathbf{T}_k^{(2)} \quad \cdots \quad \sqrt{\delta_L}\mathbf{T}_k^{(L)} \quad \mathbf{0}\cdots\ \mathbf{0}\right], \end{aligned} \tag{16}$$

where $\mathbf{T}_k^{(j)}$ is the j-th column of $\mathbf{T}_k$ that corresponds to the j-th largest generalized eigenvalue of $\left(\mathbf{H}_k^*\mathbf{H}_k, \frac{M}{P_k}\mathbf{I}+\tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k\right)$, with $\mathbf{T}_k$ being the nonsingular matrix that simultaneously diagonalizes $\mathbf{H}_k^*\mathbf{H}_k$ and $\frac{M}{P_k}\mathbf{I}+\tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k$, arranging the generalized eigenvalues in decreasing order. As $\epsilon \to 0$, we have that $\delta_1, \ldots, \delta_L \to 0$ and hence $\mathbf{W}^o_{\mathsf{SLNR},P2,k} \to \mathbf{W}^o_{\mathsf{SLNR},P1,k}$.

Proof: See Appendix A. □

Corollary 2 shows that, adding a minimum rank constraint of L has no fundamental effect on the maximum SLNR and the SLNR-maximizing precoder design. The SLNR can still be rendered arbitrarily close to $\lambda_{\max}$, which is the maximum SLNR without rank constraint. The construction in the proof (see Appendix B) suggests that the maximum SLNR is achieved when the precoder $\mathbf{W}_k$ is highly "skewed", in the sense that *almost* all the available transmit power is allocated to one stream, while the other streams have only negligible powers so as to obey the rank constraint. Such precoder design, although satisfying the rank constraint, is numerically unstable, and is likely to result in rather poor decoding performance as the worst case stream SINR is extremely low.

Theorem 1 and Corollary 2 highlight the issue with the SLNR-based precoder design in a full MIMO setting which exploits the degree of freedom gain. The SLNR is a single-variable performance metric summarizing the power strengths among streams, and only reflects the difference between the power strengths of the signal and the leakage plus noise; the total power constraint also does not enforce any per-stream requirement. Consequently, the SLNR-maximizing precoder is always (or arbitrarily close to) a single-stream beamformer, and thus cannot effectively exploit the available spatial degrees of freedom in an MU-MIMO system.

IV. SIGNAL-OVER-LEAKAGE CAPACITY WITH MATRIX POWER CONSTRAINT

Motivated by the issue of exploiting the spatial multiplexing gain of an SLNR-based precoder design, in this section we propose a new design criterion called *signal-over-leakage capacity* (SLC), and proceed to establish the SLC-based optimal precoder design. Following a similar philosophy as the SLNR in [18], herein we focus on maximizing the difference between the impact of user k's precoded signal on its intended receiver (user k) and that on the remaining interfered receivers (all the other users $i \neq k$). However, instead of using the metric of the SLNR which only captures the difference in terms of the received *power*, in the SLC-based design we use the difference in terms of the *achieved rates* (i.e., *capacity* under specified precoders), between the intended link and the interfered links.

A. DEFINITION OF SLC

To begin with, let us first consider a virtual link, from the transmitter to the receiver at the intended user k, which is a single-user MIMO channel as

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{X}_k + \mathbf{z}_k = \mathbf{H}_k\mathbf{W}_k\mathbf{s}_k + \mathbf{z}_k. \tag{17}$$

We may imagine that a genie perfectly removes all the precoded signals intended for all the other users, from the actual signal model (5).

Under i.i.d. Gaussian input distribution for $\mathbf{s}_k$ as we have already assumed in the system model, the capacity of (17), which contains no interference, can be written as

$$C_k = \log \left|\mathbf{I}_M + \mathbf{H}_k \mathbf{W}_k \mathbf{W}_k^* \mathbf{H}_k^*\right|. \tag{18}$$

We call C_k the *signal capacity* for user k.

Next let us introduce the *leakage capacity*, which roughly speaking quantifies the aggregate achieved rates for all the other users $i \neq k$ from the transmission of user k's signal $\mathbf{X}_k$. Consider the following virtual link,

$$\tilde{\mathbf{y}}_k = \tilde{\mathbf{H}}_k \mathbf{W}_k \mathbf{s}_k + \tilde{\mathbf{z}}_k, \tag{19}$$

where

$$\tilde{\mathbf{y}}_k = [\mathbf{y}_1, \cdots, \mathbf{y}_{k-1}, \mathbf{y}_{k+1}, \cdots, \mathbf{y}_K]^T, \tag{20}$$
$$\tilde{\mathbf{z}}_k = [\mathbf{z}_1, \cdots, \mathbf{z}_{k-1}, \mathbf{z}_{k+1}, \cdots, \mathbf{z}_K]^T. \tag{21}$$

So $\tilde{\mathbf{y}}_k$ is the collection of all the received signals at the interfered users $i \neq k$, due to the precoded signal $\mathbf{W}_k \mathbf{s}_k$ intended for user k. Again we may imagine that a genie perfectly removes all the precoded signals intended for all the other users. The leakage capacity is thus given by

$$\tilde{C}_k = \log \left|\mathbf{I}_{(K-1)M} + \tilde{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^* \tilde{\mathbf{H}}_k^*\right|, \tag{22}$$

implicitly assuming that all the interfered receivers $i \neq k$ can jointly perform decoding. Note that this is a worst-case scenario for interference.

Finally the SLC for user k is defined as the gap between C_k and $\tilde{C}_k$,

$$\begin{aligned} C_{\mathsf{SL},k} &\doteq C_k - \tilde{C}_k \\ &= \log \frac{\left|\mathbf{I}_M + \mathbf{H}_k \mathbf{W}_k \mathbf{W}_k^* \mathbf{H}_k^*\right|}{\left|\mathbf{I}_{(K-1)M} + \tilde{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^* \tilde{\mathbf{H}}_k^*\right|}. \end{aligned} \tag{23}$$

We note that the SLC in (23) is a metric for quantifying the difference between the quality of signaling in the intended link and the capability of creating interference in the interfered links, for each considered user. Clearly, a large SLC implies that the quality of signaling in the intended link is high, and/or that the quality of signaling by the precoded signal in all the interfered links is low, either of which is definitely desirable for effective MU-MIMO transmission. Admittedly, just like the SLNR, the SLC is also a heuristic metric, which, to the best of our knowledge at present, cannot be readily rendered a physically measurable interpretation (such as SINR or achievable rates), because both C_k and $\tilde{C}_k$ are defined for virtual links; establishing appropriate interpretation for SLNR and SLC is an interesting research topic for future study. Nevertheless, as our numerical experiments show, precoders designed based on maximizing the SLC tend to perform quite well, compared with existing techniques, especially in terms of achieving spatial multiplexing gains in the system.

B. SLC-OPTIMAL PRECODER DESIGN

We can formulate the SLC maximization problem with a general matrix power constraint as follows.

Problem 3: Select an $N \times \min(M, N)$ precoder matrix $\mathbf{W}_k$ such that the SLC of user k is maximized with a matrix power constraint of a positive semidefinite matrix $\mathbf{P}_k$, i.e., solve the following problem:

$$\begin{aligned} &\underset{\mathbf{W}_k}{\text{maximize}}\ C_{\mathsf{SL},k} \\ &\text{subject to}\ \mathbf{W}_k \mathbf{W}_k^* \preceq \mathbf{P}_k. \end{aligned} \tag{24}$$

The expression of $C_{\mathsf{SL},k}$ is the difference between two log-determinants, and thus is not concave with $\mathbf{W}_k$ in general. However, by identifying an upper bound on $C_{\mathsf{SL},k}$ and constructing a specific choice of $\mathbf{W}_k$ that exactly achieves the upper bound, we can solve this optimization problem with a closed-form solution, as discussed in the remainder of this section.

For ease of exposition, let us introduce the following auxiliary variables. Denote $\mathbf{S}_1 = \mathbf{H}_k^* \mathbf{H}_k$ and $\mathbf{S}_2 = \tilde{\mathbf{H}}_k^* \tilde{\mathbf{H}}_k$. Since $\mathbf{P}_k$ is a positive semidefinite Hermitian matrix, it has a Hermitian square root matrix $\mathbf{G}$ that satisfies $\mathbf{P}_k = \mathbf{G}^2 = \mathbf{G}\mathbf{G}^* = \mathbf{G}^*\mathbf{G}$. Now consider the generalized eigenvalue problem of

$$\left(\mathbf{I}_N + \mathbf{G}^* \mathbf{S}_1 \mathbf{G}, \mathbf{I}_N + \mathbf{G}^* \mathbf{S}_2 \mathbf{G}\right). \tag{25}$$

Since both $\mathbf{I}_N + \mathbf{G}^* \mathbf{S}_1 \mathbf{G}$ and $\mathbf{I}_N + \mathbf{G}^* \mathbf{S}_2 \mathbf{G}$ are positive definite, there exists a nonsingular matrix $\mathbf{Q}$ that simultaneously diagonalizes both matrices:

$$\mathbf{Q}^* \left(\mathbf{I}_N + \mathbf{G}^* \mathbf{S}_1 \mathbf{G}\right) \mathbf{Q} = \mathbf{\Omega}, \tag{26}$$
$$\mathbf{Q}^* \left(\mathbf{I}_N + \mathbf{G}^* \mathbf{S}_2 \mathbf{G}\right) \mathbf{Q} = \mathbf{I}_N, \tag{27}$$

where the entries of $\mathbf{\Omega}$ are arranged in decreasing order:

$$\omega_1 \geq \cdots \geq \omega_n > 1 \geq \omega_{n+1} \geq \cdots \geq \omega_N > 0. \tag{28}$$

Theorem 3: The optimal precoder that solves Problem 3 is given by

$$\mathbf{W}_{\mathsf{SLC},k}^o = \Big[\underbrace{\mathbf{G}\mathbf{Q}_1 \left(\mathbf{Q}_1^* \mathbf{Q}_1\right)^{-\frac{1}{2}}}_{N \times n} \quad \underbrace{\mathbf{0}}_{N \times (\min(M,N)-n)} \Big], \tag{29}$$

where $\mathbf{Q}_1$ is a matrix consisting of the first n columns of $\mathbf{Q}$.

Proof: See Appendix B. □

Inspecting the form of the SLC-maximizing solution in Theorem 3, it is clear that up to n streams are used for transmission, where the value of n is determined by the generalized eigendecomposition of a pair of matrices which involve the intended link channel and the interfered link channels, respectively. Comparing to the SLNR-optimal solution, the resulting precoding is capable of exploiting the available spatial degrees of freedom in the system, in a systematic and controlled manner.

As a final remark, it is worth noting that the computational complexity of implementing the SLC-maximizing precoder is $\mathcal{O}(N^3)$ (assuming that MK is on the same order of magnitude of N), which is qualitatively the same as the SLNR-based design [18] and the ZF-based design [12].

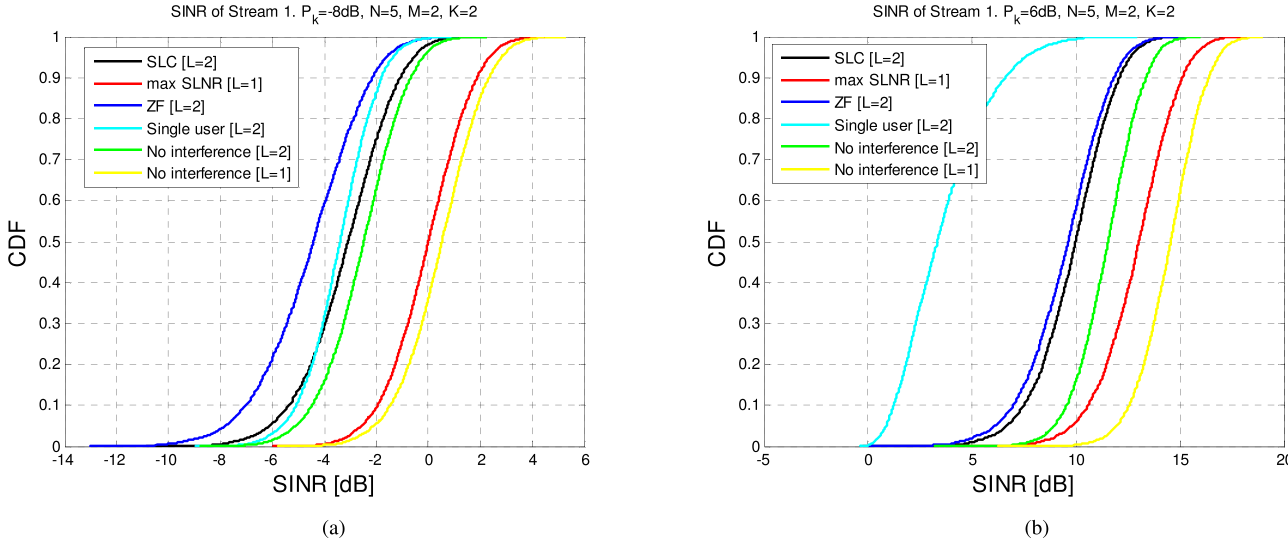


FIGURE 2. CDF curves for SINR. $(N, M, K) = (5, 2, 2)$. (a) Average receive SNR $= -8$ dB. (b) Average receive SNR $= 6$ dB.

V. SIMULATION RESULTS

Analytically quantifying the performance of the SLC-based precoder is challenging since it involves the order statistics of the generalized eigenvalues of random matrices. In this section, we resort to numerical experiments to validate the proposed SLC-based precoder design, and to compare its performance to other precoder designs. In the simulation environment, a narrowband quasi-static MU-MIMO channel is used. To make comprehensive comparison among the designs, various system configurations have been simulated to reflect the different combinations of antennas and users: (N, M, K). All simulations are performed over 5000 channel realizations for producing the cumulative distribution function (CDF) curves. We consider a simple matrix power constraint which imposes the same per-antenna maximum transmit power constraint for each of the N transmit antennas, i.e., $\mathbf{P}_k$ being a scaled identity matrix. Since the SLC-based precoder uses a matrix power constraint and may not use up all the available signal dimensions, if we set the matrix power constraint such that its total power is equal to the total power constraint for the other precoders, the actual transmit power of the SLC-based precoder may be smaller. We thus first compute the actual transmit power of the SLC-based precoder, and use it as the constraint for other precoders, in order to guarantee a fair comparison.

A. SINR COMPARISON

Neither SLNR nor SLC is directly meaningful to the practical system design, and thus we resort to numerically evaluating the SINR performance among all precoders. The SINR is computed assuming a linear ZF receiver at each user. Note that the signal received at user k is described by (5). If we define an equivalent channel $\mathbf{H}_{\mathsf{equ}} = \mathbf{H}_k \mathbf{W}_k$, the ZF detector can be written as

$$\mathbf{G}_{\mathsf{zf}} = \mathbf{H}_{\mathsf{equ}}^{\dagger} = \left(\mathbf{H}_{\mathsf{equ}}^{*} \mathbf{H}_{\mathsf{equ}}\right)^{-1} \mathbf{H}_{\mathsf{equ}}^{*}. \quad (30)$$

After some derivation, the covariance matrix of the post-ZF noise is

$$\mathbf{B} = \mathbf{G}_{\mathsf{zf}} \left(\mathbf{I} + \mathbf{H}_k \left(\sum_{i \neq k} \mathbf{W}_k \mathbf{W}_k^{*} \right) \mathbf{H}_k^{*} \right) \mathbf{G}_{\mathsf{zf}}^{*}, \quad (31)$$

and the post-ZF SINR of stream i is

$$\mathsf{SINR}_i = \frac{1}{\mathbf{B}(i, i)}. \quad (32)$$

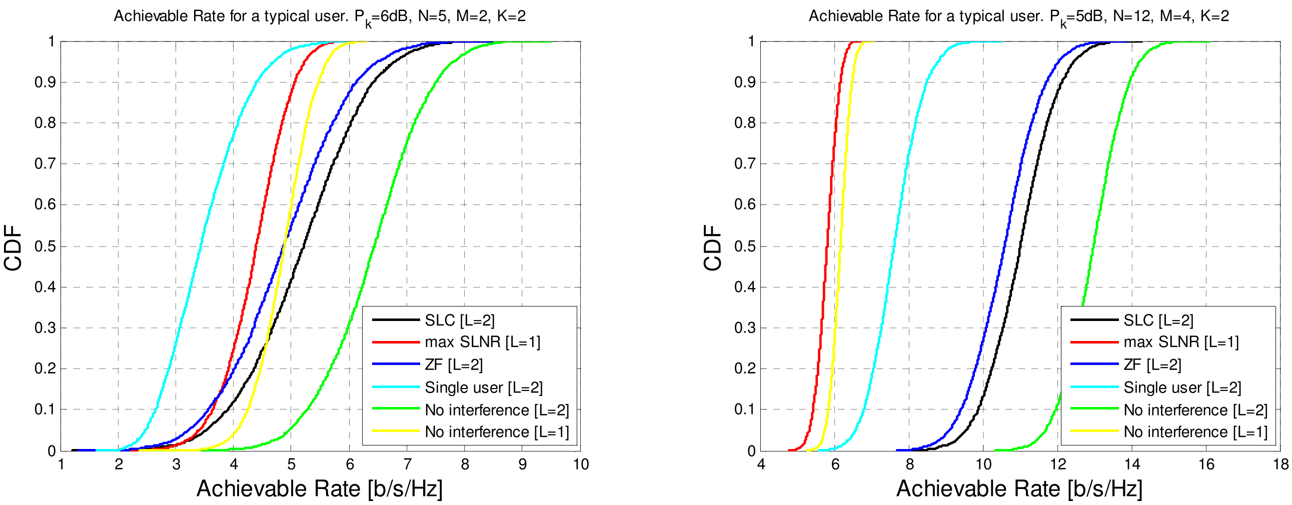
Fig. 2 plots the CDF of the SINR distribution associated with various precoders of a typical user:

- the SLC-maximizing solution (29)
- the SLNR-maximizing solution (11);
- the ZF precoder;
- the single-user precoder: the first $\min(N, M)$ eigenvectors of $\mathbf{H}_k^{*}\mathbf{H}_k$ that correspond to its $\min(N, M)$ largest eigenvalues.

We also plot the no-interference case as a benchmark, which is a genie-aided setting removing all the inter-user interference at the receiver. The allowed number of data streams L is indicated in the legends of the plots. Note that when $L > 1$ the transmitted signal may consist of multiple symbol streams, and we plot the SINR associated with the first stream.

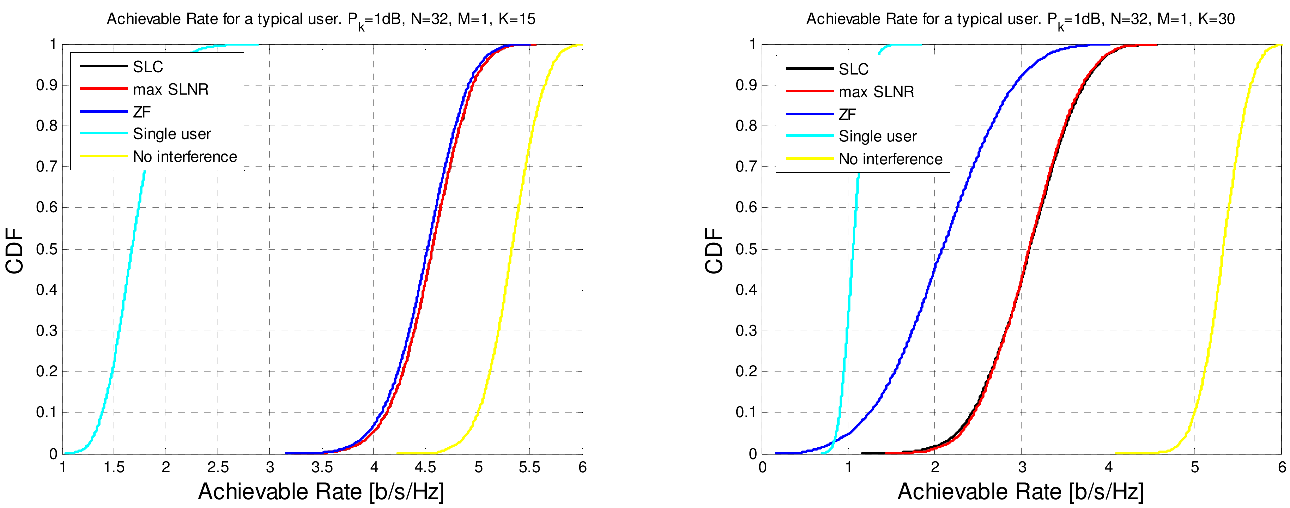
Not surprisingly, the SLNR-maximizing solution (11) achieves the best performance in terms of SINR among all precoders. This is consistent with the general rule of thumb that SLNR maximization typically leads to good SINR performance. Additionally, while all other precoders transmit two streams, the SLNR-maximizing solution (11) transmits a single stream only and hence has a power gain.

On the other hand, among all precoders with $L = 2$, the SLC-maximizing solution (29) yields the best SINR performance, consistently dominating the other precoders



(a) (b)

FIGURE 3. **CDF curves for achievable rates. (a) $(N, M, K) = (5, 2, 2)$ and average receive SNR = 6 dB. (b) $(N, M, K) = (12, 4, 2)$ and average receive SNR = 5 dB.**



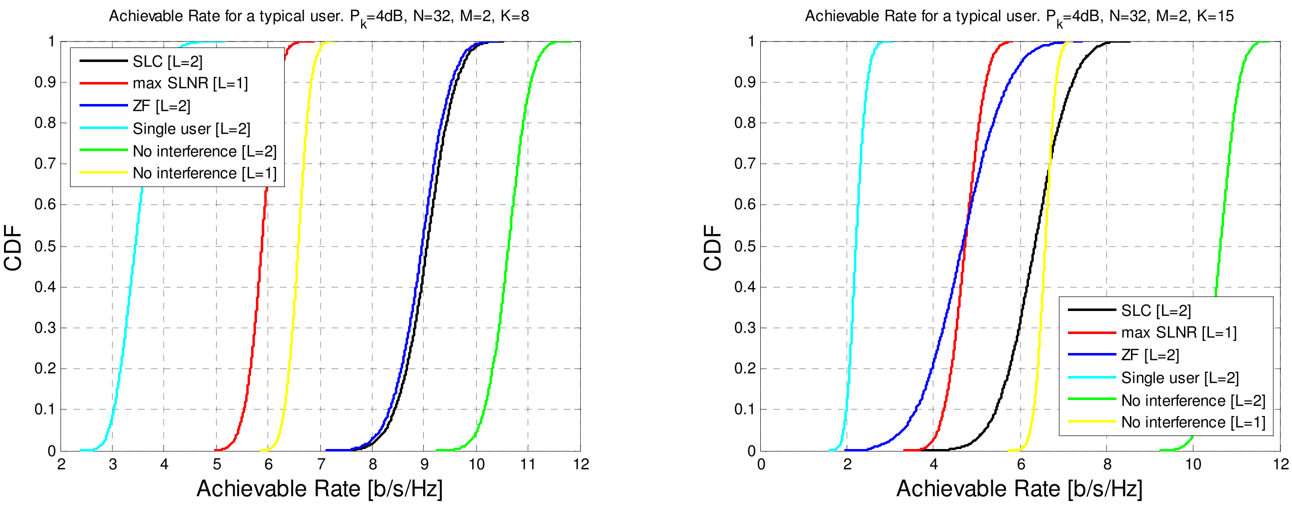
(a) (b)

FIGURE 4. **CDF curves for achievable rates with increasing number of users. $(N, M) = (32, 1)$. Average receive SNR = 1 dB. Note that SLC and max-SLNR curves are almost overlapping in both figures. (a) $(N, M, K) = (32, 1, 15)$. (b) $(N, M, K) = (32, 1, 30)$.**

(except that in Fig. 2(a) the single-user precoder exhibits a slightly thinner tail in the low-SINR region below SINR of −4 dB).

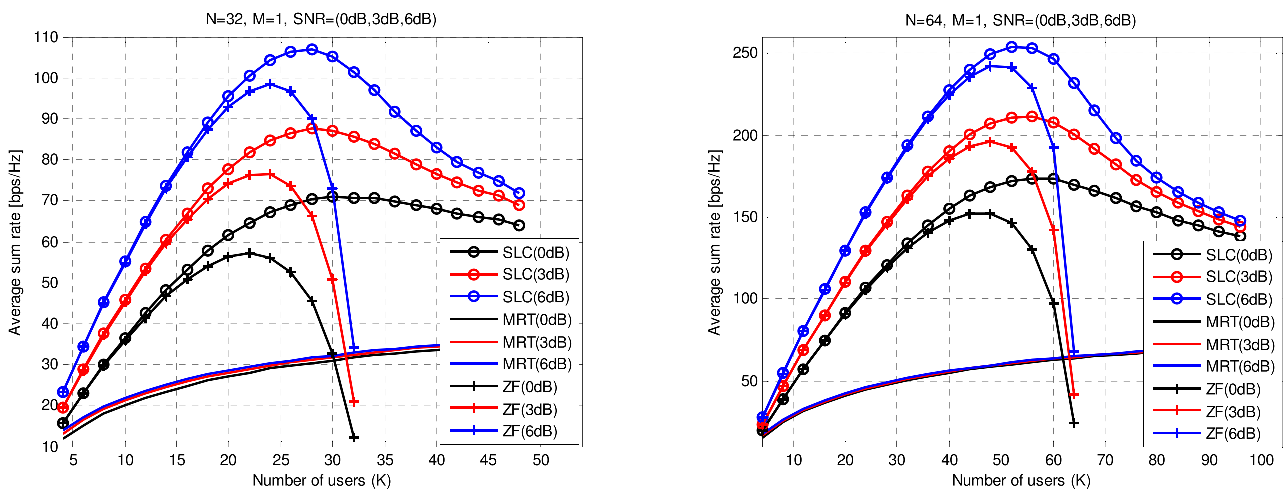
B. ACHIEVABLE RATE COMPARISON

Fig. 3(a) and Fig. 3(b) plot the CDF of a typical user's achievable rates with various precoders in the medium-to-high SNR regime, where achieving high degrees of freedom is critical for realizing the system capacity gain. Unlike in the previous subsection for SINR comparison where we assumed a ZF receiving detector, here we do not need to make such assumption and simply compute the achieved mutual information of the corresponding vector signal model. We can see that in the studied configurations, the SLC-maximizing solution (29) achieves the highest capacity among all precoders. This is due to the fact that the SLC-maximizing solution trades off, in a systematic and controlled way, the virtual signal-only link and the virtual interference-only link, and thus effectively exploits the available spatial degrees of freedom. The observation is more clear if we compare the SLC-maximizing solution to the SLNR-maximizing solution, which maximizes the SLNR by focusing all the available transmit power on a single stream and hence suffers from a rate loss due to the lack of spatial multiplexing. Note that the results for the SLC-based design plotted in Fig. 3(a) and Fig. 3(b) just correspond to a specific matrix power constraint that imposes the same per-antenna maximum transmit power constraint for each of the N transmit



(a) (b)

FIGURE 5. **CDF curves for achievable rates with increasing number of users. $(N, M) = (32, 2)$. Average receive SNR = 4 dB. (a) $(N, M, K) = (32, 2, 8)$. (b) $(N, M, K) = (32, 2, 15)$.**



(a) (b)

FIGURE 6. **Average sum rate vs. number of users. Average receive SNR varies from 0 dB to 6dB. (a) $(N, M) = (32, 1)$. (b) $(N, M) = (64, 1)$.**

antennas. Such constraint is only a subset of the total power constraint of the other precoders, and therefore via optimizing over all possible matrix power constraints that have the same total power, the achievable rate gain of the SLC-maximizing solution (29) over the other precoders may well be even larger. We do not pursue this issue further in this paper.

Finally we study the achieved rates of a typical user as the number of users changes. Fig. 4 and 5 display the CDF of the achieved rates when we fix the numbers of antennas at the transmitter and each receiver ($(N, M) = (32, 1)$ and $(N, M) = (32, 2)$, respectively), but change the number of users. Note that here with $M = 1$ all the considered precoders are single-stream ones. We note that for $M = 1$ the performance of the SLC-maximizing solution is very close to the SLNR-maximizing solution. More importantly, we can see that the SLC-maximizing solution has a higher rate gain over ZF as the number of users increases: in the setting of $(N, M) = (32, 1)$, the gain is merely 0.1 b/s/Hz for $K = 15$ and increases to nearly 1 b/s/Hz for $K = 30$. The same trend can be observed for $(N, M) = (32, 2)$ in Fig. 5. To illustrate this improvement further, Fig. 6 displays the average sum rates for a $(N, M) = (32, 1)$ and $(N, M) = (64, 1)$ with K changing from underloading to overloading the system. Comparing with the single-user solution (i.e., MRT) and the ZF solution, we observe that the SLC-maximizing solution achieves a significant system capacity gain, for the range of K simulated, and interestingly that the sum rate appears to be maximized when the loading factor K/N is pretty close to unity, namely in the heavily loaded regime.

VI. CONCLUSIONS

Pursuing the philosophy of leakage-based linear precoding for downlink MU-MIMO transmission, we studied the SLC-optimal precoder design in this paper. The leakage-based precoders can be designed in a completely decoupled and autonomous manner, for each individual user, and have a low computational complexity that is comparable with ZF-based precoders. Our study on the SLNR-based design revealed that the metric of SLNR is insufficient to enforce the requirement on precoder dimensions and thus only leads to essentially rank-one precoders, incapable of effectively exploiting the available spatial degrees of freedom in system. The proposed concept of SLC aims at striking a balance between the signal strengths in individual data streams and the spatial degrees of freedom of the overall received signal. We solved the SLC-based precoder design problem in closed form, which provides a simple, systematic, and controlled approach to exploiting the spatial degrees of freedom with multiple antennas than previous leakage-based precoder designs.

APPENDIX A
PROOF OF THEOREM 1

The proof begins similarly as that in [18, Sec. V], but diverges significantly towards the end. For facilitating the reader and rendering our paper self-contained, we provide a complete proof here.

To simplify the notation let us define:

$$\mathbf{A}_k \doteq \mathbf{H}_\mathbf{k}^*\mathbf{H}_\mathbf{k} \tag{33}$$

$$\mathbf{B}_k \doteq \frac{M}{P_k}\mathbf{I} + \tilde{\mathbf{H}}_\mathbf{k}^*\tilde{\mathbf{H}}_\mathbf{k}. \tag{34}$$

Both $\mathbf{A}_k$ and $\mathbf{B}_k$ are Hermitian and positive semidefinite. Hence, they can be simultaneously diagonalized [21], i.e., there exists a nonsingular matrix $\mathbf{T}_\mathbf{k}$ such that

$$\mathbf{T}_k^*\mathbf{A}_k\mathbf{T}_k = \mathbf{\Lambda}_k \tag{35}$$

$$\mathbf{T}_k^*\mathbf{B}_k\mathbf{T}_k = \mathbf{I}, \tag{36}$$

where the entries of $\mathbf{\Lambda}_k$ are arranged in decreasing order: $\lambda_1 \geq \cdots \geq \lambda_N \geq 0$. We also call $\{\lambda_i\}$ the generalized eigenvalues of $(\mathbf{A}_k, \mathbf{B}_k)$. The matrix $\mathbf{T}_k$ is not orthogonal in general, but it is nonsingular and its range space is the generalized eigenspace of $(\mathbf{A}_k, \mathbf{B}_k)$. We normalize $\mathbf{T}_k$ as

$$\mathbf{T}_k = \bar{\mathbf{H}}_k\mathbf{\Psi} = \bar{\mathbf{H}}_k \cdot \text{diag}\,(\psi_1, \psi_2, \cdots, \psi_N)\,, \tag{37}$$

where $\psi_j = ||\mathbf{T}_k^{(j)}|| > 0$ and $||\bar{\mathbf{H}}_k^{(j)}||^2 = 1$.

It should be emphasized that for a given set of realizations of $\{\mathbf{H}_\mathbf{k}\}_{k=1}^K$, the simultaneous diagonalization of $\mathbf{A}_\mathbf{k}$ and $\mathbf{B}_\mathbf{k}$ of (35) and (36) results in deterministic $\{\lambda_i\}_{i=1}^N$ and $\{\psi_i\}_{i=1}^N$.

Now introduce the change of variable:

$$\mathbf{W}_k = \mathbf{T}_k\mathbf{D}_k, \tag{38}$$

where $\mathbf{D}_k$ is $N \times L$. This is a one-to-one mapping between $\mathbf{W}_\mathbf{k}$ and $\mathbf{D}_\mathbf{k}$ since $\mathbf{T}_k$ is full rank. Using this change of variable, the objective function of the SLNR becomes:

$$\frac{\text{Tr}\left(\mathbf{W}_k^*\mathbf{H}_k^*\mathbf{H}_k\mathbf{W}_k\right)}{\text{Tr}\left[\mathbf{W}_k^*\left(\frac{M}{P_k}\mathbf{I} + \tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k\right)\mathbf{W}_k\right]} = \frac{\text{Tr}\left(\mathbf{D}_k^*\mathbf{\Lambda}_k\mathbf{D}_k\right)}{\text{Tr}\left(\mathbf{D}_k^*\mathbf{D}_k\right)}. \tag{39}$$

Furthermore, SVD of $\mathbf{D}_k$ gives:

$$\mathbf{D}_k = \mathbf{U}_k\begin{bmatrix}\mathbf{\Sigma}_k\\ \mathbf{0}\end{bmatrix}\mathbf{V}_k^*, \tag{40}$$

where $\mathbf{U}_k$ and $\mathbf{V}_k$ are unitary and $\mathbf{\Sigma}_k$ is an $L \times L$ diagonal matrix with non-negative entries $\{\kappa_i\}$. Without loss of generality, we assume $\{\kappa_i\}$ is arranged in decreasing order: $\kappa_1 \geq \cdots \geq \kappa_L \geq 0$.

With this change of variable, the objective function of the SLNR becomes:

$$\text{SLNR}_k = \frac{\text{Tr}\left(\mathbf{D}_k^*\mathbf{\Lambda}_k\mathbf{D}_k\right)}{\text{Tr}\left(\mathbf{D}_k^*\mathbf{D}_k\right)}. \tag{41}$$

Now turn to the objective function (41). First let us explicitly express the columns and entries of $\mathbf{U}_\mathbf{k}$ as:

$$\mathbf{U}_k = [\mathbf{u}_\mathbf{1}, \cdots, \mathbf{u}_\mathbf{N}] = \left[u_{ij}\right]_{i,j=1}^N. \tag{42}$$

The unitarity of $\mathbf{U}_k$ suggests that

$$0 \leq |u_{ij}|^2 \leq 1 \text{ and } \sum_{j=1}^N |u_{ij}|^2 = 1, \quad \sum_{i=1}^N |u_{ij}|^2 = 1. \tag{43}$$

Then, we have

$$\begin{aligned}\frac{\text{Tr}\left(\mathbf{D}_k^*\mathbf{\Lambda}_k\mathbf{D}_k\right)}{\text{Tr}\left[\mathbf{D}_k^*\mathbf{D}_k\right]} &= \frac{\text{Tr}\left(\left[\mathbf{\Sigma}_k \quad \mathbf{0}\right]\mathbf{U}_k^*\mathbf{\Lambda}_k\mathbf{U}_k\begin{bmatrix}\mathbf{\Sigma}_k\\ \mathbf{0}\end{bmatrix}\right)}{\sum_{i=1}^L \kappa_i^2}\\ &= \frac{\text{Tr}\left(\left[\mathbf{\Sigma}_k \quad \mathbf{0}\right]\left(\sum_{i=1}^N \lambda_i\mathbf{u}_\mathbf{i}\mathbf{u}_\mathbf{i}^*\right)\begin{bmatrix}\mathbf{\Sigma}_k\\ \mathbf{0}\end{bmatrix}\right)}{\sum_{i=1}^L \kappa_i^2}\\ &= \frac{\sum_{i=1}^L \kappa_i^2\left(\sum_{j=1}^N \lambda_j|u_{ji}|^2\right)}{\sum_{i=1}^L \kappa_i^2}.\end{aligned} \tag{44}$$

Due to the unitary constraint in (43) and that both $\{\lambda_i\}$ and $\{\kappa_i\}$ are in decreasing order, the objective function in (44) is maximized when one chooses

$$u_{ii} = 1 \text{ and } u_{ij} = 0 \text{ for } j \neq i, \quad i = 1, \cdots, L, \tag{45}$$

which results in

$$\frac{\text{Tr}\left(\mathbf{D}_k^*\mathbf{\Lambda}_k\mathbf{D}_k\right)}{\text{Tr}\left[\mathbf{D}_k^*\mathbf{D}_k\right]} = \frac{\sum_{i=1}^L \kappa_i^2\lambda_i}{\sum_{i=1}^L \kappa_i^2}. \tag{46}$$

Note that $\{\lambda_i\}$ are fixed, but $\{\kappa_i\}$ depends on the choice of $\mathbf{W}_k$ and hence one still needs to find the optimal $\mathbf{W}_k$ that maximizes (46). We thus convert the total power constraint on $\mathbf{W}_k$ to an equivalent constraint on $\{\kappa_i\}$. Let us write $\mathbf{D}_k$ as

$$\mathbf{D}_k = \mathbf{U}_k\begin{bmatrix}\mathbf{\Sigma}_k\\ \mathbf{0}\end{bmatrix}\mathbf{V}_k^* = \begin{bmatrix}\mathbf{I}_L & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_{N-L}\end{bmatrix}\begin{bmatrix}\mathbf{\Sigma}_k\\ \mathbf{0}\end{bmatrix}\mathbf{V}_k^* = \begin{bmatrix}\mathbf{\Sigma}_k\mathbf{V}_k^*\\ \mathbf{0}\end{bmatrix}, \tag{47}$$

where $\mathbf{C}_{N-L}$ is any unitary matrix of size $(N-L) \times (N-L)$.

For ease of notation, we write the matrix $\mathbf{T_k}$ into two block sub-matrices:

$$\mathbf{T}_k = \left[\mathbf{T}_k^{(L)} \quad \mathbf{T}_k^{(N-L)}\right], \tag{48}$$

where $\mathbf{T}_k^{(L)}$ is the first L columns of $\mathbf{T}_k$ and $\mathbf{T}_k^{(N-L)}$ is the last $(N-L)$ columns of $\mathbf{T}_k$. Combining this notation with the normalized version of $\mathbf{T}_k$ in (37), $\mathbf{W}_k$ can be written as

$$\begin{aligned}\mathbf{W}_k &= \mathbf{T}_k\mathbf{D}_k \\ &= \left[\mathbf{T}_k^{(L)} \quad \mathbf{T}_k^{(N-L)}\right]\begin{bmatrix}\boldsymbol{\Sigma}_k\mathbf{V}_k^* \\ \mathbf{0}\end{bmatrix} \\ &= \mathbf{T}_k^{(L)}\boldsymbol{\Sigma}_k\mathbf{V}_k^* \\ &= \bar{\mathbf{H}}_k^{(L)}\boldsymbol{\Psi}^{(L)}\boldsymbol{\Sigma}_k\mathbf{V}_k^*. \end{aligned} \tag{49}$$

Note that (49) is the general form of the precoder $\mathbf{W}_k$. The choice of $\mathbf{V}_k$ does not change the SLNR performance, but it can affect other metrics of the MU-MIMO system. For example, it has been shown in [18] that by choosing $\mathbf{V}_k = \mathbf{I}$, a matched filter receiver at user k will be able to decouple the multiple streams; $\mathbf{V}_k = \mathbf{I}$ is also adopted in this paper.

With (49), the total power constraint of $\mathbf{W}_k$ becomes

$$\begin{aligned}P_k &= \mathrm{Tr}\left(\mathbf{W}_k^*\mathbf{W}_k\right) \\ &= \mathrm{Tr}\left(\mathbf{V}_k\boldsymbol{\Sigma}_k\boldsymbol{\Psi}^{(L)}\bar{\mathbf{H}}_k^{(L)*}\bar{\mathbf{H}}_k^{(L)}\boldsymbol{\Psi}^{(L)}\boldsymbol{\Sigma}_k\mathbf{V}_k^*\right) \\ &= \sum_{i=1}^{L}(\psi_i\kappa_i)^2 . \end{aligned} \tag{50}$$

Define $x_i = \kappa_i^2$, $\mathbf{x} = \mathrm{vec}(x_i)$, $\boldsymbol{\lambda} = \mathrm{vec}(\lambda_i)$, $\boldsymbol{\omega} = \mathrm{vec}(\psi_i^2)$, the SLNR optimization problem of (46) can be written as:

$$\begin{aligned}&\underset{\mathbf{x}}{\text{maximize}} \ \frac{\boldsymbol{\lambda}^T\mathbf{x}}{\mathbf{1}^T\mathbf{x}} \\ &\text{subject to } \boldsymbol{\omega}^T\mathbf{x} = P_k \\ &\qquad\qquad \mathbf{x} \succeq 0. \end{aligned} \tag{51}$$

This is a standard linear-fractional programming problem [22, Sec. 4.3.2], and can be directly transformed to a linear program with efficient numerical solutions available. Indeed, it turns out that this problem has a closed-form solution, as shown below. With the change of variables $\mathbf{y} = \mathbf{x}/(\mathbf{1}^T\mathbf{x})$, $z = 1/(\mathbf{1}^T\mathbf{x})$, the following linear program is equivalent to (51):

$$\begin{aligned}&\underset{\{\mathbf{y},\mathbf{z}\}}{\text{maximize}} \ \boldsymbol{\lambda}^T\mathbf{y} \\ &\text{subject to } \boldsymbol{\omega}^T\mathbf{y} - P_k z = 0 \\ &\qquad\qquad \mathbf{1}^T\mathbf{y} = 1 \\ &\qquad\qquad \mathbf{y} \succeq 0, \quad z \geq 0. \end{aligned} \tag{52}$$

To solve this problem, we first note that z does not affect the objective function, as its only constraint is that $z = \boldsymbol{\omega}^T\mathbf{y}/P_k \geq 0$, which can be satisfied whenever $\mathbf{y} \succeq 0$. So we are looking at the optimization problem of maximizing $\boldsymbol{\lambda}^T\mathbf{y}$ when the non-negative entries of $\mathbf{y}$ sum up to 1. Due to the fact that $\lambda_1 \geq \cdots \geq \lambda_N \geq 0$, apparently the optimal solution is

$$\mathbf{y}^o = \left[1 \quad 0 \quad \cdots \quad 0\right]^T, \tag{53}$$

which means that the optimal solution to problem (51) is

$$\mathbf{x}^o = \left[P_k/\omega_1 \quad 0 \quad \cdots \quad 0\right]^T, \tag{54}$$

suggesting that $\kappa_1^o = \sqrt{P_k/\omega_1} = P_k/\psi_1$, $\kappa_i^o = 0$, $i = 2, \cdots, L$.

The solution (54) indicates that the optimal solution for the SLNR maximization problem has the following form, based on (49):

$$\mathbf{W}^o_{\mathsf{SLNR,P1},k} = \sqrt{P_k}\left[\frac{\mathbf{T}_k^{(1)}}{||\mathbf{T}_k^{(1)}||} \quad \mathbf{0} \quad \cdots, \quad \mathbf{0}\right], \tag{55}$$

which means that the optimal precoder design is to use only one single stream that is in the direction of the eigenmode corresponding to the largest generalized eigenvalue of $\left(\mathbf{H}_k^*\mathbf{H}_k, \frac{M}{P_k}\mathbf{I} + \tilde{\mathbf{H}}_k^*\tilde{\mathbf{H}}_k\right)$.

Applying (54) back to the SLNR expression (46), the maximum achievable SLNR is

$$\mathsf{SLNR}_k^o = \lambda_1. \tag{56}$$

This completes the proof.

APPENDIX B
PROOF OF COROLLARY 2

Comparing to Problem 1, Problem 2 adds a minimum rank constraint. The proof of Corollary 2 follows the same procedure as that in Appendix A until step (51), as no rank constraint has been used until then. Now, the optimization problem becomes

$$\begin{aligned}&\underset{\mathbf{x}}{\text{maximize}} \ \frac{\boldsymbol{\lambda}^T\mathbf{x}}{\mathbf{1}^T\mathbf{x}} \\ &\text{subject to } \boldsymbol{\omega}^T\mathbf{x} = P_k \\ &\qquad\qquad \mathbf{x} \succeq 0 \\ &\qquad\qquad \mathrm{rank}\left(\mathrm{diag}\left(\mathbf{x}\right)\right) \geq L, \end{aligned} \tag{57}$$

where the last constraint enforces a minimum rank L for $\mathbf{W}_k$. Similar to Appendix A, with the change of variables $\mathbf{y} = \mathbf{x}/(\mathbf{1}^T\mathbf{x})$ and $z = 1/(\mathbf{1}^T\mathbf{x})$, we have an equivalent optimization problem:

$$\begin{aligned}&\underset{\{\mathbf{y},\mathbf{z}\}}{\text{maximize}} \ \boldsymbol{\lambda}^T\mathbf{y} \\ &\text{subject to } \boldsymbol{\omega}^T\mathbf{y} - P_k z = 0 \\ &\qquad\qquad \mathbf{1}^T\mathbf{y} = 1 \\ &\qquad\qquad \mathbf{y} \succeq 0, \quad z \geq 0 \\ &\qquad\qquad \mathrm{rank}\left(\mathrm{diag}\left(\mathbf{y}\right)\right) \geq L. \end{aligned} \tag{58}$$

Again, we note that z does not affect the value of the objective function; its only constraint is that $z = \boldsymbol{\omega}^T\mathbf{y}/P_k \geq 0$, which can be satisfied whenever $\mathbf{y} \succeq 0$. So we examine the optimization problem of maximizing $\boldsymbol{\lambda}^T\mathbf{y}$ when the non-negative entries of $\mathbf{y}$ sum up to 1, and $\mathbf{y}$ has at least L non-zeros entries.

We have already proved in Appendix A that, if the rank constraint is removed from (58), the optimal solution is given by (53). Now, for any $\epsilon > 0$, we want to prove that we can find $\delta_i > 0, \forall i = 1, \cdots, L$, such that $|\mathsf{SLNR}_k - \lambda_{\max}| \leq \epsilon$. This is accomplished by choosing

$$\mathbf{y} = \begin{bmatrix} 1-\delta_1 & \delta_2 & \cdots & \delta_L & 0 & \cdots & 0 \end{bmatrix}^T, \quad (59)$$

where $\delta_1 = \sum_{i=2}^{L} \delta_i$, and this translates into

$$\mathbf{x} = \frac{P_k}{\omega_1(1-\delta_1) + \sum_{i=2}^{L} \omega_i \delta_i} \times \begin{bmatrix} (1-\delta_1) & \delta_2 & \cdots & \delta_L & 0 & \cdots & 0 \end{bmatrix}^T, \quad (60)$$

where $\omega_i = \psi_i^2 = ||\mathbf{T}_k^{(i)}||^2$.

Finally, we have

$$\kappa_1 = \sqrt{\frac{(1-\delta_1)P_k}{\omega_1(1-\delta_1) + \sum_{i=2}^{L} \omega_i \delta_i}} \quad (61)$$

$$\kappa_i = \sqrt{\frac{\delta_i P_k}{\omega_1(1-\delta_1) + \sum_{i=2}^{L} \omega_i \delta_i}}, \quad i = 2, \cdots, L \quad (62)$$

$$\kappa_i = 0, \quad i = L+1, \cdots, \min(M, N). \quad (63)$$

and the corresponding precoder is

$$\mathbf{W}_k = \begin{bmatrix} \kappa_1 \mathbf{T}_k^{(1)} & \kappa_2 \mathbf{T}_k^{(1)} & \kappa_L \mathbf{T}_k^{(L)} & \mathbf{0} \cdots & \mathbf{0} \end{bmatrix}. \quad (64)$$

Note that this is the same as (11).

The achieved SLNR is

$$\mathsf{SLNR}_k = \boldsymbol{\lambda}^T \mathbf{y} = (1-\delta_1)\lambda_1 + \sum_{i=2}^{L} \delta_i \lambda_i, \quad (65)$$

and hence if we choose

$$\delta_1 = \frac{(L-1)\epsilon}{L\lambda_1}; \quad \delta_i = \frac{\epsilon}{L\lambda_1}, \; i = 2, \cdots, L, \quad (66)$$

we have

$$|\mathsf{SLNR}_k - \lambda_{\max}| = |\delta_1 \lambda_1 - \sum_{i=2}^{L} \delta_i \lambda_i| \leq \frac{L-1}{L}\epsilon \leq \epsilon. \quad (67)$$

This completes the proof.

APPENDIX C
PROOF OF THEOREM 3

We prove Theorem 3 by first deriving an upper bound on $C_{\mathsf{SL},k}$, and then showing that the precoder (29) exactly achieves this upper bound. In the following, the proof is carried out in two phases. First, we assume that both $\mathbf{S}_1$ and $\mathbf{S}_2$ are strictly positive definite, i.e., $\mathbf{S}_1 \succ 0$ and $\mathbf{S}_2 \succ 0$, and prove Theorem 3 in Appendix C-A. Then, in Appendix C-B we show via a perturbation argument how to extend the proof to the general case, thus completing the proof.

A. $\mathbf{S}_1 \succ 0$ AND $\mathbf{S}_2 \succ 0$

As a note, $\mathbf{S}_1 \succ 0$ means rank $(\mathbf{H}_k) = N$, which implies that $N \leq M$; similarly from $\mathbf{S}_2 \succ 0$ we have rank $\left(\tilde{\mathbf{H}}_k\right) = N$ and $N \leq (K-1)M$.

We define a new matrix $\mathbf{S}_3$ that satisfies

$$\mathbf{Q}^* \left(\mathbf{I}_N + \mathbf{G}^* \mathbf{S}_3 \mathbf{G}\right) \mathbf{Q} = \tilde{\mathbf{\Omega}}, \quad (68)$$

where $\tilde{\mathbf{\Omega}} = \operatorname{diag}(\omega_1, \cdots, \omega_n, 1, \cdots, 1)$. It can be verified that $0 \prec \mathbf{S}_1 \preceq \mathbf{S}_3$ and $0 \prec \mathbf{S}_2 \preceq \mathbf{S}_3$. Hence we have:

$$\begin{aligned} C_{\mathsf{SL},k} &= \max_{0 \preceq \mathbf{W}_k \mathbf{W}_k^* \preceq \mathbf{P}_k} \log \frac{\left|\mathbf{I}_M + \mathbf{H}_k \mathbf{W}_k \mathbf{W}_k^* \mathbf{H}_k^*\right|}{\left|\mathbf{I}_{(K-1)M} + \tilde{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^* \tilde{\mathbf{H}}_k^*\right|} \\ &= \max_{0 \preceq \mathbf{W}_k \mathbf{W}_k^* \preceq \mathbf{P}_k} \log \frac{\left|\mathbf{I}_N + \mathbf{W}_k^* \mathbf{S}_1 \mathbf{W}_k\right|}{\left|\mathbf{I}_N + \mathbf{W}_k^* \mathbf{S}_2 \mathbf{W}_k\right|} \\ &\leq \max_{0 \preceq \mathbf{W}_k \mathbf{W}_k^* \preceq \mathbf{P}_k} \log \frac{\left|\mathbf{I}_N + \mathbf{W}_k^* \mathbf{S}_3 \mathbf{W}_k\right|}{\left|\mathbf{I}_N + \mathbf{W}_k^* \mathbf{S}_2 \mathbf{W}_k\right|} \\ &\leq \log \frac{|\mathbf{I}_N + \mathbf{G}^* \mathbf{S}_3 \mathbf{G}|}{|\mathbf{I}_N + \mathbf{G}^* \mathbf{S}_2 \mathbf{G}|} \\ &= \log |\tilde{\mathbf{\Omega}}|. \end{aligned} \quad (69)$$

With the upper bound (69) on $C_{\mathsf{SL},k}$, now let us prove that (29) can achieve this upper bound. From (29), we have

$$\mathbf{W}_{\mathsf{SLC},k}^{o} \mathbf{W}_{\mathsf{SLC},k}^{o*} = \mathbf{G}\mathbf{Q}_1 \left(\mathbf{Q}_1^* \mathbf{Q}_1\right)^{-1} \mathbf{Q}_1^* \mathbf{G}^*. \quad (70)$$

Thus we have

$$\log \frac{\left|\mathbf{I}_M + \mathbf{H}_k \mathbf{W}_{\mathsf{SLC},k}^{o} \mathbf{W}_{\mathsf{SLC},k}^{o*} \mathbf{H}_k^*\right|}{\left|\mathbf{I}_{(K-1)M} + \tilde{\mathbf{H}}_k \mathbf{W}_{\mathsf{SLC},k}^{o} \mathbf{W}_{\mathsf{SLC},k}^{o*} \tilde{\mathbf{H}}_k^*\right|} = \log \frac{\left|\mathbf{I}_N + \mathbf{W}_{\mathsf{SLC},k}^{o} \mathbf{W}_{\mathsf{SLC},k}^{o*} \mathbf{S}_1\right|}{\left|\mathbf{I}_N + \mathbf{W}_{\mathsf{SLC},k}^{o} \mathbf{W}_{\mathsf{SLC},k}^{o*} \mathbf{S}_2\right|}. \quad (71)$$

Let us first study the numerator inside the logarithm of (71):

$$\begin{aligned} &\left|\mathbf{I}_N + \mathbf{W}_{\mathsf{SLC},k}^{o} \mathbf{W}_{\mathsf{SLC},k}^{o*} \mathbf{S}_1\right| \\ &= \left|\mathbf{I}_N + \mathbf{G}\mathbf{Q}_1 \left(\mathbf{Q}_1^* \mathbf{Q}_1\right)^{-1} \mathbf{Q}_1^* \mathbf{G}^* \mathbf{S}_1\right| \\ &= \left|\mathbf{I}_n + \left(\mathbf{Q}_1^* \mathbf{Q}_1\right)^{-1} \mathbf{Q}_1^* \mathbf{G}^* \mathbf{S}_1 \mathbf{G} \mathbf{Q}_1\right| \\ &= \left|\mathbf{Q}_1^* \mathbf{Q}_1\right|^{-1} |\tilde{\mathbf{\Omega}}|. \end{aligned} \quad (72)$$

Performing a similar derivation on the denominator inside the logarithm of (71) yields

$$\left|\mathbf{I}_N + \mathbf{W}_{\mathsf{SLC},k}^{o} \mathbf{W}_{\mathsf{SLC},k}^{o*} \mathbf{S}_2\right| = \left|\mathbf{Q}_1^* \mathbf{Q}_1\right|^{-1} |\mathbf{I}_N|. \quad (73)$$

Substituting (72) and (73) into (71) results in

$$\log \frac{\left|\mathbf{I}_M + \mathbf{H}_k \mathbf{W}_{\mathsf{SLC},k}^{o} \mathbf{W}_{\mathsf{SLC},k}^{o*} \mathbf{H}_k^*\right|}{\left|\mathbf{I}_{(K-1)M} + \tilde{\mathbf{H}}_k \mathbf{W}_{\mathsf{SLC},k}^{o} \mathbf{W}_{\mathsf{SLC},k}^{o*} \tilde{\mathbf{H}}_k^*\right|} = \log |\tilde{\mathbf{\Omega}}|. \quad (74)$$

Hence, we have proved that (29) can achieve the upper bound (69) on $C_{\mathsf{SL},k}$.

B. GENERAL CASE: $\mathbf{S}_1 \succeq 0$ AND $\mathbf{S}_2 \succeq 0$

Let us focus on the case

$$\mathbf{S}_1 \succeq 0 \text{ but } |\mathbf{S}_1| = 0. \tag{75}$$

The same argument can be applied to $\mathbf{S}_2$. Note that there are two possible scenarios for (75):

1) CASE 1: $N > M$

In this case, let us define an equivalent channel matrix

$$\hat{\mathbf{H}}_k = \begin{bmatrix} \mathbf{H}_k \\ \mathbf{0} \end{bmatrix}. \tag{76}$$

We further have

$$\hat{\mathbf{H}}_k^* \hat{\mathbf{H}}_k = \mathbf{H}_k^* \mathbf{H}_k. \tag{77}$$

So the effective $\mathbf{S}_1$ does not change when we use $\hat{\mathbf{H}}_k$ to replace $\mathbf{H}_k$. Note that $\hat{\mathbf{H}}_k$ is a square matrix of size $N \times N$. Applying SVD on $\hat{\mathbf{H}}_k$, we have

$$\hat{\mathbf{H}}_k = \mathbf{U}_k \Xi_k \mathbf{V}_k^*, \tag{78}$$

where $\mathbf{U}_k$ and $\mathbf{V}_k$ are $N \times N$ unitary matrices, and Ξ_k is a diagonal matrix. Let us then define a new MIMO channel matrix

$$\check{\mathbf{H}}_k = \mathbf{U}_k \left(\Xi_k + \alpha \mathbf{I}_N\right) \mathbf{V}_k^*, \tag{79}$$

where $\alpha > 0$. It is clear that $\check{\mathbf{H}}_k$ is non-singular. Define a corresponding matrix $\check{\mathbf{S}}_1$ as

$$\check{\mathbf{S}}_1 = \check{\mathbf{H}}_k \check{\mathbf{H}}_k^*. \tag{80}$$

2) CASE 2: $N \leq M$ and $\text{rank}\left(\mathbf{H}_k\right) < N$

In this case, let us define an equivalent channel matrix

$$\hat{\mathbf{H}}_k = \begin{bmatrix} \mathbf{H}_k & \mathbf{0} \end{bmatrix}. \tag{81}$$

We further have

$$\hat{\mathbf{H}}_k^* \hat{\mathbf{H}}_k = \begin{bmatrix} \mathbf{H}_k^* \mathbf{H}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{82}$$

So the effective $\mathbf{S}_1$ does not change when we use $\hat{\mathbf{H}}_k$ to replace $\mathbf{H}_k$ (only the dimension expands with zero sub-matrices). Note that $\hat{\mathbf{H}}_k$ is a square matrix of size $M \times M$. Applying SVD on $\hat{\mathbf{H}}_k$, we have

$$\hat{\mathbf{H}}_k = \mathbf{U}_k \Xi_k \mathbf{V}_k^*, \tag{83}$$

where $\mathbf{U}_k$ and $\mathbf{V}_k$ are $M \times M$ unitary matrices, and Ξ_k is a diagonal matrix. Let us then define a new MIMO channel matrix

$$\check{\mathbf{H}}_k = \mathbf{U}_k \left(\Xi_k + \alpha \mathbf{I}_M\right) \mathbf{V}_k^*, \tag{84}$$

where $\alpha > 0$. It is clear that $\check{\mathbf{H}}_k$ is non-singular. Define a corresponding matrix $\check{\mathbf{S}}_1$ as

$$\check{\mathbf{S}}_1 = \check{\mathbf{H}}_k \check{\mathbf{H}}_k^*. \tag{85}$$

3) PROOF

In both cases above, we have found a perturbed channel matrix that results in a new matrix $\check{\mathbf{S}}_1$. We can apply the same procedure to $\tilde{\mathbf{H}}_k$, with the same perturbation parameter α, and obtain the corresponding matrix $\check{\mathbf{S}}_2$. Note that both $\check{\mathbf{S}}_1$ and $\check{\mathbf{S}}_2$ are non-singular, and hence the same technique in Appendix C-A can be directly applied. We denote the corresponding SLC as $C_{\mathsf{SL,k}}\left(\check{\mathbf{S}}_1, \check{\mathbf{S}}_2\right)$ and the corresponding precoder as $\mathbf{W}^o_{\mathsf{SLC},k}\left(\check{\mathbf{S}}_1, \check{\mathbf{S}}_2\right)$, which are clearly continuous with α. Finally, letting $\alpha \to 0$, we have

$$\check{\mathbf{S}}_1 \to \mathbf{S}_1 \tag{86}$$
$$\check{\mathbf{S}}_2 \to \mathbf{S}_2 \tag{87}$$

and therefore

$$C_{\mathsf{SL},k}\left(\check{\mathbf{S}}_1, \check{\mathbf{S}}_2\right) \to C_{\mathsf{SL},k}\left(\mathbf{S}_1, \mathbf{S}_2\right) \tag{88}$$
$$\mathbf{W}^o_{\mathsf{SLC},k}\left(\check{\mathbf{S}}_1, \check{\mathbf{S}}_2\right) \to \mathbf{W}^o_{\mathsf{SLC},k}\left(\mathbf{S}_1, \mathbf{S}_2\right). \tag{89}$$

This completes the proof.

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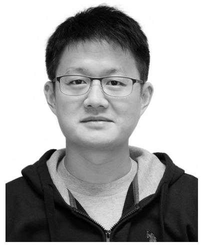

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