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An Iterative Method for Moving Target Localization Using TDOA and FDOA Measurements

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ABSTRACT For moving targets localization, incorporating frequency-difference-of-arrival (FDOA) measurements in the commonly used time-difference-of-arrival (TDOA) positioning systems will improve performance. Such an approach still has unresolved technical challenges. The commonly used maximum likelihood estimator (MLE) is nonconvex and highly nonlinear, and the parameters to be estimated are mutually coupled in the positioning process. The goal of this paper is to develop an effective iterative method that resolves these challenges for moving target localization using TDOA and FDOA. Specifically, a semidefinite programming (SDP) method is proposed to transform the MLE problem into a convex optimization problem. To improve the performance further, we develop an iterative method that uses the position and velocity estimates obtained using the SDP method as the initial values. This iterative method includes two steps: update of the velocity by using a weighted least squares method and update of the position by using SDP. The major advantage of the proposed scheme is that it significantly outperforms existing methods at moderate to high noise levels, which is validated via extensive numerical results.

INDEX TERMS Moving target localization, time-difference-of-arrival (TDOA), frequency-difference-of-arrival (FDOA), semidefinite programming (SDP).

I. INTRODUCTION

Location information has been widely applied in a number of applications such as radar, sonar, wireless sensor networks (WSNs), and wireless communications [1]–[6]. For stationary sources, a commonly used localization technique is to measure the time-difference-of-arrival (TDOA) of the source's signal to spatially separated receivers [7]–[9]. When the receivers are moving and the source is stationary, frequency-difference-of-arrival (FDOA) measurements can be used to improve localization performance and reduce the minimum number of receivers needed [10]. When the receivers and source are all moving, both TDOA and FDOA measurements can be used to determine the position and velocity [11]–[15]. The challenges of localizing a moving source using TDOA and FDOA measurements lie in the high nonlinearity and nonconvexity of the maximum likelihood estimation (MLE) problem as well as the mutual

coupling among the to-be-estimated parameters [11], [16]–[18]. Exhaustive search in the solution space is straightforward, but is computationally very expensive and inefficient, making real-time processing difficult [19]. The Taylor-series method [20] needs an initial estimate. Besides, it cannot guarantee convergence to the global optimum solution. In [19], Ho and Xu proposed an algebraic solution, which is a two-step weighted least-squares (2SWLS) method. This method first transforms the TDOA and FDOA equations into a set of linear equations by introducing two nuisance parameters. Then, it applies the linear weighted least-squares (WLS) to determine the source position, velocity, and the two nuisance parameters introduced. Finally, the nuisance parameters are eliminated through another linear WLS minimization to improve the accuracy of the estimates. It is shown that the accuracies of the source position and velocity estimates with the 2SWLS method could approach

the Cramér-Rao lower bound (CRLB) for Gaussian TDOA and FDOA noises at a moderate noise level. In [21], Wei *et al.* proposed a multidimensional scaling (MDS) method, which was shown to perform better than the 2SWLS method. In [17], Wang *et al.* proposed a semidefinite relaxation (SDR) method to approximately solve the MLE problem. This scheme somewhat resembles the 2SWLS method: first, it also transforms the TDOA and FDOA equations into a set of linear equations by introducing two nuisance parameters; then, it uses an SDR method to convert the nonconvex constraints existing in nuisance parameters into convex constraints. The methods discussed in [17], [19], and [21] apply linear approximations to the nonlinear localization problem; as a result, their performances degrade rapidly as the measurement noise level increases.

Semidefinite programming (SDP) has also been applied in localization systems recently [8], [10], [22]. The method by Yang *et al.* [10] first relaxes the MLE problem to obtain a convex SDP problem. An assumption made is that the source is stationary. When the source is moving, to the best of the authors' knowledge, the method by Wang *et al.* [17] is the only SDP solution for both position and velocity estimation. As the noise level increases, the performance of this approach degrades rapidly. The goal of this paper is to develop an effective SDP-based iterative method for localization using TDOA and FDOA measurements assuming both the sensors and the target are moving. The main difference between [17] and the algorithm proposed here is that the former approach relaxes the WLS problem, which is an approximate to the MLE problem, while the proposed scheme in the current paper directly relaxes the MLE problem. The proposed scheme also resolves the non-convexity issue of the MLE. Additionally, as the noise level increases, it outperforms existing methods, and the gain is substantial when the noise level is high.

The TDOA and FDOA measurement model will be described in Sec. II, together with the formulation of MLE problem. Sec. III presents the main contribution of this paper: an SDP technique to convert the nonlinear and nonconvex MLE problem into a convex problem and an iterative method to estimate both velocity and position. Simulation results of location and velocity estimates with the proposed scheme are presented in Sec. IV and compared with the existing methods, followed by concluding remarks in Sec. V.

Notation: The following notations are used throughout the paper. Bold uppercase and bold lowercase letters denote matrices and vectors, respectively. \mathbf{I}_m is the $m \times m$ identity matrix and $\mathbf{1}_m$ is an $m \times 1$ vector whose elements are all 1's. $\mathbf{0}_{m,n}$ is an $m \times n$ zero matrix. $\mathbb{E}(\cdot)$ denotes expectation and $\|\cdot\|$ is the l_2 norm. $\mathbf{A}(:, i)$ denotes the i th column of \mathbf{A} , and $\mathbf{A}(i, j)$ denotes the (i, j) th element of \mathbf{A} . $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is positive semidefinite.

II. MEASUREMENT MODEL AND PROBLEM FORMULATION

Consider a network with M moving sensors and one moving source in a three-dimensional (3-D) space. The position and

velocity of the m th moving sensor are known and denoted by \mathbf{s}_i and $\dot{\mathbf{s}}_i$, respectively. The position and velocity of the source are unknown and denoted by \mathbf{u} and $\dot{\mathbf{u}}$, respectively. The range-difference measurements and their rates are, respectively, given by [17]

$$\begin{aligned} r_{i1} &= \|\mathbf{u} - \mathbf{s}_i\| - \|\mathbf{u} - \mathbf{s}_1\| + n_{i1}, \\ &= d_i - d_1 + n_{i1}, \quad i = 2, \dots, M, \end{aligned} \quad (1)$$

and

$$\begin{aligned} \dot{r}_{i1} &= \frac{(\dot{\mathbf{u}} - \dot{\mathbf{s}}_i)^T(\mathbf{u} - \mathbf{s}_i)}{\|\mathbf{u} - \mathbf{s}_i\|} - \frac{(\dot{\mathbf{u}} - \dot{\mathbf{s}}_1)^T(\mathbf{u} - \mathbf{s}_1)}{\|\mathbf{u} - \mathbf{s}_1\|} + \dot{n}_{i1}, \\ &= \dot{d}_i - \dot{d}_1 + \dot{n}_{i1}, \quad i = 2, \dots, M \end{aligned} \quad (2)$$

where

$$d_i = \|\mathbf{u} - \mathbf{s}_i\|, \quad \dot{d}_i = \frac{(\dot{\mathbf{u}} - \dot{\mathbf{s}}_i)^T(\mathbf{u} - \mathbf{s}_i)}{\|\mathbf{u} - \mathbf{s}_i\|}, \quad i = 1, \dots, M,$$

and n_{i1} and \dot{n}_{i1} are the range-difference measurement noise and range-difference-rate measurement noise, respectively. The TDOA and FDOA measurements are expressed as [17]

$$t_{i1} = r_{i1}/c, \quad f_{i1} = f_0 \dot{r}_{i1}/c, \quad i = 2, \dots, M \quad (3)$$

where c is the signal propagation speed and f_0 is the carrier frequency.

We derive the proposed method by using the range-difference measurements and their rates in (1) and (2). Assume that n_{i1} and \dot{n}_{i1} are independent zero-mean Gaussian random variables, and let $\mathbf{n} = [n_{21}, \dots, n_{M1}]^T$, $\dot{\mathbf{n}} = [\dot{n}_{21}, \dots, \dot{n}_{M1}]^T$, $\mathbf{Q} = \mathbb{E}(\mathbf{nn}^T)$, and $\dot{\mathbf{Q}} = \mathbb{E}(\dot{\mathbf{n}}\dot{\mathbf{n}}^T)$. Further define the following notations:

$$\begin{aligned} \mathbf{r} &= [r_{21}, \dots, r_{M1}]^T, \quad \dot{\mathbf{r}} = [\dot{r}_{21}, \dots, \dot{r}_{M1}]^T, \\ \mathbf{d} &= [d_1, \dots, d_M]^T, \quad \dot{\mathbf{d}} = [\dot{d}_1, \dots, \dot{d}_M]^T. \end{aligned} \quad (4)$$

The ML estimation of \mathbf{u} and $\dot{\mathbf{u}}$ is expressed as

$$\min_{\mathbf{u}, \dot{\mathbf{u}}, d_i, \dot{d}_i} (\mathbf{r} - \mathbf{A}\mathbf{d})^T \mathbf{Q}^{-1}(\mathbf{r} - \mathbf{A}\mathbf{d}) + (\dot{\mathbf{r}} - \mathbf{A}\dot{\mathbf{d}})^T \dot{\mathbf{Q}}^{-1}(\dot{\mathbf{r}} - \mathbf{A}\dot{\mathbf{d}}) \quad (5a)$$

$$\text{s.t. } d_i = \|\mathbf{u} - \mathbf{s}_i\|, \quad (5b)$$

$$\dot{d}_i = \frac{(\dot{\mathbf{u}} - \dot{\mathbf{s}}_i)^T(\mathbf{u} - \mathbf{s}_i)}{\|\mathbf{u} - \mathbf{s}_i\|}, \quad i = 1, \dots, M \quad (5c)$$

where $\mathbf{A} = [-\mathbf{1}_{M-1} \quad \mathbf{I}_{M-1}]$.

III. LOCALIZATION ALGORITHM

A. SDP FOR INITIAL POSITION AND VELOCITY ESTIMATES

The nonconvex MLE problem is very difficult to solve directly. An SDP solution that transforms the nonconvex MLE problem into a convex problem is developed here to solve problem (5), which will generate the initial position and velocity estimates. Define $\mathbf{h} = [\mathbf{d}^T \quad \dot{\mathbf{d}}^T]^T$, $\mathbf{A}_1 = \mathbf{A}[\mathbf{I}_M \quad \mathbf{0}_{M,M}]$ and $\mathbf{A}_2 = \mathbf{A}[\mathbf{0}_{M,M} \quad \mathbf{I}_M]$. The problem given in (5) can be written as

$$\min_{\mathbf{u}, \dot{\mathbf{u}}, \mathbf{h}} (\mathbf{r} - \mathbf{A}_1 \mathbf{h})^T \mathbf{Q}^{-1}(\mathbf{r} - \mathbf{A}_1 \mathbf{h}) + (\dot{\mathbf{r}} - \mathbf{A}_2 \mathbf{h})^T \dot{\mathbf{Q}}^{-1}(\dot{\mathbf{r}} - \mathbf{A}_2 \mathbf{h}) \quad (6a)$$

$$\text{s.t. } h_i = \|\mathbf{u} - \mathbf{s}_i\|, \quad (6b)$$

$$h_{M+i} = \frac{(\dot{\mathbf{u}} - \dot{\mathbf{s}}_i)^T(\mathbf{u} - \mathbf{s}_i)}{\|\mathbf{u} - \mathbf{s}_i\|}, \quad i = 1, \dots, M. \quad (6c)$$

Rewrite the objective function in (6) for minimization as

$$\text{tr}[(\mathbf{A}_1^T \mathbf{Q}^{-1} \mathbf{A}_1 + \mathbf{A}_2^T \dot{\mathbf{Q}}^{-1} \mathbf{A}_2) \mathbf{H}] - 2\mathbf{h}^T (\mathbf{A}_1^T \mathbf{Q}^{-1} \mathbf{r} + \mathbf{A}_2^T \dot{\mathbf{Q}}^{-1} \dot{\mathbf{r}}) \quad (7)$$

where $\mathbf{H} = \mathbf{h}\mathbf{h}^T$ and $\text{tr}(\cdot)$ represents the trace of a matrix. The two constant terms in (7) can be discarded without affecting the results. Note that the above objective function is a linear function of \mathbf{H} and \mathbf{h} , but the constraints in (6) are nonconvex. Next the nonconvex constraints are relaxed into convex constraints that remain tightly connected with the original constraints. Let $\mathbf{X} = [\mathbf{u} \ \dot{\mathbf{u}}]$ and $\mathbf{Y} = \mathbf{X}^T \mathbf{X}$. The constraint $h_i = \|\mathbf{u} - \mathbf{s}_i\|, i = 1, \dots, M$, can be written as

$$H_{i,i} = h_i^2 = Y(1, 1) - 2\mathbf{X}(:, 1)^T \mathbf{s}_i + \mathbf{s}_i^T \mathbf{s}_i, \quad i = 1, \dots, M. \quad (8)$$

Similar to the approach used in [23], applying the Cauchy-Schwartz inequality yields

$$H_{i,j} \geq |Y(1, 1) - \mathbf{X}(:, 1)^T (\mathbf{s}_i + \mathbf{s}_j) + \mathbf{s}_i^T \mathbf{s}_j|, \quad 1 \leq i < j \leq M. \quad (9)$$

The constraint in (6c) can be written as

$$h_i h_{M+i} = (\dot{\mathbf{u}} - \dot{\mathbf{s}}_i)^T (\mathbf{u} - \mathbf{s}_i), \quad i = 1, \dots, M. \quad (10)$$

Also, the above nonconvex constraint can be expressed as

$$H_{i,M+i} = Y(1, 2) - \mathbf{X}(:, 2)^T \mathbf{s}_i - \mathbf{X}(:, 1)^T \dot{\mathbf{s}}_i + \mathbf{s}_i^T \dot{\mathbf{s}}_i. \quad (11)$$

Further applying the Cauchy-Schwartz inequality to (6c) yields

$$|h_{M+i}| \leq \|\dot{\mathbf{u}} - \dot{\mathbf{s}}_i\|. \quad (12)$$

Squaring both sides of (12) results in the following relationship:

$$H_{M+i,M+i} \leq Y(2, 2) - 2\mathbf{X}(:, 2)^T \dot{\mathbf{s}}_i + \dot{\mathbf{s}}_i^T \dot{\mathbf{s}}_i. \quad (13)$$

At this point, two nonconvex constraints remain: $\mathbf{H} = \mathbf{h}\mathbf{h}^T$ and $\mathbf{Y} = \mathbf{X}^T \mathbf{X}$. By using the SDR method [24], these two constraints can be relaxed into convex inequalities $\mathbf{H} \succeq \mathbf{h}\mathbf{h}^T$ and $\mathbf{Y} \succeq \mathbf{X}^T \mathbf{X}$, which can be expressed as linear matrix inequalities (LMI):

$$\begin{bmatrix} 1 & \mathbf{h}^T \\ \mathbf{h} & \mathbf{H} \end{bmatrix} \succeq \mathbf{0}, \quad \begin{bmatrix} \mathbf{I}_3 & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0}. \quad (14)$$

It is easy to prove that the rank of $(\mathbf{A}_1^T \mathbf{Q}^{-1} \mathbf{A}_1 + \mathbf{A}_2^T \dot{\mathbf{Q}}^{-1} \mathbf{A}_2)$ is equal to $2M - 2$. Here, similar to the technique used in [25], two penalty terms, $\eta_1 \text{tr}(\mathbf{H}(1 : M, 1 : M))$ and $\eta_2 \text{tr}(\mathbf{H}(M + 1 : 2M, M + 1 : 2M))$, where $\eta_1 > 0, \eta_2 > 0$, are introduced into the objective function. The second-order cone (SOC) constraints are expressed as

$$\|\mathbf{X}(:, 1) - \mathbf{s}_i\| \leq h_i, \quad i = 1, \dots, M. \quad (15)$$

Algorithm 1 Proposed algorithm for moving target localization using TDOA and FDOA.

Require:

- TDOA/FDOA measurements: r_{i1}, r'_{i1} ;
- Sensor positions and velocities: $\mathbf{s}_i, \dot{\mathbf{s}}_i$;
- Covariance matrix of TDOA/FDOA measurement noises: $\mathbf{Q}, \dot{\mathbf{Q}}$;
- Number of iterations: L ;

Ensure:

- Target position and velocity estimates: $\hat{\mathbf{u}}_n$ and $\hat{\dot{\mathbf{u}}}_n$;
- 1: Solving the SDP problem (16) without the two penalty terms and the SOC constraints, then using the estimates (\mathbf{u} and $\dot{\mathbf{u}}$) to calculate (17) and (18);
- 2: Solving the SDP problem (16) with the above calculated η_1 and η_2 , obtaining the initial estimates: $\hat{\mathbf{u}}_0$ and $\hat{\dot{\mathbf{u}}}_0$;
- 3: Using $\hat{\mathbf{u}}_0$ and $\hat{\dot{\mathbf{u}}}_0$ to update η_1 and η_2 ;
- 4: For $n \leq L$;
- 5: Solving the WLS problem (19), obtaining the new velocity estimate: $\hat{\dot{\mathbf{u}}}_n$;
- 6: Solving the SDP problem (23), obtaining the new position estimate: $\hat{\mathbf{u}}_n$;
- 7: Using $\hat{\mathbf{u}}_n$ and $\hat{\dot{\mathbf{u}}}_n$ to update η_1 and η_2 ;
- 8: End;

TABLE 1. Positions and velocities of the sensors.

| Sensor no. | x | y | z | \dot{x} | \dot{y} | \dot{z} |
|------------|------|------|------|-----------|-----------|-----------|
| 1 | 300 | 100 | 150 | 30 | -20 | 20 |
| 2 | 400 | 150 | 100 | -30 | 10 | 20 |
| 3 | 300 | 500 | 200 | 10 | -20 | 10 |
| 4 | 350 | 200 | 100 | 10 | 20 | 30 |
| 5 | -100 | -100 | -100 | -20 | 10 | 10 |

TABLE 2. The average running time [s] of the algorithms compared. CPU: Intel Core 3 2.4 GHz.

| Algorithms | Running Time |
|------------|--------------|
| proposed | 5.0021 |
| SDP | 0.6549 |
| MDS | 0.0134 |
| 2SWLS | 0.0064 |

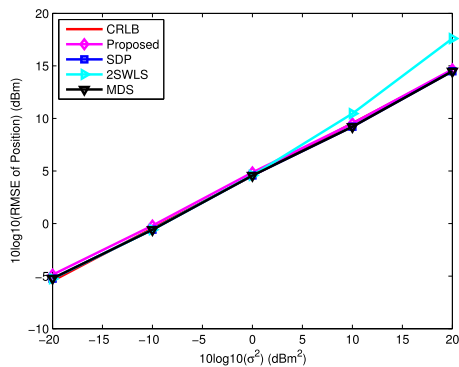
The two penalty terms and the SOC constraints ensure that all constraints are tight for an improved localization accuracy.

The above discussions lead to the proposed SDP algorithm as follows.

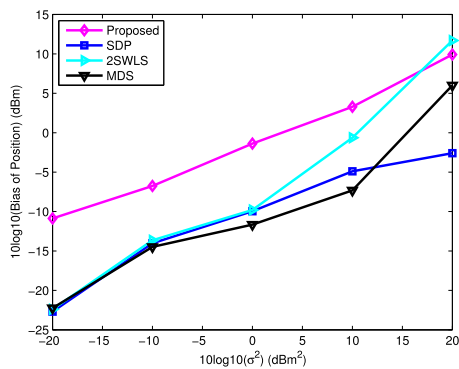
$$\begin{aligned} \min_{\mathbf{h}, \mathbf{H}, \mathbf{X}, \mathbf{Y}} \quad & \text{tr}[(\mathbf{A}_1^T \mathbf{Q}^{-1} \mathbf{A}_1 + \mathbf{A}_2^T \dot{\mathbf{Q}}^{-1} \mathbf{A}_2) \mathbf{H}] - \\ & 2\mathbf{h}^T (\mathbf{A}_1^T \mathbf{Q}^{-1} \mathbf{r} + \mathbf{A}_2^T \dot{\mathbf{Q}}^{-1} \dot{\mathbf{r}}) \\ & + \eta_1 \text{tr}(\mathbf{H}(1 : M, 1 : M)) \\ & + \eta_2 \text{tr}(\mathbf{H}(M + 1 : 2M, M + 1 : 2M)) \end{aligned} \quad (16a)$$

$$\text{s.t. } H_{i,j} \geq |Y(1, 1) - \mathbf{X}(:, 1)^T (\mathbf{s}_i + \mathbf{s}_j) + \mathbf{s}_i^T \mathbf{s}_j|, \quad (16b)$$

$$\begin{aligned} 1 \leq i < j \leq M, \\ H_{i,M+i} = Y(1, 2) - \mathbf{X}(:, 2)^T \mathbf{s}_i - \mathbf{X}(:, 1)^T \dot{\mathbf{s}}_i + \mathbf{s}_i^T \dot{\mathbf{s}}_i, \end{aligned} \quad (16c)$$



(a)



(b)

FIGURE 1. Position estimation comparison ($\mathbf{u} = [285, 325, 275]^T m$, $\dot{\mathbf{u}} = [-20, 15, 40]^T m/s$): (a) RMSE, (b) Bias.

$$H_{M+i, M+i} \leq Y(2, 2) - 2\mathbf{X}(:, 2)^T \dot{\mathbf{s}}_i + \dot{\mathbf{s}}_i^T \dot{\mathbf{s}}_i, \quad (16d)$$

$$H_{i,i} = Y(1, 1) - 2\mathbf{X}(:, 1)^T \mathbf{s}_i + \mathbf{s}_i^T \mathbf{s}_i, \quad (16e)$$

$$\|\mathbf{X}(:, i) - \mathbf{s}_i\| \leq h_i, \quad i = 1, \dots, M \quad (16f)$$

$$\begin{bmatrix} 1 & \mathbf{h}^T \\ \mathbf{h} & \mathbf{H} \end{bmatrix} \succeq \mathbf{0} \quad (16g)$$

$$\begin{bmatrix} \mathbf{I}_3 & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0} \quad (16h)$$

A feasible approach for choosing the two positive parameters η_1 and η_2 is given as:

$$\eta_1 = \frac{1}{\sum_{i=1}^M \|\mathbf{u} - \mathbf{s}_i\|^2} \quad (17)$$

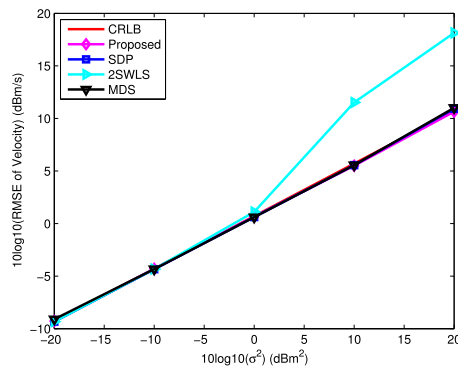
and

$$\eta_2 = \frac{1}{\sum_{i=1}^M \left(\frac{(\dot{\mathbf{u}} - \dot{\mathbf{s}}_i)^T (\mathbf{u} - \mathbf{s}_i)}{\|\mathbf{u} - \mathbf{s}_i\|} \right)^2} \quad (18)$$

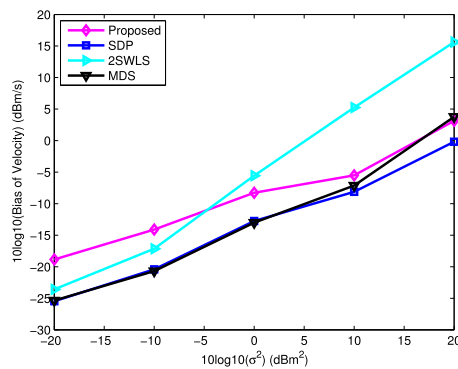
where \mathbf{u} and $\dot{\mathbf{u}}$ are obtained by computing the proposed (16) without the two penalty terms and the SOC constraints. Next, computing (16) will result in the initial estimates $\hat{\mathbf{u}}_0 = X(:, 1)$ and $\hat{\dot{\mathbf{u}}}_0 = X(:, 2)$.

B. ITERATIVE ESTIMATION OF POSITION AND VELOCITY

Because the MLE is highly nonlinear and nonconvex, the above proposed SDP method that builds upon the MLE is



(a)



(b)

FIGURE 2. Velocity estimation comparison ($\mathbf{u} = [285, 325, 275]^T m$, $\dot{\mathbf{u}} = [-20, 15, 40]^T m/s$): (a) RMSE, (b) Bias.

inefficient. Besides, the position and velocity in FDOA measurements are mutually coupled. To resolve these issues, we propose an algorithm to estimate the position and velocity separately using an iterative process next.

First, the initial values of the position and velocity estimates are used to update the velocity. Let $\hat{\mathbf{u}}_0$ be the initial position estimate. The weighted least-squares (WLS) estimate of the velocity is expressed as

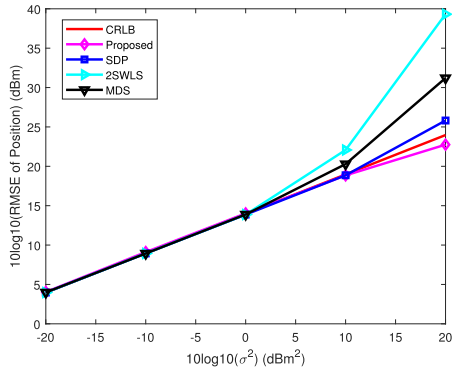
$$\hat{\dot{\mathbf{u}}} = (\mathbf{G}^T \mathbf{W}_1^{-1} \mathbf{G})^{-1} \mathbf{G}^T \mathbf{W}_1^{-1} \mathbf{g} \quad (19)$$

where

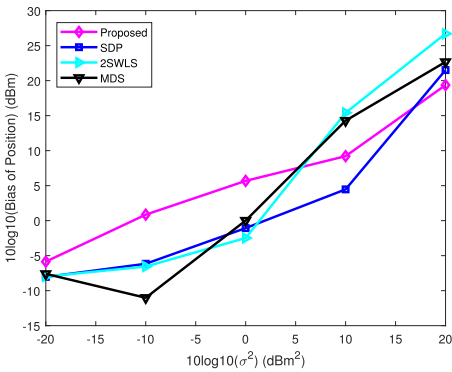
$$\mathbf{G} = \begin{bmatrix} \frac{(\hat{\mathbf{u}}_0 - \mathbf{s}_2)^T}{\|\hat{\mathbf{u}}_0 - \mathbf{s}_2\|} & - \frac{(\hat{\mathbf{u}}_0 - \mathbf{s}_1)^T}{\|\hat{\mathbf{u}}_0 - \mathbf{s}_1\|} \\ \vdots & \vdots \\ \frac{(\hat{\mathbf{u}}_0 - \mathbf{s}_M)^T}{\|\hat{\mathbf{u}}_0 - \mathbf{s}_M\|} & - \frac{(\hat{\mathbf{u}}_0 - \mathbf{s}_1)^T}{\|\hat{\mathbf{u}}_0 - \mathbf{s}_1\|} \end{bmatrix}, \quad (20)$$

$$\mathbf{g} = \begin{bmatrix} r_{21} + \frac{(\hat{\mathbf{u}}_0 - \mathbf{s}_2)^T \dot{\mathbf{s}}_2}{\|\hat{\mathbf{u}}_0 - \mathbf{s}_2\|} - \frac{(\hat{\mathbf{u}}_0 - \mathbf{s}_1)^T \dot{\mathbf{s}}_1}{\|\hat{\mathbf{u}}_0 - \mathbf{s}_1\|} \\ \vdots \\ r_{M1} + \frac{(\hat{\mathbf{u}}_0 - \mathbf{s}_M)^T \dot{\mathbf{s}}_M}{\|\hat{\mathbf{u}}_0 - \mathbf{s}_M\|} - \frac{(\hat{\mathbf{u}}_0 - \mathbf{s}_1)^T \dot{\mathbf{s}}_1}{\|\hat{\mathbf{u}}_0 - \mathbf{s}_1\|} \end{bmatrix}, \quad (21)$$

and $\mathbf{W}_1 = \dot{\mathbf{Q}} + \mathbf{F}_1 \mathbf{Q}_u \mathbf{F}_1^T$. The details of \mathbf{W}_1 are given in Appendix A.



(a)



(b)

FIGURE 3. Position estimation comparison ($\mathbf{u} = [600, 650, 550]^T m$, $\hat{\mathbf{u}} = [-20, 15, 40]^T m/s$): (a) RMSE, (b) Bias.

The next step is to use the velocity from (19) and the position from (16) to update the position estimate. Using $\hat{\mathbf{u}}$, instead of $\hat{\mathbf{u}}$ in (6), we have

$$\min_{\mathbf{u}, \mathbf{h}} (\mathbf{r} - \mathbf{A}_1 \mathbf{h})^T \mathbf{Q}^{-1} (\mathbf{r} - \mathbf{A}_1 \mathbf{h}) + (\hat{\mathbf{r}} - \mathbf{A}_2 \mathbf{h})^T \mathbf{W}_2^{-1} (\hat{\mathbf{r}} - \mathbf{A}_2 \mathbf{h}) \quad (22a)$$

$$\text{s.t. } h_i = \|\mathbf{u} - \mathbf{s}_i\|, \quad (22b)$$

$$h_{M+i} = \frac{(\hat{\mathbf{u}} - \dot{\mathbf{s}}_i)^T (\mathbf{u} - \mathbf{s}_i)}{\|\mathbf{u} - \mathbf{s}_i\|}, \quad i = 1, \dots, M \quad (22c)$$

where $\mathbf{W}_2 = \dot{\mathbf{Q}} + \mathbf{F}_2 \mathbf{Q}_i \mathbf{F}_2^T$ and the details for \mathbf{W}_2 are given in Appendix B.

Similar to the previous relaxation method, the SDP method for the position estimate is described as follows.

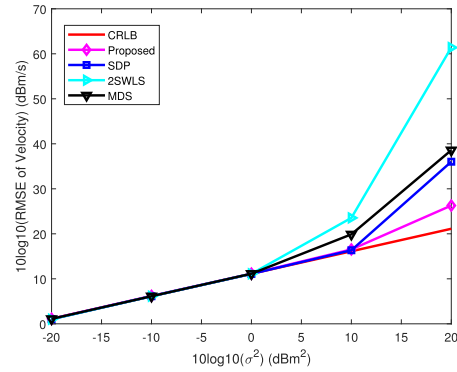
$$\begin{aligned} \min_{\mathbf{h}, \mathbf{H}, \mathbf{u}, y_s} & \text{tr}[(\mathbf{A}_1^T \mathbf{Q}^{-1} \mathbf{A}_1 + \mathbf{A}_2^T \mathbf{W}_2^{-1} \mathbf{A}_2) \mathbf{H}] \\ & - 2\mathbf{h}^T (\mathbf{A}_1^T \mathbf{Q}^{-1} \mathbf{r} + \mathbf{A}_2^T \mathbf{W}_2^{-1} \hat{\mathbf{r}}) \\ & + \eta_1 \text{tr}(\mathbf{H}(1 : M, 1 : M)) \\ & + \eta_2 \text{tr}(\mathbf{H}(M + 1 : 2M, M + 1 : 2M)) \end{aligned} \quad (23a)$$

$$\text{s.t. } H_{i,j} \geq |y_s - \mathbf{u}^T (\mathbf{s}_i + \mathbf{s}_j) + \mathbf{s}_i^T \dot{\mathbf{s}}_j|, \quad (23b)$$

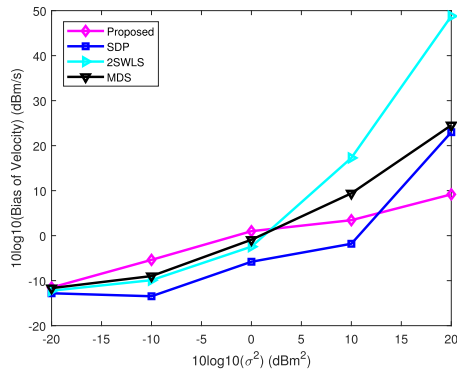
$$1 \leq i < j \leq M, \quad (23b)$$

$$H_{i,M+i} = \mathbf{u}^T \hat{\mathbf{u}} - \hat{\mathbf{u}}^T \mathbf{s}_i - \mathbf{u}^T \dot{\mathbf{s}}_i + \mathbf{s}_i^T \dot{\mathbf{s}}_i, \quad (23c)$$

$$|h_{M+i}| \leq \|\hat{\mathbf{u}} - \dot{\mathbf{s}}_i\|, \quad (23d)$$



(a)



(b)

FIGURE 4. Velocity estimation comparison ($\mathbf{u} = [600, 650, 550]^T m$, $\hat{\mathbf{u}} = [-20, 15, 40]^T m/s$): (a) RMSE, (b) Bias.

$$H_{i,i} = y_s - 2\mathbf{u}^T \mathbf{s}_i + \mathbf{s}_i^T \mathbf{s}_i, \quad (23e)$$

$$\|\mathbf{u} - \mathbf{s}_i\| \leq h_i, \quad i = 1, \dots, M \quad (23f)$$

$$\begin{bmatrix} 1 & \mathbf{h}^T \\ \mathbf{h} & \mathbf{H} \end{bmatrix} \succeq \mathbf{0} \quad (23g)$$

$$\begin{bmatrix} \mathbf{I}_3 & \mathbf{u} \\ \mathbf{u}^T & y_s \end{bmatrix} \succeq \mathbf{0}. \quad (23h)$$

The SDP method in (23) will generate the updated $\hat{\mathbf{u}}$. The updated $\hat{\mathbf{u}}$ and $\hat{\mathbf{u}}$ could be used in (19) and (23) again for improved accuracy. The proposed algorithm for moving target localization performances using TDOA and FDOA measurements is summarized in **Algorithm 1**.

IV. SIMULATION RESULTS

A number of numerical simulations are obtained to assess the performance of the proposed **Algorithm 1**. The proposed algorithm (labeled as ‘Proposed’) is compared with the SDP method by Wang *et al.* [17], 2SWLS [19], MDS [21], and CRLB. The proposed algorithm and Wang’s SDP algorithm are implemented by CVX toolbox [26] using SeDuMi as a solver [27] with precision set to ‘best’, which is same to the setting adopted in [17]. The performance metric adopted is the root mean-squared error (RMSE), which is defined as $\text{RMSE} = \sqrt{\frac{1}{N} \sum_{j=1}^N \|\hat{\mathbf{x}}_j - \mathbf{x}\|^2}$, where \mathbf{x} is the true position or velocity, $\hat{\mathbf{x}}_j$ is the estimated source position or velocity

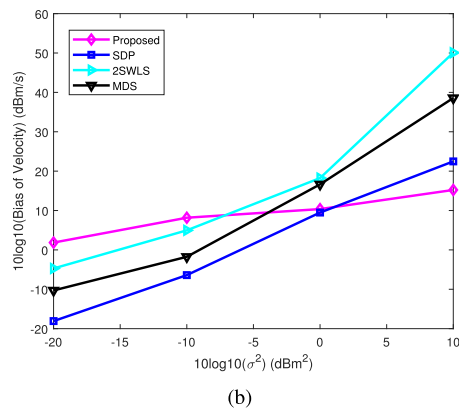
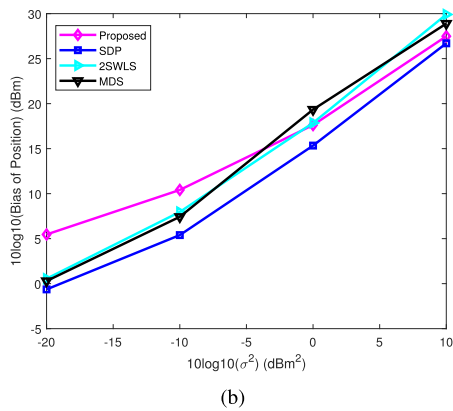
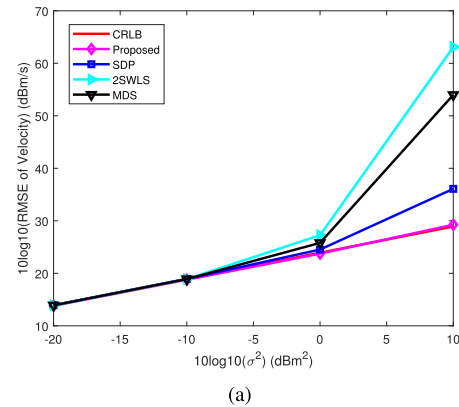
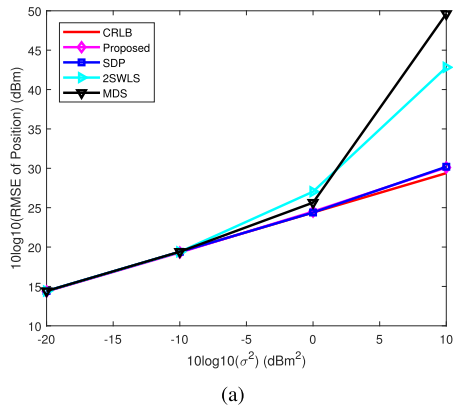


FIGURE 5. Position estimation comparison ($u = [1000, 1500, 2000]^T m$, $\dot{u} = [-20, 15, 40]^T m/s$): (a) RMSE, (b) Bias.

FIGURE 6. Velocity estimation comparison ($u = [1000, 1500, 2000]^T m$, $\dot{u} = [-20, 15, 40]^T m/s$): (a) RMSE, (b) Bias.

in the j th run, and N is the number of Monte Carlo runs ($N = 1000$ is chosen in the simulation next). The simulation configuration is as follows. There are five moving sensors and one moving source, the same configuration as adopted in [17]. The positions and velocities of the sensors are listed in Table 1. The TDOA and FDOA measurement noises are assumed to be independent Gaussian random variables, and their covariance matrices are $\mathbf{Q} = \sigma^2 \Sigma$ and $\dot{\mathbf{Q}} = 0.1\sigma^2 \Sigma$, where σ^2 represents the measurement noise level, and the diagonal elements of Σ equal 1 while its off-diagonal elements are equal to 0.5 [17]. The number of iteration is $L = 2$.

The results in Fig. 1 and Fig. 2 assume that the source is located at $[285, 325, 275]^T m$ with velocity $[-20, 15, 40]^T m/s$. In this geometry, from the two figures, we can see that: 2SWLS has a ‘threshold effect’; i.e., when noise level is low, the algorithm can reach the CRLB; as the noise level reaches a certain value, its performance starts to degrade rapidly. The proposed algorithm, SDP and MDS could reach the CRLB.

For Figs. 3 and 4, the source is assumed to be located at $[600, 650, 550]^T m$ with velocity $[-20, 15, 40]^T m/s$. In this geometry, the two figures show that both 2SWLS, MDS and SDP have a ‘threshold effect’, whereas SDP performs better than MDS, and MDS is better than 2SWLS. This is as expected since in [17], [19], and [21] linear approximations

are made to the nonlinear localization problem. Hence, their performance may degrade rapidly as the measurement noises increase. An interesting result with the proposed algorithm is that the RMSE of position is below the CRLB at high noise levels. The reason is that the proposed algorithm is a biased estimator, and the figures also show that.

In Figs. 5 and 6, the source is assumed to be located at $[1000, 1500, 2000]^T m$ with velocity $[-20, 15, 40]^T m/s$. From the two figures, it is observed that both 2SWLS, MDS and SDP have a ‘threshold effect’, and the proposed algorithm still performs well when the noise level is high. Table II lists the average running time of the algorithms considered. It is observed that the proposed algorithm is more computationally expensive than other algorithms compared. Nevertheless, for applications for which performance is more critical than computational complexity, the proposed algorithm can be applied.

Fig. 7 shows the performances of various methods for a deployment scenario where the four sensors are located at positions 1, 2, 3, and 5 as shown in Table 1. The source is assumed to be located at $[285, 325, 275]^T m$ with velocity $[-20, 15, 40]^T m/s$. It is observed that 2SWLS, MDS and SDP do not work for this case because the number of measurement equations are less than the variables required by these methods. However, the proposed algorithm still has an excellent performance (close to the CRLB).

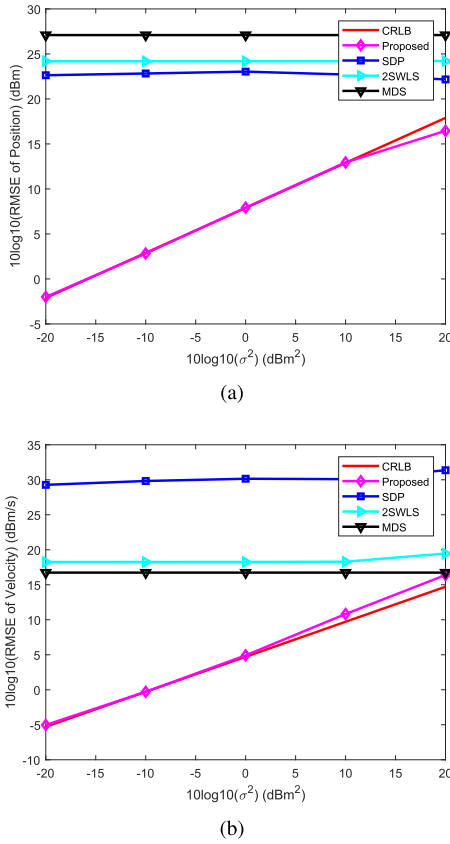


FIGURE 7. RMSE comparison ($\mathbf{u} = [285, 325, 275]^T m$, $\dot{\mathbf{u}} = [-20, 15, 40]^T m/s$, with four sensor nodes): (a) Position, (b) Velocity.

V. CONCLUSIONS

Moving target localization using TDOA and FDOA measurements is studied in this paper. We first develop an SDP technique to transform the nonconvex MLE problem into a convex problem. This SDP algorithm cannot provide a good performance because it is built upon the highly nonlinear and nonconvex MLE problem. We then propose an iterative method that uses the solutions of the SDP method as initial values to improve the accuracy of the velocity and position estimates. Extensive simulation results show that, compared with existing schemes, the proposed algorithm achieves a significant performance gain as the measurement noise level increases.

APPENDIX A
WEIGHTING MATRIX \mathbf{W}_1

The weighing matrix \mathbf{W}_1 is derived as follows. With the first-order Talor series and substituting $\hat{\mathbf{u}}$ into (2) yield

$$\dot{r}_{i1} \approx f_i(\hat{\mathbf{u}}) + \nabla f_i(\hat{\mathbf{u}})^T \Delta \mathbf{u} + \dot{n}_{i1}, \quad i = 2, \dots, M, \quad (24)$$

where

$$f_i(\hat{\mathbf{u}}) = \frac{(\hat{\mathbf{u}} - \hat{\mathbf{s}}_i)^T (\hat{\mathbf{u}} - \mathbf{s}_i)}{\|\hat{\mathbf{u}} - \mathbf{s}_i\|} - \frac{(\hat{\mathbf{u}} - \hat{\mathbf{s}}_1)^T (\hat{\mathbf{u}} - \mathbf{s}_1)}{\|\hat{\mathbf{u}} - \mathbf{s}_1\|}, \quad (25)$$

$$\nabla f_i(\hat{\mathbf{u}}) = \frac{(\hat{\mathbf{u}} - \hat{\mathbf{s}}_i)}{\|\hat{\mathbf{u}} - \mathbf{s}_i\|} - \frac{(\hat{\mathbf{u}} - \mathbf{s}_i)(\hat{\mathbf{u}} - \hat{\mathbf{s}}_i)^T (\hat{\mathbf{u}} - \mathbf{s}_i)}{\|\hat{\mathbf{u}} - \mathbf{s}_i\|^3} - \frac{(\hat{\mathbf{u}} - \hat{\mathbf{s}}_1)}{\|\hat{\mathbf{u}} - \mathbf{s}_1\|} + \frac{(\hat{\mathbf{u}} - \mathbf{s}_1)(\hat{\mathbf{u}} - \hat{\mathbf{s}}_1)^T (\hat{\mathbf{u}} - \mathbf{s}_1)}{\|\hat{\mathbf{u}} - \mathbf{s}_1\|^3}, \quad (26)$$

$$\Delta \mathbf{u} = \hat{\mathbf{u}} - \mathbf{u}. \quad (27)$$

Note that in (26), the calculation of $\nabla f_i(\hat{\mathbf{u}})$ uses $\hat{\mathbf{u}}$, instead of \mathbf{u} . Let

$$\epsilon_i = \nabla f_i(\hat{\mathbf{u}})^T \Delta \mathbf{u} + \dot{n}_{i1}, \quad (28)$$

$$\boldsymbol{\epsilon} = [\epsilon_2, \dots, \epsilon_M]^T, \quad (29)$$

and

$$\mathbf{F}_1 = \begin{bmatrix} \nabla f_2(\hat{\mathbf{u}})^T \\ \vdots \\ \nabla f_M(\hat{\mathbf{u}})^T \end{bmatrix}. \quad (30)$$

Then $\boldsymbol{\epsilon}$ is written as

$$\boldsymbol{\epsilon} = \mathbf{F}_1 \Delta \mathbf{u} + \dot{\mathbf{n}}. \quad (31)$$

The weighting matrix is obtained by

$$\mathbf{W}_1 = \mathbb{E}(\boldsymbol{\epsilon} \boldsymbol{\epsilon}^T). \quad (32)$$

Under the assumption that $\Delta \mathbf{u}$ is uncorrelated with $\dot{\mathbf{n}}$, we have

$$\mathbf{W}_1 = \mathbf{F}_1 \mathbf{Q}_u \mathbf{F}_1^T + \dot{\mathbf{Q}} \quad (33)$$

where

$$\mathbf{Q}_u = \mathbb{E}(\Delta \mathbf{u} \Delta \mathbf{u}^T). \quad (34)$$

Note that \mathbf{Q}_u is unknown. Nevertheless, if the estimator for \mathbf{u} is efficient, then the CRLB of \mathbf{u} can be used, instead of \mathbf{Q}_u , where $\hat{\mathbf{u}}$ and $\hat{\dot{\mathbf{u}}}$ are used to replace \mathbf{u} and $\dot{\mathbf{u}}$ for the calculation of the CRLB of \mathbf{u} .

APPENDIX B
WEIGHTING MATRIX \mathbf{W}_2

The weighing matrix \mathbf{W}_2 is derived as follows. With the first-order Talor series and substituting $\hat{\mathbf{u}}$ into (2) yield

$$\dot{r}_{i1} \approx q_i(\hat{\mathbf{u}}) + \nabla q_i(\hat{\mathbf{u}})^T \Delta \dot{\mathbf{u}} + \dot{n}_{i1}, \quad i = 2, \dots, M, \quad (35)$$

where

$$q_i(\hat{\mathbf{u}}) = \frac{(\hat{\mathbf{u}} - \hat{\mathbf{s}}_i)^T (\mathbf{u} - \mathbf{s}_i)}{\|\mathbf{u} - \mathbf{s}_i\|} - \frac{(\hat{\mathbf{u}} - \hat{\mathbf{s}}_1)^T (\mathbf{u} - \mathbf{s}_1)}{\|\mathbf{u} - \mathbf{s}_1\|}, \quad (36)$$

$$\nabla q_i(\hat{\mathbf{u}}) = \frac{\mathbf{u} - \mathbf{s}_i}{\|\mathbf{u} - \mathbf{s}_i\|} - \frac{\mathbf{u} - \mathbf{s}_1}{\|\mathbf{u} - \mathbf{s}_1\|}, \quad (37)$$

$$\Delta \dot{\mathbf{u}} = \hat{\dot{\mathbf{u}}} - \dot{\mathbf{u}}. \quad (38)$$

Note that in the calculation of $\nabla q_i(\hat{\mathbf{u}})$ with (37), \mathbf{u} is replaced by $\hat{\mathbf{u}}$. Let

$$\theta_i = \nabla q_i(\hat{\mathbf{u}})^T \Delta \dot{\mathbf{u}} + \dot{n}_{i1}, \quad (39)$$

$$\boldsymbol{\theta} = [\theta_2, \dots, \theta_M]^T, \quad (40)$$

and

$$\mathbf{F}_2 = \begin{bmatrix} \nabla q_2(\hat{\mathbf{u}})^T \\ \vdots \\ \nabla q_M(\hat{\mathbf{u}})^T \end{bmatrix}. \quad (41)$$

Then $\boldsymbol{\theta}$ can be represented as

$$\boldsymbol{\theta} = \mathbf{F}_2 \Delta \hat{\mathbf{u}} + \hat{\mathbf{n}}. \quad (42)$$

The weighting matrix is obtained by

$$\mathbf{W}_2 = \mathbb{E}(\boldsymbol{\theta}\boldsymbol{\theta}^T). \quad (43)$$

Under the assumption that $\Delta \hat{\mathbf{u}}$ is uncorrelated with $\hat{\mathbf{n}}$, we have

$$\mathbf{W}_2 = \mathbf{F}_2 \mathbf{Q}_i \mathbf{F}_2^T + \hat{\mathbf{Q}}. \quad (44)$$

where

$$\mathbf{Q}_i = \mathbb{E}(\Delta \hat{\mathbf{u}} \Delta \hat{\mathbf{u}}^T). \quad (45)$$

Note again that \mathbf{Q}_i is unknown. Nevertheless, if the estimator for $\hat{\mathbf{u}}$ is efficient, then the CRLB of $\hat{\mathbf{u}}$, instead of \mathbf{Q}_i , can be used, where $\hat{\mathbf{u}}$ and $\hat{\mathbf{n}}$ are used, instead of \mathbf{u} and \mathbf{n} , for calculating the CRLB of $\hat{\mathbf{u}}$.

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