IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received November 2, 2017, accepted December 8, 2017, date of publication December 18, 2017, date of current version March 16, 2018.

Digital Object Identifier 10.1109/ACCESS.2017.2783320

Dependent Evidence Combination Based on Shearman Coefficient and Pearson Coefficient

HONGHUI XU[1,2] AND YONG DENG[1,3,4]

[1]Institution of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu 610054, China
[2]School of Computer Science and Engineer, University of Electronic Science and Technology of China, Chengdu 610054, China
[3]School of Computer and Information Science, Southwest University, Chongqing 400715, China
[4]Big Data Decision Institute, Jinan University, Guangzhou 510632, China

Corresponding author: Yong Deng (dengentropy@uestc.edu.cn)

This work was supported by the National Natural Science Foundation of China under Grant 61573290 and Grant 61503237.

ABSTRACT Dempster–Shafer evidence theory is efficient to deal with uncertain information. One assumption of evidence theory is that the source of information should be independent when combined by Dempster's rule for evidence combination. However, the assumption does not coincide with the reality. A lot of works are done to solve the problem about the independence. The existing method based on the statistical parameter Pearson correlation coefficient discount is one of the feasible methods. However, the Pearson correlation coefficient is only used to characterize the linear correlation between the attributes of the normal distribution. In this paper, a new method is proposed, the Pearson correlation coefficient and Shearman correlation coefficient to generate the discounting factor. Taking the parametric statistic and nonparametric statistic into consideration, the proposed method is more efficient. The experiments on wine data set are illustrated to show the efficiency of our proposed method.

INDEX TERMS Dempster-Shafer theory, dependent evidence combination, Pearson coefficient, Shearman coefficient, total coefficient.

I. INTRODUCTION

How to deal with the uncertain information in real applications is still an open issue [29], [30]. One of the most used math tools, Dempster-Shafer evidence theory [1], [2], is efficient to model and to combine uncertain information, which is widely used in many engineering systems, such as risk analysis and reliability [31], uncertainty measure [33] and decision making [30], [33].

However, it is worth mentioning that the Dempster's rule assumes the independence between the evidence [2]. Some researchers argue that there is a disagreement about the fusion result without considering the correlation between the evidence [3], [4]. The dependence issue has been partially addressed so that the results can be more reasonable and effective [5], [6]. In order to combine the evidence efficiently, previous researches were broadly divided into two categories: the first one was to modify Dempster's rule; the second was to modify the data of evidence. For example, Cattaneo modified the combination rules, and interprets ''independence'' as ''minimal conflict'' [7]. Modification of the source of evidence has been paid great attention [8]. Yager [9] proposed a method, considering a relatively independent degree as a discounting factor. Then Su *et al.* [12] proposed the model based on the Pearson correlation coefficient of the statistical parameters. In addition, some other works were also presented [5], [16]–[26].

It should be mentioned that the Pearson correlation coefficient just efficiently characterizes the linear correlation between the attributes of the normal distribution [28]. But, it is not efficient to handle the nonparametric situations. In this paper, a new discounting coefficient based on Shearman correlation coefficient from the point of view of nonparametric statistical [11]. Since not only the Pearson correlation coefficient but also the Shearman correlation coefficient are taken into consideration, the proposed method is efficient to find more comprehensive relationship between attributes, which helps to improve the performance of dependent evidence combination.

The paper is organized as follows. Section 2 is the brief introduction of D-S evidence theory, Pearson correlation coefficient and Shearman correlation coefficient; Section 3 presents the dependent evidence fusion method based on Pearson correlation coefficient and the Shearman correlation coefficient; The experimental simulation is shown in Section 4; Section 5 ends the paper with short conclusion.

II. PRELIMINARIES

In this section, some preliminaries, such as D-S evidence theory [1], [2], Pearson correlation coefficient and Shearman correlation coefficient, are briefly introduced.

D-S Evidence Theory: In the D-S evidence theory, $\Theta = (A_1, A_2, A_3, \cdots, A_N)$ is an identification framework. $A_i(1 \leq i \leq N)$ represents the identification of a focal element in the framework. N is to identify the number of elements in the framework.

Basic Belief Assignment(BBA), a mass function is one of the most basic and important definitions of D-S evidence theory. Θ is known to identify the framework. There are 2^Θ subsets of the framework Θ. Each subset's mapping is BBA. BBA has two features: $m(\emptyset) = 0$ and $\sum_{A \subseteq \Theta} m(A) = 1$.

Assuming the identification framework is Θ, $m_1, m_2, m_3, \cdots m_n$ are N BBAs which are all independent. According to the Dempster combination rule, the result is presented as follows [27]:

$$m = m_1 \bigoplus m_2 \bigoplus m_3 \bigoplus \cdots \bigoplus m_n, \quad \alpha \in [0, 1] \quad (1)$$

($\bigoplus$ means the direct sum.)

where

$$m(A) = \begin{cases} 0 & \textit{if } A = \emptyset; \\ K^{-1} \sum_{\bigcap A_j = A} \prod_{i=1}^{n} m_i(A_j) & \textit{otherwise}, \end{cases} \quad (2)$$

K is the normalization factor, defined as follow:

$$K = 1 - \sum_{\bigcap A_j = \emptyset} \prod_{i=1}^{n} m_i(A_j) \quad (3)$$

Given the discounting factor $\alpha(\alpha \in [0, 1])$, m is one of BBAs on the identification frame Θ. ${}^\alpha m$ is defined as a discounted mass function, shown as follows [10]:

$${}^\alpha m(A) = \begin{cases} \alpha m(A) & \textit{if } A \subset \Theta, \quad A \neq \Theta; \\ 1 - \alpha + \alpha m(\Theta) & \textit{otherwise}, \end{cases} \quad (4)$$

Pearson correlation coefficient: The Pearson correlation coefficient is a linear correlation coefficient, which is used to reflect the linear correlation of two normal continuous variables. Assume X and Y are two samples: the sample X contains n sample observations $(x_1, x_2, x_3, \cdots, x_n)$ and sample Y contains n sample observations $(y_1, y_2, y_3, \cdots, y_n)$. Then the Pearson correlation coefficient is defined as follows [28]:

$$r = \frac{(N \sum x_i y_i - \sum x_i \sum y_i)}{\sqrt{N x_i^2 - (\sum x_i)^2} \sqrt{N y_i^2 - (\sum y_i)^2}} \quad (5)$$

The value of r is in the interval $[-1, 1]$. And the greater the value is, the higher X, Y linear correlation rate will be. When $r = 1$, X and Y are completely positive correlation. When $r = -1$, X and Y are completely negative correlation. When $r = 0$, the linear correlation between X and Y is not obvious.

Shearman correlation coefficient [11]: The Shearman correlation coefficient is also called the rank correlation coefficient. It is a nonparametric parameter and does not rely on the distribution of the samples. Therefore, the rank correlation coefficient can be used to describe the correlation between variables when the sample variables do not strictly follow the normal distribution. Similarly, assume X and Y are two samples. The sample X contains n sample observations $(x_1, x_2, x_3, \cdots, x_n)$ and the sample Y contains n sample observations $(y_1, y_2, y_3, \cdots, y_n)$. The coordinates of X, Y are in accordance with the order from large to small (or from small to large). x_i', y_i' are used to record the position of x_i, y_i after the arrangement. Set $d_i = x_i' - y_i'$, then the Shearman correlation coefficient is defined as follows [15]:

$$r_S = 1 - 6 \sum_{i=1}^{n} d_i^2 / (n(n^2 - 1)) \quad (6)$$

With the Pearson correlation coefficient analysis, the value of r_S is in the interval $[-1, 1]$. The greater the value of r_S is, the closer X, Y is to the strict monotonous in function. When $r_S = 1$, X and Y are strictly monotonically increasing in function. When $r_S = -1$, X and Y are strictly monotonically decreasing in function. When $r_S = 0$, the monotonic relationship of X and Y is not obvious in function.

III. PROPOSED METHOD

To improve the efficiency of the nonparametric case, a new method, taking the Shearman correlation coefficient into consideration, is presented in this section.

Assume the collected evidence $S_1, S_2, S_3, S_4, \cdots, S_N$ are obtained by the sensors. Given two evidence S_i, S_j, the Pearson correlation coefficient is $r^P_{S_i,S_j}(i, j = 1, 2, 3, \cdots, N)$, and the Shearman correlation coefficient is $r^S_{S_i,S_j}(i, j = 1, 2, 3, \cdots, N)$. In this paper, we ignore the sign of the correlation coefficient. The main reason is that both positive and negative correlation mean the dependent degree of evidence, to some degree. We use $d^P_{S_i,S_j}$ and $d^S_{S_i,S_j}$ to model as the dependent degree of evidence S_i, S_j.

In this situation, set $(S_1, S_2), (S_1, S_3), (S_1, S_4), (S_1 S_5), \cdots, (S_N, S_N)$ as N^2 pairs of sources of evidence, and then calculate their Pearson correlation coefficient and Shearman correlation coefficient.

Definition 1: Given $d^P_{S_i,S_j}$, the corresponding Pearson correlation coefficient matrix is defined as follows:

$$\Omega = \begin{bmatrix} d^P_{S_1,S_1} & \cdots & d^P_{S_1,S_N} \\ \ddots & \vdots & \ddots \\ d^P_{S_N,S_1} & \cdots & d^P_{S_N,S_N} \end{bmatrix} \quad (7)$$

Definition 2: Given $d^P_{S_i,S_j}$, the corresponding Shearman correlation coefficient matrix is defined as follows:

$$\Psi = \begin{bmatrix} d^S_{S_1,S_1} & \cdots & d^S_{S_1,S_N} \\ \ddots & \vdots & \ddots \\ d^S_{S_N,S_1} & \cdots & d^S_{S_N,S_N} \end{bmatrix} \quad (8)$$

Since there exists correlation between one source of evidence and the others, for each source of evidence, the

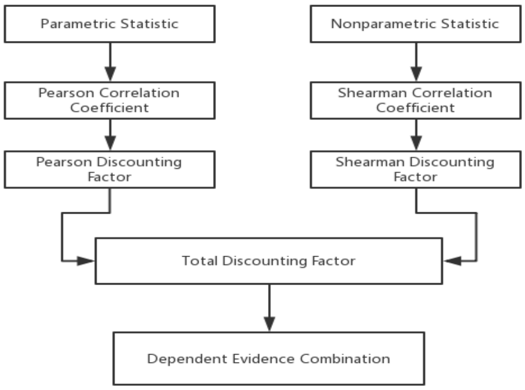

FIGURE 1. The flow chart of the proposed method.

rate of correlation with other sources of evidence is totally calculated.

Definition 3: The total correlation rate of each source of evidence based on the Pearson correlation coefficient is defined as follows:

$$W_i^P = \sum_{k=1}^{n} d^P_{S_i,S_k} \tag{9}$$

Definition 4: The total correlation rate of each source of evidence based on the Shearman correlation coefficient is defined as follows:

$$W_i^S = \sum_{k=1}^{n} d^S_{S_i,S_k} \tag{10}$$

The next step is to define the discounting factor. The function of the discounting factor is to minimize the correlation between the sources of evidence to achieve the "independence" requirement.

Definition 5: Given the discounting Pearson correlation coefficient, its corresponding discounting factor is defined as follows:

$$\pi^i = 1/(W_i^P) \tag{11}$$

Definition 6: Given the discounting Shearman correlation coefficient, its corresponding discounting factor is defined as follows:

$$\phi_i = 1/(W_i^S) \tag{12}$$

Definition 7: The total correlation coefficient corresponding to the discounting factor is defined as:

$$\varpi_i = \pi_i * \phi_i \tag{13}$$

According to the evidence of the discounting combination rules Eq. (4): If only the Pearson correlation coefficient is considered, then $\alpha = \pi_i$. If only the Shearman correlation coefficient is considered, then $\alpha = \phi_i$. If both the Pearson correlation coefficient and the Shearman correlation coefficient are taken into account, then $\alpha = \varpi_i$.

TABLE 1. The value of 13BBAs.

The Ingredient of the wine	Basic Belief Assignment
Alcohol	m{1}=0.5309 m{1,3}=0.3669 m{1,2,3}=0.1022
Malic acidm	m{1}=0.5156 m{1,2}=0.3777 m{1,2,3}=0.1067
Ash	m{3}=0.4319 m{1,3}=0.3171 m{1,2,3}=0.2438
Alcalinity of ash	m{1}=0.3363 m{1,3}=0.3360 m{1,2,3}=0.3277
Magnesium	m{1}=0.4184 m{1,3}=0.3438 m{1,2,3}=0.2378
Total phenols	m{1}=0.7579 m{1,2}=0.2327 m{1,2,3}=0.0094
Flavanoids	m{1}=0.9049 m{1,2}=0.0951 m{1,2,3}=0.0000
Nonflavanoid phenols	m{1}=0.5161 m{1,2}=0.3345 m{1,2,3}=0.1494
Proanthocyanins	m{1}=0.5700 m{1,2}=0.2931 m{1,2,3}=0.1369
Color intensity	m{1}=0.5041 m{1,3}=0.4941 m{1,2,3}=0.0018
Hue	m{1}=0.6414 m{1,2}=0.3575 m{1,2,3}=0.0011
OD280/OD315	m{1}=0.5099 m{1,2}=0.4892 m{1,2,3}=0.0000
Proline	m{1}=1.0000 m{1,2}=0.0000 m{1,2,3}=0.0000

The flow chart of the proposed method is illustrated in Fig. 1.

The algorithm of the proposed method applied to dataset is as follows:

TABLE 2. Pearson correlation coefficient.

	Alcohol	Malic acidm	Ash
Alcohol	1.0000	0.1862	0.2704
Malic acidm	0.1862	1.0000	0.3121
Ash	0.2704	0.3121	1.0000
Alcalinity of ash	0.2059	0.3673	0.4281
Magnesium	0.2519	0.1102	0.3630
Total phenols	0.3827	0.3712	0.1662
Falvanoids	0.3678	0.4062	0.1065
Nonflavanoid	0.0938	0.2635	0.0943
Proanthocyanins	0.2765	0.2674	0.1001
Color intensity	0.5373	0.4232	0.2247
Hue	0.0313	0.5818	0.0869
OD280/OD315	0.2181	0.3359	0.0663
Proline	0.5405	0.0515	0.2417
	Alcalinity of ash	Magnesium	Total phenols
Alcohol	0.2059	0.2519	0.3827
Malic acidm	0.3673	0.1102	0.3712
Ash	0.4281	0.3630	0.1662
Alcalinity of ash	1.0000	0.0925	0.3872
Magnesium	0.0925	1.0000	0.1925
Total phenols	0.3872	0.1925	1.0000
Falvanoids	0.4554	0.2353	0.8498
Nonflavanoid	0.2986	0.2475	0.3914
Proanthocyanins	0.2530	0.1751	0.6413
Color intensity	0.0938	0.1861	0.0323
Hue	0.3419	0.0079	0.5416
OD280/OD315	0.3848	0.0693	0.7175
Proline	0.3620	0.4258	0.4819
	Falvanoids	Nonfalvanoids	Proanthocyanins
Alcohol	0.3678	0.0938	0.2765
Malic acidm	0.4062	0.2635	0.2674
Ash	0.1065	0.0943	0.1001
Alcalinity of ash	0.4554	0.2986	0.2530
Magnesium	0.2353	0.2475	0.1751
Total phenols	0.8498	0.3914	0.6413
Falvanoids	1.0000	0.5352	0.7123
Nonflavanoid	0.5352	1.0000	0.3577
Proanthocyanins	0.7123	0.3577	1.0000
Color intensity	0.0892	0.1355	0.0066
Hue	0.6363	0.2726	0.3725
OD280/OD315	0.7538	0.4820	0.5713
Proline	0.4974	0.1562	0.3811

TABLE 3. Shearman correlation coefficient.

	Alcohol	Malic acidm	Ash
Alcohol	1.0000	0.0958	0.2477
Malic acidm	0.0958	1.0000	0.2463
Ash	0.2477	0.2463	1.0000
Alcalinity of ash	0.2277	0.3331	0.4774
Magnesium	0.1647	0.0441	0.2601
Total phenols	0.3762	0.4316	0.1485
Falvanoids	0.3800	0.5034	0.1009
Nonflavanoid	0.0974	0.3461	0.1302
Proanthocyanins	0.2681	0.3156	0.0902
Color intensity	0.4481	0.3550	0.2246
Hue	0.0456	0.6093	0.1123
OD280/OD315	0.2077	0.4513	0.0586
Proline	0.5909	0.2115	0.2343
	Alcalinity of ash	Magnesium	Total phenols
Alcohol	0.2277	0.1647	0.3762
Malic acidm	0.3331	0.0441	0.4316
Ash	0.4774	0.2601	0.1485
Alcalinity of ash	1.0000	0.0398	0.3237
Magnesium	0.0398	1.0000	0.1834
Total phenols	0.3237	0.1834	1.0000
Falvanoids	0.4238	0.2015	0.8588
Nonflavanoid	0.2804	0.2857	0.4056
Proanthocyanins	0.2076	0.2938	0.5984
Color intensity	0.1583	0.0504	0.0597
Hue	0.3057	0.0673	0.5293
OD280/OD315	0.3339	0.1102	0.7104
Proline	0.3580	0.3120	0.5497
	Falvanoids	Nonfalvanoids	Proanthocyanins
Alcohol	0.3800	0.0974	0.2681
Malic acidm	0.5034	0.3461	0.3156
Ash	0.1009	0.1302	0.0902
Alcalinity of ash	0.4238	0.2804	0.2076
Magnesium	0.2015	0.2857	0.2938
Total phenols	0.8588	0.4056	0.5984
Falvanoids	1.0000	0.5609	0.6592
Nonflavanoid	0.5609	1.0000	0.3308
Proanthocyanins	0.6592	0.3308	1.0000
Color intensity	0.1707	0.1879	0.0482
Hue	0.6495	0.3213	0.3232
OD280/OD315	0.7784	0.5108	0.5186
Proline	0.5974	0.2389	0.4141

Step 1: Considering the wine data set of attributes as the sources of evidence, use the method from literature [13] to generate BBAs.

Step 2: Obtain the Pearson correlation coefficient matrix and the Shearman correlation coefficient matrix according to Eqs. (5-8).

Step 3: Use the Eqs. (9-13) mentioned above to get the total discounting factor.

Step 4: Use the total discounting factor for combination Eq. (4).

Step 5: Use the pignistic probability transformation formula to convert it into probability [12].

IV. EXPERIMENT

Wine data set is used to illustrate the efficiency of the proposed method (archive.ics.uci.edu/ml/datasets/Irism).

TABLE 4. Pearson correlation coefficient (continued table).

	Color intensity	Hue
Alcohol	0.5373	0.0313
Malic acidm	0.4232	0.5818
Ash	0.2247	0.0869
Alcalinity of ash	0.0938	0.3419
Magnesium	0.1861	0.0079
Total phenols	0.0323	0.5416
Falvanoids	0.0892	0.6363
Nonflavanoid	0.1355	0.2726
Proanthocyanins	0.0066	0.3725
Color intensity	1.0000	0.5569
Hue	0.5569	1.0000
OD280/OD315	0.3438	0.6229
Proline	0.2526	0.1686
	OD280/OD315	Proline
Alcohol	0.2181	0.5405
Malic acidm	0.3359	0.0515
Ash	0.0663	0.2417
Alcalinity of ash	0.3848	0.3620
Magnesium	0.0693	0.4258
Total phenols	0.7175	0.4819
Falvanoids	0.7538	0.4974
Nonflavanoid	0.4820	0.1562
Proanthocyanins	0.5713	0.3811
Color intensity	0.3438	0.2526
Hue	0.6229	0.1686
OD280/OD315	1.0000	0.4014
Proline	0.4014	1.0000

The wine data set contains three categories: wine1, wine2, wine3. And wine1, wine2 and wine3 all have 13 properties, naming Alcohol, Malic acid, Ash, Alcalinity of ash, Magnesium, Total phenols, Flavanoids, Nonflavanoid phenols, Proanthocyanins, Color intensity, Hue, OD280 / OD315 of diluted wines and Proline. Select 40 samples from each category for this experiment.

The steps of the experiment are shown as follows:

Step1: Considering the wine data set of attributes as 13 sources of evidence, use the method from literature [13] to generate 13 BBAs, shown in Tab. 1.

Step2: Obtain the 13-order Pearson correlation coefficient matrix, shown in Tab. 2 and Tab. 4 and 13-order the Shearman correlation coefficient matrix according to Eqs. (5-8), shown in Tab. 3 and Tab. 5.

Step3: Use the Eqs. (9-13) mentioned above to get the total discounting factor, shown in Tab.6.

Step4: Use the total discounting factor for combination Eq. (4).

TABLE 5. Shearman correlation coefficient (continued table).

	Color intensity	Hue
Alcohol	0.4481	0.0456
Malic acidm	0.3550	0.6093
Ash	0.2246	0.1123
Alcalinity of ash	0.1583	0.3057
Magnesium	0.0504	0.0673
Total phenols	0.0597	0.5293
Falvanoids	0.1707	0.6495
Nonflavanoid	0.1879	0.3213
Proanthocyanins	0.0482	0.3232
Color intensity	1.0000	0.6300
Hue	0.6300	1.0000
OD280/OD315	0.4006	0.6749
Proline	0.1648	0.2474
	OD280/OD315	Proline
Alcohol	0.2077	0.5909
Malic acidm	0.4513	0.2115
Ash	0.0586	0.2343
Alcalinity of ash	0.3339	0.3580
Magnesium	0.1102	0.3120
Total phenols	0.7104	0.5497
Falvanoids	0.7784	0.5974
Nonflavanoid	0.5108	0.2389
Proanthocyanins	0.5186	0.4141
Color intensity	0.4006	0.1648
Hue	0.6749	0.2474
OD280/OD315	1.0000	0.4317
Proline	0.4317	1.0000

TABLE 6. Discounting factors.

	Pearson	Shearman	Total
Alcohol	0.2292	0.2410	0.0552
Malic acidm	0.2138	0.2023	0.0433
Ash	0.2890	0.3002	0.0868
Alcalinity of ash	0.2141	0.2238	0.0479
Magnesium	0.2979	0.3319	0.0989
Total phenols	0.1624	0.1619	0.0263
Falvanoids	0.1505	0.1453	0.0219
Nonflavanoid	0.2310	0.2130	0.0492
Proanthocyanins	0.1955	0.1973	0.0386
Color intensity	0.2576	0.2565	0.0661
Hue	0.1915	0.1813	0.0347
OD280/OD315	0.1676	0.1616	0.0271
Proline	0.2016	0.1869	0.0377

Step5: Use the pignistic probability transformation formula to convert it into probability [12].

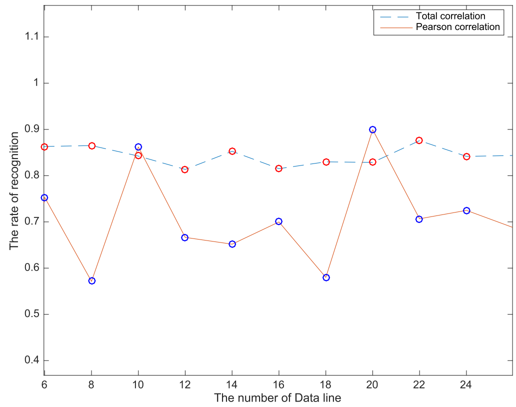

FIGURE 2. The comparison between the Pearson correlation and the total correlation about the rate of the recognition.

The result is shown in Fig. 2. The solid line is the result curve based on the Pearson discounting factor, and the dotted line is the result curve based on the total discounting factor. As can be seen from Fig. 2, the result of our proposed method is illustrated to be more stable based on the total discounting factor. In addition, the correct rate of recognition is 0.87 of our proposed method, while is 0.5 of Pearson method. The main reason of the advantage is that not only the Pearson correlation coefficient but also the Shearman correlation coefficient are taken into consideration, which is more efficient to handle nonparametric information.

V. CONCLUSION

One open issue of evidence theory is the dependent evidence combination. The existing method based on Pearson correlation coefficient is not efficient to deal with nonparametric data. To address this issue, a new method combined with Shearman correlation coefficient is presented to determine the discounting coefficient in dependent evidence combination. The result show that, compared with Pearson correlation coefficient, the proposed method has better stable and higher accuracy of recognition of objects. The merit of the proposed method is that it can efficiently deal with both the parametric and nonparametric data.

REFERENCES

[1] A. P. Dempster, "Upper and lower probabilities induced by a multivalued mapping," *Ann. Math. Stat.*, vol. 38, no. 2, pp. 325–339, 1967.

[2] G. Shafer, *A Mathematical Theory of Evidence*. Princeton, NJ, USA: Princeton Univ. Press, 1976.

[3] R. Fung and C. Chong, "Metaprobability and Dempster–Shafer in evidential reasoning," in *Proc. 1st Conf. Annu. Conf. Uncertainty Artif. Intell. (UAI)*, 1985, pp. 76–83.

[4] F. Voorbraak, " On the justification of Dempster's rule of combination," *Artif. Intell.*, vol. 48, no. 2, pp. 171–197, 1991.

[5] H. Altincay, "On the independence requirement in Dempster–Shafer theory for combining classifiers providing statistical evidence," *Appl. Intell.*, vol. 25, no. 1, pp. 73–90, 2006.

[6] P. Smets, "Belief functions: The disjunctive rule of combination and the generalized Bayesian theorem," *Int. J. Approx. Reasoning*, vol. 9, no. 1, pp. 1–35, 1993.

[7] M. E. G. V. Cattaneo, "Combining belief functions issued from dependent sources," in *Proc. 3rd Int. Symp. Imprecise Probab. Their Appl. (ISIPTA)*, 2003, pp. 1–14.

[8] P. Smets and P. Kennes, "The concept of distinct evidence," in *Proc. 4th Conf. Inf. Process. Manage. Uncertainty Knowl.-Based Syst. (IPMU)*, 1992, pp. 789–794.

[9] R. R. Yager, "On the fusion of non-independent belief structures," *Int. J. Gen. Syst.*, vol. 38, no. 5, pp. 505–531, 2009.

[10] X. Su, S. Mahadevan, P. Xu, and Y. Deng, "Handling of dependence in Dempster–Shafer theory," *Int. J. Intell. Syst.*, vol. 30, no. 4, pp. 441–467, 2015.

[11] W. Xing, *Nonparametric Statistics*. Beijing, China: Renmin Univ. China, 2005.

[12] X. Su, P. Xu, S. Mahadevan, and Y. Deng, "On consideration of dependence and reliability of evidence in Dempster–Shafer theory," *J. Inf. Comput. Sci.*, vol. 11, no. 14, pp. 4901–4910, 2014.

[13] P. Xu, Y. Deng, X. Su, and S. Mahadevan, "A new method to determine basic probability assignment from training data," *Knowl.-Based Syst.*, vol. 46, no. 1, pp. 69–80, 2013.

[14] P. Smets, Y.-T. Hsia, A. Saffiotti, R. Kennes, H. Xu, and E. Umkehren, "The transferable belief model," in *Proc. Eur. Conf. Symbolic Quant. Approaches Reasoning Uncertainty*, 1991, pp. 91–96.

[15] X. Wang, *Nonparametric Statistic*. Beijing, China: Tsinghua Univ. Press, 2005.

[16] D. Geriger, T. Verma, and J. Pearl, "Identifying independence in Bayesian networks," *Networks*, vol. 20, no. 5, pp. 507–534, 1990.

[17] L. M. de Campos and J. F. Huete, "Independence concepts in possibility theory: Part I," *Fuzzy Sets Syst.*, vol. 103, no. 1, pp. 127–152, 1990.

[18] F. E. White, "A model for data fusion," in *Proc. 1st Nat. Symp. Sensor Fusion*, Orlanda, FL, USA, 1988, pp. 149–158.

[19] D. L. Hall and J. Llinas, "An introduction to multisensor data fusion," *Proc. IEEE*, vol. 85, no. 1, pp. 6–23, Jan. 1997.

[20] J.-B. Yang and M. G. Singh, "An evidential reasoning approach for multiple-attribute decision making with uncertainty," *IEEE Trans. Syst., Man, Cybern.*, vol. 24, no. 1, pp. 1–18, Jan. 1994.

[21] L. A. Zhden, "A simple view of the Dempster–Shafer theory of evidence and its implication for the rule of combination," *AI Mag.*, vol. 7, no. 2, pp. 85–90, 1986.

[22] W. Liu, "Analyzing the degree of conflict among belief function," *Artif. Intell.*, vol. 170, no. 11, pp. 909–924, 2006.

[23] E. Lefever, O. Colot, and P. Vannoorenberghe, "A classification method based on the Dempster–Shafer's theory and information criteria," in *Proc. 2nd Int. Conf. Inf. Fusion*, 1999, pp. 1–8.

[24] Y.-G. Wu, J.-Y. Yang, and L.-J. Liu, "On the evidence inference theory," *Inf. Sci.*, vol. 89, nos. 3–4, pp. 245–260, 1996.

[25] S. Destercke, D. Dubois, and E. Chojnacki, "Cautious conjunctive merging of belief functions," in *Symbolic and Quantitative Approaches to Reasoning With Uncertainty*. Berlin, Germany: Springer, 2007, pp. 332–343.

[26] S. Destercke and D. Dubois, "Idempotent conjunctive combination of belief functions: Extending the minimum rule of possibility theory," *Inf. Sci.*, vol. 181, no. 18, pp. 3925–3945, 2011.

[27] N. Yuan, D. Y. Yang, and Y. Lin, "Multiple sensor information fusion and its application to industrial control," *J. Zhejiang Univ. Technol.*, 1999.

[28] G. S. Rogers, "A course in theoretical statistics," *Technometrics*, vol. 11, no. 4, pp. 840–841, 1969.

[29] Q. Zhang, M. Li, and Y. Deng, "Measure the structure similarity of nodes in complex networks based on relative entropy," *Phys. A, Stat. Mech. Appl.*, vol. 491, pp. 749–763, Feb. 2018.

[30] W. Jiang and J. Zhan, "A modified combination rule in generalized evidence theory," *Int. J. Fuzzy Syst.*, vol. 46, no. 3, pp. 630–640, 2017.

[31] W. Jiang, C. Xie, M. Zhuang, and Y. Tang, "Failure mode and effects analysis based on a novel fuzzy evidential method," *Appl. Soft Comput.*, vol. 57, pp. 672–683, Aug. 2017.

[32] W. Jiang and S. Wang, "An uncertainty measure for interval-valued evidences," *Int. J. Comput. Commun. Control*, vol. 12, no. 5, pp. 631–644, 2017.

[33] T. Liu, Y. Deng, and F. Chan, "Evidential supplier selection based on DEMATEL and game theory," *Int. J. Fuzzy Syst.*, no. 2, pp. 1–13, 2017.

[34] X. Zhang, S. Mahadevan, and X. Deng, "Reliability analysis with linguistic data: An evidential network approach," *Rel. Eng. Syst. Saf.*, vol. 162, pp. 111–121, Jun. 2017.

[35] X. Zhang and S. Mahadevan, "Aircraft re-routing optimization and performance assessment under uncertainty," *Decision Support Syst.*, vol. 96, pp. 76–82, Apr. 2017.

HONGHUI XU is currently pursuing the bachelor's degree with the School of Computer Science and Engineering, University of Electronic Science and Technology of China. In 2016, he was appointed as a Research Assistant with the Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China. In 2017, he entered the top-notch talent training program with the School of Computer Science and Engineering, the University of Electronic Science and Technology of China. In 2017, he successfully passed the YINXING HUANG innovation and entrepreneurship project open reply at the School of Computer Science and Engineering, University of Electronic Science and Technology of China, where he was selected as a URP student in 2017. In summer 2017 he participated in the summer school with the Future Media Laboratory, University of Electronic Science and Technology of China, studying the basic knowledge of machine learning. In 2017 he became an outstanding student of the URP program. He is engaged in the research of evidence theory and the basic theory of machine learning.

YONG DENG received the Ph.D. degree in precise instrumentation from Shanghai Jiao Tong University, Shanghai, China, in 2003. From 2005 to 2011, he was an Associate Professor with the Department of Instrument Science and Technology, Shanghai Jiao Tong University. Since 2010, he has been a Professor with the School of Computer and Information Science, Southwest University, Chongqing, China. Since 2012, he has been a Visiting Professor with Vanderbilt University, Nashville, TN, USA. Since 2016, he has been a Professor with the School of Electronic and Information Engineering, Xi'an Jiaotong University, Xi'an, China. Since 2017, he has been a Full Professor with the Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu, China. He has authored over 100 papers in referred journals, such as *Decision Support Systems*, the *European Journal of Operational Research*, and *Scientific Reports*. His research interests include evidence theory, decision making, information fusion, and complex system modeling. He has been a program member of many conferences, such as the International Conference on Belief Functions. He has served on many Editorial Board, including as an Academic Editor of the *PLOS ONE*. He served as a Reviewer for more than 30 journals, such as the IEEE TRANSACTION ON FUZZY SYSTEMS. He has received numerous honors and awards, including the Elsevier Highly Cited Scientist in China, in 2014, 2015, and 2016.