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# Stabilizing the Buck Converter With a First-Order-Filter-Based Time Delay Feedback Controller

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**ABSTRACT** A first-order-filter-based time delay feedback (TDF) controller is proposed to stabilize the buck converter when operating with non-linear phenomena, such as bifurcation and chaos. Compared with the existing second-order filter method, the proposed controller needs only two parameters. This is similar to the ideal TDF controller and it makes the design of the controller simpler. The discrete model of the converter is used to compute one of the two parameters, which is known as the feedback gain. The frequency domain comparison of the proposed controller and the ideal TDF controller shows that the proposed controller is almost non-invasive, because it does not change the dc component of the converter and only changes slightly the harmonic at the switching frequency. The proposed controller has another structure called the extended controller, which needs three parameters. By increasing the third parameter, the feedback gain needs a larger value to stabilize the converter. The benefit of the proposed controller lies in the fact that it does not need the digital circuit that was thought to be necessary for the TDF control of the converter. Simulated and experimental waveforms verify the computation results. The added feedback signal is small enough when the switch turns on.

**INDEX TERMS** Analog circuit, bifurcation, chaos, controller, dc-dc power converter, delay system, first order filter, nonlinearity, time delay feedback.

#### I. INTRODUCTION

Power converters are prone to exhibit non-linear phenomena, which are usually undesirable and should be avoided because they are detrimental to the normal operation of converters and lead to instabilities [1]. In DC/DC power converters, there are mainly two types of instabilities [2]–[7]. The first of these is the so-called fast-scale instability, resulting from the period-doubling bifurcation. The fast-scale instability leads to subharmonic oscillations and the converter has a greater ripple amplitude compared to the normal operation. The second is the so-called slow-scale instability, resulting from the Neimark-Sacker bifurcation. In this paper, the control of fast-scale instability is studied. During the last few decades, controlling the fast-scale instability of power converters has interested many researchers and some methods have been proposed. The resonant parametric perturbation involves nonfeedback control and it needs an external control signal adding to the converter [8], [9]. The resonant parametric perturbation is usually used in converters with a fixed input voltage, where the duty cycle is fixed. Therefore, this method is limited in engineering. In all existing feedback control methods, the OGY (Ott, Gebogi and Yorke) method is complicated for realization, so more research is focused on the Time Delay Feedback (TDF) control method [10]. However, in DC/DC converters, it is commonly believed that the analog implementation of TDF controller is challenging [11]. Therefore, alternative methods, based on a second-order filter, are proposed [12]. In fact, the controllers in these alternative methods has a similar low frequency characteristic as the TDF controller. In most cases, the second-order filter is a notch filter, where three important parameters have to be computed in order to control the fast-scale instability [13]-[17]. The first one is its gain, while the second one is the quality factor. The quality factor will be selected so that the notch filter and

the TDF controller have similar low frequency characteristic. The third parameter is the notch frequency, which can be easily determined. On the other hand, only two parameters are needed in the traditional TDF controller [10]. The first one is the gain, while the second one is the delay time. Generally, the delay time is the switching period. Therefore, only the gain has to be computed.

The TDF controller is a simpler design process effort. However, no effective circuits have been proposed to implement this controller. Recently, there has been progress in the dynamics of time-delayed chaotic systems, which has provided a clue about using a first-order filter to implement the TDF controller [18], [19]. The methodology used in these time-delayed chaotic systems cannot be generalized directly to power converters due to the piece-wise smooth characteristics of converters. Therefore, the appropriate model of the power converter under the TDF controller should be adopted. In this study, the TDF controller is used to stabilize the Buck converter and the design process of the controller is given. The computation of the only one parameter, the gain, is based on the discrete model of the converter. Simulated and experimental observations agree well with the analytical results.



FIGURE 1. Buck converter.

# II. BUCK CONVERTER WITH A FIRST-ORDER-FILTER-BASED TDF CONTROLLER

#### A. FIRST-ORDER-FILTER-BASED TDF CONTROLLER

The Buck converter is shown in Fig. 1. The component values used are:  $R = 22\Omega$ , A = 8.4, L = 20 mH,  $C = 47\mu$ F, *Vref* = 11.3 V, Vu = 8.2 V, Vl = 3.8V, and  $T = 400\mu$ s. The converter operates in continuous conduction mode and it has been shown that the converter may exhibit bifurcation and chaos when the input voltage *E* varies in the range of approximately 24 to 35 V. This converter has been stabilized with other methods so it is used for the comparison of the first-order-filter-based TDF controller and other controllers.

Pyragas proposed a TDF method to control bifurcation and chaos in non-linear systems [10]. The idea of TDF control uses one delayed state variable as a feedback signal. When the TDF controller is used in power converters, the delay time is normally the switching period, because it is expected that the state variable values at the beginning of the (N+1)th period



FIGURE 2. The first-order-filter-based TDF (Time Delay Feedback) controller.

coincide with that at the beginning of the Nth period. It was believed that digital implementation of the TDF controller is only feasible in power converters. The digital implementation makes the control circuit more complicated, as it needs A/D and D/A converters. The exact analog implementation of the TDF controller has been a challenge in last few decades. Recently, there have been some methods applied in studying the dynamics of time-delayed chaotic systems. Among them, the first-order filter is an effective but simple implementation of the analog time delay. In the Buck converter, it is convenient to delay the control signal  $v_{con}$ , so the first-order-filter-based TDF controller is shown in Fig.2. The controller consists of a series of two filters. Each filter can be described by

$$H_1(s) = \frac{1 - R_f C_f s}{1 + R_f C_f s} = -\frac{s - \omega_0}{s + \omega_0},$$
(1)

where  $\omega_0 = 1/R_f C_f$  is the frequency at which the phase shift of the filter is  $\pi/2$ . Each filter contributes a delay of  $T_1 \approx R_f C_f$ , so in this converter,  $R_f = 10 k\Omega$  and  $C_f =$ 20nF is selected. The frequency domain representation in Fig. 3 shows that the series of two filters and the ideal delay unit,  $e^{-Ts}$ , are similar in low frequency of under switching frequency.



**FIGURE 3.** Bode diagram of the series of two first order filters and the ideal delay unit.

### **B. DISCRETE MODEL OF CONVERTER**

The only parameter unknown in the proposed controller is the gain,  $\gamma$ . The computation of this parameter needs knowledge of the discrete model of the converter. Let  $x = [x_1x_2x_3x_4]^T = [i_Lvx_3x_4]^T$  be the state vector. At the beginning of *Nth* switching period, the state vector is  $x(NT) = [x_1(NT)x_2(NT)x_3(NT)x_4(NT)]$ . The switch is off and the converter is described by

$$\dot{x} = A_1 x + B_1 E, \qquad (2)$$

where

$$A_{1} = \begin{pmatrix} 0 & -1/L & 0 & 0\\ 1/C & -1/RC & 0 & 0\\ 0 & kA & -k & 0\\ 0 & -kA & 2k & -k \end{pmatrix}, B_{1} = \begin{pmatrix} 0\\ 0\\ -k\frac{AV_{ref}}{E}\\ k\frac{AV_{ref}}{E} \end{pmatrix},$$
$$k = 1/R_{f}C_{f}.$$

The switch S turns on when the control signal  $v'_{con}$  equals to the ramp  $v_{ramp}$ . Therefore, the switching instant is decided by

$$v'_{con} - \gamma \left( v'_{con} - \left( x_4 - R_f C_f \frac{dx_4}{dt} \right) \right) = 3.8 + 4.4d, \quad (3)$$

where d is the duty ratio. The switching surface is described by

 $u(x, d) = zx \left( (N+d) T \right) - AV_{ref} - 3.8 - 4.4d = 0, \quad (4)$ 

where  $z = (0 \text{ A} - 2\gamma 2\gamma)^{\text{T}}$ . In equation (4), x((N + d)T) is computed from equation (2).

$$x((N+d)T) = e^{A_1 dT} x(NT) + A_1^{-1} \left( e^{A_1 dT} - I \right) B_1 E.$$
 (5)

Therefore, the switching surface becomes

$$u(x(NT), d) = z \left( e^{A_1 dT} x(NT) + A_1^{-1} \left( e^{A_1 dT} - I \right) B_1 E \right)$$
  
-AV<sub>ref</sub> - 3.8 - 4.4d  
= 0. (6)

After the switch turns on, the converter is described by

$$\dot{x} = A_2 x + B_2 E,\tag{7}$$

where

$$A_{2} = \begin{pmatrix} 0 & -1/L & 0 & 0 \\ 1/C & -1/RC & 0 & 0 \\ 0 & kA & -k & 0 \\ 0 & -kA & 2k & -k \end{pmatrix}, B_{2} = \begin{pmatrix} \frac{1}{L} \\ 0 \\ -k \frac{AV_{ref}}{E} \\ k \frac{AV_{ref}}{E} \end{pmatrix}.$$

At the beginning of the (N+1)th switching period, the state vector is

$$x((N+1)T) = e^{A_2 dT} x((N+d)T) + A_2^{-1} \left(e^{A_2 dT} - I\right) B_2 E.$$
 (8)

From equations (2) and (7), one obtains

$$x ((N + 1) T) = f (x(NT), d)$$
  
=  $e^{A_2 dT} \left( e^{A_1 dT} x(NT) + A_1^{-1} \left( e^{A_1 dT} - I \right) B_1 E \right)$   
+  $A_2^{-1} \left( e^{A_2 dT} - I \right) B_2 E.$  (9)

The Jacob of the converter is

$$J = \frac{\partial f}{\partial x(NT)} - \frac{\partial f}{\partial d} \left(\frac{\partial u}{\partial d}\right)^{-1} \frac{\partial u}{\partial x(NT)}.$$
 (10)

In most engineering applications, x ((N + 1)T) should equal to x (NT), which means the converter is stable in the switching frequency with no bifurcation and chaos. This is satisfied when all eigenvalues of the Jacob lie within the unit circle. From this viewpoint, one obtains the critical values of  $\gamma$ , as shown in Fig. 4. The simulation waveforms, depicted in Fig. 5, shows that the Buck converter operates from period-2 to period-1 when the proposed controller is switched on at t = 0.54s. Furthermore, the added feedback signal  $v_{af}$ is small enough.



**FIGURE 4.** Critical values of  $\gamma$  under various input voltages.



**FIGURE 5.** Simulation waveforms of Gate Pulse, control signal, and the added feedback signal  $v_{af}$  when E = 27V and  $\gamma = 0.15$ .

# C. FREQUENCY DOMAIN COMPARISON OF PROPOSED CONTROLLER AND IDEAL TDF CONTROLLER

The first-order-filter-based TDF controller can be expressed by

$$G_{FOF}(s) = 1 - \gamma \left( 1 - \left( \frac{s - \omega_0}{s + \omega_0} \right)^2 \right).$$
(11)

The ideal TDF controller is expressed by

$$G_{FOF}(s) = 1 - \gamma \left(1 - e^{-Ts}\right).$$
 (12)

The Bode diagram of these two controllers are shown in Fig. 6. Apparently, both controllers have attenuation at



**FIGURE 6.** Bode diagram of the first-order-filter-based TDF controller and the ideal TDF controller when (a)  $\gamma = 0.2$  and ( b)  $\gamma = 0.4$ .

half of the switching frequency. When  $\gamma$  increases, the attenuation also increases. Furthermore, the diagram depicted in Fig.6 shows that the ideal TDF controller does not change the DC component and the harmonic at the switching frequency, so it has a noninvasive nature. On the other hand, the first-order-filter-based TDF controller does not change the DC component, but it changes slightly the harmonic at the switching frequency. Considering that the change is slight, the proposed controller is almost non-invasive in the control circuit. It is important to note that in Fig. 5, when the switch turns on, the value of feedback signal  $v_{af}$  is almost zero.

# III. EXTENDED FIRST-ORDER-FILTER-BASED TDF CONTROLLER

Another TDF method, the extended TDF controller, is also feasible to be used as the proposed filter. As shown in Fig. 7, another parameter,  $\beta$ , is of great importance in affecting



FIGURE 7. The extended first-order-filter-based TDF controller.

the critical values of  $\gamma$ . The converter is also described by equations (2) and (7), but with

$$A_1 = A_2 = \begin{pmatrix} 0 & -1/L & 0 & 0\\ 1/C & -1/RC & 0 & 0\\ 0 & kA & -k(1+\beta) & 2k\beta\\ 0 & -kA & 2k & -k(1+\beta) \end{pmatrix},$$
  
$$k = 1/R_f C_f (1-\beta).$$

It is important to note that the switching surface is not changed. The critical values of  $\gamma$  for different  $\beta$  is shown in Fig. 8.



**FIGURE 8.** Critical values of  $\gamma$  under various  $\beta$ .

The extended first-filter-based TDF controller can be expressed by

$$G_{EFOF}(s) = 1 - \frac{4\gamma\omega_0 s}{(1-\beta) s^2 + 2(1+\beta)\omega_0 s + (1-\beta)\omega_0^2}.$$
 (13)

The extended ideal TDF controller is expressed by

$$G_{ETDF}(s) = 1 - \gamma \frac{1 - e^{-Ts}}{1 - \beta e^{-Ts}}.$$
 (14)

The Bode diagram of these two controllers are shown in Fig. 9. Apparently, by increasing  $\beta$ , the attenuation at half of the switching frequency decreases, as stated in a previous study [20]. This requires the use of higher values of  $\gamma$  compared to the first-order-filter-based controller. Fig. 7 also proves this conclusion. It is important to note that the proposed controller slightly changes the harmonic at the switching frequency. Simulation waveforms in Fig. 10 show



**FIGURE 9.** Bode diagram of the extended first-order-filter-based TDF controller and the extended ideal TDF controller.

that the Buck converter operate from chaos to period-1 when the proposed controller is switched on at t = 0.54s. The added feedback is almost zero when the switch turns on.

### **IV. EXPERIMENTAL RESULTS**

#### A. VERIFICATION OF TWO PARAMETERS $\beta$ AND $\gamma$

Based on the diagrams shown in Fig.1 and Fig.7, an experimental system was built. The proposed controller, including the first-order filter and the gain, is implemented with LM324. The experimental systems verified the data in Fig.8. Specifically, the experimental waveform shown in Fig.11 are tested when  $\beta = 0.3$  and  $\gamma = 0.2$ . It is important note that when the voltage across the diode turns from 0 V to 30 V (the MOSFET turns on), the added feedback  $v_{af}$  is less than 0.05 V. Compared to the control voltage  $v''_{con}$ , it is small enough.



**FIGURE 10.** Simulation waveforms of Gate Pulse, control signal, and the added feedback signal when E = 32V,  $\beta = 0.2$ , and  $\gamma = 0.2$ .



**FIGURE 11.** Experimental waveforms under E = 30V,  $\beta = 0.3$ , and  $\gamma = 0.2$ . CH1: the control voltage  $v'_{con}$ ; CH2: the diode voltage; CH3: the ramp; CH4: the added feedback  $v_{af}$ .



**FIGURE 12.** Experimental waveforms under E = 30V,  $\beta = 0.3$ , and  $\gamma = 0.2$ . CH1: the control voltage  $v''_{con}$ ; CH2: the resistor change signal; CH4: the converter output voltage v.

# B. DYNAMIC PERFORMANCE OF THE PROPOSED CONTROLLER IN FRONT OF LOAD CHANGES

The control and converter output voltages are shown in Fig. 12. In this figure, a step change in the load resistor from 22  $\Omega$  to 11  $\Omega$  and vice-versa was conducted. Apparently, the converter operates stably under both load cases. In fact, without the proposed controller, the converter will exhibit nonlinear phenomena when the load is 22  $\Omega$ . Fig.12 also shows that after a short transient time, the system reaches a steady state.

### **V. CONCLUSION**

A first-order-filter-based TDF controller has been proposed to stabilize the Buck converter working in continuous conduction mode. There are three parameters in the existing second-order-filter-based method, but only two parameters in the proposed controller. The discrete model based on the stroboscopic map is used in the computation process. Unlike the ideal TDF controller, the proposed controller does not change the DC component of the converter, but it changes slightly the harmonic at the switching frequency. An extended first-order-filter-based TDF controller needs three parameters. The proposed controller does not need the digital circuit to delay the signal, so it can be easily implemented in engineering. The converter in this study is a buck converter, but the proposed controller can be used in other DC/DC converters. It is important to note that the converter in this paper adopts a P controller. In the case of PI controller, the computation process does not apply, because some matrices are non-invertible. However, other computation methods can be adopted. Details will be given in future work.

#### REFERENCES

- A. El Aroudi, "A new approach for accurate prediction of subharmonic oscillation in switching regulators—Part I: Mathematical derivations," *IEEE Trans. Power Electron.*, vol. 32, no. 7, pp. 5651–5665, Jul. 2017.
- [2] M. Huang, Y. Peng, C. K. Tse, Y. Liu, J. Sun, and X. Zha, "Bifurcation and large-signal stability analysis of three-phase voltage source converter under grid voltage dips," *IEEE Trans. Power Electron.*, vol. 32, no. 11, pp. 8868–8879, Nov. 2017.
- [3] L. Cheng, W.-H. Ki, F. Yang, P. K. T. Mok, and X. Jing, "Predicting subharmonic oscillation of voltage-mode switching converters using a circuit-oriented geometrical approach," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 64, no. 3, pp. 717–730, Mar. 2017.
- [4] Y. Al-Turki et al., "Nonaveraged control-oriented modeling and relative stability analysis of DC-DC switching converters," Int. J. Circuit Theory Appli., to be published. [Online]. Available: http://onlinelibrary. wiley.com/doi/10.1002/cta.2387/full, doi: 10.1002/cta.2387.
- [5] A. El Aroudi, "A new approach for accurate prediction of subharmonic oscillation in switching regulators—Part II: Case studies," *IEEE Trans. Power Electron.*, vol. 32, no. 7, pp. 5835–5849, Jul. 2017.
- [6] E. Rodriguez, F. Guinjoan, A. El Aroudi, and E. Alarcon, "A ripple-based design-oriented approach for predicting fast-scale instability in DC–DC switching power supplies," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 59, no. 1, pp. 215–227, Jan. 2012.
- [7] D. Giaouris, S. Banerjee, B. Zahawi, and V. Pickert, "Stability analysis of the continuous-conduction-mode buck converter via Filippov's method," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 55, no. 4, pp. 1084–1096, May 2008.
- [8] Y. Zhou, C. K. Tse, S.-S. Qiu, and F. C. M. Lau, "Applying resonant parametric perturbation to control chaos in the buck DC/DC converter with phase shift and frequency mismatch considerations," *Int. J. Bifurcation Chaos*, vol. 13, pp. 3459–3471, Nov. 2003.
- [9] C. K. Tse, Y. Zhou, F. C. M. Lau, and S. S. Qiu, "Intermittent chaos in switching power supplies due to unintended coupling of spurious signals," in *Proc. Power Electron. Spec. Conf.*, Acapulco, Mexico, Jun. 2003, pp. 642–647.
- [10] K. Pyragas, "Continuous control of chaos by self-controlling feedback," *Phys. Lett. A*, vol. 170, no. 6, pp. 421–428, 1992.

- [11] A. El Aroudi, R. Haroun, A. Cid-Pastor, and L. Martinez-Salamero, "Suppression of line frequency instabilities in PFC AC-DC power supplies by feedback notch filtering the pre-regulator output voltage," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 60, no. 3, pp. 796–809, Mar. 2013.
- [12] T. Pyragienė, A. Tamaševičius, G. Mykolaitis, and K. Pyragas, "Noninvasive control of synchronization region of a forced self-oscillator via a second order filter," *Phys. Lett. A*, vol. 361, pp. 323–331, Feb. 2007.
- [13] C. Cai, Z. Xu, W. Xu, and B. Feng, "Notch filter feedback control in a class of chaotic systems," *Automatica*, vol. 38, no. 4, pp. 695–701, 2002.
- [14] A. M. Athalye and W. J. Grantham, "Notch filter feedback control of a chaotic system," in *Proc. Amer. Control Conf.*, Seattle, WA, USA, Jun. 21–23, 1995, pp. 837–841.
- [15] W.-G. Lu, L.-W. Zhou, Q.-M. Luo, and J.-K. Wu, "Non-invasive chaos control of DCŰDC converter and its optimization," *Int. J. Circuit Theory Appl.*, vol. 39, no. 2, pp. 159–174, 2011.
- [16] W.-G. Lu, L.-W. Zhou, Q.-M. Luo, J.-K. Wu, and X.-F. Zhang, "Filter based non-invasive control of chaos in buck converter," *Phys. Lett. A*, vol. 372, no. 18, pp. 3217–3222, 2008.
- [17] E. Rodriguez, H. Iu, A. El Aroudi, and E. Alarcón, "A frequency domain approach for controlling chaos in switching converters," in *Proc. IEEE ISCAS*, May 2010, pp. 2928–2931.
- [18] D. Biswas and T. Banerjee, "A simple chaotic and hyperchaotic time-delay system: Design and electronic circuit implementation," *Nonlinear Dyn.*, vol. 83, no. 4, pp. 2331–2347, 2016.
- [19] T. Banerjee, D. Biswas, and B. C. Sarkar, "Design and analysis of a first order time-delayed chaotic system," *Nonlinear Dyn.*, vol. 70, no. 1, pp. 721–734, 2012.
- [20] E. Rodriguez, E. Alarcón, H. H. C. Iu, and A. El Aroudi, "A frequency domain approach for controlling fast-scale instabilities in switching power converters," *Int. J. Bifurcation Chaos*, vol. 25, pp. 1550141-1–1550141-11, Nov. 2015.



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