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Distributed Optimal Reactive Power Control of Power Systems

IRFAN KHAN[1], (Student Member, IEEE), YINLIANG XU[2,3], (Member, IEEE), HONGBIN SUN[2,3], (Fellow, IEEE), AND VIKRAM BHATTACHARJEE[1]

[1]Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA
[2]Smart Grid and Renewable Energy Laboratory of Environmental Science and New Energy Technology Center, Tsinghua-Berkeley Shenzhen Institute, Shenzhen 518055, China
[3]Department of Electrical Engineering, State Key Laboratory of Power Systems, Tsinghua University, Beijing 100084, China

Corresponding author: Yinliang Xu (xu.yinliang@sz.tsinghua.edu.cn)

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ABSTRACT To accommodate the increasing penetration level of distributed generators (DGs) in the electrical energy power system, appropriate reactive power control of DGs, which can lead to the voltage profile improvement and power loss minimization, should be addressed. This paper proposed a consensus-based distributed algorithm for the reactive power control of DGs in the power system to optimize the multi-objective function, which includes power loss, voltage deviation, and cost of the reactive power generation of DGs. The formulated problem is proved to be convex. The proposed algorithm is tested on 6- and 34-bus systems to validate its effectiveness and scalability. The proposed algorithm is also compared with the centralized technique particle swarm optimization (PSO), which demonstrates the effectiveness of the proposed distributed algorithm.

INDEX TERMS Reactive power control, distributed optimization, consensus, distributed control, multi-objective optimization.

NOMENCLATURE

W_1, W_2, W_3	Weight coefficients.
K	Iteration count.
P_{loss}	Power loss.
D_V	Voltage deviation.
C_Q	Cost of reactive power generation.
Y_{ij}	Magnitude of the Y_{ij} element from Y bus matrix.
θ_{ij}	Angle of the Y_{ij} from Y bus matrix.
V_i	Voltage magnitudes of *i-th* bus.
δ_j	Voltage phase angle of *j-th* bus.
α_{qi}	Reactive power utilization ratio.
P_{Gi}	Real power generation at the *i-th* bus.
P_{Li}	Real power load at the *i-th* bus.
G_{ii}	Conductance of Y_{ii}.
V_i^*	Reference voltage for the *i-th* bus.
k_i	Cost coefficient for reactive power generation.
Q_{Gi}	Reactive power generation at the *i-th* bus.
Q_{Di}	Load of the *i-th* bus.
Q_i	Scheduled reactive power from the *i-th* bus.

I. INTRODUCTION

Distributed renewable energy sources are being deployed rapidly in the electrical energy power system to reduce carbon emission, lower environmental impact and improve energy diversity [1], [2] and economic dispatch cost [3]. However, integration of DGs poses challenging control issues to the system due to the intermittent nature of the renewable energy sources [4], [5]. One important issue is the bus voltage deviation from the acceptable limits. As a result, prolonged voltage deviation can cause dire consequences to the equipment connected with deviated voltage source [6]. Active power loss is another important issue which should be minimized. Reactive power generation has been widely used to control the voltage of the buses as well as minimize the power loss in the conventional power system [7].

When the active power generation by the inverters of DGs is less than their actual power ratings, the remaining capacity of DGs generation may be used to produce reactive power [8], [9]. The appropriate control of DGs reactive power may result in the improvement of system voltage profile and the loss minimization [10]. However, the optimal injection of reactive power to minimize the power loss and voltage deviation has always been a challenging task, especially for power network with high penetration level of DGs.

Power loss and voltage regulation can be improved in islanded microgrids by uniformly sharing reactive power loads among parallel-connected inverters proportionally. Many power sharing control strategies have been proposed in literature e.g. uniform power sharing among parallel inverters is achieved by selecting controller gains proportionally [11] and a Linear Matrix Inequality (LMI) based decentralized feedback control is designed to achieve power sharing among different generation units [12]. A consensus-based distributed voltage control strategy is proposed in [13] for uniform reactive power sharing in autonomous microgrids with dominantly inductive power lines. Finally, Han *et al.* [14] reviewed and categorize various power sharing control approaches.

The primary objective of the distributed generator is to produce real power generation, thus, if reactive power generation cost is not included in the objective function, generator may generate unnecessary reactive power generation and thus, will deviate from its primary function of providing real power generation. Therefore, it is suggested to include reactive power cost as one of the term in the minimization objective function. Also, some generators have a limit on the reactive power generation. For such generators, if reactive power cost is not included in the minimization objective function, generator may keep on operating at its maximum reactive power capacity even in the steady state and will not have surplus reactive power generation capacity to handle transient conditions.

According to [15], during the holidays, a great amount of capacitive reactive power will be injected from generators in East China Grid, which exceeds the maximum regulation capacity of the voltage control methods in the AC grid. To deal with this problem, one of the possible solution to strictly control the reactive power generation and to avoid its overflow from generators in East China Grid is to include reactive power generation cost in the objective function to be minimized. Researchers in [16] investigated the effect of penetration of solar power generation in the power system considering minimization of power loss as an objective function. Penetration of massive renewable energy generation may produce unnecessary reactive power generation if cost of reactive power generation is not included. To address this issue, the reactive power generation cost along with the power loss and operation cost was minimized considering the wind power generation [17].

MAS are complex systems composed of several autonomous agents with only local knowledge, but are able to interact in order to achieve a global objective [18]. In recent years, consensus-based control theory has been extensively explored in various fields. MAS framework has been adopted by many researchers to describe the communication topology among multi-agents in a physical system. These communication topologies may be different for different physical systems. Some researchers assume that agents can exchange information locally [19] while others may have their own definition of communication topology.

In [20], authors proposed a fully decentralized control algorithm to minimize the power loss of a microgrid only. They pointed out that the performance of the proposed algorithm without communication might deteriorate noticeably compared to the case with communication. Similar decentralized study to minimize power loss for smart power network is proposed in [21]. In [7], the authors proposed a cooperative distributed optimization and control technique using local communication to minimize the voltage deviations within a grid connected microgrid. They proved that minimizing the voltage deviation naturally reduced the power loss. However, the power loss is not directly minimized, and the scalability is also not investigated. In [22], the authors proposed MAS based optimal reactive power control algorithm to minimize the voltage deviation and power loss. It is a generic subgradient control strategy, which requires the information of voltage angle difference between two neighboring buses to calculate the power loss of the system. However, firstly, finding the voltage angle difference requires special techniques [22], which may lead to lengthy calculations. Secondly, considering the features of power system with high penetration level of DGs, the cost of reactive power generation should be considered. Thirdly, it should be emphasized that all the DGs contribute reactive power uniformly based on their capacities such that none is overloaded.

In this paper, a new distributed algorithm based on MAS framework has been proposed to minimize the power loss, voltage deviation and cost of DGs reactive power generation. To address different DGs capacities and realize equal contribution of reactive power generation, reactive power fair utilization ratio has been introduced and the gradients constructed in this paper, are reduced to be more compact. The opportunity cost of reactive power generation is very important to include in the multi-objective function otherwise reactive power generation may decrease the real power output and it may not accommodate the sudden load changes of the power system. According to the proposed algorithm, each DG is assigned with one agent which communicates with its neighboring agents only and updates its local reactive power generation according to simple rules based on consensus algorithm. The major contributions of the proposed distributed control are summarized as follows:

1) A new algorithm based on MAS framework has been proposed to address multi-objective function: the power loss, voltage deviation and the opportunity cost of DGs reactive power generation, simultaneously.
2) The formulated problem has been given the form of a convex function by shaping it as a quadratically constrained quadratic program (QCQP).
3) The proposed algorithm is distributed in nature as information sharing among neighboring buses only is required to optimize the objective function.

4) Consensus algorithm is introduced, which can achieve reactive power fair utilization ratio for all DGs with different capacities.
5) Power system is assumed as fully coupled power system which takes into account effects of voltages and its angles (both local and neighboring) on power loss.

The rest of the paper is organized as follows. Section II describes the problem formulation of multi-objective function for DGs' reactive power control. Section III proves that the formulated problem is convex. Proposed MAS based algorithm design is presented in Section IV. Section V discusses the simulation results and Section VI provides the conclusion.

II. PROBLEM FORMULATION

Optimal reactive power control of DGs plays an important role in power system control and operation, which can lead to improved voltage profile, minimal active power loss and cost of controllable reactive power generation. Therefore, the objective function to be optimized is formulized as the combination of three sub-functions.

$$\min_{Q_i, V_i, \delta_i, \alpha_i} W_1 P_{loss} + W_2 D_V + W_3 C_Q \tag{1}$$

$$\text{s.t. } P_i = V_i \sum_{j=1}^{n} V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \tag{1a}$$

$$Q_i = -V_i \sum_{j=1}^{n} V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \tag{1b}$$

$$\underline{P}_{Gi} \leq P_{Gi} \leq \overline{P}_{Gi} \tag{1c}$$

$$\underline{Q}_{Gi} \leq Q_{Gi} \leq \overline{Q}_{Gi} \tag{1d}$$

$$\underline{V}_i \leq V_i \leq \overline{V_i} \tag{1e}$$

where W_1, W_2 and W_3 are the weight coefficients, which describe the preference of the DGs suppliers. P_{loss}, D_V and C_Q are the power loss, voltage deviation and cost of reactive power generation, respectively. Y_{ij} and θ_{ij} is the magnitude and angle of the Y bus entry, V_i, V_j and δ_i, δ_j are bus voltage magnitudes and angles of i and j, respectively. P_{Gi}, P_{Li} are the generation and the load at bus i. $P_i = P_{Gi} - P_{Li}$ and $Q_i = Q_{Gi} - Q_{Li}$ are the scheduled active/reactive power from bus i, $\underline{P}_{Gi}$, $\overline{P}_{Gi}$, $\underline{Q}_{Gi}$, $\overline{Q}_{Gi}$ and $\underline{V}_i$, $\overline{V_i}$ are the minimum and maximum active power ratings of generator i minimum and maximum reactive power ratings of generator i, and upper and lower ratings of bus voltage of i-th bus respectively.

The objective function given in (1) can be minimized by using the reactive power control of DGs. However, to address different DGs capacities and realize equal contribution of reactive power generation, utilization ratio α_{qi} is defined as:

$$\alpha_{qi} = Q_i/\bar{Q}_i, \quad \text{with } \bar{Q}_i = \sqrt{S_i^2 - P_i^2} \tag{2}$$

where α_{qi} is the reactive power utilization ratio, Q_i and $\bar{Q}_i$ are the present and maximum available reactive power generation, S_i and P_i are the maximum apparent power generation capacity and the real power generation of the DG i, respectively.

Unlike traditional dispatchable generators, renewable energy sources, such as solar or wind which are intermittent in nature, are dynamic. Reactive power bounds of DGs are determined by using power triangle equation (2) where apparent and real power are measured or predicted for renewable sources and reactive power bounds are then calculated. Reactive power fair utilization ratio is included in the formulation of ORPC so that every distributed generator should be equally loaded in terms of reactive power generation. If reactive power fair utilization is not included, some distributed generators may operate at its maximum reactive power generation capability which may not produce any further reactive power to deal with sudden load changes or other disturbances.

The first term of Eqn. (1) is related to the active power loss, which can be derived as [1], [22]:

$$P_{Gi} - P_{Li} - V_i \sum_{j=1}^{n} V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) = 0 \tag{3}$$

Eqn. (3) can be simplified in polar form as

$$P_i = V_i^2 G_{ii} + \sum_{j=1, j\neq i}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \tag{4}$$

where G_{ii} is the conductance of the Y_{ii}. In power system, total active power generation is either consumed by the loads or wasted in the form of losses, hence, the total power loss P_{Loss} can be obtained by taking the summation of Eqn. (4) on both sides for all buses as [23]

$$P_{loss} = \sum_{i=1}^{n} P_{Gi} - \sum_{i=1}^{n} P_{Li} = \sum_{i=1}^{n}\sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \tag{5}$$

Using $\delta_{ji} = \delta_j - \delta_i$, Eqn. (5) can written as Eqn. (6)

$$P_{loss} = \sum_{i=1}^{n}\sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_{ji}) \tag{6}$$

The second term of the objective function, which is the voltage deviation between voltage magnitude and its reference

$$D_V = \sum_{i=1}^{n} (V_i - V_i^*)^2 \tag{7}$$

where V_i^* is the reference voltage for bus i.

The third term of the objective function, the cost of reactive power generation, is simply approximated as [24], [25]:

$$C_Q = k_i Q_i^2 \tag{8}$$

where k_i is the cost coefficient for reactive power generation.

To optimize (1), a consensus based gradient approach is proposed which requires the gradient of the objective function. The gradient of the objective function *w.r.t* state variable α_{qi} can be determined as follows

$$\frac{\partial f}{\partial \alpha_{qi}} = W_1 \frac{\partial P_{loss}}{\partial \alpha_{qi}} + W_2 \frac{\partial D_V}{\partial \alpha_{qi}} + W_3 \frac{\partial C_Q}{\partial \alpha_{qi}} \tag{9}$$

Using the chain rule for the partial derivative, gradient for power loss can be expanded as

$$\frac{\partial P_{loss}}{\partial \alpha_{qi}} = W_1(\frac{\partial P_{loss}}{\partial V_i}\frac{\partial V_i}{\partial Q_i} + \sum_{j\in N_i}\frac{\partial P_{loss}}{\partial V_j}\frac{\partial V_j}{\partial Q_i} + \frac{\partial P_{loss}}{\partial \delta_i}\frac{\partial \delta_i}{\partial Q_i} + \sum_{j\in N_i}\frac{\partial P_{loss}}{\partial \delta_j}\frac{\partial \delta_j}{\partial Q_i})\frac{\partial Q_i}{\partial \alpha_{qi}}. \tag{10}$$

It should be emphasized here that usually in AC power flow, the effect of V_i on P_i and δ_i on Q_i is ignored. Known as decoupled power flow, it is a very strong assumption and it cannot be ignored especially when the resistance of the transmission line is not negligible [26].

As shown in Eqn. (10), the gradient of power loss w.r.t α_{qi} can be calculated as the sum of four terms, where the first term is determined as follows

$$\frac{\partial P_{loss}}{\partial V_i} = \frac{\partial}{\partial V_i}\sum_{j=1}^{n}\sum_{k=1}^{n} V_j V_k Y_{jk} cos(\theta_{jk} + \delta_{kj}) \tag{11}$$

There are three possible combinations for Eqn. (11) [22].

$$\frac{\partial P_{Loss}}{\partial V_i} = \begin{cases} 2V_i G_{ii} & for\ j = i, k = i \\ \sum_{k=1,k\neq i}^{n} V_k Y_{ik} cos(\theta_{ik} + \delta_{ki}) & for\ j = i, k \neq i \\ \sum_{j=1,j\neq i}^{n} V_j Y_{ji} cos(\theta_{ji} + \delta_{ij}) & for\ k = i, j \neq i \\ 0 & otherwise \end{cases} \tag{12}$$

Eqn. (12) can be further simplified as Eqn. (13)

$$\begin{aligned} \frac{\partial P_{loss}}{\partial V_i} &= 2V_i G_{ii} + \sum_{k=1,k\neq i}^{n} V_k Y_{ik} cos(\theta_{ik} + \delta_{ki}) \\ &\quad + \sum_{j=1,j\neq i}^{n} V_j Y_{ji} cos(\theta_{ji} + \delta_{ij}) \\ &= 2V_i G_{ii} + 2\sum_{j=1,j\neq i}^{n} V_j G_{ij} cos(\theta_{ij} + \delta_{ji}) \\ &= 2\sum_{j=1}^{n} V_j Y_{ij} cos(\theta_{ij} + \delta_{ji}) \end{aligned} \tag{13}$$

According to Eq. (4), multiplying and dividing by V_i on both nominator and denominator of right side of Eqn. (13) makes

$$\partial P_{loss}/\partial V_i = 2P_i/V_i. \tag{14}$$

$\partial V_i/\partial Q_i$ can be derived from the following [7].

$$\begin{aligned} Q_i = Q_{Gi} - Q_{Di} &= -\sum_{j=1}^{n} V_i V_j Y_{ij}\sin(\theta_{ij} + \delta_{ji}) \\ &= -V_i^2 B_{ii} - \sum_{j=1,i\neq j}^{n} V_i V_j Y_{ij}\sin(\theta_{ij} + \delta_{ji}) \end{aligned} \tag{15}$$

where Q_{Gi}, Q_{Di} are the reactive power generation and load at bus i, and Q_i is the scheduled reactive power from bus i, Eqn. (15) can be expanded as

$$\frac{\partial Q_i}{\partial V_i} = -2V_i B_{ii} - \sum_{j=1,j\neq i}^{n} V_j Y_{ij}\sin(\theta_{ij} + \delta_{ji}). \tag{16}$$

The right hand side of Eqn. (16) can be easily rewritten as

$$\frac{\partial Q_i}{\partial V_i} = \frac{-2V_i^2 B_{ii} - V_i\sum_{j=1,j\neq i}^{n} V_j Y_{ij}\sin(\theta_{ij} + \delta_{ji})}{V_i}. \tag{17}$$

The nominator on the right hand side of Eqn. (17) can be replaced by Q_i, according to Eqn. (15).

$$\frac{\partial Q_i}{\partial V_i} = \frac{Q_i}{V_i} - V_i B_{ii}. \tag{18}$$

For second term of gradient of power, as P_{loss} is calculated for the whole system, $\partial P_{loss}/\partial V_j$ will have the same relation as $\partial P_{loss}/\partial V_i$ but j will have different neighbors than i

$$\frac{\partial P_{loss}}{\partial V_j} = 2\sum_{k=1}^{n} V_k Y_{jk} cos(\theta_{jk} + \delta_{kj}). \tag{19}$$

According to Eqn. (4), multiplying and dividing by V_j on both nominator and denominator of right side Eqn. (19), makes

$$\partial P_{loss}/\partial V_j = 2P_j/V_j. \tag{20}$$

Similarly, $\partial Q_i/\partial V_j$ can be determined as follows

$$\frac{\partial Q_i}{\partial V_j} = -V_i Y_{ij}\sin(\theta_{ij} + \delta_{ji}). \tag{21}$$

The second term of gradient of power loss with respect to voltage becomes as

$$\sum_{j\in N_i}\frac{\partial P_{loss}}{\partial V_j}\frac{\partial V_j}{\partial Q_i} = \sum_{j\in N_i}\frac{2P_j}{-V_i V_j Y_{ij}\sin(\theta_{ij} + \delta_{ji})}. \tag{22}$$

Next, the third term can be determined as follows

$$\frac{\partial P_{loss}}{\partial \delta_i} = \begin{cases} 0 & if\ j = i, k = i \\ \sum_{k=1}^{n} V_i V_k Y_{ik}\sin(\theta_{ik} + \delta_{ki}) & if\ j = i, k \neq i \\ -\sum_{j=1}^{n} V_j V_i Y_{ji}\sin(\theta_{ji} + \delta_{ij}) & if\ k = i, j \neq i \end{cases} \tag{23}$$

Eqn. (23) can be further simplified as

$$\begin{aligned} \frac{\partial P_{loss}}{\partial \delta_i} &= \sum_{k=1,k\neq i}^{n} V_i V_k Y_{ik}\sin(\theta_{ik} + \delta_{ki}) \\ &\quad - \sum_{j=1,j\neq i}^{n} V_j V_i Y_{ji}\sin(\theta_{ji} + \delta_{ij}) \\ &= 2\sum_{j=1,j\neq i}^{n} V_i V_j Y_{ij}\sin(\theta_{ij} + \delta_{ji}) \end{aligned} \tag{24}$$

According to Eqns. (15), Eqn. (24) is rewritten as

$$\partial P_{loss}/\partial \delta_i = -2(Q_i + V_i^2 B_{ii}). \tag{25}$$

Now, $\partial\delta_i/\partial Q_i$ can be calculated from Q_i

$$\frac{\partial Q_i}{\partial \delta_i} = \begin{cases} 0 & if\ j = i \\ \sum_{j=1, j\neq i}^{n} V_i V_j Y_{ij}\cos(\theta_{ij}+\delta_{ji}) & otherwise \end{cases} \tag{26}$$

According to Eqn. (4), Eqn. (26) becomes

$$\partial Q_i/\partial\delta_i = P_i - V_i^2 G_{ii}. \tag{27}$$

Last term of gradient of power loss can be determined as follows

$$\frac{\partial P_{loss}}{\partial \delta_j} = 2\sum_{k=1,k\neq j}^{n} V_j V_k Y_{jk}\sin(\theta_{jk}+\delta_{kj}) \tag{28}$$

According to Eqn. (15), Eqn. (28) becomes as

$$\partial P_{loss}/\partial\delta_j = -2(Q_j + V_j^2 B_{jj}). \tag{29}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -V_i V_j Y_{ij}\cos(\theta_{ij}+\delta_{ji}) \tag{30}$$

Thus, last term of gradient of power loss will be

$$\sum_{j\in N_i}\frac{P_{loss}}{\partial\delta_j}\frac{\partial\delta_j}{\partial Q_i} = 2\sum_{j\in N_i}\frac{Q_j + V_j^2 B_{jj}}{V_i V_j Y_{ij}\cos(\theta_{ij}+\delta_{ji})} \tag{31}$$

$\partial Q_i/\partial\alpha_{qi}$ can be derived from Eqn. (2) as

$$\frac{\partial Q_i}{\partial \alpha_{qi}} = \bar{Q}_i. \tag{32}$$

The derivative of D_V *w.r.t* state variable α_{qi} can be determined as follows

$$\frac{\partial D_V}{\partial \alpha_{qi}} = W_2(\frac{\partial D_V}{\partial V_i}\frac{\partial V_i}{\partial Q_i}\frac{\partial Q_i}{\partial \alpha_{qi}}) \tag{33}$$

where $\partial D_V/\partial V_i$ is determined as

$$\frac{\partial D_V}{\partial V_i} = 2(V_i - V_i^*). \tag{34}$$

$\partial V_i/\partial Q_i$ and $\partial Q_i/\partial\alpha_{qi}$ can be obtained from Eqn. (18) and (32), respectively. Thus, the gradient of voltage deviation is given as

$$\begin{aligned}\frac{\partial D_V}{\partial \alpha_{qi}} &= 2W_2(V_i - V_i^*)(\frac{V_i}{Q_i - V_i^2 B_{ii}})\bar{Q}_i \\ &= \frac{2W_2 V_i \bar{Q}_i (V_i - V_i^*)}{Q_i - V_i^2 B_{ii}}\end{aligned} \tag{35}$$

The derivative of the cost w.r.t. α_{qi} is calculated as

$$\frac{\partial C_Q}{\partial \alpha_{qi}} = W_3\frac{\partial C_Q}{\partial Q_i}\frac{\partial Q_i}{\partial \alpha_{qi}} = 2W_3 k_i Q_i \bar{Q}_i. \tag{36}$$

Finally, all the terms in Eqn. (9) are known, and Eqn. (9) can be obtained as

$$\begin{aligned}\frac{\partial f}{\partial \alpha_{qi}} &= 2W_1\bar{Q}_i\Bigg\{\frac{P_i}{Q_i - V_i^2 B_{ii}} + \sum_{j\in N_i}\frac{P_j}{-V_i V_j Y_{ij}\sin(\theta_{ij}+\delta_{ji})} \\ &\quad -\frac{Q_i + V_i^2 B_{ii}}{P_i - V_i^2 G_{ii}} + \sum_{j\in N_i}\frac{Q_j + V_j^2 B_{jj}}{V_i V_j Y_{ij}\cos(\theta_{ij}+\delta_{ji})}\Bigg\} \\ &\quad + \frac{2W_2 V_i \bar{Q}_i (V_i - V_i^*)}{Q_i - V_i^2 B_{ii}} + 2W_3 k_i Q_i \bar{Q}.\end{aligned} \tag{37}$$

It is worthy to note that only admittance of the transmission line connecting two buses and local information, such as bus voltage, voltage reference, reactive power capacity, are required to calculate the gradient. No global parameter of the power system is required.

III. CONVEXITY ANALYSIS

To express problem (1) as a quadratic cone quadratic programming (QCQP), constraints (1a) and (1b), jointly, can be reformulated as Branch Injection Model (BIM) of power flow, defined by Kirchhoff's laws as given as [27]

$$\vec{S}_i = \sum_{j=1}^{n}\vec{Y}_{ij}^H\vec{V}_i(\vec{V}_i^H - \vec{V}_j^H) \tag{38}$$

where S represents the apparent power, $\rightarrow$ represents the complex quantity and H represents its Hermitian. Eqn. (38) can be rewritten in terms of current injection from bus i as:

$$\vec{S}_i = \vec{V}_i\vec{I}_i^H = (e_i^H\vec{V})(\vec{I}^H e_i) \tag{39}$$

where e_i is the n-dimensional vector with 1 in the i^{th} entry and 0 elsewhere. Here, $\vec{I} = \vec{Y}\vec{V}$, hence, Eqn. (39) can be rewritten as:

$$\vec{S}_i = tr(e_i^H\vec{V}\vec{V}^H\vec{Y}^H e_i) = tr(\vec{Y}^H e_i e_i^H)\vec{V}\vec{V}^H = \vec{V}^H\vec{Y}_i^H\vec{V} \tag{40}$$

where $\vec{Y}_i := e_i e_i^H\vec{Y}$ is an $n\times n$ matrix with its i-th row equal to i-th row of $\vec{Y}$ matrix and all other rows equal to zero. Hermitian and skew Hermitian components of $\vec{Y}_i^H$ and $\vec{Y}^H$ are defined as:

$$\begin{aligned}\Phi_i &= \frac{1}{2}(\vec{Y}_i^H + \vec{Y}_i) \text{ and } \Psi_i = \frac{1}{2}(\vec{Y}_i^H - \vec{Y}_i) \\ \Phi &= \frac{1}{2}(\vec{Y}^H + \vec{Y}) \text{ and } \Psi = \frac{1}{2}(\vec{Y}^H - \vec{Y})\end{aligned} \tag{41}$$

Using (41), (1a) and (1b) can be reformulated as:

$$P_i = \vec{V}^H\Phi_i\vec{V} \quad \text{and } Q_i = \vec{V}^H\Psi_i\vec{V} \tag{42}$$

Let $J_i := e_i e_i^H$ denote the Hermitian matrix with a single 1 at $(i,i)^{th}$ entry and 0 elsewhere, problem formulation (1) can be written as QCQP:

$$\begin{aligned}&\vec{V}^H\left[w_1\Phi + w_2 I + w_3\Psi\right]\vec{V} - 2w_2 {V^*}^H\vec{V} \\ &\quad -1^H P_D + w_2 {V^*}^H V^* \\ &\quad s.t.\ P_i = \vec{V}^H\Phi_i\vec{V};\quad Q_i = \vec{V}^H\Psi_i\vec{V} \\ &\qquad \vec{V}^H\Phi_i\vec{V} \le \bar{P}_i;\quad \vec{V}^H(-\Phi_i)\vec{V} \le -\underline{P}_i \\ &\qquad \vec{V}^H\Psi_i\vec{V} \le \bar{Q}_i;\quad \vec{V}^H(-\Psi_i)\vec{V} \le -\underline{Q}_i \\ &\qquad \vec{V}^H J_i\vec{V} \le \bar{V}_i;\quad \vec{V}^H(-J_i)\vec{V} \le -\underline{V}_i.\end{aligned} \tag{43}$$

Even though different sets of equations define the optimization model (1) and (43) in terms of their own variables, yet both are models of the Kirchhoff's laws and both mathematical models are equivalent in the precise sense, which means any result in one model is derivable in the other [27]. In this paper, we expressed the OPF problem as QCQP using BIM. In the similar work, some other authors have attempted to express their optimization model as Semidefinite Programming (SDP) [28] or Second Order Cone Problem (SOCP) [29], depending on their own applications.

IV. PROPOSED DISTRIBUTED ALGORITHM

Each bus is equipped with a bus agent (BA) responsible for obtaining the local measurement of four state variables of P_i, Q_i, $V_i\&\delta_i$ and exchanging the information with its neighboring agents. The communication network for the MAS framework is designed in such a way that two neighboring agents (NA) communicate with each other only if their corresponding buses are electrically coupled. In addition, each DG bus is attached with a Reactive Power Control Agent (RPCA) which receives information from BA and decide its control output. Communication of state variables among various buses is graphically explained in Fig. 1 and pseudo code of the control algorithm is described in Table.1 The stability proof of the auxiliary voltage controller is similar to (27)-(36), which has been abbreviated here.

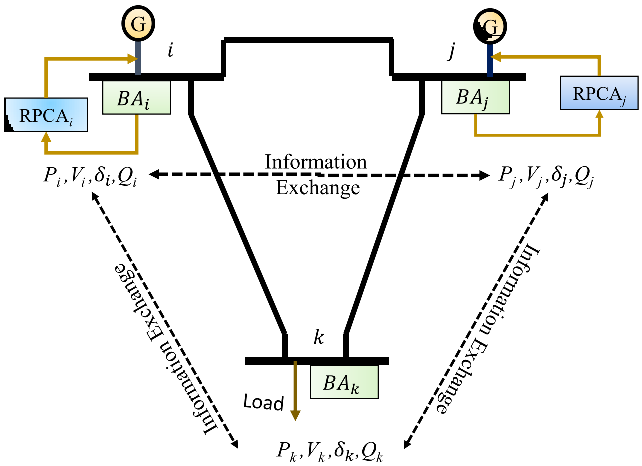


FIGURE 1. Scheme of the communications among agents.

TABLE 1. Power line data for the 6-bus system.

From	To	R(p.u.)	X(p.u.)	From	To	R(p.u.)	X(p.u.)
1	4	0.05	0.095	2	6	0.07	0.115
1	5	0.08	0.125	3	5	0.12	0.165
2	5	0.1	0.145				

The reactive power utilization ratio of every bus in the power system is discovered iteratively as follows

$$\alpha_i[k+1] = \sum d_{ij}\alpha_i[k] - \varepsilon\frac{\partial f}{\partial \alpha_{qi}} \tag{44}$$

where ε is the step size that can be adjusted to control the converging speed, d_{ij} is designed as [30].

$$d_{ij} = \begin{cases} 2/(n_i+n_j+1) & j \in N_i \\ 1 - \sum_{j\in N_i} 2/(n_i+n_j+1) & i = j \\ 0 & otherwise \end{cases} \tag{45}$$

Algorithm 1 Pseudo Code Describing the Implementation of the Proposed Algorithm

Step I: Initialization

a. Initialize variables at flat start:
$V_i[0]$, $V_j[0]$, $Q_i[0]$, $\bar{Q}_i[0]$, $P_i[0]$ are taken from local measurement and prediction.

b. Initialize Utilization Ratio
$\alpha_i[0] = Q_i[0]/\bar{Q}_i[0]$

Step II: Update Process

Calculate utilization ratio and DGs reactive power generation update procedure following steps (1)-(7)

1) $\alpha_i[k+1] = \sum d_{ij}\alpha_i[k] - \varepsilon(\partial f/\partial \alpha_{qi})$
2) $Q_i[k+1] = \alpha_i[k+1]\bar{Q}_i$
3) *if* $Q_i[k+1] \geq \bar{Q}_i$ *then* $Q_i[k+1] = \bar{Q}_i$
   if $Q_i[k+1] \leq \underline{Q}_i$ *then* $Q_i[k+1] = \underline{Q}_i$
4) Implement the control update of $Q_i[k+1]$
5) Measure $V_i[k+1]$ and $\delta_{ij}[k+1]$ and $f[k+1]$ as in Eqn. (1)
6) If f is not real, change Q_i to adjust line flows
7) Calculate $\partial f/\partial \alpha_{qi}$ according to Eqn.(37)
8) Go to step 1)

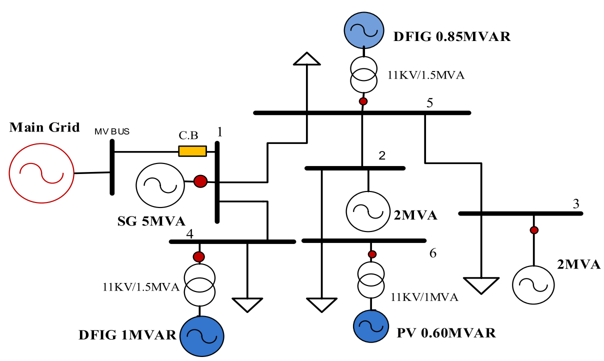


FIGURE 2. Six-bus radial power network.

where n_i and n_j are the numbers of agents connected to agents i and j, respectively. N_i denotes neighboring agent set of agent i

The utilization ratio will be updated iteratively for a given number of iterations, which is predetermined by the size and connectivity of the power system, until an optimal solution of the objective function is achieved.

V. SIMULATION STUDIES

In this section, several case studies are presented to exhibit the effectiveness of the proposed distributed control algorithm.

A. CASE STUDY 1: 6-BUS SYSTEM

The proposed algorithm is applied to 6-bus radial power network as shown in Fig. 2, in which bus 1 is a slack bus, attached to the main grid. Three DGs with reactive power ranges -0.30 to 0.56 MVAR, -0.45 to 0.75 MVAR and -0.35 to 0.60 MVAR are attached at bus 4, 5 and 6 respectively. Reactive power generation from these DGs is utilized to minimize the power loss and voltage deviation of the system, taking the reactive power generation cost into account simultaneously. Reference voltages for 6 buses is assigned as 1.04, 1.03, 1, 1, 1, and 1 in an ascending order.

Usually, weights for multi-objective function are determined by specific applications and the method of finding weights may vary for each specific problem. In our paper, three sub-functions are determined in different units: power loss and voltage deviation are determined in p.u. while reactive power cost is calculated in dollars. In order to give equal contributions to each of the three sub-functions, reactive power cost weight coefficient is divided by 10^2 to convert it into p.u. As voltage deviation is formulated by taking the square of voltage difference, its value will be of the order 10^2. Therefore, weight coefficient to the voltage deviation is given as of the order of 10^2 to give equal contributions from each sub-function. Three sub-objective functions: power loss, voltage deviation and reactive power cost are weighted as 3.5, 150 and 0.28 respectively. However, the weight coefficients can be changed to any value depending on DGs suppliers' preferences.

In the first scenario, the gradient of power loss is calculated including $\cos(\delta_{ij})$ and results are shown in shown in Fig. 3. In the second scenario, the gradient of power loss is simplified by ignoring the bus angle difference between two neighboring buses as shown in Fig. 4. Both Figs. 3 (a-d) & 4(a-d) exhibit the update of objective functions, reactive power generation, fair utilization ratio and voltage of the non-PV bus buses, respectively. To test the effectiveness of the proposed algorithm, an event sequence of load demand variation is introduced in the existing power system as given in Table. II.

TABLE 2. Event sequences of load changes on 6-bus system.

Iteration No.	Event No.	Bus No.	Load Type	Load Change
25	Event 1	[4, 5, 6]	React. Load	1.2×Initial
50	Event 2	[3, 4, 5, 6]	Real Load	1.05×Even1
75	Event 3	[4, 5, 6]	React. Load	0.80×Even2
100	Event 4	[3, 4, 5, 6]	Real Load	0.95×Even3

The objective function, along with its individual sub-functions, converges to their optimal value before 10^{th} iteration. When reactive power load demand rises at 25^{th} iteration, it is compensated by the reactive power generation sources at bus 4, 5 and 6th buses as evident from Fig. 3(b & c). Similarly, change in real power load and decrease in reactive power load is counteracted by their respective generators.

It is important to note that proposed algorithm strives to attain the reference voltages of the load buses immediately after the abrupt load changes take place on the energy system as shown in Fig. 3(d). Another important observation is to spot the behavior of the power loss in case of abrupt real power load changes. Increase/decrease of real power demand means flow of higher/lower current through lines, which increases/decreases the power loss on the lines as evident from Fig. 3(a). As included in the chapter II of formulation, each DG should contribute reactive power uniformly based upon its available capacity. Thus, Fig. 3(c) manifests that the

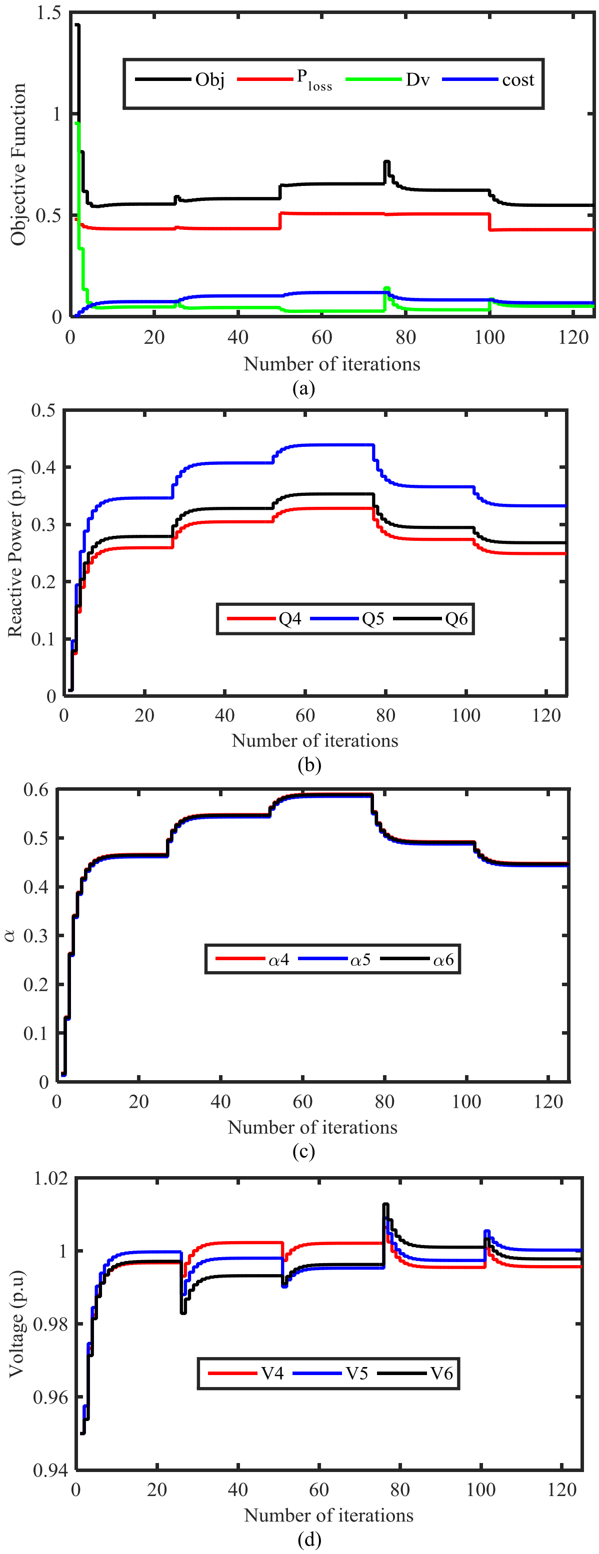


FIGURE 3. Updates of the proposed algorithm for 6-bus system using real bus angles in the power loss function. (a) Total objective and sub-objective function profiles. (b) Reactive power generation update. (c) Utilization ratio update. (d) Updates of improved voltage profile.

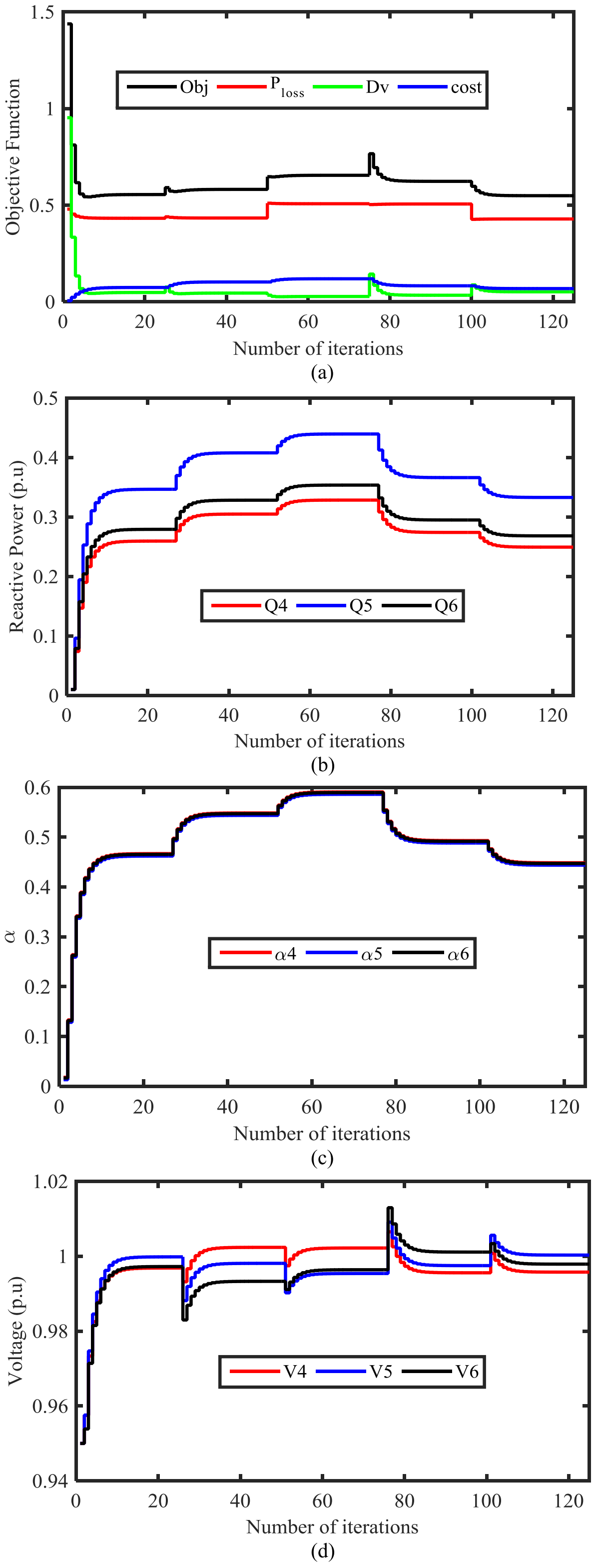


FIGURE 4. Updates of the proposed algorithm for 6-bus system using voltage angle approximation. (a) Total objective and sub-objective function profiles. (b) Reactive power generation update. (c) Utilization ratio update. (d) Updates of improved voltage profile.

reactive power utilization ratio (α) converges to a common value of 0.47 for every DG till 25th iteration, before any load change occurs.

Fig. 4 shows the simulation results of the second scenario: the angle differences between two neighboring buses are approximated to zero. Comparing of Fig.3 and Fig.4 shows that no significant difference between the two graphs is observed. After analyzing both the results closely, it is found that with approximation of $\cos(\delta_{ij}) = 1$, it conforms to the original value with a small error of only 0.22%. According to [22], special techniques are required to obtain the information of angle differences, which may make the online application very complicated and computationally expensive. From authors 'work, it is concurred that making approximation of $\cos(\delta_{ij}) = 1$ is a valid approximation and angle difference may be approximated to zero when the system is operating in the steady state condition.

To authenticate the efficacy of the proposed distributed algorithm, it is compared with centralized control technique of PSO [31]. The adopted PSO uses 20 particles and converges at 28th iteration, which means 560 calculations in total. It is noticed that results obtained from the both approaches are identical. Table III is provided to compare the values of reactive power generations, voltage magnitudes, power loss and objective function for the proposed algorithm and the centralized approach.

TABLE 3. Simulation result summary for 6-bus system.

Variables	Distributed	Centralized PSO
Q_4	0.2596	0.2589
Q_5	0.2793	0.2801
Q_6	0.3466	0.3470
V_1	1.030	1.030
V_2	1.040	1.040
V_3	1.000	1.000
V_4	1.000	1.006
V_5	0.998	0.992
V_6	0.997	1.001
Power Loss	0.4323	0.4333
Objective Function	0.5446	0.5483

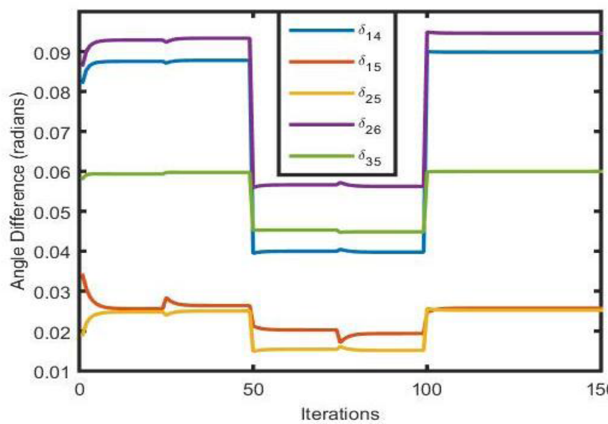


FIGURE 5. Voltage angle difference of all lines in 6-bus power network.

Voltage angle differences of lines in the 6-bus radial system is shown in Fig. 5. The angle difference changes during the load changing condition and gets restored to its previous state immediately when the external loading is removed.

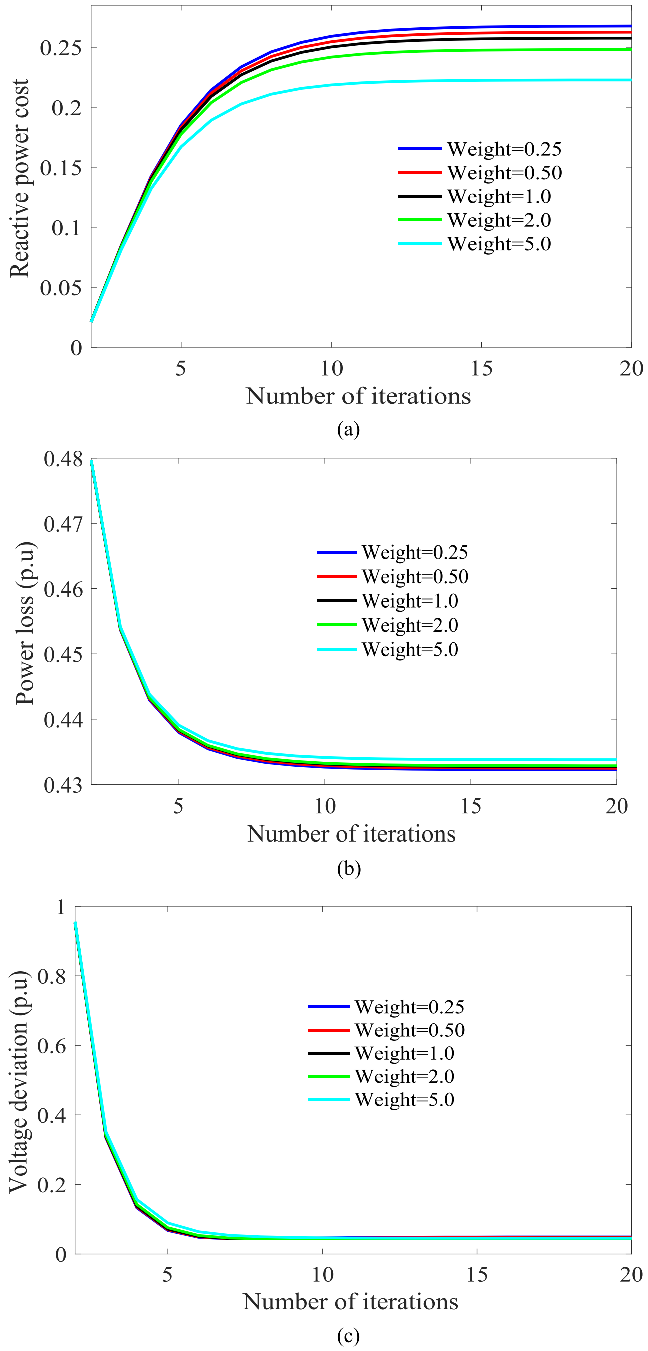


FIGURE 6. Results for the 6-bus system with different weight coefficients of reactive power generation cost. (a) Reactive power generation cost profiles for the 6-bus system with different weights of reactive power generation cost. (b) Power loss profiles for the 6-bus system with different weights of reactive power generation cost. (c) Total voltage deviation profiles for the 6-bus system with different weights of reactive power generation cost.

Fig. 5 shows that voltage angles changes much more in case of real power load change than that of reactive power load change. It is also clear from the magnitude of angle differences is close to zero and has very limited effect on the optimal solution of the proposed distributed algorithm. Thus, angle difference can be approximated to zero, which can reduce the computational burden of the controller and make the algorithm fast. Measuring the angle difference slows down the convergence rate of the algorithms and incurs an additional computational cost on the controller. It is clear from the graph that angle difference is small and the proposed algorithm may not require its measurement for decision of the control action. This substantiates the assumption of considering $\cos(\delta_{ij}) = 1$ and validates the claim that the algorithm can provide comparable results to the one without an approximation.

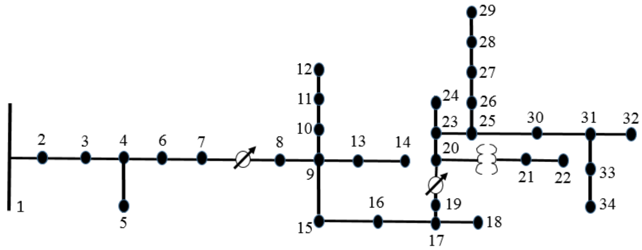


FIGURE 7. Schematic diagram of the 34-bus system.

B. DISCUSSION OF THE REACTIVE POWER GENERATION COST

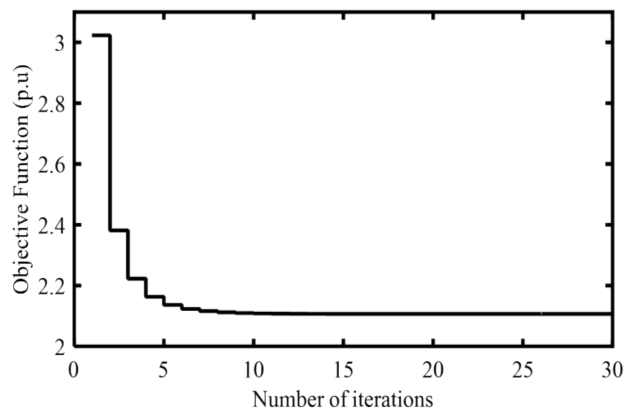
To analyze the importance of including reactive power generation cost, numerical results with different weights of the reactive power generation cost for the 6-bus system are presented Figs. 6(a)-(c) show the profiles for reactive power generation cost, power loss and voltage deviation without multiplying the weight coefficients, respectively. It shows that by increasing the weight coefficient for reactive power cost generation, reactive power generation becomes expensive and thus the proposed algorithm reduces the reactive power generation to minimize the overall objective function as evident from Fig. 6(a). Higher is the weight coefficient, lower is the total reactive power generation cost. However, on the other hand, effect of increasing reactive power cost coefficient may not change the power loss and voltage deviation significantly. Figs. 6(b), (c) shows that power loss and voltage deviation converge to almost the same value. Thus, it is recommended to include reactive power generation cost in the objective function for efficient, reliable and cost-effective operation of power system.

C. CASE STUDY 2: 34-BUS SYSTEM

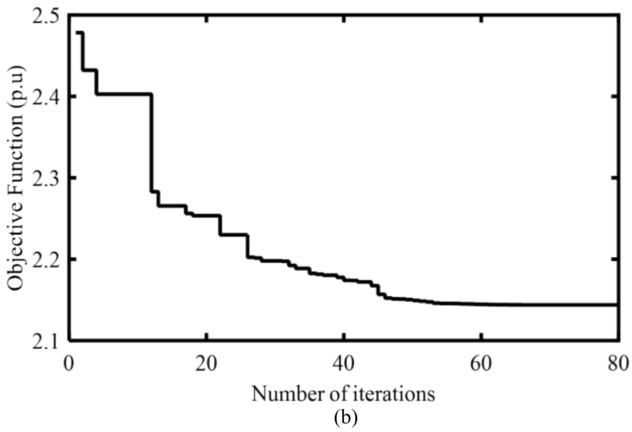
A 34-bus radial power network [32] with the topology and line data as given in Fig.7 and Table IV respectively, is being utilized for validation of the proposed algorithm. 9 DGs for reactive power control are connected at bus 4, 8, 12, 16, 20, 21, 23, 27 and 28, whose reactive power generation capacities are predicted as 0.6, 0.75, 0.75, 0.675, 0.9, 0.675, 0.75, 0.85, and 0.95 MVAR, respectively. DGs attached at bus 4, 8, 20, 23 and 27 are PV generators while DGs attached at bus 12, 16, 21 and 28 are DFIGs. Reference voltages are taken as given in [32]. Cost coefficient (k_i) for reactive power generation is set to 1 whereas coefficients for power loss, voltage

TABLE 4. Power line data for the 34-bus system.

From	To	R(p.u.)	X(p.u.)	From	To	R(p.u.)	X(p.u.)
1	2	0.117	0.1085	17	19	0.2079	0.1994
2	3	0.10725	0.09875	19	20	0.189	0.1805
3	4	0.16445	0.15595	20	21	0.189	0.1805
4	5	0.1495	0.141	21	22	0.262	0.2535
4	6	0.1495	0.141	20	23	0.262	0.2535
6	7	0.3144	0.3059	23	24	0.3144	0.3059
7	8	0.2096	0.2011	23	25	0.2096	0.2011
8	9	0.3144	0.3059	25	26	0.131	0.1225
9	10	0.2096	0.2011	26	27	0.1048	0.0963
10	11	0.131	0.1225	27	28	0.1572	0.1487
11	12	0.10148	0.09298	28	29	0.1572	0.1487
9	13	0.1572	0.1487	25	30	0.1572	0.1487
13	14	0.2096	0.2011	30	31	0.1572	0.1487
13	15	0.1048	0.0963	31	32	0.2096	0.2011
15	16	0.0524	0.0439	31	33	0.1572	0.1487
16	17	0.1794	0.1709	33	34	0.1048	0.0963
17	18	0.16445	0.15595	-	-	-	-



(a)



(b)

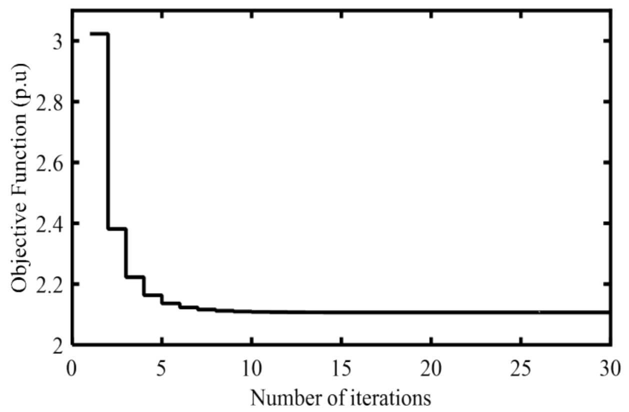
FIGURE 8. Convergence comparison for 34-bus system. (a). The proposed distributed algorithm. (b). Centralized algorithm.

deviation and reactive power cost are chosen as 2, 100 and 0.1 respectively.

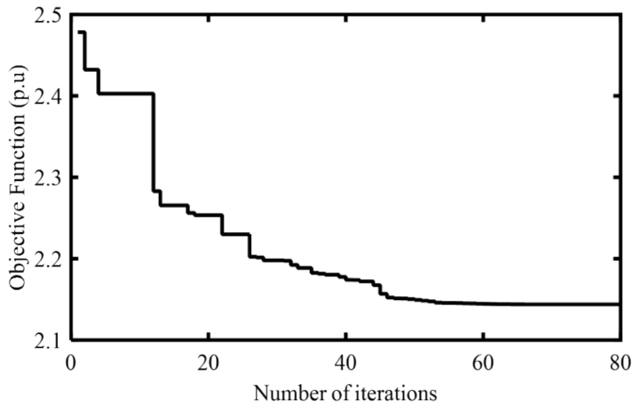
Results for the 34-bus system show that the proposed algorithm is faster than the centralized approach as visible in Figs. 8(a) & (b). It exhibits that the distributed algorithm achieves the optimal solution, equal to the centralized approach but within much less time. The detailed comparison between the proposed distributed algorithm and the centralized approach is provided in Table V, which shows that

TABLE 5. Summary for reactive power control result for 34-Bus System.

Variables	Distributed	Centralized PSO	Maximum Q
Q_4	0.510	0.514	0.600
Q_8	0.637	0.640	0.750
Q_{12}	0.637	0.637	0.750
Q_{16}	0.574	0.572	0.675
Q_{20}	0.765	0.770	0.900
Q_{21}	0.574	0.570	0.675
Q_{23}	0.638	0.638	0.750
Q_{27}	0.720	0.720	0.850
Q_{28}	0.797	0.800	0.950
Power Loss	1.7982	1.8020	--
Objective Function	2.107	2.109	--



(a)



(b)

FIGURE 9. Convergence update for 34-bus system. (a). Reactive power generation update. (b). Utilization ratio update.

power loss, objective function and reactive power updates. Figs. 9 (a) & (b) reveal the convergence of the reactive power generations for DGs and the fair utilization ratio converges α, respectively. Fair utilization ratio attains a ratio of 0.85 for each DG, which means every DG is 85% of the maximum generating reactive power generation capability.

VI. CONCLUSION

This paper proposes a distributed consensus-based optimal reactive power control algorithm for multiple DGs in a power system. The effectiveness of the proposed distributed algorithm is validated by comparing to the centralized algorithm:

PSO, for 6-bus and 34-bus systems. The proposed algorithm achieves the following four main merits.

1) The proposed distributed algorithm effectively minimizes the objective function consisted of active power loss, voltage deviation and reactive power generation opportunity cost. Only information exchange among neighboring buses is used to attain the optimal solution.
2) It has been proved that making the approximation of $\cos(\delta_{ij}) = 1$ can provide comparable results to the one without approximation, which simplifies the calculation at the cost of only a few more iterations.
3) Fair utilization ratio of reactive power generation is achieved for uniform utilization of all DGs.
4) The proposed algorithm is scalable in the sense that, the iteration number does not increase exponentially as the size of the system increases, as validated in the 34-bus system.

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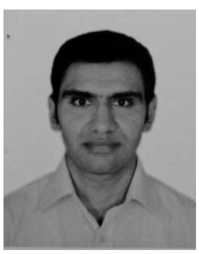

IRFAN KHAN (S'16) received the B.Sc. degree in electrical engineering from the University of Engineering and Technology, Lahore, Pakistan, in 2009, and the M.Sc. degree in electrical power engineering from the University of Greenwich, London, U.K., in 2011.

He is currently pursuing the Ph.D. degree in electrical and computer engineering from Carnegie Mellon University, Pittsburgh, PA, USA. His research interests include control and optimization of smart energy networks.

YINLIANG XU (M'13) received the B.S. and M.S. degrees in control science and engineering from the Harbin Institute of Technology, China, in 2007 and 2009, respectively, and the Ph.D. degree in electrical and computer engineering from New Mexico State University, Las Cruces, NM, USA, in 2013.

He was with the School of Electronics and Information Technology, Sun Yat-sen University. From 2013 to 2014, he was a Visiting Scholar with the Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA, where he is currently an Adjunct Faculty Member with the Department of Electrical and Computer Engineering. He is also an Assistant Professor with the Laboratory of Smart Grid and Renewable Energy, Tsinghua-Berkeley Shenzhen Institute, Shenzhen, Guangdong, China. His research interests include distributed control and optimization of power systems, renewable energy integration, and microgrid modeling and control.

HONGBIN SUN (SM'12–F'17) received the double B.S. degrees from Tsinghua University in 1992 and the Ph.D. degree from the Department of E.E., Tsinghua University in 1996.

From 2007 to 2008, he was a Visiting Professor with the School of EECS, Washington State University, Pullman. He is currently the ChangJiang Scholar Chair Professor with the Department of E.E. and the Director of the Energy Management and Control Research Center, Tsinghua University. He is currently an IET Fellow. He also serves as the Editor for the IEEE TSG, an Associate Editor for the IET RPG, and a member of the Editorial Board of four international journals and several Chinese journals. His technical areas include electric power system operation and control with specific interests on the energy management system, system-wide automatic voltage control, and energy system integration. He has authored over 400 peer-reviewed papers, within which over 60 are IEEE and IET journal papers, and four books. He has been authorized five U.S. patents of invention and over 100 Chinese patents of invention.

VIKRAM BHATTACHARJEE received the B.Tech. degree in chemical engineering from Jadavpur University, India, in 2015. He is currently pursuing the M.S. degree in electrical and computer engineering with Carnegie Mellon University, Pittsburgh, PA, USA. His research interests include control and optimization in energy storage and power systems.