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A Time-Delayed Multi-Master-Single-Slave Non-Linear Tele-Robotic System Through State Convergence

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ABSTRACT This paper presents the design of a multi-master–single-slave nonlinear tele-robotic system working in the presence of time varying delays. The structure of the proposed tele-robotic system is derived from the extended state convergence architecture and the control objective is defined as the position regulation of the slave manipulator. The desired reference value for the slave manipulator is set by the master systems according to their authority levels. To ensure that the tele-robotic system remains stable in the presence of time varying delays and the control objective is also achieved, Lyapunov-based stability analysis is carried out which results in certain guidelines to be followed for the selection of the control gains. In order to check the validity of the proposed scheme, MATLAB simulations are performed on a two degrees-of-freedom nonlinear tele-robotic system containing three master and single slave manipulators. Simulation results suggest that the proposed scheme is viable and can be deployed to control a class of multilateral nonlinear tele-robotic systems.

INDEX TERMS Tele-robotics, multi-lateral systems, non-linear dynamics, state convergence, MATLAB/simulink.

I. INTRODUCTION

Tele-operation forms an important class of robotics which deals with the control of distant processes through the use of robotic devices (such as manipulators). These devices are present both on the operator (termed as the master devices) and the remote sites (termed as the slave devices). The slave devices actually perform the desired task while the purpose of master devices is to provide the operators with a sensation of the remote environment. Based on the number of such devices, the tele-robotic system can be categorized as either bilateral (one master and one slave device) or multilateral (more than one master and/or slave devices). Besides these robotic devices, communication channel is another component of the tele-robotic system and is the main source of instability in such a system [1]. Many research efforts have been directed to address the stability issue in tele-robotic systems arising from the time delays of the communication channel. The groundbreaking work in designing the passive bilateral tele-robotic systems has appeared in [2] and [3] where the transmission line and wave variable theories are used to overcome the destabilizing effect of the communication time delays. It is later found that this novel passivity control algorithm is conservative due to the dissipation of excessive energy. Thus, passivity observers [3], [4] are proposed which allow the activation of the passivity controller on an as-needed basis thereby alleviating the over dissipation phenomenon. It is also found that wave-based passivity controllers suffer from degraded position tracking performance for which various solutions are also proposed [6], [7]. A comprehensive treatment of passivity techniques in bilateral tele-robotic systems can be found in [8]. In addition to passivity based approaches, other control algorithms are also developed for bilateral tele-robotic systems. For example, adaptive control schemes are explored to estimate the input-output disturbances and uncertainties such as gravity effects on local and remote sites [9]–[11], sliding mode techniques are introduced to reduce the effect of parametric uncertainties and time delays [12], [13], model predictive controllers are designed to reproduce the remote environment at the local site [14], [15], disturbance observer based controllers are developed to decouple the position and force sub-systems [16], [17], state convergence theory is introduced to achieve the desired dynamic behaviour of the linear tele-robotic system [18], fuzzy logic schemes with their universal approximation capability are utilized to compensate for the uncertainties in robot dynamics and environment [19]–[22], and Lyapunov-Krasovskii functions are used to prove the system's stability against the time delays [23], [24]. Few efforts have also been made to compare different bilateral control algorithms [25]–[27] which can serve as a starting point for the researchers who are interested in tele-robotic systems.

More recently, there is a growing interest in multilateral tele-robotic systems due to their advantages over the conventional bilateral systems including the task-sharing capability, increased loading capacity and improved robustness owing to have redundancy. These potential benefits, however, make the controller design process difficult and existing bilateral control algorithms cannot be easily extended for multilateral tele-robotic systems as pointed out by the researchers. As an instance, it is mentioned in [28] that wave reflection phenomenon associated with passivity based control scheme results in significant loss of transparency when directly applied to multilateral tele-robotic system operating in the presence of time varying delays. Therefore, a modified form of the wave-variable controller is proposed which is then used to develop a multilateral wave controller for a single-master-multiple-slave tele-robotic system. In another work on multi-master-single-slave tele-robotic system designed for both the cooperative and training tasks [29], it is pointed out that unlike a bilateral system; a multilateral tele-robotic system with the aforementioned objectives is no longer passive. An impedance approach is therefore introduced as a remedy and passivity condition is shown to hold through the use of small gain theorem. Time domain passivity and power based time domain passivity approaches have also been used to develop multi-master-multi-slave tele-robotic systems [30], [31] and it is shown that energy and power flows in case of multilateral networks are not as easy to manage as in their bilateral counterparts. Theory of adaptive control is employed in [32] and [33] to design multilateral controller when the uncertainties exist in both the local and remote environments. These two adaptive controllers are different in the sense that instead of direct force transmission as in [32], estimated environmental parameters are transmitted in [33]. The use of disturbance observer to design multilateral controller is demonstrated in [34] and [35]. It is shown that this methodology provides a natural choice for designing tele-robotic systems with the same or different degrees-of-freedom master/slave manipulators. Moreover, through the introduction of novel concepts of modal space and quarry matrices, a high performance cooperative motion of the multilateral tele-robotic system is achieved. Sliding mode based controllers are presented in [36] and [37] for dual-master-single-slave tele-robotic systems. The control gains are shown to be independent of time delays and desired impedance behavior is assigned to the master/slave systems. Moreover, chattering free operation is ensured in [37] through the use of higher-order sliding algorithm. The design of cooperating robotic manipulators based on fuzzy logic scheme is discussed in [38]. Here, the universal approximation property of fuzzy systems is combined with linear matrix inequality techniques to design the robust synchronization controller for multilateral tele-robotic system operating in the presence of dynamics uncertainties, disturbances and random network delays. The design of trilateral tele-robotic systems with the aim of achieving both the cooperative and training tasks is reported in [39] and [40]. The stability of these systems is analyzed with the help of Lyapunov-Krasovskii functional and it is shown that the tele-robotic systems remain stable in the presence of asymmetric time varying delays while the desired objectives are also achieved.

The above literature review reveals that the control schemes originally proposed for bilateral tele-robotic systems are being extended for multilateral tele-robotic systems with different set of objectives. This has motivated us to design the extended version of the state convergence scheme which can be used for multilateral tele-robotic systems [41]. However, in our earlier work, we have only considered the linear multilateral tele-robotic system without time delays. The aim of this paper is, therefore, to address these limitations of the extended state convergence scheme and to propose a state convergence based multi-master-single-slave nonlinear tele-robotic system which can be operated in the presence of asymmetric time varying delays. To achieve this, we first eliminate some design parameters associated with the extended state convergence scheme while keeping the desired objective to be the same i.e., the convergence of the slave position to the reference value (which is formed by the weighted combination of the masters' positions). The remaining control gains of the extended state convergence scheme are then selected based on a Lyapunov-Krasovskii analysis which also helps in establishing the stability of the proposed multilateral tele-robotic system. Further analysis reveals that the desired objective is also achieved. Finally, the feasibility of the proposed multilateral tele-robotic scheme is analyzed through simulations in MATLAB/Simulink environment where a non-linear robotic system containing three master and single slave manipulators each of which has two degrees-of-freedom is simulated in the presence of time varying delays. It is found that the multilateral tele-robotic system remains stable and position regulation of the slave manipulator is achieved.

The paper is structured as follows. Section II includes the modeling of the multilateral nonlinear tele-robotic system and some preliminaries. Section III presents the simplified extended state convergence architecture for multi-master-single-slave tele-robotic system. Section IV describes the stability analysis and control design procedure based on Lyapunov-Krasovskii approach. Section V encompasses the simulation results and Section VI concludes the paper along with some future directions.

II. MODELING OF MULTILATERAL NONLINEAR TELE-ROBOTIC SYSTEM

We consider an n degrees-of-freedom tele-robotic system which is comprised of k-local (master) and 1-remote (slave) manipulators and posses the following nonlinear dynamics:

$$M_l^j\left(q_l^j\right)\ddot{q}_l^j + C_l^j\left(q_l^j,\dot{q}_l^j\right)\dot{q}_l^j + g_l^j\left(q_l^j\right) = \tau_l^j + F_l^j, \forall j = 1, 2, ..., k \quad (1)$$

$$M_r\left(q_r\right)\ddot{q}_r + C_r\left(q_r,\dot{q}_r\right)\dot{q}_r + g_r\left(q_r\right) = \tau_r - F_r \quad (2)$$

Where the subscript l stands for the local while subscript r stands for the remote systems. The tele-robotic system parameters are: $\left(M_l^j, M_r\right) \in \mathbb{R}^{n\times n}$, $\left(C_l^j, C_r\right) \in \mathbb{R}^{n\times n}$, $\left(g_l^j, g_r\right) \in \mathbb{R}^{n\times 1}$ which denotes the inertia matrices, coriolis/centrifugal matrices and gravity vectors of local and remote manipulators respectively . The other quantities in (1) and (2) are

$$\left(q_l^j, q_r\right) \in \mathbb{R}^{n\times 1}, \left(\dot{q}_l^j, \dot{q}_r\right) \in \mathbb{R}^{n\times 1}, \left(\ddot{q}_l^j, \ddot{q}_r\right) \in \mathbb{R}^{n\times 1},$$
$$\left(\tau_l^j, \tau_r\right) \in \mathbb{R}^{n\times 1}, \left(F_l^j, F_r\right) \in \mathbb{R}^{n\times 1},$$

which denotes the position, velocity, acceleration, torque and external force signals in local and remote manipulators respectively. The dynamic representation of the tele-robotic system given by (1) and (2) posses following properties which will be utilized in Section 4 to prove the system's stability:

P1: The inertia matrices are positive definite, symmetric and bounded i.e. under the existence of two positive constants ε_1 and ε_2, the inequality $0 < \varepsilon_1 I < M(q) < \varepsilon_2 I \leq \infty$ holds.

P2: The inertia and coriolis/centrifugal matrices have a skew-symmetric relation which exists in the form $a^T\left(\dot{M}(q) - 2C\left(q,\dot{q}\right)\right)a = 0, \forall a \in \mathbb{R}^n$.

P3: The coriolis/centrifugal vectors are bounded i.e. under the existence of a positive constant ε_3, the inequality $\left\|C\left(q,\dot{q}\right)\dot{q}\right\| \leq \varepsilon_3\left\|\dot{q}\right\|$ holds.

P4: If the velocity and acceleration signals are bounded, then the time derivative of coriolis/centrifugal matrices is also bounded.

Along with the above properties P1-P4, the following assumptions A1-A2 and lemmas L1-L2 will be used in the paper:

A1: The human operators and remote environment are passive i.e. there exists positive constants υ_l^j and υ_r such that the inequalities $-\upsilon_l^j < \int_0^{t_f} -F_l^j\dot{q}_l^j\,dt \,\&\, -\upsilon_r < \int_0^{t_f} -F_r^j\dot{q}_r\,dt$ hold. Also, the environment is modeled by a spring-damper system i.e. $F_r = K_{re}q_r + B_{re}\dot{q}_r$ with $K_{re} \in \mathbb{R}^{n\times n}$ and $B_{re} \in \mathbb{R}^{n\times n}$ being positive definite diagonal matrices.

A2: The gravity loading vectors of local and remote manipulators are known.

L1: Let $a, b \in \mathbb{R}^n$ be any vector signals, $K \in \mathbb{R}^{n\times n}$ be a positive definite diagonal matrix and γ be a positive constant. Then for any time varying continuously differentiable function $T_i(t)$ with a known upper bound T_i^+, the following inequality holds:

$$-2\int_0^{t_f}\dot{a}^T K\int_0^{T_i(t)}\dot{b}(t-\sigma)d\sigma\,dt \leq \gamma\int_0^{t_f}\dot{a}^T K a dt + \frac{T_i^{+2}}{\gamma}\int_0^{t_f}\dot{b}^T K b dt$$

L2: Let $a \in \mathbb{R}^n$ be any vector signal and $T_i(t)$ be a time varying function with a known upper bound T_i^+, then the following inequality holds: $a(t - T_i(t)) - a(t) = \int_0^{T_i(t)}\dot{a}(t-\sigma)\,d\sigma \leq T_i^{+1/2}\left\|\dot{a}\right\|_2$

III. SIMPLIFIED EXTENDED STATE CONVERGENCE SCHEME

We have proposed an extended state convergence scheme in [41] for delay-free linear multilateral tele-robotic system where cooperative control of l-remote robotic systems by k-local robotic systems is established. In this paper, we will show that the presented scheme can indeed be applied to control nonlinear multilateral tele-robotic system when asymmetric time varying delays exist in the communication links. To achieve it, a simplified version of the extended state convergence architecture is proposed with a view of controlling a multi-master-single-slave nonlinear tele-robotic system. The proposed architecture differs slightly from its standard counterpart [41] in that the control gains associated with the direct transmission of operators' forces to slave robotic system are eliminated. The modified architecture is shown in Fig. 1 and is comprised of the following parameters:

F_l^j : This scalar parameter denotes the force applied by the j^{th} operator onto the j^{th} local manipulator.

α_j : This scalar parameter denotes the authority level of the j^{th} operator in the desired remote tracking task and all such authority factors are aggregated to be unity i.e. $\sum_j \alpha_j = 1$.

$T_{lj}(t)$: This scalar parameter denotes the time varying delay faced by the motion signals as transmitted by the j^{th} local system across the communication channel towards the remote system.

$T_{rj}(t)$: This scalar parameter denotes the time varying delay faced by the motion signals as transmitted by the remote system across the communication channel towards the j^{th} local system.

$K_l^j = \left[K_{l1}^j\ K_{l2}^j\right]$: This $n \times 2n$ matrix parameter defines the position $\left(K_{l1}^j \in \mathbb{R}^{n\times n}\right)$ and velocity $\left(K_{l2}^j \in \mathbb{R}^{n\times n}\right)$ feedback gains for the j^{th} local manipulator. Both the constituent parameters will be found as a part of the design procedure.

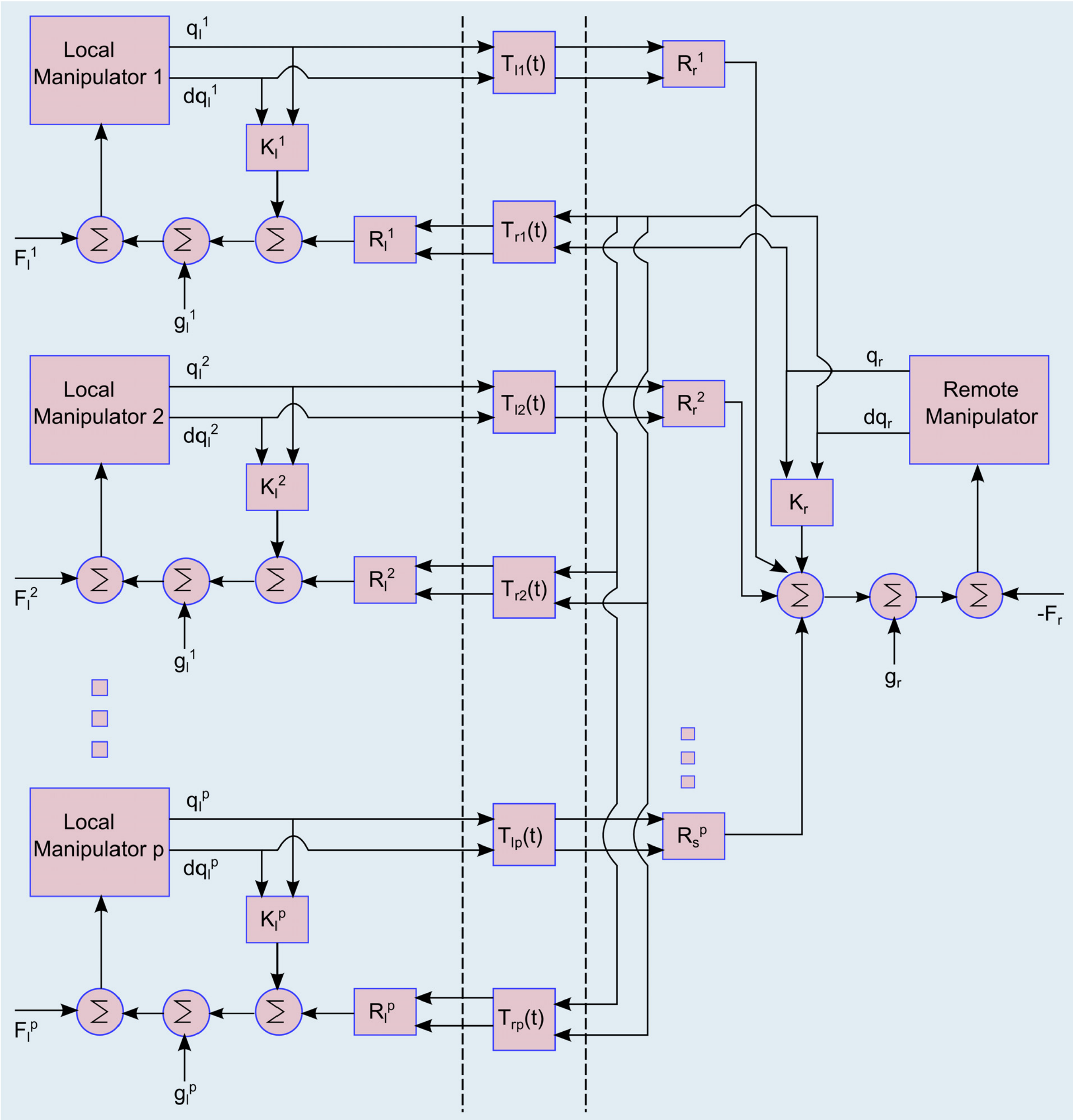


FIGURE 1. **Modified extended state convergence architecture for multi-master-single-slave tele-robotic system.**

$K_r = \left[K_{r1} \; K_{r2} \right]$: This $n \times 2n$ matrix parameter defines the position $\left(K_{r1} \in \mathbb{R}^{n \times n}\right)$ and velocity $\left(K_{r2} \in \mathbb{R}^{n \times n}\right)$ feedback gains for the remote manipulator. Similar to K_l^j, both the constituent parameters of K_r will be found as a part of the design procedure.

$R_r^j = \left[R_{r1}^j \; R_{r2}^j \right]$: This $n \times 2n$ matrix parameter models the effect of j^{th} local manipulator's motion onto the remote manipulator where both the constituent parameters $R_{r1}^j \in \mathbb{R}^{n \times n}$ and $R_{r2}^j \in \mathbb{R}^{n \times n}$ will be determined as a part of the design procedure.

$R_l^j = \left[R_{l1}^j \; R_{l2}^j \right]$: This $n \times 2n$ matrix parameter models the effect of the remote manipulator's motion onto j^{th} local manipulator where both the constituent parameters $R_{l1}^j \in \mathbb{R}^{n \times n}$ and $R_{l2}^j \in \mathbb{R}^{n \times n}$ will be determined as a part of the design procedure.

IV. STABILITY ANALYSIS AND CONTROL DESIGN

The goal of this section is to establish that the proposed multilateral tele-robotic system, as depicted in Fig. 1, can maintain stability in the presence of time varying delays and under an appropriate selection of the control gains, the remote manipulator can follow the reference set by the local manipulators according to their authority levels i.e. $\lim_{t\to\infty}\left(q_r(t)-\sum_{j=1}^{k}\alpha_j q_l^j(t)\right)=0$. To achieve these goals, we proceed as follows:

Theorem 1: Let γ_{lj},γ_{rj} be positive scalar constants, $K\in\mathbb{R}^{n\times n}$, $K_1\in\mathbb{R}^{n\times n}$ be positive definite diagonal matrices and T_{lj}^{+},T_{rj}^{+} be the bounds on time varying delays. Now, if the control gains of the multilateral tele-robotic system (1)-(2) are selected as in (3)-(4) and $k+1$ inequalities in (5) are also satisfied, then the proposed multilateral tele-robotic system remains stable in the presence of time varying delays i.e.,

$$\lim_{t\to\infty}=\dot{q}_l^j=\lim_{t\to\infty}=\dot{q}_r=\lim_{t\to\infty}=\ddot{q}_l^j=\lim_{t\to\infty}=\ddot{q}_r=0,\quad\forall j=1,2,\ldots,k.$$

$$\begin{aligned}
K_{l1}^j&=-K,\,K_{l2}^j=-2K_1-\alpha_jK_{ld}^j,\,R_{l1}^j=\alpha_jK,\\
R_{l2}^j&=2\alpha_jK_{ld}^j,\,\forall j=1,2,\ldots,k\\
K_{r1}&=-K,\,K_{r2}=-2K_1-\sum_{j=1}^{k}\alpha_jK_{rd}^j,\,R_{r1}^j=\alpha_jK,\\
R_{r2}^j&=2\alpha_jK_{rd}^j,\,\forall j=1,2,\ldots,k
\end{aligned}\tag{3}$$

$$K_{ld}^j=\left(1-\dot{T}_r^j(t)\right)K_1\quad,K_{rd}^j=\left(1-\dot{T}_{lj}(t)\right)K_1,\quad\forall j=1,2,\ldots,k\tag{4}$$

$$\begin{aligned}
&(2-\alpha_j)K_1-\frac{\alpha_j\gamma_{rj}}{2}K-\frac{\alpha_jT_{lj}^{+^2}}{2\gamma_{lj}}K>0,\quad\forall j=1,2,\ldots,k\\
&K_1-\sum_{j=1}^{k}\frac{\alpha_j\gamma_{lj}}{2}K-\sum_{j=1}^{k}\frac{\alpha_jT_{rj}^{+^2}}{2\gamma_{rj}}K>0.
\end{aligned}\tag{5}$$

Proof: Consider the multilateral tele-robotic system given by (1)-(2). The control inputs τ_l^j,τ_r for this tele-robotic system can be written by observing the extended state convergence architecture of Fig. 1 as

$$\begin{aligned}
\tau_l^j&=g_l^j\left(q_l^j\right)+K_{l1}^jq_l^j+K_{l2}^j\dot{q}_l^j+R_{l1}^jq_r\left(t-T_{rj}(t)\right)\\
&\quad+R_{l2}^j\dot{q}_s\left(t-T_{rj}(t)\right),\forall j=1,2,\ldots,k
\end{aligned}\tag{6}$$

$$\begin{aligned}
\tau_r&=g_r(q_r)+K_{r1}q_r+K_{r2}\dot{q}_r+\sum_{j=1}^{k}R_{r1}^jq_l^j\left(t-T_{lj}(t)\right)\\
&\quad+\sum_{j=1}^{k}R_{r2}^j\dot{q}_l^j\left(t-T_{lj}(t)\right)
\end{aligned}\tag{7}$$

By substituting the control law (6) in (1) and (7) in (2) and by considering the model of the remote environment, the closed loop multilateral tele-robotic system is obtained as:

$$\begin{aligned}
M_l^j\ddot{q}_l^j+C_l^j\dot{q}_l^j&=K_{l1}^jq_l^j+K_{l2}^j\dot{q}_l^j+R_{l1}^jq_r\left(t-T_{rj}(t)\right)\\
&\quad+R_{l2}^j\dot{q}_s\left(t-T_{rj}(t)\right)+F_l^j,\quad\forall j=1,2,\ldots,k
\end{aligned}\tag{8}$$

$$\begin{aligned}
M_r\ddot{q}_r+C_r\dot{q}_r&=K_{r1}q_r+K_{r2}\dot{q}_r+\sum_{j=1}^{k}R_{r1}^jq_l^j\left(t-T_{lj}(t)\right)\\
&\quad+\sum_{j=1}^{k}R_{r2}^j\dot{q}_l^j\left(t-T_{lj}(t)\right)-K_{re}q_r-B_{re}\dot{q}_r
\end{aligned}\tag{9}$$

Now, we define the following Lyapunov-Krasovskii function to analyze the stability of the closed loop tele-robotic system (8)-(9) with the control gains given in (3)-(4):

$$\begin{aligned}
&V\left(\dot{q}_l^j,\dot{q}_r,q_l^j-q_r,q_r,q_l^j\right)\\
&=\frac{1}{2}\sum_{j=1}^{k}\dot{q}_l^{jT}M_l^j\dot{q}_l^j\\
&\quad+\frac{1}{2}\dot{q}_r^TM_r\dot{q}_r+\frac{1}{2}\sum_{j=1}^{k}(1-\alpha_j)q_l^{jT}Kq_l^j+\frac{1}{2}q_r^TK_{re}q_r\\
&\quad+\sum_{j=1}^{k}\int_0^t-\dot{q}_l^{jT}(\xi)F_l^j(\xi)d\xi+\sum_{j=1}^{k}\upsilon_l^j\\
&\quad+\frac{1}{2}\sum_{j=1}^{k}\alpha_j\left(q_l^j-q_r\right)^TK\left(q_l^j-q_r\right)\\
&\quad+\sum_{j=1}^{k}\alpha_j\int_{t-T_{lj}(t)}^{t}\dot{q}_l^{jT}(\xi)K_1\dot{q}_l^j(\xi)d\xi\\
&\quad+\sum_{j=1}^{k}\alpha_j\int_{t-T_{rj}(t)}^{t}\dot{q}_r^T(\xi)K_1\dot{q}_r(\xi)d\xi.
\end{aligned}\tag{10}$$

By taking the time derivative of (10) and using the closed loop tele-robotic system (8)-(9) along with the property P2 and assumption A1, we have:

$$\begin{aligned}
\dot{V}&=\sum_{j=1}^{k}\dot{q}_l^{jT}K_{l1}^jq_l^j+\sum_{j=1}^{k}\dot{q}_l^{jT}K_{l2}^j\dot{q}_l^j\\
&\quad+\sum_{j=1}^{k}\dot{q}_l^{jT}R_{l1}^jq_r\left(t-T_{rj}(t)\right)\\
&\quad+\sum_{j=1}^{k}\dot{q}_l^{jT}R_{l2}^j\dot{q}_r\left(t-T_{rj}(t)\right)\\
&\quad+\dot{q}_r^TK_{r1}q_r+\dot{q}_r^TK_{r2}\dot{q}_r+\dot{q}_r^T\sum_{j=1}^{k}R_{r1}^jq_l^j\left(t-T_{lj}(t)\right)\\
&\quad+\dot{q}_r^T\sum_{j=1}^{k}R_{r2}^j\dot{q}_l^j\left(t-T_{lj}(t)\right)-\dot{q}_r^TK_{re}q_r-\dot{q}_r^TB_{re}\dot{q}_r\\
&\quad+\sum_{j=1}^{k}(1-\alpha_j)\dot{q}_l^{jT}Kq_l^j+\dot{q}_r^TK_{re}q_r
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{k}\alpha_j \dot{q}_l^{jT} K\left(q_l^j-q_r\right)+\sum_{j=1}^{k}\alpha_j \dot{q}_r^{T} K\left(q_r-q_l^j\right)\\
&+\sum_{j=1}^{k}\alpha_j \dot{q}_l^{jT} K_1 \dot{q}_l^j+\sum_{j=1}^{k}\alpha_j \dot{q}_r^{T} K_1 \dot{q}_r\\
&-\sum_{j=1}^{k}\alpha_j \dot{q}_l^{jT}\left(t-T_{lj}(t)\right)\left(1-\dot{T}_{lj}(t)\right)K_1 \dot{q}_l^j\left(t-T_{lj}(t)\right)\\
&-\sum_{j=1}^{k}\alpha_j \dot{q}_r^{T}\left(t-T_{rj}(t)\right)\left(1-\dot{T}_{rj}(t)\right)K_1 \dot{q}_r\left(t-T_{rj}(t)\right)
\end{aligned}
\tag{11}
$$

By defining $K_{ld}^j=\left(1-\dot{T}_{rj}(t)\right)K_1$, $K_{rd}^j=\left(1-\dot{T}_{lj}(t)\right)K_1$ and grouping the terms in (11) and simplifying further, we obtain:

$$
\begin{aligned}
\dot{V}=&\sum_{j=1}^{k}\dot{q}_l^{jT}\left(K_{l1}^j+K\right)q_l^j\\
&+\sum_{j=1}^{k}\dot{q}_l^{jT}\left(R_{l1}^j q_r\left(t-T_{rj}(t)\right)-\alpha_j K q_r(t)\right)\\
&+\dot{q}_r^{T}\left(K_{r1}+\sum_{j=1}^{k}\alpha_j K\right)q_r\\
&+\sum_{j=1}^{k}\dot{q}_r^{T}\left(R_{r1}^j q_l^j\left(t-T_{lj}(t)\right)-\alpha_j K q_l^j\right)\\
&+\sum_{j=1}^{k}\dot{q}_l^{jT}\left(K_{l2}^j+\alpha_j K_1\right)\dot{q}_l^j\\
&+\dot{q}_r^{T}\left(K_{r2}+\sum_{j=1}^{k}\alpha_j K_1\right)\dot{q}_r\\
&-\dot{q}_r^{T} B_{re}\dot{q}_r+\sum_{j=1}^{k}\dot{q}_l^{jT} R_{l2}^j \dot{q}_r\left(t-T_{rj}(t)\right)\\
&+\sum_{j=1}^{k}\dot{q}_r^{T} R_{r2}^j \dot{q}_l^j\left(t-T_{lj}(t)\right)\\
&-\sum_{j=1}^{k}\alpha_j \dot{q}_l^{jT}\left(t-T_{lj}(t)\right)K_{rd}^j \dot{q}_l^j\left(t-T_{lj}(t)\right)\\
&-\sum_{j=1}^{k}\alpha_j \dot{q}_r^{T}\left(t-T_{rj}(t)\right)K_{ld}^j \dot{q}_r\left(t-T_{rj}(t)\right)
\end{aligned}
\tag{12}
$$

By plugging the control gains (3) in (12) and on simplifying the resultant expression, we get:

$$
\begin{aligned}
\dot{V}=&\sum_{j=1}^{k}\alpha_j \dot{q}_l^{jT} K\left(q_r\left(t-T_{rj}(t)\right)-q_r\right)\\
&+\sum_{j=1}^{k}\alpha_j \dot{q}_r^{T} K\left(q_l^j\left(t-T_{lj}(t)\right)-q_l^j\right)\\
&-\sum_{j=1}^{k}\dot{q}_l^{jT}\left(2-\alpha_j\right)K_1\dot{q}_l^j-\dot{q}_r^{T}K_1\dot{q}_r-\sum_{j=1}^{k}\alpha_j \dot{q}_l^{jT} K_{ld}^j \dot{q}_l^j\\
&-\sum_{j=1}^{k}\alpha_j \dot{q}_r^{T}\left(t-T_{rj}(t)\right)K_{ld}^j \dot{q}_r\left(t-T_{rj}(t)\right)\\
&+\sum_{j=1}^{k}2\alpha_j \dot{q}_l^{jT} K_{ld}^j \dot{q}_r\left(t-T_{rj}(t)\right)-\dot{q}_r^{T}B_{re}\dot{q}_r\\
&-\sum_{j=1}^{k}\alpha_j \dot{q}_r^{T} K_{rd}^j \dot{q}_r-\sum_{j=1}^{k}\alpha_j \dot{q}_l^{jT}\left(t-T_{lj}(t)\right)\\
&\times K_{rd}^j \dot{q}_l^j\left(t-T_{lj}(t)\right)\\
&+\sum_{j=1}^{k}2\alpha_j \dot{q}_r^{T} K_{rd}^j \dot{q}_l^j\left(t-T_{lj}(t)\right)
\end{aligned}
\tag{13}
$$

Let us now define the error signals relevant to the proposed multilateral tele-robotic system as:

$$
\begin{aligned}
e_{q_r^j}&=q_r-q_l^j\left(t-T_{lj}(t)\right), \quad \forall j=1,2,\ldots,k\\
e_{q_l^j}&=q_l^j-q_r\left(t-T_{rj}(t)\right), \quad \forall j=1,2,\ldots,k
\end{aligned}
\tag{14}
$$

We can write (13) in terms of the integral equality $q\left(t-T_i(t)\right)-q(t)=-\int_0^{T_i(t)}\dot{q}(t-\sigma)d\sigma$ and error signals (14) as:

$$
\begin{aligned}
\dot{V}=&-\sum_{j=1}^{k}\alpha_j \dot{q}_l^{jT} K\int_0^{T_{rj}(t)}\dot{q}_r(t-\sigma)d\sigma\\
&-\sum_{j=1}^{k}\alpha_j \dot{q}_r^{T} K\int_0^{T_{lj}(t)}\dot{q}_l^j(t-\sigma)d\sigma\\
&-\sum_{j=1}^{k}\dot{q}_l^{jT}\left(2-\alpha_j\right)K_1\dot{q}_l^j\\
&-\dot{q}_r^{T}K_1\dot{q}_r-\dot{q}_r^{T}B_{re}\dot{q}_r\\
&-\sum_{j=1}^{k}\alpha_j \dot{e}_{q_r^j}^{T} K_{rd}^j \dot{e}_{q_r^j}-\sum_{j=1}^{k}\alpha_j \dot{e}_{q_l^j}^{T} K_{ld}^j \dot{e}_{q_l^j}
\end{aligned}
\tag{15}
$$

By integrating (15) over the time interval $\left[0,t_f\right]$ and using lemma L1, we have:

$$
\begin{aligned}
&\int_0^{t_f}\dot{V}\,ds\\
&\le\sum_{j=1}^{k}\alpha_j\left(\frac{\gamma_{rj}}{2}\int_0^{t_f}\dot{q}_l^{jT}K\dot{q}_l^j ds+\frac{T_{rj}^{+2}}{2\gamma_{rj}}\int_0^{t_f}\dot{q}_r^{T}K\dot{q}_r ds\right)\\
&\quad-\sum_{j=1}^{k}\int_0^{t_f}\dot{q}_l^{jT}\left(2-\alpha_j\right)K_1\dot{q}_l^j ds
\end{aligned}
$$

$$+\sum_{j=1}^{k}\alpha_j\left(\frac{\gamma_{lj}}{2}\int_0^{t_f}\dot{q}_r^T K\dot{q}_r\,ds+\frac{T_{lj}^{+^2}}{2\gamma_{lj}}\int_0^{t_f}\dot{q}_l^{j^T} K\dot{q}_l^j\,ds\right)$$
$$-\int_0^{t_f}\dot{q}_r^T K_1\dot{q}_r\,ds-\int_0^{t_f}\dot{q}_r^T B_{re}\dot{q}_r\,ds$$
$$-\sum_{j=1}^{k}\alpha_j\int_0^{t_f}\dot{e}_{q_r^j}^T K_{rd}^j\dot{e}_{q_r^j}\,ds-\sum_{j=1}^{k}\alpha_j\int_0^{t_f}\dot{e}_{q_l^j}^T K_{ld}^j\dot{e}_{q_l^j}\,ds \quad (16)$$

The inequality (16) can be further reduced as:

$$V(t_f)-V(0)\le$$
$$-\sum_{j=1}^{k}\mu\left((2-\alpha_j)K_1-\frac{\alpha_j\gamma_{rj}}{2}K-\frac{\alpha_j T_{lj}^{+^2}}{2\gamma_{lj}}K\right)\left\|\dot{q}_l^j\right\|_2^2$$
$$-\sum_{j=1}^{k}\mu\left(\alpha_j K_{rd}^j\right)\left\|\dot{e}_{q_r^j}\right\|_2^2-\sum_{j=1}^{k}\mu\left(\alpha_j K_{ld}^j\right)\left\|\dot{e}_{q_l^j}\right\|_2^2$$
$$-\mu\left(K_1-\sum_{j=1}^{k}\left(\frac{\alpha_j\gamma_{lj}}{2}K-\frac{\alpha_j T_{rj}^{+^2}}{2\gamma_{rj}}K\right)\right)\left\|\dot{q}_s\right\|_2^2$$
$$-\mu(B_e)\left\|\dot{q}_r\right\|_2^2 \quad (17)$$

Where $\mu(X)$ specifies the minimum Eigen value of X. Taking the limit as $t_f\to\infty$ in (17) and on the satisfaction of the inequalities in (5), it can be concluded that the signals

$$\left\{\dot{q}_l^j,\dot{q}_r,q_l^j-q_r,q_r,q_l^j\right\}\in L_\infty$$

and

$$\left\{\dot{q}_l^j,\dot{q}_r,\dot{e}_{q_r^j},\dot{e}_{q_l^j}\right\}\in L_2.$$

Now, it is left to show the zero convergence of velocity and acceleration signals for proving the system's stability. The zero convergence of velocity signals is achieved if the acceleration signals remain bounded. Thus, we first analyze the acceleration signals of (8) and (9) by disregarding the external forces and rewriting them as:

$$\ddot{q}_l^j=\left(M_l^j\right)^{-1}\begin{bmatrix}-C_l^j\dot{q}_l^j+K_{l1}^j q_l^j+K_{l2}^j\dot{q}_l^j\\+R_{l1}^j q_r\left(t-T_{rj}(t)\right)\\+R_{l2}^j\dot{q}_r\left(t-T_{rj}(t)\right)\end{bmatrix} \quad (18)$$

$$\ddot{q}_r=M_r^{-1}\begin{bmatrix}-C_r\dot{q}_r+K_{r1}q_r+K_{r2}\dot{q}_r\\+\sum_{j=1}^{k}R_{r1}^j q_l^j\left(t-T_{lj}(t)\right)\\+\sum_{j=1}^{k}R_{r2}^j\dot{q}_l^j\left(t-T_{lj}(t)\right)\end{bmatrix} \quad (19)$$

If we analyze (18) and (19) along with the control gains (3) of the tele-robotic system, we are left to show that the signals

$$\left\{q_l^j-\alpha_j q_r\left(t-T_{rj}(t)\right),q_r-\sum_{j=1}^{k}\alpha_j q_l^j\left(t-T_{lj}(t)\right)\right\}\in L_\infty$$

since it has already been shown that $\left\{\dot{q}_l^j,\dot{q}_r\right\}\in L_2$ and $\left\{\dot{q}_l^j,\dot{q}_r,q_l^j-q_r,q_r,q_l^j\right\}\in L_\infty$ by virtue of $\int_0^\infty \dot{V}\,ds\le 0$. We can write the left-over signals as:

$$q_l^j-\alpha_j q_r\left(t-T_{rj}(t)\right)=\overbrace{\left(q_l^j-\alpha_j q_r\right)}^{1}+\alpha_j\overbrace{\left(q_r-q_r\left(t-T_{rj}(t)\right)\right)}^{2} \quad (20)$$

$$q_r-\sum_{j=1}^{k}\alpha_j q_l^j\left(t-T_{lj}(t)\right)=\overbrace{\left(q_r-\sum_{j=1}^{k}\alpha_j q_l^j\right)}^{1}+\sum_{j=1}^{k}\alpha_j\overbrace{\left(q_l^j-\alpha_j q_l^j\left(t-T_{lj}(t)\right)\right)}^{2} \quad (21)$$

The first part of the signals in (20)-(21) are bounded since $\left\{\dot{q}_l^j,\dot{q}_r,q_l^j-q_r\right\}\in L_\infty$ while the second part of the signals are bounded by virtue of lemma L2 and $\left\{\dot{q}_l^j,\dot{q}_r\right\}\in L_\infty$. This implies that the left hand sides of (20)-(21) are also bounded. By using the properties P1 and P3 of the robot dynamics and the result

$$\left\{\begin{matrix}\dot{q}_l^j,\dot{q}_r,q_l^j-q_r,q_r,q_l^j,q_l^j-\alpha_j q_r\left(t-T_{rj}(t)\right),\\ q_r-\sum_{j=1}^{k}\alpha_j q_l^j\left(t-T_{lj}(t)\right)\end{matrix}\right\}\in L_\infty,$$

it can be concluded that the signals $\left\{\ddot{q}_l^j,\ddot{q}_r\right\}$ are bounded. Since the signals $\left\{\dot{q}_l^j,\dot{q}_r\right\}$ also belong to L_2, then by Barbalat's lemma, we have: $\lim_{t\to\infty}\dot{q}_l^j=\lim_{t\to\infty}\dot{q}_r=\lim_{t\to\infty}\dot{e}_{q_l^j}=\lim_{t\to\infty}\dot{e}_{q_r^j}=0$. To show the convergence of acceleration signals, we consider the time-derivative of (18)-(19):

$$\frac{d\ddot{q}_l^j}{dt}=\frac{d}{dt}\left(M_l^j\right)^{-1}\begin{bmatrix}-C_l^j\dot{q}_l^j+K_{l1}^j q_l^j+K_{l2}^j\dot{q}_l^j+\\R_{l1}^j q_r\left(t-T_{rj}(t)\right)\\+R_{l2}^j\dot{q}_r\left(t-T_{rj}(t)\right)\end{bmatrix}+\left(M_l^j\right)^{-1}\frac{d}{dt}\begin{bmatrix}-C_l^j\dot{q}_l^j+K_{l1}^j q_l^j+K_{l2}^j\dot{q}_l^j+\\R_{l1}^j q_r\left(t-T_{rj}(t)\right)+\\R_{l2}^j\dot{q}_r\left(t-T_{rj}(t)\right)\end{bmatrix} \quad (22)$$

$$\frac{d\ddot{q}_r}{dt} = \frac{d}{dt}\left(M_r^{-1}\right)\begin{bmatrix} -C_r\dot{q}_r + K_{r1}q_r + K_{r2}\dot{q}_r + \\ \sum_{j=1}^{k} R_{r1}^{j} q_l^{j}\left(t - T_{lj}(t)\right) + \\ \sum_{j=1}^{k} R_{r2}^{j} \dot{q}_l^{j}\left(t - T_{lj}(t)\right) \end{bmatrix} + M_s^{-1}\frac{d}{dt}\begin{bmatrix} -C_r\dot{q}_r + K_{r1}q_r + K_{r2}\dot{q}_r + \\ \sum_{j=1}^{k} R_{r1}^{j} q_l^{j}\left(t - T_{lj}(t)\right) + \\ \sum_{j=1}^{k} R_{r2}^{j} \dot{q}_l^{j}\left(t - T_{lj}(t)\right) \end{bmatrix} \tag{23}$$

By combining the earlier result

$$\left\{ \begin{matrix} \dot{q}_l^j, \dot{q}_r, q_l^j - q_r, q_r, q_l^j, q_l^j - \alpha_j q_r\left(t - T_{rj}(t)\right), \\ q_r - \sum_{j=1}^{k} \alpha_j q_l^j\left(t - T_{lj}(t)\right) \end{matrix} \right\} \in L_\infty$$

with the properties P3 and P4 of the robot dynamics, it can be concluded that the second derivative terms in (22) and (23) are bounded. The first derivative terms in (22) and (23) are also bounded owing to the boundedness of the signals $\left\{\dot{q}_l^j, \ddot{q}_l^j, \dot{q}_r, \ddot{q}_r\right\}$, properties P1 and P2 of the robot dynamics, and considering $\dot{M}^{-1} = -M^{-1}\dot{M}M^{-1} = -M^{-1}\left(C + C^T\right)M^{-1}$. Thus the right hand sides of (22) and (23) remain bounded implying that the signals $\left\{\dddot{q}_l^j, \dddot{q}_r\right\} \in L_\infty$ are uniformly continuous. Signal continuity further implies that the integral exists and is bounded. Thus, based on the previous result: $\lim_{t\to\infty}\dot{q}_l^j = \lim_{t\to\infty}\dot{q}_r = 0$, we have $\lim_{t\to\infty}\int_0^t \ddot{q}_l^j\, dt = -\dot{q}_l^j(0)$, $\lim_{t\to\infty}\int_0^t \ddot{q}_r\, dt = -\dot{q}_r(0)$ and by Barbalat's lemma, it can be concluded that: $\lim_{t\to\infty}\ddot{q}_l^j = \lim_{t\to\infty}\ddot{q}_r = 0$. The proof is now completed.

Theorem 2: In the absence of environmental force, the remote manipulator achieves the desired position in equilibrium state i.e. $\lim_{t\to\infty}\left(q_r(t) - \sum_{j=1}^{k}\alpha_j q_l^j(t)\right) = 0$ when the control gains of the tele-robotic system are set according to (3).

Proof: It has been shown in theorem 1 that the closed loop multilateral tele-robotic system (8)-(9) remains stable under the control gains of (3). Then, by plugging (3) in (9) and using the result from theorem 1, $\lim_{t\to\infty}\dot{q}_r = \lim_{t\to\infty}\ddot{q}_r = 0$, we have:

$$\lim_{t\to\infty}\left\| q_r - \sum_{j=1}^{k}\alpha_j q_l^j\left(t - T_{lj}(t)\right)\right\| = 0 \tag{24}$$

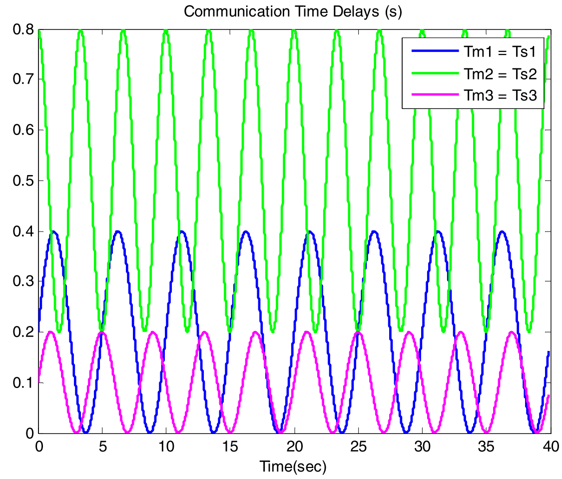


FIGURE 2. Time varying delays of the communication channel.

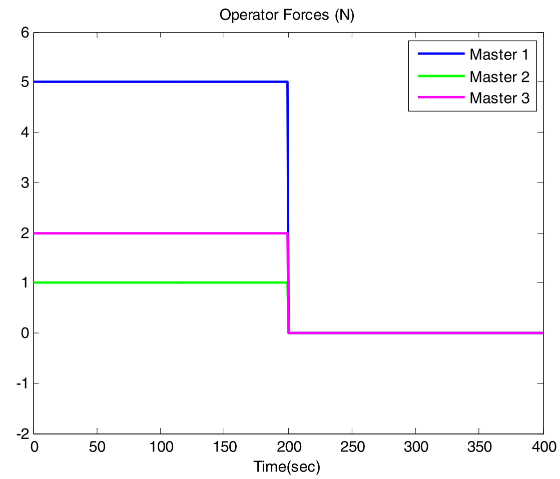
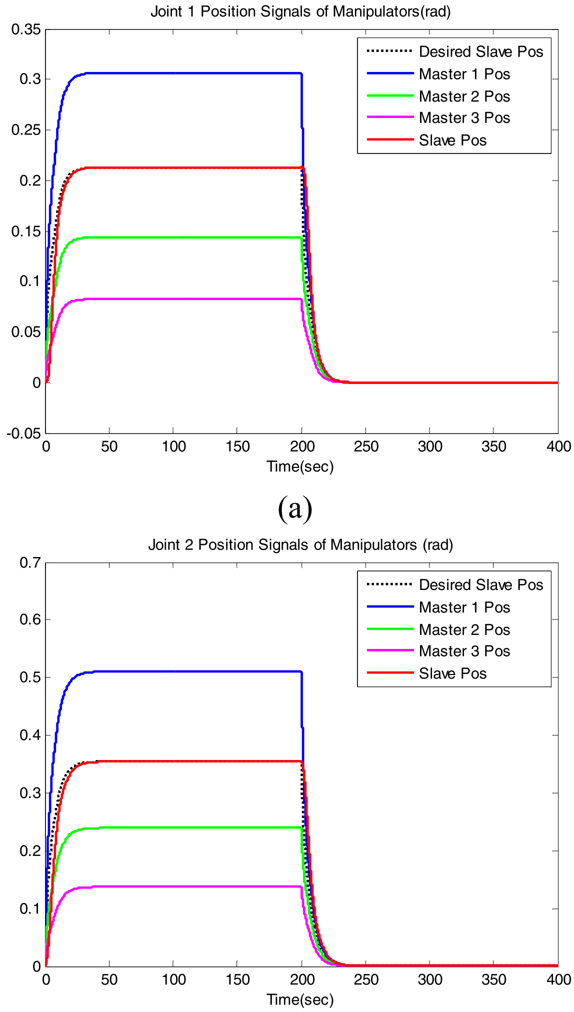


FIGURE 3. Profile of operators' forces.

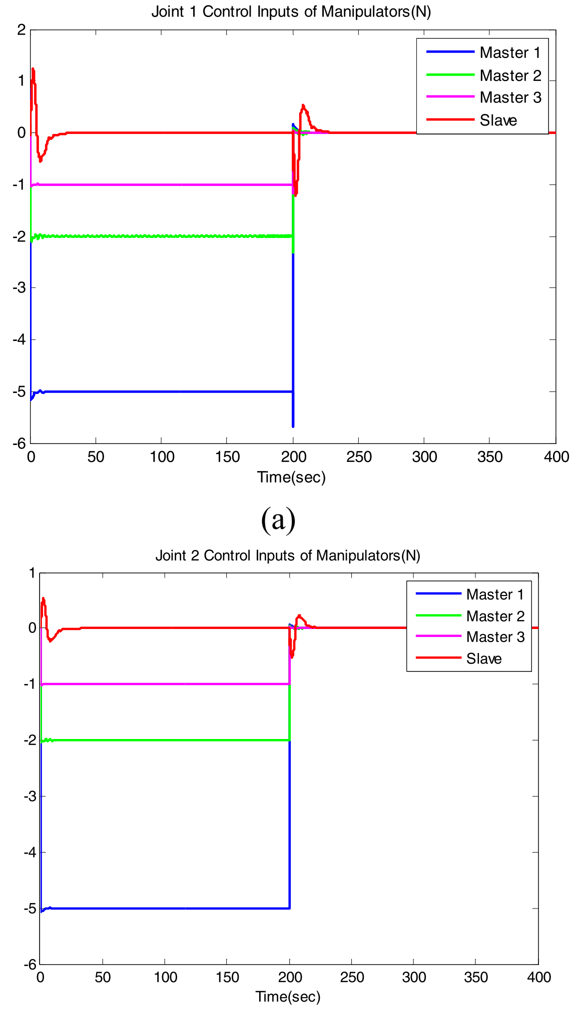
Through the use of integral equality $q_l^j\left(t - T_{lj}(t)\right) = q_l^j - \int_{t-T_{lj}(t)}^{t}\dot{q}_l^j(\xi)\,d\xi$, and the earlier result on velocity convergence, $\lim_{t\to\infty}\dot{q}_l^j = 0$, we can write (24) as $\lim_{t\to\infty}\left\| q_r - \sum_{j=1}^{k}\alpha_j q_l^j\right\| = 0$. Thus, the remote manipulator achieves the desired position in free motion when the equilibrium state of the tele-robotic system is reached. This completes the proof.

Remark 1: The velocity control gains of the proposed multilateral tele-robotic system depend on the derivative of the time varying delays as can be seen from (3). These gains, therefore, are unrealizable since only the upper bounds on the communication delays are known. As a remedy, extra ramp signals $r\left(t - T_{lj}(t)\right), r\left(t - T_{rj}(t)\right)$ are transmitted across the communication channel and their time-derivatives are



(b)

FIGURE 4. Position signals of the manipulators. (a) Joint 1 trajectories. (b) Joint 2 trajectories.



(b)

FIGURE 5. Torque inputs of the manipulators. (a) Joint 1 control inputs. (b) Joint 2 control inputs.

used to implement the velocity control gains as:

$$K_{ld}^{j} = \dot{r}\left(t - T_{rj}(t)\right) K_1, K_{rd}^{j} = \dot{r}\left(t - T_{lj}(t)\right) K_1, \quad \forall j = 1, 2, \ldots, k \tag{25}$$

Remark 2: It is usual to have force feedback in tele-robotic system which is believed to improve the task performance. The proposed multilateral tele-robotic system also provides force feedback to the operators when the remote manipulator comes in contact with the environment. It is not difficult to show that in the proposed multilateral tele-robotic system, static environmental force is related to the operators' forces as $F_e = \sum_{j=1}^{k} \alpha_j F_l^j - \left(1 - \sum_{j=1}^{k} \alpha_j^2\right) K q_r$.

V. SIMULATION RESULTS

In order to validate the proposed multilateral tele-robotic system, simulations are performed in MATLAB/Simulink environment where three local manipulators are driving a single remote manipulator each of which has two degrees-of-freedom. The dynamical system representation of these manipulators is given by (1)-(2) with the following description of inertia matrices, coriolis/centrifugal matrices and gravity vectors:

$$M(q) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \quad C\left(q, \dot{q}\right) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix},$$

$$g(q) = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{26}$$

$$\begin{aligned} m_{11} &= m_2 l^2 + (m_1 + m_2) l^2 + 2 m_2 l^2 \cos(q_2) \\ m_{12} &= m_{21} = m_2 l^2 + m_2 l^2 \cos(q_2) \\ m_{22} &= m_2 l^2 \end{aligned} \tag{27}$$

$$c_{11} = -\dot{q}_2 m_2 l^2 \sin(q_2), \quad c_{12} = -\left(\dot{q}_1 + \dot{q}_2\right) m_2 l^2 \sin(q_2)$$

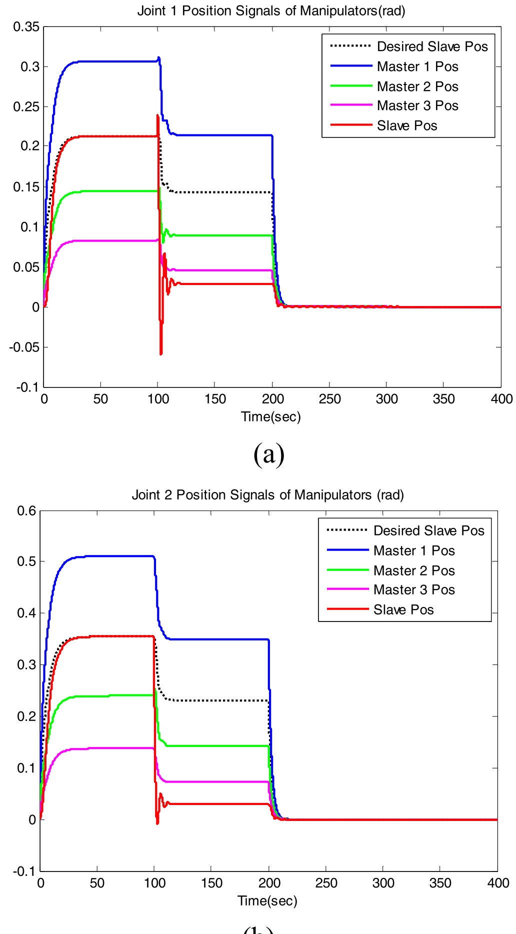


FIGURE 6. **Position signals of the manipulators during free ($t < 100$) and contact ($100 < t < 200$) motion. (a) Joint 1 position trajectories. (b) Joint 2 position trajectories.**

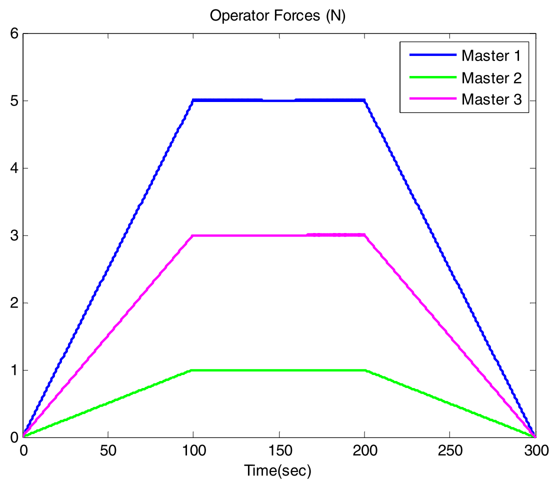


FIGURE 7. **Time varying operators' forces.**

$$c_{21} = \dot{q}_1\, m_2 l^2 \sin(q_2), \quad c_{22} = 0 \tag{28}$$

$$g_1 = a_g m_2 l \sin(q_1 + q_2) + a_g (m_1 + m_2)\, l \sin(q_1),$$
$$g_2 = a_g m_2 l \sin(q_1 + q_2) \tag{29}$$

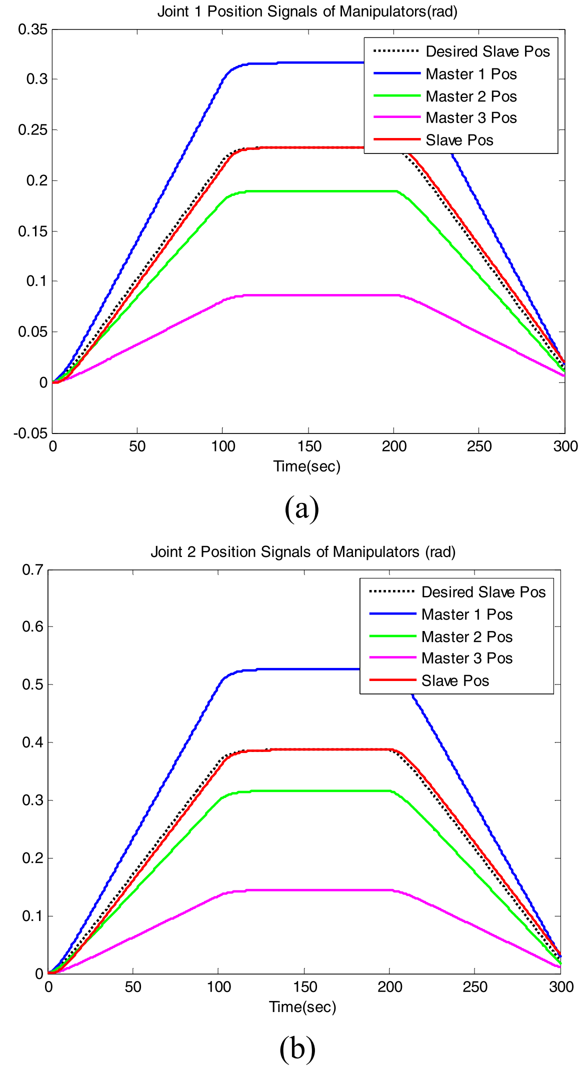


FIGURE 8. **Position trajectories of the manipulators under time varying applied forces. (a) Joint 1 position signals. (b) Joint 2 position signals.**

Where m_1, m_2 are the masses of links 1 and 2 respectively; $l_1 = l_2 = l$ are the lengths of links and a_g is the acceleration due to gravity. The numerical values of these parameters are assumed to be the same for all the local manipulators: $m_{1m} = m_{2m} = 1kg, l_m = 1m$. However, remote manipulator has more inertia than the local manipulators: $m_{1s} = m_{2s} = 5kg, l_s = 2m$. These manipulators communicate over a communication channel which offers time varying delays as shown in Fig. 2. The only information which is known to the designer about these delays is their upper bounds: $T_{l1}^+ = T_{r1}^+ = 0.4, T_{l2}^+ = T_{r2}^+ = 0.8, T_{l3}^+ = T_{r2}^+ = 0.2$. By assigning the authority factors to the operators as $\alpha_1 = 0.5, \alpha_2 = 0.3, \alpha_3 = 0.2$ and solving the inequalities in (5) and using (3), we obtain the following control gains for the tele-robotic system:

$$K = \begin{pmatrix} 25.0 & 0 \\ 0 & 15.0 \end{pmatrix}, K_1 = \begin{pmatrix} 31.25 & 0 \\ 0 & 18.75 \end{pmatrix} \tag{30}$$

$$R_{l1}^1 = R_{r1}^1 = \begin{pmatrix} 12.5 & 0 \\ 0 & 7.5 \end{pmatrix}, \quad R_{l1}^2 = R_{r1}^2 = \begin{pmatrix} 7.5 & 0 \\ 0 & 4.5 \end{pmatrix},$$
$$R_{l1}^3 = R_{r1}^3 = \begin{pmatrix} 5.0 & 0 \\ 0 & 3.0 \end{pmatrix} \quad (31)$$

Under the control gains of (30)-(31), we first simulate the behavior of the tele-robotic system in free motion when operators apply constant forces. These force profiles are shown in Fig. 3 while the resultant position trajectories of the local and remote manipulators are shown in Fig. 4. It can be observed that the proposed tele-robotic system remains stable in the presence of time varying delays and the remote manipulator displays the desired response. The corresponding control inputs of the manipulators are also shown in Fig. 5. We have also investigated the operation of tele-robotic system when the remote manipulator comes in contact with the environment. For this purpose, the parameters of the remote environment are assumed as:

$$K_{re} = \begin{pmatrix} 100.0 & 0 \\ 0 & 100.0 \end{pmatrix}, B_{re} = \begin{pmatrix} 10.0 & 0 \\ 0 & 10.0 \end{pmatrix} \quad (32)$$

The interaction of the remote manipulator with the environment exists for 100sec in simulations starting at $t = 150s$ as can be seen in Fig. 6. It can be observed that during the interaction period, tele-robotic system is able to maintain its stability as the position signals do not diverge. However, remote manipulator fails to follow the desired position references. In fact, the analysis of (9) in steady state reveals that the position error is inevitable during the contact motion. In addition, the local manipulators do receive the force feedback from the remote environment as their position signals show a decrease when the remote manipulator hits the environment.

The response of the time-delayed tele-robotic system is also observed under the application of more realistic operators' forces which are shown in Fig. 7. The results for this simulation are shown in Fig. 8 which clearly demonstrates that the tele-robotic system remains stable and the remote manipulator successfully tracks the desired position references.

VI. CONCLUSIONS

We have presented the design of a state convergence based multilateral tele-robotic system where remote manipulator can track the combined reference position of the local manipulators in the presence of time varying delays. Using a simplified form of the extended state convergence architecture and Lyapunov-Krasovskii theory, a set of design inequalities are obtained which along with the known information on the bounds of time varying delays and authority factors are solved to get control gains of the tele-robotic system. MATLAB simulations are finally carried out on a two degrees-of-freedom tri-master-single-slave nonlinear tele-robotic system and the results have shown the validity of the proposed multilateral control scheme. Future work involves improving the operators' perception of the remote environment and extending the proposed scheme to cover any number of remote manipulators. Experimental results are also planned as a part of future study.

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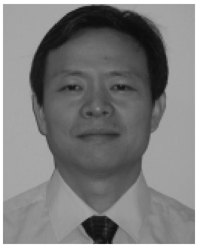

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