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Energy-Efficiency Maximization for Secure Multiuser MIMO SWIPT Systems With CSI Uncertainty

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ABSTRACT This paper studies the secure transmission issue for simultaneous wireless information and power transfer in a multiuser multiple-input-multiple-output system with multiple external eavesdroppers, where the transmitter broadcasts independent confidential messages to different legitimate receivers. Each receiver can be seen as an internal eavesdropper intended by other receivers. Our objective is to achieve the robust beamforming design under imperfect channel state information, in which the total transmission power is minimized with constraints on the achievable secrecy rate and the energy harvesting. Since the problem is nonconvex, we propose a two-level optimization scheme. For the inner problem, we investigate two conservative relaxation approaches, large-deviation inequality and Bernstein-type inequality (BTI), to reformulate the outage secrecy rate constraints into some convex ones, yielding a convex optimization by semidefinite programming (SDP) relaxation. For the outer problem, it is a K-variable optimization problem, which can be solved via the novel line-dimensional-like search method. Moreover, we characterize the rank profile of SDP relaxed solution for these two approaches. Specifically, the optimal solution is proved to be rank-one. Numerical results are provided to verify the performance of the proposed algorithms, where the LDI-based scheme outperforms the BTI-based scheme in terms of energy efficiency.

INDEX TERMS Imperfect CSI, multiuser MIMO, simultaneous wireless information and power transfer (SWIPT), secure transmission.

I. INTRODUCTION

In recent years, simultaneous wireless information and power transfer (SWIPT) has been considered as a promising solution to mitigating the energy scarcity problem in wireless communication networks. With the SWIPT technology, the available radio frequency (RF) signal transmitted by base station (BS) is not only decoded by the receivers for information but also used to charge the receivers' batteries. In most existing works, two practical receiver schemes, time switching (TS) [1] and power splitting (PS) [2], have been investigated for SWIPT. In TS, the receiver performs information decoding (ID) and energy harvesting (EH) in different time slots. In PS, the receiver splits the received signal into two partions with a power ratio so as to decode the information and harvest the energy. In general, PS presents a better performance in terms of rate-energy trade-off than TS [2]–[4]. Thus, we consider PS in this paper.

Unlike conventional wireless communication networks, the transmitter in SWIPT systems needs to increase the transmission power of the information-bearing signal to meet the requirements of energy harvesting at the receivers. This may cause the potential information leakage and makes the wireless information more susceptible to the external eavesdropping due to the broadcast nature of the wireless channel. Also, to further improve the energy harvesting efficiency, the energy receivers (ERs) are often placed closer to the transmitter than the co-located receivers (CRs) [5]. This leads to an unexpected security risk that the transmitted confidential message intended for CRs may be eavesdropped by the ERs.

Thus, it is important to guarantee the security in SWIPT systems. On one hand, the energy harvesting requirements of the ERs have to be satisfied for efficient harvesting. On the other hand, the transmit beamforming should be optimally designed to ensure that the confidential information could be securely delivered to the CRs even in the presence of potential eavesdropping from the ERs [6]. There have been a great deal of encryption algorithms to increase the security performance of the wireless networks. However, they might not be applicable to the SWIPT systems as they increase the computional complexity and also may underestimate the computational capability at the adversaries [7].

Recently, physical layer security as an alternative technology has been investigated to ensure the security of SWIPT [6], [8], [9]. Liu *et al.* [6] investigated a MISO SWIPT systems, where they formulated two optimization problems: 1) the secrecy rate maximization for the information receiver subject to individual harvested energy constraints of energy receivers, 2) the weighted sum energy maximization for the energy receivers subject to a secrecy rate constraint on information receiver. Both problems could be relaxed via semidefinite programming (SDP) relaxation. The corresponding optimal transmit beamforming and power allocation were designed. In [8], the secrecy rate maximization was achieved by jointly designing AN-aided transmission and PS scheme subject to the transmission power constraint and energy harvesting constraint. Zhang *et al.* [9] focused on the total transmission power minimization while guaranteeing the secrecy rate and energy harvesting constraints at each receiver in multiuser SWIPT MISO systems, where they proposed a low-complexity suboptimal algorithm based on the genetic algorithm to solve the problem.

All these works assume that the transmitter has perfect CSI. However, this may not be possible in some applications. For example, due to the channel delay, the estimated CSI at the transmitter is often outdated. Hence, Chu *et al.* [10] and Ng *et al.* [11] considered the channel uncertainty model for SWIPT. Chu *et al.* [10] considered the MISO SWIPT scenario and studied the secrecy rate maximization problem by designing the optimal transmit beamforming in two cases: with AN and without AN. In [11], the security with SWIPT was addressed in the presence of passive eavesdroppers and potential eavesdroppers (idle legitimate receivers). Note that the existing works [6], [8]–[11] consider the case where the transmitter only sends a single confidential message. In multiuser MIMO SWIPT downlink, this is not the case. Furthermore, other works studied the worst-case robust approaches [12]–[14], in which the channel errors were assumed to be specified within a deterministic bounded set. This might not always be feasible when considering the randomness of the channel estimation error. Hence, the transmitter may not obtain the deterministic channel model perfectly.

For multiuser MIMO SWIPT broadcast systems, all other users could be seen as the internal eavesdroppers. Also, most existing works focus on the external eavesdroppers. However, few works take into account potential eavesdropping from inside, especially in homogeneous network. This brings a new challenge to guarantee the security of the multiuser MIMO SWIPT system.

In this paper, we consider a secure multiuser MIMO SWIPT system, where a user broadcasts independent confidential messages to different receivers and attempts to keep the internal/external eavesdroppers from the secret messages. The main contributions can be summarized:

1) First, we aim to achieve the robust beamforming design with CSI uncertainty. We minimize the total transmission power with constraints on the secrecy rate and the energy harvesting. For the inaccurate channel estimation, statistical information of channel uncertainty is available at the transmitter.
2) We propose a two-level optimization scheme to tackle with the nonconvex minimization problem. It first can be relaxed via SDP relaxation. Then, we consider two conservative approximation approaches: LDI and BTI, to reformulate the nonconvex constraints into the linear matrix inequalities (LMI) and the second-order cone (SOC) constraint. The outer problem is a K-variable optimization problem, which can be solved via novel line-dimensional-like search (LDLS) method. Moreover, we characterize the rank profile of the SDP relaxed solution for these two approaches to show the tightness of SDP relaxation.

The rest of the paper is organized as follows: Section II presents the system model and the problem formulation. The robust beamforming design is given in Section III. Section IV presents the simulation results to evaluate the proposed scheme. Conclusions are drawn in Section V.

Notation: In this paper, the upper case boldface letters and the lower case boldface represent the matrices and the vectors, respectively. $\mathbb{C}^N$ stands for N-dimensional complex vector. $\mathbb{H}$ is the set of $N \times N$ complex Hermitian matrix. $(\cdot)^H$ is the conjugate transpose. $\mathrm{Tr}(\cdot)$ stands for the trace of a matrix and $\mathbb{E}(\cdot)$ is the expectation operation. $\|\cdot\|$ is the Euclidean norm and $\|\cdot\|_F$ denotes the Frobenius norm. $\mathrm{Vec}(\mathbf{A})$ is the vector-version of the matrix A and $\otimes$ is the Kronecker product. $\mathbf{A} \succeq \mathbf{0}$ denotes that $\mathbf{A}$ is a positive semidefinite matrix. $\mathbf{I}$ represents the identity matrix and $\lambda_{\max}(\cdot)$ is the maximum eigenvalue. $\Re\{\cdot\}$ stands for the real part of a complex number. $\mathbf{x} \sim \mathcal{CN}(\mathbf{s}, \Sigma)$ means the random vector $\mathbf{x}$ follows from the complex circularly symmetric Gaussian distribution with the mean vector $\mathbf{s}$ and the variance matrix Σ.

II. SYSTEM MODEL

A. SYSTEM MODEL

We consider SWIPT in a multiuser MIMO broadcast system as shown in Figure 1, which consists of one transmitter equipped with N_t ($N_t > 1$) antennas, K single-antenna CRs and L single-antenna ERs. In this model, it is assumed that the ERs are closer to the transmitter than the CRs so as to harvest energy from the transmitter well. We consider the

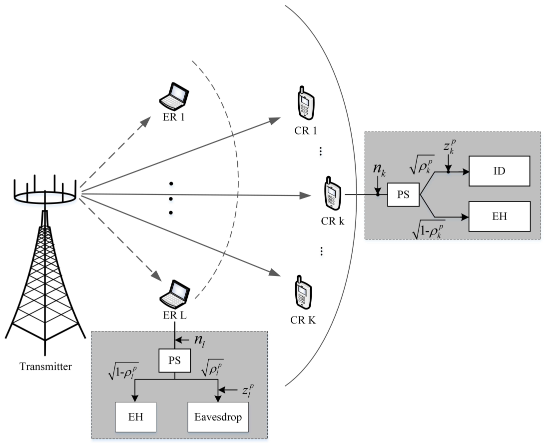

FIGURE 1. **Illustration of a multiuser MIMO SWIPT system.**

homogeneous users. The transmitter simultaneously broadcasts K independent confidential messages to the kth CR only. In this case, each CR could be seen as a internal eavesdropper interested in the messages for others. While harvesting the energy, the ERs could be seen as the external eavesdroppers, and behave passively to eavesdrop the confidential messages. For convenience, we denote $\mathcal{C} = \{1, \ldots, K\}$ and $\mathcal{L} = \{1, \ldots, L\}$ as the index sets of all the CRs and ERs, Therefore, the received signals at the kth CR and the lth ER can be expressed as:

$$y_k = \mathbf{h}_k^H \sum_{i=1}^{K} \mathbf{q}_i s_i + n_k, \quad \text{for } k \in \mathcal{C}. \tag{1}$$

$$y_l = \mathbf{h}_l^H \sum_{i=1}^{K} \mathbf{q}_i s_i + n_l, \quad \text{for } l \in \mathcal{L}, \tag{2}$$

where $\mathbf{h}_k \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{h}_l \in \mathbb{C}^{N_t \times 1}$ represent the channel vectors from the transmitter to the kth CR and to the lth ER, respectively. s_i denotes the confidential message intended for the ith CR with $\mathbb{E}\{\|s_i\|^2\} = 1$, and $\mathbf{q}_i \in \mathbb{C}^{N_t \times 1}$ is the corresponding precoding vector. $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ and $n_l \sim \mathcal{CN}(0, \sigma_l^2)$ represent the additive white Gaussian noise vectors at the kth CR and the lth ER, respectively.

In this system, each CR is assumed to adopt PS to decode the information and harvest energy from the received signal. Specifically, for each CR, the received signal is split by the PS ratio ρ_k^p $(0 < \rho_k^p < 1)$, ρ_k^p portion of the signal power for ID and the remaining $1 - \rho_k^p$ for EH. Thus, the equivalent signal for ID at the kth CR are:

$$y_k^{\text{ID}} = \sqrt{\rho_k^p}\left(\mathbf{h}_k^H \sum_{i=1}^{K} \mathbf{q}_i s_i + n_k\right) + z_k^p, \quad \text{for } k \in \mathcal{C}, \tag{3}$$

where $z_k^p \sim \mathcal{CN}(0, \sigma_{p,k}^2)$ is the additive Gaussian noise caused by the power splitting. Hence, the SINR at the kth CR is written as:

$$\Gamma_k = \frac{\rho_k^p |\mathbf{h}_k^H \mathbf{q}_k|^2}{\rho_k^p (\sum_{i \neq k}^{K} |\mathbf{h}_k^H \mathbf{q}_i|^2 + \sigma_k^2) + \sigma_{p,k}^2}. \tag{4}$$

Due to the inherent openness of the wireless channel, the internal CR eavesdroppers could jointly perform the eavesdropping. This may occur when the confidential message intended for the kth CR is eavesdropped by the other $K - 1$ CRs. In this paper, to guarantee the secrecy, we consider the worst-case scenario. First, all the $i \neq k$ CRs cooperate maliciously to eavesdrop the message s_k. This means the other $K - 1$ CRs can be equivalently treated as a single eavesdropper $\tilde{k}$ with $K - 1$ receive antennas [15]. In addition, the eavesdropper doesn't perform the EH process, i.e., $\rho_{\tilde{k}}^p = 1$, to increase its eavesdropping capability. Second, it is assumed that the collusive eavesdropper $\tilde{k}$ can remove the inter-user interference by using the successive interference cancellation [9], [11], [16]. Therefore, the signal $\tilde{\mathbf{y}}_k$ at the eavesdropper $\tilde{k}$ can be equivalently expressed as:

$$\mathbf{y}_{\tilde{k}} = \mathbf{H}_{\tilde{k}}^H \mathbf{q}_k s_k + \mathbf{n}_{\tilde{k}}, \tag{5}$$

where $\mathbf{H}_{\tilde{k}} = [\mathbf{h}_1, \ldots, \mathbf{h}_{k-1}, \mathbf{h}_{k+1}, \ldots, \mathbf{h}_K]$ represents the channel matrix for the equivalent eavesdropper $\tilde{k}$, and $\mathbf{n}_{\tilde{k}} = [n_1 + n_1^p, \ldots, n_{k-1} + n_{k-1}^p, n_{k+1} + n_{k+1}^p, \ldots, n_K + n_K^p]$ is the Gaussian noise vector. Thus, the SINR $\Gamma_{\tilde{k}}$ for the internal CR eavesdropper $\tilde{k}$ is given by:

$$\Gamma_{\tilde{k}} = \frac{\|\mathbf{H}_{\tilde{k}}^H \mathbf{q}_k\|^2}{\mathrm{Tr}(\mathbf{W})}, \tag{6}$$

where $\mathrm{Tr}(\mathbf{W}) = \mathrm{Diag}[\sigma_1^2 + \sigma_{p,1}^2, \ldots, \sigma_{k-1}^2 + \sigma_{p,k-1}^2, \sigma_{k+1}^2 + \sigma_{p,k+1}^2, \ldots, \sigma_K^2 + \sigma_{p,K}^2]$.

In addition, the energy signal harvested by the kth CR can be expressed as:

$$y_k^{\text{EH}} = \sqrt{1 - \rho_k^p}\left(\mathbf{h}_k^H \sum_{i=1}^{K} \mathbf{q}_i s_i + n_k\right), \quad \text{for } k \in \mathcal{C}. \tag{7}$$

On the other hand, due to the broadcast nature of the wireless channel, the confidential messages can also be overheard by the ERs. In this system, we assume each ER adopts the power splitting with the PS ratio ρ_l^p $(0 < \rho_l^p < 1)$ to eavesdrop the received signal while harvesting energy. In this case, the received signal at the lth ER in (2) can be rewritten as:

$$y_l^{\text{Eav}} = \sqrt{\rho_l^p}\left(\mathbf{h}_l^H \sum_{i=1}^{K} \mathbf{q}_i s_i + n_l\right) + z_l^p, \quad \text{for } l \in \mathcal{L}, \tag{8}$$

$$y_l^{\text{EH}} = \sqrt{1 - \rho_l^p}\left(\mathbf{h}_l^H \sum_{i=1}^{K} \mathbf{q}_i s_i + n_l\right), \quad \text{for } l \in \mathcal{L}, \tag{9}$$

where $z_l^p \sim \mathcal{CN}(0, \sigma_{p,l}^2)$ is the additive Gaussian noise caused by the power splitting. Thus, for the confidential message intended for the kth CR, the SINR at the lth ER can be equivalently expressed as:

$$\Gamma_l^k = \frac{|\mathbf{h}_l^H \mathbf{q}_k|^2}{\sum_{i \neq k}^{K} |\mathbf{h}_l^H \mathbf{q}_i|^2 + \sigma_l^2 + \frac{\sigma_{p,l}^2}{\rho_l^p}}, \quad \forall l. \tag{10}$$

Similar to [17] and [18], the achievable secrecy rate at the kth CR can be expressed as

$$R_k = \left[\log_2(1+\Gamma_k) - \log_2\left(1+\max_{\forall \tilde{k},\forall l}(\Gamma_{\tilde{k}}, \Gamma_l^k)\right)\right]^+. \quad (11)$$

The harvested energy at the kth CR and the lth ER is:

$$E_{p,k} = \eta_k(1-\rho_k^p)\left(\sum_{i=1}^{K} |\mathbf{h}_k^H \mathbf{q}_i|^2 + \sigma_k^2\right), \quad (12)$$

$$E_l = \xi_l(1-\rho_l^p)\left(\sum_{i=1}^{K} |\mathbf{h}_l^H \mathbf{q}_i|^2 + \sigma_l^2\right), \quad (13)$$

respectively, where $0 < \eta_k \leq 1$ and $0 < \xi_l \leq 1$ represent the harvesting efficiency at the kth CR and the lth ER, respectively, which are set to be 1 for simplicity.

For SWIPT, a commonly used criteria for secure communication is to minimize the total transmission power subject to the constraints on the achievable secrecy rate and the energy harvesting at all the receivers as [11]:

$$\begin{aligned} \min_{\mathbf{q}_k, \rho_k^p, \forall k} \quad & \sum_{k=1}^{K} \|\mathbf{q}_k\|^2 \\ \text{s.t.} \quad & R_k \geq \bar{R}_k, \quad \forall k \in \mathcal{C}, \\ & E_i \geq \bar{E}_i \quad \forall i \in \{\mathcal{C}, \mathcal{L}\}, \\ & 0 < \rho_k^p < 1, \quad \forall k, \quad \sum_{k=1}^{K} \|\mathbf{q}_k\|^2 \leq P, \end{aligned} \quad (14)$$

where $\bar{R}_k > 0$, $\bar{E}_i > 0$ are the corresponding secrecy and energy requirements at the receivers, and P is the maximum allowable power.

Remark: Note that our optimization aims to minimize the transmission power with guaranteed secrecy rate in an unreliable network. In this case, since other users are hostile, there is no need to minimize the interference to other users for the kth user and hence to keep the preceding vectors orthogonal. In some previous works, a reliable network was considered so that the minimization of interference or the orthogonality of the precoding vectors were more important. This is a different optimization problem with different objectives, compared with ours. Also, we assume that the other $K-1$ users can adopt interference cancellation if they want. Thus, the orthogonality of the precoding vectors is not considered in (14).

B. CSI UNCERTAINTY AND PROBLEM FORMULATION

One factor that could significantly affect the above transmission power minimization is the availability of CSI between the transmitter and the receiver. Due to the limitation of the channel estimation, it is usually hard to achieve perfect CSI in practical applications [19]–[21]. As a result, the CSI uncertainty or error must be taken into consideration to achieve optimal beamforming for secure communications. In this paper, we use the statistical channel uncertainties for each receiver, which are determined as:

$$\mathbf{h}_k = \bar{\mathbf{h}}_k + \Delta\mathbf{h}_k, \quad \forall k, \quad (15)$$

$$\mathbf{h}_l = \bar{\mathbf{h}}_l + \Delta\mathbf{h}_l, \quad \forall l, \quad (16)$$

where $\bar{\mathbf{h}}_k \in \mathbb{C}^{N_t \times 1}$ and $\bar{\mathbf{h}}_l \in \mathbb{C}^{N_t \times 1}$ are the estimated channel vectors for the kth CR and lth ER, respectively. $\Delta\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_k)$ and $\Delta\mathbf{h}_l \sim \mathcal{CN}(\mathbf{0}, \mathbf{P}_l)$ are the corresponding statistical errors, where $\mathbf{R}_k$ and $\mathbf{P}_l$ are positive semidefinite matrices.

For each internal eavesdropper $\tilde{k}$, the channel uncertainty is defined as:

$$\mathbf{H}_{\tilde{k}} = \bar{\mathbf{H}}_{\tilde{k}} + \Delta\mathbf{H}_{\tilde{k}}, \quad \forall \tilde{k}, \quad (17)$$

where $\bar{\mathbf{H}}_{\tilde{k}} \in \mathbb{C}^{N_t \times (K-1)}$ is the estimated channel vectors, and $\bar{\mathbf{H}}_{\tilde{k}} = [\bar{\mathbf{h}}_1, \ldots, \bar{\mathbf{h}}_{k-1}, \bar{\mathbf{h}}_{k+1}, \ldots, \bar{\mathbf{h}}_K]$. $\Delta\mathbf{H}_{\tilde{k}}$ is the corresponding channel error, and $\Delta\mathbf{H}_{\tilde{k}} = [\Delta\mathbf{h}_1, \ldots, \Delta\mathbf{h}_{k-1}, \Delta\mathbf{h}_{k+1}, \ldots, \Delta\mathbf{h}_K]$.

In general, imperfect CSI knowledge would definitely increase the difficulty of a transmitter design. It may not always achieve the secure constraints. Fortunately, occasional outage without affecting users' service can be considered as an alternative solution. Therefore, it is reasonable to design the optimal robust beamforming to meet the secrecy rate requirements with high probability. Thus, we aim to minimize the total transmission power while maintaining the outage constraints under the imperfect CSI:

$$\min_{\mathbf{q}_k, \rho_k^p, \forall k} \quad \sum_{k=1}^{K} \|\mathbf{q}_k\|^2 \quad (18a)$$

$$\text{s.t.} \quad \Pr\{R_k \geq \bar{R}_k\} \geq 1-\rho_k, \quad \forall k \in \mathcal{C}, \quad (18b)$$

$$\Pr\{E_i \geq \bar{E}_i\} \geq 1-\rho_i, \quad \forall i \in \{\mathcal{C}, \mathcal{L}\}, \quad (18c)$$

$$(15), (16), (17), \quad (18d)$$

$$0 < \rho_k \leq 1, \quad 0 < \rho_i \leq 1, \quad (18e)$$

$$0 < \rho_k^p < 1, \quad \forall k, \quad \sum_{k=1}^{K} \|\mathbf{q}_k\|^2 \leq P, \quad (18f)$$

where (18b) represents the outage secrecy rate for the information decoding at each CR, (18c) is the outage energy harvesting constraint to ensure energy harvested successfully at the CRs and the ERs. ρ_k, ρ_i are the maximum allowable outage secrecy probabilities for (18b), (18c), respectively. In addition, the constraint (18b) guarantees that the kth CR could reliably decode the information at least $(1-\rho_k) \times 100\%$ of the time. Similarly, each CR/ER could successfully harvest the minimum power at least $(1-\rho_i) \times 100\%$ of the time under the constraint (18c).

Note that, due to the coupled optimization variables $\mathbf{q}_k$, ρ_k^p, the complicated outage constraints and the channel errors as well as the quadratic terms of $\mathbf{q}_k$, the problem shown in (18) is not convex and intractable.

III. ROBUST BEAMFORMING DESIGN

In this section, we present the robust beamforming design, for which we develop a two-level optimization scheme to deal with the non-convex problem in (18). We first use two approximation approaches to reformulate the inner problem into a tractable convex one, and then propose the LDLS scheme to achieve the robust beamforming design.

A. TWO-LEVEL OPTIMIZATION PROBLEM

By introducing the slack variables $\eta_k, \forall k$ and fixing $\mathbf{Q}_k = \mathbf{q}_k \mathbf{q}_k^H$, the problem in (18) can be expressed as:

$$\min_{\mathbf{Q}_k, \rho_k^p, \forall k} \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{Q}_k) \tag{19a}$$

$$\text{s.t. } \Pr\left\{ \frac{\mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k}{\sum_{i \neq k}^{K} \mathbf{h}_k^H \mathbf{Q}_i \mathbf{h}_k + \sigma_k^2 + \frac{\sigma_{p,k}^2}{\rho_k^p}} \geq t_k \right\} \geq 1 - \rho_k, \quad \forall k \tag{19b}$$

$$\frac{\mathrm{Tr}\left(\mathbf{H}_{\tilde{k}}^H \mathbf{Q}_k \mathbf{H}_{\tilde{k}}\right)}{\mathrm{Tr}(\mathbf{W})} \leq \eta_k - 1, \quad \forall \tilde{k} \tag{19c}$$

$$\frac{\mathbf{h}_l^H \mathbf{Q}_k \mathbf{h}_l}{\sum_{i \neq k}^{K} \mathbf{h}_l^H \mathbf{Q}_i \mathbf{h}_l + \sigma_l^2 + \frac{\sigma_{p,l}^2}{\rho_l^p}} \leq \eta_k - 1, \quad \forall l, \tag{19d}$$

$$\Pr\left\{ \sum_{i=1}^{K} \mathbf{h}_k^H \mathbf{Q}_i \mathbf{h}_k + \sigma_k^2 \geq \frac{\bar{E}_{p,k}}{1 - \rho_k^p} \right\} \geq 1 - \rho_{p,k}, \quad \forall k \tag{19e}$$

$$\Pr\left\{ \sum_{i=1}^{K} \mathbf{h}_l^H \mathbf{Q}_i \mathbf{h}_l + \sigma_l^2 \geq \frac{\bar{E}_l}{1 - \rho_l^p} \right\} \geq 1 - \rho_l, \quad \forall l \tag{19f}$$

$$(18e), \quad 0 < \rho_k^p < 1, \ \sum_{i=1}^{K} \mathrm{Tr}(\mathbf{Q}_k) \leq P, \tag{19g}$$

$$(15), (16), (17), \mathrm{rank}(\mathbf{Q}_k) = 1, \quad \mathbf{Q}_k \succeq \mathbf{0}, \ \forall k, \tag{19h}$$

where $t_k = 2^{\bar{R}_k} \eta_k - 1$. Furthermore, $\eta_k, \forall k$ represents the maximum achievable rate for the internal and external eavesdroppers. Thus, $\eta_k, \forall k$ plays a crucial role to balance the system security.

Nevertheless, the problem in (19) is still not convex because of the quadratic-over-quadratic constraints, the outage constraints, the rank-one restrictions and the channel errors. To make this problem tractable, we propose a two-level optimization algorithm. Specifically, the problem (19) can be equivalently recast into:

$$\max_{\eta_1, \ldots, \eta_K} \begin{cases} \min_{\mathbf{Q}_k, \rho_k^p, \forall k} \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{Q}_k) \\ \text{s.t. } (19b) - (19h). \end{cases} \tag{20}$$

In the first level, we use two convex approximate approaches: LDI-based approach and BTI-based approach, to solve the inner minimization problem in (20) for any given sets of $\eta_k, \forall k$. In the second level, we propose a line-dimensional-like search scheme which leads to the solution to the maximization in (20).

B. SAFE CONVEX APPROXIMATIONS FOR THE INNER PROBLEM

In this subsection, we turn our attention to the inner minimization problem based on imperfect CSI, for which we propose two conservative approaches: LDI-based approach and BTI-based approach. For any given sets of $\eta_k, \forall k$, we first drop the rank-one constraints and replace $\mathbf{h}_k$, $\mathbf{h}_l$ and $\mathbf{H}_{\tilde{k}}$ with (15), (16), (17), respectively. Consequently, the inner optimal problem can be further rewritten as:

$$\min_{\mathbf{Q}_k, \rho_k^p, \forall k} \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{Q}_k) \tag{21a}$$

$$\text{s.t. } \Pr\Big\{ \Delta\mathbf{h}_k^H \Theta_k \Delta\mathbf{h}_k + 2\Re(\Delta\mathbf{h}_k^H \Theta_k \bar{\mathbf{h}}_k) + \bar{\mathbf{h}}_k^H \Theta_k \bar{\mathbf{h}}_k - t_k \sigma_k^2 - \frac{t_k \sigma_{p,k}^2}{\rho_k^p} \geq 0 \Big\} \geq 1 - \rho_k, \quad \forall k \tag{21b}$$

$$\frac{\mathrm{Tr}\left[\left(\bar{\mathbf{H}}_{\tilde{k}} + \Delta\mathbf{H}_{\tilde{k}}\right)^H \mathbf{Q}_k \left(\bar{\mathbf{H}}_{\tilde{k}} + \Delta\mathbf{H}_{\tilde{k}}\right)\right]}{\mathrm{Tr}(\mathbf{W})} \leq \eta_k - 1, \quad \forall \tilde{k} \tag{21c}$$

$$\Delta\mathbf{h}_l^H \Phi_k \Delta\mathbf{h}_l + 2\Re(\Delta\mathbf{h}_l^H \Phi_k \bar{\mathbf{h}}_l) + \bar{\mathbf{h}}_l^H \Phi_k \bar{\mathbf{h}}_l - (\sigma_l^2 + \frac{\sigma_{p,l}^2}{\rho_l^p})(\eta_k - 1) \leq 0, \quad \forall l \tag{21d}$$

$$\Pr\Big\{ \sum_{i=1}^{K} \Delta\mathbf{h}_k^H \mathbf{Q}_i \Delta\mathbf{h}_k + 2\Re(\sum_{i=1}^{K} \Delta\mathbf{h}_k^H \mathbf{Q}_i \bar{\mathbf{h}}_k) + \sum_{i=1}^{K} \bar{\mathbf{h}}_k^H \mathbf{Q}_i \bar{\mathbf{h}}_k + \sigma_k^2 \geq \frac{\bar{E}_{p,k}}{1 - \rho_k^p} \Big\} \geq 1 - \rho_{p,k}, \quad \forall k \tag{21e}$$

$$\Pr\Big\{ \sum_{i=1}^{K} \Delta\mathbf{h}_l^H \mathbf{Q}_i \Delta\mathbf{h}_l + 2\Re(\sum_{i=1}^{K} \Delta\mathbf{h}_l^H \mathbf{Q}_i \bar{\mathbf{h}}_l) + \sum_{i=1}^{K} \bar{\mathbf{h}}_l^H \mathbf{Q}_i \bar{\mathbf{h}}_l + \sigma_l^2 \geq \frac{\bar{E}_l}{1 - \rho_l^p} \Big\} \geq 1 - \rho_l, \quad \forall l. \tag{21f}$$

$$(19g), \quad \mathbf{Q}_k \succeq \mathbf{0}, \ \forall k, \tag{21g}$$

where $\Theta_k = \mathbf{Q}_k - t_k \sum_{i \neq k}^{K} \mathbf{Q}_i$, $\Phi_k = \mathbf{Q}_k - (\eta_k - 1) \sum_{i \neq k}^{K} \mathbf{Q}_i$. The problem in (21) is still non-convex due to the outage constraints and the channel errors.

Since $\Delta\mathbf{h}_k$ is given by $\Delta\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_k)$, it can be derived as $\Delta\mathbf{h}_k = \mathbf{R}_k^{\frac{1}{2}} \mathbf{u}_k$, where $\mathbf{u}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. Hence, (21b) can be further transformed as:

$$\Pr\Big\{ \mathbf{u}_k^H (\mathbf{R}_k^{\frac{1}{2}} \Theta_k \mathbf{R}_k^{\frac{1}{2}}) \mathbf{u}_k + 2\Re\big[\mathbf{u}_k^H (\mathbf{R}_k^{\frac{1}{2}} \Theta_k \bar{\mathbf{h}}_k)\big] + \bar{\mathbf{h}}_k^H \Theta_k \bar{\mathbf{h}}_k - t_k(\sigma_k^2 + \frac{\sigma_{p,k}^2}{\rho_k^p}) \geq 0 \Big\} \geq 1 - \rho_k, \quad \forall k. \tag{22}$$

Similarly, (21e) can be reformulated as:

$$\Pr\Big\{\mathbf{u}_k^H\Big[\mathbf{R}_k^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\mathbf{R}_k^{\frac{1}{2}}\Big]\mathbf{u}_k + 2\Re\Big[\mathbf{u}_k^H\mathbf{R}_k^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\bar{\mathbf{h}}_k\Big] + \sum_{i=1}^{K}\bar{\mathbf{h}}_k^H\mathbf{Q}_i\bar{\mathbf{h}}_k + \sigma_k^2 - \frac{\bar{E}_{p,k}}{1-\rho_k^p} \geq 0\Big\} \geq 1-\rho_{p,k}, \quad \forall k. \tag{23}$$

For $\Delta\mathbf{h}_l \sim \mathcal{CN}(\mathbf{0},\mathbf{P}_l)$, we have $\Delta\mathbf{h}_l = \mathbf{P}_l^{\frac{1}{2}}\mathbf{v}_l$, where $\mathbf{v}_l \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$. Thus, (21f) and (21d) can be rewritten as, respectively

$$\Pr\Big\{\mathbf{v}_l^H\Big[\mathbf{P}_l^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\mathbf{P}_l^{\frac{1}{2}}\Big]\mathbf{v}_l + 2\Re\Big[\mathbf{v}_l^H\mathbf{P}_l^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\bar{\mathbf{h}}_l\Big] + \sum_{i=1}^{K}\bar{\mathbf{h}}_l^H\mathbf{Q}_i\bar{\mathbf{h}}_l + \sigma_l^2 - \frac{\bar{E}_l}{1-\rho_l^p} \geq 0\Big\} \geq 1-\rho_l, \quad \forall l, \tag{24}$$

$$\mathbf{v}_l^H(\mathbf{P}_l^{\frac{1}{2}}\Phi_k\mathbf{P}_l^{\frac{1}{2}})\mathbf{v}_l + 2\Re\Big[\mathbf{v}_l^H(\mathbf{P}_l^{\frac{1}{2}}\Phi_k\bar{\mathbf{h}}_l)\Big] + \bar{\mathbf{h}}_l^H\Phi_k\bar{\mathbf{h}}_l - (\sigma_l^2 + \frac{\sigma_{p,l}^2}{\rho_l^p})(\eta_k - 1) \leq 0, \quad \forall l. \tag{25}$$

In the following, we tackle with these outage constraints by using some safe convex approximations based on the LDI-based approach and the BTI-based approach.

1) LDI-BASED APPROACH

In this subsection, we develop convex restriction approximations by using the decomposition of LDI-based approach ([22, Lemma 2]), which is very useful for complex Gaussian quadratic functions. Using this approach and performing some mathematical manipulations, (22) can be relaxed into:

$$\mathrm{Tr}\Big(\mathbf{R}_k^{\frac{1}{2}}\Theta_k\mathbf{R}_k^{\frac{1}{2}}\Big) + \bar{\mathbf{h}}_k^H\Theta_k\bar{\mathbf{h}}_k - t_k(\sigma_k^2 + \frac{\sigma_{p,k}^2}{\rho_k^p}) \geq 2\sqrt{-\ln(\rho_k)}(\tau_k + \psi_k), \tag{26a}$$

$$\frac{1}{\sqrt{2}}\Big\|\mathbf{R}_k^{\frac{1}{2}}\Theta_k\bar{\mathbf{h}}_k\Big\| \leq \tau_k, \tag{26b}$$

$$\theta_k\Big\|\mathbf{R}_k^{\frac{1}{2}}\Theta_k\mathbf{R}_k^{\frac{1}{2}}\Big\|_F \leq \psi_k, \quad \forall k, \tag{26c}$$

where τ_k, ψ_k and θ_k are the slack variables. Here, $\theta_k \in (\frac{1}{\sqrt{2}}, \infty)$ satisfys the equation $\big[1 - 1/(2\theta_k^2)\big]\theta_k = \sqrt{-\ln(\rho_k)}$. Interestingly, the constraint (26) is tractable and computationally efficient in terms of the LMIs and the SOC constraints.

Using a similar approach and the mathematical analysis, the constraint (23) can be expressed as:

$$\mathrm{Tr}\Big[\mathbf{R}_k^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\mathbf{R}_k^{\frac{1}{2}}\Big] + \bar{\mathbf{h}}_k^H\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\bar{\mathbf{h}}_k + \sigma_k^2 - \frac{\bar{E}_{p,k}}{1-\rho_k^p} \geq 2\sqrt{-\ln(\rho_{p,k})}(\tau_k^p + \psi_k^p), \tag{27a}$$

$$\frac{1}{\sqrt{2}}\Big\|\mathbf{R}_k^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\bar{\mathbf{h}}_k\Big\| \leq \tau_k^p, \tag{27b}$$

$$\theta_k^p\Big\|\mathbf{R}_k^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\mathbf{R}_k^{\frac{1}{2}}\Big\|_F \leq \psi_k^p, \quad \forall k, \tag{27c}$$

where τ_k^p, ψ_k^p and θ_k^p are the slack variables.

Similarly, we express the constaint (24) as:

$$\mathrm{Tr}\Big[\mathbf{P}_l^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\mathbf{P}_l^{\frac{1}{2}}\Big] + \bar{\mathbf{h}}_l^H\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\bar{\mathbf{h}}_l + \sigma_l^2 - \frac{\bar{E}_l}{1-\rho_l^p} \geq 2\sqrt{-\ln(\rho_l)}(\tau_l + \psi_l), \tag{28a}$$

$$\frac{1}{\sqrt{2}}\Big\|\mathbf{P}_l^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\bar{\mathbf{h}}_l\Big\| \leq \tau_l, \tag{28b}$$

$$\theta_l\Big\|\mathbf{P}_l^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\mathbf{P}_l^{\frac{1}{2}}\Big\|_F \leq \psi_l, \quad \forall l, \tag{28c}$$

where τ_l, ψ_l and θ_l are the slack variables.

The constraints (23) and (24) already violate the intractable outage restrictions and are computationally efficient using (27) and (28). Next, we consider the constraints (21c) and (25) for the internal/external eavesdroppers. Intuitively, they are hard to be reformulated by the LDI-based approach as they are not probabilistic outage constraints. In order to remove the channel errors, we first take *Sphere Bounding* method in [22] to specify the channel uncertainty region.

Thus, (22) can be expressed as:

$$\mathbf{u}_k^H\Big(\mathbf{R}_k^{\frac{1}{2}}\Theta_k\mathbf{R}_k^{\frac{1}{2}}\Big)\mathbf{u}_k + 2\Re\Big[\mathbf{u}_k^H\Big(\mathbf{R}_k^{\frac{1}{2}}\Theta_k\bar{\mathbf{h}}_k\Big)\Big] + s_k \geq 0, \quad \mathbf{u}_k^H\mathbf{u}_k - \xi_k^2 \leq 0, \quad \forall k, \tag{29}$$

where $s_k = \bar{\mathbf{h}}_k^H\Theta_k\bar{\mathbf{h}}_k - t_k(\sigma_k^2 + \frac{\sigma_{p,k}^2}{\rho_k^p})$. Interestingly, specifying the set of the channel uncertainty region $\mathcal{S} = \{\mathbf{u}_k|\Pr(\mathbf{u}_k^H\mathbf{u}_k \leq \xi_k^2) \geq 1-\rho_k\}, \forall k$ is sufficient to guarantee the outage constraint (22). Since $\mathbf{u}_k$ is defined as $\mathbf{u}_k \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$, it can be easily verified that $\|\mathbf{u}_k\|^2$ follows a *Chi-square* distribution with $2N_t$ degrees of freedom. Thus the channel uncertainty region holds for $\xi_k = \sqrt{\frac{F^{-1}_{\chi^2_{2N_t}}(1-\rho_k)}{2}}, \forall k$ [23], where $F^{-1}_{\chi^2_m}(a)$ denotes the inverse cumulative distribution function of a *Chi-square* random variable a with $m = 2N_t$ degrees of freedom. That is to say, the channel uncertainty region for the CRs can be specified into the ball $\mathcal{S}$ with a radius of ξ_k.

The constraint (21c) is non-convex due to the channel errors $\Delta\mathbf{H}_{\tilde{k}}, \forall\tilde{k}$. However, it is hard to remove the channel uncertainty $\Delta\mathbf{H}_{\tilde{k}}$ in (21c) using the LDI-based approach due to its non-probabilistic style. So we exploit the following matrix identities:

$$(\mathbf{A}\otimes\mathbf{B})^T = \mathbf{A}^T\otimes\mathbf{B}^T, \tag{30}$$

$$\mathrm{Tr}(\mathbf{A}^T\mathbf{B}) = \mathrm{Vec}(\mathbf{A})^T\mathrm{Vec}(\mathbf{B}), \quad (31)$$
$$\mathrm{Vec}(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})\mathrm{Vec}(\mathbf{X}), \quad (32)$$

to transform the constraint (21c). After some mathematical manipulations, we obtain:

$$\Delta\mathbf{h}_{\tilde{k}}^H(\mathbf{I}\otimes\mathbf{Q}_k)\Delta\mathbf{h}_{\tilde{k}} + 2\Re\left[\mathbf{h}_{\tilde{k}}^H(\mathbf{I}\otimes\mathbf{Q}_k)\Delta\mathbf{h}_{\tilde{k}}\right] + z_{\tilde{k}} \le 0, \quad (33)$$

where $\mathbf{h}_{\tilde{k}} = \mathrm{Vec}(\mathbf{H}_{\tilde{k}})$, $\Delta\mathbf{h}_{\tilde{k}} = \mathrm{Vec}(\Delta\mathbf{H}_{\tilde{k}})$, and $z_{\tilde{k}} = \mathbf{h}_{\tilde{k}}^H(\mathbf{I}\otimes\mathbf{Q}_k)\mathbf{h}_{\tilde{k}} - (\eta_k - 1)\mathrm{Tr}(\mathbf{W})$.

However, (33) is still non-convex due to the channel uncertainty $\Delta\mathbf{h}_{\tilde{k}}$. To deal with the challenge, it is necessary to specify the channel uncertainty region of $\Delta\mathbf{h}_{\tilde{k}}$, for which we consider the following definition of $\mathrm{Vec}(\Delta\mathbf{H}_{\tilde{k}})$, which is

$$\Delta\mathbf{h}_{\tilde{k}} = \mathrm{Vec}(\Delta\mathbf{H}_{\tilde{k}}) = \begin{bmatrix} \Delta\mathbf{h}_1 \\ \cdots \\ \Delta\mathbf{h}_{k-1} \\ \Delta\mathbf{h}_{k+1} \\ \cdots \\ \Delta\mathbf{h}_K \end{bmatrix}. \quad (34)$$

Considering the channel uncertainty region $\mathcal{S}$ and applying the Cauchy-Schwarz inequality, we obtain:

$$\Delta\mathbf{h}_{\tilde{k}}^H\Delta\mathbf{h}_{\tilde{k}} = \mathrm{Vec}(\Delta\mathbf{H}_{\tilde{k}})^H\mathrm{Vec}(\Delta\mathbf{H}_{\tilde{k}}) = \sum_{i\ne k}^{K}\Delta\mathbf{h}_i^2$$
$$= \sum_{i\ne k}^{K}\|\mathbf{R}_i^{\frac{1}{2}}\mathbf{u}_i\|^2 \le \sum_{i\ne k}^{K}\mathrm{Tr}(\mathbf{R}_i)\xi_i^2, \quad \forall\tilde{k}. \quad (35)$$

To further transform (33) into a tractable linear matrix inequality, the S-*procedure* method in [24] is helpful. Based on that, (33) can be reformulated as:

$$\begin{bmatrix} \mu_{\tilde{k}}\mathbf{I}_{(K-1)N_t} - (\mathbf{I}\otimes\mathbf{Q}_k) & -(\mathbf{I}\otimes\mathbf{Q}_k)\mathbf{h}_{\tilde{k}} \\ -\mathbf{h}_{\tilde{k}}^H(\mathbf{I}\otimes\mathbf{Q}_k) & -\mu_{\tilde{k}}\sum_{i\ne k}^{K}\mathrm{Tr}(\mathbf{R}_i)\xi_i^2 - z_{\tilde{k}} \end{bmatrix} \succeq \mathbf{0}, \quad \forall\tilde{k}, \quad (36)$$

where $\mu_{\tilde{k}} \ge 0$.

Next, we consider the constraint (25). Based on the *Sphere Bounding* method in [22], the channel uncertainty region for the ERs can be bounded by a ball region $\mathcal{A}$ to satisfy the outage constraint (24), where $\mathcal{A} = \{\mathbf{v}_l|\Pr(\mathbf{v}_l^H\mathbf{v}_l \le \gamma_l^2) \ge 1-\rho_l\}, \forall l$, and $\gamma_l = \sqrt{\frac{F^{-1}_{\chi^2_{2N_t}}(1-\rho_l)}{2}}, \forall l$. Therefore, by adopting the *S-procedure* method and performing a similar analysis to (33), there exists $\mu_l \ge 0$ such that (25) can be recast into:

$$\begin{bmatrix} \mu_l\mathbf{I} - \mathbf{P}_l^{\frac{1}{2}}\Phi_k\mathbf{P}_l^{\frac{1}{2}} & -\mathbf{P}_l^{\frac{1}{2}}\Phi_k\bar{\mathbf{h}}_l \\ -\bar{\mathbf{h}}_l^H\Phi_k\mathbf{P}_l^{\frac{1}{2}} & -\mu_l\gamma_l^2 - \bar{\mathbf{h}}_l^H\Phi_k\bar{\mathbf{h}}_l + q_l(\eta_k - 1) \end{bmatrix} \succeq \mathbf{0},$$
$$\mu_l \ge 0, \quad \forall l. \quad (37)$$

where $q_l = (\sigma_l^2 + \frac{\sigma_{p,l}^2}{\rho_l^p})$.

Combining the above analysis, the problem in (21) can be equivalently reformulated as:

$$\min_{\mathbf{Q}_k,\rho_k^p,\forall k} \sum_{k=1}^{K}\mathrm{Tr}(\mathbf{Q}_k) \quad (38a)$$
$$\text{s.t. } (26),\ (27),\ (28),\ (36),\ (37), (19g), \mathbf{Q}_k \succeq \mathbf{0},\ \forall k. \quad (38b)$$

The problem (38) has been relaxed into a convex one and can be efficiently solved using the interior-point method [25]. It is noted that we drop the rank-one condition before the safe approximation. Hence, in order to guarantee that the optimal solution $\mathbf{Q}_k$ to the problem (38) is also the optimal solution to the relaxed problem (21), the following theorem is presented to study the tightness of the relaxation.

Theorem 1: Suppose the problem (21) *is feasible with a given group of* $\eta_k, \forall k$*, the relaxed problem* (38) *yields the optimal solution satisfying rank*$(\mathbf{Q}_k) = 1, \forall k$.

Proof: Please refer to Appendix A. ■

2) BTI-BASED APPROACH

In this subsection, we present another conservative approximation, BTI-based approach [26], for the outage probabilistic constraints of the power minimization problem (21).

Let's first consider the outage probabilistic constraints (22), (23) and (24). Based on the BTI-based approach and using some suitable slack variables, the outage constraint (22) could be equivalently re-expressed into:

$$\mathrm{Tr}\left(\mathbf{R}_k^{\frac{1}{2}}\Theta_k\mathbf{R}_k^{\frac{1}{2}}\right) - \sqrt{-2\ln(\rho_k)}\bar{\tau}_k + \ln(\rho_k)\bar{\psi}_k + s_k \ge 0, \quad (39a)$$

$$\left\|\begin{bmatrix} \mathrm{Vec}\left(\mathbf{R}_k^{\frac{1}{2}}\Theta_k\mathbf{R}_k^{\frac{1}{2}}\right) \\ \sqrt{2}\left(\mathbf{R}_k^{\frac{1}{2}}\Theta_k\bar{\mathbf{h}}_k\right) \end{bmatrix}\right\| \le \bar{\tau}_k, \quad (39b)$$

$$\bar{\psi}_k\mathbf{I} + \mathbf{R}_k^{\frac{1}{2}}\Theta_k\mathbf{R}_k^{\frac{1}{2}} \succeq \mathbf{0},\ \bar{\psi}_k \ge 0, \quad \forall k, \quad (39c)$$

where $\bar{\tau}_k$, $\bar{\psi}_k$ are the slack variables, and $s_k = \bar{\mathbf{h}}_k^H\Theta_k\bar{\mathbf{h}}_k - t_k(\sigma_k^2 + \frac{\sigma_{p,k}^2}{\rho_k^p})$.

One sees that (39) only has K linear constraints, K SOC constraints and K convex SDP constraints, which denotes (22) is efficiently computable. That is, (39) is jointly convex in $\mathbf{Q}_k$, $\bar{\tau}_k$, $\bar{\psi}_k$. So it can be tractable. By adopting a similar analysis, the outage constraint (23) can be reformulated as

$$\mathrm{Tr}\left[\mathbf{R}_k^{\frac{1}{2}}\left(\sum_{i=1}^{K}\mathbf{Q}_i\right)\mathbf{R}_k^{\frac{1}{2}}\right] - \sqrt{-2\ln(\rho_{p,k})}\bar{\tau}_k^p + \ln(\rho_{p,k})\bar{\psi}_k^p$$
$$+ \bar{\mathbf{h}}_k^H(\sum_{i=1}^{K}\mathbf{Q}_i)\bar{\mathbf{h}}_k + \sigma_k^2 - \frac{\bar{E}_{p,k}}{1-\rho_k^p} \ge 0, \quad (40a)$$

$$\left\|\begin{bmatrix} \mathrm{Vec}\left(\mathbf{R}_k^{\frac{1}{2}}(\sum_{i=1}^{K}\mathbf{Q}_i)\mathbf{R}_k^{\frac{1}{2}}\right) \\ \sqrt{2}\left(\mathbf{R}_k^{\frac{1}{2}}(\sum_{i=1}^{K}\mathbf{Q}_i)\bar{\mathbf{h}}_k\right) \end{bmatrix}\right\| \le \bar{\tau}_k^p, \quad (40b)$$

$$\bar{\psi}_k^p\mathbf{I} + \mathbf{R}_k^{\frac{1}{2}}\left(\sum_{i=1}^{K}\mathbf{Q}_i\right)\mathbf{R}_k^{\frac{1}{2}} \succeq \mathbf{0}, \quad \bar{\psi}_k^p \ge 0,\ \forall k, \quad (40c)$$

where $\bar{\tau}_k^p$, $\bar{\psi}_k^p$ are slack variables.

Similarly, we transform the constraint (24) into:

$$\mathrm{Tr}\Big[\mathbf{P}_l^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\mathbf{P}_l^{\frac{1}{2}}\Big] - \sqrt{-2\ln(\rho_l)}\bar{\tau}_l + \ln(\rho_l)\bar{\psi}_l + s_l \geq 0, \tag{41a}$$

$$\left\| \begin{bmatrix} \mathrm{Vec}\Big(\mathbf{P}_l^{\frac{1}{2}}\big(\sum_{i=1}^{K}\mathbf{Q}_i\big)\mathbf{P}_l^{\frac{1}{2}}\Big) \\ \sqrt{2}\Big(\mathbf{P}_l^{\frac{1}{2}}\big(\sum_{i=1}^{K}\mathbf{Q}_i\big)\bar{\mathbf{h}}_l\Big) \end{bmatrix} \right\| \leq \bar{\tau}_l, \tag{41b}$$

$$\bar{\psi}_l\mathbf{I} + \mathbf{P}_l^{\frac{1}{2}}\Big(\sum_{i=1}^{K}\mathbf{Q}_i\Big)\mathbf{P}_l^{\frac{1}{2}} \succeq \mathbf{0}, \quad \bar{\psi}_l \geq 0,\ \forall l, \tag{41c}$$

where $\bar{\tau}_l$, $\bar{\psi}_l$ are slack variables, and $s_l = \bar{\mathbf{h}}_l^H(\sum_{i=1}^{K}\mathbf{Q}_i)\bar{\mathbf{h}}_l + \sigma_l^2 - \frac{\bar{E}_l}{1-\rho_l^p}$.

Next, we focus on the safe approximations of (21c), (25) for the eavesdroppers. They cannot be reformulated using the proposed BTI-based approach because of the non-probabilistic form. Consequently, we attempt to tackle with the constraints by exploiting the same method as shown in the subsection III-B-1. Based on the BTI-based approach, (21) can be equivalently reformulated into:

$$\min_{\mathbf{Q}_k,\rho_k^p,\forall k} \sum_{k=1}^{K}\mathrm{Tr}(\mathbf{Q}_k) \tag{42a}$$

$$\text{s.t. } (39),\ (40),\ (41),\ (36),\ (37),(19\text{g}),\ \mathbf{Q}_k \succeq \mathbf{0},\ \forall k. \tag{42b}$$

Now the problem (42) is relaxed into a convex one, and can be efficiently solved using the interior-point scheme [25]. The following theorem states that the relaxed problem (42) based on the BTI-based approach indeed yields the rank-one optimal solution.

Theorem 2: Suppose the problem (21) *is feasible with a given group of* $\eta_k, \forall k$*, the relaxed problem* (42) *yields the optimal solution satisfying rank*$(\mathbf{Q}_k) = 1, \forall k$.

Proof: The proof is similar to that of Theorem 1. Hence, it is omitted for brevity. ■

C. A LINE-DIMENSIONAL-LIKE SEARCH FOR THE OUTER PROBLEM

Based on the problem (20), the outer problem can be re-expressed as:

$$\max_{\eta_k,\forall k} \ \Phi(\eta_1, \eta_2, \ldots, \eta_k)$$
$$\text{s.t. } \eta_{k,\min} \leq \eta_k \leq \eta_{k,\max}, \tag{43}$$

where $\Phi(\eta_1, \ldots, \eta_k)$ denotes the outer optimal solutions, $\eta_{k,\min}$ and $\eta_{k,\max}$ are the lower bound and upper bound for $\eta_k, \forall k$, respectively. With the aforementioned analysis of the slack variables $\eta_k, \forall k$, it is easy to verify that $\eta_{k,\min} = 1, \forall k$, whereas the $\eta_{k,\max}$ can be specified by the following theorem.

Theorem 3: To satisfy the feasible solutions of the problem (43), $\eta_{k,\max}, \forall k$ *can be obtained by*

$$\eta_{k,\max} = 1 + \frac{1}{K-1+A_k}, \quad \forall k \tag{44}$$

where $A_k = \frac{K(\sigma_k^2+\sigma_{p,k}^2)}{PTr(\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H+\mathbf{R}_k)}$.

Proof: Please refer to Appendix B. ■

Algorithm 1 LDLS Algorithm for the Outer Problem

1: **Initialize**: given $a_k = \eta_{k,\min}$, $b_k = \eta_{k,\max}$, $\forall k$, and the accuracy $\epsilon > 0$.
2: Calculate $c_k = a_k + 0.382(b_k - a_k)$, $d_k = a_k + 0.618(b_k - a_k)$, $\forall k$. Solve the problem (38) (or the problem (42)), and obtain $t_1 = \Phi(c_1, c_2, \ldots, c_K)$, $t_2 = \Phi(d_1, d_2, \ldots, d_K)$, respectively.
3: **while** $b_k - a_k > \epsilon, \forall k$ **do**
4:   **if** $t_1 < t_2$ **then**
5:     $a_k = c_k, c_k = d_k, t_1 = t_2, d_k = a_k + 0.618(b_k - a_k)$, $\forall k$. Solve the probem (38) (or the problem (42)), and obtain $t_2 = \Phi(d_1, d_2, \ldots, d_K)$.
6:   **else**
7:     $b_k = d_k, d_k = c_k, t_2 = t_1, c_k = a_k + 0.382(b_k - a_k)$, $\forall k$. Solve the probem (38) (or the problem (42)), and obtain $t_1 = \Phi(c_1, c_2, \ldots, c_K)$.
8:   **end if**
9: **end while**
10: Set $\zeta_k = (a_k + b_k)/2, \forall k$. Solve the probem (38) (or the problem (42)), and obtain the optimal $(\mathbf{Q}_k, \rho_k^p), \forall k$.

To achieve the optimal solution to the problem (43), we first need to calculate $\eta_{k,\min}$ and $\eta_{k,\max}$, and then perform the LDLS algorithm. For the detailed process, we summarize it in Algorithm 1.

IV. NUMERICAL RESULTS AND DISSCUSSION

In this section, we study the performance of our proposed schemes for SWIPT in the multiuser MIMO systems using numerical examples. Without loss of generality, we consider a flat Rayleigh fading channel model, where the fading gains follow independent and identically distributed (i.i.d.) complex Gaussian distributions $\mathcal{CN}(0, 1)$. We set $N_t = 4, K = 3$, $L = 2$ and $P = 50$dBm. In particular, we consider the case in which $\rho_k = \rho_{p,k} = \rho_l = 0.1$. In addition, we define the CSI error variance matrices as $\mathbf{R}_k = \epsilon_k^2\mathbf{I}, \forall k$ and $\mathbf{P}_l = \epsilon_l^2\mathbf{I}, \forall l$ for all the CRs and ERs, where ϵ_k^2 and ϵ_l^2 represent the channel error variances at the kth CR and the lth ER, respectively. For simplicity, it is assumed that $\epsilon_k^2 = \epsilon_l^2 = \epsilon^2$. In our simulation, we compare the proposed LDI-based approach and BTI-based approach with the non-robust method in which the channel errors are not considered.

Figure 2 presents the required transmission power versus the target secrecy rate, where it is assumed that the noise power for all the CRs and ERs is -20dBm, i.e., $\sigma_k^2 = \sigma_{p,k}^2 = \sigma_l^2 = \sigma^2 = -20$dBm, $\forall k, l$, and the threshold of the energy harvesting at each receiver satisfies $\bar{E}_{p,k} = -20$dBm, $\forall k$ and $\bar{E}_l = -15$dBm, $\forall l$, respectively. It can be observed that the total transmission power for all the schemes monotonically increases as the target secrecy rate increases. This happens because higher target secrecy rate leads to more stringent secrecy performance so that the transmitter has to allocate more transmission power to meet the outage constraints. It can

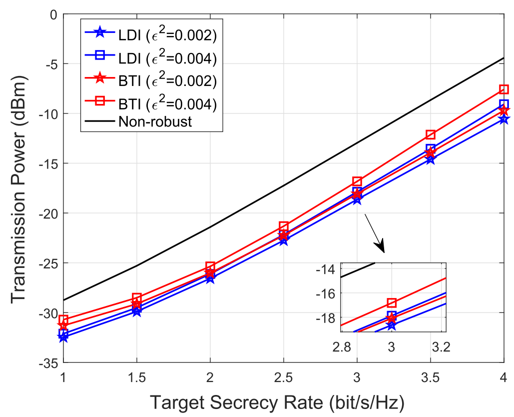

FIGURE 2. Transmission power v.s. target secrecy rate.

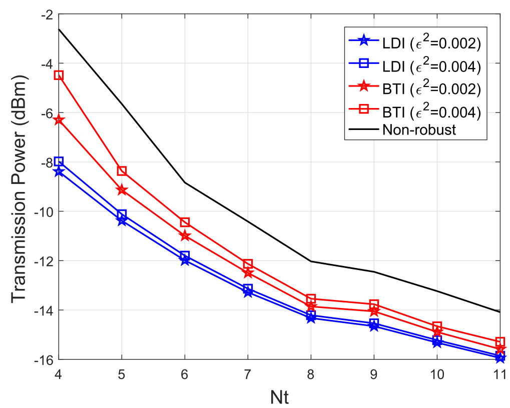

FIGURE 4. Transmission power v.s. Nt.

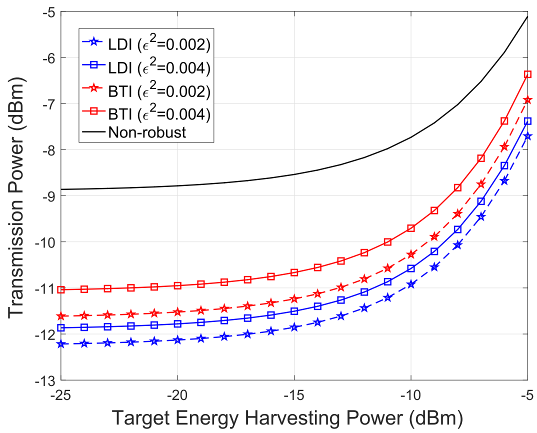

FIGURE 3. Transmission power v.s. target energy harvesting power.

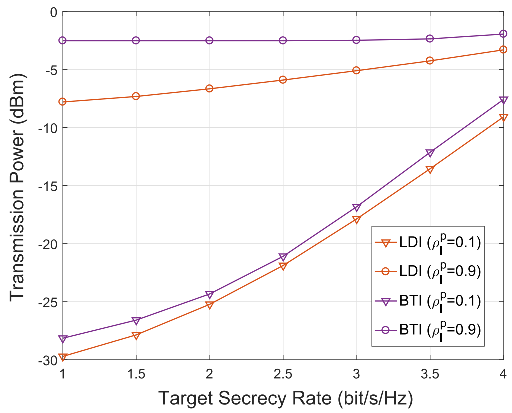

FIGURE 5. Transmission power v.s. target secrecy rate with $\epsilon^2 = 0.004$, $\rho_l^p = 0.1$ and $\rho_l^p = 0.9$.

also be observed from Figure 2 that, compared to the non-robust scheme, the proposed ones based on LDI and BTI need less transmission power.

Figure 3 depicts the transmission power versus the target harvested power at each CR. Here, we assume that $\sigma_k^2 = \sigma_{p,k}^2 = -10$dBm, $\sigma_l^2 = -10$dBm, $\bar{R}_k = 1.3$bit/s/Hz and $\bar{E}_l = -30$dBm, $\forall k, l$. It can be seen that the total transmission power for all the schemes is an increasing function in terms of the target energy harvesting power. This is because with the increased target energy harvesting power at each CR, the energy outage requirement at each CR is higher. Hence, all the algorithms need more transmission power to satisfy the secrecy requirements. In addition, it can be observed that all the proposed schemes outperform the non-robust one in terms of the target energy harvesting power.

Figure 4 describes the transmission power versus the number of the transmit antenna, where we set $\sigma_k^2 = -15$dBm, $\sigma_{p,k}^2 = -20$dBm, $\sigma_l^2 = -30$dBm, $\bar{R}_k = 1.4$bit/s/Hz, and $\bar{E}_{p,k} = \bar{E}_l = -30$dBm, $\forall k, l$. Different from the results in Figure 2 and Figure 3, the total required transmission power for all the schemes substantially decreases as the number of the transmit antennas increases. This is due to the fact that a larger number of available transmit antennas at the transmitter increases the extra degrees of the freedom, which can be well exploited for the beamforming design. We can also observe that all the proposed schemes need less transmission power than the non-robust one, which is similar to the results in Figure 2 and Figure 3.

On the other hand, we can also see from Figure 2, Figure 3 and Figure 4 that the proposed schemes need to consume more transmission power when the channel error increases. Furthermore, it can also been seen that the proposed LDI-based approach yields better transmission power performance than the BTI-based approach in terms of the target secrecy rate, the target energy harvesting power and the number of the transmit antennas.

Figure 5 shows the transmission power versus the target secrecy rate with $\epsilon^2 = 0.004$ and different ρ_l^p, i.e., $\rho_l^p = 0.1$ and $\rho_l^p = 0.9$, in which we take the same parameters as that in Figure 2. It is expected that the total transmission power increases with increasing the target secrecy rate. This translates to a higher demand of the target secrecy rate requirement facilitating the more power allocation at the transmitter, which is similar to the result in Figure 2. In addition, a larger portion of power is allocated at the transmitter with $\rho_l^p = 0.9$ than that with $\rho_l^p = 0.1$. The reason behind this is that

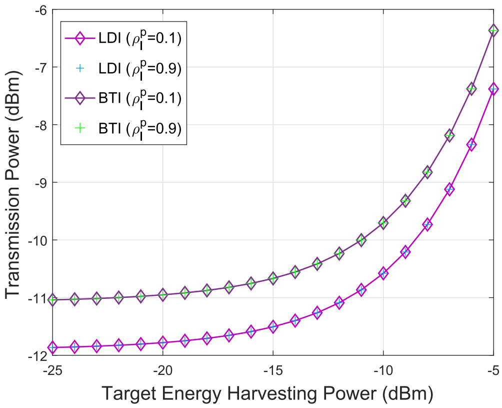

FIGURE 6. **Transmission power v.s. target energy harvesting power with $\epsilon^2 = 0.004$, $\rho_l^p = 0.1$ and $\rho_l^p = 0.9$.**

the smaller ρ_l^p means that the portion of the power used for eavesdropping by ERs is less. Thus, the transmitter don't need to generate a higher transmission power to guarantee communication secrecy. Besides, it is interesting to note that there is big gap between the total transmission power with $\rho_l^p = 0.1$ and that with $\rho_l^p = 0.9$, whereas the gap gets shirnking as the target secrecy rate increases. The result suggests that the secure constraints dominate the system performance as the target secrecy rate requirements become more stringent.

Figure 6 shows the transmission power versus the target energy harvesting power with $\epsilon^2 = 0.004$ and different ρ_l^p, i.e., $\rho_l^p = 0.1$ and $\rho_l^p = 0.9$, where $\sigma_k^2 = \sigma_{p,k}^2 = \sigma_l^2 = \sigma^2 = -10$dBm, $\forall k, l$, $\bar{R}_k = 1.3$bit/s/Hz and $\bar{E}_l = -30$dBm, $\forall l$. It can be observed that the total transmission power increases with the target energy harvesting power. Since a higher target energy harvesting power requirement can lead to a higher energy level at the CRs for energy harvesting, it demands a higher amount of transmission power, which presents the same result as in Figure 3. Furthermore, the total transmission power with $\rho_l^p = 0.1$ keeps consistent in that with $\rho_l^p = 0.9$ when the target energy harvesting power increases. That is because the target secrecy rate at CRs is fixed so that the portion of power used for ID keeps unchanged. The transmitter only increases the transmission power to satisfy the increasing portion of power for EH at CRs, which is not related to the ρ_l^p increase.

V. CONCLUSION

In this paper, we have investigated the energy efficiency for secure communications in the multiuser MIMO SWIPT system with multiple internal/external eavesdroppers. We have formulated the power optimization problem by considering the outage constraints on the achievable secrecy rate and the harvested energy under imperfect CSI. We have proposed a two-level optimization algorithm. For the inner problem, we have used two safe approximations: LDI-based approach and BTI-based approach, to convert the outage constraints into a series of LMIs and SOCs. For the outer problem, we have proposed a novel LDLS algorithm. We have verified the tightness of the relaxation by showing that the optimal solutions were indeed rank-one. Simulation results have been presented to validate the performance of our proposed schemes.

APPENDIX A
PROOF OF THEOREM 1

We provide the proof of the rank-one solution for the power minimization problem (38). In order to show the optimal solution is rank-one, we first focus on the constraints (26b), (26c), (27b), (27c), (28b), (28c), which could be equivalently rewritten without $\mathbf{Q}_k$. For the constraint (26b), it can be expressed as:

$$\begin{aligned}&\frac{1}{\sqrt{2}}\|\mathbf{R}_k^{\frac{1}{2}}\Theta_k\bar{\mathbf{h}}_k\|\\ &\overset{(a)}{\leq} \frac{1}{\sqrt{2}}\left(\|\mathbf{R}_k^{\frac{1}{2}}\mathbf{Q}_k\bar{\mathbf{h}}_k\| + \|t_k\sum_{i\neq k}^{K}\mathbf{R}_k^{\frac{1}{2}}\mathbf{Q}_i\bar{\mathbf{h}}_k\|\right) \leq \gamma_k,\\ &\overset{(b)}{\Longrightarrow} \|\mathbf{Q}_k\|^2 \leq \bar{x}_k^2,\\ &\overset{(c)}{\Longrightarrow} \lambda_{\max}(\mathbf{Q}_k\mathbf{Q}_k^H) \leq \mathrm{Tr}(\mathbf{Q}_k\mathbf{Q}_k^H) \leq \bar{x}_k^2,\\ &\Longrightarrow \mathbf{Q}_k\mathbf{Q}_k^H \preceq \bar{x}_k^2\mathbf{I}_{N_t}, \Longrightarrow \begin{bmatrix}\bar{x}_k\mathbf{I} & \mathbf{Q}_k\\ \mathbf{Q}_k^H & \bar{x}_k\mathbf{I}\end{bmatrix} \succeq \mathbf{0}, \qquad (45)\end{aligned}$$

where $\bar{x}_k = \sqrt{2\left(\frac{\gamma_k - \|t_k\sum_{i\neq k}^{K}\mathbf{R}_k^{\frac{1}{2}}\mathbf{Q}_i\|}{\|\bar{\mathbf{h}}_k\|\|\mathbf{R}_k^{\frac{1}{2}}\|}\right)}$, (a) holds due to the basic property of the matrix norm which is $\|\mathbf{A}+\mathbf{B}\| \leq \|\mathbf{A}\| + \|\mathbf{B}\|$ for any matrices $\mathbf{A}$ and $\mathbf{B}$, (b) holds because of the another property of the matrix form which is $\|\mathbf{AB}\| \leq \|\mathbf{A}\|\|\mathbf{B}\|$ for any matrices $\mathbf{A}$ and $\mathbf{B}$, and (c) can be achieved by the fact that $\lambda_{\max}(\mathbf{A}) \leq \mathrm{Tr}(\mathbf{A})$ for any $\mathbf{A} \succeq \mathbf{0}$.

For a further analysis, (45) can be equivalently rewritten as the following LMI:

$$\begin{aligned}\begin{bmatrix}\bar{x}_k\mathbf{I} & \mathbf{Q}_k\\ \mathbf{Q}_k^H & \bar{x}_k\mathbf{I}\end{bmatrix} &\succeq \mathbf{0}\\ &\Longrightarrow \begin{bmatrix}\bar{x}_k\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \bar{x}_k\mathbf{I}\end{bmatrix} \succeq \begin{bmatrix}\mathbf{I}\\ \mathbf{0}\end{bmatrix}\mathbf{Q}_k\begin{bmatrix}\mathbf{0} & -\mathbf{I}\end{bmatrix}\\ &\quad + \begin{bmatrix}\mathbf{0}\\ -\mathbf{I}\end{bmatrix}\mathbf{Q}_k^H\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix},\\ \|\mathbf{Q}_k\| &\leq \bar{x}_k. \qquad (47)\end{aligned}$$

In order to further analyze the above LMI in (47), we take Proposition 2 in [27]. The merit of the proposition comes from the fact that the complex matrix inequalities can be equivalently transformed into the deterministic LMI ones without the matrix variables.

Hence, (47) can be equivalently rewritten as the following LMI based on the proposition:

$$\begin{aligned}&\begin{bmatrix}\begin{bmatrix}\bar{x}_k\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \bar{x}_k\mathbf{I}\end{bmatrix} - a_k\begin{bmatrix}\mathbf{0}\\ -\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{0} & -\mathbf{I}\end{bmatrix} & -\bar{x}_k\begin{bmatrix}\mathbf{I}\\ \mathbf{0}\end{bmatrix}\\ -\bar{x}_k\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix} & a_k\mathbf{I}\end{bmatrix} \succeq \mathbf{0},\\ &a_k > 0. \qquad (48)\end{aligned}$$

According to (48), we readily claim that (26b) is irrelevant of $\mathbf{Q}_k, \forall k$. By the similar analysis, (26c), (27b), (27c), (28b), (28c) can also be equivalently reformulated into LMIs which are not involving $\mathbf{Q}_k, \forall k$.

In addition, we can eqivalently re-express (36) into the sum of two sections:

$$\Pi_{\tilde{k}} = \mathbf{A}_{\tilde{k}} - \hat{\mathbf{H}}_{\tilde{k}}^H(\mathbf{I}\otimes\mathbf{Q}_k)\hat{\mathbf{H}}_{\tilde{k}}, \quad \forall\tilde{k} \tag{49}$$

where $\mathbf{A}_{\tilde{k}} = \begin{bmatrix} \mu_{\tilde{k}}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mu_{\tilde{k}}\sum_{i\neq k}^{K}\mathrm{Tr}(\mathbf{R}_i)\xi_i^2 + (\eta_k - 1)\mathrm{Tr}(\mathbf{W}) \end{bmatrix}$ and $\hat{\mathbf{G}}_{\tilde{k}} = [\mathbf{I}\ \mathbf{h}_{\tilde{k}}]$. Interestingly, the first part $\mathbf{A}_{\tilde{k}}$ is without the matrix variables $\mathbf{Q}_k, \forall k$.

Similarly, the constraint (37) can be further transformed into the following identity:

$$\Lambda_l = \mathbf{B}_l - \hat{\mathbf{K}}_l^H\Phi_k\hat{\mathbf{K}}_l, \quad \forall l \tag{50}$$

where $\mathbf{B}_l = \begin{bmatrix} \mu_l\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mu_l\zeta_l^2 + (\sigma_l^2 + \frac{\sigma_{p,l}^2}{\rho_l^p})(\eta_k - 1) \end{bmatrix}$ and $\hat{\mathbf{K}}_l = [\mathbf{P}_l^{\frac{1}{2}}\ \bar{\mathbf{h}}_l]$. Obviously, the first parts $\mathbf{B}_l$ is also rewritten without the matrix variables $\mathbf{Q}_k, \forall l$.

In the following, we consider the lagrangian function of problem (38) as shown in (46), where $\mathbf{Z}_k$, $\mathbf{T}_{\tilde{k}}$, $\bar{a}_k$, $\bar{b}_k$, $\bar{c}_l$ and $\mathbf{Y}_l$ are the lagrangian dual variables with respect to the variables $\mathbf{Q}_k$ and the constraints of (36), (26a), (27a), (28a) and (37), respectively, and Δ represents the collections of the terms only involving the variables but not being relevant of the proof. Based on (46), the lagrangian dual problem can be expressed as follows:

$$\max_{\substack{\mathbf{Z}_k,\mathbf{T}_{\tilde{k}},\bar{a}_k,\\ \bar{b}_k,\bar{c}_l,\mathbf{Y}_l}}\ \min_{\mathbf{Q}_k,\rho_k^p} \mathcal{L}(\mathbf{Q}_k, \mathbf{Z}_k, \mathbf{T}_{\tilde{k}}, \bar{a}_k, \bar{b}_k, \bar{c}_k, \mathbf{Y}_l) \tag{51}$$

According to (38), the optimal inner problem (21) has been relaxed into the convex one, which satisfies the Slater's condition [24]. Hence, there is no duality gap between the primal and dual optimization problems. We can solve the inner primal problem (38) by its dual problem (51). The related Karush-Kuhn-Tucker (KKT) condition of the lagrangian function can be given by:

$$\frac{\partial\mathcal{L}}{\partial\mathbf{Q}_k^*} = 0, \quad \forall k, \tag{52a}$$

$$\frac{\partial\mathcal{L}}{\partial\rho_k^{p*}} = 0, \quad \forall k, \tag{52b}$$

$$\mathbf{Z}_k^*\mathbf{Q}_k^* = \mathbf{0}, \quad \mathbf{Q}_k^* \succeq \mathbf{0},\ \mathbf{Z}_k^* \succeq \mathbf{0},\ \forall k, \tag{52c}$$

$$\bar{a}_k^* \geq 0, \quad \bar{b}_k^* \geq 0,\ \bar{c}_l^* \geq 0,\ \forall k, l, \tag{52d}$$

where $(\mathbf{Q}_k^*, \rho_k^{p*})$ and $(\mathbf{Z}_k, \mathbf{T}_{\tilde{k}}, \bar{a}_k, \bar{b}_k, \bar{c}_l, \mathbf{Y}_l)$ are the primal and dual optimal points. According to (52b), we can obtain:

$$\rho_k^{p*} = \frac{\sqrt{t_k\bar{a}_k^*\delta_{p,k}}}{\sqrt{t_k\bar{a}_k^*\delta_{p,k}} + \sqrt{\bar{b}_k^*\bar{E}_{p,k}}} \tag{53}$$

It is easy to claim that there exists at least one $\bar{a}_k^*$, $\bar{b}_k^*$, $\forall k$ such that $\bar{a}_k^* > 0$, $\bar{b}_k^* > 0$ to satisfy $t_k > 0$ and $\bar{E}_{p,k} > 0$. That is because if $\bar{a}_k^* = 0$, then $\rho_k^{p*} = 0$, which will lead to no power used for the information decoding at the CRs and can't satisfy the secrecy rate constraints. Similarly, $\bar{b}_k^* = 0$ results in $\rho_k^{p*} = 1$, which can't satisfy the energy harvesting constraints at the CRs.

According to (52a), we have

$$\mathbf{Z}_k^* = \mathbf{I}_k - (\bar{a}_k^* + \bar{b}_k^*)(\mathbf{R}_k + \bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H) - \sum_{i\neq 1}^{K}\bar{b}_i^*(\mathbf{R}_i + \bar{\mathbf{h}}_i\bar{\mathbf{h}}_i^H) - \sum_{l=1}^{L}\bar{c}_l^*(\mathbf{P}_l + \bar{\mathbf{h}}_l\bar{\mathbf{h}}_l^H) + \hat{\mathbf{K}}_l\mathbf{Y}_l\hat{\mathbf{K}}_l^H + \sum_{k=1}^{K}\sum_{n=1}^{K-1}\mathbf{G}_{\tilde{k}}^{(n,n)}, \tag{54}$$

where $\mathbf{G}_{\tilde{k}}^{(n,n)} \in \mathbb{H}_+^{N_t}$ is the block submatrix of $\hat{\mathbf{H}}_{\tilde{k}}\mathbf{T}_{\tilde{k}}\hat{\mathbf{H}}_{\tilde{k}}^H$, which is defined as

$$\hat{\mathbf{H}}_{\tilde{k}}\mathbf{T}_{\tilde{k}}\hat{\mathbf{H}}_{\tilde{k}}^H = \begin{bmatrix} \mathbf{G}_{\tilde{k}}^{(1,1)} & \cdots & \mathbf{G}_{\tilde{k}}^{(1,K-1)} \\ \vdots & \ddots & \vdots \\ \mathbf{G}_{\tilde{k}}^{(K-1,n)} & \cdots & \mathbf{G}_{\tilde{k}}^{(K-1,K-1)} \end{bmatrix}.$$

For convenience, we assume $\Xi_k = \mathbf{I}_k - (\bar{a}_k^* + \bar{b}_k^*)\mathbf{R}_k - \sum_{i\neq 1}^{K}\bar{b}_i^*(\mathbf{R}_i + \bar{\mathbf{h}}_i\bar{\mathbf{h}}_i^H) - \sum_{l=1}^{L}\bar{c}_l^*(\mathbf{P}_l + \bar{\mathbf{h}}_l\bar{\mathbf{h}}_l^H) + \hat{\mathbf{K}}_l\mathbf{Y}_l\hat{\mathbf{K}}_l^H + \sum_{k=1}^{K}\sum_{n=1}^{K-1}\mathbf{G}_{\tilde{k}}^{(n,n)}$. Thus we have $\mathbf{Z}_k^* = \Xi_k - (\bar{a}_k^* + \bar{b}_k^*)\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H$. To keep the information-bearing signal availability and to satisfy the secrecy constraint, it must hold that $\mathbf{Q}_k^* \neq \mathbf{0}$, or equivalently, $\mathrm{rank}(\mathbf{Q}_k^*) \geq 1$. Consequently, it follows that $\mathrm{rank}(\mathbf{Z}_k^*) \leq N_t - 1$ based on the KKT condition of $\mathbf{Z}_k^*\mathbf{Q}_k^* = \mathbf{0}$ in (52c).

Let $\mathrm{rank}(\Xi_k^*) = \bar{r}_k$ denotes the rank of Ξ_k^*. By the fact that for any two matrices $\mathbf{A}$ and $\mathbf{B}$, it holds that $\mathrm{rank}(\mathbf{A} - \mathbf{B}) \geq \mathrm{rank}(\mathbf{A}) - \mathrm{rank}(\mathbf{B})$. Thus it is easy to obtain that $\mathrm{rank}(\mathbf{Z}_k^*) \geq \mathrm{rank}(\Xi_k) - \mathrm{rank}[(\bar{a}_k^* + \bar{b}_k^*)\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H] = \bar{r}_k - 1$.

$$\begin{aligned}
&\mathcal{L}(\mathbf{Q}_k, \mathbf{Z}_k, \mathbf{T}_{\tilde{k}}, \bar{a}_k, \bar{b}_k, \bar{c}_l, \mathbf{Y}_l) \\
&= \sum_{k=1}^{K}\mathrm{Tr}(\mathbf{Q}_k) - \sum_{k=1}^{K}\mathrm{Tr}(\mathbf{Z}_k\mathbf{Q}_k) + \sum_{k=1}^{K}\mathrm{Tr}\Big[\hat{\mathbf{H}}_{\tilde{k}}\mathbf{T}_{\tilde{k}}\hat{\mathbf{H}}_{\tilde{k}}^H(\mathbf{I}\otimes\mathbf{Q}_k)\Big] - \sum_{k=1}^{K}\bar{a}_k\Big[\mathrm{Tr}\Big((\mathbf{R}_k + \bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H)\mathbf{Q}_k - \frac{t_k\delta_{p,k}^2}{\rho_k^p}\Big)\Big] \\
&\quad - \sum_{k=1}^{K}\bar{b}_k\Big[\mathrm{Tr}\Big((\mathbf{R}_k + \bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H)(\sum_{i=1}^{K}\mathbf{Q}_k) - \frac{\bar{E}_{p,k}}{1-\rho_k^p}\Big)\Big] - \sum_{l=1}^{L}\bar{c}_l\Big[\mathrm{Tr}\Big((\mathbf{P}_l + \bar{\mathbf{h}}_l\bar{\mathbf{h}}_l^H)(\sum_{i=1}^{K}\mathbf{Q}_k)\Big)\Big] + \sum_{k=1}^{K}\mathrm{Tr}\Big[\hat{\mathbf{K}}_l\mathbf{Y}_l\hat{\mathbf{K}}_l^H\mathbf{Q}_k\Big] + \Delta
\end{aligned} \tag{46}$$

Since $\mathbf{Z}_k^* \succeq \mathbf{0}$ and $-(\bar{a}_k^* + \bar{b}_k^*)\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H \preceq \mathbf{0}$, it follows that $\Xi_k \succeq \mathbf{0}$. In the following part, we consider two cases of Ξ_k. For the case of Ξ_k being positive-definite, i.e., $\Xi_k \succ \mathbf{0}$, it holds that $\bar{r}_k = N_t$ and $\text{rank}(\mathbf{Z}_k^*) \geq N_t - 1$. If $\text{rank}(\mathbf{Z}_k^*) = N_t$, it implies that $\mathbf{Z}_k^*$ is full-rank. From the KKT condition of (52c), it holds that $\mathbf{Q}_k^* = \mathbf{0}$, which means no any information-bearing signal could be transmitted at the transmitter and can't be an optimal solution. As a result, we have $\text{rank}(\mathbf{Z}_k^*) = N_t - 1$. With the KKT condition of (52c), it readily holds $\text{rank}(\mathbf{Q}_k^*) = 1$.

Next we consider the another case of $\text{rank}(\Xi_k) = \bar{r}_k \neq N_t$. Without loss of generality, let $\mathbf{F}_k = \mathbf{f}_{k,1}, \ldots, \mathbf{f}_{k,N_t-\bar{r}_k}$ denotes the orthogonal basis of the nulll of Ξ_k. So we have $\Xi_k\mathbf{F}_k = \mathbf{0}$ and $\text{rank}(\mathbf{F}_k) = N_t - \bar{r}_k$. Furthermore, by considering the KKT condition $\mathbf{Z}_k^* \succeq \mathbf{0}$ in (52c), we could obtain

$$\begin{aligned}\mathbf{f}_k^H\mathbf{Z}_k^*\mathbf{f}_k &= \mathbf{f}_k^H[\Xi_k - (\bar{a}_k^* + \bar{b}_k^*)\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H]\mathbf{f}_k \\ &= -(\bar{a}_k^* + \bar{b}_k^*)|\bar{\mathbf{h}}_k^H\mathbf{f}_k|^2 \geq 0 \end{aligned} \tag{55}$$

It should be noted that based on the aforementioned analysis, we have $\bar{a}_k^* + \bar{b}_k^* > 0$. Hence, it is intutive to obtain that

$$\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H\mathbf{F}_k = \mathbf{0} \tag{56}$$

Consequently, we can get

$$\mathbf{Z}_k^*\mathbf{F}_k = \mathbf{0} \tag{57}$$

Obviously, all $\mathbf{f}_k$ lies in the null space of $\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H$ and $\mathbf{Z}_k^*$. To satisfy the KKT condition of $\mathbf{Z}_k^*\mathbf{Q}_k^* = \mathbf{0}$ in (52c), the information-bearing signal should be transmitted by the transmitter in the null space of $\mathbf{Z}_k^*$, i.e., $\mathbf{f}_k$. That is to say, no confidential message could be transferred to the CRs in this case since $\mathbf{f}_k$ also lies in the null space of $\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H$. Hence, it must hold that $\text{rank}(\Xi_k) = N_t$, which indicates that $\text{rank}(\mathbf{Z})_k^* \geq N_t - 1$.

Combining this result with $\text{rank}(\mathbf{Z}_k^*) \leq N_t - 1$ in the former analysis, it is easy to verify that $\text{rank}(\mathbf{Z}_k^*) = N_t - 1$. Accordingly, it can be obtained that

$$\text{rank}(\mathbf{Q}_k^*) = \text{rank}(\text{null}(\mathbf{Z}_k^*)) = N_t - \text{rank}(\mathbf{Z}_k^*) = 1. \tag{58}$$

This completes the proof.

APPENDIX B
PROOF OF THEOREM 3

Based on (19b), to satisfy the secrecy rate constraints at the CRs, η_k should be bounded by:

$$\eta_k \leq 1 + \frac{\rho_k^p(\mathbf{h}_k^H\mathbf{Q}_k\mathbf{h}_k)}{\rho_k^p\left(\sum_{i\neq k}^K \mathbf{h}_k^H\mathbf{Q}_i\mathbf{h}_k + \sigma_k^2\right) + \sigma_{p,k}^2}, \quad \forall k \tag{59}$$

By considering the imperfect CSI, we have

$$\begin{aligned}\eta_k &\overset{(a)}{\leq} 1 + \frac{\text{Tr}(\mathbf{Q}_k)\|\bar{\mathbf{h}}_k + \mathbf{R}_k^{\frac{1}{2}}\mathbf{u}_k\|^2}{\sum_{i\neq k}^K \text{Tr}(\mathbf{Q}_i)\|\bar{\mathbf{h}}_k + \mathbf{R}_k^{\frac{1}{2}}\mathbf{u}_k\|^2 + \sigma_k^2 + \sigma_{p,k}^2} \\ &\overset{(b)}{\leq} 1 + \frac{\frac{P}{K}(\|\bar{\mathbf{h}}_k\|^2 + \|\mathbf{R}_k^{\frac{1}{2}}\mathbf{u}_k\|^2)}{\frac{P}{K}(K-1)(\|\bar{\mathbf{h}}_k\|^2 + \|\mathbf{R}_k^{\frac{1}{2}}\mathbf{u}_k\|^2) + \sigma_k^2 + \sigma_{p,k}^2} \\ &\leq 1 + \frac{\frac{P}{K}\text{Tr}(\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H + \mathbf{R}_k)}{\frac{P}{K}(K-1)\text{Tr}(\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H + \mathbf{R}_k) + \sigma_k^2 + \sigma_{p,k}^2} \\ &= 1 + \frac{1}{K - 1 + A_k} \triangleq \eta_{k,\max} \end{aligned} \tag{60}$$

where (a) follows from the fact that $\mathbf{h}_k^H\mathbf{Q}_k\mathbf{h}_k \leq \text{Tr}(\mathbf{Q}_k)\|\mathbf{h}_k\|^2, \forall k$ for any $\mathbf{Q}_k \succeq \mathbf{0}$ [28]; (b) holds due to the fact that $\text{Tr}(\mathbf{Q}_k) \leq \frac{P}{K}, \forall k$, and the basic property of the matrix norm which is $\|\mathbf{A} + \mathbf{B}\| \leq \|\mathbf{A}\| + \|\mathbf{B}\|$ for all matrices $\mathbf{A}$ and $\mathbf{B}$ [28].

This completes the proof.

REFERENCES

[1] J. Xu, L. Liu, and R. Zhang, "Multiuser MISO beamforming for simultaneous wireless information and power transfer," *IEEE Trans. Signal Process.*, vol. 62, no. 18, pp. 4798–4810, Sep. 2014.

[2] Q. Shi, L. Liu, W. Xu, and R. Zhang, "Joint transmit beamforming and receive power splitting for MISO SWIPT systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 6, pp. 3269–3280, Jun. 2014.

[3] L. Liu, R. Zhang, and K.-C. Chua, "Wireless information transfer with opportunistic energy harvesting," *IEEE Trans. Wireless Commun.*, vol. 12, no. 1, pp. 288–300, Jan. 2013.

[4] L. Liu, R. Zhang, and K.-C. Chua, "Wireless information and power transfer: A dynamic power splitting approach," *IEEE Trans. Commun.*, vol. 61, no. 9, pp. 3990–4001, Sep. 2013.

[5] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 1989–2001, May 2013.

[6] L. Liu, R. Zhang, and K.-C. Chua, "Secrecy wireless information and power transfer with MISO beamforming," *IEEE Trans. Signal Process.*, vol. 62, no. 7, pp. 1850–1863, Apr. 2014.

[7] L. Zhou and Z. J. Haas, "Securing ad hoc networks," *IEEE Netw.*, vol. 13, no. 6, pp. 24–30, Nov. 1999.

[8] X. Zhao, J. Xiao, Q. Li, Q. Zhang, and J. Qin, "Joint optimization of AN-aided transmission and power splitting for MISO secure communications with SWIPT," *IEEE Commun. Lett.*, vol. 19, no. 11, pp. 1969–1972, Nov. 2015.

[9] H. Zhang, C. Li, Y. Huang, and L. Yang, "Secure beamforming for SWIPT in multiuser MISO broadcast channel with confidential messages," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1347–1350, Aug. 2015.

[10] Z. Chu, Z. Zhu, M. Johnston, and S. Y. L. Goff, "Simultaneous wireless information power transfer for MISO secrecy channel," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 6913–6925, Sep. 2016.

[11] D. W. K. Ng, E. S. Lo, and R. Schober, "Robust beamforming for secure communication in systems with wireless information and power transfer," *IEEE Trans. Wireless Commun.*, vol. 13, no. 8, pp. 4599–4615, Aug. 2014.

[12] H. Zhang, C. Li, Y. Huang, and L. Yang, "Simultaneous wireless information and power transfer in a MISO broadcast channel with confidential messages," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2015, pp. 1–6.

[13] R. Feng, Q. Li, Q. Zhang, and J. Qin, "Robust secure transmission in MISO simultaneous wireless information and power transfer system," *IEEE Trans. Veh. Technol.*, vol. 64, no. 1, pp. 400–405, Jan. 2015.

[14] M. R. A. Khandaker and K.-K. Wong, "Robust secrecy beamforming with energy-harvesting eavesdroppers," *IEEE Wireless Commun. Lett.*, vol. 4, no. 1, pp. 10–13, Feb. 2015.

[15] G. Geraci, S. Singh, J. G. Andrews, J. Yuan, and I. B. Collings, "Secrecy rates in broadcast channels with confidential messages and external eavesdroppers," *IEEE Trans. Wireless Commun.*, vol. 13, no. 5, pp. 2931–2943, May 2014.

[16] H. Zhang, Y. Huang, C. Li, and L. Yang, "Secure beamforming design for SWIPT in MISO broadcast channel with confidential messages and external eavesdroppers," *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7807–7819, Nov. 2016.

[17] A. S. Mansour, R. F. Schaefer, and H. Boche, "The individual secrecy capacity of degraded multi-receiver wiretap broadcast channels," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Jun. 2015, pp. 4181–4186.

[18] Y. Chen, O. O. Koyluoglu, and A. Sezgin, "Individual secrecy for the broadcast channel," *IEEE Trans. Inf. Theory*, vol. 63, no. 9, pp. 5981–5999, Sep. 2017.
[19] W. Feng, Y. Wang, N. Ge, J. Lu, and J. Zhang, "Virtual MIMO in multicell distributed antenna systems: Coordinated transmissions with large-scale CSIT," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 10, pp. 2067–2081, Oct. 2013.
[20] Y. Wang, W. Feng, L. Xiao, Y. Zhao, and S. Zhou, "Coordinated multicell transmission for distributed antenna systems with partial CSIT," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1044–1047, Jul. 2012.
[21] W. Feng, Y. Wang, D. Lin, N. Ge, J. Lu, and S. Li, "When mmWave communications meet network densification: A scalable interference coordination perspective," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 7, pp. 1459–1471, Jul. 2017.
[22] K.-Y. Wang, A. M.-C. So, T.-H. Chang, W.-K. Ma, and C.-Y. Chi, "Outage constrained robust transmit optimization for multiuser MISO downlinks: Tractable approximations by conic optimization," *IEEE Trans. Signal Process.*, vol. 62, no. 21, pp. 5690–5705, Nov. 2014.
[23] Q. Li and W.-K. Ma, "Spatially selective artificial-noise aided transmit optimization for MISO multi-eves secrecy rate maximization," *IEEE Trans. Signal Process.*, vol. 61, no. 10, pp. 2704–2717, May 2013.
[24] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
[25] M. Grant, S. Boyd, and Y. Ye, "Cvx: MATLAB software for disciplined convex programming," CVX Res., Inc. Stanford, CA, USA, 2008.
[26] I. Bechar. (Sep. 2009). "A Bernstein-type inequality for stochastic processes of quadratic forms of Gaussian variables." [Online]. Available: https://arxiv.org/abs/0909.3595
[27] Y. C. Eldar and N. Merhav, "A competitive minimax approach to robust estimation of random parameters," *IEEE Trans. Signal Process.*, vol. 52, no. 7, pp. 1931–1946, Jul. 2004.
[28] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 2012.

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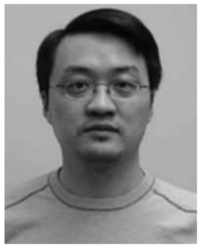

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