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Root-Based Nonlinear Companding Technique for Reducing PAPR of Precoded OFDM Signals

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ABSTRACT Orthogonal frequency division multiplexing (OFDM) signals are characteristically independent and identically distributed Gaussian random variables that follow Rayleigh distribution. The signals also exhibit high peak-to-average power ratio (PAPR) problem due to the infinitesimal amplitude component distributed above the mean of the Rayleigh distribution plot. Since the amplitudes are nonlinearly and non-monotonically increasing, applying roots to the amplitude distribution is shown in this paper to change the probability density function (PDF) and thus reduces the PAPR. We exemplify these by imposing this constraint on standard μ-law companding (MC) technique in reducing PAPR of OFDM signals, which is known to expand the amplitudes of low power signals only without impacting the higher amplitude signals. This limits the PAPR reduction performance of the MC scheme. Since companding involves simultaneously compressing/expanding high/low amplitude OFDM signals, respectively, in this paper, we refer to the new method as a root-based MC (RMC) scheme that simultaneously expands and compresses OFDM signal amplitudes unlike MC. In addition, we express a second transform independent of the MC model. The results of the two proposed schemes outperform four other widely used companding techniques [MC, log-based modified (LMC), hyperbolic arc-sine companding (HASC) and exponential companding (EC)]. Besides these, we precode the OFDM signals using discrete Hartley transform (DHT) in order to further reduce the PAPR limits achieved by RMC by distorting the phase. While preserving the BER, DHT-precoded RMC outperforms all the four other companding schemes (MC, EC, HASC, LMC) in terms of PAPR.

INDEX TERMS Discrete Hartley transform (DHT), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), DHT precoding, companding, high power amplifier (HPA).

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an integral part of modern communication technologies such as the long-term evolution (LTE) and the upcoming LTE-Advanced [1]. Besides its ability to efficiently manage spectrum, its capability to partition widebands into narrowbands makes it robust over fading channels. However, its major drawback is that the OFDM signals are characterized by high amplitude that complicate analog-to-digital (A/D) conversion. The high amplitudes also lead to high power consumption by power amplifiers. Although it can be argued that the peak-to-average power ratio (PAPR) problem can be combated by resetting the back-off, this leads to inefficiency of power amplifiers [2]. Currently, there are several methods proposed in literature for PAPR reduction [3]–[5] and can be grouped into three, namely, signal distortion techniques, multiple signaling and probabilistic techniques, and coding techniques. Reducing the PAPR will enhance high power amplifier (HPA) performance, reduce power consumption, reduce signal distortion by HPA and improve bit error ratio (BER) performance. PAPR reduction is also useful in mitigating impulsive noise in powerline systems [6].

Among the widely discussed amplitude distortion PAPR techniques, iterative clipping and filtering (ICF) is one of the simplest techniques [7], [8]. However, ICF expends the energy of communication systems in the order of J iterations to drive the fast Fourier transform (FFT) and inverse FFT blocks [9]. Also, ICF techniques are severed by high out-of-band interference (OBI) due to "hard" clipping of OFDM signals [10]. A solution to the significant OBI radiation is to "soft" compress and expand the OFDM signal peaks [10]. The soft compressing and expanding of OFDM signals is usually referred to as companding and does not involve $2J + 1$ FFT/IFFT blocks required in ICF and exhibits well reduced OBI compared to the uncompanded OFDM signals [10]–[12].

Sufficiently, companding PAPR techniques outperform clipping schemes [13]. In addition, due to its lightweight and good PAPR performances, companding is one of the most widely explored PAPR reduction schemes and has been recently combined with ICF [14] to reduce the PAPR of OFDM signals. Though combining ICF and companding techniques requires driving the IFFT/FFT many times to achieve a target PAPR, this expends the system power and increases communication latency. In this paper, we propose the combination of OFDM signal precoding and companding to reduce the PAPR of OFDM signals. While precoding reduces the PAPR of OFDM signals to a limit, we also show that precoding preserves the BER performance.

Precoding spreads the deep fading in OFDM signals across the whole bandwidth to enhance frequency diversity [15], [16]. In addition, precoding scrambles the signal phase to reduce the PAPR of the system. Consequently, since precoding the OFDM signals in the frequency domain improves the PAPR, we adopt discrete Hartley transform (DHT) precoding which demonstrates better PAPR reduction other precoding styles [15]. Then, we pass the resulting signal through IFFT block to generate time-domain OFDM signals and compand them to further reduce the PAPR by distorting the signal amplitudes.

In literature, different companding techniques exists [5], [10], [12], [13], [17], [18]. These are characteristically, based on the fact that OFDM signals have three components, namely, average amplitude, low amplitude and high amplitudes [6], [19]. In [10], airy function-based companding transform was proposed to reduce PAPR but requires the construction of a look-up table to decompand the received signals. This technique is realistically and computationally burdensome to OFDM system and energy inefficient. On the other hand, the low amplitude signals are expanded only in μ-law companding (MC) [20] without adequate compression of the higher amplitude ones as will be shown in Section II-B. This involves unfairly increasing the output power of companded signal power leading to unfair BER performance improvement when compared to the uncompanded signals [21]. In this study, we propose a root-based MC (RMC) PAPR scheme that overcomes the aforementioned limitations of the conventional MC scheme. We are aware of the attempt in [12] to solve the problem by using logarithimic function, however, we will show shortly that the scheme achives sub-optimal results.

Precoding is further also applied to achieve PAPR reduction while preserving the BER. The precoding converts the PAPR behaviour of a multicarrier system towards a single-carrier system [22]. This technique provides significant reduction in the PAPR while preserving the BER of the OFDM system [18] as will be shown in Section III. Afterwards, we compand the output time-domain OFDM signal to reduce the PAPR through amplitude distortion. The combination of DHT precoding and companding reduce the PAPR while maintaining low BER. We also compare our proposed RMC results with four different companding techniques in terms of PAPR and BER. These include exponential companding (EC) [23], hyperbolic arc-sine companding (HASC) [24], log-based modified MC (LMC) [12] and the standard MC [20] schemes. Finally, since each technique reduces the PAPR of an OFDM systems to a certain limit, we combine the precoding and companding to further reduce the PAPR beyond these limits while preserving the BER. In addition, we implement the precoding over the four different companding techniques to further improve their PAPR reductions beyond their respective limits. We found that the proposed RMC technique combined with DHT precoding achieves 9dB better PAPR performance than using original symbols and also outperforms the precoded LMC (PLMC) and MC (PMC) by 1dB at 10^{-3} complementary cumulative distribution function (CCDF). For the unprecoded symbols, the RMC outperforms LMC at 10^{-5} BER by 1dB and EC by 5dB at 10^{-3} BER, respectively.

The remaining parts of this paper are organized as follows. In Section II, we describe the proposed system model and present the performance evaluation in Section III followed by the conclusion.

II. SYSTEM MODEL

The goal of this study is to achieve PAPR reduction both before and after OFDM modulation using DHT precoding and companding, respectively. Precoding is a computationally convenient PAPR reduction scheme existing among other pre-OFDM modulation PAPR reduction schemes such as partial transmit sequence (PTS) and selective mapping (SLM). On the other hand, companding is a post-OFDM modulation PAPR reduction scheme that achieves better performance than clipping and does not expend system power in driving $2J + 1$ IFFT/FFT blocks in the order of J iterations. Thus, consider N randomly generated data which can be modulated using quadrature phase-shift keying (QPSK) to realize $d(n) = [d_0, d_1, \cdots, d_{N-1}]$ at the transmitter. Now, let the Hartley transform be [25]

$$H_s(\omega) = \int_{-\infty}^{\infty} f(t)\text{cas}(\omega t)\, dt \tag{1}$$

where $\text{cas}(\omega t) = \cos(\omega t) + \sin(\omega t)$ and $\omega = 2\pi f$. The inverse of (1) can be expressed as

$$f(t) = \int_{-\infty}^{\infty} H_s(\omega)\, \text{cas}(\omega t)\, d\omega. \tag{2}$$

To reduce the number of function computations, $\text{cas}(\omega t)$ can be expressed as $\text{cas}(\omega t) = \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right) = \sqrt{2}\cos\left(\omega t - \frac{\pi}{4}\right)$ [25]. Then, in discrete sense, given the time (n) and frequency (k) indices of the Hartley transform, (1) can be rewritten for the output QPSK symbol $s(n)$ as [26]

$$\begin{aligned} H_s(k) &= \frac{1}{\sqrt{N}} \sum_{n=0}^{NL-1} s(n)\text{cas}\left(\frac{2\pi nk}{N}\right) \\ &= \frac{\sqrt{2}}{\sqrt{N}} \sum_{n=0}^{NL-1} s(n)\cos\left(\frac{2\pi nk}{N} - \frac{\pi}{4}\right), \\ &\qquad \forall k = 0, 1, \cdots, N-1 \end{aligned} \tag{3}$$

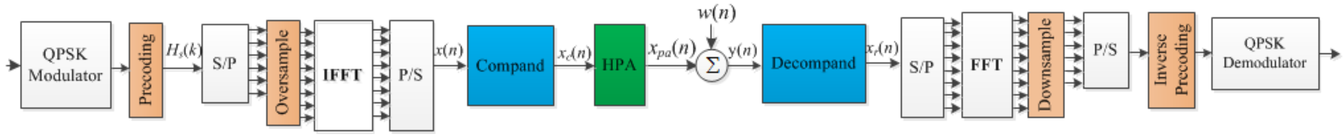

FIGURE 1. System model for precoded OFDM system with companding and decompanding for reducing the PAPR of OFDM signals.

where $1/\sqrt{N}$ is introduced as a normalization parameter. $H_s(k)$ may be described as the frequency Hartley-domain signal which rotates the phase of the input symbols. Besides the fundamental PAPR problem in OFDM systems, OFDM is also sensitive to spectral null problem over a frequency-selective fading channel [16]. The spread appeals mostly to symbols transmitted in the vicinity of deep fading which are usually overwhelmed by noise that makes the symbols become irrecoverable [15], [16]. Using Hartley transform, the nulls can be spread over the entire bandwidth to increase the probability of correctly recovering the transmitted symbols. Precoding of a multicarrier system has been exemplified in [27] and motivates the architecture used in this study as depicted in Fig. 1.

By using precoding, the PAPR performance of an OFDM multicarrier system can be restored to that of single-carrier systems [27]. Also, it spreads the input symbols such that when a signal among a modulated symbol is deeply corrupted, it does not damage the entire symbol [27].

Meanwhile, oversampling is performed at the baseband to make the PAPR of OFDM signals sampled at Nyquist frequency at baseband equivalent to that of the continuous-time signal [13]. Thus, an oversampling factor of $\ell \geq 4$ correctly approximates the true PAPR at baseband [13], [28]. The output frequency Hartley-domain signal $H_s(k)$ is oversampled as

$$S(k) = [H_s(0), H_s(1), \cdots, H_s(N-1), 0_N, \cdots, 0_{\ell N-1}] \tag{4}$$

where $S(k)$ is the k^{th} vector of a certain discrete input symbol and $0_{1\times(\ell N-N)}$ represents the zero-padding used for oversampling. This set of frequency domain symbols can be spread in phase using the Hartley transform. To realize the time-domain OFDM symbol at the transmitter, the Hartley-domain symbols are now transformed into time domain using the computationally efficient equivalent of the discrete Fourier transform (DFT), namely the IFFT as

$$s_p(n) = \frac{1}{\sqrt{\ell N}} \sum_{k=0}^{\ell N-1} S(k) e^{j2\pi \frac{kn}{\ell N}} \quad \forall k, n = 0, 1, \cdots, \ell N-1 \tag{5}$$

Since the components of (5) are complex, the amplitudes of OFDM signal can be extracted as follows

$$|s_p(n)| = \sqrt{s_r(n)^2 + s_i(n)^2} \tag{6}$$

where $|\cdot|$ represents the absolute value operator, $s_r(n)$ and $s_i(n)$ are from $s_p(n) = s_r(n) + js_i(n)$. This is useful in the computation and analyses of the PAPR behaviour of the signal. For example, when ℓN is sufficiently large, the central limit theorem provides that both $s_r(n)$ and $s_i(n)$ parts of (5) are asymptotically independent and identical Gaussian distributed variables [29], [30]. In other words, the amplitudes ($|s(n)|$) of OFDM symbols follow a Rayleigh distribution as [30]

$$f_{|s_p(n)|}(s_0) = \frac{s_0^2}{\sigma_s^2} \exp\left(-\frac{s_0^2}{2\sigma_s^2}\right), \quad \forall n = 0, 1, \cdots, \ell N-1 \tag{7}$$

where s_0 is the discrete-time envelope of (5) and $\sigma_s^2 = \frac{1}{2}\mathbb{E}\left\{|s_p(n)|^2\right\}$ is the variance; $\mathbb{E}\{\cdot\}$ represents the expectation value of $\{\cdot\}$. Given the signal amplitudes in (6), the complementary cumulative density function (CCDF) of $|s_p(n)|$ that measures the PAPR of the OFDM signal can then be expressed as

$$CC_{|s_p(n)|}(s_0) = \Pr\{\text{PAPAR} > \text{PAPR}_0\} \tag{8a}$$

$$= \left[1 - (1 - \exp(-\text{PAPR}_0))^{\ell N}\right] \tag{8b}$$

where $\Pr\{\cdot\}$ represents the probability of $\{\cdot\}$, PAPR_0 is the desired PAPR target and the system PAPR can be expressed as

$$\text{PAPR} = \frac{\max\limits_{n=0,1,\cdots,\ell N-1} \left(|x(n)|^2\right)}{\frac{1}{\ell N}\sum\limits_{n=0}^{\ell N-1}\left(|x(n)|^2\right)}. \tag{9}$$

Recall that the DHT precoding distorts phase of the signal while companding distorts the amplitudes; both towards high level PAPR reduction while preserving the BER performance. Thus, the resulting DHT-precoded OFDM symbols are companded using different companding transforms including the proposed RMC scheme described later in Section II-A as

$$s_c(n) = \mathcal{F}\left\{s_p(n)\right\} \quad \forall n = 0, 1, \cdots, \ell N-1 \tag{10}$$

where $\mathcal{F}\{\cdot\}$ is the companding transform applied to the time-domain OFDM symbols; in Section II-A, we will also show that $\mathcal{F}\{\cdot\}$ only impacts the amplitude.

Note that in Fig. 1 no cyclic prefix is used because the model does not include any fading (e.g. multipath) channel. In this case, the received signal over an additive white Gaussian noise (AWGN) channel can be expressed as

$$\boldsymbol{y}(n) = \boldsymbol{s}_c(n) + \boldsymbol{w}(n) \quad \forall n = 0, 1, \cdots, \ell N-1 \tag{11}$$

where $\boldsymbol{w}(n)$ is modeled as a circularly symmetric Gaussian random variable [31] with zero mean and variance $\sigma_w^2 = \frac{1}{2}\mathbb{E}\left\{|w(n)|^2\right\}$. The received signal over the AWGN channel is then decompanded as

$$\begin{aligned}\boldsymbol{r}(n) &= \mathcal{F}^{-1}\left\{\boldsymbol{y}(n)\right\} \\ &= \mathcal{F}^{-1}\left\{\mathcal{F}\left\{\boldsymbol{s}_p(n)\right\} + w(n)\right\} \\ &= \mathcal{F}^{-1}\left\{s_c(n) + w(n)\right\} \\ &= s_p(n) + \mathcal{F}^{-1}\left\{w(n)\right\} \quad \forall n = 0, 1, \cdots, \ell N - 1 \quad (12)\end{aligned}$$

where $\mathcal{F}^{-1}\{\cdot\}$ is the decompanding transform. At the receiver, the BER performance of the companded signals depend on the ability of the decompanding transform, $\mathcal{F}^{-1}\{\cdot\}$, to correctly recover transmitted companded signals (in addition to the degree of amplitude distortion at the transmitter which amplifies the background noise, thus trading the BER for PAPR gain). In fact, since companding/decompanding is a nonlinear process that involves the distortion of the input signal amplitudes, the BER depends on the ability of $\mathcal{F}^{-1}\left\{\mathcal{F}\left\{\boldsymbol{s}(n)\right\}\right\}$ to correctly decompand received signals at the receiver after being impacted by the system noise.

From (12), if $\mathbb{E}\left\{|s_c(n)|^2\right\}$ is the average received signal power and $\mathbb{E}\left\{|\tilde{w}(n)|^2\right\}$ is the average noise power, where $\tilde{w}(n) = \mathcal{F}^{-1}\left\{w(n)\right\}$, by letting $\gamma = \frac{\mathbb{E}\left\{|s_c(n)|^2\right\}}{\mathbb{E}\left\{|\tilde{w}(n)|^2\right\}}$ be the received signal-to-noise ratio, then the decompanding function enhances the noise power. Thus, while the PAPR is reduced, the BER is impaired due to the degree of amplitude distortion which impacts the background noise. The BER can be restored to an extent depending on the ability of the decompanding transform to recover the transmitted companded signal.

Then, the forward FFT is performed to transform the processed signal back into Hartley-frequency domain as follows

$$\hat{S}(k) = \sum_{n=0}^{\ell N-1} \boldsymbol{r}(n) e^{-j2\pi\frac{kn}{lN}} \quad \forall k = 0, 1, \cdots, \ell N - 1 \quad (13)$$

The received signal transformed into frequency domain still contains the oversampling components and are then downsampled as illustrated in Fig. 1. Next, we perform the inverse DHT (IDHT) to recover the original transmitted symbols. From (1) and (2), the HT has a special property that it is the inverse of itself so that the inverse of (3) can be realized as

$$\hat{s}(n) = \frac{1}{N}\sum_{k=0}^{N-1} \hat{S}(k) \cos\left(\frac{2\pi nk}{N} - \frac{\pi}{4}\right), \quad \forall n = 0, 1, \cdots, N - 1 \quad (14)$$

It can be summarized that the IDHT is the transform of itself with slightly reduced complexity by using $\sqrt{2}\cos\left(\frac{2\pi nk}{NL} - \frac{\pi}{4}\right)$ instead of $\text{cas}\left(\frac{2\pi nk}{NL}\right) = \cos\left(\frac{2\pi nk}{NL}\right) + \sin\left(\frac{2\pi nk}{NL}\right)$. In addition to significantly reducing the PAPR of OFDM system, the DHT provides an excellent BER performance.

A. PROPOSED RMC COMPANDING TRANSFORMS

It is widely known that PDF of (z) is related to the CDF as $F_z(u) = \int_0^u f_z(u)du$, where $F_z(u)$ is the CDF and $f_z(u)$ is the PDF. Notably, these can be summarized by the following identity [32]

$$\mathcal{F}(\boldsymbol{x}(n)) = F_{x_c}^{-1}\left(F_x\left(x\left(n\right)\right)\right) \quad (15)$$

where $F_{x_c}^{-1}(\cdot)$ is the inverse CDF of companded signal and $F_x(\cdot)$ is the CDF of the uncompanded signal expressed as

$$F_x(\cdot) = 1 - \exp\left(-\frac{x_0^2}{\sigma_x^2}\right), \quad x_0 > 0 \quad (16)$$

where x_0 is the discrete envelope of $x(n)$. The resulting model $\mathcal{F}(\boldsymbol{x}(n))$ is the required companding transform that converts the PDF of conventional OFDM to a desired distribution. Since $|x_c(n)|$ is close to the desired uniform distribution, then the CDF is written as [11]

$$F_{x_c(n)}(x_c) = \frac{x_c}{2A} + \frac{1}{2}, \quad 0 \le x_c \le A_{EC}. \quad (17)$$

By manipulating (15), the companding transform can be expressed as

$$\mathcal{F}_{EC}(\boldsymbol{x}(n)) = \text{sgn}(x) \cdot A_{EC} \cdot \text{erf}\left(\frac{|x|}{\sqrt{2\sigma_x^2}}\right), \quad 0 \le x \le 1 \quad (18)$$

where $A_{EC} = \sqrt{3\sigma_x^2}$. (18) is the fundamental error-function based companding transform. Before this, the MC was introduced based on [20]

$$\begin{aligned}s_c(n) &= \mathcal{F}(s_p(n)) \\ &= \text{sgn}\left(s_p(n)\right) \frac{A_s \times \log\left(1 + \mu\left|\frac{s_p(n)}{A_s}\right|\right)}{\log(1+\mu)} \quad (19)\end{aligned}$$

where $\text{sgn}(x(n)) = \frac{x(n)}{|x(n)|}$ is the phase and $A_s = \max(|s_p(n)|)$. We enumerate three fundamental problems with the standard MC technique; 1) it expands the amplitudes of the lower power symbols only; and the PAPR is poor compared to other companding techniques. Lastly, since MC expands the amplitude of weaker symbols, then the companded signal output power will be higher than the original input power leading to an "unfair" improvement of BER when compared with the uncompanded signals; this was also pointed out by [21] and addressed in [33]. Since the higher amplitude signals are not compressed, the idea of PAPR reduction to reduce HPA smearing of the signals (leading to poor BER performance after passing through HPA) and the consequent high power consumption are defeated.

To overcome this problem, consider a nonlinear transform, namely

$$x_{nlin} = \mathcal{F}\left\{x\right\} = a_1 x^{\beta} + b_1 \quad (20)$$

where a_1 is the attenuation factor and b_1 is the distortion noise. If $1 \le \beta \le \infty$, then $|x_{nlin}(n)|^2$ increases as $\beta \Rightarrow \infty$, where x is the OFDM symbol; this is the default case of the standard MC. Similarly, if β is bounded by $0 < \beta \leq 1$, then $|x_{nlin}(n)|^2$ approaches a constant as $\beta \Rightarrow 0$. Based on this, we propose a new transform such that in addition to increasing the amplitudes of smaller signals like the classical MC, it also compresses amplitudes of the higher energy signals; while maintaining good BER and PAPR performances simultaneously. Now, recall the conventional MC we rewrite (19) as

$$\mathcal{F}(s_p(n)) = A_s \times \text{sgn}(s_p(n)) \frac{\log\left(1 + \mu \frac{|s_p(n)|^{\frac{1}{\beta}}}{A_s}\right)}{\log(1+\mu)} \quad (21)$$

where α is a normalization factor that enables the transform to work with equal output power as the uncompanded signal $s_p(n)$, namely $E_{s_p} = E_{s_c}$ and $E_{s_c} = \mathbb{E}\left\{|s_c|^2\right\}$. We know that by compressing large amplitude signals and expanding weaker ones, both the PAPR reduction and immunity of small signals from noise can be achieved [13]. Based on the PAPR modulating parameter, $1/\beta$ in (21), the proposed is referred to as root-based MC (RMC). For the normalization of output power of $s_c(n)$, authors in [21] suggest that

$$\alpha = \frac{\log(1+\mu)}{\mu} \quad (22)$$

should be used as the normalization parameter with (19); this was later used by [33]. In this work, we propose a more intuitive method, namely

$$\alpha\mathbb{E}\left\{|s_c(n)|^2\right\} = \mathbb{E}\left\{\left|s_p(n)\right|^2\right\} \quad (23)$$

where $\mathbb{E}\left\{|s_c(n)|^2\right\}$ and $\mathbb{E}\left\{\left|s_p(n)\right|^2\right\}$ are the average powers of the companded and input (uncompanded) signals respectively. It follows that,

$$\alpha = \sqrt{\frac{\mathbb{E}\left\{\left|s_p(n)\right|^2\right\}}{\mathbb{E}\left\{\left|\mathcal{F}(s_p(n))\right|^2\right\}}}. \quad (24)$$

This allows for fair comparison with uncompanded signals [21] and efficient operation of the power amplifier [33]. Consequently, the well-normalized companded signal becomes

$$s_c^{new}(n) = \alpha \times \mathcal{F}(s_p(n)). \quad (25)$$

For example, OFDM systems designed to operate in the perfectly linear region of the amplifier suggest that they operate at power levels well below the maximum available power [33]. Thus, with companding however, the HPA of an OFDM system can also benefit from the nonlinear region with higher power.

In Fig. 2, the PDF of OFDM signal amplitude distributions show the concentration of the converted amplitude distribution to uniform distribution with all the amplitudes distributed around the mean amplitude. It can also be observed that the amplitudes of the companded signals are more uniformly distributed than the conventional amplitudes of the unmodified OFDM signals. This will impact the PAPR performance as will be shown in Section III. In fact, since RMC shows better concentration of amplitudes around the mean amplitude than MC and RC, its PAPR performance will be better than the previous two.

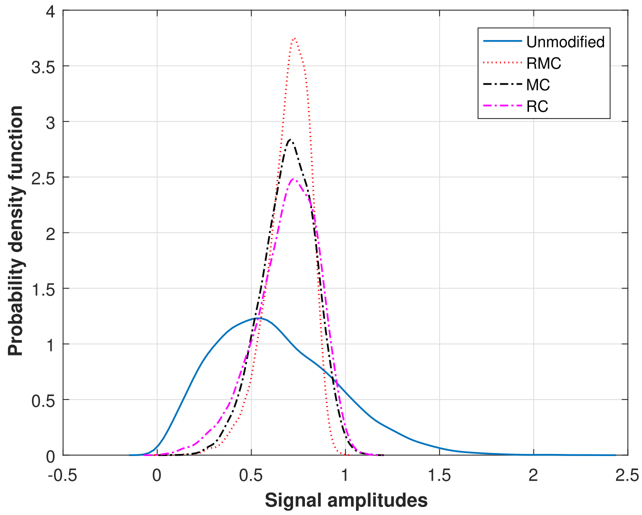

FIGURE 2. Investigation of PDF performances of DHT precoded symbols modulated using OFDM and companded using different companding methods.

The decompanding transform for recovering the transmitted signal is derived as follows

$$\mathcal{F}^{-1}\left\{\mathcal{F}\left\{s_c(n)\right\}\right\} = \text{sgn}(s_c(n)) \times \left(\frac{A_s}{\mu}\left[(\log(1+\mu))^{\frac{|s_c(n)|}{\sqrt{\alpha}A_s}} - 1\right]\right)^{\beta} \quad (26)$$

knowing that $\log_Q P = \frac{\log_a P}{\log_a Q}$. A variant of (26) can be expressed as

$$\mathcal{F}^{-1}\{\mathcal{F}\{s_c(n)\}\} = \text{sgn}(s_c(n)) \times \left(\frac{A_s}{\mu}\left[\exp\left(\frac{|s_c(n)|}{\sqrt{\alpha}A_s} \times \log(1+\mu)\right) - 1\right]\right)^{\beta}. \quad (27)$$

In Section II-B, we present the behaviours of the proposed companding technique in comparison to other companding styles [12], [23], [33], [34]. We stress the compressing and expanding abilities of the proposed scheme which the default MC [20] does not have.

Since (6) is nonlinear and non-monotonically increasing, in addition to (21), one can similarly achieve companded signal by imposing the proposed constraint on (20), for example

$$x_{nlin}^c(n) = \mathcal{F}\left\{s_p(n)\right\} = |x(n)|^{\frac{1}{\beta}} \frac{x(n)}{|x(n)|} \quad (28)$$

where the corresponding decompanding transform can be achieved by

$$x(n) = \left|\mathcal{F}\left\{s_p(n)\right\}\right|^{\beta} \text{sgn}(x(n)). \quad (29)$$

Notice that if $y = \mathcal{F}\{x(n)\} = s \times \text{sgn}(x)$, then $\text{sgn}(x) \simeq \text{sgn}(y)$ representing the phase since the companding transform impacts the amplitude only. Meanwhile, the output in (28) must be scaled to ensure equal power output with the original signal.

B. PERFORMANCE OF THE PROPOSED COMPANDING TRANSFORM

From the foregoing discussions, it is well established by now that the proposed companding transform is derived from the MC. A well-known problem with the MC is that it only expands the lower amplitude symbols of an OFDM system.

In Fig. 3, five different companding transforms are compared including the proposed RMC companding transform. It is observed that the MC expands the lower amplitude symbols while the EC and LMC both compress and expand the lower and higher amplitude symbols. The distortions of these symbols (with lower and higher amplitude symbols inclusive) will result in significant PAPR reduction. Now, on the proposed RMC scheme, it can be seen that the amplitudes of all inputs symbols are distorted which further reduces the PAPR of an MC technique. Lastly, the HASC scheme does not alter the amplitudes of lower energy signals; this will lead to better BER performance.

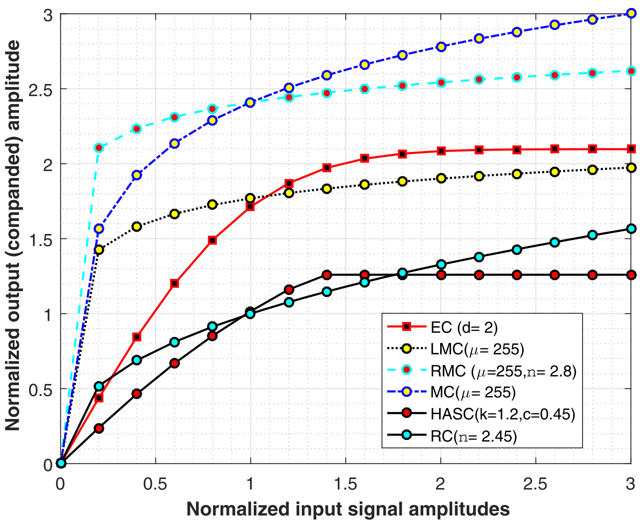

FIGURE 3. Comparison of the proposed transform with other companding transform methods.

Besides the root, $1/\beta$, the μ parameter also affects the amplitude of the OFDM signals as shown in Fig. 4 for the proposed RMC. By varying the roots $(1/n)$ as shown in Fig. 4, the smaller amplitudes are expanded and the higher amplitudes are compressed; this achieves amplitude distortion that leads towards a uniform distribution which significantly affects the PAPR.

Thus, the performance of an OFDM system based on the proposed RMC scheme must be carefully adjusted using the best design choice of β and μ for best performance, where $1 < \beta \leq \infty$. It is a game of design choice on what to sacrifice in terms of the PAPR and the BER. Clearly, from Fig. 4,

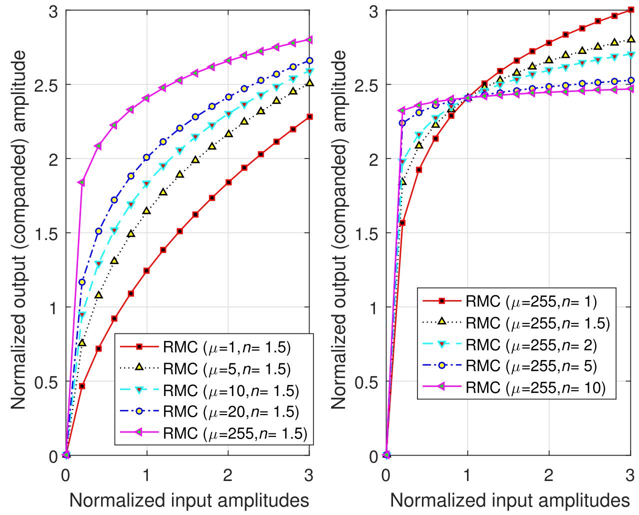

FIGURE 4. Performance of output (companded) signal with μ using the RMC transform for different roots $(1/n)$.

it can be seen that the root is responsible for higher symbol amplitude compression which complements the expanding parameter, μ. Thus, combining μ and $1/\beta$ enables the MC conventional companding transform to both compress and expand OFDM symbols simultaneously. Since HPAs are sensitive to, and also distort amplitudes residing within the nonlinear region [13], the root $(1/\beta)$ must be chosen cautiously to minimize the degree of compressing (clipping) of higher amplitude signals and the aforementioned effects.

Recall that in Section I, we discussed that OFDM signal amplitudes can be classified into three, namely, average, low or high amplitudes. By definition, companding is expected to compress high amplitude signals and also expand the low amplitudes. Unfortunately, the MC scheme only expands low energy signals using μ values, with no impacts on the high amplitudes which limits its PAPR performance. Consequently, in Fig. 5, it can be seen that the proposed RMC $(\beta > 1)$ significantly reduces the PAPR of the conventional MC (i.e. $\beta = 1$) for all μ compressing values. In terms of PAPR, it can be inferred that the proposed technique considerably reduces the PAPR of OFDM signals compared to the conventional MC technique. Also, it shows that any μ value can be used for well-behaving PAPR performance. In dB, the gain is demonstrated in Fig. 5. It is seen that as $\beta \Rightarrow \infty$, the PAPR gain increases. Specifically, comparing the MC and the proposed RMC ($\beta = 1+\epsilon$, where ϵ is non-negative), the proposed RMC outperforms the conventional MC for all values.

C. NONLINEAR TRANSMISSIONS OVER HIGH POWER AMPLIFIER

To evaluate the performance of the proposed model over an AM-based HPA, we invoke the SSPA-HPA [35]

$$\boldsymbol{x}_{pa}(n) = \frac{g_0\left(\hat{\boldsymbol{x}}(n)\right)}{\left(1+\left(\frac{g_0(\hat{\boldsymbol{x}}(n))}{A_{sat}}\right)^{2M}\right)^{\frac{1}{2M}}} e^{j\phi_n} \tag{30}$$

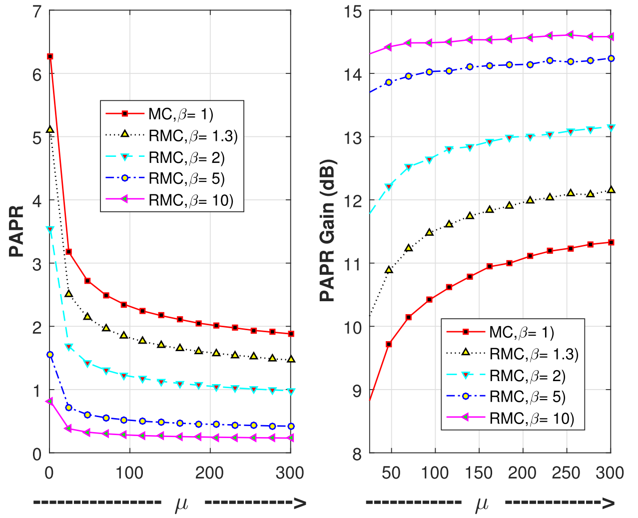

FIGURE 5. Performance evaluation of proposed RMC transform ($\beta > 1$) in comparison with the conventional MC scheme ($\beta = 1$) for different OFDM amplitude compressing values (μ).

where $g_0\left(\hat{x}(n)\right)$ is the input gain and A_{sat} is the input saturation level and M determines the output sharpness parameter. From (6), the HPA distorts only the amplitudes of the PAPR reduced signal in its nonlinear region. Since $M = [2, 3]$ in practical HPAs, we use $M = 3$ as in [36]. We illustrate in Fig. 6, three different saturation levels namely $A_{sat} = 0.5$, $A_{sat} \simeq 0.63$ (average amplitude of the signal after companding) and $A_{sat} = 1.0$. Clearly, at 0.63, the companded signal is far below the HPA saturation level. However, only the sharpness parameter significantly affects the signals. After passing through the HPA as expressed in (30), the received signal can be written as

$$\begin{aligned} \boldsymbol{r}(n) &= \alpha_{pa}\boldsymbol{x}_{pa}(n) + \boldsymbol{d}_{pa}(n) + \boldsymbol{w}(n) \\ &= \alpha_{pa}\boldsymbol{x}_{pa}(n) + \boldsymbol{w}_d(n) \end{aligned} \tag{31}$$

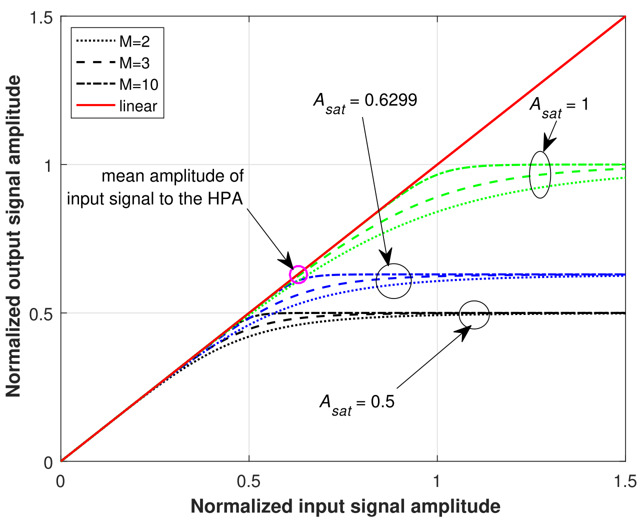

FIGURE 6. An example of the performance of nonlinear SSPA at varying saturation levels including the mean amplitude of PAPR reduced signal.

where $\boldsymbol{w}_d(n) = \boldsymbol{d}_{pa}(n) + \boldsymbol{w}(n)$ and α_{pa} is the combined attenuation due to companding and HPA. If α_{pa} is known (where α_{pa} depends on the amplitude attenuation induced by a companding transform), then compensating for this before transmission can performed as

$$\hat{\boldsymbol{x}}_{pa}(n) = \frac{\boldsymbol{x}_{pa}(n)}{\alpha_{pa}} = \hat{\boldsymbol{x}}(n) + \frac{\boldsymbol{d}_{pa}(n)}{\alpha_{pa}} \tag{32}$$

Furthermore, suppose that the distortion noise due to HPA negligible due to PAPR reduction, we can enhance the signal integrity using the decompanding transform as

$$\bar{\boldsymbol{x}}(n) = \mathcal{F}^{-1}\left(\hat{\boldsymbol{x}}_{pa}(n)\right) = \hat{\boldsymbol{x}}_{pa}(n) - \frac{\boldsymbol{d}_{pa}(n)}{\alpha_{pa}} \tag{33a}$$

$$= \hat{\boldsymbol{x}}(n) + \frac{\boldsymbol{d}_{pa}(n)}{\alpha_{pa}} - \frac{\boldsymbol{d}_{pa}(n)}{\alpha_{pa}} \tag{33b}$$

$$\Rightarrow \bar{\boldsymbol{x}}(n) = \boldsymbol{x}(n) + \boldsymbol{d}(n). \tag{33c}$$

Unfortunately, applying compensation after passing the signal through AWGN channel amplifies the noise as follows

$$\bar{\boldsymbol{x}}_{pa}(n) = \frac{\boldsymbol{r}(n)}{\alpha_{pa}} \tag{34}$$

$$= \boldsymbol{x}_{pa}(n) + \frac{1}{\alpha_{pa}}\left(\boldsymbol{d}_{pa}(n) + \boldsymbol{w}(n)\right). \tag{35}$$

One of the ways of improving the signal purity relies on finding the correlation coefficient using the distorted and original signal, $\mathcal{R}_{pa}$, after passing the signal through the HPA. This minimizes the error floor $\mathbb{E}\left[\left|\boldsymbol{x}(n) - \mathcal{R}_{pa}\bar{\boldsymbol{x}}^*(n)\right|^2\right]$ [37], where $(\cdot)^*$ represents complex conjugate and

$$\mathcal{R}_{pa} = \frac{\mathbb{E}\left[\boldsymbol{x}(n) \cdot \bar{\boldsymbol{x}} * (n)\right]}{\mathbb{E}\left[|\boldsymbol{x}(n)|^2\right]}. \tag{36}$$

In (36), it can be said that $\bar{\boldsymbol{x}}(n)$ is the HPA output signal compensated after the SSPA-HPA before passing it over the AWGN channel thereby minimizing the error floor due to nonlinear distortion.

D. TRANSMISSION OF THE PAPR REDUCED SIGNAL OVER MULTIPATH FADING CHANNELS

We further send the PAPR reduced signal over fading channels involving Rayleigh multipath channel as in [38]. When passed through the channel, the signal undergoes fading due to the amplitude attenuation induced by the channel impulse response, $h(\tau)$ with channel response $H(f)$. Unlike [38] that uses zero-forcing equalization (ZFE) scheme, we adopt minimum mean square error (MMSE) equalization scheme [39] which minimizes the error floor due to noise usually expanded by ZFE. The MMSE model can be expressed as

$$\hat{x} = \frac{H(f)^* X(k)}{|H(f)|^2 + SNR^{-1}} \tag{37}$$

where $X(k)$ represents the frequency domain content of the received signal after decompanding and signal transformation into frequency domain. This MMSE scheme appeals to companding PAPR reduction model as decompanding at the receiver might expand the noise overhead.

III. PAPR AND BER RESULTS AND DISCUSSIONS

In this section, we describe the computer simulation used to evaluate the performance of the proposed design. The design involves QPSK modulation of some randomly generated symbols which are precoded using DHT and then oversampled 4 times as depicted in Fig. 1. As we suggested in Section II-A, the proposed power normalization parameter α is used to ensure that the input power of the companded symbols are equal to the output power of the output companded symbols. The simulation results are provided for the proposed systems as well as the conventional OFDM systems for comparison. At first, we modulate $N = 128$ random symbols using QPSK which are then normalized such that $E_s = \frac{1}{2}\mathbb{E}\left\{|s(n)|^2\right\} = 1$. Then, these symbols are oversampled 4-times resulting in a total of 512 IFFT/FFT multicarrier points for the input symbols. To assess the system performance with respect to the unprecoded and uncompanded symbols, we compute the PAPR of the original (non-precoded and uncompanded) symbols first. Subsequently, we process the input symbols over the proposed RMC companding transform in comparison to four other well-known techniques, namely MC, EC, LMC and HASC. The results are presented and discussed in Section III-A. Finally, we precode the input symbols and process them using these four companding methods; the results are discussed in Section III-B.

A. COMPANDING TRANSFORM (ONLY) RESULT ANALYSES

For a QPSK oversampled signal which are companded using EC, MC, LMC, HASC and RMC, we compare their performances with the uncompanded signals. From Fig. 7, all the transforms significantly reduced the PAPR of the OFDM system with the EC ($d = 2$) outperforming the remaining techniques. However, it is clear that while the proposed RMC outperforms all other transforms including the EC scheme it (proposed RMC) also outperforms the original symbol by 8dB at 10^{-4} CCDF.

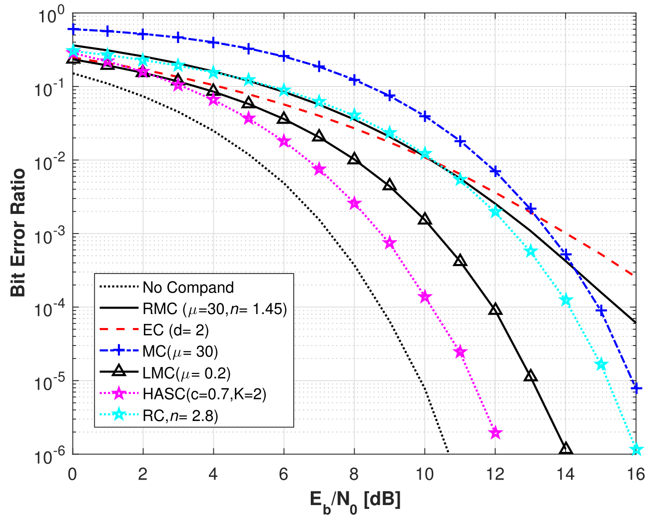

FIGURE 8. Comparison of BER of different OFDM symbols over different companding schemes ($\ell = 4$, $N = 128$).

Well-performing PAPR techniques come with BER penalties. This is exemplified in Fig. 8 for the foregoing discussions of the different companding schemes. We find that using the normalized style described in Section II, the HASC has best-performing BER next to the original symbols. However, the proposed RMC achieves better BER performance than EC scheme at low BER thresholds.

Based on these results, we uphold the RMC for OFDM system design for good PAPR with BER performances.

B. COMPANDED DHT PRECODED SYMBOL ANALYSES

In Sections I and II, we described that precoding OFDM symbols using DHT preserves the BER while significantly

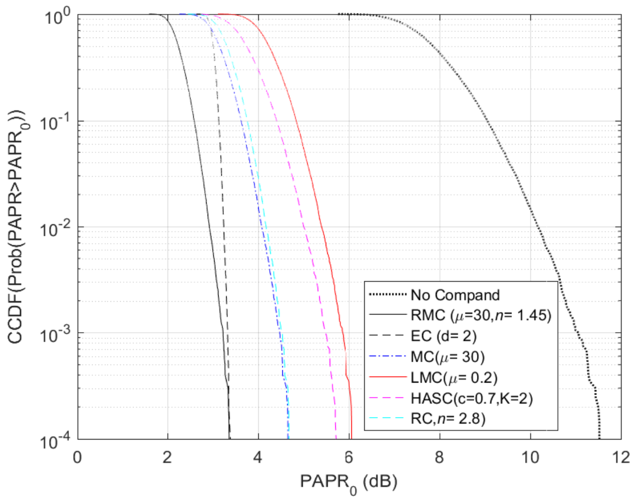

FIGURE 7. PAPR comparison of OFDM symbols over different companding schemes oversampled by $\ell = 4$ times, $N = 128$.

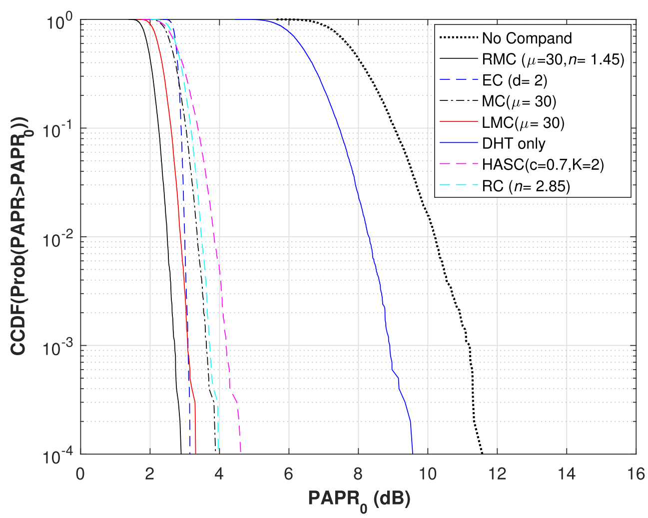

FIGURE 9. PAPR of DHT precoded OFDM symbols over different companding schemes ($\ell = 4$, $N = 128$).

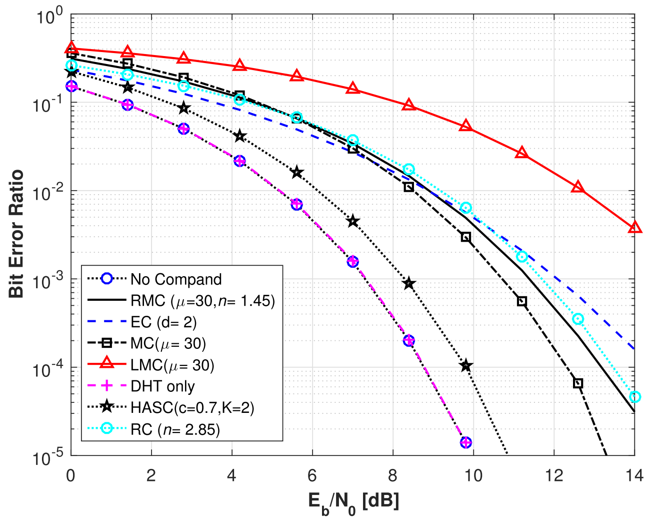

FIGURE 10. BER of DHT precoded OFDM symbols over different companding schemes $\ell = 4$, $N = 128$.

reducing the PAPR; these are demonstrated in Figs. 9 and 10. Precoding the input symbols spreads the spectral nulls over the entire bandwidth of an OFDM system evenly. We find from Fig. 9 that the OFDM symbols that are only precoded (e.g. using DHT) reduces the PAPR by 3dB. Then, for the remaining PAPR techniques, for example, comparing Figs. 9 and 7, the DHT precoding improves the companding techniques (MC, RMC, HASC, EC and LMC) beyond their performance limits by at least 1.2dB. Notice that our results for OFDM symbols precoded with DHT only (without companding) also agrees with the ones reported in [40].

Previously, we described that precoding the input symbols preserves the BER and this is corroborated in Fig. 10 as all the BERs of all companding techniques are reduced compared to the one shown in Fig. 8. While the performances of RMC, LMC and MC can be further improved both in terms of BER and PAPR by varying μ (and β for the RMC only), it follows therefore that DHT is not a good hybrid for EC given the investigated $d = 2$ version.

Lastly, while the companding technique is a lightweight one in OFDM systems and demonstrates excellent PAPR performance, DHT proposed here preserves the BER. For future studies, we encourage the interested reader to explore the implementation of DHT over other companding schemes. Although the DHT preserves the BER by spreading OFDM symbols, its impact when the OFDM symbols are companded is also noticeable; the error vector magnitude (EVM) depends on the degree of symbol amplitude distortion and the ability to recover companded symbols by the decompander for companded signals. Then the spread by the DHT improves the EVM such that the BER is improved; for example, comparing Figs. 10 and 8, PEC achieved 2dB gain at 10^{-3} BER over EC. Similarly, PRMC, PLMC, PMC and PHASC (where "P" represents precoded) achieved at least 1dB gain over RMC, LMC, MC and HASC respectively at 10^{-4} BER.

Thus, in terms of both PAPR and BER, the proposed RMC scheme performs well. In addition, by considering the proposed RMC scheme for fair comparison with the original OFDM signals using the proposed power normalization metric, it is observed that MC does not need to operate at $\mu = 255$ for optimal performance as earlier studies suggested. Finally, the proposed method of power normalization can be extended to any known companding transform to ensure equivalent power dissipation as the original OFDM signal.

C. BER PERFORMANCE OVER HIGHER ORDER SIGNAL CONSTELLATION

In this section, we further demonstrate the BER performance of our proposed system for higher order constellation size, for example, with the use of 16-QAM as shown in Fig. 11. While the BER performance of the precoded OFDM signal is not diminished, the BER of the HASC scheme achieves the best performance among all the companding schemes due to the fairly slight amplitude distortion it induces on the signals up to 10^{-3} BER where a severe irreducible error was seen. However, the proposed RMC model achieves better BER performance than LMC and EC schemes. BER degradation is the prize paid by reduction the PAPR of OFDM signals due to in-band noise from amplitude distortion of the signals. In addition, the second proposed RC PAPR reduction achieved better BER performance than all other companding models except HASC scheme.

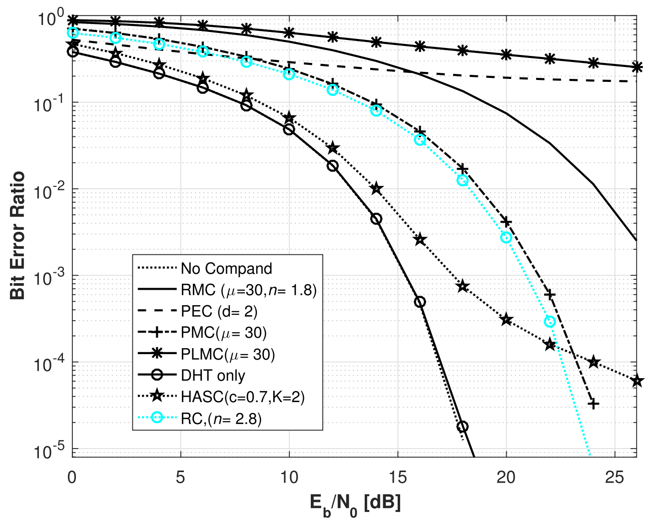

FIGURE 11. BER performance of proposed PAPR reduction scheme using 16-QAM in comparison with other PAPR reduction schemes.

D. BER PERFORMANCE EVALUATION OF PROPOSED COMPANDING SCHEMES OVER HPA

In Fig. 12, the impact of the proposed companding transforms on OFDM signal performances over HPA are presented in comparison with other companding schemes. Recall, however, that the PAPR of an OFDM system is usually reduced to minimize the signal occurrence within the nonlinear region of the HPA and thus, reduces the amplitude smearing which leads to in-band distortion and BER degradation. The companded signal is passed through an HPA characterized in Section II-C with a saturation level of 1. Then considering the unmodified signal, the signal is severely impaired compared to all companded output signals as the companding PAPR reduction suppresses the peaks such that they fall below the nonlinear region. We emphasize, however, that MMSE compensation factor is estimated respective to prevailing signal frame to compensate for nonlinear distortion induced both by the companding and the HPA. The result presented in Fig. 12 suffices for OFDM system operated without phase distortion through precoding. To examine the effects of both amplitude and phase distortion in PAPR reduction before passing the signal through HPA, we precode the signal using the foregoing DHT precoding model then distorting the signal amplitude using companding schemes. The results shown in Fig. 13 show BER improvement when considering unmodified OFDM signal. However, for remaining companding schemes including RMC and RC, the phase distortion due to DHT precoding before carrying out companding marginally improves the BER performance with slight BER degradation due to the HPA effects.

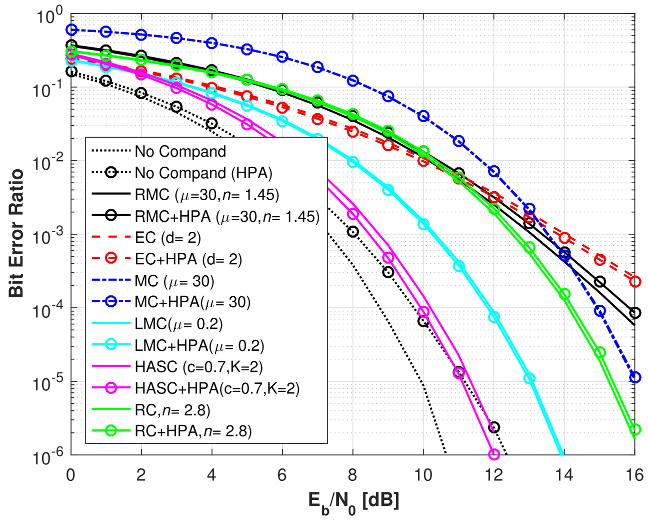

FIGURE 12. BER Performances proposed PAPR reduction techniques over HPA (all with nonlinear MMSE compensations).

E. PERFORMANCE OF RECEIVED COMPANDED SIGNAL OVER MULTIPATH FADING CHANNELS

Over the Rayleigh fading channel, the received signal decoded after MMSE equalization as shown in Figs. 14 and 15. At the receiver, we assumed that the channel state information is present at the receiver so that no feedback system is applied. In Fig. 14, it is observed that the precoded-only signals perform reasonably akin to the undistorted signal due to the fact that it only phase distortion is induced - this is usually not as severe as amplitude distortion. Notwithstanding, when distorted in terms of amplitudes by using companding there exist slight BER degradation due to the in-band noise. This slight BER degradation can be explained on the premises that the decompanding transform is able to correctly restore signal amplitudes that have been companded at the transmitter. Secondly, the slight deviation in performance in comparison to the original (undistorted) signals can be explained on the leverage of in-band noise induced during companding at the transmitter which are, at the receiver, collectively restored by the MMSE equalization scheme and correct decompanding. Meanwhile, notice that EC is worst degraded at low BER measure. It is interesting to find that by applying DHT precoding, the severed BER performance of EC scheme is improved as shown in Fig. 15. This is found among other companding models when comparing Figs. 14 and 15. It is also observed that the DHT precoding enhances the BER performance in Fig. 15 when compared to Fig. 14. The phenomenon can be explained on the premise that DHT frequency domain precoding helps to spread the dip fade

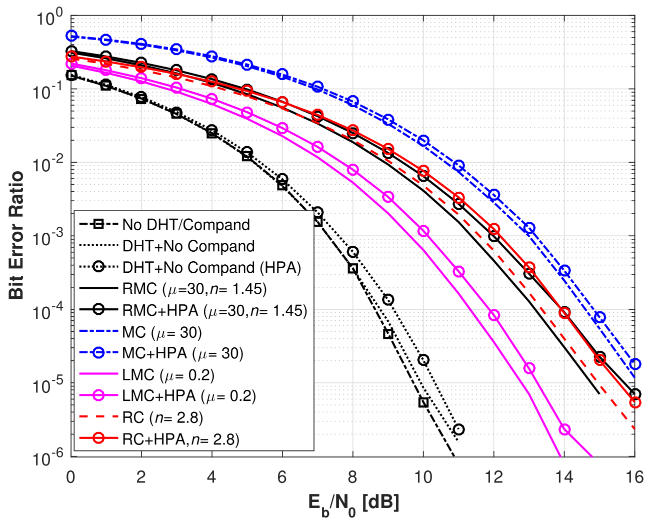

FIGURE 13. BER Performances of DHT precoded OFDM over HPA for the two proposed PAPR reduction methods (all with nonlinear MMSE compensations).

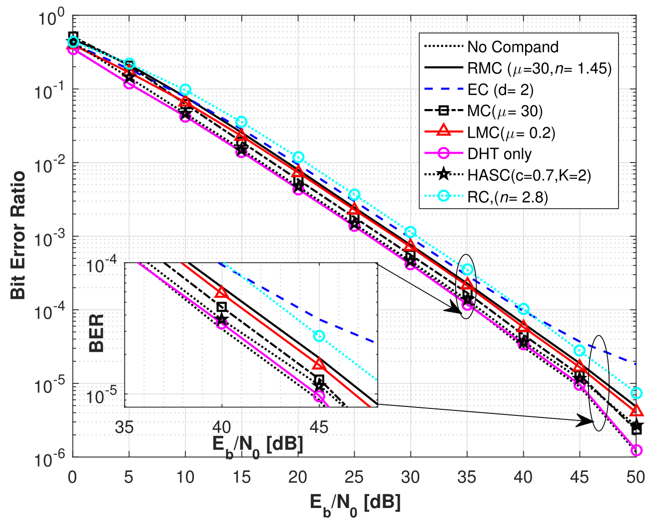

FIGURE 14. BER performance of proposed companding PAPR reduction schemes over multipath fading channels without DHT precoding ($N = 128$, QPSK, CP length = 25%, $L = 4$).

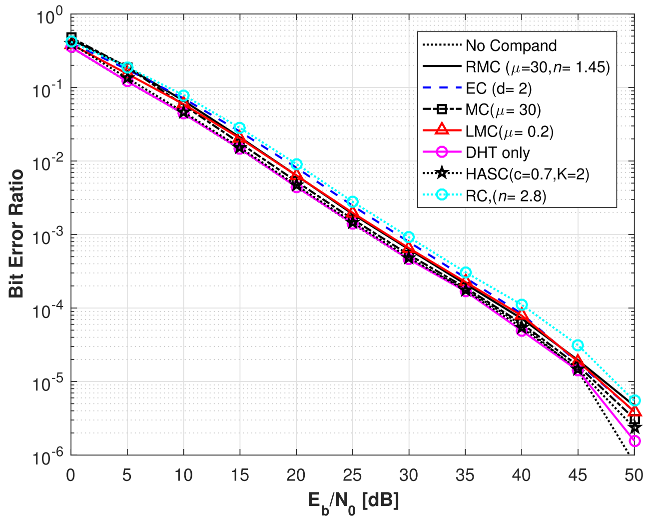

FIGURE 15. BER performance of proposed companded signal over multipath fading channels with DHT precoding ($N = 128$, QPSK, CP length = 25%, $L = 4$).

across the entire bandwidth thus enhancing the likelihood of recovering originally transmitted signal.

IV. CONCLUSION

In this study, we have proposed two new companding transforms that benefit from the roots of Rayleigh distributed envelope of Gaussian random variable characteristic of OFDM signals. These overcome the limitation of the standard $\mu-$law companding transform which only expands the low amplitudes of OFDM signals without compression of high amplitude signals. The implication is that the fundamental problem with the standard μ-law companding technique is overcome. Furthermore, we introduced the concept of phase distortion using DHT precoding before the amplitude distortion achieved through companding. While the DHT reduced the PAPR to some extent, the BER performance is still preserved. Introducing companding PAPR criteria on this, achieves the dual gain of PAPR reduction and excellent BER performance. The result of proposed DHT-precoded RMC companding transform achieved better PAPR reduction when compared to five other companding schemes whose signals were precoded with DHT. The second root-based PAPR reduction scheme, RC, achieved 7.8dB PAPR reduction of original signal signal and performed better than HASC when precoded. Since DHT significantly reduces PAPR while preserving BER, one of the ways of improving the system performance is by implementing low-complexity DHT to increase computational efficiency. We also infer that based on the investigated companding models, companding is ideally a robust PAPR reduction technique for multicarrier signals traversing multipath fading channels.

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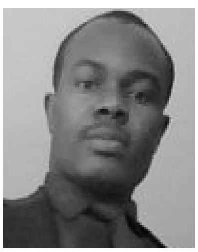

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