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Mean-Square Asymptotic Synchronization Control of Discrete-Time Neural Networks With Restricted Disturbances and Missing Data

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ABSTRACT The problem of controller design is investigated to achieve the mean-square asymptotic synchronization of discrete-time neural networks with time-varying delay and restricted disturbances. The unreliable communication links between the neural networks, which are modeled as stochastic dropouts satisfying the Bernoulli distributions, are taken into account. By applying the Lyapunov function, a synchronization controller design method is proposed in the form of linear matrix inequalities. The design method is also extended to neural networks including modeling uncertainties. Two numerical examples are given to illustrate the effectiveness of the proposed methods.

INDEX TERMS Asymptotic synchronization, controller design, discrete-time neural networks, time-varying delay, disturbance constraints, uncertainty, missing data.

I. INTRODUCTION

The past few decades have witnessed a number of successful applications of recurrent neural networks, such as parallel computing, associative memory, pattern recognition, fault tolerance, and combinatorial optimization. Neural networks with time delays have attracted attention for several years, see, e.g. [1]–[13] and the references therein. Time delays are frequently encountered due to the finite switching speed of electronic components, which leads to unstable and poor performance [14]. In [3], delay-dependent criteria were applied to the robust stability of recurrent neural networks with delays and uncertainties; these criteria were established using the free-weighting matrices method. In [6], the robust mean-square stability for stochastic discrete-time recurrent neural networks with mixed time-varying delays was derived via the Lyapunov functional method and linear matrix inequality (LMI) technology.

With the pioneering working by Pecora and Carroll [15], the synchronization problem has gained substantial attention due to its promising applications in secure communication, image processing, artificial intelligence, etc. [16], [17]. In [16], the output synchronization problem is studied for a heterogeneous network by the robust output regulation theory and the adaptive control theory. Output synchronization problem in heterogeneous network of nonidentical uncertain agents subject to an uncertain leader is considered in [17]. A novel control scheme based on hierarchical decomposition is proposed to guarantee the network synchronization. Delayed neural networks have been found to have some complicated characteristics, such as stable equilibria and chaotic attractors. Hence, great effort has been made to investigate the synchronization of delayed neural networks [18]–[23]. The problems of synchronization were studied in [18] and [19] for continuous-time and discrete-time neural networks with time-varying delays and multiagent systems via an event-based leader-following strategy. In [20] and [21], the synchronization problems of stochastic delayed neural networks and neural networks with mixed delays were respectively resolved with the sufficient conditions. In [22] and [23], a sampled-data controller and stochastic controller were individually designed to guarantee global synchronization.

Overall, the control schemes for the synchronization of neural networks can be realized by physical mechanisms. The transmission of signals relies on communication channels in the practical applications of many synchronization control schemes. It is assumed in [24] and [25] that communication between the controller and the physical plant is normal. In practice, this assumption may be invalid. Dropout may occur between a physical plant and its controller due to unreliable communication, which is more prevalent in the networked control systems (NCSs) [12], [26]. In network-based synchronization systems, signal transmission often refers to time delays, packet dropouts, environment disturbances, and so on. In past years, data packet dropout has been studied for NCSs, and a large quantity of results have been reported in the literature [27]–[33]. Very recently, in [34], the problem of the synchronization of discrete-time neural networks with mixed time delays, actuator saturation and failures was investigated, and packet dropout between neural networks was considered. The packet dropout probability depended on the synchronization criterion given using the Lyapunov approach. It is easy to see that disturbances always exist, which may cause instability and poor performance in real physical systems [35]–[38]. Therefore, how to reduce the effect of disturbances in the synchronization process for chaotic systems has become an important issue. To the best of our knowledge, the delay-dependent synchronization problem of discrete-time delayed neural networks with packet dropouts and disturbances has not been investigated, which motivates the current study.

The synchronization problem of discrete-time delayed neural networks with restricted disturbances and dropouts is considered in this paper. Dropout is modeled as a stochastic process satisfying a Bernoulli distribution. We construct a series of Lyapunov functions and design a feedback controller to ensure that the master-slave system is asymptotically synchronous with respect to mean-square by means of the LMI approach. Furthermore, the result is extended to uncertain neural networks with disturbance and dropouts. Finally, two numerical examples are presented to illustrate the effectiveness of the proposed methods.

Notation: Throughout this paper, $\mathbb{R}$ denotes real numbers and $\mathbb{N}$ denotes non-negative integers. $\mathbb{R}^n$ denotes the n-dimensional real vector space. $\|x\|$ denotes the Euclidean norm of the vector $x \in \mathbb{R}^n$. $\mathcal{E}\{\alpha\}$ stands for the expectation of stochastic variable α. $\mathbb{R}^{n\times m}$ is the set of real matrices of $n \times m$ dimension. A real matrix $P > 0 (\geq 0)$ means that P is a positive definite (positive semi-definite) matrix, and $A > B\,(A \geq B)$ means $A - B > 0\,(A - B \geq 0)$. I denotes an identity matrix with the appropriate dimensions. The superscript "T"represents the transpose, and the symmetric terms in a symmetric matrix are denoted by "$*$". Matrices, if not explicitly stated, are assumed to have compatible dimensions.

A. PROBLEM FORMULATION

In this paper, we consider the following discrete n-neuron neural networks with time-varying delays:

$$\begin{cases} x(k+1) = Cx(k) + Af(x(k)) + Bf(x(k-d(k))) \\ x(s) = \psi_1(s) \quad s = -d_M, -d_M+1, \ldots, 0 \end{cases} \tag{1}$$

where $x(k) = [x_1(k)\; x_2(k)\; \cdots\; x_n(k)]^T \in \mathbb{R}^n$ represents the state vector associated with the n neurons at time $k \in \mathbb{N}$, vector $\psi_1(s) \in \mathbb{R}^n$ is the initial conditions, integer $d(k)$ is the time-varying delay that satisfies $0 < d_m \leq d(k) \leq d_M$, and d_m and d_M are positive integers, respectively. $C = \mathrm{diag}\{c_1, c_2, \ldots, c_n\} \in \mathbb{R}^{n\times n}$ is the state feedback coefficient matrix, $A \in \mathbb{R}^{n\times n}$ is the connection weight matrix and $B \in \mathbb{R}^{n\times n}$ is the time-delay connection weight matrix. The nonlinear vector-valued function $f(x(k)) = [f_1(x_1(k))\; f_2(x_2(k))\; \cdots\; f_n(x_n(k))]^T \in \mathbb{R}^n$ is the activation function satisfying the following assumption.

Assumption 1: Each activation function $f_i(.)$ is continuous, and there exist constants γ_i and σ_i such that

$$\gamma_i \leq \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq \sigma_i, i = 1, 2, \cdots, n \tag{2}$$

for any $\alpha_1, \alpha_2 \in \mathbb{R}$ and $\alpha_1 \neq \alpha_2$.

In this paper, we consider system (1) as the master system; a slave system for (1) can be expressed as follows:

$$\begin{cases} y(k+1) = Cy(k) + Af(y(k)) + Bf(y(k-d(k))) \\ +u(k) + \delta(e(k)) \\ y(s) = \psi_2(s) \quad s = -d_M, -d_M+1, \ldots, 0 \end{cases} \tag{3}$$

where $u(k) \in \mathbb{R}^n$ is the control input to the slave system, $\psi_2(s)$ is the initial conditions, and $e(k) = y(k) - x(k)$ is the synchronization error. $\delta(e(k)) \in \mathbb{R}^n$ is the system disturbance satisfying the following assumption.

Assumption 2: There exist symmetric matrix $Q \in \mathbb{R}^{n\times n}$ and positive number β such that

$$\delta^T(z)\,[\delta(z) - Qz] \leq 0 \tag{4}$$

$$\left[\delta^T(z) - \delta^T(\hat{z})\right]\left[\beta\delta(z) - \beta\delta(\hat{z}) - (z - \hat{z})\right] \leq 0 \tag{5}$$

for any $z, \hat{z} \in \mathbb{R}^n$.

Then, the error system can be obtained as follows:

$$\begin{cases} e(k+1) = Ce(k) + Ag_x(e(k), k) \\ \quad +Bg_x(e(k-d(k)), k-d(k)) \\ +u(k) + \delta(e(k)) \\ e(s) = \psi_2(s) - \psi_1(s) \quad s = -d_M, -d_M+1, \ldots, 0 \end{cases} \tag{6}$$

where $g_x(z,k) = \begin{bmatrix} g_{x1}(z_1,k) \\ g_{x2}(z_2,k) \\ \vdots \\ g_{xn}(z_n,k) \end{bmatrix} = f(x(k)+z) - f(x(k))$ with $z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \in \mathbb{R}^n$. According to Assumption 1, it follows that

$$\gamma_i \leq \frac{f_i(x_i(k)+z_i) - f_i(x_i(k))}{z_i} \leq \sigma_i, i = 1, 2, \cdots, n$$

then, we get

$$\gamma_i z_i \leq g_{xi}(z_i, k) \leq \sigma_i z_i, \quad i = 1, 2, \cdots, n \tag{7}$$

The control input $u(k)$ takes the following form:

$$u(k) = \theta(k)Ke(k) \tag{8}$$

where $K \in \mathbb{R}^{n\times n}$ is the control gain matrix to be determined, and $\theta(k)$ models the unreliable nature of the communication links as $\theta(k) = 0$ when the transmission fails (that is, the data are lost) and $\theta(k) = 1$ when the transmission is successful. In this paper, $\theta(k)$ is a Bernoulli-type missing data process whose probability mass function is

$$\Pr\{\theta(k) = 0\} = \tilde{\theta} \tag{9}$$

$$\Pr\{\theta(k) = 1\} = 1 - \tilde{\theta} \tag{10}$$

with a given number $\tilde{\theta} \in (0, 1)$.

Remark 1: The advantage of control scheme with the missing data process which is modeled as a Bernoulli distribution from other controllers such as linear feedback controller is that this control scheme adequately considers the realistic data transmission of NCSs. In realistic NCSs, dropout is very common and the Bernoulli-type missing data process is a special case of Markov chain process [23]. But in design of linear feedback controller, the control input is $u(k) = Ke(k)$, which does not take the dropout into account.

Substituting (8) into (6) gives the following closed-loop system:

$$\begin{cases} e(k+1) = (C + \theta(k)K)e(k) + Ag_x(e(k), k) \\ +Bg_x(e(k-d(k)), k-d(k)) + \delta(e(k)) \\ e(s) = \psi_2(s) - \psi_1(s) \quad s = -d_M, -d_M+1, \ldots, 0 \end{cases} \tag{11}$$

Definition 1 [39]: The master system (1) and slave system (3) under control input (8) are said to be asymptotically synchronized with respect to the mean square if the error of the system satisfies

$$\lim_{k\to+\infty} \mathcal{E}\{\|e(k)\|^2\} = 0 \tag{12}$$

Lemma 1 [40]: Let X, Y and Z be real matrices of appropriate dimensions, with $Z > 0$. Then

$$X^TZY + Y^TZX \le X^TZX + Y^TZY \tag{13}$$

Lemma 2 [41]: Given constant matrices Ω_1, Ω_2, and Ω_3, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 > 0$,

$$\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$$

if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0$$

B. MAIN RESULT

For simplicity, in the following, we denote

$$\begin{aligned} h &= d_M - d_m + 1 \\ G_1 &= \mathrm{diag}\{\gamma_1\sigma_1, \gamma_2\sigma_2, \cdots, \gamma_n\sigma_n\} \\ G_2 &= \mathrm{diag}\{\frac{\gamma_1+\sigma_1}{2}, \frac{\gamma_2+\sigma_2}{2}, \cdots, \frac{\gamma_n+\sigma_n}{2}\} \end{aligned}$$

Theorem 1: Under Assumptions 1∼ 2, suppose there exist positive-definite matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $S_3 > 0$, and $S_4 > 0$, matrices R and X and diagonal positive-definite matrices $T > 0$ and $L > 0$ such that the following LMIs hold:

$$W = \begin{bmatrix} S_3 & R \\ * & S_4 \end{bmatrix} > 0 \tag{14}$$

$$\begin{bmatrix} \Upsilon & \Gamma_2 & \Gamma_3 \\ * & \Gamma_1 & 0 \\ * & * & -P \end{bmatrix} < 0 \tag{15}$$

Then, the error system (11) is asymptotically stable with respect to mean square with $K = P^{-1}X$, where

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} \\ * & * & \Upsilon_{33} & \Upsilon_{34} \\ * & * & * & \Upsilon_{44} \end{bmatrix}$$

$$\begin{aligned}
\Upsilon_{11} &= C^TPC + \tilde{\theta}C^TX + \tilde{\theta}X^TC - P + S_1 + S_2 + hS_3 - G_1T \\
\Upsilon_{12} &= C^TPA + \tilde{\theta}X^TA + hR + G_2T \\
\Upsilon_{13} &= C^TPB + \tilde{\theta}X^TB \\
\Upsilon_{14} &= C^TP + \tilde{\theta}X^T + Q/2 + I/2 \\
\Upsilon_{22} &= A^TPA + hS_4 - T \\
\Upsilon_{23} &= A^TPB \\
\Upsilon_{24} &= A^TP \\
\Upsilon_{33} &= B^TPB - S_4 - L \\
\Upsilon_{34} &= B^TP \\
\Upsilon_{44} &= P - (\beta+1)I
\end{aligned}$$

$$\Gamma_1 = \begin{bmatrix} -S_3 - G_1L & 0 & 0 & I/2 \\ * & -S_1 & 0 & 0 \\ * & * & -S_2 & 0 \\ * & * & * & -\beta I \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} 0 & 0 & 0 & -I/2 \\ 0 & 0 & 0 & 0 \\ -R^T + LG_2^T & 0 & 0 & 0 \\ -I/2 & 0 & 0 & \beta I \end{bmatrix}, \quad \Gamma_3 = \begin{bmatrix} \tilde{\theta}X^T \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Proof: From Lemma 2 and (15), it is easy to see that

$$\begin{bmatrix} \Upsilon + \Gamma_3P^{-1}\Gamma_3^T & \Gamma_2 \\ * & \Gamma_1 \end{bmatrix} < 0 \tag{16}$$

We consider the following Lyapunov-Krasovskii function of system (11):

$$V(k) = \sum_{i=1}^{4} V_i(k) \tag{17}$$

where

$$V_1(k) = e^T(k)Pe(k)$$

$$V_2(k) = \sum_{i=k-d_M}^{k-1} e^T(i)S_1e(i)$$

$$V_3(k) = \sum_{i=k-d_m}^{k-1} e^T(i)S_2e(i)$$

$$V_4(k) = \sum_{j=-d_M+1}^{-d_m+1} \sum_{i=k-1+j}^{k-1} \begin{bmatrix} e(i) \\ g_x(e(i), i) \end{bmatrix}^T W \begin{bmatrix} e(i) \\ g_x(e(i), i) \end{bmatrix}$$

Defining $\Delta V(k) = V(k+1) - V(k)$ and $\xi = [e^T(k) \quad g_x^T(e(k), k) \quad g_x^T(e(k-d(k)), k-d(k)) \quad \delta^T(e(k))]^T$, it is easy to see that

$$\mathcal{E}\{\Delta V_1(k)\} = \mathcal{E}\left\{e^T(k+1)Pe(k+1) - e^T(k)Pe(k)\right\} = \xi^T \Pi \xi \tag{18}$$

$$\mathcal{E}\{\Delta V_2(k)\} = e^T(k)S_1e(k) - e^T(k-d_M)S_1e(k-d_M) \tag{20}$$

$$\mathcal{E}\{\Delta V_3(k)\} = e^T(k)S_2e(k) - e^T(k-d_m)S_2e(k-d_m) \tag{21}$$

$$\begin{aligned}\mathcal{E}\{\Delta V_4(k)\} &= h\begin{bmatrix} e(k) \\ g_x(e(k), k) \end{bmatrix}^T W \begin{bmatrix} e(k) \\ g_x(e(k), k) \end{bmatrix} - \sum_{i=k-d_M}^{k-d_m} \begin{bmatrix} e(i) \\ g(e(i)) \end{bmatrix}^T W \begin{bmatrix} e(i) \\ g(e(i)) \end{bmatrix} \\ &\le h\begin{bmatrix} e(k) \\ g_x(e(k), k) \end{bmatrix}^T W \begin{bmatrix} e(k) \\ g_x(e(k), k) \end{bmatrix} - \begin{bmatrix} e(k-d(k)) \\ g_x(e(k-d(k)), k-d(k)) \end{bmatrix}^T W \\ &\quad \times \begin{bmatrix} e(k-d(k)) \\ g_x(e(k-d(k)), k-d(k)) \end{bmatrix}\end{aligned} \tag{22}$$

when (14) is true. On the other hand, it is clear from Assumption 1 that

$$\theta_1(k) = \begin{bmatrix} e(k) \\ g_x(e(k), k) \end{bmatrix}^T \begin{bmatrix} G_1T & -G_2T \\ * & T \end{bmatrix} \begin{bmatrix} e(k) \\ g_x(e(k), k) \end{bmatrix} \le 0$$

$$\theta_2(k) = \begin{bmatrix} e(k-d(k)) \\ g_x(e(k-d(k)), k-d(k)) \end{bmatrix}^T \times \begin{bmatrix} G_1L & -G_2L \\ * & L \end{bmatrix} \times \begin{bmatrix} e(k-d(k)) \\ g_x(e(k-d(k)), k-d(k)) \end{bmatrix} \le 0 \tag{23}$$

According to Assumption 2, the following inequalities are true:

$$\theta_3(k) = \begin{bmatrix} e(k) \\ \delta(e(k)) \end{bmatrix}^T \begin{bmatrix} 0 & -Q/2 \\ * & I \end{bmatrix} \begin{bmatrix} e(k) \\ \delta(e(k)) \end{bmatrix} \le 0$$

$$\theta_4(k) = \begin{bmatrix} e(k)) \\ e(k-d(k)) \\ \delta(e(k)) \\ \delta(e(k-d(k))) \end{bmatrix}^T \begin{bmatrix} 0 & 0 & -I/2 & I/2 \\ * & 0 & I/2 & -I/2 \\ * & * & \beta I & -\beta I \\ * & * & * & \beta I \end{bmatrix} \times \begin{bmatrix} e(k)) \\ e(k-d(k)) \\ \delta(e(k)) \\ \delta(e(k-d(k))) \end{bmatrix} \le 0 \tag{24}$$

Then, we obtain from (18)~(24) and $K = P^{-1}X$ that

$$\mathcal{E}\{\Delta V(k)\} = \mathcal{E}\left\{\sum_{i=1}^{4} \Delta V_i(k)\right\} \le \mathcal{E}\left\{\sum_{i=1}^{4} \Delta V_i(k)\right\} - \sum_{i=1}^{4} \theta_i(k) = \psi^T(k) \begin{bmatrix} \Upsilon + \Gamma_3 P^{-1} \Gamma_3^T & \Gamma_2 \\ * & \Gamma_1 \end{bmatrix} \psi(k) < 0 \tag{25}$$

where

$$\psi(k) = [e^T(k)\ g_x^T(e(k), k)\ g_x^T(e(k-d(k)), k-d(k))\ \delta^T(e(k))\ e^T(k-d(k))\ e^T(k-d_M)\ e^T(k-d_m)\ \delta^T(e(k-d(k)))\ ]^T$$

The proof is completed. □

Theorem 1 gives a design method for a synchronization controller. Using MATLAB, it is convenient to check whether the LMIs in Theorem 1 have a feasible solution. If the LMIs have a feasible solution, we get a synchronization controller by $K = P^{-1}X$.

In engineering, it is inevitable that synchronization suffers from modeling uncertainties. Therefore, for master-slave systems with modeling uncertainties, we will discuss the design method of the synchronization controller. When modeling uncertainties occur, the master-slave system can be rewritten as

$$\begin{cases} x(k+1) = (C + \Delta C(k))x(k) + (A + \Delta A(k))f(x(k)) \\ +(B + \Delta B(k))f(x(k-d(k))) \\ x(s) = \psi_1(s) \quad s = -d_M, -d_M+1, \ldots, 0 \end{cases} \tag{26}$$

$$\begin{cases} y(k+1) = (C + \Delta C(k))y(k) + (A + \Delta A(k))f(y(k)) \\ +(B + \Delta B(k))f(y(k-d(k))) + u(k) + \delta(e(k)) \\ y(s) = \psi_2(s) \quad s = -d_M, -d_M+1, \ldots, 0 \end{cases} \tag{27}$$

It is then easy to get the error system:

$$\begin{cases} e(k+1) = (C + \Delta C(k) + \theta(k)K)e(k) \\ +(A + \Delta A(k))g_x(e(k), k) \\ +(B + \Delta B(k))g_x(e(k-d(k)), k-d(k)) + \delta(e(k)) \\ e(s) = \psi_2(s) - \psi_1(s) \quad s = -d_M, -d_M+1, \ldots, 0 \end{cases} \tag{28}$$

$$\Pi = \begin{bmatrix} C^TPC + \tilde{\theta}C^TPK + \tilde{\theta}K^TPC + \tilde{\theta}^2K^TPK - P & C^TPA + \tilde{\theta}K^TPA & C^TPB + \tilde{\theta}K^TPB & C^TP + \tilde{\theta}K^TP \\ * & A^TPA & A^TPB & A^TP \\ * & * & B^TPB & B^TP \\ * & * & * & P \end{bmatrix} \tag{19}$$

We assume that the matrices $\Delta A(k)$, $\Delta B(k)$, and $\Delta C(k)$ satisfy:

$$[\Delta C(k) \quad \Delta A(k) \quad \Delta B(k)] = MF(k)[N_1 \quad N_2 \quad N_3] \tag{29}$$

where M, N_1, N_2, and N_3 are known constant matrices with proper dimensions and

$$M^T M \leq I \tag{30}$$
$$F^T(k)F(k) \leq I, \quad k \in \mathbb{N}. \tag{31}$$

To simplify the equations, we write $\Delta A(k)$, $\Delta B(k)$, and $\Delta C(k)$ as ΔA, ΔB, and ΔC.

Theorem 2: Under Assumptions 1∼2, suppose there exist positive-definite matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $S_3 > 0$, and $S_4 > 0$, matrices R and X, and diagonal positive-definite matrices $T > 0$ and $L > 0$, such that the following LMIs hold:

$$W = \begin{bmatrix} S_3 & R \\ * & S_4 \end{bmatrix} > 0 \tag{32}$$

$$\begin{bmatrix} \Xi & \Gamma_2 & \Gamma_3 & \Gamma_4 \\ * & \Gamma_1 & 0 & 0 \\ * & * & -P & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{33}$$

$$M^T PM < I \tag{34}$$

Then, the error system (28) is asymptotically stable with respect to mean square, with $K = P^{-1}X$, where

$$\Xi = \begin{bmatrix} \Xi_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} \\ * & \Xi_{22} & \Upsilon_{23} & \Upsilon_{24} \\ * & * & \Xi_{33} & \Upsilon_{34} \\ * & * & * & \Xi_{44} \end{bmatrix}$$

$$\Xi_{11} = \Upsilon_{11} + 3C^T PC + (7 + \tilde{\theta})N_1^T N_1$$
$$\Xi_{22} = \Upsilon_{22} + 3A^T PA + (7 + \tilde{\theta})N_2^T N_2$$
$$\Xi_{33} = \Upsilon_{33} + 3B^T PB + (7 + \tilde{\theta})N_3^T N_3$$
$$\Xi_{44} = \Upsilon_{44} + 3P$$
$$\Gamma_4 = \begin{bmatrix} \sqrt{3\tilde{\theta}}X^T \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Proof: From Lemma 2 and (33), it is easy to see that

$$\begin{bmatrix} \Xi + \Gamma_3 P^{-1}\Gamma_3^T + \Gamma_4\Gamma_4^T & \Gamma_2 \\ * & \Gamma_1 \end{bmatrix} < 0 \tag{35}$$

Using the same Lyapunov-Krasovskii function $V(k)$ and the same deduction as in the proof of Theorem 1, we have

$$\mathcal{E}\{\Delta V(k)\} \leq \psi^T(k) \begin{bmatrix} \Upsilon' + \Gamma_3 P^{-1}\Gamma_3^T & \Gamma_2 \\ * & \Gamma_1 \end{bmatrix} \psi(k) \tag{36}$$

It is noted that we use Υ' to represent the corresponding term in (15) with uncertainties. Replacing A, B, and C with $(A + \Delta A)$, $(B + \Delta B)$, and $(C + \Delta C)$ in (15) yields that

$$\begin{aligned} \Upsilon'_{11} &= (C + \Delta C)^T P(C + \Delta C) + \tilde{\theta}(C + \Delta C)^T X \\ &\quad + \tilde{\theta}X^T(C + \Delta C) + \tilde{\theta}^2 X^T P^{-1} X - P + S_1 \\ &\quad + S_2 + hS_3 - G_1 T \\ &= \Upsilon_{11} + Y_1 + \tilde{\theta}Y_2 \end{aligned} \tag{37}$$

where

$$\begin{aligned} Y_1 &= C^T P\Delta C + (\Delta C)^T PC + (\Delta C)^T P\Delta C \\ Y_2 &= (\Delta C)^T X + X^T \Delta C \end{aligned}$$

It follows from Lemma 1 and (34) that

$$\begin{aligned} Y_1 &\leq C^T PC + 2(\Delta C)^T P\Delta C \\ &= C^T PC + 2N_1^T F^T(k)M^T PMF(k)N_1 \\ &< C^T PC + 2N_1^T N_1 \end{aligned} \tag{38}$$

Similarly, we obtain

$$\tilde{\theta}Y_2 < \tilde{\theta}X^T X + \tilde{\theta}N_1^T N_1 \tag{39}$$

Similar to (38), we get

$$\begin{aligned} \Upsilon'_{22} &= (A + \Delta A)^T P(A + \Delta A) + hS_4 - T \\ &< \Upsilon_{22} + A^T PA + 2N_2^T N_2 \end{aligned} \tag{40}$$
$$\begin{aligned} \Upsilon'_{33} &= (B + \Delta B)^T P(B + \Delta B) - S_4 - L \\ &< \Upsilon_{33} + B^T PB + 2N_3^T N_3 \end{aligned} \tag{41}$$

On the other hand, it is easy to verify that

$$\begin{aligned} \Upsilon'_{12} &= (C + \Delta C)^T P(A + \Delta A) + hR + G_2 T \\ &\quad + \tilde{\theta}X^T(A + \Delta A) \\ &= \Upsilon_{12} + Y_3 + \tilde{\theta}Y_4 \end{aligned}$$

where

$$\begin{aligned} Y_3 &= C^T P\Delta A + (\Delta C)^T PA + (\Delta C)^T P\Delta A \\ Y_4 &= X^T \Delta A \end{aligned}$$

Considering the symmetry of Υ'_{12} and Υ'_{21}, we have

$$\begin{aligned} &e^T(k)C^T P\Delta A g_x(e(k), k) + g_x^T(e(k), k)(\Delta A)^T PCe(k) \\ &\leq e^T(k)C^T PCe(k) + g_x^T(e(k), k)(\Delta A)^T P\Delta A g_x(e(k), k) \\ &< e^T(k)C^T PCe(k) + g_x^T(e(k), k)N_2^T N_2 g_x(e(k), k) \end{aligned} \tag{42}$$
$$\begin{aligned} &e^T(k)(\Delta C)^T PA g_x(e(k), k) + g_x^T(e(k), k)A^T P\Delta Ce(k) \\ &< e^T(k)N_1^T N_1 e(k) + g_x^T(e(k), k)A^T PA g_x(e(k), k) \end{aligned} \tag{43}$$
$$\begin{aligned} &e^T(k)(\Delta C)^T P\Delta A g_x(e(k), k) + g_x^T(e(k), k)(\Delta A)^T P\Delta Ce(k) \\ &< e^T(k)N_1^T N_1 e(k) + g_x^T(e(k), k)N_2^T N_2 g_x(e(k), k) \end{aligned} \tag{44}$$

With regard to $\tilde{\theta}Y_4$, it follows that

$$\begin{aligned} &e^T(k)\tilde{\theta}X^T \Delta A g_x(e(k), k) + g_x^T(e(k), k)\tilde{\theta}(\Delta A)^T Xe(k) \\ &< e^T(k)\tilde{\theta}X^T Xe(k) + g_x^T(e(k), k)\tilde{\theta}N_2^T N_2 g_x(e(k), k) \end{aligned} \tag{45}$$

Considering the symmetry of Υ'_{13} and Υ'_{31}, it is easy to compute that

$$\begin{aligned} &e^T(k)C^T P\Delta B g_x(e(k - d(k)), k - d(k)) \\ &\quad + g_x^T(e(k - d(k)), k - d(k))(\Delta B)^T PCe(k) \end{aligned}$$

$$
\begin{aligned}
&< e^T(k)C^T PCe(k) \\
&\quad + g_x^T(e(k-d(k)), k-d(k)) \\
&\quad \times N_3^T N_3 g_x(e(k-d(k)), k-d(k)) \qquad (46)
\end{aligned}
$$

$$
\begin{aligned}
&e^T(k)(\Delta C)^T PBg_x(e(k-d(k)), k-d(k)) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))B^T P\Delta Ce(k) \\
&< e^T(k)N_1^T N_1 e(k) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))B^T \\
&\quad \times PBg_x(e(k-d(k)), k-d(k)) \qquad (47)
\end{aligned}
$$

$$
\begin{aligned}
&e^T(k)(\Delta C)^T P\Delta Bg_x(e(k-d(k)), k-d(k)) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))(\Delta B)^T P\Delta Ce(k) \\
&< e^T(k)N_1^T N_1 e(k) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))N_3^T \\
&\quad \times N_3 g_x(e(k-d(k)), k-d(k)) \qquad (48)
\end{aligned}
$$

$$
\begin{aligned}
&e^T(k)\tilde{\theta} X^T \Delta Bg_x(e(k-d(k)), k-d(k)) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))\tilde{\theta}(\Delta B)^T Xe(k) \\
&< e^T(k)\tilde{\theta} X^T Xe(k) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))\tilde{\theta} N_3^T \\
&\quad \times N_3 g_x(e(k-d(k)), k-d(k)) \qquad (49)
\end{aligned}
$$

Considering the uncertainty in Υ'_{14} and Υ'_{41}, we obtain

$$
\begin{aligned}
&e^T(k)(\Delta C)^T P\delta(e(k)) + \delta^T(e(k))P\Delta Ce(k) \\
&\quad < e^T(k)N_1^T N_1 e(k) + \delta^T(e(k))P\delta(e(k)) \qquad (50)
\end{aligned}
$$

Considering the symmetry of Υ'_{23} and Υ'_{32}, it is easy to compute that

$$
\begin{aligned}
&g_x^T(e(k), k)A^T P\Delta Bg_x(e(k-d(k)), k-d(k)) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))(\Delta B)^T PAg_x(e(k), k) \\
&< g_x^T(e(k), k)A^T PAg_x(e(k), k) \\
&\quad + g_x^T(e(k-d(k)), k-d(k)) \\
&\quad \times N_3^T N_3 g_x(e(k-d(k)), k-d(k)) \qquad (51)
\end{aligned}
$$

$$
\begin{aligned}
&g_x^T(e(k), k)(\Delta A)^T PBg_x(e(k-d(k)), k-d(k)) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))B^T P\Delta Ag_x(e(k), k) \\
&< g_x^T(e(k), k)N_2^T N_2 g_x(e(k), k) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))B^T \\
&\quad \times PBg_x(e(k-d(k)), k-d(k)) \qquad (52)
\end{aligned}
$$

$$
\begin{aligned}
&g_x^T(e(k), k)(\Delta A)^T P\Delta Bg_x(e(k-d(k)), k-d(k)) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))(\Delta B)^T P\Delta Ag_x(e(k), k) \\
&< g_x^T(e(k), k)N_2^T N_2 g_x(e(k), k) \\
&\quad + g_x^T(e(k-d(k)), k-d(k))N_3^T \\
&\quad \times N_3 g_x(e(k-d(k)), k-d(k)) \qquad (53)
\end{aligned}
$$

Considering the uncertainty in Υ'_{24} and Υ'_{42}, we get

$$
\begin{aligned}
&g_x^T(e(k), k)(\Delta A)^T P\delta(e(k)) + \delta^T(e(k))P\Delta Ag_x(e(k), k) \\
&\quad < g_x^T(e(k), k)N_2^T N_2 g_x(e(k), k) + \delta^T(e(k))P\delta(e(k)) \qquad (54)
\end{aligned}
$$

Considering the uncertainty in Υ'_{34} and Υ'_{43}, we have

$$
\begin{aligned}
&g_x^T(e(k-d(k)), k-d(k))(\Delta B)^T P\delta(e(k)) \\
&\quad + \delta^T(e(k))P\Delta Bg_x(e(k-d(k)), k-d(k)) \\
&< g_x^T(e(k-d(k)), k-d(k))N_3^T \\
&\quad \times N_3 g_x(e(k-d(k)), k-d(k)) \\
&\quad + \delta^T(e(k))P\delta(e(k)) \qquad (55)
\end{aligned}
$$

From (35)∼(55), it is known that

$$
\begin{aligned}
\mathcal{E}\{\Delta V(k)\} &\leq \psi^T(k) \begin{bmatrix} \Upsilon' + \Gamma_3 P^{-1}\Gamma_3^T & \Gamma_2 \\ * & \Gamma_1 \end{bmatrix} \psi(k) \\
&\leq \psi^T(k) \begin{bmatrix} \Xi + \Gamma_3 P^{-1}\Gamma_3^T + \Gamma_4^T\Gamma_4 & \Gamma_2 \\ * & \Gamma_1 \end{bmatrix} \psi(k) \\
&< 0
\end{aligned}
$$

Hence, Theorem 2 is obtained. □

C. NUMERICAL EXAMPLES

In this section, two numerical examples are provided to illustrate the effectiveness of the proposed method.

Example 1:

We consider the discrete recurrent neural networks system (1) and (3) with the following parameters:

$$
C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad A = \begin{bmatrix} 0.01 & 0.1 \\ 0 & 0.1 \end{bmatrix}
$$

$$
B = \begin{bmatrix} -0.1 & 0.1 \\ -0.2 & -0.1 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}
$$

The activation functions are $f_1(x_1(k)) = \tanh(x_1(k))$, $f_2(x_2(k)) = \tanh(x_2(k))$, which satisfy Assumption 1 with $\gamma_1 = 0, \gamma_2 = 0$, $\sigma_1 = 1$, $and \sigma_2 = 1$. Let $\beta = 100$, $\tilde{\theta} = 0.3$, $d_m = 6, d_M = 10$, and disturbance $\delta(e(k)) = 0.01\tanh(e(k))$. The initial conditions are $x(k) = [-0.5\ 1]^T, y(k) = [0\ 0.5]^T$, and $k = -10, -9, \ldots, 0$. By applying Theorem 1, it can be checked by MATLAB that (14) and (15) are feasible, and we can also obtain the controller

$$
K = P^{-1}X = \begin{bmatrix} -0.3908 & -0.1918 \\ -0.0282 & -0.5699 \end{bmatrix}
$$

The unreliable communication link is plotted in Fig. 1. In Fig. 1, $\theta(k)$=0 means that the data is lost whereas $\theta(k)$=1 denotes the packet is receieved successfully. The time-varying delay is depicted in Fig. 2, and the dynamics of the master and slave system are shown in Fig. 3 and Fig. 4, respectively. The state trajectories of the closed-loop error dynamic system with the above control input is given in Fig. 5. From simulation results, we can conclude that the synchronization errors converge asymptotically to zero.

Example 2: We consider the discrete recurrent neural networks system (26) and (27) with the following uncertainty parameters:

$$
M = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad N1 = \begin{bmatrix} 0.02 & 0.01 \end{bmatrix}
$$

$$
N2 = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}, \quad N3 = \begin{bmatrix} 0.01 & 0.02 \end{bmatrix}
$$

The other parameters are the same as those in Example 1. By applying Theorem 2, it can be checked by MATLAB

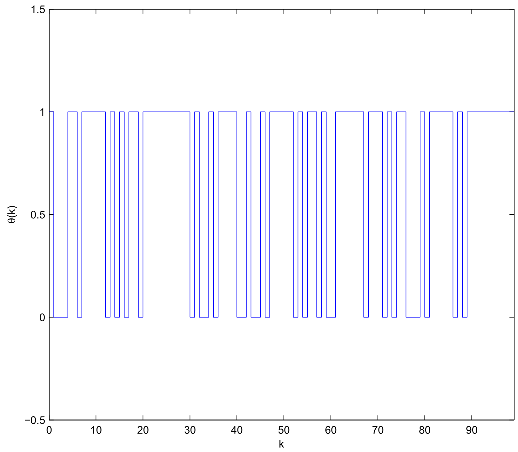

FIGURE 1. $\theta(k)$ **in Example 1.**

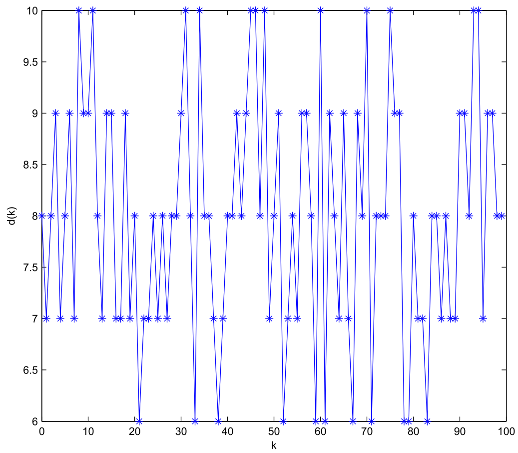

FIGURE 2. **Time-varying delay in Example 1.**

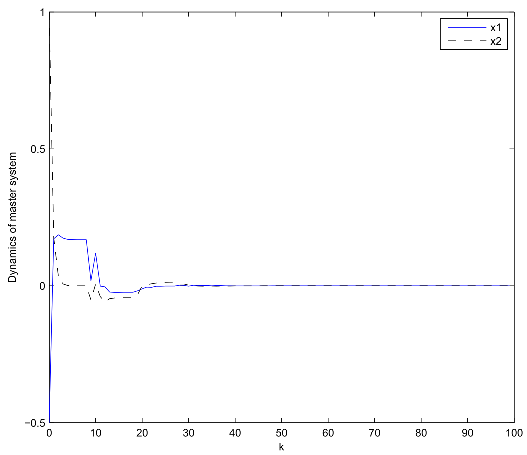

FIGURE 3. **Dynamics of master system.**

that (32), (33) and (34) are feasible, and we can also obtain the controller:

$$K = P^{-1}X = \begin{bmatrix} -0.4121 & -0.1142 \\ -0.1459 & -0.7786 \end{bmatrix}$$

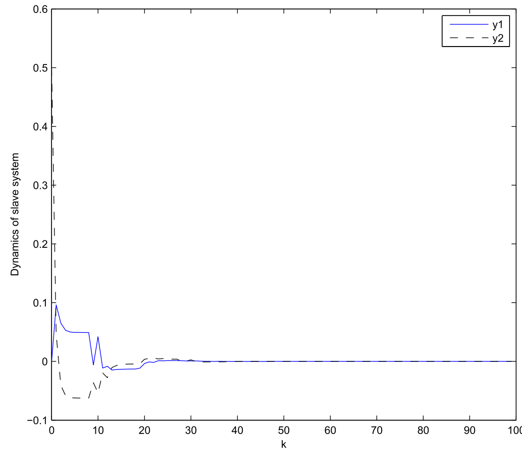

FIGURE 4. **Dynamics of slave system.**

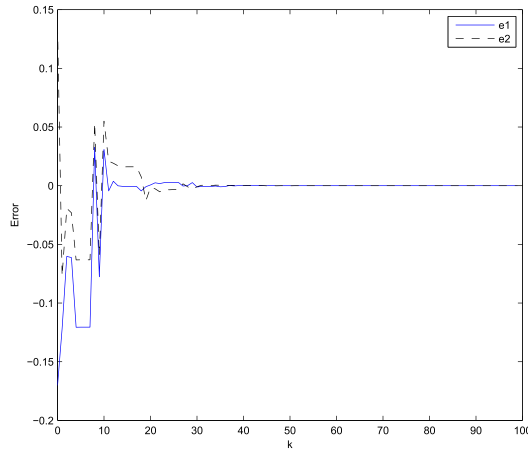

FIGURE 5. **State response of error system (11).**

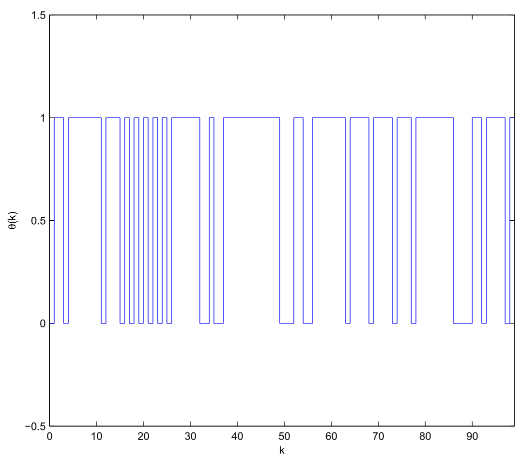

FIGURE 6. $\theta(k)$ **in Example 2.**

The missing data process $\theta(k)$ is plotted in Fig. 6, the time-varying delay is displayed in Fig. 7, and the dynamics of the master and slave system with uncertainties are shown in Fig. 8 and Fig. 9, respectively. Comparing Fig. 8 and Fig. 9, we can see that the slave system resembles the master system under the proposed control input (8). The synchronization

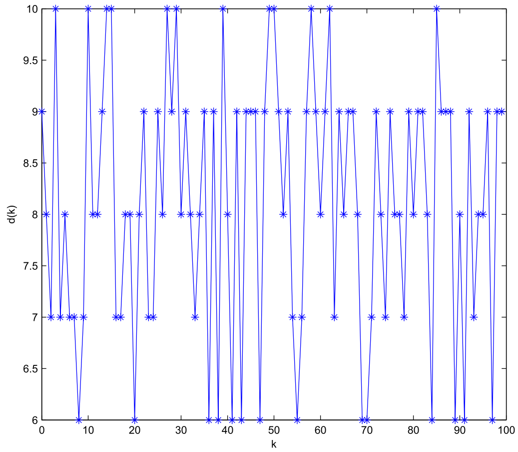

FIGURE 7. Delays in Example 2.

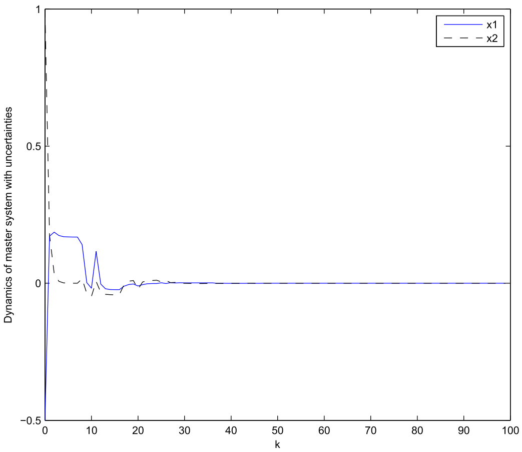

FIGURE 8. Dynamics of master system with uncertainties.

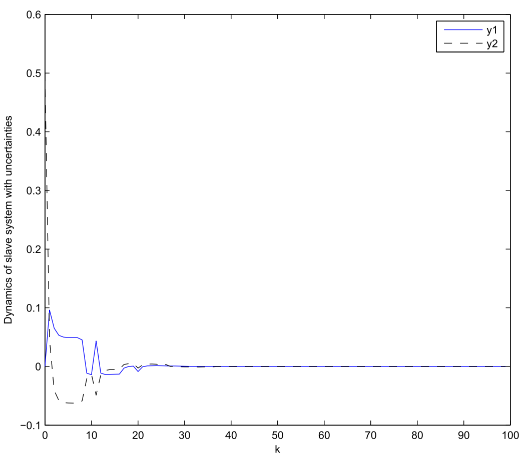

FIGURE 9. Dynamics of slave system with uncertainties.

errors curve is given in Fig. 10. Clearly, the master-slave system with uncertainties (28) reaches synchronization.

Remark 2: Due to full consideration of dropout in unreliable communication in NCSs, the master-slave system with or without uncertainties is synchronized under the proposed control input, respectively. Compared with the common linear feedback control method, the presented method has the advantage of simplicity and feasibility.

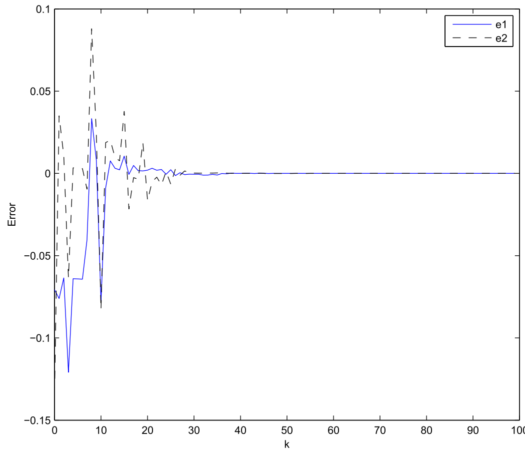

FIGURE 10. State response of error system (28).

II. CONCLUSIONS

This paper has studied the mean-square asymptotic synchronization control problem for time-varying delay neural networks with disturbances and missing stochastic data. A stochastic process that satisfies the Bernoulli distribution is given to model the random missing data. A sufficient criterion is derived to ensure the error system is asymptotically mean-square stable, and a controller design method is presented. The error system with uncertainties is also discussed. Finally, two numerical examples are provided to demonstrate the effectiveness of the obtained results.

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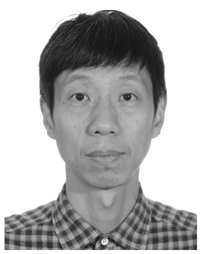

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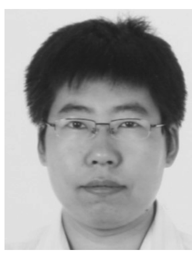

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