IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received November 5, 2017, accepted November 28, 2017, date of publication December 4, 2017, date of current version February 14, 2018.

Digital Object Identifier 10.1109/ACCESS.2017.2779171

A New Nested Array Configuration With Increased Degrees of Freedom

HUIPING HUANG[1], (Student Member, IEEE), BIN LIAO[1], (Senior Member, IEEE), XIAOYE WANG[1], XIANSHENG GUO[2], (Member, IEEE), AND JIANJUN HUANG[1], (Member, IEEE)

[1]College of Information Engineering, Shenzhen University, Shenzhen 518060, China
[2]Department of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 610051, China

Corresponding author: Bin Liao (binliao@szu.edu.cn)

This work was supported in part by the Natural Science Foundation of China under Grant 61771316, in part by the Foundation of Department of Education of Guangdong under Grant 2016KTSCX125, and in part by the Foundation of Shenzhen under Grant JCYJ20170302150044331.

ABSTRACT In this paper, a new linear array configuration based on the concept of two-level nested array is proposed. Specifically, the proposed array configuration consists of two uniform linear arrays (ULAs) plus a separate sensor with appropriate spacing apart. Compared with the original two-level nested array, the degrees of freedom of our proposed array configuration can be increased by $2L - 6$ for an array with L physical sensors. Moreover, the virtual array aperture can be enlarged by nearly $(L^2/2 - L - 2)d_1$, where d_1 is the inter-element spacing of the first ULA. Owing to these advantages, our proposed array configuration has enhanced resolution and achieves better performance in parameter estimation, such as direction-of-arrival (DOA) estimation. Numerical simulations of DOA estimation exhibit the effectiveness and superiority of this array configuration.

INDEX TERMS Nested array, array configuration, degrees of freedom, virtual array aperture, parameter estimation, direction-of-arrival.

I. INTRODUCTION

Array signal processing is known to be a time-honored and valuable research topic, which has attracted much interest. Usually, it is assumed that the array sensors are arranged in linear or circular. Furthermore, uniform linear array (ULA) and uniform circular array (UCA) are often considered in both theoretical research and engineering applications due to their symmetrical and simple array geometries. Classical techniques such as spatial smoothing [1] and the estimation of signal parameters via rotational invariant techniques (ESPRIT) [2] rigidly require symmetrical array geometries and therefore they can be successfully applied to ULAs. A great amount of their follow-up methods concentrated straightly on ULAs and have deepened the research on their particular aspects [3]–[11].

During the recent years, two kinds of unprecedented non-uniform linear array configurations have been proposed by Vaidyanathan and Pal in [12]–[15]. These two kinds of array configurations are named as coprime and nested arrays, respectively. They benefit from significantly increased degrees of freedom (DOF), and stem from the difference set. The difference set is related to the Khatri-Rao (KR) subspace [16] of the physical array manifold and is generated by a so-called co-array. The co-array combines two independent uniform linear arrays in different manners, i.e., coprime relationship for coprime array and nesting relationship for nested array. It can lead to a virtual linear array with locations of virtual sensors marked by the difference set. Note that nested arrays include two-level and more-level nested arrays, where two-level nested array is relatively much simple and hence has drawn more attention since they were introduced.

There exist several modified array configurations based on the coprime array or two-level nested array [17]–[23]. For instance, the unfolded coprime array [18] is a reformative configuration of the coprime array. It can achieve $2MN - 1$ DOF with $M+N$ physical sensors, while the original coprime array [13] can only achieve $MN + M + N - 2$ DOF with the same number of physical sensors. A few extensions of the two-level nested array have been proposed, including [20] (named as Iizuka-Nested array), [21] (named as Yang-Nested array), [22] (named as Yang-NMRA), and [23] (named as Liu-Nested array). These array configurations are able to increase the DOF as well as virtual array apertures (VAAs) compared with the original two-level nested array [14]. For instance, the DOF of Iizuka-Nested array, Yang-Nested array and Liu-Nested array are increased by 2, about $L-2$ and about $L-2$ respectively, compared with that of the original two-level nested array, where L is the total number of physical sensors. The VAAs of these three modified nested arrays are increased, respectively, by nearly Ld_1, nearly $(L-2)d_1$ and nearly $(L-2)d_1$, compared with that of the original two-level nested array, where d_1 denotes the inter-element spacing of sensors in the first ULA. It should be mentioned that the DOF of these modified array configurations are still far lower than the theoretical maximum $L(L-1)+1$ [14].

For the motivation of further increasing the DOF and VAAs, we propose a new nest-based array configuration in this paper. In brief, the proposed nested array configuration is composed of two independent ULAs plus an additional sensor, and all sensors are arranged in linear. Assuming the total number of physical sensors is L and the inter-element spacing of the first ULA is d_1, it will be shown that the DOF and VAAs of the proposed array configuration are, respectively, $2L-6$ and nearly $(\frac{L^2}{2}-L-2)d_1$ larger than those of the original two-level nested array. Such increase in both DOF and VAAs would lead to enhanced resolution, which enables us to estimate the signal parameters such as direction-of-arrival (DOA) more accurately. Representative numerical examples are provided to verify the effectiveness of the proposed array configuration.

The remainder of this paper is organized as follows. Some necessary preliminaries are introduced in Section II. Section III describes the proposed array configuration in detail and gives the comparisons of both DOF and VAAs between our proposed array configuration and other existing commonly-used or related linear array configurations. Simulations and conclusions are presented in Section IV and Section V, respectively.

Notations: in this paper, we use boldfaced uppercase and lowercase letters to represent matrices and vectors, respectively. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ stand for transpose, conjugate transpose and complex conjugate, respectively. $E\{\cdot\}$, $\text{vec}(\cdot)$ and $\text{diag}\{\cdot\}$ represent statistical expectation, vectorization and diagonalization operators, respectively. $\odot$ denotes Khatri-Rao product and $\lfloor\cdot\rfloor$ is the integral part of the rational number in the square brackets.

II. PRELIMINARIES

To facilitate the presentation of the proposed array configuration for DOA estimation, we shall briefly provide some preliminaries of the difference set, two-level nested array, and KR subspace based DOA estimation in this section.

A. DIFFERENCE SET AND DEGREES OF FREEDOM

Let us consider two integer number sets: $\{n_1, n_2, \cdots, n_N\}$ and $\{m_1, m_2, \cdots, m_M\}$, the self-difference set is defined as

$$SD=\{n_i-n_j \mid 1\leq i,j\leq N\}\cup\{m_i-m_j \mid 1\leq i,j\leq M\} \quad (1)$$

and the cross-difference set is defined as

$$CD=\{\pm(n_i-m_j) \mid 1\leq i\leq N, 1\leq j\leq M\}. \quad (2)$$

The difference set (denoted as D) is the union of the self-difference and the cross-difference set, i.e.,

$$D=SD\cup CD. \quad (3)$$

If it is allowed repetition of the elements in the difference set D [14], the DOF is equal to the total number of the distinct elements. For example, the difference set of $\{0, 1\}$ and $\{2, 5, 8\}$ is $\{0, -1, 1, 0, 0, -3, -6, 3, 0, -3, 6, 3, 0, -2, -5, -8, -1, -4, -7, 2, 1, 5, 4, 8, 7\}$, where there are 17 distinct elements, i.e., $-8, -7, \cdots, -1, 0, 1, \cdots, 7, 8$. As a result, the corresponding DOF is 17.

B. TWO-LEVEL NESTED ARRAY

The two-level nested array is a co-array composed of two ULAs, whose sensor positions are, respectively, given by the following two sets [14]:

$$S_M=\{md_1 \mid m=0,1,\cdots,M-1\} \quad (4)$$
$$S_N=\{Md_1+nd_2 \mid n=0,1,\cdots,N-1\} \quad (5)$$

where d_1 and M are the inter-element spacing and the number of sensors of the first ULA, respectively, while d_2 and N are those of the second ULA. Moreover, d_1 and d_2 satisfy

$$d_2=(M+1)d_1. \quad (6)$$

This co-array is able to produce a virtual array with sensor locations corresponding to the difference set of S_M and S_N.

It should be pointed out that the difference set of S_M and S_N is a set of continuous integer numbers, and its DOF is $2N(M+1)-1$. In what follow, the VAA (normalized by d_1) is quantified as $\text{VAA}=\max\{D\}-\min\{D\}$, where D is the difference set of S_M and S_N.

C. DOA ESTIMATION BASED ON KR SUBSPACE

Suppose that a linear array with L sensors observes K uncorrelated sources with DOAs θ_k, $k=1,2,\cdots,K$ and that the sources emit quasi-stationary signals. The observation vector of the sensor array can thus be modeled as

$$\mathbf{x}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{n}(t) \quad (7)$$

where $\mathbf{A}=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_K)]$, $\mathbf{s}(t)$ and $\mathbf{n}(t)$ represent steering matrix, signal vector and noise vector, respectively. $\mathbf{a}(\theta_k)$ is the steering vector associated with the k-th signal. Particulary, we have $\mathbf{a}(\theta_k)=[\beta^{v_1}(\theta_k),\beta^{v_2}(\theta_k),\cdots,\beta^{v_L}(\theta_k)]^T$, where $\beta(\theta_k)=e^{-j\frac{2\pi}{\lambda}\sin(\theta_k)}$ and v_l, $l=1,2,\cdots,L$, denotes the distance between the l-th sensor and the reference point.

The covariance matrix of the f-th frame of quasi-stationary signals is defined as [16]

$$\mathbf{R}_f=E\{\mathbf{x}(t)\mathbf{x}^H(t)\},\quad \forall t\in[(f-1)T, fT-1] \quad (8)$$

for $f=1,2,\ldots,F$, where F and T are the number of frames and the length of each frame, respectively. Assuming that the noise is uncorrelated with the signals, then (8) can be expressed as

$$\mathbf{R}_f = \mathbf{A}\mathbf{R}_{\mathbf{s}f}\mathbf{A}^H + \mathbf{R}_\mathbf{n} \tag{9}$$

where $\mathbf{R}_{\mathbf{s}f} = E\{\mathbf{s}_f(t)\mathbf{s}_f^H(t)\} = \mathrm{diag}\{\sigma_{f1}^2, \sigma_{f2}^2, \cdots, \sigma_{fK}^2\}$ and $\mathbf{R}_\mathbf{n} = E\{\mathbf{n}(t)\mathbf{n}^H(t)\}$ are the signal and noise covariance matrices, respectively, $\sigma_{f1}^2, \sigma_{f2}^2, \cdots, \sigma_{fK}^2$ are the signal powers in the f-th frame.

The vectorization of $\mathbf{R}_f$ is given by

$$\mathbf{y}(f) = \mathrm{vec}(\mathbf{R}_f) = (\mathbf{A}^* \odot \mathbf{A})\mathbf{p}(f) + \mathrm{vec}(\mathbf{R}_\mathbf{n}) \tag{10}$$

where $\mathbf{p}(f) = [\sigma_{f1}^2, \sigma_{f2}^2, \cdots, \sigma_{fK}^2]^T$ and the columns of $\mathbf{A}^* \odot \mathbf{A}$ span the so-called KR subspace. By stacking $\mathbf{y}(1)$, $\mathbf{y}(2), \cdots, \mathbf{y}(F)$ as $\mathbf{Y} \triangleq [\mathbf{y}(1), \mathbf{y}(2), \cdots, \mathbf{y}(F)]$, we have

$$\mathbf{Y} = (\mathbf{A}^* \odot \mathbf{A})\mathbf{P} + \mathrm{vec}(\mathbf{R}_\mathbf{n})\mathbf{1}_F^T \tag{11}$$

where $\mathbf{P} = [\mathbf{p}(1), \mathbf{p}(2), \cdots, \mathbf{p}(F)]$. It can be seen from (10) that $\mathbf{y}(f)$ behaves as the observation of a virtual array whose sensor locations are determined by the physical array. In other words, the steering vector of the virtual array corresponds to the difference set of the steering vector $\mathbf{a}(\theta)$ and its conjugate $\mathbf{a}^*(\theta)$. After eliminating the noise component in (11) through projection [16], the space that is orthogonal to $\mathbf{A}^* \odot \mathbf{A}$ can be achieved through singular value decomposition. Thus, DOA estimation can be performed by using the MUSIC algorithm [24].

III. PROPOSED ARRAY CONFIGURATION

It is found from $\mathbf{A}^* \odot \mathbf{A}$ in (10) that the number of distinct virtual sensors is directly related to the arrangement of the physical sensors. For instance, the number of distinct virtual sensors is $2L - 1$ when L physical sensors are arranged in uniformly linear, and thus there exist $L^2 - 2L + 1$ redundancies. For the motivation of producing as many distinct virtual sensors as possible (in other words, reducing the redundancy of the virtual array), a novel array configuration is presented in this section. Similar to those existing methods, our proposed array configuration is based on the two-level nested array. However, the main difference is that it is composed of two ULAs plus a single separate sensor. This composition is shown to be able to increase the DOF and enlarge the VAAs.

Assume that the number of physical sensors is $L = M + N$. In particular, we choose M and $N - 1$ sensors to form the first and the second ULAs with inter-element spacings of d_1 and d_2, respectively. The remaining sensor is set as a separate sensor apart from the last sensor of the second ULA with a distance of $d_1 + d_2$. The gap between the first and the second ULAs is $\left\lfloor \frac{M+N-1}{2} \right\rfloor d_1$ wide, and d_1 and d_2 are related with each other as

$$d_2 = \left(M + \left\lfloor \frac{M+N-1}{2} \right\rfloor\right) d_1. \tag{12}$$

It should be emphasized that $\left\lfloor \frac{M+N-1}{2} \right\rfloor$ denotes the integral part of $\frac{M+N-1}{2}$. That is to say, the sensor locations of the first and the second ULAs are given by S_M and S_{N-1}, respectively, which are represented as follows

$$S_M = \{md_1 \mid m = 0, 1, \cdots, M-1\} \tag{13a}$$
$$S_{N-1} = \{(d_2 - 1)d_1 + nd_2 \mid n = 0, 1, \cdots, N-2\}. \tag{13b}$$

The separate sensor is located at

$$S_1 = \{d_1 d_2 + (N-1)d_2\}. \tag{14}$$

From another point of view, the proposed array configuration is composed of a ULA with sensor positions given by S_M and a nonuniform linear array with sensor positions denoting by

$$S_N = S_{N-1} \cup S_1. \tag{15}$$

Hence, the positions of the physical sensors are given by $S_M \cup S_N$. In order to have an intuitive perspective of the proposed array configuration, the sensor locations are shown in Fig. 1. As a result, the entries of the difference set of S_M and S_N correspond to the sensor positions of the virtual array.

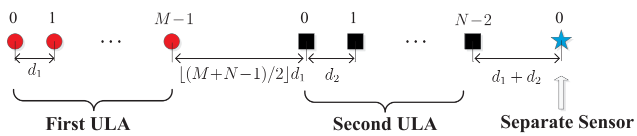


FIGURE 1. Illustration of the proposed array configuration.

It can be readily verified that the proposed array configuration is capable of producing increased DOF and enlarged VAAs over the existing geometries. More precisely, we can obtained the following conclusions that, if $L \geq 5$ and:

- If L is even, choosing $M = \frac{L}{2} - 1$ and $N = \frac{L}{2} + 1$, then we have

$$\mathrm{DOF} = \frac{L^2 - 2}{2} + 3L - 6 \tag{16a}$$
$$\mathrm{VAA} = L^2 - 4. \tag{16b}$$

- If L is odd, choosing $M = \frac{L-1}{2}$ and $N = \frac{L+1}{2}$, then we have

$$\mathrm{DOF} = \frac{L^2 - 1}{2} + 3L - 6 \tag{17a}$$
$$\mathrm{VAA} = L^2 - 1. \tag{17b}$$

Obviously, it can be found that the proposed configuration has increased DOF compared with existing configurations such as two-level nested array [14], whose DOF is $\frac{L^2-2}{2} + L$ if L is even, and is $\frac{L^2-1}{2} + L$ if L is odd. Furthermore, it should be pointed out that the above formulas are not applicable to the cases of $L \leq 4$. As a matter of fact, it can be simply calculated that if $L = 4$ we have DOF $= 11$ and VAA $= 12$, and if $L = 3$ we have DOF $= 7$ and VAA $= 8$.

Now, let us take a simple example for illustration. Assume that $M = 2$ and $N = 3$ (hence, $L = 5$), and without loss of generality, we set $d_1 = 1$. According to (12)–(15), we have $d_2 = 4$, $S_M = \{0, 1\}$ and $S_N = \{3, 7, 12\}$. It can be readily

TABLE 1. Summary of DOFs of different array configurations.

Number of sensors ($L = M + N$)		KR ULA	Coprime	Unfolded Coprime	Nested	Iizuka-Nested	Yang-Nested	Yang-NMRA	Liu-Nested (ANAI-1)	Proposed
5 = 2 + 3		9	9	11	17	19	19	NA	19	21
7 = 3 + 4		13	17	23	31	33	35	NA	35	39
8 = 3 + 5		15	21	29	39	41	45	27	45	49
12 = 5 + 7		23	45	69	83	85	93	91	93	101
13 = 6 + 7		25	53	83	97	99	107	NA	107	117
16 = 7 + 9		31	77	125	143	145	157	169	157	169
DOF	L is even	$2L-1$	$MN+M$ $+N-2$	$2MN-1$	$\frac{L^2-2}{2}+L$	$\frac{L^2-2}{2}+L+2$	$\frac{L^2-2}{2}+2L-2$	NA	$\frac{L^2-2}{2}+2L-2$	$\frac{L^2-2}{2}+3L-6$
	L is odd				$\frac{L^2-1}{2}+L$	$\frac{L^2-1}{2}+L+2$	$\frac{L^2-1}{2}+2L-3$		$\frac{L^2-1}{2}+2L-3$	$\frac{L^2-1}{2}+3L-6$

TABLE 2. Summary of VAAs of different array configurations.

Number of sensors ($L = M + N$)		KR ULA	Coprime	Unfolded Coprime	Nested	Iizuka-Nested	Yang-Nested	Yang-NMRA	Liu-Nested (ANAI-1)	Proposed
5 = 2 + 3		8	8	14	16	22	18	NA	18	24
7 = 3 + 4		12	18	34	30	38	34	NA	34	48
8 = 3 + 5		14	24	44	38	46	44	26	44	60
12 = 5 + 7		22	60	116	82	94	92	90	92	140
13 = 6 + 7		24	72	142	96	110	106	NA	106	168
16 = 7 + 9		30	112	220	142	158	156	168	156	252
VAA	L is even	$2L-2$	$2MN-(M+N)+2$	$4MN-$ $2(M+N)$	$\frac{L^2-2}{2}+L-1$	$\frac{L^2-2}{2}+2L-1$	$\frac{L^2-2}{2}+2L-3$	NA	$\frac{L^2-2}{2}+2L-3$	L^2-4
	L is odd		$2MN-(M+N)+1$		$\frac{L^2-1}{2}+L-1$	$\frac{L^2-1}{2}+2L$	$\frac{L^2-1}{2}+2L-4$		$\frac{L^2-1}{2}+2L-4$	L^2-1

verified that the difference set of S_M and S_N (excluding the redundant entries) is given by

$$D = \{-12, -11, -9, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 9, 11, 12\}. \quad (18)$$

It is seen that this set includes 21 distinct elements and the difference between the maximal and minimal numbers is 24. The results are coincident with the result in (17) that DOF = 21 and VAA = 24.

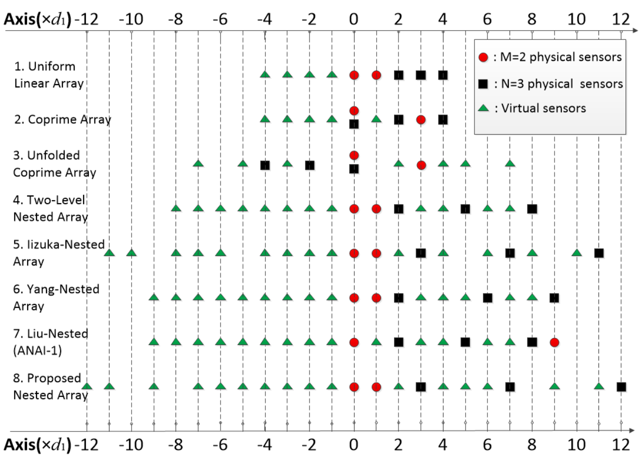


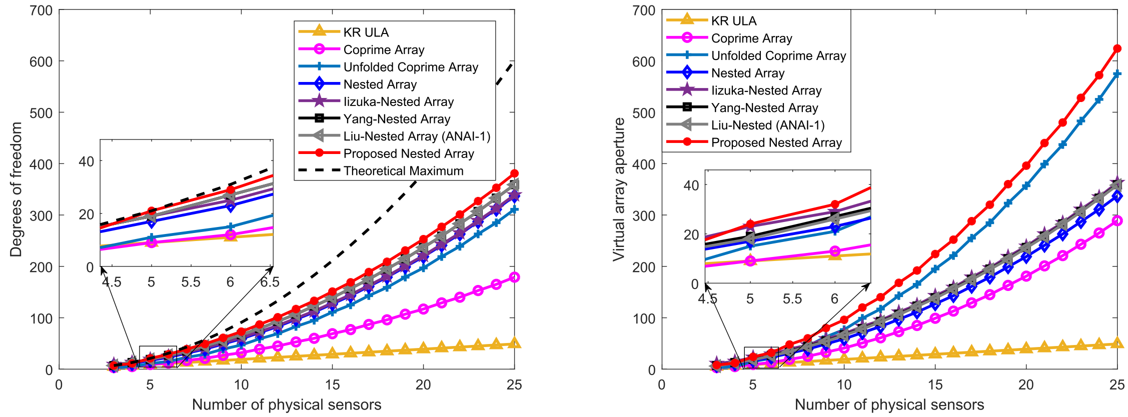
FIGURE 2. Demonstration of sensor locations of various array configurations (the number of physical sensors is $L = 5$).

In order to make a clearer view on the physical and virtual sensor locations of different array configurations, we again take $L = 5$ sensors ($M = 2$ and $N = 3$) for example, and draw the sensor locations of eight different array configurations in Fig. 2. For comparison, the other seven existing array configurations are also given: 1) uniform linear array, 2) coprime array [13], 3) unfolded coprime array [18], 4) two-level nested array [14], 5) Iizuka-nested array [20], 6) Yang-nested array [21], and 7) Liu-nested array [23] (Liu-nested array includes two kinds, i.e., ANAI-1 and ANAI-2. ANAI-1 has larger DOF if not considering the mutual coupling, and therefore it is taken for comparison in this paper). Obviously, it is seen that the proposed array configuration has the largest DOF and VAA. It should be noted that Yang-NMRA [22] is not depicted in Fig. 2, since it is not worked out for the case of $L = 5$.

Now, we summarize and compare the formulas of the DOFs and VAAs of these array configurations, which are abbreviated as KR ULA [16], Coprime [13], Unfolded Coprime [18], Nested [14], Iizuka-Nested [20], Yang-Nested [21], Yang-NMRA [22], Liu-Nested (ANAI-1) [23], and Proposed, respectively. The comparisons of DOFs are represented in TABLE 1 and Fig. 3(a), while the comparisons of VAAs are displayed in TABLE 2 and Fig. 3(b). Since there are certain cases with special values of L that are not worked out for Yang-NMRA, they are displayed as 'NA' in the corresponding table. Note that, there is a theoretical maximum of DOF for a given number of physical sensors [14]. Hence, for the purpose of analysis and comparison, the theoretical maximum of DOF is also calculated and drawn in Fig. 3(a).

It can be seen from Table 1 and Fig. 3(a) that, Yang-nested array and Liu-nested (ANAI-1) share the same closed-form expressions for DOF and VAA. When $L \geq 5$, the proposed nest-based array configuration has the most DOF among all the array configurations listed. On the other hand, it can be concluded from Table 2 and Fig. 3(b) that, when $L \geq 5$, the proposed nest-based array configuration owns larger VAA than that of any others.

It is worth mentioning that, in our proposed array configuration, the difference set produced by S_M and S_N leads



(a) (b)

FIGURE 3. Comparisons of the DOFs and VAAs of different array configurations. (a) Comparisons of DOFs. (b) Comparisons of VAAs.

to a virtual linear array which consists of a filled ULA in the middle of the whole virtual array and some holes at both ends of the whole virtual array. It is known the holes may cause ambiguity in parameter estimation, such as DOA estimation. Nevertheless, the larger virtual array aperture is capable of providing better performance of DOA estimation when the number of sources is no larger than the number of sensors of the middle filled ULA. If the number of sources is larger than the number of consecutive virtual sensors, positive-definite Toeplitz completion techniques could be utilized. Since this is out of the scope of this work, the interested reader is referred to [25] and related references for more details.

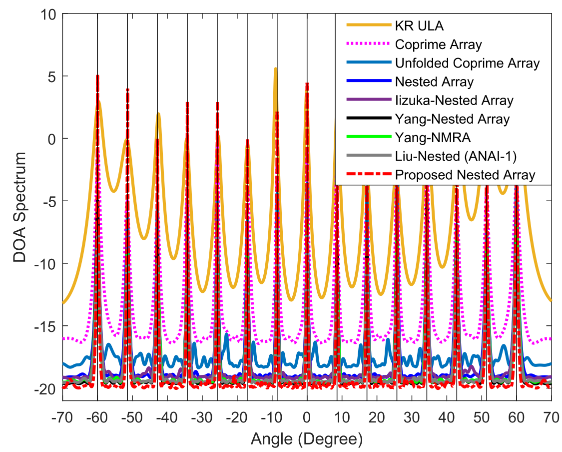
IV. SIMULATIONS

In this section, we utilize the proposed nested array configuration to perform DOA estimation in the scenario of quasi-stationary signal, where the KR subspace-based MUSIC algorithm (which was introduced in Section II-C) can be employed. The KR subspace-based ULA [16], coprime array [13], unfolded coprime array [18], two-level nested array [14], Iizuka-nested array [20], Yang-nested array [21], Yang-NMRA [22], and Liu-nested (ANAI-1) [23] are meanwhile experimented for comparison.

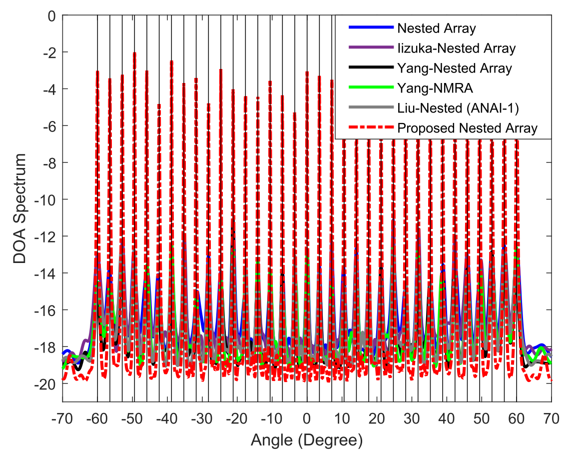
A. SPATIAL SPECTRUM COMPARISON

Example 1: Assume that $K = 15$ uncorrelated sources with DOAs uniformly distributed in $[-60^\circ, 60^\circ]$ emit quasi-stationary signals onto a linear array with $L = 12$ ($M = 5$, $N = 7$) physical sensors. The number of frames is set to be $F = 50$, while the length of each frame is $T = 500$. The signal-to-noise ratio (SNR) is set as 0 dB. The simulation results are drawn in Fig. 4(a).

Example 2: We raise the number of sources from 15 in the first example to 35 (with DOAs uniformly distributed in $[-60^\circ, 60^\circ]$), and keep other simulation settings unchanged. In this example, we only test the six nest-based array configurations, namely, two-level nested array, Iizuka-nested array, Yang-nested array, Yang-NMRA, Liu-nested (ANAI-1), and the proposed nested array. The results are shown in Fig. 4(b).



(a)



(b)

FIGURE 4. The spatial spectra with SNR = 0 dB. (a) 15 sources. (b) 35 sources.

It can be observed from Fig. 4 that, the proposed nest-based array configuration provides high-quality spatial spectrum for DOA estimation. In particular, it has the sharpest peaks among all these array configurations. This is because both the DOF and the VAA of the proposed array configuration are larger than those of any other array geometries (see Table 1 and Table 2). Hence, the proposed nested array possesses of the capacity of shaping the sharpest peaks.

B. RMSE OF DOA ESTIMATION VERSUS SNR

We evaluate the performance in terms of root mean squared error (RMSE) of DOA estimation, which is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{KQ}\sum_{k=1}^{K}\sum_{q=1}^{Q}(\hat{\theta}_{k,q} - \theta_k)^2} \tag{19}$$

where $\hat{\theta}_{k,q}$ represents the DOA estimate of kth source in the qth Monte Carlo trial and Q is the total number of Monte Carlo trials.

Example 3: In this example, $K = 15$ sources are examined for all the array configurations listed afore. The number of physical sensors is $L = 12$ ($M = 5$ and $N = 7$), the number of frames is $F = 50$, the length of each frame is $T = 500$, and the number of Monte Carlo trials is $Q = 500$. The SNRs are varied uniformly from -18 dB to 10 dB with a stepsize of 2 dB.

Example 4: To further test and compare the performances of six nest-based array configurations, in this example we assume $K = 20$ sources and other settings remain the same as those in the third example.

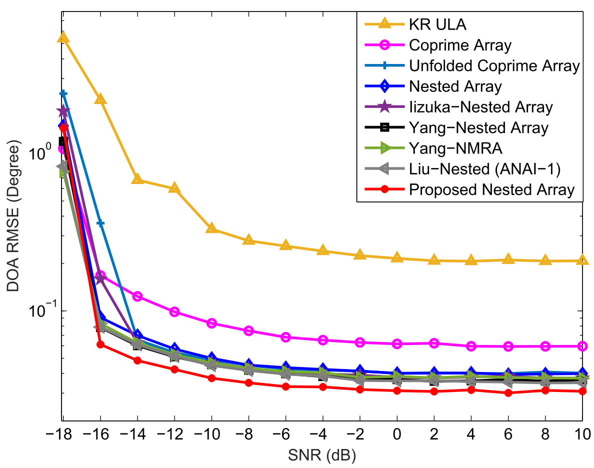


FIGURE 5. RMSE of DOA estimation versus SNR of nine array configurations with 12 physical sensors and 15 sources.

The simulation results of these two examples are illustrated in Fig. 5 and Fig. 6, respectively, from which it is seen that, the proposed nest-based array geometry achieves lower RMSE of DOA estimation than those of other array configurations (including ULA, coprime-based arrays, and nest-based arrays).

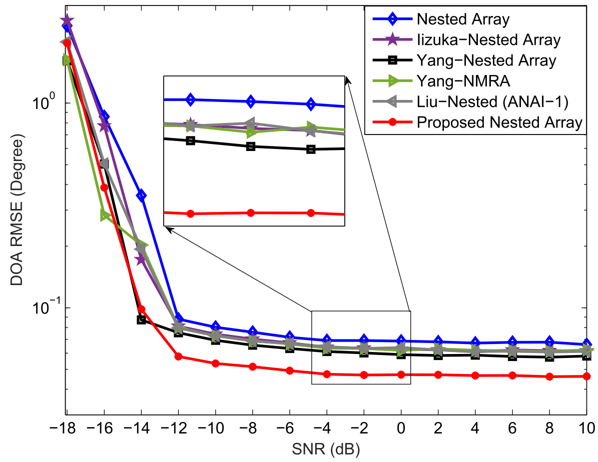


FIGURE 6. RMSE of DOA estimation versus SNR of six nest-based configurations with 12 physical sensors and 20 sources.

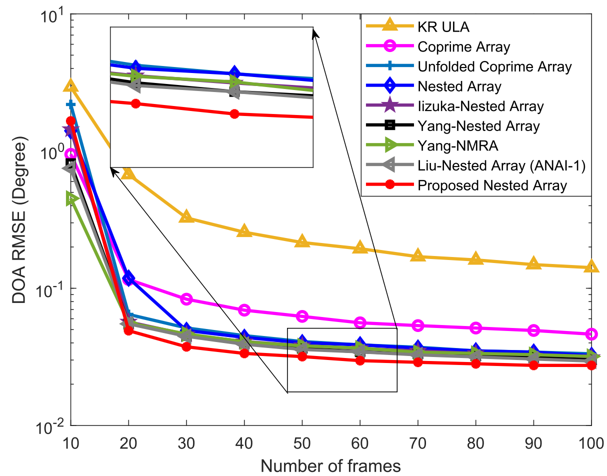


FIGURE 7. RMSE of DOA estimation versus number of frames of nine array configurations with SNR = 0 dB.

C. DOA RMSE VERSUS THE NUMBER OF FRAMES

Example 5: The RMSE of DOA estimation versus the number of frames is examined in this example. It is assumed that $L = 12$ ($M = 5$ and $N = 7$), $K = 10$, $T = 500$, the SNR is set to be 0 dB, and $Q = 500$ Monte Carlo trials are run. The results are demonstrated in Fig. 7. As expected, it is observed from Fig. 7 that the proposed array configuration has the lowest RMSE among all the array structures when the number of frames is greater than or equal to 20. Even in the case when the number of frames is small, the proposed nested array configuration owns little RMSE of DOA estimation.

V. CONCLUSIONS

We proposed a novel nest-based array configuration in this paper by extending the concept of two-level nested array. The proposed array configuration is able to achieve more degrees of freedom and larger virtual array aperture than those of existing configurations. As a result, signal parameter estimation such as DOA estimation can be improved based on the proposed configuration. Simulation results indicated the effectiveness and good performance of our proposed array configuration.

REFERENCES

[1] T.-J. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-33, no. 4, pp. 806–811, Apr. 1985.

[2] R. Roy and T. Kailath, "Esprit-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-37, no. 7, pp. 984–995, Jul. 1989.

[3] V. Reddy, A. Paulraj, and T. Kailath, "Performance analysis of the optimum beamformer in the presence of correlated sources and its behavior under spatial smoothing," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-35, no. 7, pp. 927–936, Jul. 1987.

[4] S. Reddi, "On a spatial smoothing technique for multiple source location," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-35, no. 5, p. 709, May 1987.

[5] Z. Ye and X. Xu, "DOA estimation by exploiting the symmetric configuration of uniform linear array," *IEEE Trans. Antennas Propag.*, vol. 55, no. 12, pp. 3716–3720, Dec. 2007.

[6] E. H. Bae, J. S. Kim, B. W. Choi, and K. K. Lee, "Decoupled parameter estimation of multiple distributed sources for uniform linear array with low complexity," *Electron. Lett.*, vol. 44, no. 10, pp. 649–650, May 2008.

[7] Z. Ye, J. Dai, X. Xu, and X. Wu, "DOA estimation for uniform linear array with mutual coupling," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 45, no. 1, pp. 280–288, Jan. 2009.

[8] L. Zhang, W. Liu, and R. J. Langley, "A class of constrained adaptive beamforming algorithms based on uniform linear arrays," *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3916–3922, Jul. 2010.

[9] B. Liao and S. C. Chan, "Direction finding with partly calibrated uniform linear arrays," *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 922–929, Feb. 2012.

[10] B. Liao and S.-C. Chan, "Direction finding in partly calibrated uniform linear arrays with unknown gains and phases," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 1, pp. 217–227, Jan. 2015.

[11] K. Liu, Z. Zheng, H. Yang, H. Meng, and Y. Ge, "An approach to robust beamforming in the presence of mutual coupling for uniform linear arrays," in *Proc. 2nd IEEE Int. Conf. Comput. Commun. (ICCC)*, Oct. 2016, pp. 1870–1874.

[12] P. P. Vaidyanathan and P. Pal, "Sparse sensing with coprime arrays," in *Proc. Conf. Rec. 44th Asilomar Conf. Signals, Syst. Comput.*, Nov. 2010, pp. 1405–1409.

[13] P. Pal and P. P. Vaidyanathan, "Coprime sampling and the MUSIC algorithm," in *Proc. 14th IEEE DSP/SPE Workshop*, Sedona, AZ, USA, Jan. 2011, pp. 289–294.

[14] P. Pal and P. P. Vaidyanathan, "Nested arrays: A novel approach to array processing with enhanced degrees of freedom," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4167–4181, Aug. 2010.

[15] P. Pal and P. P. Vaidyanathan, "Multiple level nested array: An efficient geometry for 2qth order cumulant based array processing," *IEEE Trans. Signal Process.*, vol. 60, no. 3, pp. 1253–1269, Mar. 2012.

[16] W.-K. Ma, T.-H. Hsieh, and C.-Y. Chi, "DOA estimation of quasi-stationary signals with less sensors than sources and unknown spatial noise covariance: A Khatri–Rao subspace approach," *IEEE Trans. Signal Process.*, vol. 58, no. 4, pp. 2168–2180, Apr. 2010.

[17] S. Qin, Y. D. Zhang, and M. G. Amin, "Generalized coprime array configurations," in *Proc. IEEE 8th Sensor Array Multichannel Signal Process. Workshop (SAM)*, Jun. 2014, pp. 529–532.

[18] J. Li and X. Zhang, "Direction of arrival estimation of quasi-stationary signals using unfolded coprime array," *IEEE Access*, vol. 5, pp. 6538–6545, 2017.

[19] H. Duan, T. Tuo, and Y. Yin, "Real-valued Khatri–Rao subspace approaches on the ULA and a new nested array," in *Proc. IEEE Int. Conf. Digit. Signal Process. (DSP)*, Oct. 2016, pp. 398–402.

[20] Y. Iizuka and K. Ichige, "Extension of two-level nested array with larger aperture and more degrees of freedom," in *Proc. Int. Symp. Antennas Propag. (ISAP)*, Oct. 2016, pp. 442–443.

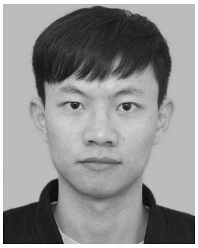
[21] M. Yang, L. Sun, X. Yuan, and B. Chen, "Improved nested array with hole-free DCA and more degrees of freedom," *Electron. Lett.*, vol. 52, no. 25, pp. 2068–2070, Aug. 2016.

[22] M. Yang, A. M. Haimovich, B. Chen, and X. Yuan, "A new array geometry for DOA estimation with enhanced degrees of freedom," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, Mar. 2016, pp. 3041–3045.

[23] J. Liu, Y. Zhang, Y. Lu, S. Ren, and S. Cao, "Augmented nested arrays with enhanced dof and reduced mutual coupling," *IEEE Trans. Signal Process.*, vol. 65, no. 21, pp. 5549–5563, Nov. 2017.

[24] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. AP-34, no. 3, pp. 276–280, Mar. 1986.

[25] Y. I. Abramovich, N. K. Spencer, and A. Y. Gorokhov, "Positive-definite Toeplitz completion in DOA estimation for nonuniform linear antenna arrays. II. Partially augmentable arrays," *IEEE Trans. Signal Process.*, vol. 47, no. 6, pp. 1502–1521, Jun. 1999.



HUIPING HUANG (S'16) received the B.Eng. degree in electronic and information engineering from Shenzhen University, Shenzhen, China, in 2015, where he is currently pursuing the M.Eng. degree in electronic and communication engineering. His research interests include sensor array processing and optimization algorithm

BIN LIAO (S'09–M'13–SM'16) received the B.Eng. and M.Eng degrees from Xidian University, Xian, China, in 2006 and 2009, respectively, and the Ph.D. degree from The University of Hong Kong, Hong Kong, in 2013. He was with the Department of Electrical and Electronic Engineering, The University of Hong Kong, as a Research Assistant from 2013 to 2014, and a Research Scientist in 2016. He is currently an Associate Professor with the College of Information Engineering, Shenzhen University, Shenzhen, China. His research interests include sensor array processing, adaptive filtering, convex optimization, with applications to radar, navigation and communications. He is a recipient of the Best Paper Award at the 21st International Conference on Digital Signal Processing in 2016 and 22nd International Conference on Digital Signal Processing in 2017.

Dr. Liao is an Associate Editor of the IEEE Transactions on Aerospace and Electronic Systems,the *IET Signal Processing*, the *Multidimensional Systems and Signal Processing*, and the IEEE Access.

XIAOYE WANG received the B.S. degree in electronic information engineering from the Hunan University of Science and Engineering, Hunan, China, in 2010, and the M.S. degree in communication and information system from Shenzhen University, Shenzhen, China, in 2013, where she is currently pursuing the Ph.D. degree in signal and information processing with the College of Information Engineering. From 2013 to 2015, she was an Assistant teacher with the Shenzhen Institute of Information Technology. Her current research interests include array signal processing, space-time adaptive processing, and compressive sensing.

XIANSHENG GUO (S'07–M'11) received the B.Eng. degree from Anhui Normal University, Wuhu, China, in 2002, the M.Eng. degree from the Southwest University of Science and Technology, Mianyang, China, in 2005, and the Ph.D. degree from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 2008. From 2008 to 2009, he was a Research Associate with the Department Electrical and Electronic Engineering, The University of Hong Kong. From 2012 to 2014, he was a Research Fellow with the Department of Electronic Engineering, Tsinghua University. He was a Research Scholar with the Advanced Networking Laboratory, Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ, USA, from 2016 to 2017. He is currently an Associate Professor with the Department of Electronic Engineering, UESTC. His research interests include array signal processing, wireless localization, machine learning, information fusion, and software radio design.

JIANJUN HUANG (M'03) received the B.Eng. and Ph.D. degrees from Xidian University, Xi'an China, in 1992 and 1997, respectively. From 1997 to 1999, he was a Post-Doctoral Researcher with the Department of Computer Science, Northwestern Polytechical University, Xi'an. Since 1999, he has been with the College of Information Engineering, Shenzhen University, where he is currently a Professor. His current research interests include signal detection, adaptive filtering, compressive sensing, and target recognition.