IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received October 9, 2017, accepted November 13, 2017, date of publication November 21, 2017, date of current version February 14, 2018.

Digital Object Identifier 10.1109/ACCESS.2017.2776121

Passivity and Fault Alarm-Based Hybrid Control for a Markovian Jump Delayed System With Actuator Failures

JIAN-NING LI, YU-FEI XU, WEN-DONG BAO, AND XIAO-BIN XU
Institute of Systems Science and Control Engineering, School of Automation, Hangzhou Dianzi University, Hangzhou 310018, China

Corresponding author: Jian-Ning Li (ljn@hdu.edu.cn)

This work was supported by the National Natural Science Foundation of China under Grant 61733009, Grant 61403113, Grant 61433001, Grant U1509203, and Grant U170920062.

ABSTRACT In this paper, a passivity and fault alarm-based hybrid controller is designed for a Markovian jump delayed system with actuator failures. First, a passive condition is given, and a type of hybrid controller that combines with robust and fault-tolerant controllers is presented to ensure that both the normal system and the fault system are robustly stochastically passive. Next, a fault alarm signal is proposed by choosing the alarm threshold, and this signal is used to invoke the fault-tolerant controller. Finally, a numerical example is provided to show the effectiveness of the method.

INDEX TERMS Fault alarm, Markovian jump system, passivity, linear matrix inequality (LMI), fault-tolerant control, actuator fault.

I. INTRODUCTION

Markovian jump systems are used to describe the systems that experience abrupt changes in their structure and parameters caused by component failures or repairs, changing subsystem interconnections, abrupt environmental disturbances and so on [1]. There are two components in the hybrid systems. The first component is a Markovian process, which is defined in the finite space as a mode of the system. The second component is the state of the systems, in which the state in each mode is described by a stochastic differential equation. In recent decades, numerous relevant research results have been reported. For example, the relationships among the second moment stability properties of jump linear systems have been discussed in [2]. For Markovian jump systems with an uncertain model and an external disturbance, robust control theory plays an important role. The problems of robust $\mathcal{H}_2$-control and robust stabilization of Markovian jump linear systems have been investigated in [3] and [4], respectively. The network induced problems of networked Markovian jump systems have been discussed in [5]–[8]. However, the transition probabilities may not be measurable exactly or may be only partly known in most practical systems. Under this assumption, a number of relevant results have been reported [9]–[14].

Passivity, a concept that originated from electrical networks, has a deep physical meaning. In addition, the close relation of passivity with the Lyapunov function indicates that it plays an important role in analysing the stability of nonlinear systems. With the development of passivity theory, many results have been produced. For example, in [15], the definition of stochastic passivity for Markovian jump systems is given, and the passive controllers are designed. A robust passivity controller is designed for 2-D uncertain Markovian jump linear systems in [16]. In [17], the problem of feedback passivity for the networked control systems with packet drops has been analysed. Moreover, it is an unavoidable fact that time-delays occur frequently in many practical systems, thus causing instability and poor performance. Reference [18] has studied the problem of control for discrete time delay linear Markovian jump systems. The fault-tolerant control problem was discussed for a class of uncertain networked control systems with induced delays and actuator saturation in [19]. Wu provided some excellent results in [20]–[27]. In [28], the problem of observer-based passive control of a class of uncertain linear systems with delayed state and parameter uncertainties was investigated. To the best of the authors' knowledge, the research for passivity and fault alarm-based hybrid controller designing algorithm is rare, thus, we provide a kind of designing algorithm in this work.

In practice, using the fault-tolerant controllers directly is much more conservative because of the invariance of the controller's gain. Thus, to increase the performance of the closed-loop systems, we must design a switching method, namely, the robust controllers are chosen to operate when the systems run without fault and the fault-tolerant controllers will replace the robust ones if the fault occurs. First, a Markovian jump systems model is given. Next, the sufficient passive condition is established. Moreover, based on this condition and inspired by [19], we can obtain the controller gains of robust controllers and fault-tolerant controllers by LMI technology. Next, we design a switching method by selecting a suitable threshold to achieve the aim above, which is the most difficult problem we have to solve. Finally, a numerical example is given to demonstrate the availability of the proposed methods.

Notation: In this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the $n-$dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. $X > 0$ (or $X < 0$) indicates that matrix X is a real symmetric positive definite (or negative definite). The superscript T and + denote the matrix transposition and the pseudo inverse, respectively. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. $\varepsilon\{\cdot\}$ denotes the mathematical expectation.

II. PROBLEM FORMULATION

In this paper, consider Markovian jump systems with time-varying delays on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ as follows:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + A_d(r(t))x(t-d_1(t)) + W(r(t))\omega(t) \\ \qquad + B(r(t))u^A(t) + B_d(r(t))u^B(t-d_2(t)), \\ y(t) = C(r(t))x(t), \\ x(t) = \varphi(t), \quad \forall t \in [-\bar{d}, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector; $u^A(t)$ and $u^B(t) \in \mathbb{R}^m$ are the control input vectors from the actuator to the plant; $\omega(t) \in \mathbb{R}^q$ is the exogenous disturbance input that belongs to $\mathfrak{L}_2[0,\infty)$; $y(t) \in \mathbb{R}^p$ is the control output; $\varphi(t)$ is the initial condition defined on $[-\bar{d}, 0]$; $d_1(t)$ and $d_2(t)$ are the time-varying state delay and the time-varying control delay of systems, respectively, and satisfy

$$\begin{cases} 0 \le d_1(t) \le \bar{d}_1 < \infty, \quad \dot{d}_1(t) \le h_x < 1, \\ 0 \le d_2(t) \le \bar{d}_2 < \infty, \quad \dot{d}_2(t) \le h_u < 1, \\ \bar{d} = max[\bar{d}_1, \bar{d}_2]. \end{cases} \tag{2}$$

$\{r(t)\}$ is a continuous-time Markov process with continuous trajectories and takes values in a finite set $S = \{1, 2, 3, \ldots, s\}$. Moreover, the mode transition probabilities of $\{r(t)\}$ satisfies

$$Pr\{r(t+\Delta)\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j, \\ 1 + \pi_{ij}\Delta + o(\Delta), & i = j, \end{cases}$$

where $\Delta > 0$, $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta} = 0$, and π_{ij} is the transition rate from mode i to mode j at time $t+\Delta$, which satisfies

$$\pi_{ij} \ge 0, \quad \forall i, j, i \neq j,$$

$$\sum_{j=1}^{s} \pi_{ij} = 0, \quad \forall i \in S.$$

$A(r(t)) \in \mathbb{R}^{n\times n}$, $A_d(r(t)) \in \mathbb{R}^{n\times n}$, $B(r(t)) \in \mathbb{R}^{n\times m}$, $B_d(r(t)) \in \mathbb{R}^{n\times m}$, $W(r(t)) \in \mathbb{R}^{n\times q}$ and $C(r(t)) \in \mathbb{R}^{p\times n}$ are known real constant matrices for each $r(t) \in S$. For notational simplicity, A_i, A_{di}, B_i, B_{di}, W_i and C_i are used to define $A(r(t)), A_d(r(t)), B(r(t)), B_d(r(t)), W(r(t))$ and $C(r(t))$, respectively. Thus, the system (1) can be rewritten as

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di}x(t-d_1(t)) + W_i\omega(t) \\ \qquad + B_i u^A(t) + B_{di}u^B(t-d_2(t)), \\ y(t) = C_i x(t), \\ x(t) = \varphi(t), \quad \forall t \in [-\bar{d}, 0]. \end{cases} \tag{3}$$

The controllers considered in this paper are described by:

$$u^A(t) = \bar{K}_1 x(t), \quad u^B(t) = \bar{K}_2 x(t), \tag{4}$$

where $\bar{K}_1, \bar{K}_2 \in \mathbb{R}^{m\times n}$ are controller gain matrices.

Substituting (4) into (3), system (3) can be rewritten as:

$$\begin{cases} \dot{x}(t) = (A_i + B_i\bar{K}_1)x(t) + A_{di}x(t-d_1(t)) + W_i\omega(t) \\ \qquad + B_{di}\bar{K}_2 x(t-d_2(t)), \\ y(t) = C_i x(t), \\ x(t) = \varphi(t), \quad \forall t \in [-\bar{d}, 0]. \end{cases} \tag{5}$$

For deriving the main results of this paper, the following definition and lemmas are required.

Definition: System (5) is said to be robustly stochastically passive if there exists a positive scalar $\gamma > 0$ under zero initial condition for any $\omega \in \mathfrak{L}_2[0,\infty)$, all solutions of (5) with $\varphi(t) = 0$, $t \in [\bar{d}, 0]$ and any $\bar{K}_1, \bar{K}_2$, such that

$$\varepsilon\{2\int_0^t y^T(s)\omega(s)\,ds\} \ge -\gamma\int_0^t \omega^T(s)\omega(s)ds, \quad \forall t > 0. \tag{6}$$

Lemma 1 (Schur Complement): For a given symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, the following statements are equivalent:

(i) $S < 0$;

(ii) $S_{11} < 0, \quad S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(iii) $S_{22} < 0, \quad S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

Lemma 2 [19]: Given matrices Γ , Λ and symmetric matrix Υ with appropriate dimensions,

$$\Upsilon + \Gamma F(k)\Lambda + \Lambda^T F^T(k)\Gamma^T < 0$$

holds for any $F^T(k)F(k) \le I$, if and only if there exists a scale $\alpha > 0$ such that

$$\Upsilon + \alpha\Gamma\Gamma^T + \alpha^{-1}\Lambda^T\Lambda < 0.$$

Lemma 3 [24]: For any constant symmetric matrix $X \in \mathbb{R}^{n\times n}$, $X = X^T > 0$, scalar $\gamma > 0$, vector function $\omega : [0, \gamma] \to \mathbb{R}^n$, the integrations are satisfied as follows:

$$[\int_0^\gamma \omega(s)\,ds]^T X[\int_0^\gamma \omega(s)\,ds] \le \gamma[\int_0^\gamma \omega^T(s)X\omega(s)\,ds].$$

III. STOCHASTIC PASSIVITY ANALYSIS

In this section, a sufficient condition for stochastic passivity of system (5) is given as follows:

Theorem 1: For given positive scalars δ_1, δ_2 and δ_3, system (5) is stochasticly passive if there exist matrices $P_i > 0$, $Q_x > 0$, $Q_u > 0$, $R_x > 0$, $R_u > 0$, invertible matrix N with appropriate dimension, and a scalar $\gamma > 0$, such that for all $i \in S$

$$\Xi = \begin{bmatrix} \Phi_{1,1} & \Phi_{1,2} & \Phi_{1,3} & \Phi_{1,4} & \Phi_{1,5} \\ * & \Phi_{2,2} & \Phi_{2,3} & \Phi_{2,4} & \Phi_{2,5} \\ * & * & \Phi_{3,3} & \Phi_{3,4} & \Phi_{3,5} \\ * & * & * & \Phi_{4,4} & \Phi_{4,5} \\ * & * & * & * & \Phi_{5,5} \end{bmatrix} < 0 \quad (7)$$

where

$$\begin{aligned} \Phi_{1,1} &= \sum_{j=1}^{s} \pi_{ij} P_j + Q_x + Q_u - R_x - R_u + \delta_1 N A_i \\ &\quad + \delta_1 A_i^T N^T + \delta_1 N B_i \bar{K}_1 + \delta_1 \bar{K}_1^T B_i^T N^T, \\ \Phi_{1,2} &= P_i - \delta_1 N + A_i^T N^T + \bar{K}_1^T B_i^T N^T, \\ \Phi_{1,3} &= R_x + \delta_1 N A_{di} + \delta_2 A_i^T N^T + \delta_2 \bar{K}_1^T B_i^T N^T, \\ \Phi_{1,4} &= R_u + \delta_1 N B_{di} \bar{K}_2 + \delta_3 A_i^T N^T + \delta_3 \bar{K}_1^T B_i^T N^T, \\ \Phi_{1,5} &= \delta_1 N W_i - C_i, \\ \Phi_{2,2} &= \bar{d}_1^2 R_x + \bar{d}_2^2 R_u - N - N^T, \\ \Phi_{2,3} &= N A_{di} - \delta_2 N^T, \\ \Phi_{2,4} &= N B_{di} \bar{K}_2 - \delta_3 N^T, \\ \Phi_{2,5} &= N W_i, \\ \Phi_{3,3} &= -(1 - h_x) Q_x - R_x + \delta_2 N A_{di} + \delta_2 A_{di}^T N^T, \\ \Phi_{3,4} &= \delta_2 N B_{di} \bar{K}_2 + \delta_3 A_{di}^T N^T, \\ \Phi_{3,5} &= \delta_2 N W_i, \\ \Phi_{4,4} &= -(1 - h_u) Q_u - R_u + \delta_3 N B_{di} \bar{K}_2 + \delta_3 \bar{K}_2^T B_{di}^T N^T, \\ \Phi_{4,5} &= \delta_3 N W_i, \\ \Phi_{5,5} &= -\gamma I. \end{aligned}$$

Proof: We define a Lyapunov-Krasovskii functional for system (5) as follows:

$$V(r(t), t) = \sum_{m=1}^{5} V_m(r(t), t), \qquad r(t) = i \in S \quad (8)$$

where

$$\begin{aligned} V_1(r(t), t) &= x^T(t) P_i x(t), \\ V_2(r(t, t)) &= \int_{t-d_1(t)}^{t} x^T(s) Q_x x(s)\, ds, \\ V_3(r(t), t) &= \bar{d}_1 \int_{-\bar{d}_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_x \dot{x}(s)\, ds\, d\theta, \\ V_4(r(t), t) &= \int_{t-d_2(t)}^{t} x^T(s) Q_u x(s)\, ds, \\ V_5(r(t), t) &= \bar{d}_2 \int_{-\bar{d}_2}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_u \dot{x}(s)\, ds\, d\theta, \end{aligned}$$

where $P_i > 0$, $Q_u > 0$,$Q_x > 0$, $R_u > 0$ and $R_x > 0$, we can find out that $V(r(t), t) > 0$. Then, for any $i \in S$, we have

$$\mathfrak{A} V(r(t), t) = \sum_{m=1}^{5} \mathfrak{A} V_m(r(t), t), \quad (9)$$

$$\begin{aligned} \mathfrak{A} V_1(r(t), t) &= x^T(t) P_i \dot{x}(t) + \dot{x}^T(t) P_i x(t) \\ &\quad + x^T(t)[\sum_{j=1}^{s} \pi_{ij} P_j] x(t), \\ \mathfrak{A} V_2(r(t), t) &= x^T(t) Q_x x(t) \\ &\quad - (1 - \dot{d}_1(t)) x^T(t - d_1(t)) Q_x x(t - d_1(t)) \\ &\le x^T(t) Q_x x(t) \\ &\quad - (1 - h_x) x^T(t - d_1(t)) Q_x x(t - d_1(t)), \\ \mathfrak{A} V_3(r(t), t) &= \bar{d}_1^2 \dot{x}^T(t) R_x \dot{x}(t) - \bar{d}_1 \int_{t-\bar{d}_1}^{t} \dot{x}^T(s) R_x \dot{x}(s)\, ds \\ &\le \bar{d}_1^2 \dot{x}^T(t) R_x \dot{x}(t) \\ &\quad - d_1(t) \int_{t-d_1(t)}^{t} \dot{x}^T(s) R_x \dot{x}(s)\, ds, \\ \mathfrak{A} V_4(r(t), t) &= x^T(t) Q_u x(t) \\ &\quad - (1 - \dot{d}_2(t)) x^T(t - d_2(t)) Q_u x(t - d_2(t)) \\ &\le x^T(t) Q_u x(t) \\ &\quad - (1 - h_u) x^T(t - d_2(t)) Q_u x(t - d_2(t)), \\ \mathfrak{A} V_5(r(t), t) &= \bar{d}_2^2 \dot{x}^T(t) R_u \dot{x}(t) - \bar{d}_2 \int_{t-\bar{d}_2}^{t} \dot{x}^T(s) R_u \dot{x}(s)\, ds \\ &\le \bar{d}_2^2 \dot{x}^T(t) R_u \dot{x}(t) \\ &\quad - d_2(t) \int_{t-d_2(t)}^{t} \dot{x}^T(s) R_u \dot{x}(s)\, ds, \end{aligned} \quad (10)$$

where $\mathfrak{A}$ is the weak infinitesimal operator of the random process $\{x(t), r(t), t \ge 0\}$.

Accoding to **Lemama 3**, we can obtain

$$\begin{aligned} \mathfrak{A} V_3(r(t), t) &\le \bar{d}_1^2 \dot{x}^T(t) R_x \dot{x}(t) \\ &\quad - [\int_{t-d_1(t)}^{t} \dot{x}^T(s)\, ds] R_x [\int_{t-d_1(t)}^{t} \dot{x}(s)\, ds] \\ &= \bar{d}_1^2 \dot{x}^T(t) R_x \dot{x}(t) - [x^T(t) \quad x^T(t - d_1(t))] \\ &\quad \cdot \begin{bmatrix} R_x & -R_x \\ * & R_x \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - d_1(t)) \end{bmatrix}, \\ \mathfrak{A} V_5(r(t), t) &\le \bar{d}_2^2 \dot{x}^T(t) R_u \dot{x}(t) \\ &\quad - [\int_{t-d_2(t)}^{t} \dot{x}^T(s)\, ds] R_u [\int_{t-d_2(t)}^{t} \dot{x}(s)\, ds] \\ &= \bar{d}_2^2 \dot{x}^T(t) R_u \dot{x}(t) - [x^T(t) \quad x^T(t - d_2(t))] \\ &\quad \cdot \begin{bmatrix} R_u & -R_u \\ * & R_u \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - d_2(t)) \end{bmatrix}. \end{aligned}$$

Simultaneously, for invertible matrix N with appropriate dimension, positive scalars δ_1, δ_2 and δ_3, we employ the free-weighting matrix approach to get the following equation

$$\begin{aligned} 0 &= 2[-x^T(t) \delta_1 N - \dot{x}^T(t) N - x^T(t - d_1(t)) \delta_2 N \\ &\quad - x^T(t - d_2(t)) \delta_3 N] \cdot [\dot{x}(t) - A_i x(t) - B_i \bar{K}_1 x(t) \\ &\quad - A_{di} x(t - d_1(t)) - B_{di} \bar{K}_2 x(t - d_2(t)) - W_i \omega(t)] \end{aligned} \quad (11)$$

Denote $\xi(t) = [x^T(t)\ \dot{x}^T(t)\ x^T(t-d_1(t))\ x^T(t-d_2(t))\ \omega^T(t)]$, thus, we can obtain the following result from above analysis

$$\mathfrak{A}V(r(t),t) \leq \xi(t)\bar{\Xi}\xi^T(t), \tag{12}$$

where

$$\bar{\Xi} = \begin{bmatrix} \Phi_{1,1} & \Phi_{1,2} & \Phi_{1,3} & \Phi_{1,4} & \delta_1 NW_i \\ * & \Phi_{2,2} & \Phi_{2,3} & \Phi_{2,4} & \Phi_{2,5} \\ * & * & \Phi_{3,3} & \Phi_{3,4} & \Phi_{3,5} \\ * & * & * & \Phi_{4,4} & \Phi_{4,5} \\ * & * & * & * & 0 \end{bmatrix}$$

We denote that

$$H(r(t),t) = \mathfrak{A}V(r(t),t) - 2y^T(t)\omega(t) - \gamma\omega^T(t)\omega(t),$$

the following inequation is ture

$$H(r(t),t) \leq \xi(t)\Xi\xi^T(t) < 0.$$

Obviously, when $\omega(t) = 0$, system (5) is robustly stable. Moreover, under the zero initial condition for any $t > 0$,

$$\begin{aligned} &\varepsilon\{2\int_0^t y^T(s)\omega(s)\,ds\} \\ &\quad = \varepsilon\{\int_0^t [\mathfrak{A}V(r(s),s) - \gamma\omega^T(s)\omega(s) \\ &\qquad - H(r(s),s)]\,ds\} \\ &\quad \geq \varepsilon\{\int_0^t [\mathfrak{A}V(r(s),s) - \gamma\omega^T(s)\omega(s)]\,ds\} \\ &\quad = \varepsilon\{V(r(t),t)\} - \varepsilon\{V(r(0),0)\} \\ &\qquad - \gamma\int_0^t \omega^T(s)\omega(s)\,ds \\ &\quad \geq -\gamma\int_0^t \omega^T(s)\omega(s)\,ds \end{aligned} \tag{13}$$

According to **Definition**, system (5) is stochastically passive.

IV. HYBRID CONTROLLER DESIGN

In this section, the controller gain matrices of the robust controllers and the fault-tolerant controllers can be obtained by LMI technology.

- **Case 1:** For systems (5) without actuator faults, the controller gain matrices are designed as $\bar{K}_1 = K_{i1}$ and $\bar{K}_2 = K_{i2}$, thus, the systems (5) can be rewritten as

$$\begin{cases} \dot{x}(t) = (A_i + B_iK_{i1})x(t) + A_{di}x(t-d_1(t)) + W_i\omega(t) \\ \qquad + B_{di}K_{i2}x(t-d_2(t)), \\ y(t) = C_ix(t), \\ x(t) = \varphi(t), \quad \forall t \in [-\bar{d}, 0]. \end{cases} \tag{14}$$

Theorem 2: For given positive scalars δ_1, δ_2 and δ_3, systems (14) are stochasticly passive if there exist matrices $\hat{P}_i > 0$, $\hat{Q}_x > 0$, $\hat{Q}_u > 0$, $\hat{R}_x > 0$, $\hat{R}_u > 0$, invertible matrix $\hat{N}$ with appropriate dimension, and a scalar $\gamma > 0$, such that for all $i \in S$

$$\bar{\Xi} = \begin{bmatrix} \bar{\Phi}_{1,1} & \bar{\Phi}_{1,2} & \bar{\Phi}_{1,3} & \bar{\Phi}_{1,4} & \bar{\Phi}_{1,5} \\ * & \bar{\Phi}_{2,2} & \bar{\Phi}_{2,3} & \bar{\Phi}_{2,4} & \bar{\Phi}_{2,5} \\ * & * & \bar{\Phi}_{3,3} & \bar{\Phi}_{3,4} & \bar{\Phi}_{3,5} \\ * & * & * & \bar{\Phi}_{4,4} & \bar{\Phi}_{4,5} \\ * & * & * & * & \Phi_{5,5} \end{bmatrix} < 0 \tag{15}$$

where

$$\begin{aligned} \bar{\Phi}_{1,1} &= \sum_{j=1}^{s}\pi_{ij}\hat{P}_j + \hat{Q}_x + \hat{Q}_u - \hat{R}_x - \hat{R}_u + \delta_1 A_i\hat{N}^T \\ &\quad + \delta_1\hat{N}A_i^T + \delta_1 B_iY_{i1} + \delta_1 Y_{i1}^TB_i^T, \\ \bar{\Phi}_{1,2} &= \hat{P}_i - \delta_1\hat{N}^T + \hat{N}A_i^T + Y_{i1}^TB_i^T, \\ \bar{\Phi}_{1,3} &= \hat{R}_x + \delta_1 A_{di}\hat{N}^T + \delta_2\hat{N}A_i^T + \delta_2 Y_{i1}^TB_i^T, \\ \bar{\Phi}_{1,4} &= \hat{R}_u + \delta_1 B_{di}Y_{i2} + \delta_3\hat{N}A_i^T + \delta_3 Y_{i1}^TB_i^T, \\ \bar{\Phi}_{1,5} &= \delta_1 W_i - \hat{N}C_i, \\ \bar{\Phi}_{2,2} &= \bar{d}_1^2\hat{R}_x + \bar{d}_2^2\hat{R}_u - \hat{N} - \hat{N}^T, \\ \bar{\Phi}_{2,3} &= A_{di}\hat{N}^T - \delta_2\hat{N}, \\ \bar{\Phi}_{2,4} &= B_{di}Y_{i2} - \delta_3\hat{N}, \\ \bar{\Phi}_{2,5} &= W_i, \\ \bar{\Phi}_{3,3} &= -(1-h_x)\hat{Q}_x - \hat{R}_x + \delta_2 A_{di}\hat{N}^T + \delta_2\hat{N}A_{di}^T, \\ \bar{\Phi}_{3,4} &= \delta_2 B_{di}Y_{i2} + \delta_3\hat{N}A_{di}^T, \\ \bar{\Phi}_{3,5} &= \delta_2 W_i, \\ \bar{\Phi}_{4,4} &= -(1-h_u)\hat{Q}_u - \hat{R}_u + \delta_3 B_{di}Y_{i2} + \delta_3 Y_{i2}^TB_{di}^T, \\ \bar{\Phi}_{4,5} &= \delta_3 W_i, \end{aligned}$$

with robust controller gain matrices are $K_{i1} = Y_{i1}\hat{N}^{-T}$ and $K_{i2} = Y_{i2}\hat{N}^{-T}$.

Proof: substituting $\bar{K}_1 = K_{i1}$ and $\bar{K}_2 = K_{i2}$ into ineuqation (7), then, pre- and post-multiplying by $diag\{\hat{N}, \hat{N}, \hat{N}, \hat{N}, I\}$ and $diag\{\hat{N}^T, \hat{N}^T, \hat{N}^T, \hat{N}^T, I\}$, respectively. Denote

$$\begin{aligned} \hat{N} &= N^{-1}, \quad \hat{N}P_i\hat{N}^T = \hat{P}_i, \quad \hat{N}Q_x\hat{N}^T = \hat{Q}_x, \\ \hat{N}Q_u\hat{N}^T &= \hat{Q}_u, \quad \hat{N}R_x\hat{N}^T = \hat{R}_x, \quad \hat{N}R_u\hat{N}^T = \hat{R}_u, \end{aligned}$$

we can obtain (15), easily. According to **Theorem 1**, system (14) is stochastically passive, the proof is complete.

- **Case 2:** For systems (5) with actuator faults, the controller gain matrices are designed as $\bar{K}_1 = M_1K_{i3}$ and $\bar{K}_2 = M_2K_{i4}$, where $M_1 = diag\{m_{1,1}, m_{1,2}, \ldots, m_{1,m}\}$, $M_2 = diag\{m_{2,1}, m_{2,2}, \ldots, m_{2,m}\}$ are actuator failure matrices and satify

$$0 \leq m_{ji}^{min} \leq m_{ji} \leq m_{ji}^{max} \leq 1, \quad j = 1, 2,\ i = 1, 2, \ldots, m \tag{16}$$

when $m_{ji} = 1$, the i^{th} actuator of j^{th} controller is running under normal conditions, $m_{ji} = 0$ represents that the i^{th} actuator of j^{th} controller is completely disabled, $0 < m_{ji} < 1$ means the i^{th} actuator of j^{th} controller has partial failure. Because M_1 and M_2 are unknown matrices, we define

$M_j = M_{0j}(I+G_j)$and $|G_j| \leq H_j \leq I$ to relax the conservatism, where

$$\begin{aligned} m_{0ji} &= \frac{m_{ji}^{max} + m_{ji}^{min}}{2}, \quad h_{ji} = \frac{m_{ji}^{max} + m_{ji}^{min}}{m_{ji}^{max} - m_{ji}^{min}}, \\ g_{ji} &= \frac{m_{ji} - m_{0ji}}{m_{0ji}}, \quad j = 1, 2, \ i = 1, 2, \ldots, m, \\ M_{0j} &= diag\{m_{0j1}, m_{0j2}, \ldots, m_{0jm}\}, \\ H_j &= diag\{h_{j1}, h_{j2}, \ldots, h_{jm}\}, \\ G_j &= diag\{g_{j1}, g_{j2}, \ldots, g_{jm}\}, \\ |G_j| &= diag\{|g_{j1}|, |g_{j2}|, \ldots, |g_{jm}|\}, \end{aligned} \tag{17}$$

then the systems (5) can be rewritten as

$$\begin{cases} \dot{x}(t) = (A_i + B_i M_1 K_{i3})x(t) + A_{di}x(t - d_1(t)) \\ \qquad + B_{di}M_2 K_{i4}x(t - d_2(t)) + W_i\omega(t), \\ y(t) = C_i x(t), \\ x(t) = \varphi(t), \quad \forall t \in [-\bar{d}, 0]. \end{cases} \tag{18}$$

Theorem 3: For given postive scalars δ_1,δ_2 and δ_3, systems (18) are stochastically passive if there exist matrices $\hat{P}'_i > 0$, $\hat{Q}'_x > 0$, $\hat{Q}'_u > 0$, $\hat{R}'_x > 0$, $\hat{R}'_u > 0$, invertible matrix $\hat{N}'$ with appropriate dimension, scalars $\gamma > 0$ and $\varepsilon_m > 0$, $m = 1, 2, \ldots, 10$, such that for all $i \in S$

$$\bar{\bar{\Xi}} = \begin{bmatrix} \Theta_{1,1} & \Theta_{1,2} & \Theta_{1,3} \\ * & -\Theta_{2,2} & 0^{5\times 5} \\ * & * & -\Theta_{3,3} \end{bmatrix} < 0 \tag{19}$$

where

$$\Theta_{1,1} = \begin{bmatrix} \tilde{\Phi}_{1,1} & \tilde{\Phi}_{1,2} & \tilde{\Phi}_{1,3} & \tilde{\Phi}_{1,4} & \tilde{\Phi}_{1,5} \\ * & \tilde{\Phi}_{2,2} & \tilde{\Phi}_{2,3} & \tilde{\Phi}_{2,4} & \bar{\Phi}_{2,5} \\ * & * & \tilde{\Phi}_{3,3} & \tilde{\Phi}_{3,4} & \bar{\Phi}_{3,5} \\ * & * & * & \tilde{\Phi}_{4,4} & \bar{\Phi}_{4,5} \\ * & * & * & * & \tilde{\Phi}_{5,5} \end{bmatrix},$$

$$\Theta_{1,2} = \begin{bmatrix} \tilde{\Phi}_{1,6} & \tilde{\Phi}_{1,7} & \tilde{\Phi}_{1,8} & \tilde{\Phi}_{1,9} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Theta_{1,3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \tilde{\Phi}_{4,11} & \tilde{\Phi}_{4,12} & \tilde{\Phi}_{4,13} & \tilde{\Phi}_{4,14} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} \Theta_{2,2} &= diag\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}, \\ \Theta_{3,3} &= diag\{\varepsilon_6, \varepsilon_7, \varepsilon_8, \varepsilon_9, \varepsilon_{10}\}, \\ \tilde{\Phi}_{1,1} &= \sum_{j=1}^{s} \pi_{ij}\hat{P}'_j + \hat{Q}'_x + \hat{Q}'_u - \hat{R}'_x - \hat{R}'_u + \delta_1 A_i \hat{N}'^T \\ &\quad + \delta_1 \hat{N}' A_i^T + \delta_1 B_i M_{01} Y_{i3} + \delta_1 Y_{i3}^T M_{01}^T B_i^T \\ &\quad + \varepsilon_1 B_i H_1 H_1^T B_i^T + \varepsilon_6 B_{di} H_2 H_2^T B_{di}^T, \\ \tilde{\Phi}_{1,2} &= \hat{P}'_i - \delta_1 \hat{N}'^T + \hat{N}' A_i^T + Y_{i3}^T M_{01}^T B_i^T, \\ \tilde{\Phi}_{1,3} &= \hat{R}'_x + \delta_1 A_{di} \hat{N}'^T + \delta_2 \hat{N}' A_i^T + \delta_2 Y_{i3}^T M_{01}^T B_i^T, \\ \tilde{\Phi}_{1,4} &= \hat{R}'_u + \delta_1 B_{di} M_{02} Y_{i4} + \delta_3 \hat{N}' A_i^T + \delta_3 Y_{i3}^T M_{01}^T B_i^T, \\ \tilde{\Phi}_{1,5} &= \delta_1 W_i - \hat{N}' C_i, \\ \tilde{\Phi}_{2,2} &= \bar{d}_1^2 \hat{R}'_x + \bar{d}_2^2 \hat{R}'_u - \hat{N}' - \hat{N}'^T \\ &\quad + \varepsilon_2 B_i H_1 H_1^T B_i^T + \varepsilon_7 B_{di} H_2 H_2^T B_{di}^T, \\ \tilde{\Phi}_{2,3} &= A_{di} \hat{N}'^T - \delta_2 \hat{N}', \\ \tilde{\Phi}_{2,4} &= B_{di} M_{02} Y_{i4} - \delta_3 \hat{N}', \\ \tilde{\Phi}_{3,3} &= -(1 - h_x)\hat{Q}'_x - \hat{R}'_x + \delta_2 A_{di} \hat{N}'^T + \delta_2 \hat{N}' A_{di}^T \\ &\quad + \varepsilon_3 B_i H_1 H_1^T B_i^T + \varepsilon_8 B_{di} H_2 H_2^T B_{di}^T, \\ \tilde{\Phi}_{3,4} &= \delta_2 B_{di} M_{02} Y_{i4} + \delta_3 \hat{N}' A_{di}^T, \\ \tilde{\Phi}_{4,4} &= -(1 - h_u)\hat{Q}'_u + \delta_3 B_{di} M_{02} Y_{i4} + \delta_3 Y_{i4}^T M_{02}^T B_{di}^T \\ &\quad + \varepsilon_4 B_i H_1 H_1^T B_i^T + \varepsilon_9 B_{di} H_2 H_2^T B_{di}^T - \hat{R}'_u, \\ \tilde{\Phi}_{5,5} &= -\gamma I + \varepsilon_5 B_i H_1 H_1^T B_i^T + \varepsilon_{10} B_{di} H_2 H_2^T B_{di}^T, \\ \tilde{\Phi}_{1,6} &= \delta_1 Y_{i3}^T M_{01}^T, \quad \tilde{\Phi}_{1,7} = Y_{i3}^T M_{01}^T, \\ \tilde{\Phi}_{1,8} &= \delta_2 Y_{i3}^T M_{01}^T, \quad \tilde{\Phi}_{1,9} = \delta_3 Y_{i3}^T M_{01}^T, \\ \tilde{\Phi}_{4,11} &= \delta_1 Y_{i4}^T M_{02}^T, \quad \tilde{\Phi}_{4,12} = Y_{i4}^T M_{02}^T, \\ \tilde{\Phi}_{4,13} &= \delta_2 Y_{i4}^T M_{02}^T, \quad \tilde{\Phi}_{4,14} = \delta_3 Y_{i4}^T M_{02}^T, \end{aligned}$$

with the fault-tolerant controller gain matrices are $K_{i3} = Y_{i3}\hat{N}'^{-T}$ and $K_{i4} = Y_{i4}\hat{N}'^{-T}$.

Proof: We substitute $\bar{K}_1 = M_1 K_{i3}$ and $\bar{K}_2 = M_2 K_{i4}$ into ineuqation (7). In order to avoid confusion, replace N by N', then, pre- and post-multiply by $diag\{\hat{N}', \hat{N}', \hat{N}', \hat{N}', I\}$ and $diag\{\hat{N}'^T, \hat{N}'^T, \hat{N}'^T, \hat{N}'^T, I\}$, respectively. Denote

$$\begin{aligned} &\hat{N}' = N'^{-1}, \quad \hat{N}' P_i \hat{N}'^T = \hat{P}'_i, \quad \hat{N}' Q_x \hat{N}'^T = \hat{Q}'_x, \\ &\hat{N}' Q_u \hat{N}'^T = \hat{Q}'_u, \quad \hat{N}' R_x \hat{N}'^T = \hat{R}'_x, \quad \hat{N}' R_u \hat{N}'^T = \hat{R}'_u, \end{aligned}$$

we obtain

$$\breve{\Xi} = \begin{bmatrix} \check{\Phi}_{1,1} & \check{\Phi}_{1,2} & \check{\Phi}_{1,3} & \check{\Phi}_{1,4} & \check{\Phi}_{1,5} \\ * & \check{\Phi}_{2,2} & \check{\Phi}_{2,3} & \check{\Phi}_{2,4} & \bar{\Phi}_{2,5} \\ * & * & \check{\Phi}_{3,3} & \check{\Phi}_{3,4} & \bar{\Phi}_{3,5} \\ * & * & * & \check{\Phi}_{4,4} & \bar{\Phi}_{4,5} \\ * & * & * & * & \Phi_{5,5} \end{bmatrix} < 0 \tag{20}$$

where

$$\begin{aligned} \check{\Phi}_{1,1} &= \sum_{j=1}^{s} \pi_{ij}\hat{P}'_j + \hat{Q}'_x + \hat{Q}'_u - \hat{R}'_x - \hat{R}'_u + \delta_1 A_i \hat{N}'^T \\ &\quad + \delta_1 \hat{N}' A_i^T + \delta_1 B_i M_1 Y_{i3} + \delta_1 Y_{i3}^T M_1^T B_i^T, \\ \check{\Phi}_{1,2} &= \hat{P}'_i - \delta_1 \hat{N}'^T + \hat{N}' A_i^T + Y_{i3}^T M_1^T B_i^T, \\ \check{\Phi}_{1,3} &= \hat{R}'_x + \delta_1 A_{di} \hat{N}'^T + \delta_2 \hat{N}' A_i^T + \delta_2 Y_{i3}^T M_1^T B_i^T, \\ \check{\Phi}_{1,4} &= \hat{R}'_u + \delta_1 B_{di} M_2 Y_{i4} + \delta_3 \hat{N}' A_i^T + \delta_3 Y_{i3}^T M_1^T B_i^T, \\ \check{\Phi}_{1,5} &= \delta_1 W_i - \hat{N}' C_i, \\ \check{\Phi}_{2,2} &= \bar{d}_1^2 \hat{R}'_x + \bar{d}_2^2 \hat{R}'_u - \hat{N}' - \hat{N}'^T, \\ \check{\Phi}_{2,3} &= A_{di} \hat{N}'^T - \delta_2 \hat{N}', \\ \check{\Phi}_{2,4} &= B_{di} M_2 Y_{i4} - \delta_3 \hat{N}', \\ \check{\Phi}_{3,3} &= -(1 - h_x)\hat{Q}'_x - \hat{R}'_x + \delta_2 A_{di} \hat{N}'^T + \delta_2 \hat{N}' A_{di}^T, \\ \check{\Phi}_{3,4} &= \delta_2 B_{di} M_2 Y_{i4} + \delta_3 \hat{N}' A_{di}^T, \\ \check{\Phi}_{4,4} &= -(1 - h_u)\hat{Q}'_u - \hat{R}'_u + \delta_3 B_{di} M_2 Y_{i4} + \delta_3 Y_{i4}^T M_2^T B_{di}^T. \end{aligned}$$

Because $M_1 = M_{01}(I + G_1)$, $M_2 = M_{02}(I + G_2)$, $|G_1| \le H_1 \le I$ and $|G_2| \le H_2 \le I$, we obtain the following inequation

$$\check{\Xi} \le \Theta_{1,1} + \Gamma_1 F(k)\Lambda_1 + \Lambda_1^T F^T(k)\Gamma_1^T + \Gamma_2 F(k)\Lambda_2 + \Lambda_2^T F^T(k)\Gamma_2^T \quad (21)$$

where

$$\Gamma_1^T = diag\{H_1^T B_i^T, H_1^T B_i^T, H_1^T B_i^T, H_1^T B_i^T, H_1^T B_i^T\},$$
$$\Gamma_2^T = diag\{H_2^T B_{di}^T, H_2^T B_{di}^T, H_2^T B_{di}^T, H_2^T B_{di}^T, H_2^T B_{di}^T\},$$
$$\Lambda_1^T = \Theta_{1,2}, \Lambda_2^T = \Theta_{1,3}, F(k) = I.$$

According to **Lemma 2**, for $F^T(k)F(k) \le I$, if and only if there exist scalar matrices $\Theta_{2,2} > 0$ and $\Theta_{3,3} > 0$, such that $\Theta_{1,1} + \Theta_{2,2}\Gamma_1\Gamma_1^T + \Theta_{2,2}^{-1}\Lambda_1^T\Lambda_1 + \Theta_{3,3}\Gamma_2\Gamma_2^T + \Theta_{3,3}^{-1}\Lambda_2^T\Lambda_2 < 0$, $\check{\Xi} \le \Gamma_1 F(k)\Lambda_1 + \Lambda_1^T F^T(k)\Gamma_1^T + \Gamma_2 F(k)\Lambda_2 + \Lambda_2^T F^T(k)\Gamma_2^T < 0$ holds.

Thus, based on **Lemma 1**, inequation (19) can be obtained. The proof is complete.

V. DESIGN OF A SWITCHING SYSTEM WITH A FAULT ALARM

To invoke the fault-tolerant controller timely and accurately, we design a residual observer to obtain the real-time estimated value of $y(t)$ under normal condition in this section as follows:

$$\begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + A_{di}\hat{x}(t-\bar{d}_1) + B_i u^a(t) \\ \qquad + B_{di}u^b(t-d_2(t)) + L_i(y(t) - C_i\hat{x}(t)), \\ \hat{y}(t) = C_i\hat{x}(t), \\ \hat{x}(t) = 0, \quad \forall t \in [-\bar{d}, 0]. \end{cases} \quad (22)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the observer state, $u^a(t)$ and $u^b(t) \in \mathbb{R}^m$, which can be measured, are inputs from the controller to the actuator, $L_i \in \mathbb{R}^{n\times p}$ is the gain matrix of the observer and $\hat{y}(t) \in \mathbb{R}^p$ is the observer output.

Thus, we define the state residual value of the observer as

$$\tilde{x}(t) = x(t) - \hat{x}(t), \quad (23)$$

and from system (5) and system (22), we obtain

$$\dot{\tilde{x}}(t) = (A_i - L_iC_i)\tilde{x}(t) + A_{di}(x(t-d_1(t)) - \hat{x}(t-\bar{d}_1)) + B_i(u^A(t) - u^a(t)) + W_i\omega(t) + B_{di}(u^B(t-d_2(t)) - u^b(t-d_2(t))). \quad (24)$$

When the actuators run normally, i.e., $u^A(t) = u^a(t)$ and $u^B(t) = u^b(t)$, equation (24) can be rewritten as follows:

$$\dot{\tilde{x}}(t) = (A_i - L_iC_i)\tilde{x}(t) + A_{di}(x(t-d_1(t)) - \hat{x}(t-\bar{d}_1)) + W_i\omega(t). \quad (25)$$

Under the normal condition, the system state converges to a small interval around zero; thus, we can consider $A_{di}(x(t-d_1(t))$ as a type of disturbance input. The design algorithm that we employ to obtain the value of L_i, is similar to **Theorem 2**. Thus, the passive observer can be established.

By contrast, if the actuator fault occurs, in other words, $u^A(t) = M_1u^a(t)$ and $u^B(t) = M_2u^b(t)$, then the state residual value of observer will be described by

$$\dot{\tilde{x}}(t) = (A_i - L_iC_i)\tilde{x}(t) + A_{di}(x(t-d_1(t)) - \hat{x}(t-\bar{d}_1)) + B_i(M_1 - I)u^a(t) + B_{di}(M_2 - I)u^b(t-d_2(t)) + W_i\omega(t). \quad (26)$$

When partial failure of actuators become sufficiently problematic, the value of $\tilde{x}(t)$ will diverge, namely, the system state will not be under control, which leads to damage to the system. Therefore, it is necessary to select a threshold to detect fault.

Considering equation (25), when the system state converges to a small interval around zero, $A_{di}(x(t-d_1(t)) - \hat{x}(t-\bar{d}_1))$ can be replaced by $A_{di}\tilde{x}(t)$. Thus, we can obtain the critical relationship between $\tilde{x}(t)$ and $\omega(t)$ as follows:

$$(L_iC_i - A_i - A_{di})\tilde{x}(t) = W_i\bar{\omega}, \quad (27)$$

where $\bar{\omega}$ is the upper bound of $\omega(t)$. Premultiplying by $(L_iC_i - A_i - A_{di})^+$, we obtain

$$\|e(t)\| \le \|\gamma_{th_i}\bar{\omega}\| \le \|\gamma_{th_i}\|\|\bar{\omega}\|, \quad (28)$$

where $\gamma_{th_i} = C_i(L_iC_i - A_i - A_{di})^+W_i$, $e(t) = y(t) - \hat{y}(t)$. Define $J_{th} = \sup_{i\in S}\|\gamma_{th_i}\|\|\bar{\omega}\|$ as the so-called threshold, and use the following logical algorithm for fault detection:

$$S_w(t) = \begin{cases} 0, & \|e(t)\| \le J_{th}, \ normal, \\ 1, & \|e(t)\| > J_{th}, \ switch. \end{cases}$$

Therefore, based on $S_w(t)$ ïijŇthe switching signal, the switching system can be given by

$$u^a(t) = (1 - S_w(t))K_{i1}x(t) + S_w(t)K_{i3}x(t),$$
$$u^b(t) = (1 - S_w(t))K_{i2}x(t) + S_w(t)K_{i4}x(t). \quad (29)$$

Obviously, between robust controllers and fault-tolerant controllers, only one type of controllers could function at one moment.

In addition, we can design an alarm signal to increase the reliability of the system. Employing the Chapman-Kolmogorov equation, we can obtain the probability distribution of all system modes. Using the weighted average method, we can obtain a suitable alarm signal as follows:

$$A_{th} = (\sum_{i=1}^{s} Pr_i\|\gamma_{th_i}\|)\|\bar{\omega}\|, \quad (30)$$

where Pr_i is the weighting of i^{th} mode, and the working principle can be given as follows:

$$A_l(t) = \begin{cases} normal, & \|e(t)\| \le A_{th}, \\ alarm, & \|e(t)\| > A_{th}. \end{cases}$$

When we receive the alarm signal, it is probable that the system (3) is undergoing an actuator fault. Thus, we will pay more attention to the system and distinguish whether it is just a false alarm or not.

VI. NUMERICAL EXAMPLE

In this section, a numerical example will be described to demonstrate the effectiveness of the methods described in the previous sections. Consider the following systems:

$$A_1 = \begin{bmatrix} -0.6 & 1.6 \\ 0 & 1.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.05 & -1.2 \\ -1 & 0.21 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 0.13 & 0.13 \\ 0.25 & 0.2 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -0.6 & 1.3 \\ 0 & -0.9 \end{bmatrix},$$
$$B_{d1} = \begin{bmatrix} -0.04 & -0.6 \\ -0.4 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.8 & 1.7 \\ 0 & 1.5 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} -0.03 & -1.2 \\ -1 & 0.2 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.17 & 0.13 \\ 0.21 & 0.23 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} -0.4 & 1.5 \\ 0 & -0.8 \end{bmatrix}, \quad B_{d2} = \begin{bmatrix} -0.01 & -0.8 \\ -0.6 & 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.7 & 2 \\ 0 & 1.5 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -0.03 & -1 \\ -1 & 0.1 \end{bmatrix},$$
$$W_3 = \begin{bmatrix} 0.17 & 0.1 \\ 0.1 & 0.23 \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} -0.5 & 1.5 \\ 0 & -0.7 \end{bmatrix},$$
$$B_{d3} = \begin{bmatrix} -0.02 & -0.8 \\ -0.6 & 0 \end{bmatrix}, \quad C_1 = C_2 = C_3 = I,$$

The transition rate matrix of the stochastic process $r(t)$ is given as

$$Tr = \begin{bmatrix} -0.2 & 0.1 & 0.1 \\ 0.3 & -0.5 & 0.2 \\ 0.4 & 0.2 & -0.6 \end{bmatrix}.$$

The output response of open-loop system is shown in FIGURE 1; we find that the open-loop system is unstable.

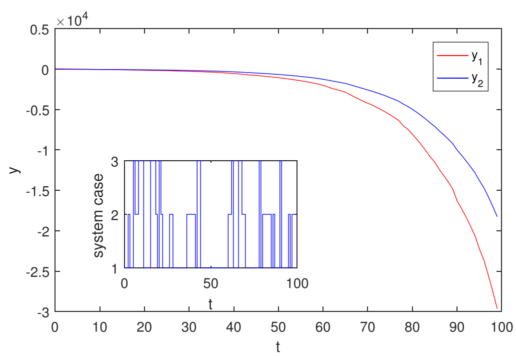


FIGURE 1. Out response of an open-loop system.

By **Theorem 2**, we can obtain the following robust controller gains:

$$K_{11} = \begin{bmatrix} -0.553 & 4.613 \\ 1.080 & 1.873 \end{bmatrix}, \quad K_{12} = \begin{bmatrix} 0.097 & 0.122 \\ 0.207 & 0.053 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} -0.468 & 4.711 \\ 0.863 & 2.073 \end{bmatrix}, \quad K_{22} = \begin{bmatrix} 0.052 & 0.081 \\ 0.176 & 0.047 \end{bmatrix},$$
$$K_{31} = \begin{bmatrix} -0.868 & 4.485 \\ 0.989 & 2.244 \end{bmatrix}, \quad K_{32} = \begin{bmatrix} -0.016 & 0.285 \\ 0.220 & -0.038 \end{bmatrix}.$$

Given $M_{01} = diag\{0.1, 0.1\}$, $M_{02} = diag\{0.3, 0.3\}$, $H_1 = diag\{0.25, 0.25\}$, and $H_2 = diag\{0.33, 0.33\}$, according to **Theorem 3**, the fault-tolerant controller gains for the system with an actuator fault can be obtained as follows:

$$K_{13} = \begin{bmatrix} -1.547 & 29.600 \\ 2.870 & 0.519 \end{bmatrix}, \quad K_{14} = \begin{bmatrix} 0.059 & -0.373 \\ 0.019 & 0.363 \end{bmatrix},$$
$$K_{23} = \begin{bmatrix} -1.613 & 32.226 \\ 2.388 & 2.072 \end{bmatrix}, \quad K_{24} = \begin{bmatrix} 0.041 & -0.254 \\ 0.226 & 0.461 \end{bmatrix},$$
$$K_{33} = \begin{bmatrix} -1.584 & 34.976 \\ 2.707 & 4.950 \end{bmatrix}, \quad K_{34} = \begin{bmatrix} 0.035 & -0.158 \\ 0.299 & 0.627 \end{bmatrix}.$$

With the robust controllers, the output response of the closed-loop system is shown in FIGURE 2. Moreover, when an actuator fault occurs, the closed-loop system becomes unstable.

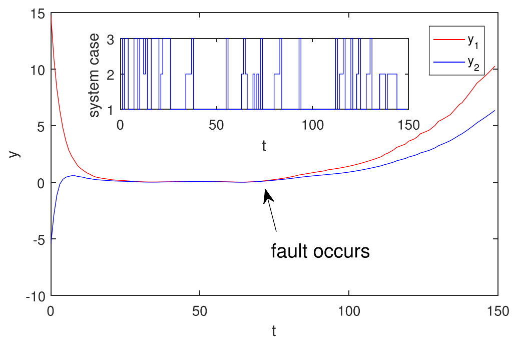


FIGURE 2. Output response of a closed-loop system.

When an actuator fault occurs, with the robust controllers, fault-tolerant controllers and switching signal, the output response of the closed-loop system is exhibited in FIGURE 3.

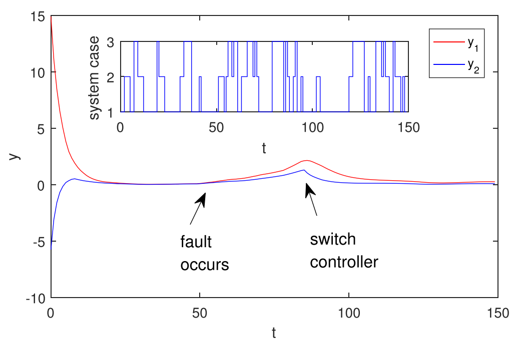


FIGURE 3. Output response of a closed-loop system.

Remark 1: Comparing FIGURE 3 with FIGURE 2, the hybrid control method can effect even if the actuator fault occurs, however, in the same situation, the robust one can not stable the systems. The advantage of hybrid control method is prominented.

Remark 2: To make the switching process more conspicuous, the threshold is increased appropriately. In fact, this process can be completed in very short time after the occurance of the actuator fault and can not be found by vision easily.

VII. CONCLUSION

This paper has investigated passivity and fault-alarm based hybrid control problem for a Markovian jump system with actuator failures. The key to this problem is to design appropriate state feedback controllers that can guarantee that the closed-loop system is robustly stochastically passive. According to the sufficient passive condition, which is established by constructing time-dependent Lyapunov-Krasovskii functional, the controller gains of robust controllers and fault-tolerant controllers can be obtained by employing LMI technology. In addition, a fault alarm-based switching method was designed to switch the controller when an actuator fault occurs, thus maintaining the level of system performance and advoiding the issues of fault-tolerant controllers. Finally, a numerical example was presented to demonstrate the effectiveness of the obtained results.

REFERENCES

[1] X. Mao, A. Matasov, and A. B. Piunovskiy, "Stochastic differential delay equations with Markovian switching," *Bernoulli*, vol. 6, no. 1, pp. 73–90, Feb. 2000.

[2] X. Feng, K. A. Loparo, Y. Ji, and H. J. Chizeck, "Stochastic stability properties of jump linear systems," *IEEE Trans. Autom. Control*, vol. 37, no. 1, pp. 38–53, Jan. 1992.

[3] J. Dong and G.-H. Yang, "Robust H_2 control of continuous-time Markov jump linear systems," *Automatica*, vol. 44, no. 5, pp. 1431–1436, May 2008.

[4] E. K. Boukas, P. Shi, and K. Benjelloun, "On stabilization of uncertain linear systems with jump parameters," *Int. J. Control*, vol. 72, no. 9, pp. 842–850, Jun. 1999.

[5] K. Liang *et al.*, "L_2-L_∞ synchronization for singularly perturbed complex networks with semi-Markov jump topology," *Appl. Math. Comput.*, to be published, doi: 10.1016/j.amc.2017.10.039.

[6] X. Song, Y. Men, J. Zhou, J. Zhao, and H. Shen, "Event-triggered H_∞ control for networked discrete-time Markov jump systems with repeated scalar nonlinearities," *Appl. Math. Comput.*, vol. 298, pp. 123–132, Apr. 2017.

[7] H. Shen, Y. Zhu, L. Zhang, and J. H. Park, "Extended dissipative state estimation for Markov jump neural networks with unreliable links," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 2, pp. 346–358, Feb. 2017.

[8] H. Shen, Y. Men, Z.-G. Wu, and H. P. Ju, "Nonfragile H_∞ control for fuzzy Markovian jump systems under fast sampling singular perturbation," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2017.2758381.

[9] L. Zhang, E. K. Boukas, and J. Lam, "Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities," *IEEE Trans. Autom. Control*, vol. 53, no. 10, pp. 2458–2464, Nov. 2008.

[10] L. Zhang and E. K. Boukas, "Stability and stabilization of Markovian jump linear systems with partly unknown transition probabilities," *Automatica*, vol. 45, no. 2, pp. 463–468, Feb. 2009.

[11] J. Xiong, J. Lam, H. Gao, and D. W. C. Ho, "On robust stabilization of Markovian jump systems with uncertain switching probabilities," *Automatica*, vol. 41, no. 5, pp. 897–903, May 2005.

[12] L. Zhang and E. K. Boukas, "Discrete-time Markovian jump linear systems with partly unknown transition probabilities: H_∞ filtering problem," in *Proc. ACC*, Seattle, WA, USA, Jun. 2008, pp. 2272–2277.

[13] B. Du, J. Lam, Y. Zou, and Z. Shu, "Stability and stabilization for Markovian jump time-delay systems with partially unknown transition rates," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 60, no. 2, pp. 341–351, Feb. 2013.

[14] J. Shi, Y. Yin, and F. Liu, "Robust fault detection for nonlinear discrete-time Markovian jump systems with partly unknown transition probabilities," in *Proc. WCICA*, Guilin, China, Jun. 2016, pp. 721–726.

[15] W.-H. Chen, Z.-H. Guan, and X. Lu, "Passive control synthesis for uncertain Markovian jump systems with multiple mode-dependent time-delays," *Asian J. Control*, vol. 7, no. 2, pp. 135–143, Jun. 2005.

[16] Z. Li, T. Zhang, C. Ma, H. Li, and X. Li, "Robust Passivity Control for 2-D Uncertain Markovian Jump Linear Discrete-Time Systems," *IEEE Access*, vol. 5, pp. 12176–12184, Jun. 2017.

[17] Y. Wang, M. Xia, V. Gupta, and P. J. Antsaklis, "On feedback passivity of discrete-time nonlinear networked control systems with packet drops," *IEEE Trans. Autom. Control*, vol. 60, no. 9, pp. 2434–2439, Sep. 2015.

[18] P. Shi, E. K. Boukas, and R. K. Agarwal, "Control of Markovian jump discrete-time systems with norm bounded uncertainty and unknown delay," *IEEE Trans. Autom. Control*, vol. 44, no. 11, pp. 2139–2144, Nov. 1999.

[19] J.-N. Li, Y.-J. Pan, H.-Y. Su, and C.-L. Wen, "Stochastic reliable control of a class of networked control systems with actuator faults and input saturation," *Int. J. Control, Autom., Syst.*, vol. 12, no. 3, pp. 564–571, Jun. 2014.

[20] Z. Wu, H. Su, and J. Chu, "Delay-dependent H_∞ filtering for singular Markovian jump systems with time delay Systems," *Signal Process.*, vol. 90, no. 6, pp. 1815–1824, Jun. 2010.

[21] Z. Wu, P. Shi, H. Su, and J. Chu, "Delay-dependent stability analysis for switched neural networks with time-varying delay," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 41, no. 6, pp. 1522–1530, Nov. 2011.

[22] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time delays," *IEEE Trans. Neural Netw.*, vol. 22, no. 10, pp. 1566–1575, Oct. 2011.

[23] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Reliable H_∞ control for discrete-time fuzzy systems with infinite-distributed delay," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 1, pp. 22–31, Feb. 2012.

[24] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Exponential synchronization of neural networks with discrete and distributed delays under time-varying sampling," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 9, pp. 1368–1376, Sep. 2012.

[25] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Sampled-data synchronization of chaotic Lur'e systems with time delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 3, pp. 410–421, Mar. 2013.

[26] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Dissipativity analysis for discrete-time stochastic neural networks with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 3, pp. 345–355, Mar. 2013.

[27] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Sampled-data exponential synchronization of complex dynamical networks with time-varying coupling delay," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 8, pp. 1177–1187, Aug. 2013.

[28] B. T. Cui and M. Hua, "RETRACTED: Observer-based passive control of linear time-delay systems with parametric uncertainty," *Chaos, Solitons Fractals*, vol. 32, no. 1, pp. 160–167, Apr. 2007.

[29] Y.-M. Fu and G.-R. Duan, "Stochastic stabilizability and passive control for time-delay systems with Markovian jumping parameters," in *Proc. ICARCV*, Kunming, China, Dec. 2004, pp. 1757–1761.

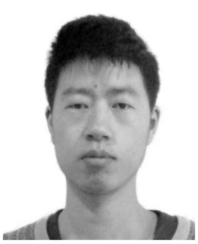
[30] Y. Q. Wu, H. Su, R. Lu, Z. G. Wu, and Z. Shu, "Passivity-based non-fragile control for Markovian jump systems with aperiodic sampling," *Syst. Control Lett.*, vol. 84, pp. 35–43, Oct. 2015.

[31] Y. Wu, H. Su, P. Shi, Z. Shu, and Z.-G. Wu, "Consensus of multiagent systems using aperiodic sampled-data control," *IEEE Trans. Cybern.*, vol. 46, no. 9, pp. 2132–2143, Sep. 2016.

[32] Z.-G. Wu, P. Shi, Z. Shu, H. Su, and R. Lu, "Passivity-based asynchronous control for Markov jump systems," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 2020–2025, Apr. 2016.

JIAN-NING LI received the Ph.D. degree in control science and engineering from Zhejiang University in 2013. He then joined the Institute of Systems Science and Control Engineering, Hangzhou Dianzi University. In 2012, he was a Visiting Student with the Department of Mechanical, Dalhousie University. His research interests include robust control, neural network, networked control system, and fault-tolerant control.

YU-FEI XU received the B.E. degree in automation from Hangzhou Dianzi University, Hangzhou, China, in 2016. He is currently pursuing the M.S. degree in control science and engineering from Hangzhou Dianzi University. His research interests include robust control, neural network, networked control system, and fault-tolerant control.



WEN-DONG BAO received the B.E. degree in electrical engineering and automation from Quzhou University in 2015. He then joined the Institute of Systems Science and Control Engineering, Hangzhou Dianzi University, as a Graduate Student. His research interests include robust control, neural network, networked control system, and fault-tolerant control.

XIAO-BIN XU is currently a Professor with the Department of Automation, Belt and Road Information Technology Research Institute, Hangzhou Dianzi University. His research interests include fuzzy set theory, DS evidence theory and its applications in the processing of uncertain information, the reliability analysis, safety evaluation, and fault diagnosis of complex industrial Systems.