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FORM and Out-Crossing Combined Time-Variant Reliability Analysis Method for Ship Structures

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ABSTRACT In view of the varying characteristics of material properties, environmental conditions, and loading effects randomly with long time, the random and temporal characters of different variables should take into account when the structural reliability analysis methods are adopted to estimate the reliability of ship structures. Therefore, a time-variant reliability analysis method combined the out-crossing approach and the first-order reliability method is proposed. In this paper, the research is conducted on the time-variant feature of ship structures under the corrosive action according to the environmental testing data. Furthermore, the limit state function is derived based on the time-variant feature analysis results in which the strength and stress of ship structures are both regarded as variables with respect to time. Then, the time-dependent reliability model of ship structures is established based on this combined method, and the solving process of parameters in this model is illustrated. Finally, a case of a ship grillage structure under marine environment is given to prove that the proposed method has a good application in evaluating the reliability of ship structures. The comparison analysis using different methods is also conducted to study the accuracy and efficiency of this method.

INDEX TERMS Loading and environmental effects, ship structures, time-variant reliability analysis, out-crossing rate, FORM.

NOMENCLATURE

LIST OF ACRONYMS

FORM	First-Order Reliability Method
FOSM	First-Order Second moment Method
SORM	Second-Order Reliability Method
RSM	Response Surface Method
PDF	Probability Density Function
CDF	Cumulative Distribution Function
SSC	Ship Structure Committee
IACS	International Association of Classification Societies
FoS	Factor of Safety

LIST OF SYMBOLS

$G(t, X)$	Time-dependent Limit State Function
$P_{f,i}$	Instantaneous Failure Probability
$P_{f,c}$	Cumulative Failure Probability
$v^+(t)$	Out-crossing Rate
β	Reliability Index
$\boldsymbol{\alpha}(t)$	Unit Normal Vector of the Limit-state Surface
$\mathbf{u}$	Standard Normal Space
φ	Normal Probability Density Function
Φ	Normal Cumulative Distribution Function
δ	Thickness of I-shaped Section (mm)
B	Width of the Upper and Lower Edge Strip of I-shaped Section (mm)
H	Web Height of I-shaped Section (mm)
b	Web Breadth of I-shaped Section (mm)
M_S	Maximum Still Water Bending Moment (KN)
M_w	Maximum Wave Bending Moment (KN)
M_c	Combined Bending Moment (KN)
L_0	Ship Length (m)
B_0	Ship Molded Breadth (m)
C_B	Block Coefficient
C_w	Wave Coefficient
φ_t	Error Estimation Factor
φ_w	Load Reduction Factor
P	Strength of Ship Structures (MPa)
P_0	Initial Strength of Ship Structures (MPa)

$p(t)$	Time Variability of $P(t)$
S	Stress of Ship Structures (MPa)
W	Section Modulus of Ship Structures (mm^3)
W_0	Initial Section Modulus of Ship Structures (mm^3)
$w(t)$	Time Variability of $W(t)$
μ	expectation of random variables
σ	standard deviation of random variables

I. INTRODUCTION

During the service life of ship structures, loading and natural environment effects, geometry size and material properties are not only random and heterogeneous but also time-variant. For instance, material properties may decay over time and can be presented as the degradation mechanisms. On the other hand, the loading and environmental effects may vary randomly over time and can be described as the stochastic process. Therefore, for more accurate reliability assessment of ship structures, the randomness and time-dependent characteristic of material properties, environmental and loading effects should be taken into account.

Affected by the complex loading conditions and the ocean environment during the life cycle, ship structures are vulnerable to corrosion damage. As a result, the thickness of ship structures will be reduced with time, which leads to the decrease of effective section modulus. Furthermore, the bending strength of the materials will also be reduced over time, which results in the flexural failure of ship structures. All of these factors will cause the significant reduction of the reliability of ship structures over time. Consequently, the section modulus and bending strength of ship structures shall be regarded as the random variables, while the time-variant reliability method should be adopted for this analysis.

In the traditional design of ship structures, safety factor method [1] that is a term to describe the structural capacity of a system beyond the expected load or actual load has been mainly adopted for reliability evaluation, which, however, cannot actually reflect the practical reliability and the results are too conservative. What's more, several analytical algorithms, such as the first-order reliability method (FORM) [2], the first-order second moment method (FOSM) [3], the second-order reliability method (SORM) [4], the maximum entropy method [5] and the response surface method (RSM) [6] have been utilized to analyze the reliability of structural systems. Collectively known as first-order reliability methods based on the explicit definition of performance functions or limit state functions, these methods are commonly used to calculate the failure probability of structural systems [7]–[12]. Among them, FORM is utilized to calculate the reliability via adopting the first-order Taylor series approximation of a performance function, where all probabilistic design variables are transformed into the independent and standard normal distributions [2]–[3]. However, the temporal elements are not fully taken into consideration in those traditional methods, which means that the strength and stress of ship structures might be time-invariant.

Focusing on the time-variant reliability problems, the out-crossing approach has been widely utilized and improved over these years. According to this theory, the probability of failure is related to the out-crossings' mean number of the random process through the limit surface and the key of this approach is to calculate the out-crossing rate [13]. There are several methods for the calculation of out-crossing rate: analytical method with Rice's formula, asymptotic method [14], and an alternative approach based on system reliability. The system reliability theory, introduced by Hagen and Tvedt [15], suggests a parallel system reliability formulation for computing the out-crossing rate. Andrieu-Renaud et al. [16] performed the time-variant reliability analysis with the PHI2 method which enables the classical time-independent reliability methods, such as FORM and SORM, to solve the time-dependent problems. However, the disadvantages of PHI2 are that only a limited upper bound of the system reliability is provided and computational error could occur because of the nonlinearity of limit states. Sudret et al. [17], [18] developed a new formula based on the original PHI2 method to stabilize the step size effect in the calculation of time-variant reliability. However, the accuracy of this method largely depends on the selection of time interval. In order to facilitate this choice, Mejri et al. [19] came up with a modified PHI2 method, which is applied in this paper. In addition, many other scholars have also devoted themselves to the time-varying problems in different structures and systems by using other methods, such as SVD-based approach [20] and wavelet methods [21], [22]. Moreover, Hawchar et al. [23], [24] put forward the polynomial chaos expansion method to approximate the time-dependent limit state function of the system with a polynomial surrogate model. However, the Monte-Carlo simulation is also needed to carry out, which may increase the computational cost. Hu and Mahadevan [25] proposed a single-loop Kriging surrogate modeling approach for time-variant reliability analysis and significantly increased the analysis efficiency without sacrificing the accuracy.

This paper reviews the main points of time-varying reliability methods and provides a way to combine it with FORM and evaluates the reliability of ship structures based on time-variant feature analysis. The basic time-variant reliability concepts are illustrated and the calculation approach of out-crossing rate combined with FORM is derived in section II. In section III, the time-variant reliability analysis on ship structures is elaborated including the analysis of randomness and time-dependent characteristic, the establishment of the limit state function and the determination process of parameters. In section IV, a case study of ship grillage structure is provided to demonstrate the feasibility and possibility of this method for ship structures.

II. TIME-VARIANT RELIABILITY METHOD

A. TIME-VARIANT RELIABILITY MODEL

The reliability method for structural components is deemed to be random and time-variant. Provided that $X(x_1, x_2, \ldots, x_n)$ denotes a set of random variables utilized in solving the mechanical problems, including the uncertainties of load effects, material properties, geometry dimension and boundary conditions, the time-dependent limit state function $G(t, X)$ divides the space of outcomes into two domains: the space where $G(t, X) > 0$ is the safe domain; the space where $G(t, X) \leq 0$ is the failure domain; the space where $G(t, X) = 0$ is the boundary between this two domains, called as the limit state surface.

The instantaneous failure probability $P_{f,i}$ at time T can be represented as:

$$P_{f,i}(T) = prob\,(G(T, X) \leq 0) \tag{1}$$

The cumulative failure probability $P_{f,c}$ of the structural components within the interval $[0, T]$ can be obtained as:

$$P_{f,c}(0, T) = prob\,\{\exists t \in [0, T], G(t, X) \leq 0\}. \tag{2}$$

As a case for ship structures in this paper, this property is to validate time-dependent reliability methodologies. In other cases where the evolutions of the limit-state function cannot be acquired in advance, an alternative approach has to be considered. The most commonly used approach is out-crossing method that relies on the computation of the out-crossing rate $v^+(t)$. The out-crossing rate $v^+(t)$ is the probability when a structure works normally under load at time t and it fails at time $t + \Delta t$, which can be defined as [15]:

$$v^+(t) = \lim_{\Delta t \to 0^+} \frac{prob\,(A \cap B)}{\Delta t}$$

where

$$\begin{cases} A = \{G(t, X) > 0\} \\ B = \{G(t + \Delta t, X) \leq 0\} \end{cases} \tag{3}$$

Cumulative failure probability within time interval $[0, T]$ corresponds to either the failure at the initial instant $t = 0$ or a later out-crossing of the limit-state surface if the structural components are in the safe domain at $t = 0$. Classical arguments [26] lead to the following bounds on $P_{f,c}(0, T)$:

$$\max_{0 \leq t \leq T}\left[P_{f,i}(t)\right] \leq P_{f,c}(0, T) \leq P_{f,i}(0) + \int_0^T v^+(t)\,dt. \tag{4}$$

Provided that the out-crossing event follows Poisson distribution, the cumulative failure probability can be expressed as [19]:

$$P_{f,c}(0, T) \approx 1 - \exp\left[-\int_0^T v^+(t)\,dt\right]. \tag{5}$$

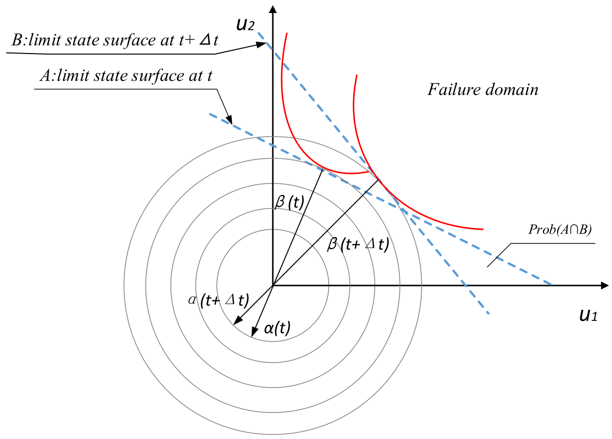

FIGURE 1. Evolution of the reliability problems during Δt with FORM approximation.

B. CALCULATION OF THE OUT-CROSSING RATE COMBINED WITH FORM

The out-crossing is crucial for the calculation of the cumulative failure probability. The assessment of probability based on (3) is regarded as a two-component parallel system analysis. With the FORM, the classical reliability variables such as the reliability index β at time t and $t + \Delta t$ (Fig.1) can be obtained after two successive analyses, which are led by the Abdo-Rackwits algorithm combined with a Newton-Raphson line search. Traditional FORM analysis corresponds to the limit-state surface approximating the hyper-plane $\boldsymbol{\alpha}(t) \cdot \mathbf{u} + \beta(t) = 0$. Therefore, $\boldsymbol{\alpha}(t)$ is the unit normal vector of the limit-state surface at t, and $\mathbf{u}$ is the standard normal space. Fig.1 shows the probability $v^+(t)\,\Delta t$ in FORM approximation in the space of standard normal space.

Through a reconsideration of (3), this method has been improved in [19] by introducing the following equation:

$$f_t(\Delta t) = prob\,(\{G(t, X) > 0\} \cap \{G(t + \Delta t, X) \leq 0\}) \tag{6}$$

Since the two events in (6) are independent with each other and, (3) can thus be rewritten into the new formulation of out-crossing rate as:

$$\begin{aligned} v^+(t) &= \lim_{\Delta t \to 0} \frac{f_t(\Delta t) - f_t(0)}{\Delta t} = \left.\frac{df_t(\Delta t)}{d\Delta t}\right|_{\Delta t = 0} \\ &= \left\|\boldsymbol{\alpha}'(t)\right\| \cdot \varphi(\beta(t)) \cdot \psi\left(\frac{\beta'(t)}{\|\boldsymbol{\alpha}'(t)\|}\right) \quad with \\ \psi(x) &= \varphi(x) - x \cdot \Phi(-x), \end{aligned} \tag{7}$$

where the notation " ′ " is used to denote the derivative of function with respect of time. What's more, φ and Φ represent the normal probability density function (PDF) and cumulative distribution function (CDF) respectively.

III. TIME-VARIANT RELIABILITY ANALYSIS FOR SHIP STRUCTURES

A. TIME-VARIANT FEATURE ANALYSIS OF SHIP STRUCTURES

There are many factors will impact the structural reliability of ship structures, such as environmental and loading effects.

What's more, due to these factors, the strength and stress of ship structures are time-dependent in a changing marine environment. Therefore, time-variant feature analysis shall be conducted on these factors before developing the limit state function of ship structures.

1) ENVIRONMENTAL EFFECTS

Ship structures are in the marine environment for a long time, which leads to a serious corrosion effect. The leading environmental effects causing the corrosion contain temperature, humidity, salt-fog and their comprehensive effect. What's more, the corrosion effect such as the general corrosion and pitting corrosion, will reduce the reliability level of ship structures. In addition, the intergranular corrosion and stress corrosion are also existed with the increase of corrosion effect. In practical engineering applications, carbon steel and aluminum alloy are two common hull structural materials. However, in severe corrosive environment, carbon steel mainly occurs general corrosion, while aluminum alloy mainly occurs pitting corrosion and exfoliation corrosion. Carbon steel is the main research material of this paper.

The influence of corrosion effect aggravates over time, which results in the decrease of the thickness, width and web height of the grillage structures, the reduction of effective section modulus of ship structures, and the deterioration of materials' mechanical properties. In order to measure the corrosion degree of ship structures, the corrosion depth of structural component is used as a quantitative index of corrosion. In this paper, the corrosion depth model of ship structures is adopted as follows: [27]

$$\delta(t) = \delta_0 \Delta(t), \tag{8}$$

where, $\delta\ (t)$ refers to the thickness at time t (unit: mm); δ_0 refers to the initial thickness (unit: mm); $\Delta\ (t)$ refers to the time variability of $\delta\ (t)$ and is an expression with respect to time based on the thickness data of the accelerated degradation test for ship structures.

2) LOADING EFFECTS

The working load features of ship structures are complicated, such as still water load and wave load. In this paper, both still water bending moment and wave bending moment are taken into account. When a ship sails, there would be a flexural failure of deck or bottom grillage structures owing to the co-effect of still water load and wave load, which greatly reduces the reliability of the ship. To better describe the flexural failure of grillage structures, Fig. 2 depicts the load conditions of a certain ship deck grillage structures. Fig. 2(a) refers to the grillage structures bearing uniformly distributed load, while Fig. 2(b) represents the deformation of the grillage structures after loading.

For still water bending moment, the statistical analysis of United States Ship Structure Committee (SSC) [28] assumes that it could follow a normal distribution, and the maximum bending moment of grillage structures can be calculated as [14]:

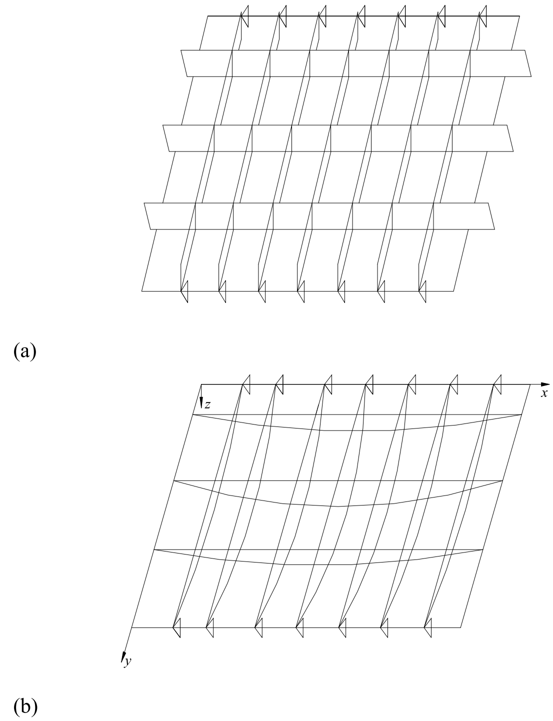

FIGURE 2. **Load conditions of a certain ship deck grillage. (a) Before loading. (b) Deformation after loading.**

$$M_S = \begin{cases} -0.065 C_w L_0^2 B_0 (C_B + 0.7), & sagging \\ C_w L_0^2 B_0 (0.1225 - 0.015 C_B), & hogging, \end{cases} \tag{9}$$

where, M_S refers to the maximum still water bending moment (unit: KN); L_0 stands for the ship length (unit: m); B_0 indicates the molded breadth (unit: m); C_B means the block coefficient and C_w represents the wave coefficient.

Compared with still water bending moment, wave bending moment is provided with greater variability and the randomness of it mainly comes from the storm. According to the research [10], wave load varies mainly with wave length, wave height, ship size and the relative location of ship and wave. At present, the empirical formula provided by IACS-URS11 (International Association of Classification Societies) is widely used [14]:

$$M_w = \begin{cases} -0.11 C_w L_0^2 B_0 (C_B + 0.7), & sagging \\ 0.19 C_w L_0^2 B_0 C_B, & hogging, \end{cases} \tag{10}$$

where, M_w refers to the maximum wave bending moment (unit: KN).

Combined bending moment is composed of still water bending moment and wave bending moment. The relationship between the peak value of combined bending moment M_c and the peak value of still water bending moment M_s and wave moment M_w in T years is [13]:

$$M_c = \varphi_t (M_s + M_w) = M_s + \varphi_w M_w, \tag{11}$$

where, φ_t stands for the error estimation factor of direct addition M_s and M_w, and φ_w stands for the load reduction factor.

$$\varphi_w = \frac{0.83M_w - 0.17M_s}{M_w}. \tag{12}$$

B. LIMIT STATE FUNCTION OF SHIP STRUCTURES

In order to calculate the parameters in (7), the limit state function needs to be established. The limited state function of ship structures consists of their own internal strength and the stress caused by the external load as the two main variables.

The strength of ship structures P refers to the capability of sustaining effectiveness, such as capability of damage and deformation resistance. Besides, the strength of ship structures is decided by various factors, such as material properties, geometry shape, and environmental effects. The raw data of ship structures strength, obtained from the experiment in mechanics, are always discrete random variables and obey to some specific distributions. Nevertheless, the strength of ship structures will degenerate with the impact of environmental effects, which is a time-variant variable. That is to say, the strength P is a variable both random and time-dependent. The strength P of ship structures can be defined as:

$$P(t) = P_0 p(t), \tag{13}$$

where, P_0 stands for the initial strength of ship structures, and $p(t)$ stands for the time variability of $P(t)$ that is an expression with respect to time based on the bending strength data of the accelerated degradation test for ship structures.

The stress of ship structures S consists of both the internal stress and the environmental stress, which is influenced by environmental load and geometry shape directly. Furthermore, the stress of ship structures is determined by the longitudinal bending moment and section modulus of ship structures. Through the analysis, section modulus is a random variable with degradation over time due to the section shape changing with time. The stress S of ship structures can be expressed as:

$$S(t) = \frac{M}{W(t)} \quad with \quad W(t) = W_0 w(t), \tag{14}$$

where, $W(t)$ denotes the section modulus that is depend on the section shape of ship structures; W_0 refers to the initial section modulus of ship structures; $w(t)$ refers to the time variability of $W(t)$ and is an expression with respect to time based on the thickness data of the accelerated degradation test for ship structures.

Therefore, the limit state function of ship structures could be derived as:

$$G(t, X) = P(t) - S(t). \tag{15}$$

According to the randomness and time variability of the strength and stress, the variable separation is done for (15):

$$G(t, X) = P_0 p(t) - \frac{M}{W_0 w(t)}. \tag{16}$$

In addition, from the FORM method, we can know:

$$\beta(t) = \frac{\mu[G(t, X)]}{\sigma[G(t, X)]}, \tag{17}$$

where, $\mu[G(t, X)]$ is the expectation of stochastic process $G(t, X)$, and $\sigma[G(t, X)]$ is the standard deviation of $G(t, X)$. Moreover, the unit normal vector $\boldsymbol{\alpha}(t)$ is as follows:

$$\boldsymbol{\alpha}(t) = \left(-\frac{\sigma[P(t)]}{\sigma[G(t, X)]}, \frac{\sigma[S(t)]}{\sigma[G(t, X)]}\right). \tag{18}$$

By substituting (17) and (18) into (7), the final out-crossing rate can be calculated.

C. PARAMETERS DETERMINATION PROCESS

This method involves three consecutive steps and the flowchart of this method is shown in Fig. 3.

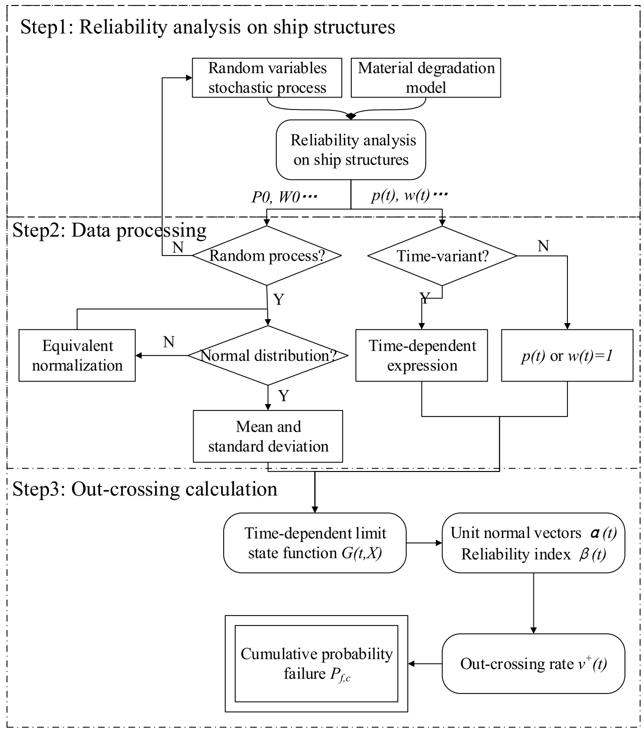

FIGURE 3. The flowchart depicting the general algorithm of this method.

1) RELIABILITY ANALYSIS ON SHIP STRUCTURES

With this step, the stochastic process of random variables and the material degradation model of ship structures can be determined, by combining with its own natural and working environment.

2) DATA PROCESSING

For the random process, the distributions are established respectively for P_0 and W_0. If some of them are not normal distributions, they should be equivalently normalized. Besides, the mean value and standard deviation is confirmed. For the time-variant variables, the time-dependent expressions of $p(t)$ and $w(t)$ are determined. If some of them are time-invariant, the expression is supposed to be constant 1.

3) OUT-CROSSING CALCULATION

Now that all the data have been collected, the time-dependent limit state function can be established consequently. FORM is utilized to figure out the unit normal vector $\boldsymbol{\alpha}(t)$ and the time-variant reliability index $\beta(t)$. Out-crossing equations are utilized to calculate the rate $v^{+}(t)$ so as to get the cumulative failure probability $P_{f,c}$ and the reliability R of ship structures.

IV. CASE STUDY

A. CASE DESCRIPTIONS & DEGRADATION TEST

A typical bottom member of a medium ship is selected as the analysis object, and the time-dependent reliability model is built. A certain type medium ship is $103.2m \times 10.8m \times 3.19m$; the deck structure is 50×5 grillage; and the I-shaped section material is carbon steel with normal strength. The major parameters of the ship grillage structure are listed in Table 1 [29], [30].

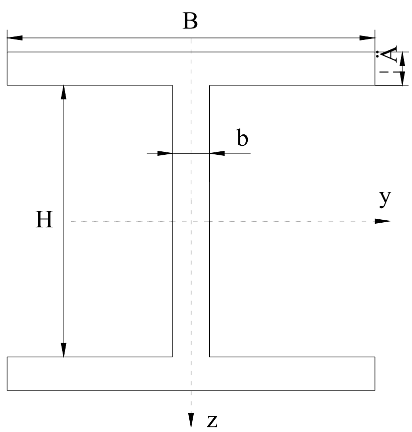

FIGURE 4. Symmetric I-shaped section.

The I-shaped section of the grillage structure is shown in Fig. 4, where B is the width of the upper and lower edge strip; δ is the thickness and δ_0 is the initial thickness; H is the web height; b is the web breadth. The thickness $\delta(t)$ of the I-shaped section is regarded as random variable with time. The time-variant model of bending section modulus and the expression of initial section modulus can be established as:

$$W(t) = \frac{(B^3 - b^3)\delta(t) + Hb^3}{12} / (B/2)$$
$$W_0 = \frac{(B^3 - b^3)\delta_0 + Hb^3}{12} / (B/2). \quad (19)$$

Therefore, $w(t)$ can be derived according to (14):

$$w(t) = W(t)/W_0 = \delta(t)/\delta_0 = \Delta(t). \quad (20)$$

The stress of different nodes of the grillage structure can be shown in Table 2 It can be seen that the bending moment of hogging condition is larger, so the bending moments of hogging condition are calculated, and the total bending moment based on (11) is calculated as:

$$M = M_c + M_{sT} + \varphi_w M_{wT} = 1.017 \times 10^7 N \cdot mm. \quad (21)$$

TABLE 1. The major parameters of the 50 × 5 grillage structure.

Parameters	Unit	Value
L	m	103.2
W	m	10.8
a	m	3.19
B	mm	142
H	mm	379
b	mm	16.5
δ_0	mm	10.5
E	MPa	235

TABLE 2. The calculation results of different bending moment under two conditions.

Bending moment	Sagging condition	Hogging condition
Structure bending moment	-2.95×10^6	3.76×10^6
Static bending moment M_{sT} $(N\cdot mm)$	-2.42×10^6	3.12×10^6
Wave bending moment M_{wT} $(N\cdot mm)$	-4.11×10^6	4.91×10^6
Loading reduction coefficient φ_w	0.73	0.67

Considering the characteristics of sea water corrosion, the ship grillage structure is affected by corrosion action in the marine environment and the mechanical properties of carbon steel will change over time. In order to get the degradation data of thickness and bending strength, five groups of indoor immersion accelerated seawater corrosion tests in different periods were designed and conducted. There were three parallel samples in each group and the fifteen samples were all $100mm \times 50mm \times 5mm$ plate shape with the same steel material. The test medium was a mixture of both 3.5% natural seawater solution and 0.05mol/L, H_2O_2 solution and the constant temperature water baths were adopted to control the corrosion medium temperature at 25°C. The thickness of each sample was measured first and then the bending strength was measured at each measuring time. The degradation data of thickness and bending strength of test samples are listed in Table 3, from which it can be demonstrated that the thickness and bending strength decreases over time in a certain way.

Through the regression analysis of the above data in Table 3, it can be known that the degradation processes of the thickness and the bending strength of this carbon steel material both obey the power function regression. The fitting outcomes of the regression analysis are separately described and listed in Fig. 5 and Fig. 6, from which it can be demonstrated that the fitting outcomes are reliable.

Furthermore, taking material performance, volume shape and uncertain experiment data into account, the initial strength P_0 of the grillage structure can be regarded as a random variable obeying to the normal distribution, and both the mean value and the standard deviation are determined by

TABLE 3. The degradation data of thickness and bending strength of test samples.

Test time (day)	Test sample number	Thickness (mm)	Mean value (mm)	Bending strength (MPa)	Mean value (MPa)
	1	4.953		816.089	
7	2	4.912	4.921	844.121	831.33
	3	4.897		833.777	
	4	4.855		834.281	
14	5	4.814	4.797	820.392	828.25
	6	4.722		830.068	
	7	4.676		845.054	
28	8	4.571	4.597	799.177	826.29
	9	4.543		834.629	
	10	4.547		828.928	
56	11	4.469	4.484	800.655	813.43
	12	4.435		810.708	
	13	4.421		791.614	
112	14	4.382	4.367	782.305	799.68
	15	4.298		825.109	

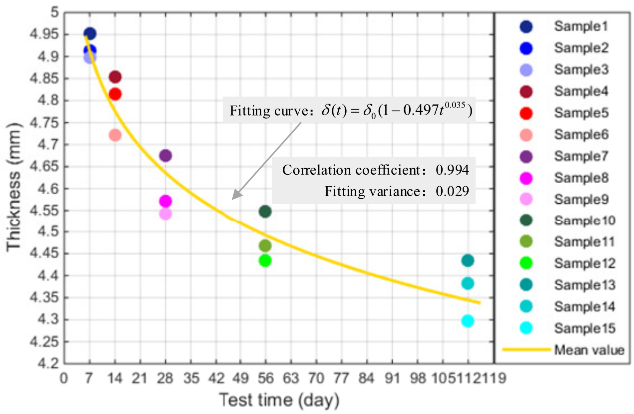

FIGURE 5. The fitting outcome of the thickness degradation data.

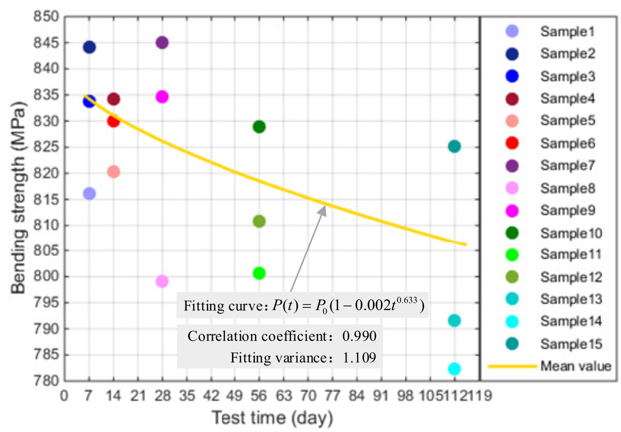

FIGURE 6. The fitting outcome of the bending strength degradation data.

the statistical analysis of raw data obtained from the tensile experiment of the test samples. Considering the manufacturing and measuring error, the major parameters of the ship grillage structure are also be deemed as random variables following the normal distribution, and the statistical data are collected by the measurement. Then, the statistical data of the initial section modulus W_0 of the grillage structure can be calculated, by regarding the relevant variables as independent of each other. The random process information of the grillage structure is shown in Table 4.

TABLE 4. The random process information of the grillage structure.

Variable	Unit	Distribution	Mean	Standard deviation	Variation coefficient
R_0	MPa	Normal	840	27.23	0.0324
δ_0	mm	Normal	10.5	0.50	0.0476
B	mm	Normal	142	0.96	0.0078
H	mm	Normal	379	8.33	0.0220
b	mm	Normal	0.36	0.36	0.0279
W_0	mm^3	/	37229.9	193.61	0.0052

B. CALCULATION OF SHIP STRUCTURES RELIABILITY

According to (16) and (17), the temporal reliability index $\beta(t)$ can be expressed as follows:

$$\beta(t)=\frac{\mu_{P_0}p(t)-\frac{M}{w(t)}\mu_{\frac{1}{W_0}}}{\Omega}\quad with$$
$$\Omega=\sqrt{\sigma_{P_0}^2 p(t)+\frac{M^2}{w(t)}\sigma_{\frac{1}{W_0}}^2}. \tag{22}$$

In addition, the unit normal vector $\boldsymbol{\alpha}(t)$ is calculated according to (18):

$$\boldsymbol{\alpha}(t)=\left(-\frac{\mu_{P_0}p(t)}{\Omega},\frac{\frac{M}{w(t)}\mu_{\frac{1}{W_0}}}{\Omega}\right). \tag{23}$$

By substituting $\boldsymbol{\alpha}(t)$ and $\beta(t)$ into (7), the out-crossing rate $v^+(t)$ can be calculated. Then by substituting $v^+(t)$

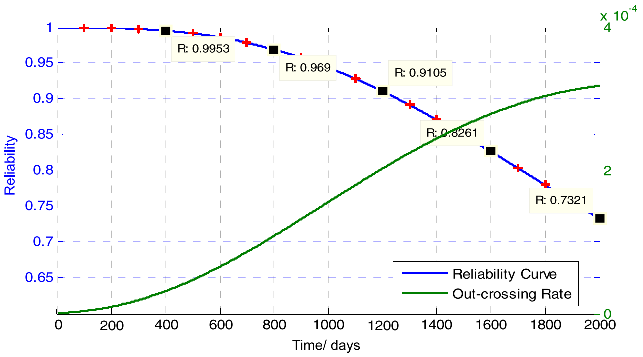

FIGURE 7. **Out-crossing rate and reliability curves.**

into (5), the cumulative failure probability $P_{f,c}$ and the reliability of the ship grillage structure can be obtained as shown in Fig. 7.

From Fig. 7, it can be demonstrated that the reliability curve of the ship structures reduced with time and the out-crossing rate curve increased with time. It is because that $v^+(t)$ is proportional to $P_{f,c}$ according to (5) and the out-crossing rate curve can reflect the trend of cumulative probability of failure. Besides, it can be seen that the reliability of the ship grillage structure stays above 0.969 within 800 days and shows a rapid downward trend after that time with the aggravation of corrosion. The reasons for the rapid decline of the reliability curve are that protective coating is damaged completely and the corrosion rate is speeding up over time.

C. DISCUSSION

In order to verify the accuracy and calculation efficiency, traditional FORM, factor of safety method and Monte-Carlo simulation are adopted to deal with the same problem.

1) TRADITIONAL FORM

With this method, time-variant process is not taken into consideration, and the reliability value is a constant $\Phi(\beta)$. The reliability index β is expressed as [30]:

$$\beta = \frac{\mu_{P_0} - \mu_{S_0}}{\sqrt{\sigma_{P_0}^2 + \sigma_{S_0}^2}}. \tag{24}$$

Then the reliability according to (24): $R = \Phi(\beta) = 0.998$.

Compared with the time-variant reliability model, the traditional structural reliability method is too conservative to reflect the changing property of actual reliability of ship structures over time in the current state.

2) FACTOR OF SAFETY METHOD (FOS METHOD)

Factor of safety is a term to describe the structural capacity of a system beyond the expected load or actual load. Essentially, how much stronger the system is than it usually needs to be for an intended load. The corresponding safety standard is [31]:

$$n \geq [n], \tag{25}$$

wherein, n is the ratio of strength to stress at the current moment, and $[n]$ is the factor of safety. In the reliability probability design, the value can be calculated as [31]:

$$[n] = \frac{1 - \Phi^{-1}(R_r)\, C_r}{1 + \Phi^{-1}(R_s)\, C_s} n_m, \tag{26}$$

wherein, C_r and C_s are the variation coefficients of strength and stress, and n_m the ratio of strength and stress mean value. R_r and R_s are the reliabilities of strength and stress respectively. According to the general mechanical structure design specification, $R_s = 0.99$ and $R_r = 0.95$. As a consequence, the calculation result is $t \leq 1044\,days$.

3) MONTE-CARLO SIMULATION

As a general method to solve reliability problems, Monte-Carlo simulation gives accurate results if the number of the samples is large enough. Matlab routines are utilized to discretize the random process and perform the simulation. 10^4 samples are collected here to compute an accurate probability of failure.

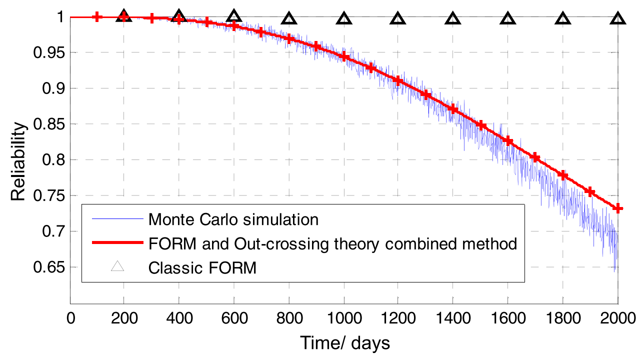

FIGURE 8. **Reliability calculation results comparison.**

The comparison results are drawn in Fig. 8. It can be seen that the reliability of the grillage structure is proved to be a constant 0.998 with the traditional FORM from Fig. 8. The result is conservative and unauthentic. FoS method reveals that it might fail after an on-going operating time of 1044 days. Meanwhile, the Monte Carlo simulation and out-crossing approach show that the reliability is above 0.9 within 1200 days. FoS method seems to be radical and keep much safety margin. By and large, the combined method of FORM and out-crossing approach for ship structures in this paper keeps small deviation compared with the simulation. However, when the simulation calls for a large number of samples, the combined method has the advantages of both accuracy and calculation efficiency. This quite realistic case study of the ship structures shows the applicability and versatility of the approach. It is essential to note that this kind of issues may be solved by introducing the time element and randomness in the definition of the reliability problems.

V. CONCLUSION

This paper has presented a reliability method for solving the time-variant reliability problems of ship structures. This approach specifically deals with the calculation of the out-crossing rate in time-variant reliability problems by combining with the traditional FORM. The strength and stress of ship structures are both time-dependent according to the time-variant feature analysis, which better demonstrate the necessity of this method when the time-variant reliability analysis is conducted for ship structures.

The main advantages of this combined method are concluded that:

(1) It only needs low numerical cost through the combination of out-crossing approach and FORM, without any software involving simulation or additional implementation.

(2) The time-dependent feature analysis of ship structures better describes the time-variant characteristic of the stress and strength in actual working conditions by fully taking many factors into account. What's more, the quantitative expressions of stress and strength that is the main variables in figuring out the reliability index and the unit normal vector are also obtained in this method.

(3) In the case study, accurate results can be obtained with this method when the comparison with a Monte Carlo simulation is possible. Finally, according to the efficiency of this time-variant reliability method, its industrial applications in evaluating the reliability of ship structures is feasible and prospective.

REFERENCES

[1] B. M. Ayyub, I. A. Assakkaf, J. E. Beach, W. M. Melton, N. Nappi, Jr., and J. A. Conley, "Methodology for developing reliability-based load and resistance factor design(LRFD) guidelines for ship structures," *Naval Eng. J.*, vol. 114, no. 2, pp. 23–42, 2002.

[2] R. Rackwitz and B. Flessler, "Structural reliability under combined random load sequences," *Comput. Struct.*, vol. 9, no. 5, pp. 489–494, 1978.

[3] A. Haldar and S. Mahadevan, "Probability, reliability and statistical methods in engineering design," *Bautechnik*, vol. 77, no. 5, p. 379, 2000.

[4] K. W. Breitung, *Asymptotic Approximations for Probability Integrals*, vol. 4, no. 4. Berlin, Germany: Springer-Verlag, 1994, pp. 187–190.

[5] A. Tagliani, "On the existence of maximum entropy distributions with four and more assigned moments," *Probabilistic Eng. Mech.*, vol. 5, no. 4, pp. 167–170, 1990.

[6] C. G. Bucher and U. Bourgund, "A fast and efficient response surface approach for structural reliability problems," *Struct. Safety*, vol. 7, no. 1, pp. 57–66, 1990.

[7] I. Lee, K. K. Choi, L. Du, and D. Gorsich, "Inverse analysis method using MPP-based dimension reduction for reliability-based design optimization of nonlinear and multi-dimensional systems," *Comput. Methods Appl. Mech. Eng.*, vol. 198, no. 1, pp. 14–27, 2008.

[8] J. Cheng, J. Zhang, C. S. Cai, and R.-C. Xiao, "A new approach for solving inverse reliability problems with implicit response functions," *Struct. Eng.*, vol. 29, no. 1, pp. 71–79, 2007.

[9] Y. Xiang and Y. Liu, "Application of inverse first-order reliability method for probabilistic fatigue life prediction," *Probabilistic Eng. Mech.*, vol. 26, no. 2, pp. 148–156, 2011.

[10] Y. H. Sung, D.-W. Kim, and D.-H. Kim, "Applying reliability assessment methods to superconducting magnetic energy storage system (SMES) designs," *IEEE Transaction Magn.*, vol. 47, no. 11, pp. 4623–4628, Nov. 2011.

[11] X. Guan, J. He, R. Jha, and Y. Liu, "An efficient analytical Bayesian method for reliability and system response updating based on Laplace and inverse first-order reliability computations," *Rel. Eng. Syst. Safety*, vol. 97, no. 1, pp. 1–13, 2012.

[12] S. Sankararaman, M. J. Daigle, and K. Goebel, "Uncertainty quantification in remaining useful life prediction using first-order reliability methods," *IEEE Trans. Rel.*, vol. 63, no. 2, pp. 603–619, Jun. 2014.

[13] A. Der Kiureghian, "The geometry of random vibrations and solutions by FORM and SORM," *Probabilistic Eng. Mech.*, vol. 15, no. 1, pp. 81–90, 2000.

[14] R. Rackwitz, "Computational techniques in stationary and non-stationary load combination—A review and some extensions," *J. Struct. Eng.*, vol. 25, no. 1, pp. 1–20, 1998.

[15] O. Hagen and L. Tvedt, "Vector process out-crossing as parallel system sensitivity measure," *J. Mech. Eng.*, vol. 117, no. 10, pp. 2201–2220, 1991.

[16] C. Andrieu-Renaud, B. Sudret, and M. Lemaire, "The PHI2 method: A way to compute time-variant reliability," *Rel. Eng. Syst. Safety*, vol. 84, no. 1, pp. 75–86, 2004.

[17] B. Sudret, "Analytical derivation of the outcrossing rate in time-variant reliability problems," *Struct. Infrastructure Eng.*, vol. 4, no. 5, pp. 353–362, 2008.

[18] B. Sudret and G. Defaux, "Time-variant finite element reliability analysis-application to the durability of cooling towers," *J. Struct. Safety*, vol. 27, no. 2, pp. 93–112, 2005.

[19] M. Mejri, M. Cazuguel, and J. Y. Cognard, "A time-variant reliability approach for ageing marine structures with non-linear behavior," *J. Comput. Struct.*, vol. 89, nos. 19–20, pp. 1743–1753, 2011.

[20] G. Ruocci *et al.*, "Damage assessment of pre-stressed structures: A SVD-based approach to deal with time-varying loading," *Mech. Syst. Signal Process.*, vol. 47, nos. 1–2, pp. 50–65, 2014.

[21] W. J. Staszewski and D. MarkWallace, "Wavelet-based frequency response function for time-variant systems—An exploratory study," *Mech. Syst. Signal Process.*, vol. 47, nos. 1–2, pp. 35–49, 2014.

[22] Y. Yang and S. Nagarajaiah, "Blind identification of damage in time-varying systems using independent component analysis with wavelet transform," *Mech. Syst. Signal Process.*, vol. 47, nos. 1–2, pp. 3–20, 2014.

[23] L. Hawchar, C. P. El Soueidy, and F. Schoefs, "Time-variant reliability analysis using polynomial chaos expansion," in *Proc. 12th Int. Conf. Appl. Statist. Probability Civil Eng. (ICASP)*, Vancouver, BC, Canada, Jul. 2015.

[24] L. Hawchar, C.-P. El Soueidy, and F. Schoefs, "Principal component analysis and polynomial chaos expansion for time-variant reliability problems," *Rel. Eng. Syst. Safety*, vol. 167, pp. 406–416, Nov. 2017.

[25] Z. Hu and S. Mahadevan, "A single-loop Kriging surrogate modeling for time-dependent reliability analysis," *J. Mech. Design*, vol. 138, no. 6, p. 061406, May 2016, doi: 10.1115/1.4033428.

[26] M. Shinozuka, "Probability of structural failure under random loading," *J. Sanitary Eng. Division*, vol. 90, no. 5, pp. 147–170, 1964.

[27] S. P. Qin, W. C. Cui, and K. Shen, "A Non-linear corrosion model for time variant reliability analysis of ship structures," (in Chinese), *J. Ship Mech.*, vol. 7, no. 1, pp. 94–103, 2003.

[28] *Ship Design Practical Handbook*, (in Chinese), 2nd ed., National Defense Industry Press, Beijing, China, 2013, pp. 752–756.

[29] *Hull Structural Design Ships With Length 100 Meters and Above*, Det Norske Veritas, Oslo, Norway, Jan. 2015. [Online]. Available: http://rules.dnvgl.com/docs/pdf/dnv/rulesship/2015-01/ts301.pdf

[30] T. Y. Ye, "Time-variant reliability model for ship structural component considering natural environment effectiveness," Ph.D. dissertation, Dept. Ind. Eng., Beihang Univ., Beijing, China, 2015.

[31] *Structural Design and Test Factors for Spaceflight Hardware*, Standard NASA-STD-5001, 2008.

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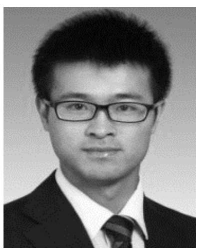

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