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Adaptive Image Contrast Enhancement by Computing Distances into a 4-Dimensional Fuzzy Unit Hypercube

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ABSTRACT A new fuzzy procedure for adaptive gray-level image contrast enhancement (CE) is presented in this paper. Starting from the pixels belonging to a normalized gray-level image, an appropriate smooth S-shaped fuzzy membership function (MF) is considered for gray-scale transformation and is adaptively developed through noise reduction and information loss minimization. Then, a set of fuzzy patches is extracted from the MF, and for each support of each patch, we compute four ascending-order statistics that become points inside a 4-D fuzzy unit hypercube after a suitable fuzzification step. CE is performed by computing the distances among the above points and the points of maximum darkness and maximum brightness (special vertexes in the hypercube), and by determining the rotation of the tangent line to the MF around a crucial point where fuzzy patches and the MF coexist. The proposed procedure enables high CE in all the treated images with performance that is fully comparable with that obtained by three more sophisticated fuzzy techniques and by standard histogram equalization.

INDEX TERMS Image processing, adaptive contrast enhancement, fuzzy logic, fuzzy unit hypercubes, over-/under-enhancement.

I. INTRODUCTION

Within *image processing (IP)*, *contrast enhancement (CE)* improves image data by stretching the distribution of the gray levels using adaptive techniques that directly and automatically process the extracted features [1]–[3]. Many *CE* techniques have been developed based on the type of information to be highlighted. Mathematical morphology, for example, is a mature technique for extracting shape and size information from an image that involves configuration of a set of nonlinear operators (dilation and erosion) that act on images using structuring elements [4], [5]. By contrast, the top hat transformation is considered to be a good technique to extract bright or dark features smaller than a given size from an uneven background [6]. Both single-scale and multi-scale morphological filtering have been successfully exploited in local contrast enhancement, producing good visual results in terms of contrast [4], [6]. In [7]–[9], for example, good results were obtained by power-law transformation and saturation operators, which

resulted in good quality in the modified images. Furthermore, substantial effort has been made to remove noise while preserving the edges [10], [11]. *CE* techniques based on histogram equalization (*HE*) are the most popular because of their simplicity and the meaningful results [12]–[15]. This technique, which is based on flattening and stretching the dynamic range of the image's histogram, can achieve overall *CE* but may sometimes reduce the local details and result in over-enhancement. Accordingly, specific *HE* techniques, such as contrast-limited adaptive histogram equalization, which splits the image into tiles on which *HE* is applied and combines the neighboring tiles by bilinear interpolation to eliminate artificially induced boundaries and emphasize local contrast rather than overall contrast, have been developed [16]. In addition, taking into account the input histogram separation approach, several techniques have been developed based on the mean gray level (brightness-preserving *bi-HE*), the median gray level (dualistic sub-image *HE*), and the maximum gray level (minimum mean brightness error

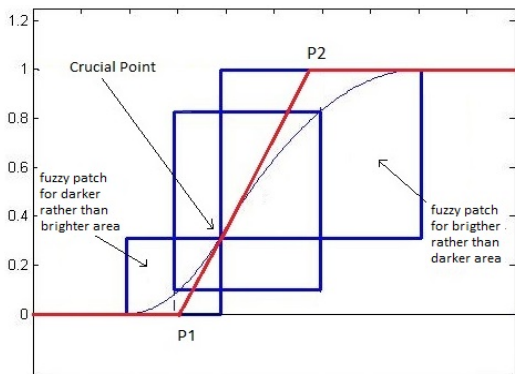


FIGURE 1. S-function covered by FP_1 , FP_2 , FP_3 (gray, dark and bright zones) and the new S – function produced by the proposed procedure (red line). Two cusps, P_1 and P_2 , are clearly shown.

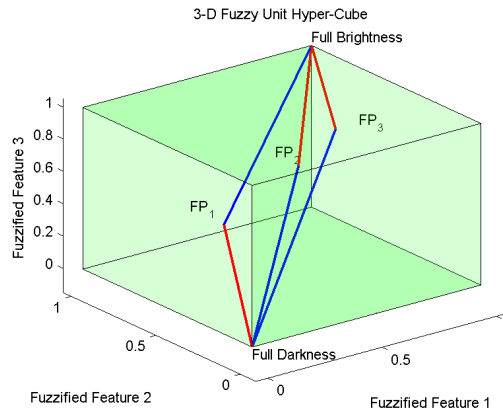


FIGURE 2. FFs inside a $n - UKC$.

Bi-HE) [1], [17]. However, the techniques mentioned above do not consider the fact that the sampling techniques and the transition zones of a gray level image often feature from uncertainty and vagueness, so it may be useful to treat images with fuzzy approaches. In particular, the logarithmic function [18], fuzzy entropy approaches [19]–[21] and fuzzy similarity indexes [22] have been successfully exploited for gray level fuzzification and to capture the neighborhood characteristics. In addition, appreciable results have been achieved through fuzzy-wavelet procedures using approximation and detail coefficients together with transformation and saturation operators to transfer images remotely [23]–[26]. In such a context, owing to the development of a geometrical fuzzy procedure that makes the approach “readable” and helpful for non-technical experts, in this paper, inspired by [27], we propose a new fuzzy approach based on the combined exploitation of a 4-dimensional fuzzy unit hyper-cube (FUHC), noise reduction and information loss minimization to enhance gray level images. A smooth S-shaped fuzzy membership function (MF), which is adaptively set by techniques based on both noise reduction and information loss minimization, fuzzifies the gray level image. Then, the input-output space of the MF is covered by a set of partially overlapped fuzzy patches (FPs) (Fig. 1) seeking within that space a crucial point (CP) (i.e., the point of maximal ambiguity with membership values falling around 0.5) on which to modify the MF. From the supports of each FP, we extract 4 statistical features (F) that, after appropriate fuzzification, become 4-dimensional points inside a FUHC (or 4-unit Kosko’s cube (4 – UKC)) [28]. Two particular points lie inside each 4 – UKC: the total brightness and total darkness (Fig. 2). If the fuzzified features (FFs) are closer to the total brightness point, they are considered brighter rather than darker, and the MF slope around the CP is increased by a factor depending on that distance. Conversely, if the FFs are closer to the total darkness point, then they are considered darker rather than brighter, and the MF slope around the CP is reduced by a factor depending on that distance (for details, see Fig. 3).

When applied to a set of low-contrast images, satisfying results, largely comparable with those obtained by using established techniques, are achieved. The rest of this paper is organized as follows. After a quick overview of fuzzy set theory for IP (Section II), the materials and methods are detailed, leading to the exploitation of $n - FUHC$ to set and edit the parameters characterizing the CE (Section III). The approach is applied to a set of low-contrast gray level images with different features, and each operational choice is justified by comparing the results with those obtained by other established fuzzy techniques, such as those presented in [18], [19], and [21], and with the standard histogram equalization procedure (Section IV). Finally, in Section V, some conclusions and future perspectives are discussed.

II. FUZZY SET THEORY FOR IMAGE PROCESSING (FIP)

Proposed as an extension of classic set theory, fuzzy set theory addresses problems with inherent vagueness and/or imprecision in the shading of the membership of each element to a given set in the range [0, 1] by a membership function (MF) that manages its fuzziness. Formally, a fuzzy set A is defined by the ordered pair (U, h(U)), where U is the universe of discourse (all possible values defining A), and $h(u) : U \rightarrow [0, 1]$, $u \in U$, defining the membership value of u to A. Fuzzy IP (FIP) usually follows three steps: fuzzification, inference engine and defuzzification. Fuzzification maps an image from the pixel plane (PP) to the fuzzy plane (FPL) over which the image enhancement is performed (inference engine). Finally, defuzzification maps the modified image from the FP to the PP. If x_{ij} is a gray-level pixel of an original image (OI) whose brightness is $L(x_{ij}) \in [0, 255]$, setting $L_{min} = 0$ and $L_{max} = 255$, then the image can be formalized as

$$\begin{cases} \sum_{i=1}^{M-1} \sum_{j=1}^{N-1} [x_{ij}, L_{norm}(x_{ij})] \\ \forall x_{ij} \in OI, \quad i = 0, \dots, M - 1, j = 0, \dots, N - 1, \end{cases}$$

where $L_{norm}(x_{ij}) = y_{ij} = \frac{L(x_{ij})}{L_{max}} \in [0, 1]$, $i = 0, 1, \dots, M - 1$, and $j = 0, 1, \dots, N - 1$ are the normalized gray

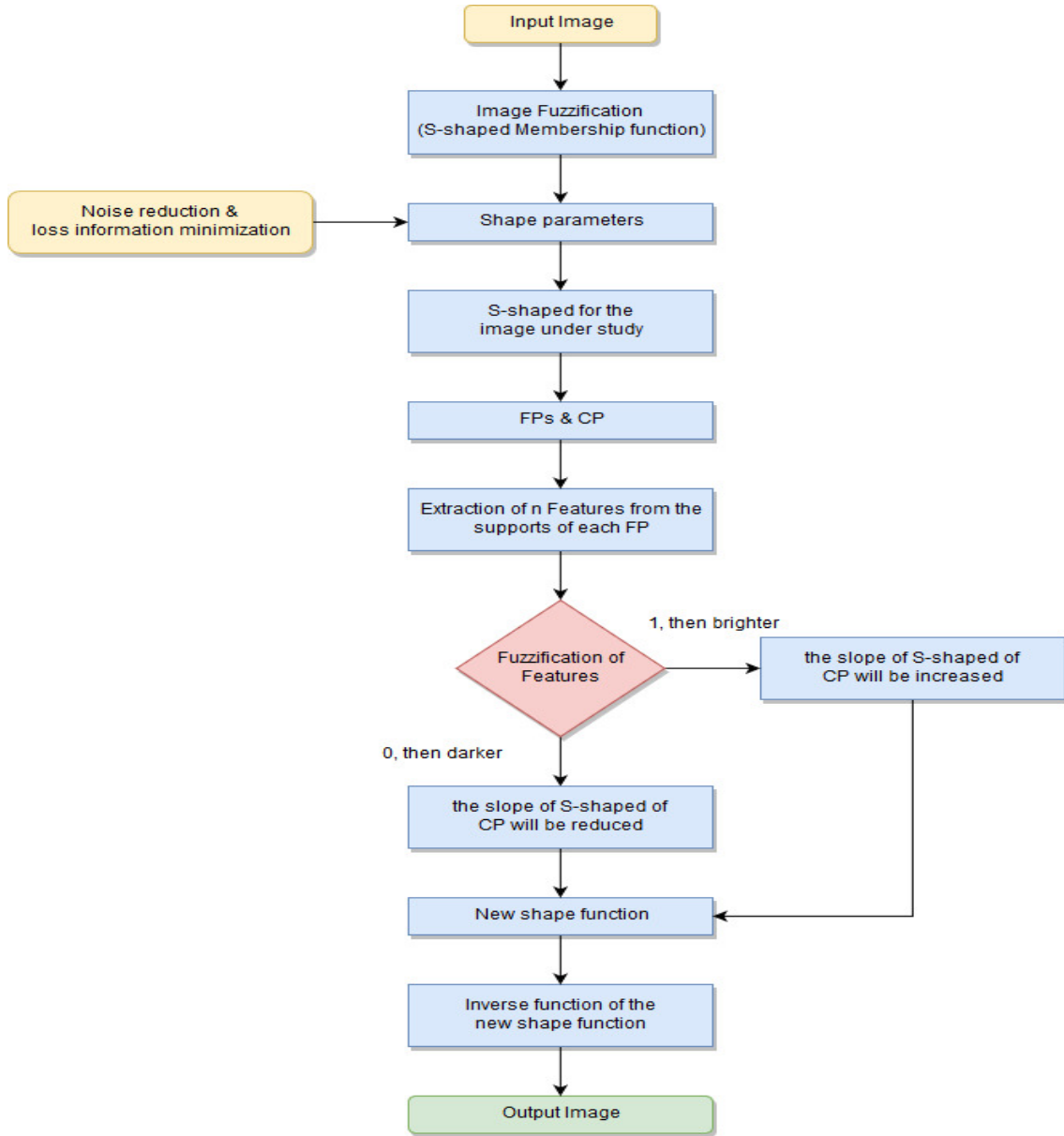


FIGURE 3. Flow chart of the proposed procedure.

levels. In addition, a fuzzy MF, $h_{ij} = h(y_{ij}) : I \rightarrow [0, 1]$, formalizes the membership of y_{ij} to OI such that:

$$\begin{cases} h_{ij} = 0 & \text{if } y_{ij} \notin OI \text{ (totally),} \\ h_{ij} = 1 & \text{if } y_{ij} \in OI \text{ (totally).} \end{cases}$$

Intermediate values of h_{ij} indicate partial membership of the related pixel. Therefore, CE can be defined as an h_{ij} transformation to obtain the modified image on the FPL .

III. MATERIALS AND METHODS

To explain the details of the proposed procedure, in this section, we introduce some definitions exploited in the following.

Definition 1 (Fuzzy Contrast Enhancement (FCE)): By considering *dark*, *light* and *gray* as instances of fuzzy quantities, we define a smooth shape function, $S(\cdot)$, that, starting from a normalized OI , fuzzifies the information content and modifies its values:

$$S = S(h_{ij}) \quad h_{ij} \in OI \text{ normalized,}$$

to obtain an image on a FPL over which CE is performed. Thus, the shape parameters, $S(h_{ij})$, have to be adaptively computed to achieve the best performance. In particular, if \underline{c} is a k -dimensional vector defining such parameters, then S is dependent on both h_{ij} and \underline{c} . In this paper, \underline{c} is computed via noise reduction and information loss minimization.

Definition 2 (Fuzzy Paches (FPs) and Supports): By FPs, we denote geometrical spots, e.g., rectangular spots that are partially superimposed and completely cover S (Fig. 1). If $|\{FPs\}| = m$, then

$$\{FPs\} = \{(FP)_j, j = 1, \dots, m \ni (FP)_j \subseteq U \times [0, 1]\}.$$

The following characterization ensures partial overlap among consecutive FPs: 1) $\{FPs\}$ are a covering of the S -function (labeled by A); and 2) consecutive FPs are partially superimposed. Formally:

$$\bigcup_j (FP)_j \supseteq A; (FP)_k \cap (FP)_l \neq \emptyset, \quad k = l + 1.$$

We orthogonally project each FP_j over both U and $[0, 1]$ and determine closed and limited $Support_j$ and u_j characterized by (Fig. 1):

$$\begin{cases} (FP)_j = Support_j \times u_j \\ Support_j \subset U \\ u_j \subseteq [0, 1]; \quad Support_k \cap Support_l \neq \emptyset; \quad u_k \cap u_l \neq \emptyset, \end{cases}$$

with $k = l + 1$.

Definition 3 (Crucial Point (CP)): For the FPL, the membership value equal to 0.5 is the most critical because it is the site of maximum uncertainty. Therefore, this point plays a major role in the proposed procedure. Formally, we define CP as the h_{ij} value corresponding to the fuzzy value equal to 0.5, i.e., $CP = h_{ij} \ni S(h_{ij}) = 0.5$. $S(h_{ij})$ is modified around this point, as detailed below. Each $Support_j$ must be fuzzified and, after the extraction of an n -dimensional vector of features, transformed into a point falling in a particular n -dimensional space ($n - UKC$).

Definition 4 (Features (Fs) of Supports): $\forall Support_j$ we extract n statistical F to characterize each by a set of real parameters. Therefore, $Features_j \in \mathbb{R}^n \quad \forall j = 1, \dots, m$ have to be fuzzified, for example, by a sigmoidal function, $Sig([\cdot])$,¹ whose argument $[\cdot]$ is n -dimensional to obtain the fuzzified features (FFs) that, inside the $n - UKC$, represent n -dimensional points so that FCE is treated by computation of distances inside the $n - UKC$ (Fig. 2). The FFs can be formalized as

$$Sig([Features_j]) = [FF_j] \in n - UKC, \quad \forall j = 1, \dots, m.$$

Definition 5 (Kosko's Hypercube $n - UKC$): Kosko's hypercube ($n - UKC$) is a unitary orthogonal n -dimensional cube [28]: $n - UKC = [0, 1]_1 \times [0, 1]_2 \times \dots \times [0, 1]_n \subseteq \mathbb{R}^n$. Each $[FF_j]$ inside $n - UKC$ is a point, FF_j , such that:

- 1) the nearer FF_j is to point zero (0_{n-UKC} , maximal darkness point), the darker FP_j is considered to be;
- 2) the nearer FF_j is to the unitary point (1_{n-UKC} , maximal brightness point), the brighter FP_j is.

Therefore, both 0_{n-UKC} and 1_{n-UKC} strongly influence the shape function so that $distance(FF_j, 1_{n-UKC})$ indicates how

¹Other types of fuzzifying function can be considered depending on the application under study.

far FF_j is from total brightness and $distance(FF_j, 0_{n-UKC})$ measures how far FF_j is from total darkness.

Since CP is the maximal fuzziness point, FCE is obtained after considering, on CP, the tangent line to the S -shaped membership function ($S - SMF$) (t -line for short), whose slope increases (or decreasing) according to the following criterion: if FF_j is close to $1_{n-UKC} \Rightarrow FP_j$, it is brighter:

$$Slope_{new} = Slope_{old} \times \frac{distance(FF_j, 1_{n-UKC})}{distance(FF_j, 0_{n-UKC})}; \quad (1)$$

conversely, if FF_j is close to $0_{n-UKC} \Rightarrow FP_j$, it is darker:

$$Slope_{new} = Slope_{old} \times \frac{distance(FF_j, 0_{n-UKC})}{distance(FF_j, 1_{n-UKC})}, \quad (2)$$

where $Slope_{old}$ and $Slope_{new}$ are the t -line slopes before and after (1) and (2), respectively. (1) and (2) can be taken into account iff both denominators are $\neq 0$, and it is imperative to guarantee that $Slope_{new}$ is still limited. The following theorem, whose proof is reported in the appendix, meets these requirements.

Theorem 1 (): If Eqs. (1) and (2) are satisfied, surely:

$$\begin{cases} distance(FF_j, 0_{n-UKC}) \neq 0 \\ distance(FF_j, 1_{n-UKC}) \neq 0 \\ 0 < Slope_{new} < \infty. \end{cases} \quad (3)$$

A. MAPPING OF THE OI INTO FPL

For our application, to achieve good smoothing transition quality among gray levels, let us consider an S -shaped membership function ($S - SMF$) defined as follows (Fig. 1):

$$\begin{cases} h_{ij} = 0, & \text{if } 0 \leq y_{ij} \leq c_1; \\ h_{ij} = \frac{(y_{ij} - c_1)^2}{(c_2 - c_3) \cdot (c_3 - c_1)} & \text{if } c_1 \leq y_{ij} \leq c_2; \\ h_{ij} = 1 - \frac{(y_{ij} - c_3)^2}{(c_3 - c_2) \cdot (c_3 - c_1)} & \text{if } c_2 \leq y_{ij} \leq c_3; \\ h_{ij} = 1 & \text{if } y_{ij} \geq c_3, \end{cases} \quad (4)$$

where c_1 , c_2 and c_3 are the $S - SMF$ shape parameters that are determined adaptively as described below.

B. ADAPTIVE PROCEDURE FOR SETTING THE $S - SMF$ PARAMETERS

Since c_1 and c_3 are external values, they are adaptively computed by noise reduction while c_2 is evaluated by entropy maximization (located around a gray level whose membership value falls in the neighborhood of 0.5). Starting from [19], let us consider the gray level distribution over the FPL whose histogram $H_{ist}(h)$ has s peaks, $\{H_{ist}(h_1), H_{ist}(h_2), \dots, H_{ist}(h_s)\}$, with weighted mean

$$(Y_{ist_{max}}(h))_{mean} = \frac{\sum_{i=1}^s H_{ist_{max}}(h_i) \cdot h_i}{\sum_i h_i},$$

where $Y_{istmax}(h)$ is the height of the center of gravity of the histogram.² Starting from the s peaks, we consider t peaks ($t \leq s$) larger than $(Y_{istmax}(h))_{mean}$, ruling out the others because they are less meaningful. From this new set of peaks, we select the minimum and maximum values, $(Y_{istmin}(h_1))_{mean}$ and $(Y_{istmax}(h_t))_{mean}$, so that the gray levels with h such that $h < (Y_{istmin}(h_1))_{mean}$ are considered as the background while gray levels with $h > (Y_{istmax}(h_t))_{mean}$ are considered as noise preserving. Taking into account that the loss of information of an OI occurs in the neighborhood of the endpoints of the gray level range, we identify two characteristic gray levels to determine GL_1 and GL_2 , such that $GL_1 < GL_2$, and consider the ranges $[h_{min}, GL_1]$ and $[GL_2, h_{max}]$ on which the information loss is equal to a fixed “ad hoc” value $0 < f_1 < 1$. GL_1 and GL_2 are produced by verification of the following equalities:

$$\sum_{i=h_{min}}^{GL_1} H_{ist}(i) = f_1; \quad \sum_{i=GL_2}^{h_{max}} H_{ist}(i) = f_1.$$

Finally, c_1 and c_3 are obtained by

$$c_1 = \frac{h_{max} - h_{min}}{2} + h_{min} \text{ and } c_3 = \frac{h_{max} - h_k}{2} + h_k$$

with two further conditions:

$$\text{if } (c_1 > GL_1) \rightarrow c_1 = B_1; \quad \text{if } (c_3 > GL_2) \rightarrow c_3 = B_1.$$

After computing c_1 and c_3 , if $c_2 \in (c_1, c_3)$, the entropy of an FI will depend on both FI and c as:

$$\text{Entropy} = \text{Entropy}(FI, c_1, c_2, c_3) \tag{5}$$

with

$$S(h_{min}) \leq c_1 < c_2 < c_3 \leq S(h_{max}),$$

where $c_{2_{optimal}}$ is computed by maximization of Eq. (5).

C. EDITING OF THE S-FUNCTION

We now split $[c_1, c_3]$ into three superimposed sub-ranges to create FP_1, FP_2 and FP_3 (Fig.1): FP_1 and FP_3 represent darker and brighter zones of the OI , respectively. To underline the fuzzy nature of the procedure, FP_1 and FP_3 have a common vertex located at U with fuzzy membership equal to 0.5 (maximal fuzziness) so that FP_2 is centered on a fuzzy membership equal to 0.5 characterizing the gray zones. In this way,

$$\begin{cases} \text{Support}_{FP_1} = \frac{c_2 + c_3}{2} - \frac{c_1 + c_2}{2}; \\ \text{Support}_{FP_2} = c_2 - c_1; \\ \text{Support}_{FP_3} = c_3 - c_2. \end{cases}$$

²In [19], this computation was obtained by

$$(Y_{istmax}(h))_{mean} = \frac{1}{s} \sum_{i=1}^s H_{istmax}(h_i),$$

but no weighted mean of $H_{istmax}(h_i)$ was taken into account, which could be determinant for a correct FCE .

$\forall \text{Support}_j, j = FP_1, FP_2, FP_3$, we address our attention to the fourth-order statistics exploiting F_j as a $4 - D$ vector

$$\underline{F}_j = [\text{stat}_i(\text{Support}_j)], \quad i = 1, 2, 3, 4,$$

where $\text{stat}_1(\cdot)$ is the arithmetic average; $\text{stat}_2(\cdot)$ is the variance; $\text{stat}_3(\cdot)$ is the skewness; and $\text{stat}_4(\cdot)$ is the kurtosis index. Therefore, \underline{F}_j (with $j = FP_1, FP_2, FP_3$) represents three points over \mathbb{R}^4 that, after fuzzification by a sigmoid function, for $j = FP_1, FP_2, FP_3$, gives us three $4 - D$ vectors

$$\underline{FF}_j = \left[\frac{1}{1 + e^{-u(F_j - w)}} \right] \in 4 - UKC \quad i = 1, 2, 3, 4,$$

so the new $S - SMF$ is edited by evaluating the distances between \underline{FF}_j and both $0_n - UKC$ and $1_n - UKC$. After evaluating \underline{FF}_j ($j = FP_1, FP_2, FP_3$), the mutual distances in $n - UKC$ can easily be computed as

$$\begin{cases} \text{distance}(\underline{FF}_m, \underline{FF}_n) = \|\underline{FF}_m, \underline{FF}_n\|_2 \\ m, n = FP_1, FP_2, FP_3. \end{cases}$$

Clearly, two of the following cases could be true. 1) If FP_1 represents a brighter zone rather than a darker zone, then the following inequality holds:

$$\|\underline{FF}_1 - \underline{FF}_2\|_2 > \|\underline{FF}_1 - \underline{FF}_3\|_2. \tag{6}$$

2) If FP_1 is related to a darker zone rather than a brighter zone, the inequality assumes the following form:

$$\|\underline{FF}_1 - \underline{FF}_2\|_2 < \|\underline{FF}_1 - \underline{FF}_3\|_2. \tag{7}$$

The editing of the S -function is performed through anticlockwise rotation of the tangent line ($t - line$ for short) to the $S - SMF$ in CP (point of overlap between darkness and brightness, where FP_1, FP_2, FP_3 and $S - SMF$ coexist). If FP_1 is bright rather than dark, \underline{FF}_1 is both next to $1_n - UKC$ and further away from $0_n - UKC$, increasing the slope of $t - line$ by a factor equal to:

$$\frac{\|\underline{FF}_1 - 1_n - UKC\|_2}{\|\underline{FF}_1 - 0_n - UKC\|_2}. \tag{8}$$

Similarly, if FP_1 is considered darker rather than brighter, then the $t - line$ slope is decreased by a factor equal to:

$$\frac{\|\underline{FF}_1 - 0_n - UKC\|_2}{\|\underline{FF}_1 - 1_n - UKC\|_2}. \tag{9}$$

In any case, a new $t - line$, labeled $rot - line$, is adaptively generated. Over the Cartesian plane, the CP coordinates can be written as $(c_2, (c_2 - c_1)/(c_3 - c_1))$, so the $t - line$ equation can be written as

$$t - line(h_{ij}) = \frac{c_2 - c_1}{c_3 - c_1} + \frac{2}{c_3 - c_1} (h_{ij} - c_2).$$

Taking into account both (6) and (8), the rot -line slope assumes the form

$$rot - line_{Slope} = t - line_{Slope} + t - line_{Slope}$$

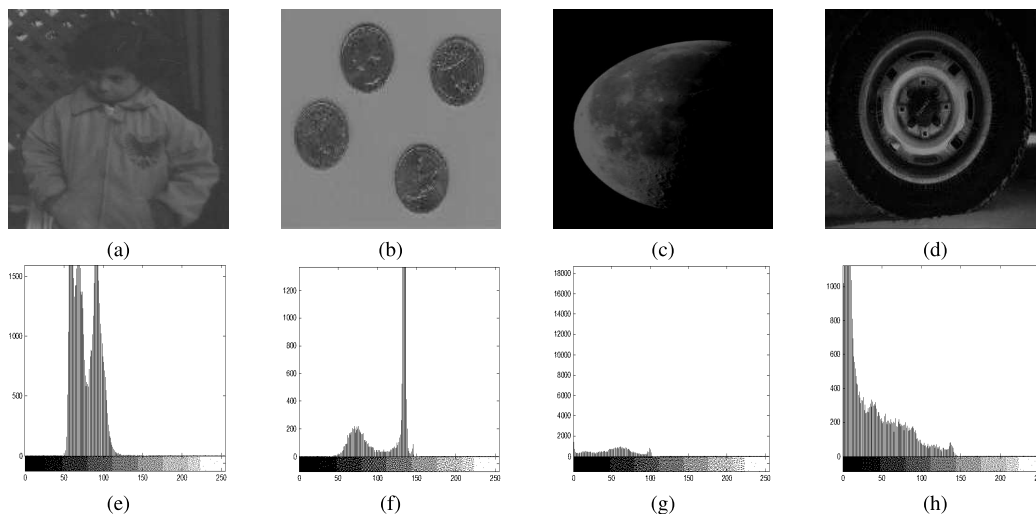


FIGURE 4. (a) Portrait representative of Class I (Image 1 in text); (b) A set of coins representing Class II (Image 2 in text); (c) Small bright areas in large dark areas are highlighted in this image chosen to represent Class III (Image 3 in text); (d) Image with high alternation of bright and dark areas chosen to represent Class IV (Image 4 in text). (e) to (h) show the histograms for (a) to (d), respectively.

$$\frac{\|FF_1 - 1_{n-UKC}\|_2}{\|FF_1 - 0_{n-UKC}\|_2} = \frac{2}{c_3 - c_1} \left(1 + \frac{\|FF_1 - 1_{n-UKC}\|_2}{\|FF_1 - 0_{n-UKC}\|_2} \right),$$

so its equation immediately follows:

$$rot - line(h_{ij}) = \frac{c_2 - c_1}{c_3 - c_1} + \frac{2}{c_3 - c_1} \cdot \left(1 + \frac{\|FF_1 - 1_{n-UKC}\|_2}{\|FF_1 - 0_{n-UKC}\|_2} \right) (h_{ij} - b).$$

Finally, to fully define $S - SMF$, we must identify points P_1 and P_2 , and the intersection of the *rot*-line with both the line of membership values equal to zero and the line of membership values equal to unity (Fig. 1):

$$P_1 = \left(\underbrace{c_2 + \frac{c_1 - c_2}{2 \cdot \left(1 + \frac{\|FF_1 - 1_{n-UKC}\|_2}{\|FF_1 - 0_{n-UKC}\|_2} \right)}}_{h_{P_1}}, 0 \right) \text{ and}$$

$$P_2 = \left(\underbrace{\left(1 - \frac{c_2 - c_1}{c_3 - c_1} \right) \cdot \frac{c_1 - c_2}{\frac{2}{c_3 - c_1} \left(1 + \frac{\|FF_1 - 1_{n-UKC}\|_2}{\|FF_1 - 0_{n-UKC}\|_2} \right)}}_{h_{P_2}}, 1 \right),$$

where the gray levels are indicated by h_{P_1} and h_{P_2} , $h_{P_1} < h_{P_2}$.³ Finally, the new $S - SMF$ function, shown in Fig. 1,

³ h_{P_1} will never be equal to h_{P_2} by virtue of the previous theorem.

can be written as follows:

$$\begin{cases} 0 & h_{ij} \leq h_{P_1} \\ \frac{c_2 - c_1}{c_3 - c_1} + \frac{2}{c_3 - c_1} \cdot \left(1 + \frac{\|FF_1 - 1_{n-UKC}\|_2}{\|FF_1 - 0_{n-UKC}\|_2} \right) \cdot (h_{ij} - p_2) & h_{P_1} \leq h_{ij} \leq h_{P_2} \\ 1 & h_{ij} \geq h_{P_2}. \end{cases} \quad (10)$$

Finally, we obtain the enhanced *OI* by defuzzification. Specifically, by applying the inverse function of (10), we obtain the fuzzy gray levels (ranging over [0, 1]); then, we convert the fuzzy gray levels into the equivalent levels ranging over [0, 255]. When patch FP_1 is considered darker rather than brighter, it is sufficient to follow the same process (including fuzzification procedure), by carefully considering the inequality (7) with decreased factor (9), to obtain similar equations.

IV. EXPERIMENTAL RESULTS

Four distinct classes of images of the same size with different contrast characteristics are considered. The database includes 35 low-contrast images belonging to four classes:

- 1) Class I contains low-contrast portraits;
- 2) Class II contains images with few areas with medium gray levels;
- 3) Class III contains images with large dark areas and small bright areas;
- 4) Class IV contains images with alternating small bright and dark areas.

The procedure is implemented on an Intel Core 2 1.47 GHz CPU in MatLab R2013. The results for the different images are comparable, so we present the results of a single image for each Class (labeled Image 1, Image 2, Image 3 and

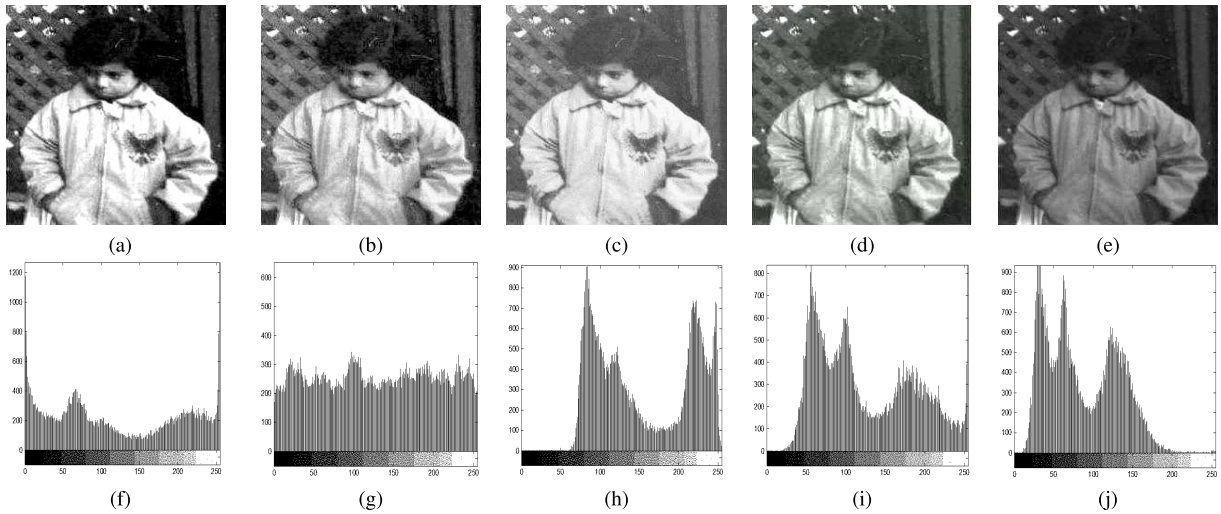


FIGURE 5. CE of Image 1 by means of (a) Prop. Proc., (b) HE, (c) R, (d) P and (e) C, and their histograms from (f) to (j), respectively.

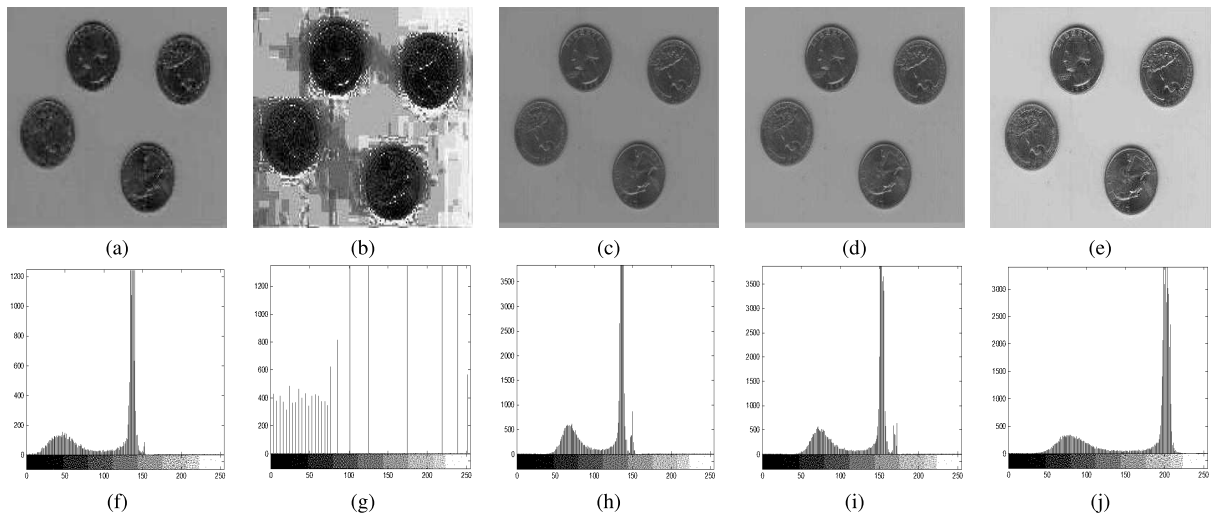


FIGURE 6. CE of Image 2 by exploiting (a) Prop. Proc., (b) HE, (c) R, (d) P and (e) C. (f) to (j) show the histograms of (a) to (e), respectively.

TABLE 1. Comparison of the results obtained by the proposed procedure and by other established techniques.

		Prop. Proc.	<i>C</i> [19]	<i>L</i> [21]	<i>R</i> [18]	<i>HE</i>		Prop. Proc.	<i>C</i> [19]	<i>L</i> [21]	<i>R</i> [18]	<i>HE</i>	
Image 1	EM	5.701	5.698	5.692	5.681	5.700	Image 3	EM	6.059	6.340	6.356	5.903	5.984
	PSNR	11.24	13.026	12.983	12.374	12.214		PSNR	12.853	13.654	13.056	12.965	12.785
	MLI	1.801	1.789	1.795	1.792	1.797		MLI	2.056	2.159	2.126	2.149	2.149
	MCI	1.312	1.301	1.306	1.302	1.310		MCI	1.698	1.623	1.642	1.645	1.621
Image 2	EM	4.112	6.018	5.991	5.983	5.975	Image 4	EM	5.956	5.873	5.823	5.793	5.916
	PSNR	7.965	9.251	9.121	9.138	9.132		PSNR	11.590	12.486	12.036	11.843	11.559
	MLI	2.601	2.114	2.120	2.122	2.119		MLI	1.773	1.745	1.769	1.765	1.764
	MCI	1.901	1.439	1.423	1.410	1.413		MCI	1.406	1.398	1.398	1.389	1.400

Image 4), as shown in Fig. 4 together with their histograms, which confirm the low-contrast characteristics of each image. Figs. 5a, 6a, 7a and 8a, obtained by setting $f_1 = 0.2$ [13],

shows the results of the proposed procedure in which, as indicated by the histograms, a substantial increase in contrast is detectable in all images, although we note some areas in

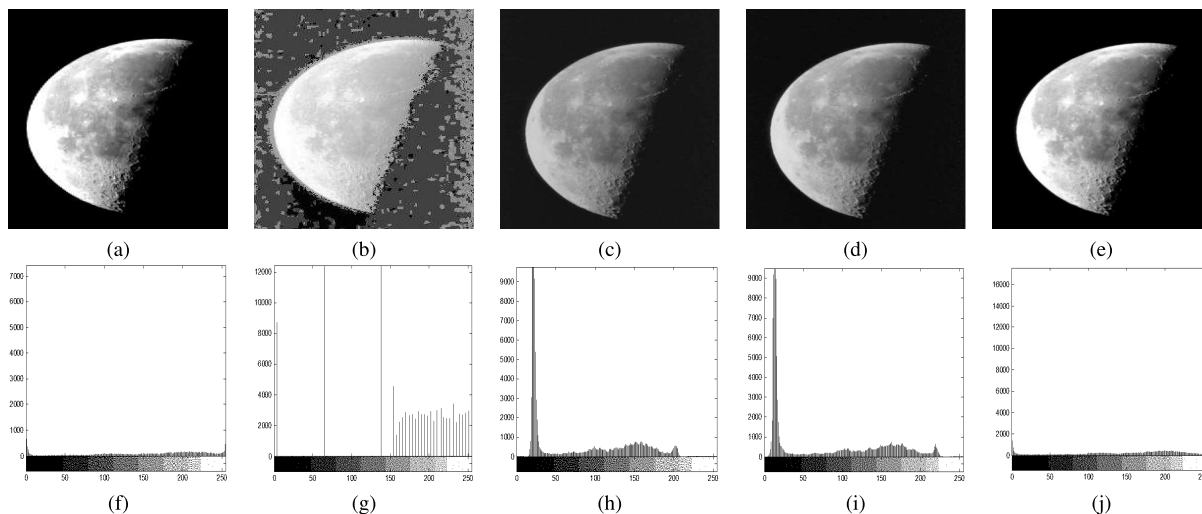


FIGURE 7. CE of Image 3 obtained by (a) Prop. Proc., (b) HE, (c) R, (d) P and (e) C, and their histograms, respectively, from (f) to (j).

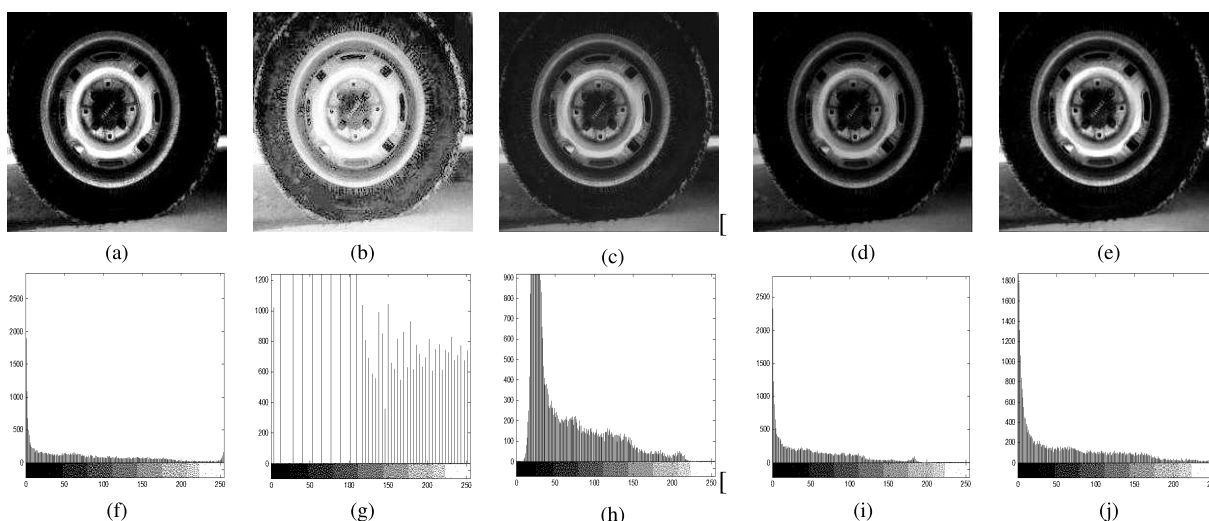


FIGURE 8. CE of Image 4 obtained by (a) Prop. Proc., (b) HE, (c) R, (d) P and (e) C for CE of Image 4, and their histograms from (f) to (j), respectively.

which over- and under-enhancement occur because the edited $S - SMF$ presents two cusps. However, the good quality of the obtained contrast compensates the presence of these small altered areas. After implementing the proposed procedure, the principal elements of the images are enhanced, and the contours, shadows and details are highlighted. The performance of the proposed procedure (*Prop. Proc.*) is evaluated by comparing the results with those of Cheng and Xu [19], Li et al. [21], Reshmalakshmi and Sasikumar [18] and histogram equalization (*HE*) approaches. Moreover, in each $4 - UKC$, the location of the individual *FP* denotes the tendency for under- and/or over-enhancement (see Fig. 2 for Image 1).⁴ The obtained *CE* is also evaluated based on

⁴The closer the *FP* is to 0_{4-UKC} , the higher the risk of under-enhancement. Conversely, the closer *FP* is to 1_{4-UKC} , the higher the risk of over-enhancement

four metrics, namely, entropy measure (*EM*), peak signal-to-noise ratio (*PSNR*), measure of luminance index (*MLI*) and measure of contrast index (*MCI*), whose numerical results are displayed in Table 1. The high values of *EM* and *MLI* together with the small values of *PSNR* indicate a high-quality contrast. However, in confirmation of the risk of under-/over-enhancement, some of the *PSNR* values are lower than those produced by the other techniques. Similarly, the *MLI* values are slightly higher than those obtained by the alternative techniques. Figs. 5b, 6b, 7b and 8b show the results of the *HE* procedure. This procedure produced good results for images belonging to Class III, in which large dark areas and small bright areas are present (Fig. 7b). The other images (Figs. 5b, 6b and 8b), although conservative in detail, are lacking in clarity, as numerically confirmed by the results reported in Table 1. Figs. 5c, 6c, 7c and 8c

present the results of Reshmalakshmi’s approach [18]. Relative to the treated images, the obtained contrast lacks in quality, especially for highly detailed images (Figs. 5c and 8c), while a good contrast is produced in the remaining images. Even in these cases, the numerical results show compliance with the qualitative results (Table 1). The following set of Figures, 5d, 6d, 7d and 8d, is produced by Li’s approach [21]. This approach does not produce high-quality images when there is a high level of detail in the *OI*, as indicated by both qualitative and numerical analysis. However, the procedure produced good results for images in Class III (Fig. 7d, Table 1). Finally, Figs. 5e, 6e, 7e and 8e show the results produced by Cheng’s method [19].

V. CONCLUSIONS AND PERSPECTIVES

In this paper, the problem of contrast enhancement for gray level images is solved with a new fuzzy procedure. Starting from a pixel-by-pixel scan of a normalized gray level image, a smooth S-shaped membership function is adaptively set according to both noise reduction and information loss minimization. A set of partially overlapped fuzzy patches cover the input-output space of the S-function; after appropriate fuzzification, a set of statistical features are extracted from the supports to identify the critical point characterized by the maximum fuzziness. This point give us the coexistence of fuzzy patches and the S-function and provides the point where the S-function will change its slope to implement contrast enhancement. Specifically, the fuzzified statistical features are considered as points inside a fuzzy unit hypercube, and fuzzy contrast enhancement is achieved by a weighted calculation of the distance among these points and the vertexes of the hypercube representing both the maximum brightness and maximum darkness. This procedure is implemented and applied to four classes of low-contrast images with different characteristics and the performance is evaluated with respect to four objective metrics and standard histogram comparisons. The experimental results are encouraging, as clearly demonstrated by comparison with other established fuzzy techniques and the standard histogram equalization approach. However, it should be noted that the proposed technique can produce over- and under-enhancement phenomena due to the non-differentiability of the S-function in two cusps. Therefore, future effort should be focused in this direction.

**APPENDIX
PROOF OF THEOREM 1**

Let us consider $(\underline{c}, <) = (\{c_1, c_2, c_3\}, <)$. Since \underline{c} is a vector of brightness values, then

$$h_{min} < c_1 = \min(\underline{c}) < c_2 < c_3 < \dots < c_{k-1} < c_k \\ = \max(\underline{c}) < h_{max}$$

holds. If $[h_{min}, c_1]$ represents the total darkness, its membership degree is null. Therefore, $Support_1 = [h_{min}, c_1]$ gains in significance and $S(Support_1) = 0$. Clearly, $[c_{j-1}, c_j]$ with $j = 1, \dots, k$ can be considered as supports: $Support_j = [c_{j-1}, c_j]$

with $j = 1, \dots, k$ and $S(Support_j) \neq 0$ (particularly, > 0). Since $S(Support_1) = 0$ and $S(Support_j) \neq 0, j = 2, \dots, k$, we extract k n -dimensional vectors of features, $F(Support_j)$, but $S(Support_j) > 0$ implies $F(Support_j) \neq 0$ and $F(Support_j) > 0$ is not necessary. Finally, $F(Support_1) = \underline{0}$ and $F(Support_j) \neq \underline{0}, j = 2, \dots, k$. The k vectors require fuzzification, which by means of sigmoidal functions, gives us the following characterization:

$$\begin{cases} FF_{Support_1} = Sig(\underline{0}) = \underline{0} \\ FF_{Support_j} = Sig(F(Support_j)) > \underline{0}, j = 2, \dots, k. \end{cases}$$

Note that $FF_{Support_j}$ is greater than zero (for $j = 2, \dots, k$) since fuzzification produces a mapping ranging into $n - UKC$. To obtain (3), it is sufficient to observe that, in $n - UKC$ and by a norm, for $j = 2, \dots, k$, the relation

$$distance(FF_j, 0) = ||Sig(F(Support_j)) - Sig(\underline{0})||$$

holds and, by the considerations specified above, gives us

$$distance(FF_j, 0) \neq 0.$$

To prove that $distance(FF_j, 1_{n-UKC}) \neq 0$, it is sufficient to repeat the same path, resulting in the following implication:

$$\begin{cases} S(Support_1) = 1 \\ S(Support_j) \neq 0 \\ j = 2, \dots, k \end{cases} \Rightarrow \begin{cases} F(Support_1) = \underline{1} \\ F(Support_j) \neq \underline{0} \\ j = 2, \dots, k \end{cases}$$

and, after fuzzification by the *Sig* function, it is easy to get

$$\begin{cases} FF_{Support_1} = Sig(\underline{1}) = \underline{1} \\ FF_{Support_j} = Sig(F(Support_j)) \neq \underline{0}, j = 2, \dots, k. \end{cases}$$

Finally, the purpose is obtained in terms of norms:

$$distance(FF_j, 1) = ||Sig(F(Support_j)) - Sig(\underline{1})|| \neq 0.$$

To prove that $0 < slope_{new} < \infty$, we observe that both $distance(FF_j, 1_{n-UKC})$ and $distance(FF_j, 0_{n-UKC})$ are inner ranges of $n - UKC$, so they suffer from limitations

$$\begin{cases} 0 < distance(FF_j, 1_{n-UKC}) < \sqrt{n} \\ 0 < distance(FF_j, 0_{n-UKC}) < \sqrt{n}, \end{cases}$$

(where \sqrt{n} represents the diagonal length in $n - UKC$), that when divided by each other give

$$0 < \frac{distance(FF_j, 1_{n-UKC})}{distance(FF_j, 0_{n-UKC})} < \infty,$$

from which the assertion follows.

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