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# Impact of Co-Channel Interference on the Outage Performance Under Multiple Type II Relay Environments

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**ABSTRACT** In this paper, through an exact analysis of the outage probability, we investigate the impact of co-channel interference (CCI) on the outage performance of type II (or user equipment) relay under multiple-relay environments considering the selection combining-based relay selection scheme with the decode-and-forward protocol. We consider the signal to interference plus noise ratio (SINR) over both independent and identically distributed and independent but non-identically distributed fading channels. To fully take into account the effect of CCI, we adopt a more practical parameter such as the CCI coefficient. The major difficulty in the analysis resides in the determination of the statistics of the output SINR. To settle this problem, we first present the general but relatively simplified expressions for the statistics and then the related outage probability in closed-form. Furthermore, to consider more practical scenario, based on the fact that the number of participating relays can be random, we investigate the average outage probability by averaging the number of participating relays.

**INDEX TERMS** Type II relay, co-channel interference (CCI), selection combining (SC), signal to interference plus noise ratio (SINR), order statistics.

### I. INTRODUCTION

In the latest cellular standards, e.g., LTE-Advanced, two types of relaying strategies, i.e., type I and type II, are defined in the 3rd Generation Partnership Project (3GPP) [1]–[4] to increase the cell coverage and the data rate without creating undue inter-cell interference [5]–[9]. In specific, as shown in Fig. 1, the type II relay, so called user equipment (UE) relay scheme, forwards the overheard messages to increase the data rate of the end user having weaker signal quality from the serving base station (or eNodeB) in case normally located near cell boundary or in indoor wireless environments while the type I relay forms an independent cell with a small coverage for the coverage extension. With the type II relay, the end user may also be the UE destination  $(\mathcal{D})$  node. Therefore, the relayto-destination  $(\mathcal{R}-\mathcal{D})$  link must be operated in an open-loop (or transparent) mode because of the absence of a dedicated control channel, which means the  $\mathcal{R}$ - $\mathcal{D}$  link channel state information (CSI) is not available at both  $\mathcal{R}s$  and  $\mathcal{D}$ , and each  $\mathcal{R}$  appears transparent to  $\mathcal{D}$ .<sup>1</sup>

Since the early 1950s, to mitigate the effects of multipath fading in wireless communication systems, one form or another of diversity combining schemes, e.g., selection combining (SC), equal-gain combining (EGC), maximalratio combing (MRC), and generalized selection combining (GSC), has been devised and used for its own purpose [10]–[14]. Among these schemes, SC has been widely used for its practical benefit. It offers a commensurable performance with the least complexity since the processing is performed on only one diversity branch with the strongest signal-to-noise ratio (SNR) among the branches and no channel information is required. These diversity schemes can also be applied to the cooperative communication systems [8].

<sup>1</sup>In what follows,  $\mathcal{R}(\text{or }\mathcal{R}s)$  means UE relay(or relays).



FIGURE 1. Example of Type I and Type II relays.

For example, the conventional SC scheme can be adopted to the relay selection schemes for the type I relay to achieve the coverage extension with the least complexity.<sup>2</sup>

In a similar way, the SC-based relay selection schemes can be adopted for the type II relay. However, the SC-based relay selection schemes developed for the type I relay cannot be directly applied to the type II relay for the following reasons. In the case of the cooperative communication systems with the type II relay [1]–[3], when the source (S) transmits a message to  $\mathcal{D}$ ,  $\mathcal{R}s$  can overhear and then retransmit this message to  $\mathcal{D}$ . However, especially under multiple-relay environments,  $\mathcal{R}$  with the best  $\mathcal{R}$ - $\mathcal{D}$  link as well as other unscheduled  $\mathcal{R}$ s may keep forwarding overheard messages to  $\mathcal{D}$  on a common channel because all UE nodes communicate with each other under the assumption of the open-loop transmission mode. As results, the desired signal at  $\mathcal{D}$  received from the best  $\mathcal{R}$  can be interfered by the signals retransmitted from all other  $\mathcal{R}$ s. These effects are called co-channel interference (CCI) leading to limiting the capacity and the system performance. Therefore, to achieve the promising gain which can be obtained by applying the diversity scheme under multiple-type II relay environments, this CCI has to be carefully controlled and, before that, the accurate analysis of the CCI effect should precede. Note that in the type II relaybased communication, although the control messages cannot be directly exchanged among  $\mathcal{R}$ s as well as between  $\mathcal{R}$  and  $\mathcal{D}$ ,  $\mathcal{R}$  can overhear the ACK/NACK signal periodically sent from  $\mathcal{D}$  to  $\mathcal{S}$  [2]–[4], [8]. Hence, it is possible to select  $\mathcal{R}$  with the best  $\mathcal{R}$ - $\mathcal{D}$  link from the estimation of the  $\mathcal{D}$ - $\mathcal{R}$  link by overhearing these reference signals (i.e., ACK/NACK) exchanged between S and D assuming that both  $\mathcal{R}$ -D and D- $\mathcal{R}$  links are reciprocal [2]. However, in practice, especially under multiple-relay environments, the received signal at  $\mathcal{D}$  through  $\mathcal{R}$  with this best  $\mathcal{R}$ - $\mathcal{D}$  link is susceptible to be interfered by the retransmitted signals from other unscheduled  $\mathcal{R}$ s.

Based on the above observations, it is important to analyze and quantify these effects on the system performance preferentially while the study of the techniques or mechanisms to achieve interference reduction in practical scenarios such as directional antenna and proper scheduling protocols needs to be performed. To consider the effect of CCI, the signalto-interference-plus-noise ratio (SINR) which measures the ratio between the desired signal power of the best  $\mathcal{R}$  and the amount of noise and interference generated by all the other  $\mathcal{R}$ s, will be considered in the performance evaluation. Note that the theoretical analyses based on the signal-tointerference ratio (SIR) are only valid in an interferencelimited environment. Therefore, to consider more practical and general cases, the investigation of SINR is more essential. In practice, these co-channel interferences should be nonidentical due to, for example, the random distribution of  $\mathcal{R}s$ between S and D, the different adjacent multipath routes with the same path loss, and the resulting unbalance among paths. As a result, the performance analysis over more practical fading conditions needs to be considered. Hence, we analyze the average outage probability by averaging the previously considered conditional outage probability at a given fixed number of participating Rs for both independent and identically distributed (i.i.d.) and independent but non-identically distributed (i.n.d.) Rayleigh fading channels.

## A. MAIN CONTRIBUTION

Recently, the impacts of CCI on the performance of the opportunistic decode-and-forward (DF) cooperative networks [16] and the DF cooperative networks with multiple relays [17] operating over i.i.d. Rayleigh fading channels have been addressed. Note that although the CCI issue in the previous works in [16] and [17] dealing with the type I relay and the fundamental problem in this paper treating the type II relay seem to be very similar, the derived results or mathematical approaches in [9] and [10] cannot be directly applied to our case under consideration. Hence, we need new analytical approach, mathematical formula, channel models, and so on. With these motivations in mind, we investigate the impact of CCI on the outage performance of the SC-based type II relay under multiple-relay environments. To fully take into account the effect of CCI under the SINR constraint, as a more practical fact, we consider the CCI coefficient.

The main contributions and points of difference between the previous works and this work are briefly summarized as follows:

• In real-life applications, the average fading power may vary because the branches are sometimes unbalanced and the communication system is sometimes operating over frequency-selective channels with a non-uniform power delay profile or channel multipath intensity profile, i.e., the average SNRs of the diversity paths are not necessarily the same. To deliberate a more

<sup>&</sup>lt;sup>2</sup>Note that, with the conventional relay selection scheme [15], the best relay will send a short flag packet to indicate its presence when its timer expires, and other relays can choose to back off as soon as they receive this flag packet. As a result, in this case, the co-channel interference (CCI) among relays may not exist. However, for the case of the type II relay, the relay and the destination cannot directly exchange a control message. Therefore, even though the best relay transmits the flag packet, other relays cannot fundamentally receive this signal and it will eventually lead CCI.

comprehensive characteristic, we develop both a closedform expression and a general/simplified expression for the outage analysis over Rayleigh fading channels as well as the generalized fading channels with not only i.i.d. but also i.n.d. assumptions.

- Our results are more general enough because our results are based on the SINR and also consider both identical and non-identical cases. Although the addressed interference problem and the related mathematical approach seem to be similar, the statistical model of the total interference-to-noise ratio in [16] and [17] is different from ours. More specifically, we define the interference term as the sum of ordered random variables (RVs) while in [16] and [17], this term is simply defined as a sum of i.i.d. RVs. The main challenging issue is that the analysis of the ordered RVs is much more complicated than that of the i.i.d. RVs due to the difficulty in deriving the joint statistics of the output SINR. So far, the required joint statistics such as the moment generating function (MGF) or the probability density function (PDF) of the output SINR, have been in general not available in closed- and even simple-form, and the MGF/PDF-based analysis becomes mathematically intractable since it apparently involves the multiple-fold integrals. This multiple-fold integration is tedious and complicated and it takes a long time to evaluate numerically, especially as the number of diversity branches increases. Moreover, for the i.n.d. fading cases, the required joint statistics are much more complicated than those of the i.i.d. cases. Hence, we need to carry out more sophisticated manipulations and introduce new mathematical representations to obtain the final results for the i.n.d. cases in a compact form. Fortunately, with the help of our recent results presented in [18] for the i.i.d. case and [19] for the i.n.d. case, we can accurately characterize these joint statistics. With these results, the closed-form expression of the outage probability over Rayleigh fading channels can be obtained.
- In general, the number of participating relays can be random. Based on this, we extend our analysis to the average outage probability by averaging the previously considered conditional outage probability at a given fixed number of participating relays for both i.i.d. and i.n.d. cases. Additionally, we consider SINR with the CCI cancellation coefficient while [16] and [17] considered the fundamental SINR definition.
- Further, we provide the analytical results for other general fading channel models. With these unified frameworks, one can easily investigate the performance over other fading assumptions. Even though there remain the integral expressions, it is still much simpler than the original multiple-fold integral formula.

# **II. SYSTEM AND CHANNEL MODELS**

We assume that multiple-relay environments with the DF protocol are considered, all nodes (S, D, and multiple UE ate in a half-duplex mode where the transmission occurs in two phases. More specifically, in the first listening phase, Sbroadcasts its message and  $\mathcal{D}$  receives and each  $\mathcal{R}$  overhears this message. If  $\mathcal{R}$  successfully decodes the overheard message, the second collaboration phase starts. During the second phase, S becomes inactive and each R forwards its message to  $\mathcal{D}$  on a common channel upon successful detection. Note that as shown in Fig. 1, we assume that  $\mathcal{R}s$  can overhear the reference signals exchanged between S and D [2]. More specifically, each  $\mathcal{R}$  can overhear the ACK/NACK signals periodically sent from  $\mathcal{D}$  to  $\mathcal{S}$  and such overheard signal can be used for estimating each  $\mathcal{R}$ - $\mathcal{D}$  link quality [2], [9], [20]. Further, the conventional relay selection scheme is assumed, so that  $\mathcal{R}$  with the best  $\mathcal{R}$ - $\mathcal{D}$  link quality based on the link quality participates in retransmission and  $\mathcal{D}$  may treat the signal transmitted from this selected  $\mathcal{R}$  as the desired signal. However, other  $\mathcal{R}s$  may keep the retransmission to  $\mathcal{D}$  on a common channel and it can eventually cause interference.

relays (or  $\mathcal{R}$ s)) employ a single antenna, and all  $\mathcal{R}$ s oper-

In addition, we assume a reliable feedback path between S and R. Further, it is assumed that the channel estimation is perfect at the receiver and the feedback to the transmitter (S) is performed upon request without any error. Finally, we adopt a block flat fading channel model. More specifically, with slowly-varying fading conditions, the different paths from users experience roughly the same or different fading conditions for the i.i.d. cases and for i.n.d. cases, respectively. In addition, the fading conditions are assumed to be independent across the paths from  $\mathcal{R}s$ .

## **III. STATISTICAL ANALYSIS OF THE OUTAGE PROBABILITY**

Wireless communication systems subject to both fading and CCI are alternatively interference-limited or power-limited depending on the instantaneous propagation and fading conditions affecting the desired and interfering signals. For such types of systems, one of the standard performance criterion characteristics of diversity systems operating over fading channels is the so-called outage probability. An overall outage is declared when the output SINR falls below a predetermined threshold, T. [11]-[14].

### A. OVER i.i.d. FADING CONDITIONS

We denote the received SNR of the *i*-th  $\mathcal{R}$  out of  $K \mathcal{R}$ s by  $\gamma_i$  (i = 1, 2, ..., K) and let  $\gamma_{(n)}$  be the *n*-th order statistics obtained by arranging K nonnegative i.i.d. RVs,  $\{\gamma_i\}_{i=1}^K$ , in decreasing order of magnitude such that  $\infty > \gamma_{(1)} \ge \gamma_{(2)} \ge$  $\gamma_{(3)} \cdots \geq \gamma_{(K)} > 0$ . The objective is to derive the probability of the output SINR involving all K ordered RVs. Based on the system model,  $\mathcal{D}$  only uses the forwarded message from the best  $\mathcal{R}$  among  $K \mathcal{R}$ s and then the interference is the sum of others. Therefore, if we let  $\gamma_{(1)}$  be the received SNR of the desired  $\mathcal{R}$ , it can be formulated as

$$\gamma_{\text{SINR}} = \frac{\gamma_{(1)}}{1 + \alpha \sum_{n=2}^{K} \gamma_{(n)}} \tag{1}$$

where  $K \ge 2$ , and  $\alpha$  ( $0 \le \alpha \le 1$ ) is defined as a CCI cancellation coefficient that quantifies the level of CCI. In particular, when CCI is perfectly canceled, that is  $\alpha = 0$ , the output SINR reverts to the output SNR. On the other hand, when  $\alpha = 1$ , CCI is fully present.<sup>3</sup>

In this paper, for convenience, we assume that the value of  $\alpha$  is fixed to a certain number in  $0 < \alpha \le 1$ . Note that, even if a fixed value of  $\alpha$  is imposed, this will lead to a higher complexity in analysis. The outage probability of the output SINR can be written as

$$P_{\text{Out}} = \Pr\left[\gamma_{\text{SINR}} = \frac{\gamma_{(1)}}{1 + \alpha \sum_{n=2}^{K} \gamma_{(n)}} < T\right]$$
(2)

where 0 < T and  $0 < \alpha \le 1$ .

If we let  $Z_1 = \gamma_{(1)}$  and  $Z_2 = \sum_{n=2}^{K} \gamma_{(n)}$ , then (2) can be calculated in terms of the 2-dimensional joint PDF of  $Z_1$  and  $Z_2$  as

$$\Pr\left[\frac{Z_1}{1+\alpha Z_2} < T\right] = \Pr\left[(Z_1 - T) - \alpha T Z_2 < 0\right]$$
$$= \int_0^\infty \int_0^{(1+\alpha w)T} p_{z_1, z_2}(y, w) dy dw.$$
(3)

## B. OVER i.n.d. FADING CONDITIONS

Similar to i.i.d. case in Sec. III-A, we consider  $K \mathcal{R}s$  and let  $\gamma_{i_l}$   $(i_l = 1, 2, ..., K)$  be the received SNR from the  $i_l$ -th  $\mathcal{R}$  and  $u_i$  (i = 1, 2, ..., K) be the order statistics obtained by arranging K  $(K \ge 2)$  nonnegative i.n.d. RVs,  $\{\gamma_{i_l}\}_{i_l=1}^K$ , in decreasing order of magnitude such that  $u_1 \ge u_2 \ge \cdots \ge u_K$ . Then, similar to (3), letting  $Z_1' = u_1$  and  $Z_2' = \sum_{n=2}^K u_n$ , we can write the outage probability of the output SINR in terms of the 2-dimensional joint PDF of  $Z_1'$  and  $Z_2'$  as

$$\Pr\left[\frac{Z_{1}'}{1+\alpha Z_{2}'} < T\right] = \int_{0}^{\infty} \int_{0}^{(1+\alpha w)T} p_{z_{1}', z_{2}'}(y, w) dy dw.$$
(4)

### C. TARGET 2-DIMENSIONAL JOINT PDF

The major difficulty in obtaining the statistics of the sum of the ordered RVs, which are the key required equations for the analyses in (3) and (4), resides in the fact that while the original RVs are independently distributed, their ordered statistics are indispensably not independent due to the inequality relations among them.

For the i.i.d. case, if we assume the original RVs { $\gamma_i$ } are i.i.d. with a common arbitrary PDF,  $p(\cdot)$ , the joint PDF in (3)

can be written in a tedious and complicated form involving *K*-fold integration as [18]

$$p_{Z_1,Z_2}(z_1, z_2) = \mathcal{L}_{S_1,S_2}^{-1} \left\{ \text{MGF}_{Z_1,Z_2}(-S_1, -S_2) \right\}$$
(5)

where  $\mathcal{L}_{S_1,S_2}^{-1}\{\cdot\}$  denotes the inverse Laplace transform with respect to  $S_1$  and  $S_2$  and

$$MGF_{Z_{1},Z_{2}}(\lambda_{1},\lambda_{2}) = K! \int_{0}^{\infty} d\gamma_{(1)} p(\gamma_{(1)}) \exp(\lambda_{2}\gamma_{(1)})$$

$$\times \int_{0}^{\gamma_{(1)}} d\gamma_{(2)} p(\gamma_{(2)}) \exp(\lambda_{2}\gamma_{(2)})$$

$$\cdots \int_{0}^{\gamma_{(K-1)}} d\gamma_{(K)} p(\gamma_{(K)}) \exp(\lambda_{2}\gamma_{(K)}).$$
(6)

Note that this multiple-fold integral expression is not suitable for the analysis even numerically, especially as *K* increases.

However, by employing the approach presented in [18], we can obtain the analytical expressions of the joint MGF of some partial sums of the ordered RVs. These new expressions are quite adequate to apply the classical inverse Laplace transform with the related Laplace transform theory [21] to derive the target joint PDF of the partial sums of the ordered RVs. The obtained joint PDF has the compact and ready-to-use closed-form expressions over i.i.d. general fading conditions as shown in (33) and, as a special case, i.i.d. Rayleigh fading conditions such that  $p(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$  where  $\bar{\gamma}$  is the common average faded SNR, as shown in (35) in Table 1.

For the i.n.d. case, if we assume that the original RVs  $\{\gamma_{i_l}\}$  are i.n.d. with a common arbitrary PDF,  $p_{i_l}(\cdot)$ , the joint PDF in (4) can be written as

$$p_{Z_{1}',Z_{2}'}(z_{1},z_{2}) = \mathcal{L}_{S_{1},S_{2}}^{-1} \left\{ \text{MGF}_{Z_{1}',Z_{2}'}(-S_{1},-S_{2}) \right\}$$
(7)

where

$$MGF_{Z_{1}',Z_{2}'}(\lambda_{1},\lambda_{2}) = \sum_{\substack{i_{1},i_{2},\cdots,i_{K}\\i_{1}\neq i_{2}\neq\cdots\neq i_{K}}}^{1,2,\cdots,i_{K}} \int_{0}^{\infty} du_{1}p_{i_{1}}(u_{1}) \exp(\lambda_{1}u_{1}) \\ \times \int_{0}^{u_{1}} du_{2}p_{i_{2}}(u_{2}) \exp(\lambda_{2}u_{2}) \\ \cdots \int_{0}^{u_{K-1}} du_{K}p_{i_{K}}(u_{K}) \exp(\lambda_{2}u_{K}).$$
(8)

Similar to the i.i.d. case, (7) also involves K-fold integration. In this case, we can obtain the required joint statistics as a compact ready-to-use form with the help of the recent result presented in [19] by carrying out more detailed manipulations and introducing a new mathematical representation compared

<sup>&</sup>lt;sup>3</sup>If CDMA is considered,  $\alpha$  can be treated as the inverse of the processing gain for matched filtering. When the minimum mean square error (MMSE) detection is assumed, a specific value of  $\alpha$  depends on the amount of the CCI cancellation. The exact characterization is beyond the scope of this paper.

TABLE 1. The joint PDF and the outage probability over i.i.d. fading channels.

	Joint PDF	Outage Probability
General Fading	$p_{Z}(z_{1}, z_{2}) = \frac{K!}{(K-1)!} p(z_{1}) \mathcal{L}_{S_{2}}^{-1} \left\{ \left[ c(z_{1}, -S_{2}) \right]^{(K-1)} \right\} $ (33) where $z_{1} = \gamma_{(1)}, z_{2} = \sum_{n=2}^{K} \gamma_{(n)}, \gamma_{(n)}$ is the received SNR from the <i>n</i> -th best relay, and $c(\gamma, \lambda) = \int_{0}^{\gamma} p(x) \exp(\lambda x) dx.$	$P_{\text{Out}} = \Pr\left[\frac{Z_1}{1 + \alpha Z_2} < T\right]$ = $\int_0^\infty \int_0^{(1+\alpha w)T} \frac{K!}{(K-1)!} p(z_1) \mathcal{L}_{S_2}^{-1} \left\{ \left[ c(z_1, -S_2) \right]^{(K-1)} \right\} dz_1 dz_2$ (34)
Rayleigh Fading	$p_{Z}(z_{1}, z_{2}) = \frac{K!}{(K-1)!(K-2)!\bar{\gamma}K}$ $\times \exp\left(-\frac{z_{1}+z_{2}}{\bar{\gamma}}\right) \sum_{j=0}^{K-1} (-1)^{j} \binom{K-1}{j}$ $\times (z_{2}-jz_{1})^{K-2}U(z_{2}-jz_{1}),$ for $z_{1} \ge 0 \& z_{2} \ge 0$ (35) where $U(\cdot)$ is an unit-step function.	$P_{\text{Out}} = \frac{K!}{(K-1)!(K-2)!\bar{\gamma}^{K}} \left[ \left(\frac{1}{\bar{\gamma}}\right)^{-K} \Gamma(K-1) \left(1 - (1+\alpha T)^{-K+1} \exp\left(-\frac{T}{\bar{\gamma}}\right) \right) \right. \\ \left. + \sum_{j=1}^{K-1} \sum_{i=0}^{K-2} (-1)^{j} \left(-K-1 \atop j \right) \left(-K-2 \atop i \right) (-j)^{i} \left(\frac{1}{\bar{\gamma}}\right)^{-i-1} (i!) \left\{ \left(\frac{1}{\bar{\gamma}}\right)^{-K+i+1} \times \Gamma(K-i-1) - \left(1 - U\left(j-\frac{1}{\alpha T}\right)\right) \exp\left(-\frac{T}{\bar{\gamma}}\right) \sum_{l=0}^{i} \frac{1}{l!} \left(\frac{T}{\bar{\gamma}}\right)^{l} \sum_{n=0}^{l} \left(-\frac{1}{n}\right) \times \alpha^{n} \left(\frac{1+\alpha T}{\bar{\gamma}}\right)^{-K-n+i+1} \Gamma\left(K+n-i-1,\frac{jT(1+\alpha T)}{\bar{\gamma}(1-j\alpha T)}\right) + \sum_{l=0}^{i} \frac{1}{l!} \left(\frac{1}{\bar{\gamma}j}\right)^{l} \times \left(\frac{1+j}{\bar{\gamma}j}\right)^{-K-l+i+1} \left(\Gamma\left(K+l-i-1,\frac{(1+j)T}{\bar{\gamma}(1-j\alpha T)}\right) \times \left(1 - U\left(j-\frac{1}{\alpha T}\right)\right) - \Gamma(K+l-i-1)\right) \right\} \right] $ (36) where $\Gamma(\cdot), \Gamma(\cdot, \cdot)$ and $\gamma(\cdot)$ are the gamma function, the upper incomplete gamma function, and the lower incomplete gamma function, respectively, defined in [22, Eq. (8.339.1), (8.352.7), and $\alpha(n, x) = (n-1)! \left[1 - e^{-x} \sum_{m=0}^{n-1} \frac{x^{m}}{m!}\right].$

to the i.i.d. case presented in [18]. Applying an interchange of multiple integrals [18], [19], we can rewrite the MGF expression as

$$MGF_{Z_{1}',Z_{2}'}(\lambda_{1},\lambda_{2}) = \sum_{i_{1}=1}^{K} \int_{0}^{\infty} du_{1}p_{i_{1}}(u_{1}) \exp(\lambda_{1}u_{1}) \\ \times \sum_{\{i_{2},...,i_{K}\}\in P_{K-1}(I_{K}-\{i_{1}\})} \int_{0}^{u_{1}} du_{2}p_{i_{2}}(u_{2}) \exp(\lambda_{2}u_{2}) \\ \cdots \int_{0}^{u_{K-1}} du_{K}p_{i_{K}}(u_{K}) \exp(\lambda_{2}u_{K}).$$
(9)

With (9), by replacing the (K - 1)-fold multiple integral expression from  $u_K$  to  $u_2$  with the multiple product of  $c_{i_l}$ and then applying inverse Laplace transform, the simplified 2-dimensional joint PDF of  $Z_1'$  and  $Z_2'$  for i.n.d. general fading conditions can be obtained as given in (37) in Table 2. Then, by adopting the derived result given in [19, eq. (20)], the ready-to-use form of the 2-dimensional joint PDF over i.n.d. Rayleigh fading conditions such that  $p_{i_l}(\gamma) = \frac{1}{\bar{\gamma}_{i_l}} \exp\left(-\frac{\gamma}{\bar{\gamma}_{i_l}}\right)$  where  $\bar{\gamma}_{i_l}$  is the average SNR of the path from the  $i_l$ -th relay, can be obtained as given in (39) in Table 2.

# IV. CLOSED-FORM EXPRESSIONS OF THE OUTAGE PROBABILITY BASED ON THE OUTPUT SINR

In this section, we present the closed-form expressions for the outage probability over both i.i.d. and i.n.d. general and Rayleigh fading channels, which are most widely used in the performance evaluation of networks and telecommunication systems. Note that even if the final results for other types of RVs do not have the closed-form expressions, our approach will still lead to much simpler results than the original expressions involving multiple-fold integration (e.g., (5) and (7)).

### A. OVER i.i.d. RAYLEIGH FADING CONDITIONS

We can apply the derived joint statistic results in the previous section to obtain the closed-form expression of the outage probability of the SC scheme based on the output SINR over i.i.d. Rayleigh fading conditions. For a valid mathematical investigation of the closed-form expression after inserting (35) into (3), we need to consider two cases, j = 0 and  $j \neq 0$ , separately. Additionally, in case of  $j \neq 0$ , we also need to consider four cases separately based on the relationships among  $\alpha$ , j, and T as i)  $w > \frac{jT}{(1-j\alpha T)}$  and  $1 \leq j \leq \text{Min} \left[K - 1, \left\lceil \frac{1}{\alpha T} \right\rceil - 1\right\rceil$ , ii)  $w \leq 0$  and  $j = \frac{1}{\alpha T}$ , and iv)  $w \geq 0$  and  $\text{Max} \left[1, \left\lfloor \frac{1}{\alpha T} \right\rfloor + 1\right] \leq j \leq K - 1$ .

# TABLE 2. The joint PDF and the outage probability over i.n.d. fading channels.

	Joint PDF	Outage Probability
General Fading	$p_{Z}(z_{1}, z_{2}) = \sum_{i_{m}=1}^{K} p_{i_{m}}(z_{1}) \sum_{\{i_{2}, \cdots, i_{K}\} \in P_{K-1}(I_{K} - \{i_{1}\})} \times \mathcal{L}_{S_{2}}^{-1} \left\{ \prod_{\substack{l=2\\ \{i_{2}, \cdots, i_{K}\}}}^{K} c_{i_{l}}(z_{1}, -S_{2}) \right\}$ where $z_{1} = u_{1}, z_{2} = \sum_{n=2}^{K} u_{n}$ , and $c_{i_{l}}(\gamma, \lambda) \stackrel{(37)}{=} \int_{0}^{\gamma} p_{i_{l}}(x) \exp(\lambda x) dx$	$P_{\text{Out}} = \int_{0}^{\infty} \int_{0}^{(1+\alpha w)T} \sum_{i_{m}=1}^{K} p_{i_{m}}(z_{1}) \sum_{\{i_{2},\cdots,i_{K}\} \in P_{K-1}(I_{K}-\{i_{1}\})} \sum_{\substack{I = 2\\\{i_{2},\cdots,i_{K}\}}} c_{i_{l}}(z_{1},-S_{2}) dz_{1}dz_{2}$ where the index set $I_{K}$ is defined as $I_{K} = \{i_{1},\cdots,i_{K}\}$ . The subset of $I_{K}$ with $n \ (n \leq K)$ elements is denoted by $\mathcal{P}_{n}(I_{K})$ . The remaining indices can be grouped in the set $I_{K} - P_{n}(I_{K})$ .
Rayleigh Fading	$p_{Z}(z_{1}, z_{2}) = \sum_{i_{1}=1}^{K} \frac{1}{\tilde{\gamma}_{i_{1}}} \exp\left(-\frac{z_{1}}{\tilde{\gamma}_{i_{1}}}\right) \sum_{\{i_{2}, i_{3}, \dots, i_{K}\} \in P_{K-1}(I_{K}-\{i_{1}\})} \sum_{\substack{q = 2 \\ \{i_{2}, \dots, i_{K}\}}}^{K} C_{q,2,K} \left(-\exp\left(-\frac{z_{2}}{\tilde{\gamma}_{i_{q}}}\right)\right) \\ \{i_{2}, \dots, i_{K}\} + \sum_{i_{1}=1}^{K} \frac{1}{\tilde{\gamma}_{i_{1}}} \exp\left(-\frac{z_{1}}{\tilde{\gamma}_{i_{1}}}\right) \\ \times \sum_{\{i_{2}, i_{3}, \dots, i_{K}\} \in P_{K-1}(I_{K}-\{i_{1}\})} \sum_{\substack{q = 2 \\ \{i_{2}, \dots, i_{K}\}}}^{K} C_{q,2,K}} \sum_{\substack{\{i_{2}, \dots, i_{K}\}}} \sum_{\substack{\{i_{2}, \dots, i_{K}\}\\ \sum_{i_{1}=1}^{K} (-1)^{h} \sum_{j_{1}=j_{0}+2}^{K-h+1} \sum_{j_{2}=j_{1}+1}^{K-h+2} \sum_{j_{3}=j_{2}+1}^{K-h+3} \dots} \sum_{\substack{j_{h}=j_{h-1}+1}}^{K} \exp\left(-\sum_{m=1}^{h} \frac{z_{1}}{\tilde{\gamma}_{i_{m}}}\right) \\ \times \left(-\exp\left(-(z_{2}-hz_{1})\frac{1}{\tilde{\gamma}_{i_{q}}}\right)\right) U(z_{2}-hz_{1})\right]$	$\begin{split} P_{\text{Out}} &= \sum_{i_{1}=1}^{K} \frac{1}{\tilde{\gamma}_{i_{1}}} \sum_{\{i_{2},i_{3},\dots i_{K}\} \in P_{K-1}(I_{K}-\{i_{1}\})} \sum_{\substack{q=2\\\{i_{2},\cdots,i_{K}\}}}^{K} C_{q,2,K} \tilde{\gamma}_{i_{1}} \left(\exp\left(-\frac{T}{\tilde{\gamma}_{i_{1}}}\right)\right) \\ &\times \frac{1}{\left(\frac{\alpha T}{\tilde{\gamma}_{i_{1}}} + \frac{1}{\tilde{\gamma}_{i_{q}}}\right)} - \tilde{\gamma}_{i_{q}}\right) + \sum_{i_{1}=1}^{K} \frac{1}{\tilde{\gamma}_{i_{1}}} \sum_{\{i_{2},i_{3},\dots i_{K}\} \in P_{K-1}(I_{K}-\{i_{1}\})} \sum_{\substack{q=2\\\{i_{2},\cdots,i_{K}\}}}^{K} C_{q,2,K} \\ &\sum_{h=1}^{K-1} (-1)^{h} \sum_{j_{1}=j_{0}+2j_{2}=j_{1}+1}^{K-h+2} \sum_{j_{h}=j_{h-1}+1}^{K} \frac{1}{M_{q,h}} \left[ \left(\frac{1}{\left(\frac{M_{q,h}}{h} - \frac{1}{\tilde{\gamma}_{i_{q}}}\right) + \tilde{\gamma}_{i_{q}}}\right) \\ &+ \left(G\left(\frac{M_{q,h}}{h} - \frac{1}{\tilde{\gamma}_{i_{q}}}\right) - \exp(M_{q,h}T) G\left(M_{q,h}\alpha T - \frac{1}{\tilde{\gamma}_{i_{q}}}\right)\right) \left\{U\left(h - \frac{1}{\alpha T}\right) - 1\right\} \right] \\ &\text{where} \end{split} $ $M_{q,h} &= \frac{h}{\tilde{\gamma}_{i_{q}}} - \sum_{m=1}^{h} \frac{1}{\tilde{\gamma}_{i_{m}}} - \frac{1}{\tilde{\gamma}_{i_{1}}} \text{ and } G(x) = \frac{\exp\left(x\left(\frac{Th}{1-h\alpha T}\right)\right)}{x}, \\ C_{l,n_{1},n_{2}} &= \frac{1}{\prod_{l=n_{1}}^{n_{2}} (-\tilde{\gamma}_{i_{l}}) F'\left(\frac{1}{\tilde{\gamma}_{i_{l}}}\right)}, \\ F'(x) &= \left[\sum_{l=1}^{n_{2}-n_{1}} (n_{2} - n_{1} - l + 1)x^{n_{2}-n_{1}-l}(-1)^{l} \sum_{j_{1}=j_{0}+n_{1}}^{n_{2}-l+1} \cdots \sum_{j_{l}=j_{l-1}+1}^{n_{2}} \prod_{l=1}^{l} \frac{1}{\tilde{\gamma}_{i_{lm}}}} \right] \\ &+ (n_{2} - n_{1} + l)x^{n_{2}-n_{1}} \end{cases} $

After careful derivations and manipulations as shown in Appendix A, the final outage probability can be obtained as shown in (36) in Table 1. In addition, the simplified version of the outage probability for i.i.d. general fading conditions is also given in (34) which involves only a double-integration and can be easily and accurately evaluated numerically while the original outage probability expression, (3), involves (K + 2)-fold integrations.

# B. OVER i.n.d. RAYLEIGH FADING CONDITIONS

Similar to the i.i.d Rayleigh fading case, for mathematically investigating the valid closed-form expression of (4) with (39), the following three cases need to be separately considered based on the relationships among  $\alpha$ , h, and T as i)  $h \geq \frac{1}{\alpha T}$ , ii)  $h < \frac{1}{\alpha T}$  and  $0 \leq w < \frac{T \cdot h}{1 - h\alpha T}$ , and iii)  $h < \frac{1}{\alpha T}$  and  $\frac{T \cdot h}{1 - h\alpha T} \leq w$ . By considering the above three conditions, we can obtain the closed-form expression of the outage probability for i.n.d. Rayleigh fading

conditions as shown in (40) in Table 2 (see Appendix B for details). Similar to the i.i.d. case, the simplified expression for i.n.d. general fading conditions is also given in (38) which also involves a double integration while the original outage probability expression, (4), involves (K + 2)-fold integrations.

### V. AVERAGE OUTAGE PROBABILITY

In the previous section, we derived the outage probability for a fixed number of participating  $\mathcal{R}$ s. However, in general, the number of participating  $\mathcal{R}$ s is random and the outage probability eventually depends on this number. Therefore, the analysis of the conditional outage probability can be made more generally by conditioning on *k* as

$$P_{\text{Out}}(k) = \Pr\left[\gamma_{\text{SINR}} = \frac{\gamma_{(1)}}{1 + \alpha \sum_{n=2}^{k} \gamma_{(n)}} < T\right].$$
 (10)

Conditioned on the number of participating  $\mathcal{R}$ s, k, the average outage probability,  $\overline{P_{\text{Out}}}$ , can be evaluated as

$$\overline{P_{\text{Out}}} = \sum_{k=0}^{N} P_{\text{Out}}(k) P(K=k).$$
(11)

Note that the closed-form expressions of the conditional outage probability,  $P_{\text{Out}}(k)$ , for i.i.d. and i.n.d. Rayleigh fading assumptions can be obtained by replacing K with k in (36) and (40), respectively.

In (11), for the i.i.d. case, the number of participating  $\mathcal{R}s$  can be modeled as a binomial random variable. If  $\mathcal{R}$  decodes successfully, this  $\mathcal{R}$  will help. Since this is the same for all  $\mathcal{R}s$ , the probability of being *k* participating  $\mathcal{R}s$  out of *K*  $\mathcal{R}s$  follows the binomial distribution as

$$P(K=k) = \binom{K}{k} p^k (1-p)^{K-k}$$
(12)

where p is the probability of success. As a result, we can generalize the result by averaging K over its binomial distribution.

For the i.n.d. case, the number of participating  $\mathcal{R}s$  can be modeled as a Poisson binomial random variable [23]. Therefore, the probability of successful decoding at  $\mathcal{R}$  can be written as

$$P(K = k) = \sum_{A \in F_k} \prod_{i \in A} p_i \prod_{j \in A^c} (1 - p_j)$$
(13)

where  $F_k$  is the set of all subsets of k integers that can be selected from  $\{1, 2, 3, ..., K\}$  and  $A^c$  is the complement of set A, i.e.,  $A^c = \{1, 2, ..., K\} \setminus A$ . Note that the Poisson binomial distribution becomes the ordinary binomial distribution when all success probabilities are the same, that is  $p_1 = p_2 = \cdots = p_K = p$ , as shown in (12).



**FIGURE 2.** Outage probability based on SINR and SIR in terms of *K* over i.i.d. Rayleigh fading conditions when  $\bar{y} = 7.78$  dB,  $\alpha = 0.5$ , and T = 2.

### VI. NUMERICAL RESULTS AND DISCUSSION

As a validation of our analytical formula, we compare in the following figures the analytical results (the marker) with the simulation results (the line) obtained via Monte-Carlo simulation. Figs. 2 and 3 show the outage probability subject



**FIGURE 3.** Outage probability based on SINR in terms of *T* for various values of  $\alpha$  over i.i.d. Rayleigh fading conditions when  $\bar{\gamma} = 0$  dB and K = 6.

to CCI over i.i.d. Rayleigh fading conditions. From Fig. 2, we can clearly observe that the outage probability performance degrades as the number of interferers, K, increases. Fig. 3 shows the effect of  $\alpha$  such that as  $\alpha$  increases, the outage probability increases which results in a degraded performance due to the increased interference.



**FIGURE 4.** Outage probability based on SINR and SIR in terms of *T* for various values of  $\bar{y}$  and  $\alpha$  over i.i.d. Rayleigh fading conditions when K = 3.

In Figs. 2 and 4, the performance comparisons based on SINR and SIR are also presented over i.i.d. Rayleigh fading conditions. Note that, for a fair comparison, in the SIR based performance results,  $\alpha$  was considered. The numerical results show that i) as  $\bar{\gamma}$  increases or  $\alpha$  increases with the same number of interferers (Fig. 4) or ii) as the number of interferers (Fig. 2), the performance gap between SINR and SIR decreases, and the SINR based results can be viewed as an upper bound on the performance compared to the SIR based results because the theoretical analyses based on SIR are valid only in an interference-limited environment while the theoretical analyses based on SINR consider more practical/general cases.

In Fig. 5, we present some selected numerical results to examine the outage probability over i.n.d. Rayleigh fading channels. Specifically, we assume that the channel has an



**FIGURE 5.** Outage probability based on SINR in terms of *T* for various values of  $\delta$  over i.n.d. Rayleigh fading conditions when  $\bar{\gamma} = 0$  dB, K = 6, and  $\alpha = 0.3$ .

exponential multipath intensity profile (MIP) with  $\bar{\gamma}_i = \bar{\gamma} \exp(-\delta(i-1))$ , (i = 1, ..., K), where  $\bar{\gamma}$  is the strongest average SNR (or the average SNR of the first path) and  $\delta$  is the power decay factor. It is interesting to see that for the i.n.d. case, the effect of the interference of the more heavily decayed MIP is smaller than that of the smaller decay exponents. As a result, if  $\delta$  decreases, the interference level increases which leads to the system performance degradation.

In this paper, for the analytical tractability, we assumed that  $\alpha$  is a constant value. However, our analytical results which assume the smallest and largest possible values of  $\alpha$ (between 0 and 1) can provide the upper and lower bounds of the performance. Note that this value quantifies the level of CCI. It is also notable that we can assume this value as a RV and then through some computer simulation and analytical results, we can easily assess the effect of a RV.

Note that the dramatic impact of CCI on the outage performance mainly depends on increasing the interfered relays and the level of CCI as well as the traffic intensity. In particular, our analytical results can provide the upper and lower bounds on the performance in the presence of CCI. Note also that our analytical and simulation results show that the study of the technique or mechanism to achieve interference reduction for the type II relay in practical scenarios under multiple-relay environments, such as directional antenna and proper scheduling protocols, must be performed. For an applicable example, each  $\mathcal{R}$  can overhear the reference signals (i.e., ACK/NACK) periodically sent from  $\mathcal{D}$  to  $\mathcal{S}$  and such overheard signal can be used for estimating each  $\mathcal{R}$ - $\mathcal{D}$  link quality. Based on the link quality feedback information,  $\mathcal S$  may select  $\mathcal R$  with the best  $\mathcal R$ - $\mathcal D$  link quality and this information may be shared with  $\mathcal{D}$  and multiple  $\mathcal{R}$ s. Then, only  $\mathcal{R}$  with the best  $\mathcal{R}$ - $\mathcal{D}$  link quality will start retransmission while other unscheduled  $\mathcal{R}s$  will stop retransmission. As a result,  $\mathcal{D}$  may treat the signal transmitted from this selected  $\mathcal{R}$  as the desired signal. Additionally, we also may apply the relaying protocol proposed for the type II relay including transmission and single or multiple relays selection strategy based on rateless code [9].

# APPENDIX A DERIVATION OF THE CLOSED-FORM EXPRESSION OF THE OUTAGE PROBABILITY OVER i.i.d. RAYLEIGH FADING CONDITIONS

By inserting (35) into (3), we can rewrite (3) as

$$\int_{0}^{\infty} \int_{0}^{(1+\alpha w)T} \frac{K!}{(K-1)! (K-2)! \bar{\gamma}^{K}} \exp\left(-\frac{z_{1}+z_{2}}{\bar{\gamma}}\right) \\ \times \sum_{j=0}^{K-1} (-1)^{j} {\binom{K-1}{j}} [z_{2}-jz_{1}]^{K-2} U (z_{2}-jz_{1}) dy dw.$$
(14)

For a valid mathematical investigation of the closed-form expression of (14), we need to consider two cases A) j = 0 and B)  $j \neq 0$ , separately.

A) For j = 0,

In this case, we start from the following equation

$$\int_0^\infty w^{K-2} \exp\left(-\frac{w}{\bar{\gamma}}\right) U(w) \int_0^{(1+\alpha w)T} y^i \exp\left(-\frac{y}{\bar{\gamma}}\right) dy dw.$$
(15)

After some manipulations and with the help of the result in [22, eq. (3.381)-(4)], we can derive the following closed form of (15) as

$$\left(\frac{1}{\bar{\gamma}}\right)^{-K} \Gamma \left(K-1\right) \left[1-(1+\alpha T)^{-K+1} \exp\left(-\frac{T}{\bar{\gamma}}\right)\right].$$
(16)

B) For  $j \neq 0$ ,

In this case, we need to consider four cases separately based on the relationships among  $\alpha$ , *j*, and *T* as follows:

i) 
$$w > \frac{jT}{(1-j\alpha T)}$$
 and  $1 \le j \le Min\left[K-1, \left\lceil \frac{1}{\alpha T} \right\rceil - 1\right]$   
We start from the following equation

$$\int_{0}^{\infty} w^{K-i-2} \exp\left(-\frac{w}{\bar{\gamma}}\right) \left[1 - U\left(\frac{jT}{1-j\alpha T} - w\right)\right] \\ \times \int_{0}^{(1+\alpha w)T} y^{i} \exp\left(-\frac{y}{\bar{\gamma}}\right) dy dw.$$
(17)

After some manipulations and with the help of the results in [22, eqs. (3.381-1), (8.352-1), (1.111), and (3.381-3)], we can derive the closed form of (17) as

$$\left(\frac{1}{\bar{\gamma}}\right)^{-i-1} (i!) \left[ \left(\frac{1}{\bar{\gamma}}\right)^{-K+i+1} \Gamma\left(K-i-1,\frac{j\cdot T}{\bar{\gamma}\left(1-j\alpha T\right)}\right) - \exp\left(-\frac{T}{\bar{\gamma}}\right) \sum_{l=0}^{i} \frac{1}{l!} \left(\frac{T}{\bar{\gamma}}\right)^{l} \sum_{n=0}^{l} \binom{l}{n} \alpha^{n} \left(\frac{1+\alpha T}{\bar{\gamma}}\right)^{-K-n+i+1} \times \Gamma\left(K+n-i-1,\frac{j\cdot T\left(1+\alpha T\right)}{\bar{\gamma}\left(1-j\alpha T\right)}\right) \right].$$

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ii)  $w \le \frac{j}{(1-j\alpha T)}$  and  $1 \le j \le Min \left\lfloor K - 1, \left\lfloor \frac{1}{\alpha T} \right\rfloor - 1 \right\rfloor$ We start from the following equation

$$\int_{0}^{\infty} w^{K-i-2} \exp\left(-\frac{w}{\bar{\gamma}}\right) U\left(\frac{jT}{1-j\alpha T} - w\right) \\ \times \int_{0}^{\frac{w}{\bar{\gamma}}} y^{i} \exp\left(-\frac{y}{\bar{\gamma}}\right) dy dw.$$
(19)

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With (19), after some manipulations and with the help of the results in [22, eqs. (3.381-1) and (8.352-1)] we can derive the closed form of (19) as

$$\left(\frac{1}{\bar{\gamma}}\right)^{-i-1} (i!) \left[ \left(\frac{1}{\bar{\gamma}}\right)^{-K+i+1} \gamma \left(K-i-1, \frac{j \cdot T}{\bar{\gamma} (1-j\alpha T)}\right) - \sum_{l=0}^{i} \frac{1}{l!} \left(\frac{1}{\bar{\gamma} \cdot j}\right)^{l} \left(\frac{1+j}{\bar{\gamma} \cdot j}\right)^{-K-l+i+1} \gamma \left(K+l-i-1, \frac{(1+j)T}{\bar{\gamma} (1-j\alpha T)}\right) \right]$$
(20)

where  $\gamma(\cdot, \cdot)$  is the incomplete gamma function defined for a positive integer n [22, eq. (8.352.6)].

iii)  $w \ge 0$  and  $j = \frac{1}{\alpha T}$ 

We start from the following equation

$$\int_{0}^{\infty} w^{K-i-2} \exp\left(-\frac{w}{\bar{\gamma}}\right) U(w) \int_{0}^{\frac{w}{\bar{j}}} y^{i} \\ \times \exp\left(-\frac{y}{\bar{\gamma}}\right) dy dw \delta(j\alpha T - 1).$$
(21)

From (21), after some manipulations and with the help of the results in [22, eqs. (3.381-1) and (8.352-1)] we can derive the closed form as

$$\begin{pmatrix} \frac{1}{\bar{\gamma}} \end{pmatrix}^{-i-1} (i!) \left[ \left( \frac{1}{\bar{\gamma}} \right)^{-K+i+1} \Gamma \left( K-i-1 \right) \right. \\ \left. - \sum_{l=0}^{i} \frac{1}{l!} \left( \frac{1}{\bar{\gamma} \cdot j} \right)^{l} \left( \frac{1+j}{\bar{\gamma} \cdot j} \right)^{-K-l+i+1} \Gamma \left( K+l-i-1 \right) \right] \\ \times \delta \left( j\alpha T-1 \right) .$$

$$(22)$$

iv)  $w \ge 0$  and  $\operatorname{Max}\left[1, \left\lfloor \frac{1}{\alpha T} \right\rfloor + 1\right] \le j \le K - 1$ We start from the following equation

$$\int_{0}^{\infty} w^{K-i-2} \exp\left(-\frac{w}{\bar{\gamma}}\right) U(w) \int_{0}^{\frac{v}{\bar{\gamma}}} y^{i} \\ \times \exp\left(-\frac{y}{\bar{\gamma}}\right) dy dw.$$
(23)

In this case, the closed-form expression is the same as in case iii). Based on the above derived results and then after some manipulations, we can get the final closed-form expression as shown in (36).

## **APPENDIX B DERIVATION OF THE CLOSED-FORM EXPRESSION OF** THE OUTAGE PROBABILITY OVER i.n.d. RAYLEIGH **FADING CONDITIONS**

With (39), to obtain the closed-form expression of (4), we need to perform two integral terms. For the first integral term, we need to perform a double-integral as

$$\int_0^\infty \int_0^{(1+\alpha w)T} \left( -\exp\left(-\frac{w}{\bar{\gamma}_{i_q}}\right) \right) \exp\left(-\frac{y}{\bar{\gamma}_{i_1}}\right) dy dw.$$
(24)

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With the help of an ordinary integral method [22], the double-integral term in (24) can be simply shown to be equal to the following closed-form expression

$$\bar{\gamma}_{i_1}\left(\exp\left(-\frac{T}{\bar{\gamma}_{i_1}}\right)\frac{1}{\left(\frac{\alpha T}{\bar{\gamma}_{i_1}}+\frac{1}{\bar{\gamma}_{i_q}}\right)}-\bar{\gamma}_{i_q}\right).$$
(25)

Similarly, for the second integral term, we also need to perform the following double-integral

$$\int_{0}^{\infty} \int_{0}^{(1+\alpha w)T} \exp\left(-y\left(\sum_{m=1}^{h} \frac{1}{\bar{\gamma}_{i_{j_m}}} + \frac{1}{\bar{\gamma}_{i_1}} - \frac{h}{\bar{\gamma}_{i_q}}\right)\right) \times \left(-\exp\left(-\frac{w}{\bar{\gamma}_{i_q}}\right)\right) U(w-hy) \, dy dw.$$
(26)

In (26), based on the valid mathematical relationship between w and y and a valid integration region for integration over y, we need to consider the following three cases as

i)  $h \geq \frac{1}{\alpha T}$ 

In this case, with the given conditions, the valid integral region of y becomes  $0 < y < \frac{w}{h}$  and then (26) can be re-written as

$$\int_{0}^{\infty} \int_{0}^{\frac{w}{h}} \exp\left(-y\left(\sum_{m=1}^{h} \frac{1}{\bar{\gamma}_{i_{j_{m}}}} + \frac{1}{\bar{\gamma}_{i_{1}}} - \frac{h}{\bar{\gamma}_{i_{q}}}\right)\right) \times \left(-\exp\left(-\frac{w}{\bar{\gamma}_{i_{q}}}\right)\right) dy dw.$$
(27)

The closed-form expression of (27) can be obtained with the help of an ordinary integral method [22] as

$$\frac{1}{M_{q,h}} \left( \frac{1}{\left(\frac{M_{q,h}}{h} - \frac{1}{\bar{\gamma}_{iq}}\right)} + \bar{\gamma}_{iq} \right) U\left(h - \frac{1}{\alpha T}\right).$$
(28)

ii)  $h < \frac{1}{\alpha T}$  and  $0 \le z_2 < \frac{T \cdot h}{1 - h \alpha T}$ 

The valid integral regions of y and w become  $0 < y < \frac{w}{h}$  and  $0 < x < \frac{T \cdot h}{1 - h\alpha T}$ . Then, (26) can also be re-written as

$$\int_{0}^{\frac{T-h}{1-h\alpha T}} \int_{0}^{\frac{w}{h}} \exp\left(-y\left(\sum_{m=1}^{h} \frac{1}{\bar{\gamma}_{ijm}} + \frac{1}{\bar{\gamma}_{i1}} - \frac{h}{\bar{\gamma}_{iq}}\right)\right) \times \left(-\exp\left(-\frac{w}{\bar{\gamma}_{iq}}\right)\right) dy dw.$$
(29)

Therefore, the closed-form expression of (29) can be obtained as

$$\frac{1}{M_{q,h}} \left\{ -G\left(\frac{M_{q,h}}{h} - \frac{1}{\bar{\gamma}_{i_q}}\right) + \frac{1}{\left(\frac{M_{q,h}}{h} - \frac{1}{\bar{\gamma}_{i_q}}\right)} + G\left(-\frac{1}{\bar{\gamma}_{i_q}}\right) + \bar{\gamma}_{i_q} \right\} \times \left(1 - U\left(h - \frac{1}{\alpha T}\right)\right). \quad (30)$$

iii)  $h < \frac{1}{\alpha T}$  and  $\frac{1 \cdot n}{1 - h\alpha T} \le z_2$ Opposed to case ii), the valid integral region of *w* becomes  $\frac{T \cdot h}{1 - h\alpha T} < w < \infty$  and for y, the valid integral region can be

estimated as  $0 < y < (1 + \alpha w) T$ . Therefore, (26) can be re-written as

$$\int_{\frac{T\cdot h}{1-h\alpha T}}^{\infty} \int_{0}^{(1+\alpha w)T} \exp\left(-y\left(\sum_{m=1}^{h} \frac{1}{\bar{\gamma}_{i_{j_m}}} + \frac{1}{\bar{\gamma}_{i_1}} - \frac{h}{\bar{\gamma}_{i_q}}\right)\right) \times \left(-\exp\left(-\frac{w}{\bar{\gamma}_{i_q}}\right)\right) dy dw.$$
(31)

Similarly, the closed-form expression of (31) can be obtained as

$$\frac{\exp\left(M_{q,h}T\right)G\left(M_{q,h}\alpha T - \frac{1}{\bar{\gamma}_{i_q}}\right) - G\left(-\frac{1}{\bar{\gamma}_{i_q}}\right)}{M_{q,h}} \times \left(1 - U\left(h - \frac{1}{\alpha T}\right)\right). \quad (32)$$

By adopting the above four integral results in (25), (28), (30), and (32), we can finally obtain the closed-form expression as shown in (40).

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