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Robust Iterative Transceiver Beamforming for Multipair Two-Way Distributed Relay Networks

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ABSTRACT In this paper, the transceiver beamforming problem is studied for multipair two-way distributed relay networks, particularly with multi-antenna user nodes and in the presence of channel state errors. With multi-antenna setting on the user nodes, some of the usual signal processing tasks are shifted from the relay nodes to the user nodes with the proposed transceiver beamforming designs. The transmit beamforming vectors, distributed relay coefficients, and the receive beamforming vectors are obtained by iteratively solving three sub-problems, each having a closed-form solution. The tasks of maximizing desired signal power, and reducing inter-pair interference and noise are thus allocated to different iteration steps. By this arrangement, the transmit and receive beamformers of each user are responsible for improving its own performance and the distributed relay nodes with simple amplify-and-forward protocol aim at creating a comfortable communication environment for all user pairs. With respect to the channel uncertainty, two relay strategies are proposed considering two different requirements from the communication network: sum relay output power and individual relay output power. Our simulation demonstrates that the performance improvement can be very significant through cooperation of the three components, especially when the number of relay nodes is large.

INDEX TERMS Transceiver beamforming, relay networks, robust, multi-pair, two-way.

I. INTRODUCTION

Distributed relay network has its distinct advantages in exploiting spatial diversity, reducing the deployment cost and mitigating the effect of fading in wireless communications [1]–[7]. In a typical relay network, data transmission between a source and a destination is assisted by relay nodes with various protocols, among which amplify-and-forward (AF) is one of the most popular due to its simplicity [8]–[13]. On the other hand, various strategies have also been proposed with different network constructions and different tasks assigned to relay nodes.

One particular research area is the *multipair relay network*, where multiple peer-to-peer user pairs communicate with each other simultaneously, which significantly increases the overall throughput and efficiency of the relay network [14]–[18]. For such relay networks, the inter-pair interference (IPI) caused by simultaneous signal transmissions is a crucial issue. Tao and Wang [16], Suraweera *et al.* [17], and Joung and Sayed [19] used different zero-forcing (ZF) based methods to eliminate the IPI among users, while block diagonalization (BD) was employed on a central relay node with multiple antennas to reduce interferences experienced by each user pair in [20] and [21]. In [22], beamforming vectors for the multi-antenna user pairs were jointly determined to null out the IPI and maximize the effective channel gain between user pairs. In [14], an *ad hoc* network with multi-pair communications was studied with the one-way strategy. Leow [15] and Tao and Wang [16], investigated the multipair two-way relay networks, where the bi-directional transmissions are supported by a multi-antenna central relay node, while in [17], a central relay node equipped with a very large array of antennas was considered, which can substantially reduce the interference with simple signal processing techniques.

For multipair relay networks with distributed single-antenna relays [23]–[28], the desired signals at the destinations suffer from a higher level of interference, due to the assumption that the distributed relays do not share their received signals and thus cannot cooperate as effectively as the former network to suppress the interference accumulated in the source-relay transmission stage. Fazeli-Dehkordy *et al.* [23] demonstrated that such a network with destination quality-of-service constraints will lead to a non-convex beamforming problem, that can be turned into a semi-definite programming (SDP) optimization, and solved using interior point methods. Chen *et al.* [24] proposed to approximate the same relay beamforming problem by a convex second-order cone programming (SOCP) problem with drastic improvement in computational complexity. On the basis of it, a distributed optimization method with a faster convergence speed was developed in [28], based on the accelerated distributed augmented lagrangians (ADAL) algorithm [29]. Another method was proposed in [26], where ZF is used to cancel the inter-user interference with the assumption that the global channel state information (CSI) is known at every relay node. In [18], a different beamforming method was proposed when the number of relay nodes is large, where signal-to-interference-plus-noise ratio (SINR) is maximized instead of having a predefined lower bound.

On the other hand, since CSI errors can potentially lead to significant performance degradation, and such errors can hardly be avoided in distributed relay networks, due to inaccurate channel estimation, mobility of relays, and quantization errors, much work has been done for robust designs in distributed relay networks [30]–[38]. In [36], the robust distributed relay beamforming problem was investigated for single-pair one-way relay networks, and a robust relay scheme for multi-user single-destination one-way relay networks was proposed in [33] with the decode-and-forward protocol. In [37], a worst-case based distributed beamforming scheme was developed for a single communication pair with norm-bounded CSI errors. The filter and forward relay beamforming scheme was studied with spherical CSI uncertainties in [38], while in [34] ellipsoidal CSI uncertainties were considered for a multi-pair one-way communication network.

In all the aforementioned designs, user nodes are assumed to have a single antenna. However, in next generation wireless communication systems like LTE/LTE-A/5G [39], multi-antenna user equipments (UEs) are accepted as a basic system setup. With the development of multi-antenna devices and coordinated multi-point joint-transmission techniques [40]–[42], where multiple UEs collaborate and jointly steer the transmit signal, investigating the problem of how the communication of multi-antenna devices and/or virtual multi-antenna devices can benefit in a distributed relay network becomes more and more important.

In this paper, to take advantage of the multi-antenna implementations of the user nodes, we propose to utilize transceiver beamforming on the user nodes, and thus the quality-of-service (evaluated by SINR in this paper) of each user node will be jointly determined by three beamforming vectors: transmit beamforming vector, relay beamforming vector and receive beamforming vector, and the overall beamforming problem becomes more difficult than the single-antenna-user case. On the other hand, unlike in the single-antenna-user network case, where the relay nodes are responsible for almost all the signal processing tasks, in our considered network, since each user node is equipped with transceiver beamformers, the main part of the signal processing tasks can be shifted to the user side, and the relay can be relieved of their usual dominant role in suppressing interferences and noises.

With transceiver beamforming applied on the user nodes, obtaining global solutions of the three beamforming vectors that optimizes SINR of each user becomes very challenging. Thus, we decompose the problem into three sub-problems based on different roles of the three beamforming vectors in their contribution to the received SINR, each of which has a closed-form solution and determines one of the three beamforming vectors, and through an iterative process, an overall sub-optimal but satisfactory solution can be obtained. The idea of such an iterative transceiver beamforming design for multi-pair two-way distributed relay networks was preliminarily considered in [43]; however, in [43] the distributed relay nodes, based on a uniform AF protocol, has a rather limited contribution to the overall performance of the system. In this paper we propose two different relay strategies, with consideration of sum relay power constraint and individual relay power constraint, respectively, that can significantly improve the SINR performance of the system. Moreover, based on the structure, we also investigate the robustness of our proposed methods in the presence of CSI errors and propose worst-case based beamforming strategies for transmit beamformers and relay nodes, and as demonstrated by simulation results, the two proposed methods are extremely robust against CSI errors.

The rest of the paper is organized as follows. The system model under consideration is presented in Section II. Then, in Section III, the *iSINR method* proposed in our earlier work in [43] is briefly reviewed. Following that, two worst-case based robust iterative beamforming algorithms for SINR optimization are proposed in Section IV. Simulation results are provided in Section V, with concluding remarks in Section VI.

Notations: $[\cdot]^T$, $[\cdot]^H$ and $[\cdot]^*$ stand for transpose, Hermitian transpose and conjugate, respectively. $||\cdot||$ denotes the Frobenius norm of a matrix and $|\cdot|$ the absolute value of a scalar. $\mathbb{E}[\cdot]$ represents the expectation operator and Var$[\cdot]$ the variance operator. $\mathbf{I}_N$ is the $N \times N$ identity matrix.

II. SYSTEM MODEL AND PROBLEM DESCRIPTION

We consider a time-slotted dual-hop multipair two-way distributed relay network in which communications between K multi-antenna pairs (X_a, X_b) take place in two transmission phases aided by M single-antenna relay nodes, as shown in Fig. 1. Users in each group are equipped with N antennas, and the distance between the two user groups are assumed to be long enough compared to their transmission power so that there is no direct link between any user pairs. We also assume a rich-scattering environment and all the channels are independent Rayleigh fading, reciprocal and quasi-stationary, so that the channel gains remain unchanged during the two transmission phases.

In the first phase, all users transmit their information streams to the relay nodes simultaneously with transmit beamforming, whose weights are denoted by $\mathbf{a}_i$ and

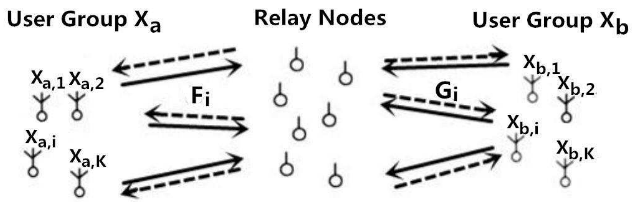

FIGURE 1. The considered time-slotted dual-hop multipair two-way distributed relay network.

$\mathbf{b}_i$ ($\in \mathbb{C}^{N\times 1}$, $i = 1,\ldots,K$) for each user from group X_a and X_b, respectively. The beamforming vectors satisfy the total transmit power constraint $||\mathbf{a}_i||^2 \le P_S$ and $||\mathbf{b}_i||^2 \le P_S$, with P_S being the upper bound. Then, the distributed relay nodes forward the information streams back to the users using low-complexity AF protocols. Following that, the received signal undergos receive beamforming, denoted by $\mathbf{c}_i$ and $\mathbf{d}_i$ ($\in \mathbb{C}^{N\times 1}$, $i = 1,\ldots,K$), at $X_{a,i}$ and $X_{b,i}$ sides, respectively. Since the receive beamforming vectors can take arbitrary absolute values without affecting SINR of each user, herein we assume all receive beamforming vectors are unit vectors ($||\mathbf{c}_i||^2 = 1$, $||\mathbf{d}_i||^2 = 1$) for convenience.

We denote the channel from $X_{a,i}$ and $X_{b,i}$ to the relay nodes by $\mathbf{F}_i$, $\mathbf{G}_i \in \mathbb{C}^{M\times N}$, respectively. And the received signal at the relay nodes can be represented by $\mathbf{r} \in \mathbb{C}^{M\times 1}$,

$$\mathbf{r} = \sum_{i=1}^{K} \mathbf{F}_i\mathbf{a}_i x_{a,i} + \sum_{i=1}^{K} \mathbf{G}_i\mathbf{b}_i x_{b,i} + \mathbf{n}_R, \tag{1}$$

where $x_{a,i}$ and $x_{b,i}$ denote the data symbol and $\mathbf{n}_R \in \mathbb{C}^{M\times 1}$ is the complex Gaussian noise vector of relay nodes with the distribution $\mathcal{CN}(\mathbf{0}, \sigma_r^2\mathbf{I})$. Then, the scaled signal using AF protocol at relay nodes can be written as

$$\mathbf{r}_T = \mathbf{W}\mathbf{r}, \tag{2}$$

where $\mathbf{W} \in \mathbb{C}^{M\times M}$ is diagonal, and we use an $M \times 1$ vector $\mathbf{w} = [w_1 w_2 \ldots w_M]^H$ to denote its diagonal entries.

Then, in the second phase, we use $y_{a,i}$ and $y_{b,i}$ to represent the signal received by $X_{a,i}$ and $X_{b,i}$, respectively, with

$$\begin{aligned} y_{a,i} = {} & \underbrace{\mathbf{c}_i^H\mathbf{F}_i^T\mathbf{W}\mathbf{G}_i\mathbf{b}_i x_{b,i}}_{\text{Desired signal}} + \underbrace{\mathbf{c}_i^H\mathbf{F}_i^T\mathbf{W}\mathbf{F}_i\mathbf{a}_i x_{a,i}}_{\text{Self Interference}} + \mathbf{c}_i^H\mathbf{F}_i^T\mathbf{W}\mathbf{n}_R \\ & + \mathbf{c}_i^H\mathbf{n}_{a,i} + \underbrace{\mathbf{c}_i^H\mathbf{W}\mathbf{F}_i^T\sum_{j\neq i}^{K}(\mathbf{F}_j\mathbf{a}_j x_{a,j} + \mathbf{G}_j\mathbf{b}_j x_{b,j})}_{\text{IPI}}, \end{aligned} \tag{3}$$

$$\begin{aligned} y_{b,i} = {} & \underbrace{\mathbf{d}_i^H\mathbf{G}_i^T\mathbf{W}\mathbf{F}_i\mathbf{a}_i x_{a,i}}_{\text{Desired signal}} + \underbrace{\mathbf{d}_i^H\mathbf{G}_i^T\mathbf{W}\mathbf{G}_i\mathbf{b}_i x_{b,i}}_{\text{Self Interference}} + \mathbf{d}_i\mathbf{G}_i^T\mathbf{W}\mathbf{n}_R \\ & + \mathbf{d}_i^H\mathbf{n}_{b,i} + \underbrace{\mathbf{d}_i^H\mathbf{G}_i^T\mathbf{W}\sum_{j\neq i}^{K}(\mathbf{F}_j\mathbf{a}_j x_{a,j} + \mathbf{G}_j\mathbf{b}_j x_{b,j})}_{\text{IPI}}, \end{aligned} \tag{4}$$

where $\mathbf{n}_{a,i}, \mathbf{n}_{b,i} \in \mathbb{C}^{N\times 1}$ are the additive white complex Gaussian noise vector at the user node, with the distribution $\mathcal{CN}(\mathbf{0}, \sigma_u^2\mathbf{I})$. Since the user knows its own transmitted signal, the self interference (SI) in (3) and (4) can be removed through some standard adaptive filtering techniques [44]. For simplicity, they are ignored in the following derivation.

III. ITERATIVE ALGORITHM FOR SINR OPTIMIZATION

In this section we briefly introduce the *iSINR method* proposed in our earlier work in [43] and use it for comparison with the robust beamforming method with relay strategies proposed in the next section. The iSINR method aims at optimizing SINR of each user node with a total relay output power constraint. In this method, relay nodes are relieved of their usual beamforming tasks, and only uniformly amplify-and-forward the received signal, and transmit and receive beamforming vectors of one user pair collaboratively ensure the SINR performance of the two user nodes. Through iteration steps, the cooperation of the transmit and receive beamformers is gradually improved, and good performance can be achieved after certain number (not large) of iterations.

As an example, consider user $X_{a,i}$. From (3), the SINR at this user can be expressed as follows,

$$SINR_{a,i} = \frac{\mathbf{c}_i^H\mathbf{F}_i^T\mathbf{Q}_{a,i}^{(S)}\mathbf{F}_i^*\mathbf{c}_i}{\sigma_u^2 + \mathbf{c}_i^H\mathbf{F}_i^T\mathbf{Q}_{a,i}^{(N)}\mathbf{F}_i^*\mathbf{c}_i + \underbrace{\mathbf{c}_i^H\mathbf{F}_i^T\mathbf{Q}_{a,i}^{(I)}\mathbf{F}_i^*\mathbf{c}_i}_{\text{IPI}}}, \tag{5}$$

where,

$$\begin{aligned} \mathbf{Q}_{a,i}^{(S)} &= Ps\cdot \mathbf{W}\mathbf{G}_i\mathbf{b}_i\mathbf{b}_i^H\mathbf{G}_i^H\mathbf{W}^H, \\ \mathbf{Q}_{a,i}^{(N)} &= \sigma_r^2\cdot \mathbf{W}\mathbf{W}^H, \\ \mathbf{Q}_{a,i}^{(I)} &= Ps\cdot \sum_{j\neq i}^{K}\mathbf{W}(\mathbf{F}_j\mathbf{a}_j\mathbf{a}_j^H\mathbf{F}_j^H + \mathbf{G}_j\mathbf{b}_j\mathbf{b}_j^H\mathbf{G}_j^H)\mathbf{W}^H. \end{aligned} \tag{6}$$

Since relay nodes uniformly amplify the received signal, $\mathbf{W}$ is a diagonal matrix with entries being $\mathbf{w} = [\lambda_R\,\lambda_R \cdots \lambda_R]$ in the iSINR method.

If maximizing $SINR_{a,i}$ is the only objective, the beamforming vectors $\mathbf{a}_j$ and $\mathbf{b}_j$ ($j = 1\cdots K, j \neq i$) could be carefully chosen to completely eliminate the IPI part, and the remaining part can be maximized by $\mathbf{c}_i$ and $\mathbf{b}_i$. However, the optimal choice of $\mathbf{a}_j$ and $\mathbf{b}_j$ for user $X_{a,i}$ will unlikely result in a sufficiently good SINR for other users, as the beamforming vectors of one user not only affects its own SINR, but also others. In fact, it is very difficult, if not impossible, to obtain an analytical solution for maximizing SINR at all user nodes for this scenario. Therefore, as an alternative, the iSINR method is designed to achieve a desirable sub-optimal SINR.

At the first iteration step, the beamforming vectors $\mathbf{c}_i$ and $\mathbf{d}_i$ are initialized as unity vectors $[\delta_N\ \delta_N\ \cdots\ \delta_N] \in \mathbb{C}^{1\times N}$, where $\delta_N = 1/\sqrt{N}$. And the initialization for the relay scalar is $\lambda_R = 1/\sqrt{M}$. Then, $\mathbf{a}_i$ and $\mathbf{b}_i$ can be updated based on maximizing the power of the desired signal received at $X_{a,i}$ and $X_{b,i}$, respectively, under a transmit power constraint. Here we continue using $X_{a,i}$ as an example, and the derivations are similar for $X_{b,i}$. From (3) and the value of $\mathbf{W}$, we can obtain the desired signal power for $X_{a,i}$ and formulate the problem as follows,

$$\max_{\mathbf{b}_i} \lambda_R^2 |\mathbf{c}_i^H \mathbf{F}_i^T \mathbf{G}_i \mathbf{b}_i|^2, \quad s.t. \ ||\mathbf{b}_i||^2 \leq P_S. \tag{7}$$

This problem has a closed-form solution, and through similar derivation we can obtain the solution for $\mathbf{a}_i$ as,

$$\mathbf{a}_i = \lambda_{a,i} \cdot \mathbf{F}_i^H \mathbf{G}_i^* \mathbf{d}_i, \quad \mathbf{b}_i = \lambda_{b,i} \cdot \mathbf{G}_i^H \mathbf{F}_i^* \mathbf{c}_i, \tag{8}$$

where $\lambda_{a,i}$ and $\lambda_{b,i}$ are the power-control scalars,

$$\lambda_{a,i} = \sqrt{\frac{P_S}{||\mathbf{F}_i^H \mathbf{G}_i^* \mathbf{d}_i||^2}}, \quad \lambda_{b,i} = \sqrt{\frac{P_S}{||\mathbf{G}_i^H \mathbf{F}_i^* \mathbf{c}_i||^2}}. \tag{9}$$

From (3) and (7), we can observe that a larger λ_R leads to a larger desired signal power and SINR. Resulting from the total relay power constraint, in the second iteration step, an optimal choice for λ_R can be expressed as

$$\lambda_R = \sqrt{\frac{P_R}{tr(\sigma_r^2 \cdot \mathbf{I}_M + \sum_{i=1}^{K} (\mathbf{F}_i \mathbf{a}_i \mathbf{a}_i^H \mathbf{F}_i^H + \mathbf{G}_i \mathbf{b}_i \mathbf{b}_i^H \mathbf{G}_i^H)}}, \tag{10}$$

where $tr(\cdot)$ denotes the trace of the matrix in the bracket.

Next, in the third iteration step, the following SINR optimization problem for user node $X_{a,i}$ can be solved locally to obtain the receive beamforming vector $\mathbf{c}_i$,

$$\max_{\mathbf{c}_i} SINR_{a,i} = \mathbf{c}_i^H \mathbf{\Theta}_{a,i} \mathbf{c}_i, \quad s.t. \ ||\mathbf{c}_i||^2 = 1, \tag{11}$$

where

$$\begin{aligned} \mathbf{\Theta}_{a,i} &= (\mathbf{\Xi}_{a,i})^{-1} \mathbf{F}_i^T \mathbf{Q}_{a,i}^{(S)} \mathbf{F}_i^*, \\ \mathbf{\Xi}_{a,i} &= \sigma_u^2 \mathbf{I}_N + \mathbf{F}_i^T \mathbf{Q}_{a,i}^{(N)} \mathbf{F}_i^* + \mathbf{F}_i^T \mathbf{Q}_{a,i}^{(I)} \mathbf{F}_i^*. \end{aligned} \tag{12}$$

The optimizing problem can be transformed to an eigenvector problem with a closed-form solution that leads to the updated value for $\mathbf{c}_i$, and similarly for $\mathbf{d}_i$ as well, as given below

$$\mathbf{c}_i = \rho\{\mathbf{\Theta}_{a,i}\}, \quad \mathbf{d}_i = \rho\{\mathbf{\Theta}_{b,i}\}, \tag{13}$$

where $\rho\{\cdot\}$ denotes the normalized principle eigenvector.

As summarized in **Iteration Steps** below, this process is repeated until a preset maximum iteration number is reached (defined by n_t) or convergence is achieved (defined by a preset small positive real number δ).

Iteration Steps: Method From [43]

1) Initialization: $\mathbf{c}_i, \mathbf{d}_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1\times N}$, where $\delta_N = 1/\sqrt{N}$, $\lambda_R = 1/\sqrt{M}$, and set t=1.
2) Update $\mathbf{a}_i$ and $\mathbf{b}_i$ based on (8).
3) Update λ_R based on (10).
4) Update $\mathbf{c}_i$ and $\mathbf{d}_i$ based on (12) and (13).
5) If $|\mathbf{x}_i^{(t)} - \mathbf{x}_i^{(t-1)}|^2 < \delta$ (considered to be converged) or $t > n_t$ ($\mathbf{x} \leftarrow \mathbf{c}$ for user $X_{a,i}$ and $\mathbf{x} \leftarrow \mathbf{d}$ for user $X_{b,i}$), iteration stops; otherwise set $t = t + 1$ and go to step 2).

Note that in practice, the initialization step may not be necessary, since the update process can continue as long as the transmission keeps going, and when the channel states change slowly, the iteration number required to achieve convergence or good SINR performance can be further reduced.

IV. WORST-CASE BASED ROBUST ITERATIVE BEAMFORMING ALGORITHM FOR SINR OPTIMIZATION

Based on the iSINR method, in this section we propose two worst-case based robust iterative beamforming algorithms for SINR optimization, with two different relay strategies, where the relay nodes are involved helping all the multi-pair transmissions in the system, and later simulation results will demonstrate that the contribution of the relay nodes can be very significant when the relay number is large. In the proposed schemes, the objective is still to optimize the SINR at each user node under total or individual relay power constraint. Furthermore, we investigate the two systems at the worst case when CSI errors exist.

Throughout this paper, we assume that the CSI is either estimated at the user or fed back to it by the relay via low rate feedback channels. Due to various reasons, such as resolution of the feedback CSI and mobility of the users and relays, the obtained CSI is likely to be imperfect, modeled as

$$\mathbf{F}_i = \hat{\mathbf{F}}_i + \mathbf{\Delta F}_i, \quad \mathbf{G}_i = \hat{\mathbf{G}}_i + \mathbf{\Delta G}_i \tag{14}$$

where $\hat{\mathbf{F}}_i$ and $\hat{\mathbf{G}}_i$ are the estimated channel matrices at the user nodes, and $\mathbf{\Delta F}_i$ and $\mathbf{\Delta G}_i$ represent the CSI error matrices. Using the uncertainty model exploited in [31], [32], and [35], we further assume that the norm of the errors are bounded by some known constants $\epsilon_{m,n}^{(i)}$ and $\beta_{m,n}^{(i)}$, i.e,

$$\begin{aligned} &|\Delta f_{m,n}^{(i)}| \leq \epsilon_{m,n}^{(i)}, \quad |\Delta g_{m,n}^{(i)}| \leq \beta_{m,n}^{(i)}, \\ &\quad m \in \{1, \ldots, M\}, \quad n \in \{1, \ldots, N\}, \end{aligned} \tag{15}$$

where $\Delta f_{m,n}^{(i)}$ and $\Delta g_{m,n}^{(i)}$ are the (m, n)-*th* element of the channel matrices $\mathbf{\Delta F}_i$ and $\mathbf{\Delta G}_i$, respectively.

According to [45], the proper values of $\epsilon_{m,n}^{(i)}$ and $\beta_{m,n}^{(i)}$ can be obtained using preliminary knowledge of the channel type. Note that even though there is an alternative way to model the uncertainty in $\hat{\mathbf{F}}_i$ and $\hat{\mathbf{G}}_i$, which is using a combined uncertainty model where the Euclidean norm of each row of $\hat{\mathbf{F}}_i$ and $\hat{\mathbf{G}}_i$ is bounded by some constant values, it will be seen that if we use this assumption in our optimization problem, the error terms will need to be decoupled and the knowledge of each $\epsilon_{m,n}^{(i)}$ and $\beta_{m,n}^{(i)}$ will still be needed.

Then, without loss of generality, again consider user $X_{a,i}$ as an example. From the expression of $y_{a,i}$, the receive SINR of $X_{a,i}$ can be derived as expressed in (5) and (6).

With the CSI errors, to maximize the minimum SINR at the user $X_{a,i}$ side, we have the following problem based on the worst-case scenario.

$$\begin{aligned}&\max_{\substack{\mathbf{a}_k,\mathbf{b}_k,\mathbf{c}_i,\lambda_R\\(k=1,\ldots,K)}}\ \min_{\Delta F_k,\Delta G_k}\ SINR_{a,i},\\&\quad s.t.\ ||\mathbf{c}_i||^2=1,\\&\qquad ||\mathbf{a}_k||^2\le P_S,\quad ||\mathbf{b}_k||^2\le P_S,\\&\qquad P_{relay}\le P_r,\quad or\ \mathbf{p}_{relay}\le\mathbf{p}_r,\\&\qquad |[\mathbf{\Delta F}_i]_{mn}|\le\epsilon_{m,n}^{(i)},\quad |[\mathbf{\Delta G}_i]_{mn}|\le\beta_{m,n}^{(i)},\\&\qquad (m\in\{1,\ldots,M\},\ n\in\{1,\ldots,N\})\end{aligned}\tag{16}$$

where P_{relay} and P_r represent the sum relay output power and the sum relay power constraint, respectively. $\mathbf{p}_{relay}=[P_{relay,1}\ P_{relay,2}\cdots\ P_{relay,M}]^T$ and $\mathbf{p}_r=[P_{r,1}\ P_{r,2}\cdots\ P_{r,M}]^T$ are the individual relay output power and the individual relay power constraint, respectively. And the two relay power constraints will be discussed in Section III-B and Section III-C, respectively.

As we can see from (16), the transmit beamforming vectors $\mathbf{a}_i$ and $\mathbf{b}_i$ have very different roles with the receive beamforming vectors $\mathbf{c}_i$ and $\mathbf{d}_i$, in maximizing the SINR. For example, by carefully choosing their coefficients, $\mathbf{c}_i$ and $\mathbf{d}_i$ can effectively reduce the IPI and propagation noise of the *i-th* user, but the same task is hard for $\mathbf{a}_i$ and $\mathbf{b}_i$, since they contribute to the IPI of all the other users except for its own. However, carefully designed $\mathbf{a}_i$ and $\mathbf{b}_i$ can directly lead to an optimal desired signal power (numerator of the SINR expression) of user $X_{a,i}$. Therefore, we decide not to jointly solve problem (16) and other $2K-1$ similar problems (for other users), where the global solution is extremely difficult, if not impossible, to obtain. As an alternative we propose to divide the problem into three sub-problems, each of which is carefully designed based on the role of the transceiver beamforming vectors and the relay coefficients in the SINR expression, and the three sub-problems are solved in three iteration steps. Note that, although the solution to the three sub-problems will very unlikely be the actual global solution of problem (16), it can provide a rather satisfactory performance. Such a strategy will also help us find a solution that can mitigate the quality-of-service reduction caused by channel errors as well as meeting the power constraint.

A. ITERATION STEP I: MAXIMIZING THE OVERALL GAIN

In the first step of our iterative design, $\mathbf{c}_i$, $\mathbf{d}_i$ and $\mathbf{W}$ are fixed to either initial values or previously updated values and we try to optimize $\mathbf{a}_i$ and $\mathbf{b}_i$ to maximize the overall gain of the desired signal, which is also the power of the desired signal, under a transmit power constraint in the case of imperfectly known CSI. We will also demonstrate that in our designed scheme, the choice of $\mathbf{a}_i$ and $\mathbf{b}_i$ in the first iteration step leads to an optimal desired signal power not only when the CSI is precisely measured, but also in the worst-case situation. Now we formulate the transmit beamforming problem with CSI errors as follows

$$\begin{aligned}&\max_{\mathbf{b}_i}\ \min_{\Delta F_i,\Delta G_i}\ |\mathbf{c}_i^H\mathbf{F}_i^T\mathbf{W}\mathbf{G}_i\mathbf{b}_i|^2,\\&s.t.\ ||\mathbf{b}_i||^2\le P_S,\\&\qquad |[\mathbf{\Delta F}_i]_{mn}|\le\epsilon_{m,n}^{(i)},\quad |[\mathbf{\Delta G}_i]_{mn}|\le\beta_{m,n}^{(i)},\\&\qquad (m\in\{1,\ldots,M\},\ n\in\{1,\ldots,N\})\\&\max_{\mathbf{a}_i}\ \min_{\Delta F_i,\Delta G_i}\ |\mathbf{d}_i^H\mathbf{G}_i^T\mathbf{W}\mathbf{F}_i\mathbf{a}_i|^2,\\&s.t.\ ||\mathbf{a}_i||^2\le P_S.\\&\qquad |[\mathbf{\Delta F}_i]_{mn}|\le\epsilon_{m,n}^{(i)},\quad |[\mathbf{\Delta G}_i]_{mn}|\le\beta_{m,n}^{(i)}.\\&\qquad (m\in\{1,\ldots,M\},\ n\in\{1,\ldots,N\})\end{aligned}\tag{17}$$

Denote $\mathfrak{f}_i=\mathbf{W}^T\mathbf{F}_i\mathbf{c}_i^*$ and $\mathfrak{g}_i=\mathbf{W}^T\mathbf{G}_i\mathbf{b}_i^*$, where $\mathfrak{f}_i$, $\mathfrak{g}_i\in\mathbb{C}^{M\times1}$. We have

$$\begin{aligned}|\mathbf{c}_i^H\mathbf{F}_i^T\mathbf{W}\mathbf{G}_i\mathbf{b}_i|^2&=|\mathfrak{f}_i^T\mathbf{G}_i\mathbf{b}_i|^2,\\|\mathbf{d}_i^H\mathbf{G}_i^T\mathbf{W}\mathbf{F}_i\mathbf{a}_i|^2&=|\mathfrak{g}_i^T\mathbf{F}_i\mathbf{a}_i|^2.\end{aligned}\tag{18}$$

From the CSI uncertainty expression (14), we can rewrite the two vectors as

$$\begin{aligned}\mathfrak{f}_i&=\hat{\mathfrak{f}}_i+\Delta\mathfrak{f}_i=\mathbf{W}^T\hat{\mathbf{F}}_i\mathbf{c}_i^*+\mathbf{W}^T\Delta\mathbf{F}_i\mathbf{c}_i^*,\\\mathfrak{f}_i&=\hat{\mathfrak{g}}_i+\Delta\mathfrak{g}_i=\mathbf{W}^T\hat{\mathbf{G}}_i\mathbf{d}_i^*+\mathbf{W}^T\Delta\mathbf{G}_i\mathbf{d}_i^*.\end{aligned}\tag{19}$$

Using $\mathfrak{f}_m^{(i)}$ and $\mathfrak{g}_m^{(i)}$ to represent the *m-th* element of $\mathfrak{f}_i$ and $\mathfrak{g}_i$, respectively, we have

$$\begin{aligned}\mathfrak{f}_m^{(i)}&=\sum_{n=1}^{N}c_{i,n}^*\hat{f}_{m,n}^{(i)}w_m+c_{i,n}^*\Delta f_{m,n}^{(i)}w_m,\\\mathfrak{g}_m^{(i)}&=\sum_{n=1}^{N}d_{i,n}^*\hat{g}_{m,n}^{(i)}w_m+d_{i,n}^*\Delta g_{m,n}^{(i)}w_m,\end{aligned}\tag{20}$$

where $\hat{g}_{m,n}^{(i)}$, $\hat{f}_{m,n}^{(i)}$, $\Delta f_{m,n}^{(i)}$ and $\Delta g_{m,n}^{(i)}$ are the (m,n)*-th* element of the channel matrices $\hat{\mathbf{G}}_i$, $\hat{\mathbf{F}}_i$, $\mathbf{\Delta G}_i$ and $\mathbf{\Delta F}_i$, respectively. And $c_{i,n}$ and $d_{i,n}$ represents the *n-th* element of $\mathbf{c_i}$ and $\mathbf{d_i}$, respectively.

Without loss of generality, consider user $X_{a,i}$ as an example. From (20) and the channel error constraint, the absolute value of the $m-th$ element of $\mathbf{\Delta}\mathfrak{f}_i$ can be expressed by

$$|\Delta\mathfrak{f}_m^{(i)}|=|\sum_{n=1}^{N}c_{i,n}^*\Delta f_{m,n}^{(i)}w_m|\le\sum_{n=1}^{N}\epsilon_{m,n}^{(i)}|c_{i,n}^*w_m|\triangleq\xi_{\mathfrak{f}_m}^{(i)}.\tag{21}$$

The upper bound of $|\Delta\mathfrak{f}_m^{(i)}|$ is reached when $|\Delta f_{m,n}^{(i)}|=\epsilon_{m,n}^{(i)}$ for $n=1,\ldots,N$, and all the values of $c_{i,n}^*\Delta f_{m,n}^{(i)}w_m$ have the same phases. From the expression we can also notice that the phase of $\Delta\mathfrak{f}_m^{(i)}$ can be arbitrary.

Now denote the matrix product of $\mathbf{c}_i^H\mathbf{F}_i^T\mathbf{W}\mathbf{G}_i$ in (17) by $\mathbf{h}_{FG}^{(i)}=\hat{\mathbf{h}}_{FG}^{(i)}+\Delta\mathbf{h}_{FG}^{(i)},\in\mathbb{C}^{1\times N}$, where $\hat{\mathbf{h}}_{FG}^{(i)}$ is related to the estimated value of the channel matrix, and

$$\Delta\mathbf{h}_{FG}^{(i)} = \hat{\mathfrak{f}}_i^T \Delta\mathbf{G}_i + \Delta\mathfrak{f}_i^T \hat{\mathbf{G}}_i + \Delta\mathfrak{f}_i^T \Delta\mathbf{G}_i, \tag{22}$$

is the error. Then, the absolute value of the *n-th* element of $\Delta\mathbf{h}_{FG}^{(i)}$ is given by

$$\begin{aligned}|\Delta h_{FG,n}^{(i)}| &= |\sum_{m=1}^{M} (\hat{\mathfrak{f}}_m^{(i)} \Delta g_{m,n}^{(i)} + \Delta\mathfrak{f}_m^{(i)} \hat{g}_{m,n}^{(i)} + \Delta\mathfrak{f}_m^{(i)} \Delta g_{m,n}^{(i)})| \\ &\leq \sum_{m=1}^{M} (|\hat{\mathfrak{f}}_m^{(i)}|\beta_{m,n}^{(i)} + \xi_{\mathfrak{f}_m}^{(i)}|\hat{g}_{m,n}^{(i)}| + \xi_{\mathfrak{f}_m}^{(i)}\beta_{m,n}^{(i)}) \triangleq \varphi_{FG,n}^{(i)}.\end{aligned} \tag{23}$$

The equality holds when all the $\hat{\mathfrak{f}}_m^{(i)} \Delta g_{m,n}^{(i)}$, $\Delta\mathfrak{f}_m^{(i)} \hat{g}_{m,n}^{(i)}$ and $\Delta\mathfrak{f}_m^{(i)} \Delta g_{m,n}^{(i)}$ have the same phase. Moreover, $\Delta h_{FG,n}^{(i)}$ can have arbitrary phase. As a result, the error vector $\Delta\mathbf{h}_{FG}^{(i)}$ has an upper norm bound as

$$\begin{aligned}||\Delta\mathbf{h}_{FG}^{(i)}|| &= (\sum_{n=1}^{N} |\Delta h_{FG,n}^{(i)}|^2)^{\frac{1}{2}} \\ &\leq (\sum_{n=1}^{N} \varphi_{FG,n}^{(i)\,2})^{\frac{1}{2}} \triangleq \varphi_{FG}^{(i)}.\end{aligned} \tag{24}$$

Now, we can rewrite the worst-case based sub-problem (17) for user $X_{a,i}$ using $\mathbf{h}_{FG}^{(i)}$.

$$\begin{aligned}&\max_{\mathbf{b}_i} \min_{\Delta\mathbf{h}_{FG}^{(i)}} |(\hat{\mathbf{h}}_{FG}^{(i)} + \Delta\mathbf{h}_{FG}^{(i)})\mathbf{b}_i|^2, \\ &s.t. \ ||\mathbf{b}_i||^2 \leq P_S, \\ &\quad\ ||\Delta\mathbf{h}_{FG}^{(i)}|| \leq \varphi_{FG}^{(i)}.\end{aligned} \tag{25}$$

Using triangle inequality and Cauchy-Schwarz inequality, we have

$$\begin{aligned}|(\hat{\mathbf{h}}_{FG}^{(i)} + \Delta\mathbf{h}_{FG}^{(i)})\mathbf{b}_i|^2 &\geq (|\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_i| - |\Delta\mathbf{h}_{FG}^{(i)}\mathbf{b}_i|)^2 \\ &\geq (|\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_i| - ||\Delta\mathbf{h}_{FG}^{(i)}|| \cdot ||\mathbf{b}_i||)^2 \\ &\geq (|\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_i| - \varphi_{FG}^{(i)}||\mathbf{b}_i||)^2,\end{aligned} \tag{26}$$

where we have made a reasonable assumption of $|\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_i| > \varphi_{FG}^{(i)}||\mathbf{b}_i||$. It can be derived that the particular $\Delta\mathbf{h}_{FG}^{(i)}$ for the equality to hold is

$$\Delta\mathbf{h}_{FG}^{(i)} = -\varphi_{FG}^{(i)} \frac{\mathbf{b}_i}{||\mathbf{b}_i||} e^{j\theta}, \quad \theta \triangleq \text{angle}(\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_i). \tag{27}$$

Therefore, the worst-case optimization sub-problem (25) for user $X_{a,i}$ can be rewritten as

$$\begin{aligned}&\max_{\mathbf{b}_i} (|\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_i| - \varphi_{FG}^{(i)}||\mathbf{b}_i||)^2, \\ &s.t. \ ||\mathbf{b}_i||^2 \leq P_S.\end{aligned} \tag{28}$$

It can be proved that the optimal solution of $\mathbf{b}_i$ will always satisfy the upper bound determined by P_S. We prove it by contradiction. Assume the optimal $\mathbf{b}_{i,opt}$ does not satisfy the upper bound, i.e., $||\mathbf{b}_{i,opt}||^2 = P_S' = P_S/\rho$, $\rho > 1$. Then, there must exist a $\mathbf{b}_i' = \sqrt{k}\mathbf{b}_{i,opt}$ which satisfies $||\mathbf{b}_i'||^2 = P_S$, leading to

$$\begin{aligned}(|\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_i'| - \varphi_{FG}^{(i)}||\mathbf{b}_i'||)^2 &= \rho(|\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_{i,opt}| - \varphi_{FG}^{(i)}||\mathbf{b}_{i,opt}||)^2 \\ &> (|\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_{i,opt}| - \varphi_{FG}^{(i)}||\mathbf{b}_{i,opt}||)^2,\end{aligned} \tag{29}$$

which contradicts the assumption that $\mathbf{b}_{i,opt}$ is the optimal solution. Therefore, the problem (28) becomes

$$\begin{aligned}&\max_{\mathbf{b}_i} (|\hat{\mathbf{h}}_{FG}^{(i)}\mathbf{b}_i| - \varphi_{FG}^{(i)}\sqrt{P_S})^2, \\ &s.t. \ ||\mathbf{b}_i||^2 = P_S.\end{aligned} \tag{30}$$

Notice that the above problem has a closed-form solution, which is the same as the solution of (17) when $\epsilon_{m,n}^{(i)} = \beta_{m,n}^{(i)} = 0$. That is to say, the solution of our first iteration step applies to both the optimal case and the worst case of the first sub-problem. Similarly, the first sub-problem for user $X_{b,i}$ can be solved using the same procedure. And the solutions lead to the updated values of $\mathbf{a}_i$ and $\mathbf{b}_i$ as

$$\mathbf{a}_i = P_S^{\frac{1}{2}} \frac{\hat{\mathbf{F}}_i^H \hat{\mathbf{G}}_i^* \mathbf{d}_i}{|\hat{\mathbf{F}}_i^H \hat{\mathbf{G}}_i^* \mathbf{d}_i|}, \quad \mathbf{b}_i = P_S^{\frac{1}{2}} \frac{\hat{\mathbf{G}}_i^H \hat{\mathbf{F}}_i^* \mathbf{c}_i}{|\hat{\mathbf{G}}_i^H \hat{\mathbf{F}}_i^* \mathbf{c}_i|}. \tag{31}$$

B. ITERATION STEP II-1: RELAY STRATEGY 1 (SUM-RELAY POWER CONSTRAINT)

In the second iteration step of our design, the relay weights are decided based on fixed values (either initialized or updated) of $\mathbf{a}_i$, $\mathbf{b}_i$, $\mathbf{c}_i$ and $\mathbf{d}_i$. And we propose two different relay strategies based on two different relay power assumptions. In this subsection, we consider our first relay strategy when a total relay power budget is applied to the network, where the designed beamforming coefficients enable the relay nodes to jointly construct a stream transmission environment that can help the users to obtain a better QoS.

Firstly, considering perfect CSI, the following formulation is adopted to find the relay weights that optimizes the sum desired signal power received by all the user nodes.

$$\begin{aligned}&\max_{\lambda_R} \sum_{i=1}^{K} (|\mathbf{c}_i^H \mathbf{F}_i^T \mathbf{W} \mathbf{G}_i \mathbf{b}_i|^2 + |\mathbf{d}_i^H \mathbf{G}_i^T \mathbf{W} \mathbf{F}_i \mathbf{a}_i|^2), \\ &s.t. \ P_{relay} \leq P_r.\end{aligned} \tag{32}$$

Denote $\mathbf{g}_i' = \mathbf{G}_i\mathbf{d}_i^*$, $\mathfrak{f}_i' = \mathbf{F}_i\mathbf{c}_i^*$, where $\mathfrak{f}_i'$, $\mathbf{g}_i' \in \mathbb{C}^{M\times 1}$, and $\mathbf{w} = [w_1 w_2 \ldots w_M]^H$. Together with (19), we can rewrite the objective function in (32) as

$$\sum_{i=1}^{K} (|\mathbf{w}^H \mathcal{G}_i \mathfrak{f}_i'|^2 + |\mathbf{w}^H \mathcal{F}_i \mathbf{g}_i'|^2) = \mathbf{w}^H \mathbf{Q}_R \mathbf{w}, \tag{33}$$

where $\mathcal{G}_i$ and $\mathcal{F}_i \in \mathbb{C}^{M\times M}$ are diagonal matrices in which the entries of their main diagonal correspond to $\mathbf{G}_i\mathbf{b}_i$ and $\mathbf{F}_i\mathbf{a}_i$, respectively, and

$$\mathbf{Q}_R = \sum_{i=1}^{K} (\mathcal{G}_i \mathfrak{f}_i' \mathfrak{f}_i'^H \mathcal{G}_i^H + \mathcal{F}_i \mathbf{g}_i' \mathbf{g}_i'^H \mathcal{F}_i^H). \tag{34}$$

Now the sum relay power P_{relay} is given by

$$P_{relay} = \mathbf{w}^H(\sigma_r^2\mathbf{I}_M+\sum_{i=1}^{K}\mathcal{G}_i\mathcal{G}_i^H+\sum_{i=1}^{K}\mathcal{F}_i\mathcal{F}_i^H)\mathbf{w} = \mathbf{w}^H\mathbf{Q}_P\mathbf{w}, \quad (35)$$

where $\mathbf{Q}_P$ is a diagonal matrix. The problem (32) can now be rewritten as

$$\max_{\lambda_R} \mathbf{w}^H\mathbf{Q}_R\mathbf{w},$$
$$s.t.\ \mathbf{w}^H\mathbf{Q}_P\mathbf{w} \leq P_r. \quad (36)$$

It can be transformed to an eigenvector problem with a closed-form solution, which leads to the following updated value for $\mathbf{w}$ when CSI errors are not presented

$$\mathbf{w} = \lambda\rho\{\mathbf{Q}_P^{-1}\mathbf{Q}_R\}, \quad (37)$$

where λ is a power control scalar decided by P_r.

In the presence of CSI errors, we propose to maintain the power constraint of the relay system for all possible CSI errors. On the other hand, since the worst case of maximizing $|\mathbf{w}^H\mathcal{G}_i\mathbf{f}_i'|^2$ and $|\mathbf{w}^H\mathcal{F}_i\mathbf{g}_i'|^2$ for each individual user node is already considered in our first iteration step, setting the objective function here with worst-case scenario again will not be necessary, and it will lead to performance degradation. Accordingly, we keep the objective function of (32) with the channel matrices replaced by their estimated values, and transform (32) to the following problem

$$\begin{aligned}&\max_{\mathbf{w}} \sum_{i=1}^{K}(|\mathbf{w}^H\hat{\mathcal{G}}_i\hat{\mathbf{f}}_i'|^2+|\mathbf{w}^H\hat{\mathcal{F}}_i\hat{\mathbf{g}}_i'|^2),\\ &s.t.\ \max_{\Delta\mathbf{F}_i\Delta\mathbf{G}_i} P_{relay}\leq P_r,\\ &\quad |[\Delta\mathbf{F}_i]_{mn}|\leq\epsilon_{m,n}^{(i)},\quad |[\Delta\mathbf{G}_i]_{mn}|\leq\beta_{m,n}^{(i)}.\\ &\quad (m\in\{1,\ldots,M\}, n\in\{1,\ldots,N\})\end{aligned} \quad (38)$$

According to (35), the maximum value of P_{relay} for all $[\Delta\mathbf{F}_i]_{mn}$ and $[\Delta\mathbf{G}_i]_{mn}$ is obtained when all the diagonal elements of the matrix $\mathbf{Q}_P$ take their maximum values. Denote $Q_{p,m}$, $\mathcal{G}_{i,m}$ and $\mathcal{F}_{i,m}$ as the *m-th* entry of the main diagonal of $\mathbf{Q}_P$, $\mathcal{G}_i$ and $\mathcal{F}_i$, respectively, and we have

$$\begin{aligned}Q_{p,m} &= \hat{Q}_{p,m}+\Delta Q_{p,m}\\ &= \sigma_r^2+\sum_{i=1}^{K}(|\hat{\mathcal{G}}_{i,m}+\Delta\mathcal{G}_{i,m}|^2+|\hat{\mathcal{F}}_{i,m}+\Delta\mathcal{F}_{i,m}|^2),\end{aligned} \quad (39)$$

where

$$|\Delta\mathcal{G}_{i,m}| = |\sum_{n=1}^{N}\Delta g_{m,n}^{(i)}b_{i,n}| \leq \sum_{n=1}^{N}\beta_{m,n}^{(i)}|b_{i,n}| = \xi_{\mathcal{G}_m}^{(i)}. \quad (40)$$

The upper bound is reached when $|\Delta g_{m,n}^{(i)}| = \beta_{m,n}^{(i)}$ for $n = 1,\ldots,N$, and all $\Delta g_{m,n}^{(i)}b_{i,n}$ have the same phase. Similarly, we can derive the upper bound, denoted as $\xi_{\mathcal{F}_m}^{(i)}$, for $|\Delta\mathcal{F}_{i,m}|$.

Then, (39) becomes

$$\begin{aligned}Q_{p,m} \leq \sigma_r^2 &+ \sum_{i=1}^{K}(|\hat{\mathcal{G}}_{i,m}|^2+2|\hat{\mathcal{G}}_{i,m}|\xi_{\mathcal{G}_m}^{(i)}+\xi_{\mathcal{G}_m}^{2(i)}+|\hat{\mathcal{F}}_{i,m}|^2\\ &+2|\hat{\mathcal{F}}_{i,m}|\xi_{\mathcal{F}_m}^{(i)}+\xi_{\mathcal{F}_m}^{2(i)}) = Q_{p,m}'.\end{aligned} \quad (41)$$

Now construct an $M \times M$ diagonal matrix $\mathbf{Q}_P'$ with the *m-th* diagonal entries being $Q_{p,m}'$. The maximum value of P_{relay}, denoted as P_{relay}', can be expressed by

$$P_{relay}' = \mathbf{w}^H\mathbf{Q}_P'\mathbf{w}. \quad (42)$$

(38) can be rewritten as

$$\begin{aligned}&\max_{\mathbf{w}} \sum_{i=1}^{K}(|\mathbf{w}^H\hat{\mathcal{G}}_i\hat{\mathbf{f}}_i'|^2+|\mathbf{w}^H\hat{\mathcal{F}}_i\hat{\mathbf{g}}_i'|^2),\\ &s.t.\ \max_{\Delta\mathbf{F}_i\Delta\mathbf{G}_i} \mathbf{w}^H\mathbf{Q}_P'\mathbf{w}\leq P_r,\\ &\quad |[\Delta\mathbf{F}_i]_{mn}|\leq\epsilon_{m,n}^{(i)},\quad |[\Delta\mathbf{G}_i]_{mn}|\leq\beta_{m,n}^{(i)}.\\ &\quad (m\in\{1,\ldots,M\}, n\in\{1,\ldots,N\})\end{aligned} \quad (43)$$

Let $\hat{\mathbf{Q}}_R$ denote the estimated value of $\mathbf{Q}_R$, and the closed-form solution to (43) becomes

$$\mathbf{w} = \lambda'\rho\{\mathbf{Q}_P'^{-1}\hat{\mathbf{Q}}_R\} = \lambda'\bar{\mathbf{w}}, \quad (44)$$

where we use $\bar{\mathbf{w}}$ to represent the normalized principle eigenvector of $\mathbf{Q}_P'^{-1}\hat{\mathbf{Q}}_R$ and the power control scalar λ' can be obtained by

$$\lambda' = \sqrt{\frac{P_r}{\bar{\mathbf{w}}^H\mathbf{Q}_P'\bar{\mathbf{w}}}}. \quad (45)$$

C. ITERATION STEP II-2: RELAY STRATEGY 2 (INDIVIDUAL-RELAY POWER CONSTRAINT)

In this subsection, we propose our second relay strategy in the second iteration step for the case that each relay node has its own power budget. In this design, we avoid using the same objective as relay strategy 1, which is optimizes the sum desired signal power received by all the user nodes, for the reason that this will transform the problem into a series of second-order cone programming (SOCP) feasibility problems and thus the computational complexity will be greatly increased. Instead, our relay strategy 2 mainly utilizes the fundamental result from [46] that when a large number of relay nodes are involved in the network, the channels between the users and relays could be pairwisely nearly orthogonal, and relay coefficient of each user can be designed accordingly. Also, the contribution of the relay nodes in our second scheme is expected to become more clear when the number of relay nodes is large.

Here we also consider the case when perfect CSI is obtained at first. Let $\mathbf{f}_{i,m}, \mathbf{g}_{i,m} \in \mathbb{C}^{1\times N}$ represents the *m-th* row of $\mathbf{F}_i$ and $\mathbf{G}_i$, respectively. We propose the following phase rotating rule for the *m-th* relay node ($m = 1, \ldots, M$).

$$w_m = \lambda_m(\sum_{i=1}^{K} \mathbf{f}^*_{i,m}\mathbf{c}_i\mathbf{b}_i^H\mathbf{g}^H_{i,m} + \mathbf{g}^*_{i,m}\mathbf{d}_i\mathbf{a}_i^H\mathbf{f}^H_{i,m}) \\ = \lambda_m(\sum_{i=1}^{K} \bar{u}^*_{i,m}v^*_{i,m} + \bar{v}^*_{i,m}u^*_{i,m}), \quad (46)$$

where $\bar{u}_{i,m} \triangleq \mathbf{f}^*_{i,m}\mathbf{c}_i$, $u_{i,m} \triangleq \mathbf{f}_{i,m}\mathbf{a}_i$, $\bar{v}_{i,m} \triangleq \mathbf{g}^*_{i,m}\mathbf{d}_i$ and $v_{i,m} \triangleq \mathbf{g}_{i,m}\mathbf{b}_i$. And λ_m is a power-control parameter which limits the output power of each relay node, given by

$$\lambda_m = \sqrt{\frac{P_{r,m}/|\sum_{i=1}^{K} \bar{u}^*_{i,m}v^*_{i,m} + \bar{v}^*_{i,m}u^*_{i,m}|^2}{\sigma_r^2 + \sum_{i=1}^{K} |u_{i,m}|^2 + |v_{i,m}|^2}}, \quad (47)$$

where $P_{r,m}$ is the individual power budget at the *m-th* relay.

As will be observed from the updating process for $\mathbf{c}_i$ in Section III-D, the correlation of $\mathbf{c}_i$ and $\mathbf{f}_{i,m}$ is very weak in our scenario, especially when M and K are large. As a result, we can consider them as the two independent variables. Therefore, $\bar{u}_{i,m}$ has a complex normal distribution of $\mathcal{CN}(0, \Gamma^{\bar{u}}_{i,m})$, where $\Gamma^{u}_{i,m}$ is a constant value decided by the value of $\mathbf{c}_i$ and the variance of $\mathbf{f}_{i,m}$. Similarly, $v_{i,m}$, $u_{i,m}$ and $\bar{v}_{i,m}$ have distributions of $\mathcal{CN}(0, \Gamma^{v}_{i,m})$, $\mathcal{CN}(0, \Gamma^{u}_{i,m})$ and $\mathcal{CN}(0, \Gamma^{\bar{v}}_{i,m})$, respectively.

In order to provide further insight for choosing the phase rotating coefficient at the relay node, we rewrite (3) after removing the self interference part, in terms of $u_{i,m}$, $v_{i,m}$, $\hat{u}_{i,m}$ and $\hat{v}_{i,m}$.

$$\bar{y}_{a,i} = \underbrace{\sum_{m=1}^{M} \bar{u}_{i,m}w_m v_{i,m}x_{b,i}}_{\text{Desired signal}} + \underbrace{\sum_{m=1}^{M} \bar{u}_{i,m}w_m n_{R,m} + n_{a,i}}_{\text{Noise}} \\ + \underbrace{\sum_{m=1}^{M}\sum_{j\neq i}^{K} (\bar{u}_{i,m}w_m u_{j,m}x_{a,j} + \bar{u}_{i,m}w_m v_{j,m}x_{b,j})}_{\text{IPI}} \\ = \mathbb{G}^{(S)}_{a,i}x_{b,i} + \mathbb{G}^{(Noise)}_{a,i}n_{R,m} + n_{a,i} \\ + \sum_{j\neq i}^{K}(\mathbb{G}^{(IPI)}_{ab,ij}x_{a,j} + \mathbb{G}^{(IPI)}_{ba,ij}x_{b,j}), \quad (48)$$

where $\mathbb{G}^{(S)}_{a,i}$, $\mathbb{G}^{(Noise)}_{a,i}$, $\mathbb{G}^{(IPI)}_{ab,ij}$ and $\mathbb{G}^{(IPI)}_{ba,ij}$ represents the gain of each component, $n_{R,m}$ represents the complex Gaussian noise of the *m-th* relay node with the distribution $\mathcal{CN}(0, \sigma_r^2)$ and $n_{a,i}$=$\mathbf{d}_i\mathbf{n}_{b,i}$. Due to the fact that in our scheme, $\mathbf{d}_i$ is a normalized vector ($||\mathbf{d}_i||^2$=1), $n_{a,i}$ will have a distribution given by $\mathcal{CN}(0, \sigma_u^2)$.

Let $\bar{y}^{(S)}_{a,i}$, $\bar{y}^{(IPI)}_{a,i}$ and $\bar{y}^{(Noise)}_{a,i}$ denote the desired signal, IPI and noise part in (48), respectively. We have

$$\bar{y}^{(S)}_{a,i} = \sum_{m=1}^{M} \lambda_m\bar{u}_{i,m}(\sum_{i=1}^{K} \bar{u}^*_{i,m}v^*_{i,m} + \bar{v}^*_{i,m}u^*_{i,m})v_{i,m}x_{b,i}. \quad (49)$$

Since $\bar{u}_{i,m}$, $\bar{v}_{i,m}$, $\bar{u}_{i',m}(i'\neq i)$ and $\bar{u}_{i,m'}(m'\neq m)$ can be considered as zero mean mutually uncorrelated random variables, with $\mathbb{E}[x^2] = \sigma^2$, where $x \sim \mathcal{CN}(0, \sigma^2)$, we have

$$\mathbb{E}[\mathbb{G}^{(S)}_{a,i}] = \mathbb{E}[\sum_{m=1}^{M} \lambda_m||\bar{u}_{i,m}||^2||v_{i,m}||^2] = \sum_{m=1}^{M} \lambda_m\Gamma^{\bar{u}}_{i,m}\Gamma^{v}_{i,m}. \quad (50)$$

Denote $\gamma_{i,m} = \bar{u}_{i,m}w_m v_{i,m}$ for $m = 1, \ldots, M$. Since all $\gamma_{i,m}$ are independent random variables, we can apply the Tchebyshev's inequality theorem [47], and for any constant ζ obtain

$$\Pr[\left|\frac{\mathbb{G}_{a,i}}{M} - \frac{\mathbb{E}[\mathbb{G}_{a,i}]}{M}\right| \geq \zeta] \leq \frac{\text{Var}[\hat{y}^{(S)}_{a,i}]/M^2}{\zeta^2}, \quad (51)$$

where $\Pr[\cdot]$ represents the probability operator. Apparently $\bar{y}^{(S)}_{a,i}/M$ will be more likely to approach $\mathbb{E}[\bar{y}^{(S)}_{a,i}/M] = x_{a,i}\mathbb{E}[\mathbb{G}_{a,i}]/M = \overline{\lambda}_{\Gamma,m}x_{a,i}$ ($\overline{\lambda}_{\Gamma,m}$ denotes the average value of $\lambda_m\Gamma^{\bar{u}}_{i,m}\Gamma^{v}_{i,m}$) as M increases. As a result, the asymptotic value of $|\bar{y}^{(S)}_{a,i}|^2$ is proportional to M^2, when M is large.

Similarly, we can derive that $\mathbb{E}[\mathbb{G}^{(Noise)}_{a,i}] = 0$, $\mathbb{E}[\mathbb{G}^{(IPI)}_{ab,ij}] = 0$ and $\mathbb{E}[\mathbb{G}^{(IPI)}_{ba,ij}] = 0$, which yields that when M is large, $\hat{y}^{(IPI)}_{a,i}/M$ and $\hat{y}^{(Noise)}_{a,i}/M$ will have a high probability of taking a value around 0.

In another word, the $\lambda_m\hat{u}_{i,m}\hat{u}^*_{i,m}v^*_{i,m}v_{i,m}x_{b,i}$ part in $\hat{y}^{(S)}_{a,i}$ is the only component in $\hat{y}_{a,i}$ that can grow steadily through accumulation as M increases; meanwhile, the other parts will grow much more slowly. The situation is similar for $\hat{y}_{b,i}$ (received signal at $X_{b,i}$).

Now, considering the presence of CSI errors, we propose to use the same phase rotating rule for the *m-th* relay node with a new power scalar λ'_m to restrict the output power of each relay node in the worst case.

$$w_m = \lambda'_m(\sum_{i=1}^{K} \hat{\bar{u}}^*_{i,m}\hat{v}^*_{i,m} + \hat{\bar{v}}^*_{i,m}\hat{u}^*_{i,m}), \quad (52)$$

where the new notations are: $\hat{\bar{u}}_{i,m} \triangleq \hat{\mathbf{f}}^*_{i,m}\mathbf{c}_i$, $\hat{u}_{i,m} \triangleq \hat{\mathbf{f}}_{i,m}\mathbf{a}_i$, $\hat{\bar{v}}_{i,m} \triangleq \hat{\mathbf{g}}^*_{i,m}\mathbf{d}_i$ and $\hat{v}_{i,m} \triangleq \hat{\mathbf{g}}_{i,m}\mathbf{b}_i$.

Similarly, using user $X_{a,i}$ as an example, (49) can be rewritten as

$$\bar{y}'^{(S)}_{a,i} = \sum_{m=1}^{M} \lambda'_m(\hat{\bar{u}}_{i,m} + \Delta\bar{u}_{i,m}) \\ \times (\sum_{i=1}^{K} \bar{u}^*_{i,m}v^*_{i,m} + \bar{v}^*_{i,m}u^*_{i,m})(\hat{v}_{i,m} + \Delta v_{i,m})x_{b,i}, \quad (53)$$

where we have $\Delta\bar{u}_{i,m} \triangleq \Delta\mathbf{f}^*_{i,m}\mathbf{c}_i$ and $\Delta v_{i,m} \triangleq \Delta\mathbf{g}_{i,m}\mathbf{b}_i$. If we assume that $\mathbb{E}[\Delta\mathbf{f}_{i,m}] = \mathbb{E}[\Delta\mathbf{g}_{i,m}] = \mathbf{0}$, we have $\mathbb{E}[\Delta\bar{u}_{i,m}] = \mathbb{E}[\Delta v_{i,m}] = 0$, and as a result, $\mathbb{E}[\mathbb{G}^{(S)}_{a,i}]$ will stay the same as in (50). It demonstrates that our phase rotating rule will remain effective in the presence of CSI, and now we will derive the choice of λ'_m in the worst case scenario.

From the definition of $\Delta u_{i,m}$ and $\Delta v_{i,m}$, we have

$$\Delta u_{i,m} = \sum_{n=1}^{N} \Delta f_{m,n}^{(i)} a_{i,n} \leq \sum_{n=1}^{N} \epsilon_{m,n}^{(i)} |a_{i,n}| = \xi_{u_{i,m}}^{(i)},$$
$$\Delta v_{i,m} = \sum_{n=1}^{N} \Delta f_{m,n}^{(i)} b_{i,n} \leq \sum_{n=1}^{N} \beta_{m,n}^{(i)} |b_{i,n}| = \xi_{v_{i,m}}^{(i)}. \quad (54)$$

The power control scalar λ'_m that can restrict the maximum output power of the *m-th* relay in the worst case can now be derived from (47), and it is given as

$$\lambda'_m = \sqrt{\frac{P_{r,m}/|\sum_{i=1}^{K} \hat{\hat{u}}_{i,m}^{*}\hat{v}_{i,m}^{*} + \hat{\hat{v}}_{i,m}^{*}\hat{u}_{i,m}^{*}|^2}{\sigma_r^2 + \sum_{i=1}^{K}(|\hat{u}_{i,m}| + \xi_{u_{i,m}}^{(i)})^2 + (|\hat{v}_{i,m}| + \xi_{v_{i,m}}^{(i)})^2}}. \quad (55)$$

D. ITERATION STEP III: MAXIMIZING USER SINR

Now we have updated the values of the two transmit beamforming vectors $\mathbf{a}_i$ and $\mathbf{b}_i$, and the relay coefficients $\mathbf{w}$. Next are the two beamforming vectors $\mathbf{c}_i$ for user $X_{a,i}$ and $\mathbf{d}_i$ for user $X_{b,i}$.

In our first two iteration steps, the power of the desired signal at each user node and the relay output power has been considered in the worst case with specific values of $\Delta\mathbf{F}_i$ and $\Delta\mathbf{G}_i$. As a result, the first two iteration steps would have sufficiently compensated for the user SINR in extreme cases (worst case for desired signal power) along with guaranteeing that the power constraints are satisfied in the worst case. The part that remains unconsidered in the SINR expression (5) and (6) is mainly the IPI and propagation noise in the denominator of the SINR expression. However, it can be observed that the IPI part is jointly decided by the transmission channel matrices of all the other user pairs apart from user $X_{a,i}$, and thus the worst-case formulation will be too conservative and has much poorer performance. Due to these reasons, finding the lower bound on the cost function of (16) would not be as important as in the two earlier steps, and we formulate the following problem for user $X_{a,i}$ to decide its receive beamformer vector (expressions for user $X_{b,i}$ can be similarly derived).

$$\max_{\mathbf{c}_i} \frac{\mathbf{c}_i^H \hat{\mathbf{F}}_i^T \hat{\mathbf{Q}}_{a,i}^{(S)} \hat{\mathbf{F}}_i^* \mathbf{c}_i}{\sigma_u^2 + \mathbf{c}_i^H \hat{\mathbf{F}}_i^T \hat{\mathbf{Q}}_{a,i}^{(N)} \hat{\mathbf{F}}_i^* \mathbf{c}_i + \mathbf{c}_i^H \hat{\mathbf{F}}_i^T \hat{\mathbf{Q}}_{a,i}^{(I)} \hat{\mathbf{F}}_i^* \mathbf{c}_i},$$
$$s.t.\ ||\mathbf{c}_i||^2 = 1 \quad (56)$$

where,

$$\hat{\mathbf{Q}}_{a,i}^{(S)} = Ps \cdot \mathbf{W}\hat{\mathbf{G}}_i \mathbf{b}_i \mathbf{b}_i^H \hat{\mathbf{G}}_i^H \mathbf{W}^H,$$
$$\hat{\mathbf{Q}}_{a,i}^{(N)} = \sigma_r^2 \cdot \mathbf{W}\mathbf{W}^H,$$
$$\hat{\mathbf{Q}}_{a,i}^{(I)} = Ps \cdot \sum_{j \neq i}^{K} \mathbf{W}(\hat{\mathbf{F}}_j \mathbf{a}_j \mathbf{a}_j^H \hat{\mathbf{F}}_j^H + \hat{\mathbf{G}}_j \mathbf{b}_j \mathbf{b}_j^H \hat{\mathbf{G}}_j^H)\mathbf{W}^H. \quad (57)$$

In the objective function of the above formulation, the channel matrices are replaced by their estimated values. The solution $\mathbf{c}_i$ to this sub-problem can very possibly provide a satisfactory user SINR even at the presence of CSI errors. The optimization problem can be transformed to an eigenvector problem with a closed-form solution. The results are

$$\mathbf{c}_i = \rho\{\mathbf{\Theta}_{a,i}\}, \quad \mathbf{d}_i = \rho\{\mathbf{\Theta}_{b,i}\}, \quad (58)$$

where

$$\mathbf{\Theta}_{a,i} = (\mathbf{\Xi}_{a,i})^{-1} \hat{\mathbf{F}}_i^T \mathbf{Q}_{a,i}^{(S)} \hat{\mathbf{F}}_i^*,$$
$$\mathbf{\Xi}_{a,i} = \sigma_u^2 \mathbf{I}_N + \hat{\mathbf{F}}_i^T \mathbf{Q}_{a,i}^{(N)} \hat{\mathbf{F}}_i^* + \hat{\mathbf{F}}_i^T \hat{\mathbf{Q}}_{a,i}^{(I)} \hat{\mathbf{F}}_i^*. \quad (59)$$

E. SUMMARY OF THE PROPOSED ITERATION ALGORITHM

In our proposed algorithms, the SINR of each user node is collaboratively maximized by the transmit beamformer, receive beamformer and relay nodes together. The iteration process with the above three steps is repeated until reaching the stopping criterion, which is defined by a preset maximum iteration number (n'_t) or some convergence requirement (defined by a preset small positive real number δ'). In fact, supported by simulation results, our proposed algorithm does not necessarily require convergence to achieve good SINR performance, and a proper n'_t could be set with trade-off between better performance and lower computational complexity.

Iteration Steps: Sum-Relay Power Constraint

1) Initialization: $\mathbf{c}_i, \mathbf{d}_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1\times N}$, where $\delta_N = 1/\sqrt{N}$, $\mathbf{w} = [\delta_M \delta_M \cdots \delta_M]$, where $\delta_M = 1/\sqrt{M}$, and set t=1.
2) Update $\mathbf{a}_i$ and $\mathbf{b}_i$ based on (31).
3) Update $\mathbf{w}$ based on (44) and (45).
4) Update $\mathbf{c}_i$ and $\mathbf{d}_i$ based on (58) and (59).
5) If $|\mathbf{x}_i^{(t)} - \mathbf{x}_i^{(t-1)}|^2 < \delta'$ (considered to be converged) or $t > n'_t$ ($\mathbf{x} \leftarrow \mathbf{c}$ for user $X_{a,i}$ and $\mathbf{x} \leftarrow \mathbf{d}$ for user $X_{b,i}$), iteration stops; otherwise, set $t = t + 1$ and go to step 2).

Iteration Steps: Individual-Relay Power Constraints

1) Initialization: $\mathbf{c}_i, \mathbf{d}_i = [\delta_N \delta_N \cdots \delta_N] \in \mathbb{C}^{1\times N}$, where $\delta_N = 1/\sqrt{N}$, $\mathbf{w} = [\delta_M \delta_M \cdots \delta_M]$, where $\delta_M = 1/\sqrt{M}$, and set t=1.
2) Update $\mathbf{a}_i$ and $\mathbf{b}_i$ based on (31).
3) Update $\mathbf{w}$ based on (52) and (55).
4) Update $\mathbf{c}_i$ and $\mathbf{d}_i$ based on (58) and (59).
5) If $|\mathbf{x}_i^{(t)} - \mathbf{x}_i^{(t-1)}|^2 < \delta'$ (considered to be converged) or $t > n'_t$ ($\mathbf{x} \leftarrow \mathbf{c}$ for user $X_{a,i}$ and $\mathbf{x} \leftarrow \mathbf{d}$ for user $X_{b,i}$), iteration stops; otherwise, set $t = t + 1$ and go to step 2).

In practice, for continuous transmission, the initialization step is only needed at the very beginning. When the channel states change slowly, the iteration number required for good performance can be further reduced.

For one user to apply the iteration algorithm locally to determine its transmit and receive beamforming vectors,

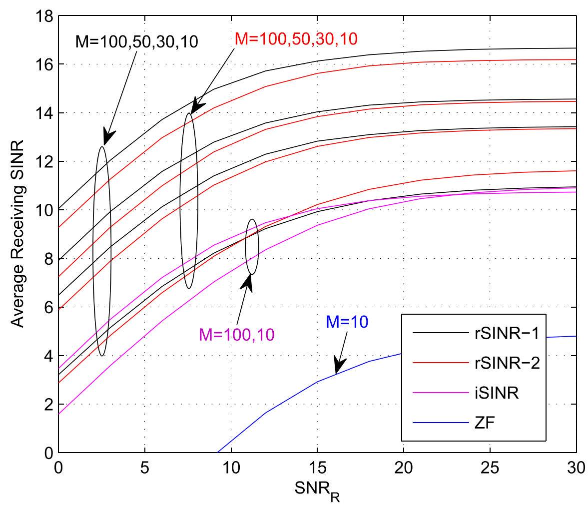

FIGURE 2. SINR performance of the proposed algorithms with different relay number settings ($\epsilon_{m,n}^{(i)} = \beta_{m,n}^{(i)} = 0$, N=5, K=3, $n_t = 5$).

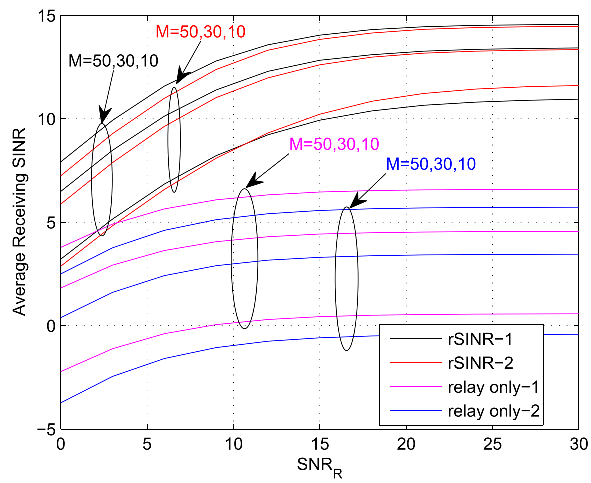

FIGURE 3. SINR performance of the proposed algorithms with relay-only methods as comparisons ($\epsilon_{m,n}^{(i)} = \beta_{m,n}^{(i)} = 0$, N=5, K=3, $n_t = 5$).

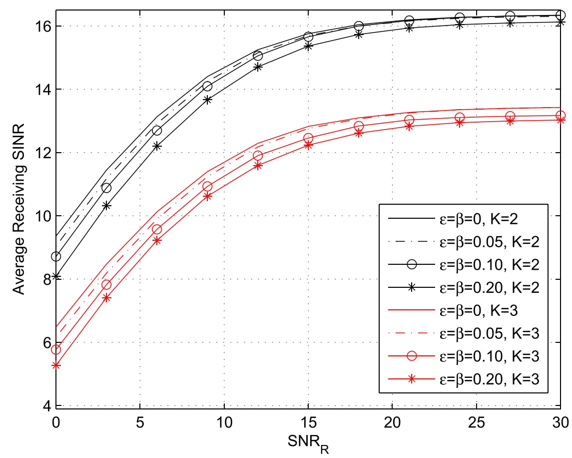

FIGURE 4. SINR performance of the proposed algorithms (with the 1st relay strategy) with different uncertainty bounds (M=30, N=5, $n_t = 5$).

knowledge of the received beamforming vectors of other users is required. This can either be calculated on this user node (assuming the initialization settings are known by each user) or shared by users from the same user group through limited backhaul resource before the next iteration loop begins. The former choice has higher computational complexity and the latter one requires intra-group communication resource. There is another way to reduce the communication complexity and the required intra-group communication resources in this scenario, which is employing a central node (it can be one of the users) for each side to perform the iteration processes and calculate the beamforming vectors for each user node in the group, and then inform them of the results.

V. SIMULATION RESULTS

In this section, simulation results are provided to evaluate the performance of the proposed method. The channels are of i.i.d. Rayleigh fading; the noise variance at any node is set at 1 ($\sigma_r^2 = \sigma_u^2 = 1$) and we set the source power at 0 dB ($P_S = 1$, compensating the unconsidered large-scale fading) and P_r is determined by $SNR_R = P_r/\sigma_r^2$. Our proposed scheme with relay strategy 1 and relay strategy 2 are referred to as *rSINR-1* and *rSINR-2* in all the figures, respectively. As for rSINR-2, we use $P_{r,m} = P_r/M$ as the individual power constraint. For a fair comparison, the sum-relay output powers of all schemes are kept the same.

Fig. 2 shows the SINR performance versus SNR_R of the proposed methods with different numbers of relay nodes, where the iSINR method in [43] and results based on a non-iterative ZF method (denoted by "ZF") used in [43] are provided for comparison. Specifically, in this ZF method, real CSI is considered, $\mathbf{a}_i$ and $\mathbf{b}_i$ are generated as the eigenvectors corresponding to the largest eigenvalues of $\mathbf{F}_i^H\mathbf{F}_i$ and $\mathbf{G}_i^H\mathbf{G}_i$, respectively, and together with $\mathbf{c}_i$ and $\mathbf{d}_i$, the IPI parts are eliminated completely without iterations. We can see from the figure that both of our proposed methods have significantly outperformed the non-iterative ZF method. We can also see that the iSINR method cannot benefit much from the number increase of the relay nodes when relay output power is large, while both of our methods yield significant SINR improvement as the relay number increases. It is also noteworthy that to achieve a certain average SINR, the total relaying power required is reduced when the number of relays increases and thus the per-relay output power decreases even more.

Fig. 3 demonstrates the effect of our proposed transceiver beamforming scheme, where two relay-only methods are used as comparison where $\mathbf{a}_i$, $\mathbf{b}_i$, $\mathbf{c}_i$ and $\mathbf{d}_i$ are fixed to their initial values. It shows that when only the two relay strategies are used in our scheme, the average SINR increases as more relay nodes are employed in the network, and our first proposed relay method has a better performance than the second one for all relay number settings. However, without the iterative transceiver beamforming steps, the performance is very limited when the relay number is small and the SINR improvement introduced by the transceiver

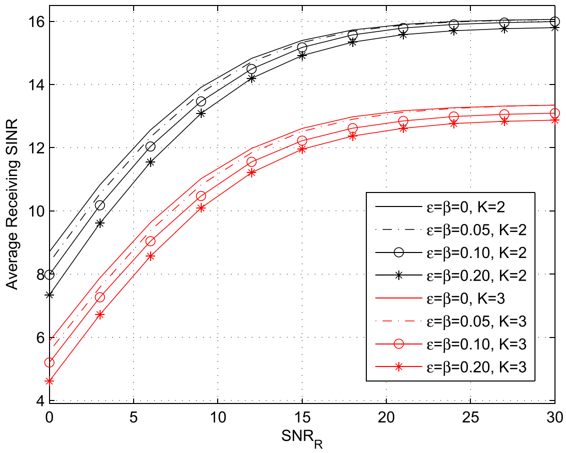

FIGURE 5. SINR performance of the proposed algorithms (with the 2nd relay strategy) with different uncertainty bounds (M=30, N=5, $n_t = 5$).

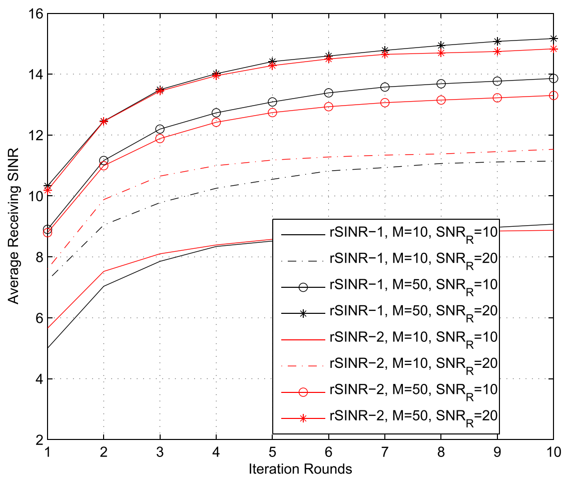

FIGURE 6. SINR performance versus iteration rounds ($\epsilon^{(i)}_{m,n} = \beta^{(i)}_{m,n} = 0$, N=5, K=3).

beamforming process is significant for all relay number settings, and the advantage becomes clearer when P_r is larger.

Now we investigate the performance in terms of the CSI uncertainty bounds. Figs. 4 and 5 present the results for rSINR-1 and rSINR-2, respectively. The situations are similar for both methods; by maintaining power constraints in their worst-case scenarios, the conservative relay strategies together with the mismatch of CSI, lead to certain degradation in SINR performance. However, the performance reduction is very limited (within 1.5dB for any P_r settings, even when $\epsilon = \beta = 0.20$), which indicates that the robustness of both of our proposed schemes is very high.

To demonstrate the cooperative performance of our proposed schemes through iteration steps, Fig. 6 illustrates the average SINR of the proposed methods after certain rounds of iterations. When the iteration round is set at 1, the three beamforming vectors can be considered as uncoordinated. As can be seen, the proposed method does not have the best performance immediately after the initialization step. However, the average SINR will quickly approach its asymptotic value after only a few rounds of iterations, and then the performance improvement becomes rather limited with further iteration. This pattern applies to different relay number settings and differen total relay power budgets. Since for both methods, computational complexity is proportional to the number of iteration rounds, trade-off should be made between average receiving SINR of each user and the predefined maximum number of iteration rounds, and the results of Fig. 6 can be used to guide the decision.

VI. CONCLUSIONS

The transceiver beamforming problem has been studied for multipair two-way distributed relay networks and in particular in the presence of channel state information errors. Iterative algorithms have been proposed where the SINR performance of each user is collaboratively optimized by the transceiver beamformers and relay nodes. For the imperfect CSI case, the robust worst-case based formulation was considered mainly in our first two iteration steps, and two different worst-case based relay strategies are proposed for the situation when total and individual relay output power is restricted, respectively. As demonstrated by simulation results, a satisfactory SINR performance has been achieved, especially when the number of relay nodes is large.

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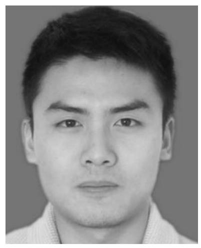

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