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Optimal Control for Zinc Electrowinning Process With Current Switching

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ABSTRACT This paper is concerned with the optimal control problem for the zinc electrowinning (EW) process during the current switching period. A mathematical model is developed to reveal the dynamic characteristics of the whole plant of the zinc EW process and an energy consumption model is established to set the expected set points of the concentrations of the zinc ion and the sulfuric acid under different current. Furthermore, an optimal control problem is constructed in the light of free initial time, free terminal time, and fixed system switching time during the zinc EW process. A novel time-scaling transformation-based control parametrization method is introduced to transform the optimal control problem into a multiple parameters optimization selection problem, which can be effectively solved by the optimization algorithm. The applications on the EW process of a zinc hydrometallurgy plant demonstrate the validity of proposed method.

INDEX TERMS Zinc electrowinning, optimal control, state transition algorithm, time-scaling transformation.

I. INTRODUCTION

Zinc electrowinning (EW) is typically the final aqueous processing step in zinc production where the zinc ions are reduced via the direct current to form metallic zinc. In most zinc hydrometallurgy plant, zinc EW is responsible for more than 75% of the total electrical energy consumption. Due to the power time-sharing policy in China, one day is classified into several load-duration periods, and the power price changes in different periods. Correspondingly, the current density, defined as the value of the current per meter square of electrodes, should be amended to make profits for this large power-consuming industrial customer [1]. More specifically, the zinc EW process should run with a low current density in the period of high power price and a high current density in the period of low price. The change of current density requires the adjustment of zinc ion concentration (CZI) and sulfuric acid (CSA) to achieve the optimum reduction conditions. However, the transition time of CSA and CZI will be much longer than that of current density. Such incompatibility between the current density, CSA and CZI will cause considerable waste of energy during the transition period. To overcome this challenge, precise mathematical models should be developed during the process control and optimization. A current efficiency estimation model based on the ratio of CZI and CSA named Warks rule was proposed in [2]. Barton and Scott [3]–[5] presented a detailed zinc EW model including the kinetics, thermodynamics, and the mass transfer effects. Mahon *et al.* [6], [7] further pursued these models and identified the conditions for achieving optimal current efficiency, energy consumption, and zinc production rate for a single cell and the entire cell house. However, it should be pointed out that, most of aforementioned models mentioned above, just considered the steady-state conditions of zinc EW process, but ignored the dynamic characteristics of the variations of CSA and CZI under different current density. Therefore, they are too difficult to be used directly for the control and optimization purpose, which motivates the work in this paper.

To the authors' best knowledge, until now, there are few results have been recorded on the control strategies for the

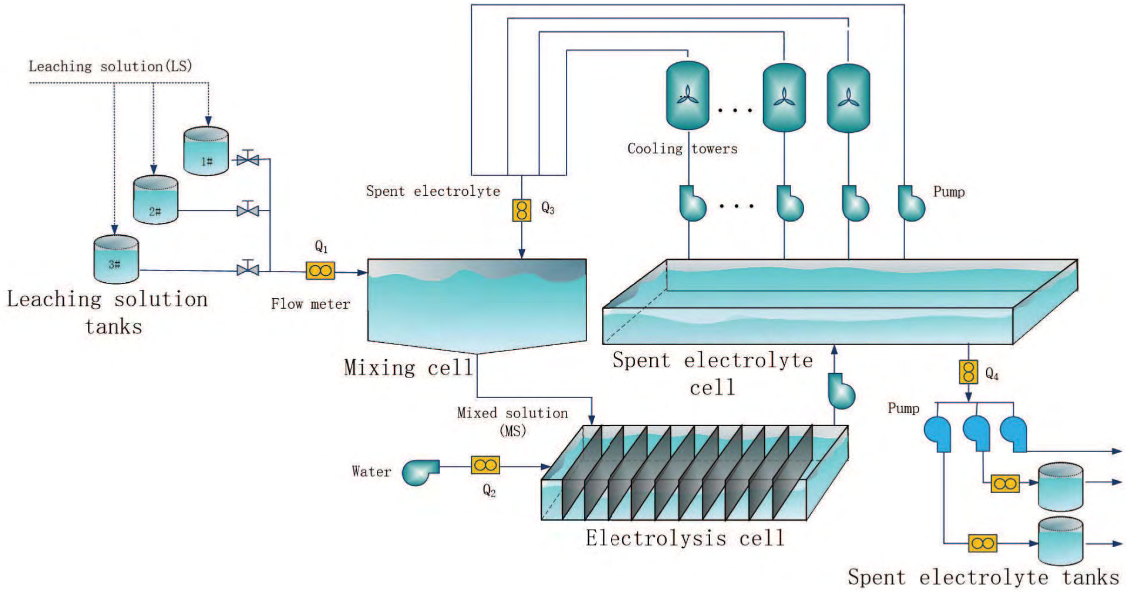

FIGURE 1. Flow sheet of zinc electrowinning process.

zinc EW process. In practice, the zinc EW process is usually controlled by the artificial experience. Nevertheless, it is difficult to control the CSA and CZI on the set points due to the complex interactions and long solution retention time. Some work for other industrial processes control, especially the predictive control, can provide references. The computation time required for this technique, however, is very high due to the diversity and complexity of the models [8]–[10]. A two-layer control problem addressed for solution purification process and a case based reasoning controller was proposed [11]. Wang [12] introduced the optimal control model for the production of 1,3-propanediol via microbial fed-batch fermentation with a fixed time terminal. Chai *et al.* [13] and Li *et al.* [14] made use of the gradient-based optimization methods to select the proper control variables in an industrial-scale evaporation process and a zinc sulphate electrolyte purification process respectively. However, these gradient-based optimization methods mentioned above, however, are only intended to find local optimal solutions which ignore the system switching. In this paper, we considered the transition cost as the objective function that takes both the transition time and energy consumption into account. The optimal control model which aim to minimize the objective function of the nonlinear dynamic model will be established for a zinc EW process with free terminal time and system switching. Due to the high nonlinearity of the governing multistage dynamic system, numerical techniques are unavoidable for solving the proposed optimal control problem. To overcome this drawback, a novel approach based on the time-scaling transformation method and state transition algorithm (STA) will be developed. This proposed control strategy has been successfully applied to a practical zinc EW process, where the transition time of CZI and CSA ,as well as the energy consumption are significant decreased during the current switching period.

The remainder of the paper is organized as follows: In Section II, the analysis of the zinc EW is described. In Section III, we present a nonlinear dynamic model to describe the variations of CSA and CZI in zinc EW process; And an energy consumption model is also developed to determine the terminal set points of CSA and CZI. Next, in Section IV, the time-scaling transformation method and the STA are also introduced in detail. Then, in Section V, the proposed control strategy is applied to study the optimal control of a practical zinc EW process. Finally, some concluding remarks are provided in Section VI.

II. ANALYSIS OF THE ZINC ELECTROWINNING PROCESS

A. PROCESS DESCRIPTION

Zinc EW is a continuous process composed of one mixing cell, a series of parallel electrolyte cells , one spent electrolyte cell, several spent electrolyte tanks and cooling towers as shown in Figure 1. Before electrowinning, purified leaching solution has been preserved in the leaching solution tanks respectively over 16-24h to deposit the impurities at bottoms. Then, the leaching solution is mixed with the spent acid solution in the mixing cell. The mixed solution is passed in to the electrolysis cell, where the zinc ions are deposited on to the cathode plate by the direct current. Electrolyte after electrodeposited is the spent electrolyte with lower zinc ion concentrations and higher sulfate acid concentrations than the electrolyte. Most of the spent electrolyte is pumped back to re-mix with the leaching solution, and the rest is preserved in the spent acid tanks. Water is added to affect the concentrations in the solution. A series of cooling towers are also built to moderate the temperature of the electrolyte. The spent electrolyte can be pumped through the cooling towers when the temperature of electrolyte is too high or into the mixing cell directly when the temperature is suitable.

To ensure a satisfactory reaction condition, the CZI and CSA of electrolyte should match with the current density. When the current density is at the steady state, the desired CZI and CSA of electrolyte can be achieved effortlessly. However, when the current density changes drastically, it is very hard to control the CZI and CSA instantaneously to the new set points. Different control will lead to different transition trajectories of the CZI and CSA with disparate energy consumption. The purpose of the optimal control for the zinc EW process is to determine the transition trajectories of the CZI and CSA with the minimum energy consumption by regulating the flow rate of leaching solution and water. Therefore, we will construct a mathematical model for the zinc EW process based on the electrochemical reaction mechanism and mass balance, resulting in an optimal control problem with free terminal time and free initial time. We then develop a computational method for the determination of the flow rate of leaching solution and water.

B. ELECTROCHEMICAL REACTION MECHANISM

In the zinc EW process, the primary reaction is the reduction of zinc ions to zinc metal at the aluminum cathode. Hydrogen reduction is also present at the cathode acting as competing reaction for zinc reduction [16]. The primary reactions at the lead-silver anode is the oxidation of water. Their reaction equations are given as follows:

$$\begin{aligned} Zn^{2+} + 2e^- &\longrightarrow Zn, \\ 2H^+ + 2e^- &\longrightarrow H_2 \uparrow, \\ 2H_2O &\longrightarrow 4H^+ + O_2 \uparrow + 4e^-, \end{aligned} \tag{1}$$

Based on the electrochemical kinetics [15], the reaction rates of the three equations are proportional to the current density and the current efficiency. Therefore, the following equations can be derived:

$$\begin{aligned} r_{Z_n} &= DS\varepsilon/(2F), \\ r_H^1 &= DS(1-\varepsilon)/F, \\ r_H^2 &= DS/F, \end{aligned} \tag{2}$$

where r_{Z_n}, r_H^1, and r_H^2 are the reduction rate of zinc ions, consumption rate, and generation rate of hydrogen ions, respectively. D is the current density. F is the faraday constant. S is the area of electrode. ε is the current efficiency, which can be calculated by the follow equation [17]:

$$\varepsilon = \frac{k_1^\varepsilon \exp(k_2^\varepsilon + k_3^\varepsilon \lg D) c_{Z_n}^{k_4^\varepsilon} c_H^{k_5^\varepsilon}}{[k_6^\varepsilon \exp(k_2^\varepsilon + k_2^\varepsilon \lg D) c_{Z_n}^{k_4^\varepsilon} c_H^{k_5^\varepsilon} + k_7^\varepsilon c_{Z_n}^{k_8^\varepsilon}]D} \tag{3}$$

where c_{Z_n} and c_H are the CZI and CSA of electrolyte, respectively; k_i^ε, $i = 1, 2, \cdots, 8$ are the parameters which need estimation.

III. MATHEMATICAL MODEL

A. MODEL OF MASS BALANCE

To reveal the dynamic characteristics of the whole plant of zinc EW, the mass balance model for the three main cells (mixing cell, electrolysis cell and spent electrolyte cell) is carried out based on the following assumptions:

Assumption 1: The temperature of the electrolyte is properly controlled in the suitable range.

Assumption 2: Impurity ions such as Co^{2+}, Cu^{2+}, Ni^{2+} et al. are not considered.

Assumption 3: CZI in the leaching solution remains stable.

Then, the mass changes in the cells can be calculated using the following formulas:

$$\begin{aligned} \frac{dC_{1.1}}{dt} &= \frac{Q_1 C_{LS} + Q_3 C_{3.1} - (Q_1 + Q_3) C_{1.1}}{V_1}, \\ \frac{dC_{1.2}}{dt} &= \frac{Q_3 C_{3.2} - (Q_1 + Q_3) C_{1.2}}{V_1}, \\ \frac{dC_{2.1}}{dt} &= \frac{(Q_1 + Q_3) C_{1.1} - (Q_1 + Q_2 + Q_3) C_{2.1} - r_{Z_n}}{V_2}, \\ \frac{dC_{2.2}}{dt} &= \frac{(Q_1 + Q_3) C_{1.2} - (Q_1 + Q_2 + Q_3) C_{2.2} + r_H}{V_2}, \\ \frac{dC_{3.1}}{dt} &= \frac{(Q_1 + Q_2 + Q_3) C_{2.1} - (Q_3 + Q_4) C_{3.1}}{V_3}, \\ \frac{dC_{3.2}}{dt} &= \frac{(Q_1 + Q_2 + Q_3) C_{2.2} - (Q_3 + Q_4) C_{3.2}}{V_3}, \end{aligned} \tag{4}$$

where $C_{i.j}$, $i = 1, 2, 3, j = 1, 2$ are the CZI and CSA of mixed solution, electrolyte and spent electrolyte respectively; C_{LS} is the CZI of leaching solution; V_i, $i = 1, 2, 3$ are the volume of the three kinds of cells; are the flow rate of leaching solution, water, recirculating spent electrolyte and the returned spent electrolyte. In practice, to guarantee the liquid level balance and prevent the overflowing of the solution, the inlet solution equal to the outlet solution, and the flow rate of the recirculating spent electrolyte remains unchanged. Therefore, the following equation can be achieved:

$$Q_4 = Q_1 + Q_2, \tag{5}$$

Let $X = [x_1, x_2, \cdots, x_6] = [C_{1.1}, C_{2.1}, \cdots, C_{3.2}]$ denote the state with six variables. Substituting (5) into (4), the zinc electrowinning system model can be expressed as a system of six differential equations. The flow rates of leaching solution and water are the control variables denoted as $U = [u_1, u_2] = [Q_1, Q_2]$.

During the mass balance model, the flow rate of the leaching solution and the water are subject to the following constraint:

$$u_{i,\min} \le u_i \le u_{i,\max}, \quad i = 1, 2, \tag{6}$$

where $u_{i,\min}$ and $u_{i,\max}$ are the low and upper bounds for the flow rates of the leaching solution and the water, respectively.

To ensure the required quality of the metallurgy zinc is achieved, there also are bounds on the CZI and CSA, which are imposed as the following states constraints:

$$x_{i,\min} \leq x_i \leq x_{i,\max}, \quad i = 1, 2, \cdots, 6, \tag{7}$$

where $x_{i,\min}$ and $x_{i,\max}$ are the low and upper concentration thresholds for the CZI and CSA, respectively. Let this system model described by (4)–(7) be referred to as system (1).

B. ENERGY CONSUMPTION MODEL

To get the proper set points of the CZI and CSA in the electrolyte under different current density, an energy consumption model is proposed in this section to reveal the nonlinear relationship between the CZI, CSA, and the current density.

The energy consumption, which is given as the electric power required per unit weight of zinc produced,can be expressed as follows:

$$W = 81960V_c/\varepsilon, \tag{8}$$

where W is the energy consumption; ε is the current efficiency as described in (3); V_c is the cell voltage, which can be calculated from (7), and

$$V_c = k_1^V - k_2^V \ln(k_3^V/c_H) - k_4^V \ln(k_5^V/c_{Z_n}) + k_6^V \lg(D) + \frac{k_7^V DL}{k_8^V + k_9^V c_H - k_{10}^V c_{Z_n}} + k_{11}^V D, \tag{9}$$

k_i^V, $1, 2, \cdots, 11$ are the parameters which need to be identified by using the real-time data.

IV. OPTIMAL CONTROL PROBLEM FORMULATION AND TRANSFORMATION

A. OPTIMAL CONTROL PROBLEM FORMULATION

The optimal control problem of the zinc EW process is to find a control such that the following tasks can be satisfied:

(1) The energy consumption is minimized during the transition period.

(2) The set points of CZI and CSA of electrolyte at the terminal time are met.

Therefore, the cost function to be minimized could be formulated as:

$$J = \Phi_0(x(t_f|u)) + \int_{t_0}^{t_s} W(x, D_1)\mathrm{d}t + \int_{t_s}^{t_f} W(x, D_2)\mathrm{d}t, \tag{10}$$

where t_0 is the initial time; t_s is the current density switching time; t_f is the terminal time. $\Phi_0(\cdot)$ is the terminal cost function given by:

$$\Phi_0(x(t_f|u)) = (\hat{x}_3 - x_3(t_f|u))^2 + (\hat{x}_4 - x_4(t_f|u))^2, \tag{11}$$

where $\hat{x}_3$ and $\hat{x}_4$ are the specified desired CZI and CSA of the electrolyte.

Therefore, the optimal control problem then can be described as follows: For a given system (4), find a control u such that the cost function (10) is minimized subject to the constraints on the state and control given by (6) and (7).

B. OPTIMAL CONTROL SOLUTION METHOD

The difficulty to solve the optimal control problem is the "free initial time and free terminal time with fixed system switching time". And the state constraints must hold at every point in the time horizon. To address this challenge, we shall present an effective solution algorithm to solve a general optimal control for the switching system with free initial time and free terminal time. And that covers the optimal control of the zinc electrowinning process during current density switching as a special case.

Consider the following switching system:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \begin{cases} f_1(t, x(t), u(t)), & t \in [t_0, t_s), \\ f_2(t, x(t), u(t)), & t \in [t_s, t_f), \end{cases}$$
$$x(t_0) = x^0, \tag{12}$$

where $x = [x_1, x_2, \cdots, x_n] \in \mathbb{R}^n$ and $u = [u_1, u_2, \cdots, u_r] \in \mathbb{R}^r$ are the state and control vectors, respectively. $x^0 = [x_1^0, x_2^0, \cdots, x_n^0] \in \mathbb{R}^n$ is a given initial state vector. t_0, t_s, and t_f are the initial time, switching time, and terminal time, respectively.

Define

$$U = u = [u_1, u_2, \cdots, u_r] \in \mathbb{R}^r$$
$$\partial_j \leq u_j \leq \beta_j, \quad j = 1, \cdots, r, \tag{13}$$

where ∂_j, β_j, $j = 1, 2, \cdots, r$ are given constants. Clearly, U is a compact and convex subset of $\mathbb{R}^r$. Any measurable function from $u = [u_1, u_2, \cdots, u_r] : [t_0, t_f] \rightarrow U$ is called an admissible control. Let $\bigcup$ be the set which consists of all the admissible controls.

Our optimal control problem with free initial time and free terminal time now be stated formally as:

Given the dynamic system (12), find a control $u \in \bigcup$ such that the cost function:

$$J_0 = \Phi_0(x(t_f|u)) + \int_{t_0}^{t_s} f_1(x, t)\mathrm{d}t + \int_{t_s}^{t_f} f_2(x, t)\mathrm{d}t, \tag{14}$$

can be minimized subject to the continuous state inequality constraints, in which, Φ_0 is the terminal constraints. This problem is referred to as the optimal control problem with multiple characteristic time points in the literatures [18]. However, the initial time and the terminal time are fixed in [18], while those in (14), are decision variables to be chosen optimally.

C. TIME-SCALING TRANSFORMATION BY SWITCHING TIME FACTOR

Firstly, we set $t_0 = 0$, therefore the switching time t_s and the terminal time t_f become free and need to be chosen optimally. Then, for each p_i, $i = 1, 2,$, let the time subintervals $[t_0, t_s]$ and $[t_s, t_f]$ be partitioned into n_{p_i} subintervals with partition points $n_{p_i} + 1$ which denote by

$$[\tau_0^{p_i}, \tau_1^{p_i}, \cdots, \tau_{n_{p_i}}^{p_i}]. \tag{15}$$

It is clearly that

$$\tau_0^{p_1} = t_0, \quad \tau_{n_{p_1}}^{p_1} = t_s,$$
$$\tau_0^{p_2} = t_s, \quad \tau_{n_{p_2}}^{p_2} = t_f, \tag{16}$$

We now approximate the control function in the form of the piecewise constant function [19] as:

$$u^p(t|\sigma^p,\tau^p)=\sum_{i=1}^{2}\sum_{k=1}^{n_{p_i}}\sigma^{p_i,k}\chi_{[\tau^{p_i}_{k-1},\tau^{p_i}_k)}(t), \tag{17}$$

where $\chi_{[\tau^{p_i}_{k-1},\tau^{p_i}_k)}$ denotes the indicator function of the interval $[\tau^{p_i}_{k-1},\tau^{p_i}_k)$, which can be defined by

$$\chi_I=\begin{cases}1, & \text{if } t\in I,\\ 0, & \text{otherwise,}\end{cases} \tag{18}$$

where σ^p are the heights of the control component and τ^p are the characteristic time points, which are defined by:

$$\begin{aligned}\sigma^p&=\{\sigma^{p_1},\sigma^{p_2}\}\\ &=\{[\sigma^{p_1,1},\cdots,\sigma^{p_1,n_{p_1}}],[\sigma^{p_2,1},\cdots,\sigma^{p_2,n_{p_2}}]\},\\ \tau^p&=\{\tau^{p_1},\tau^{p_2}\}\\ &=\{[\tau^{p_1,1},\cdots,\tau^{p_1,n_{p_1}}],[\tau^{p_2,1},\cdots,\tau^{p_2,n_{p_2}}]\}\end{aligned} \tag{19}$$

and σ^p and τ^p are decision variables satisfying:

$$\begin{aligned}t_0&=\tau^{p_1}_0\le\tau^{p_1}_1\le\cdots\le\tau^{p_1}_{n_{p_1}}=t_s,\\ t_s&=\tau^{p_2}_0\le\tau^{p_1}_1\le\cdots\le\tau^{p_2}_{n_{p_2}}=t_f,\end{aligned} \tag{20}$$

We define that the time span between interval $[\tau^{p_i,\tau^{p_i}_k}_{k-1})$ as $\Omega^{p_i}_k$ and let

$$\Omega^p=\{\Omega^{p_1},\Omega^{p_2}\}=\{[\Omega^{p_1}_1,\cdots,\Omega^{p_1}_{n_{p_1}}],[\Omega^{p_2}_1,\cdots,\Omega^{p_2}_{n_{p_2}}]\}. \tag{21}$$

Then, we introduce a switching time factor $\theta\in[0,1)$ which need to be optimally decided to ensure a proper switching time. It is clear that

$$t_s=\theta t_f, \tag{22}$$

After such transformation, the system described by the differential equations (12) takes the form:

$$\begin{aligned}\frac{\mathrm{d}x(t)}{\mathrm{d}t}&=\tilde{f}(t_t,x(t),\sigma^p,\tau^p,\theta),\\ x_{t_0}&=x^0,\end{aligned} \tag{23}$$

where

$$\tilde{f}(t,x(t),\sigma^p,\tau^p,\theta)=\begin{cases}f_1(t,x(t),\sigma^p,\tau^p,\theta)\\ f_2(t,x(t),\sigma^p,\tau^p,\theta)\end{cases}=\begin{cases}\sum_{k=1}^{n_{p_1}}\sigma^{p_1,k}\chi_{[\tau^{p_1}_{k-1},\tau^{p_1}_k)}(t))\chi[t_0,\theta_{t_f})(t),\\ \sum_{k=1}^{n_{p_2}}\sigma^{p_2,k}\chi_{[\tau^{p_2}_{k-1},\tau^{p_2}_k)}(t))\chi[\theta_{t_f},t_f)(t)\end{cases} \tag{24}$$

Let $x(t,x(t),\sigma^p,\tau^p,\theta)$ be the solution of the system (23) corresponding to the combined control parameter vector, characteristic time vector and the switching time factor (σ^p,τ^p,θ). We may now specify the approximate problem as follows:

1) APPROXIMATE PROBLEM

Subject to the switching system (23), find a combined control parameter vector, a characteristic time vector and a switching time factor (σ^p,τ^p,θ), such that the following cost functional can be minimized.

$$\begin{aligned}&\tilde{J}(\sigma^p,\tau^p,\theta)\\ &=\int_{t_0}^{\theta t_f}f_1(t,x(t),\sum_{k=1}^{n_{p_1}}\sigma^{p_1,k}\chi_{[\tau^{p_1}_{k-1},\tau^{p_1}_k]}(t))\chi[t_0,\theta_{t_f})(t))\mathrm{d}t\\ &\quad+\int_{\theta t_f}^{t_f}f_2(t,x(t),\sum_{k=1}^{n_{p_2}}\sigma^{p_2,k}\chi_{[\tau^{p_2}_{k-1},\tau^{p_2}_k]}(t))\chi[\theta_{t_f},t_f)(t))\mathrm{d}t,\end{aligned} \tag{25}$$

D. ALGORITHM DESCRIPTION

The *Approximate Problem* is a multi-parameter optimization problem which can be solved using the gradient-based optimization method in [13], [14], and [20],or using the random searching method such as particle swarm optimization [21], [22],genetic algorithm [23], [24], etc. These methods, however, are only intended to find local optimal solutions or take amount of computing time. Hence, in this section, we introduce a novel global search algorithm, which we call it the state transition algorithm (STA) [25], [26] for the *Approximate Problem*.

The STA is an efficient random search algorithm with superiority in global searching ability and computational efficiency, which regards a solution to an optimization problem as a state and the process of updating current solution as a state transition. The STA can be outlined as follows:

$$\begin{cases}\chi_{k+1}=A_k\chi_k+B_ku_k,\\ y_{k+1}=\mathrm{Obj}(\chi_{k+1}),\end{cases} \tag{26}$$

where χ_{k+1} and χ_k, corresponding to the solution of optimization problem, stand for current state and transited state, respectively; A_k and B_k are state transition matrices; u_k is a function of χ_k and historical states.

For solving continuous function optimization problems, four special state transformation operators are introduced.

(1) Rotation transformation

$$\chi_{k+1}=\chi_k+\alpha\frac{\mathrm{R}_r\chi_k}{n\|\chi_k\|_2}, \tag{27}$$

where α is a positive constant, called rotation factor; R_r is a random matrix with its entries belonging to range of $[-1,1]$, and $\|\cdot\|$ is the 2-norm of a vector.

(2) Translation transformation

$$\chi_{k+1}=\chi_k+\beta\frac{\mathrm{R}_r(\chi_{k+1}-\chi_k)}{\|\chi_{k+1}-\chi_k\|_2} \tag{28}$$

where β is a positive constant, called rotation factor; R_r is a random matrix with its entries belonging to range of $[0,1]$.

(3) Expansion transformation

$$\chi_{k+1}=\chi_k+\gamma\mathrm{R}_e\chi_k, \tag{29}$$

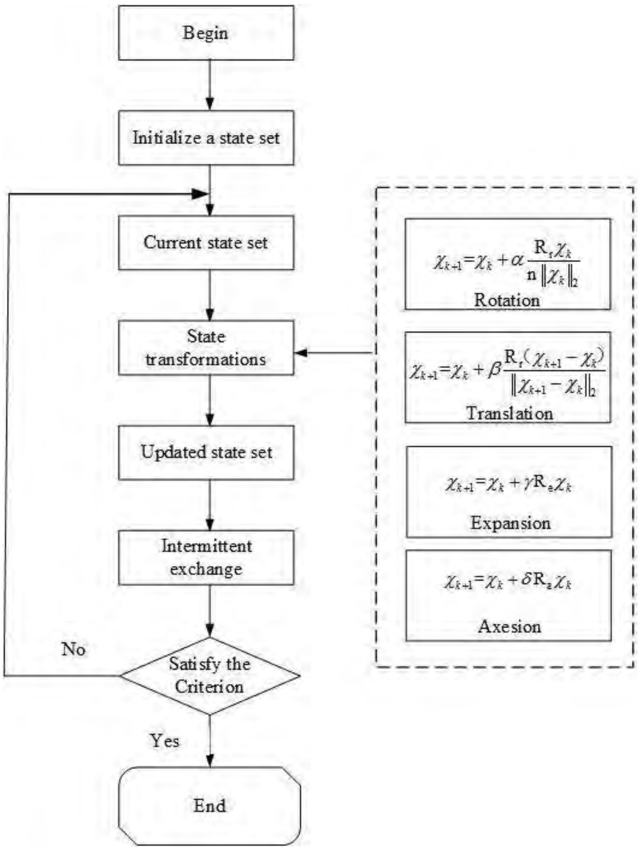

FIGURE 2. Flowchart of STA.

FIGURE 3. Electrowinning process in a zinc hydrometallurgy plant.

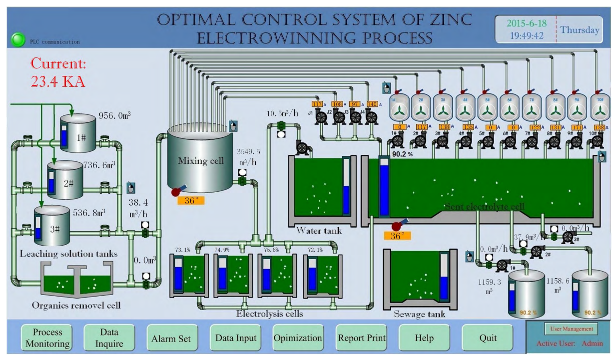

FIGURE 4. Main interface of the optimal control system.

where γ is a positive constant, called expansion factor; R_e is a random matrix with its entries obeying the Gaussian distribution.

(4) Axesion transformation

$$\chi_{k+1} = \chi_k + \delta R_a \chi_k, \tag{30}$$

where δ is a positive constant, called expansion factor; R_a is a random diagonal matrix with its entries obeying the Gaussian distribution and only one random position having nonzero value.

In the STA, the rotation transformation is for local search, the expansion transformation aims for global search, the translation transformation is designed for a line search and the axesion transformation is proposed to strengthen the single dimensional search. The four special state transformation operators are designed for improving the global searching ability and computational efficiency of random search algorithm. And the flowchart of STA is shown as Figure 2.

V. INDUSTRIAL APPLICATION

In order to evaluate the feasibility and ability of the proposed optimal control strategy, an industrial application was conducted in one of the biggest metallurgical plant in China (Figure 3). The dynamic model and the optimal control method were realized by using the software and hardware platform of the SIEMENS WINCC and S7-300 distributed control system (DCS). WINCC is used to run the online calculation and built the database. While DCS is used to realize the data acquisition and control instructions. Figure 4 shows the main operational interface of the optimal control system of the zinc EW process.

A. PARAMETER IDENTIFICATION

Before solving the optimal control problem, the values of the model constants in the current efficiency model and the cell voltage model must be determined. Corresponding to the experimental data, such parameter identification problem can be formulated as to choose the tunable model parameters k_i^{ε}, $i = 1, 2, \cdots, 8$ and k_j^V, $j = 1, 2, \cdots, 11$ to minimize (31).

$$\sqrt{\frac{1}{N}\sum_i^N(\tilde{V}_{c.i} - V_{c.i})^2}$$
$$\sqrt{\frac{1}{N}\sum_i^N(\tilde{\varepsilon}_{c.i} - \varepsilon_{c.i})^2} \tag{31}$$

where $\tilde{V}_{c.i}$ and $\tilde{\varepsilon}_{c.i}$ are the calculated values of the cell voltage and the current efficiency respectively. $V_{c.i}$ and $\varepsilon_{c.i}$ are the measured values of the cell voltage and the current efficiency, N is the sample number. Likewise, we solved the estimation problem using the proposed STA algorithm. For the

TABLE 1. Parameters setting of STA.

Parameters	SE	α	β	γ	δ	f_c
Value	32	1∼1e-4	1	1	1	2

TABLE 2. Parameter identification for current efficiency and associated 95% confidence intervals.

Parameters	k_1^{ε}	k_2^{ε}	k_3^{ε}	k_4^{ε}	k_5^{ε}	k_6^{ε}	k_7^{ε}	k_8^{ε}
Estimation	7572.45	0.27	2.33	-3.08	-0.03	-0.0006	-158.17	-3.20
Confidence interval	± 87.36	± 0.004	± 0.02	± 0.04	± 0.0005	± 0.00001	± 1.16	± 0.04

TABLE 3. Parameter identification for cell voltage and associated 95% confidence intervals.

Parameters($*10^{-3}$)	k_1^V	k_2^V	k_3^V	k_4^V	k_5^V	k_6^V	k_7^V	k_8^V	k_9^V	k_{10}^V	k_{11}^V
Estimation	11.1	1.6	1.1	120.5	0.1	402.8	6.1	580.2	58.9	0.01	0.01
Confidence interval	± 1.9	± 0.3	± 0.2	± 8.5	± 0.1	± 53.6	± 1.0	± 84.9	± 10.1	± 0.1	± 0.1

parameters in STA, we used the values in Table 1. The results of the optimal parameter estimation for the models of current efficiency and cell voltage are shown in Table 2 and Table 3.

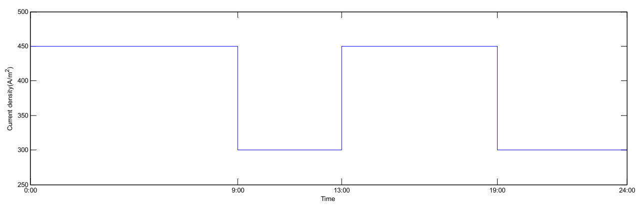

FIGURE 5. Current density distributions.

TABLE 4. Optimization of $C_{2.1}$ and $C_{2.2}$.

Current density(A/m^2)	300	450
$C_{2.1}$(g/L)	45	48
$C_{2.2}$(g/L)	162	181
Energy consumption(kWh/t)	2733.6	2693.4

B. OPTIMAL SETTING OF THE CZI AND CSA UNDER DIFFERENT CURRENT DENSITY

The variations of current density at different load-durations are shown in Figure 5. Correspondingly, the CZI and CSA should be modified to the set points as quickly as possible especially during the current densities switching period. Based on the energy consumption model, the relationship between energy consumption and the CZI, CSA and current density can be achieved as shown in Figure 6. Therefore, we can get the optimal CZI and CSA in electrolyte under different current density easily as shown in Table 4. And such optimal CZI and CSA can be regarded as the terminal constraints in *Approximate Problem*.

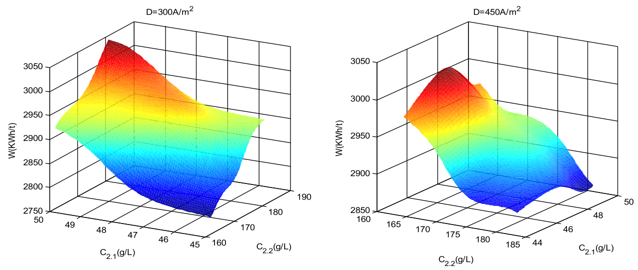

FIGURE 6. Energy consumption under different current density.

TABLE 5. Allowed domain of control variables.

Variables	Leaching solution(m^3/L)	Water(m^3/L)
Minimum value	30	0
Maximum value	200	500

C. OPTIMAL CONTROL EXPERIMENTS

Based on the dynamic model and with optimal parameters from Section V-A, the next step is to determine the optimal control variables and the switching time factor. Our implementation of the STA uses the Matlab embedded in the WINCC software.The allowed domains of the control variables are given in Table 5.

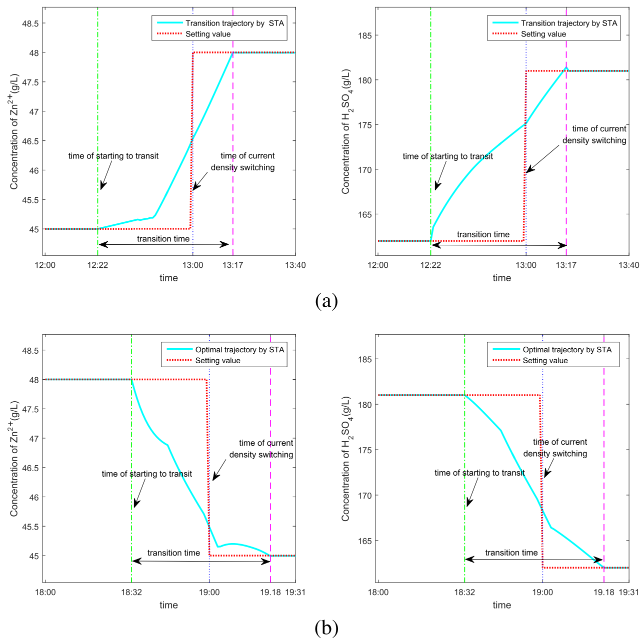

FIGURE 7. Optimal trajectory of CZI and CSA by STA. (a) Current density switching from 300 to 450A/m^2. (b) Current density switching from 450 to 300A/m^2.

Using the proposed control strategy, we solved the *Approximate Problem* for different optimization of the CZI and CSA trajectories as shown in Figure 7. For comparison, the results by the gradient-based algorithm and the artificial experience for the problem considered are shown in Figure 8 and Figure 9, respectively. For the sake of the stability of

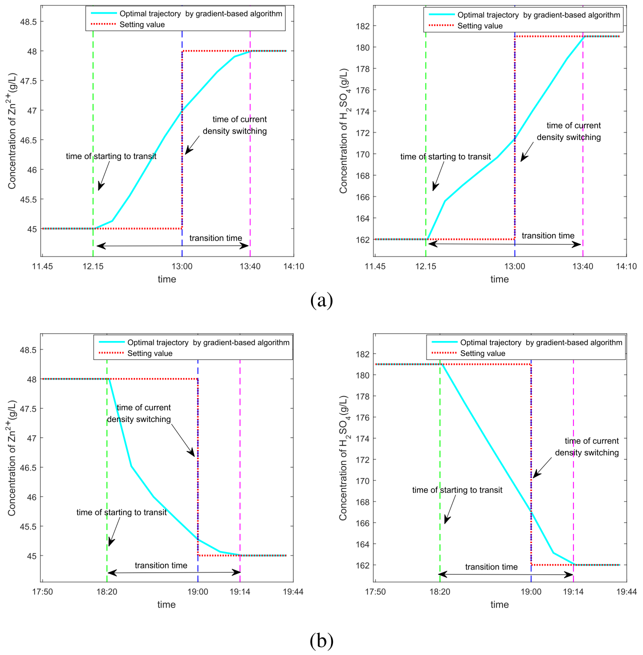

FIGURE 8. Optimal trajectory of CZI and CSA by gradient-based algorithm. (a) Current density switching from 300 to 450A/m^2. (b) Current density switching from 450 to 300A/m^2.

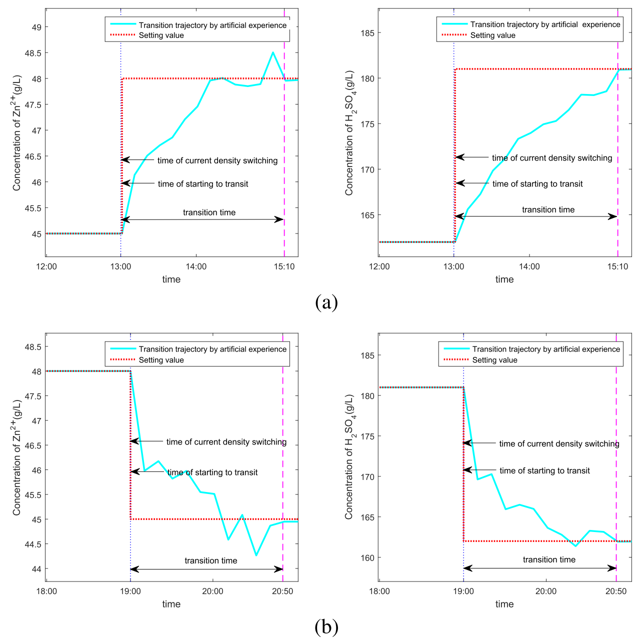

FIGURE 9. Trajectory of CZI and CSA by artificial experience. (a) Current density switching from 300 to 450A/m^2. (b) Current density switching from 450 to 300A/m^2.

the process, the fluctuations of the CZI and CSA should be minimized. Therefore, we proposed the relative fluctuation error as an index to evaluate the control performance. The relative fluctuation error can be calculated as

$$R = \frac{C_{2.1\ or\ 2.2}^{i} - C_{2.1\ or\ 2.2}^{i-1}}{C_{2.1\ or\ 2.2}^{i}}, \tag{32}$$

TABLE 6. Comparisons of the control effectiveness.

Operation	Transition time (minute) rising/dropping	Average energy consumption during transition period(kWh/t)	Max relative fluctuation error(%)		Average relative fluctuation error(%)	
			CZI	CSA	CZI	CSA
Artificial experience	130/110	2931.13/2996.87	6.25%/6.67%	10.5%/11.73%	1.31%/1.63%	3.97%/2.41%
Optimal control by gradient-based algorithm	85/54	2929.41/2870.12	6.12%/6.25%	9.52%/10.5%	2.70%/4.45%	4.45%/5.31%
Optimal control by STA	55/50	2882.34/2867.58	3.2%/5%	7.98%/6.67%	1.08%/1.27%	3.86%/2.31%

where R is relative fluctuation error, $C_{2.1\ or\ 2.2}^{i}$ is the ith measured value of $C_{2.1}$ or $C_{2.2}$. The indexes of the control effectiveness Table 6. The complexity for STA mainly consists of two parts: the search operators for generating the candidate solutions and the calculation of the fitness, which could be expressed as a function of the population size(M), maximum number of the iterations(G) and the dimension of the problem(DIM). In the STA using for the optimal control problem of Zinc EW, we set the G=1000, DIM=20 and M=50. Therefore, the complexity of STA equals $1000 \times 20 \times 50 = 10^5$. The on-line calculation time of the STA, comparing with the gradient-based algorithm, is illustrated in Table 7.

TABLE 7. On-line calculation time.

Method	Calculation time(s) rising/dropping period
STA	48.8/152.77
Gradient-based algorithm	67.48/232.11

By analyzing the statistical data, following conclusions can be drawn:

1) The average energy consumption in the industrial experiment is 2882.34/2867.58 kWh/t during the current step jumping and descent period, respectively. It is less than the corresponding result by artificial experience and the gradient-based algorithm.
2) Comparing with the 130/110 minutes of artificial experience and the 85/54 minutes of gradient-based algorithm, the regulating time using the proposed method is significantly reduced. The on-line calculation time is also decreased.
3) The volatilities of CZI and CSA of electrolyte are weaker according to the max relative fluctuation error and average relative fluctuation error.

VI. CONCLUSION

Based on the mass balance and electrochemical reaction mechanism, a nonlinear multistage dynamic model for the zinc EW process was constructed. The problem of achieving the required transition trajectory of CZI and CSA of electrolyte with minimum energy consumption is formulated as a dynamic optimal control problem governed by free initial time, free terminal time and system switching.

An efficient numerical algorithm is developed based on time-scaling transformation and state transition algorithm to solve this problem. The proposed method was successfully applied to the electrowinning process of a zinc hydrometallurgy plant in China. It demonstrated that the proposed control method has higher control performance than the manual operation. The regulating time and the amount of energy consumption had been economized under the proposed method. Meanwhile the fluctuations of CZI and CSA are reduced during the transition period. However, it should be noticed that the performance of this scheme is affected by the accuracy of the process model and based on the assumption that the disturbances of the process are sufficiently low. If the accuracy of the process model is not sufficient or the disturbances are above the certain thresholds, the performance of the scheme may deteriorate. Thus, more precise model and the disturbances such as equipment failure, excitation excitation and unreasonable operations should be taken in considered in the future work.

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The statements made herein are solely the responsibility of the authors.

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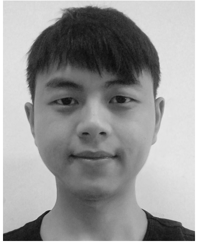

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