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A Scheduling Scheme of Linear Model Predictive Controllers for Turbofan Engines

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ABSTRACT An adaptive model predictive controller with a new scheduling scheme for turbofan engines is proposed, which can transfer engine from one working state to the others within the flight envelope. First, the flight envelope is divided into several sections according to the engine inlet parameters, and the nominal points in each section are determined, respectively. Then, considering the requirements of the turbofan engines, a constrained linear model predictive control algorithm is improved, and a series of constrained predictive controllers are designed based on the linear models at different nominal points. Furthermore, a novel scheduling scheme with two layers is constructed, where the first layer is the flight envelope scheduling layer that introduces fuzzy membership degree logic to distribute the weights of all nominal predictive controllers, and the second layer is the power scheduling layer by adopting a linear interpolation method. Simulation results show that the proposed scheduling scheme can coordinate these two layers to realize the steady-state and transition-state control of the turbofan engines at off-nominal points within the envelope, which provides an effective approach for the design of the adaptive controllers.

INDEX TERMS Adaptive model predictive control, a scheduling scheme, flight envelope, fuzzy membership degree, turbofan engine, transition state.

I. INTRODUCTION

With the development of aircraft engine technology, the complexity of engine working conditions increases, and engine controller is thus more critical to transit engine from one state to others while preventing engine from dropping into abnormal conditions, such as overspeed, overtemperature, stall/surge, etc [1]–[3]. However, traditional linear regulators [4] with Min-Max switching logic cannot handle the complicated output limits protection well due to its inherent conservativeness [5]–[7]. Moreover, with Full Authority Digital Engine Control (FADEC) system widely used in the turbofan engine controllers [8], [9], model predictive control (MPC) algorithms have been proposed to design engine controller [10]–[12], which possesses the ability to handle all kinds of limits directly and conveniently [13], [14], and are more powerful than traditional PID control method [15], [16]. Therefore, according to the control requirements of turbofan engines [17], [18], a constrained linear model predictive control approach is proposed in this paper, in order to achieve the thrust demand and limit protection simultaneously in the entire flight envelope.

In general, a model predictive controller could adapt to a wide range of disturbances and achieve satisfactory control performance, even in the case of a model mismatch [19]. However, for the nonlinear complex aero-engines, it is difficult to guarantee that a predictive controller can achieve satisfactory dynamic response in the full flight envelope [20]. Specifically, taking a predictive controller based on a fixed linear engine model for example, a series of simulations were conducted in the entire flight envelope with step inputs. The results showed that there exist large overshoot, frequent oscillations, and even instability during the transitions in some parts, which is beyond the scope of the performance requirements [21].

Therefore, due to various working conditions of turbofan engines, an adaptive predictive controller needs to be designed, and there are several methods that could be considered. One method is to modify the predictive model online to match the current engine condition through the system identification techniques [22], however, there exist some conditions that cannot meet the identification requirements. The second strategy is the parameter similarity criterion that stems from the practical engineering applications, which converts other flight condition parameters to the nominal conditions [3], while the effectiveness of the similarity criterion is not verified theoretically and necessarily not applicable to all types of turbofan engines. Another approach is parameter scheduling logic, where flight envelope and power conditions are divided in order to obtain a series of nominal points, and a scheduling logic coordinates these nominal predictive controllers. At present, in public literature, there are few works focuses on flight envelope division and scheduling logic design.

In this paper, a novel scheduling scheme with fuzzy membership degree logic is proposed to design an adaptive model predictive controller for a commercial turbofan engine. The remainder of the paper is organized as follows. Section II introduces the constrained model predictive controller design of turbofan engines at nominal points. The flight envelope division approach is described in Section III. Section IV investigates the design of a two-layer scheduling scheme to construct an adaptive model predictive controller, in which the flight envelope scheduling layer that utilizes fuzzy membership degree idea and power scheduling layer that uses linear interpolation method are described in detail. In Section V, simulations are conducted in two cases, and the proposed scheduling scheme is compared with a traditional one. The conclusions are summarized in Section VI.

II. CONSTRAINED LINEAR MODEL PREDICTIVE CONTROLLER DESIGN AT NOMINAL POINTS

Due to the wide range of flight and operation conditions for turbofan engines, a series of constrained model predictive controllers need to be well established and arranged.

For a certain type of high bypass commercial turbofan engines, a packaged component level nonlinear dynamic model is employed in Matlab/Simulink platform via dynamic link library technique [23], which was originally constructed and tested perfectly in the GasTurb software. To design the constrained model predictive controllers at nominal points, linear engine models are obtained using the fitting method at given flight conditions and power states. Note that the final adaptive model predictive controller based on a scheduling scheme is tested with the nonlinear component level engine model, although a series of linearized models are prepared for model predictive controller designs.

The purpose of an engine control system is to provide the required thrust by changing the fuel flow according to the throttle positions, while maintaining the limited-outputs in the prescribed bounds [4]. However, in practice, thrust cannot be sensed and therefore cannot be controlled directly. Generally, speeds or engine pressure (EPR) is treated to be the indicator of thrust [3]. In this paper, the control objective is to track the fan speed setpoints considering input and output constraints, and conventional constrained model predictive algorithm is improved accordingly, where tracking outputs (e.g. fan speed) and limited-outputs (e.g. temperature, surge margin) are handled in different ways. The constrained model predictive controller consists of three parts [24]: predictive model, feedback emendation, and receding horizon optimization.

A. PREDICTIVE MODEL

Linear discrete state space models of turbofan engine at nominal points are utilized as predictive models that can be expressed as:

$$\begin{cases}\hat{\boldsymbol{x}}(k+1) = \boldsymbol{A}_d\hat{\boldsymbol{x}}(k) + \boldsymbol{B}_d\hat{\boldsymbol{u}}(k)\\ \hat{\boldsymbol{y}}(k) = \boldsymbol{C}\hat{\boldsymbol{x}}(k) + \boldsymbol{D}\hat{\boldsymbol{u}}(k)\end{cases} \tag{1}$$

where $y = [\,\Delta N_f\ \Delta T_{45}\ \Delta smHPC\,]^T$, $x = [\,\Delta N_f\ \Delta N_c\,]^T$, $u = \Delta W_f$. The control variable is the deviation of fuel flow W_f in kg/s from the steady state, state variables are the deviations of fan speed N_f and core speed N_c in r/min. Three output variables are considered here, where fan speed is used for tracking and the other two outputs (high pressure compressor outlet temperature ΔT_{45} in °R and high pressure compressor stall margin $\Delta smHPC$ in %) are regarded as limited outputs. The values of matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are different corresponding to different flight conditions (e.g. flight altitude H and Mach number Ma) and power states (expressed as a percentage of the max cruise speed or actual fan speed N_f).

When the augmented state $\hat{\boldsymbol{x}}_a^T = [\hat{\boldsymbol{x}}(k)^T\ \hat{\boldsymbol{u}}(k-1)^T]$ is introduced, Formula (1) can be expressed as:

$$\begin{cases}\begin{bmatrix}\hat{\boldsymbol{x}}(k+1)\\ \hat{\boldsymbol{u}}(k)\end{bmatrix} = \begin{bmatrix}\boldsymbol{A}_d & \boldsymbol{B}_d\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{x}}(k)\\ \hat{\boldsymbol{u}}(k-1)\end{bmatrix} + \begin{bmatrix}\boldsymbol{B}_d\\ \boldsymbol{I}\end{bmatrix}\Delta\hat{\boldsymbol{u}}(k)\\ \hat{\boldsymbol{y}}(k) = \begin{bmatrix}\boldsymbol{C} & \boldsymbol{D}\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{x}}(k)\\ \hat{\boldsymbol{u}}(k-1)\end{bmatrix} + \boldsymbol{D}\Delta\hat{\boldsymbol{u}}(k)\end{cases} \tag{2}$$

B. FEEDBACK EMENDATION

Owing to the disturbances, engine component degradation and nonlinearities, predictive model is not exactly the same as engine current state, that is, there exits model mismatch. At sampling time k, define the error of engine actual output $\boldsymbol{y}_p(k)$ and predictive model output $\hat{\boldsymbol{y}}(k)$ (including tracking outputs $y_t(k)$ and limited outputs $\boldsymbol{y}_l(k)$) as $\boldsymbol{e}(k) = \boldsymbol{y}_p(k) - \hat{\boldsymbol{y}}(k)$. The corrected predicted outputs $\hat{\boldsymbol{y}}_{COR}$ can be represented as:

$$\begin{bmatrix}\hat{\boldsymbol{y}}_{COR}(k+1)\\ \hat{\boldsymbol{y}}_{COR}(k+2)\\ \vdots\\ \hat{\boldsymbol{y}}_{COR}(k+n_y)\end{bmatrix} = \begin{bmatrix}\hat{\boldsymbol{y}}(k+1)\\ \hat{\boldsymbol{y}}(k+2)\\ \vdots\\ \hat{\boldsymbol{y}}(k+n_y)\end{bmatrix} + \begin{bmatrix}h_1\\ h_2\\ \vdots\\ h_{n_y}\end{bmatrix}[\boldsymbol{y}_p(k) - \hat{\boldsymbol{y}}(k)] \tag{3}$$

where the correction factors $h_i, i = 1, 2, \ldots, n_y$ satisfy $0 < h_i \leq 1$.

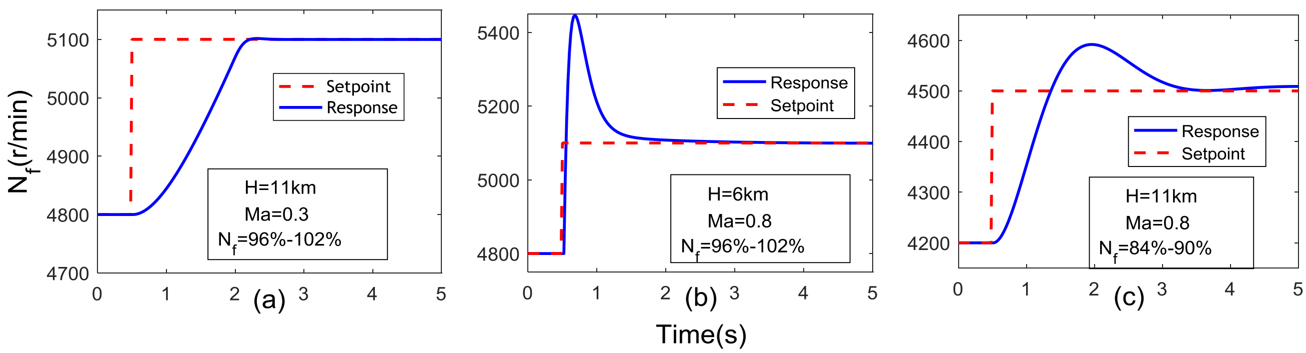

FIGURE 1. N_f **response at off-nominal points.**

C. RECEDING HORIZON OPTIMIZATION

In order to meet the control requirements, a form of cost function with constraints is shown as follows:

$$\begin{aligned} \min_{\Delta u} J &= \sum_{j=1}^{j=n_y} (\hat{y}_{tCOR}(k+j) - y_r(k+j))^2 \\ &\quad + \sum_{i=0}^{n_u-1} \boldsymbol{\lambda} \Delta\hat{\boldsymbol{u}}(k+i)^T \Delta\hat{\boldsymbol{u}}(k+i) \\ s.t.\ & \boldsymbol{u}_{\min} \leq \hat{\boldsymbol{u}}(k+i) \leq \boldsymbol{u}_{\max} \\ & \Delta\boldsymbol{u}_{\min} \leq \Delta\hat{\boldsymbol{u}}(k+i) \leq \Delta\boldsymbol{u}_{\max} \\ & \boldsymbol{y}_{l\min} \leq \hat{\boldsymbol{y}}_{lCOR}(k+j) \leq \boldsymbol{y}_{l\max} \\ & i = 0, 1, \ldots, n_u - 1 \\ & j = 1, 2, \ldots, n_y \end{aligned} \tag{4}$$

where $y_r(k + j)$ are the expected reference trajectories, $\hat{y}_{tCOR}(k + j)$ and $\hat{\boldsymbol{y}}_{lCOR}(k + j)$ are the corrected predicted tracking outputs and limited-outputs in the future j time steps respectively, $\Delta\hat{\boldsymbol{u}}(k+i)$ is the control variable increment vector and λ is the corresponding weights, n_y and n_u are the control horizon and prediction horizon respectively. $\boldsymbol{u}_{\max}$, $\boldsymbol{u}_{\min}$ and $\Delta\boldsymbol{u}_{\max}$, $\Delta\boldsymbol{u}_{\min}$ represent the maximum and minimum limits of the control variable and its increments, $\boldsymbol{y}_{l\max}$ and $\boldsymbol{y}_{l\min}$ indicate the maximum and minimum outputs limits. In formula (4), tracking output $\hat{y}_{tCOR}(k + j)$ is listed in the cost function and limited-output $\hat{\boldsymbol{y}}_{lCOR}(k + j)$ is incorporated in the constraints conditions, which meet the requirements of the turbofan engine.

Incorporate formula (1)-(3) to the cost function (4), after iteration, a quadratic programming problem with constraints could be obtained. At each sampling time, call *quadprog* function in the Matlab software to conduct the optimization, and compute the control variable sequence

$\Delta u(k), \Delta u(k+1), \ldots, \Delta u(k+n_u-1)$. However, only the first control variable $\Delta u(k)$ is applied to the turbofan engine and at next sampling time, similar optimization procedures are repeated, thus realizing receding horizon optimization.

III. ROBUSTNESS ANALYSIS OF A NOMINAL CONTROLLER AND PARTITION OF FLIGHT ENVELOPE

The robustness of a nominal model predictive controller refers to the system's ability to maintain stability with good transient performance when there exists a model mismatch between the predictive model and the actual engine state. At present, robustness analysis of model predictive control algorithm is mostly by means of simulation results, this is mainly due to the complexity of theoretical analysis.

As for turbofan engines, simulation results show that one nominal predictive controller alone cannot satisfy the requirements of dynamic performance in the entire envelope, and several examples are shown in Fig.1, where nominal point is $H = 11$km, $Ma = 0.8$, and power $= 100\%N_f$.

Fig. 1(a) shows that the response speed is too slow when Ma is far from its nominal point; Fig. 1(b) indicates that the overshoot is too large when H is off its nominal point; and bad dynamic response appears when power state is far away from its nominal point, as shown in Fig. 1(c). After all, constrained model predictive controllers are designed based on the small deviation linear dynamic models, which are only applicable to the small areas around the nominal points for better dynamic response.

As a result, different working conditions (H, Ma, N_f) correspond to different engine linear models. For better control performance, the flight envelope (H, Ma) is divided into N_1 sections with a nominal flight point determined for each section, and power states (N_f) should also be divided to generate N_2 nominal power points. In this way, a series of $N_1 \times N_2$ nominal working conditions are acquired. For simplicity, nominal power points can be determined with mean intervals from idle state to take-off state. The method for partition of the flight envelope is as follows.

For a given fuel flow supply, fan speed and the turbine expansion ratio, as well as other engine outputs are a function of flight parameters H and Ma. Furthermore, if the inlet of the turbofan engine is determined, the sensed total temperature T_1 and total pressure P_1 of inlet are a function of H and Ma, as presented in (5) and (6). Therefore, it can be concluded that the linear state space models are closely related to parameters P_1 and T_1.

When $H \leq 11$km, there exists:

$$\begin{cases} T_1 = (288.15 - 6.5 \times H) \times (1 + 0.2 \times Ma^2) - 273 \\ P_1 = 1.03323 \times (1 - \dfrac{H}{44.3})^{5.2553} \times (1 + 0.2 \times Ma^2)^{3.5} \end{cases} \tag{5}$$

When $H > 11$km, there exists:

$$\begin{cases} T_1 = 216.6 \times (1 + 0.2 \times Ma^2) - 273 \\ P_1 = 0.2314 \times e^{(\frac{11-H}{6.318})} \times (1 + 0.2 \times Ma^2)^{3.5} \end{cases} \tag{6}$$

If the sensed parameters T_1 and P_1 change within a certain small range, it is assumed that a nominal flight predictive controller can be used to regulate this section. Thus the selection rules J for section divisions can be defined as:

$$J = \sqrt{(\frac{P_{1x} - P_{11}}{P_{11}})^2 + (\frac{T_{1x} - T_{11}}{T_{11}})^2} \leq \varepsilon \tag{7}$$

where T_{11}, P_{11} and T_{1x}, P_{1x} are inlet total temperature and pressure of a nominal point and an arbitrary point x in the flight envelope respectively. If the Root-Mean-Square of the temperature and pressure change does not exceed ε, it is considered that point x is supposed to belong to the section of this nominal point.

In this paper, to verify the effectiveness of the partition method, take a part of the entire envelope, known as control domain, for example, as shown in Fig. 2.

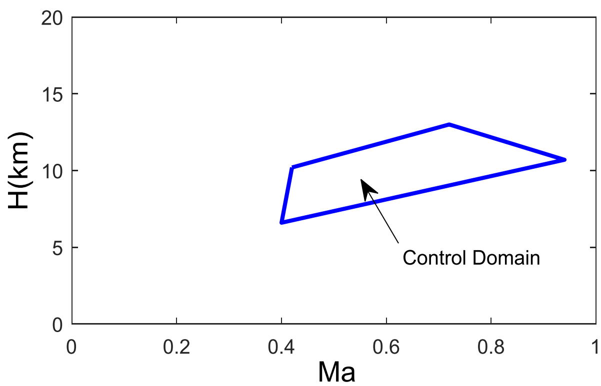

FIGURE 2. Control domain in the flight envelope.

For the turbofan engine, simulation results show that when $\varepsilon \leq 0.2$, good dynamic performance can be achieved within the entire section by the nominal controller. Here, ε is selected as $\varepsilon = 0.2$ and the nominal points need be determined so that their sections can cover the entire control domain. Through continuous attempts, three nominal points in the control domain are finally selected as ($H = 11$km, $Ma = 0.8$), ($H = 11.7$km, $Ma = 0.65$) and ($H = 9.5$km, $Ma = 0.75$), as pointed out by "$*$" in Fig. 3. Different colors in Fig.3 represent different sections. It can be seen that the constrained predictive controllers designed at these three nominal points can cover the entire control domain.

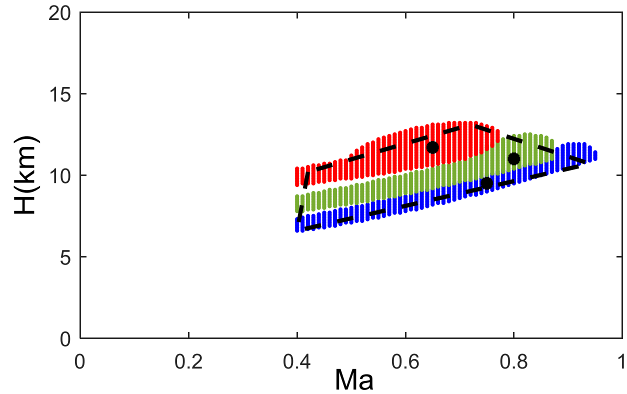

FIGURE 3. Nominal points and sections.

IV. SCHEDULING SCHEME DESCRIPTION

In general, engine characteristics change with parameters (H, Ma, N_f), so these three parameters can be used as scheduling parameters. As mentioned above, (H, Ma) determined the selection of the nominal controllers in the flight envelope. Therefore, a two-layer scheduling scheme is proposed here. Assume that the current working condition of engine is (H_x, Ma_x, N_{fx}) and $N_f(k) < N_{fx} < N_f(k+1)$ (where $N_f(k)$ and $N_f(k+1)$ are nominal power states) is satisfied. The first layer is referred as the flight envelope scheduling layer and the scheduling parameters are (H, Ma), in which control values at (H_x, Ma_x, $N_f(k)$ and (H_x, Ma_x, $N_f(k+1)$) conditions are determined through the scheduling approach of flight envelope. The second layer is called the power scheduling layer and scheduling parameter is N_f, where the final control value at (H_x, Ma_x, N_{fx}) is determined through the linear interpolation method, based on the control variables values at (H_x, Ma_x, $N_f(k)$) and (H_x, Ma_x, $N_f(k+1)$) obtained in the first layer.

An example is presented to illustrate the principle of the scheduling scheme. Suppose that an adaptive predictive controller with the above scheming scheme is designed, in order to realize the control of the turbofan engine that works from $80\%N_f$ to $104\%N_f$ in the control domain (H, Ma) of Fig. 2. Take three nominal power points $85\%N_f$, $93\%N_f$ and $100\%N_f$ for example, covering the $80\%N_f - 104\%N_f$ power states. Considering the fact that there are three nominal flight points in the control domain, a total of $3 \times 3 = 9$ nominal working condition points need to be included, as listed in Table 1.

To describe the proposed scheduling scheme in detail, the 9 nominal working conditions in Table 1 are identified in the three dimensional coordinate (H, Ma, N_f), as shown in Fig. 4.

In Fig. 4, the control domain under different nominal power states can be regarded as planes perpendicular to axis N_f. Suppose the current condition of engine is m (H_x, Ma_x, $90\%N_f$) and it is obvious that "m" lies between $85\%N_f$ and $93\%N_f$ planes. The value of W_f under the condition "m" can be obtained through the power scheduling

TABLE 1. Different Nominal Points (NP).

NP	Power	Speed (r/min)	H (km)	Ma
1	85% N_f	4250	9.5	0.75
2	85% N_f	4250	11	0.8
3	85% N_f	4250	11.7	0.65
4	93% N_f	4650	9.5	0.75
5	93% N_f	4650	11	0.8
6	93% N_f	4650	11.7	0.65
7	100% N_f	5000	9.5	0.75
8	100% N_f	5000	11	0.8
9	100% N_f	5000	11.7	0.65

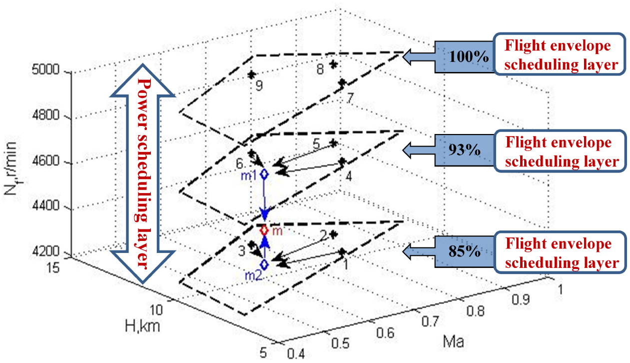

FIGURE 4. The structure of the scheduling scheme.

layer, based on the values of W_f under the conditions $m1(H_x, Ma_x, 93\%N_f)$ and $m2(H_x, Ma_x, 85\%N_f)$. Moreover, the values of W_f under the conditions "$m1$" and "$m2$" can be determined by the flight envelope scheduling layer, based on "4," "5," "6" and "1," "2," "3" nominal controllers respectively. Therefore, the value of W_f under the off-nominal condition $(H_x, Ma_x, 90\%N_f)$ can be finally determined by this two-layer scheduling scheme.

A. THE FLIGHT ENVELOPE SCHEDULING LAYER

In the first layer, (H, Ma) are used as scheduling parameters. Under a certain power state, define current flight conditions are (H_x, Ma_x), and sensed parameters (T_{1x}, P_{1x}) can then be achieved according to (5) or (6). In the same way, the sensed parameters of nominal flight points $1, 2, \ldots, N_1$ can be expressed as $(T_{11}, P_{11}), (T_{12}, P_{12}), \ldots, (T_{1N_1}, P_{1N_1})$, and outputs of these nominal predictive controllers are W_{f1}, $W_{f2}, \ldots, W_{fN_1}$ accordingly. For this layer, the proposed fuzzy membership degree scheduling approach, as well as a traditional scheduling technique used for comparison is described. The regional scheduling approach is regarded as the traditional one, which means if the current flight condition (H_x, Ma_x) lies in a section, then the nominal controller in this section takes over. The fuzzy membership degree scheduling strategy is as follows.

Define:

$$
\begin{aligned}
J_1 &= \sqrt{(\frac{P_{1x}-P_{11}}{P_{11}})^2 + (\frac{T_{1x}-T_{11}}{T_{11}})^2}\\
J_2 &= \sqrt{(\frac{P_{1x}-P_{12}}{P_{12}})^2 + (\frac{T_{1x}-T_{12}}{T_{12}})^2}\\
&\vdots\\
J_3 &= \sqrt{(\frac{P_{1x}-P_{1N_1}}{P_{1N_1}})^2 + (\frac{T_{1x}-T_{1N_1}}{T_{1N_1}})^2}
\end{aligned}
\tag{8}
$$

where $J_1, J_2, \ldots, J_{N_1}$ represent the closeness degree between current flight condition and N_1 nominal flight conditions, and the smaller the value is, the closer the two states are.

If $J_1 = 0$, the required W_{fx} for (H_x, Ma_x) is defined as $W_f = W_{f1}$, and $J_2 \ldots J_{N_1}$ are treated in the same way. If $J_1, J_2, \ldots, J_{N_1} \neq 0$, define $Q_1 = 1/J_1$, $Q_2 = 1/J_2, \ldots, Q_{N_1} = 1/J_{N_1}$, and W_{f1} can then be expressed as:

$$
\begin{aligned}
W_{fx} = &\frac{Q_1}{Q_1+Q_2+\ldots+Q_{N_1}} W_{f1} + \frac{Q_2}{Q_1+Q_2+\ldots+Q_{N_1}} W_{f2}\\
&+ \ldots + \frac{Q_{N_1}}{Q_1+Q_2+\ldots+Q_{N_1}} W_{fN_1}
\end{aligned}
\tag{9}
$$

Suppose that the current flight condition (H_x, Ma_x) belongs to section 1, it is indicated that the current state is the closest with nominal point 1, and Q_1 is bigger, thus making the coefficient $\frac{Q_1}{Q_1+Q_2+\ldots+Q_{N_1}}$ become bigger and W_{f1} plays a dominant role. Especially, if (H_x, Ma_x) and the nominal point 1 almost overlap, then $J_1 \rightarrow 0$ and $Q_1 \rightarrow \infty$, there exists $\frac{Q_1}{Q_1+Q_2+\ldots+Q_{N_1}} \approx 1$, and $W_{fx} \approx W_{f1}$. Similar conclusions can be derived for sections $2, 3, \ldots, N_1$. It is obvious that the above analysis is consistent with the practical situation.

For fuzzy membership degree scheduling technique, the control values for the off-nominal flight condition are determined by all nominal controllers with different weights, and control variable can change in a continuous way when flight conditions change. However, for the regional scheduling approach, control variable may jump when flight conditions go through different sections, because different nominal controller is activated with changes of flight conditions. Therefore, the proposed fuzzy membership degree technique is better.

B. THE POWER SCHEDULING LAYER

Now we turn our attention to the power scheduling layer as it relates to how the fan speed N_f is utilized as a scheduling parameter. In the above flight envelope scheduling layer, N_2 nominal power controllers under the current flight conditions (H_x, Ma_x) could be got. For any power status between $N_f(k)$ and $N_f(k+1)$, which are two adjacent nominal power points, the final output value of the adaptive predictive controller (fuel flow) could be obtained by the linear interpolation method, based on the control values for $(H_x, Ma_x, N_f(k))$ and $(H_x, Ma_x, N_f(k+1))$. The linear interpolation approach can be defined as:

$$u_{cmd} = W_{f\_H_x,Ma_x}(k) + \frac{N_f - N_f(k)}{N_f(k+1) - N_f(k)} \times (W_{f\_H_x,Ma_x}(k+1) - W_{f\_H_x,Ma_x}(k)) \quad (10)$$

where $W_{f\_H_x,Ma_x}(k)$ and $W_{f\_H_x,Ma_x}(k+1)$ are control variables for working conditions $(H_x, Ma_x, N_f(k))$ and $(H_x, Ma_x, N_f(k+1))$, and u_{cmd} is the final control variable after these two scheduling layers.

C. IMPLEMENTATION OF THE PROPOSED SCHEDULING SCHEME

In this part, the example mentioned above that involves 9 working status is still adopted to illustrate the implementation of an adaptive model predictive controller with the two-layer scheduling scheme, as shown in Fig. 5.

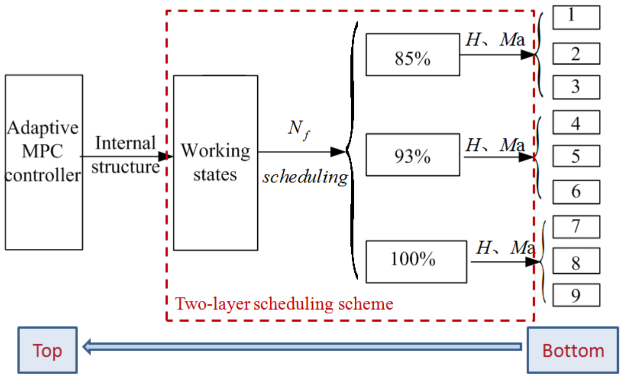

FIGURE 5. The implementation of the proposed scheduling scheme.

For the example, as shown in Fig.5, the 9 constrained model predictive controllers are the basis of this adaptive controller, which should be arranged in the bottom place. Then through the flight envelope scheduling technique and power scheduling strategy successively, as well as a packaged top layer, the expected adaptive controller can be established and realized. The inputs of this adaptive controller are composed of the fan speed N_f, the pilot's instruction (the expected power status), the flight conditions (H, Ma), input and its increment limits and output limits. The output is the main fuel flow W_f.

V. SIMULATIONS

In this part, simulations are conducted to verify the effectiveness of the proposed adaptive model predictive controller with the two-layer scheduling scheme. Two cases are included here.

A. CASE ONE

The control objective is to maintain the power states as expected when the flight conditions change. In this example, the desired power states is $90\%N_f$ (4500r/min), and the flight conditions (H, Ma) changes dramatically with time, as shown in Fig. 6 (a). In this situation, the changes of flight conditions can also be regarded as disturbances applied to the system. The input and output dynamic responses are then displayed in Fig. 6.

For an expected power states of $90\%N_f$, which is between $85\%N_f$ and $93\%N_f$ nominal power states, the final control value will be associated with six nominal model predictive controllers named "1," "2," "3," "4," "5," "6," as mentioned in Section IV. In order to show the advantage of the proposed fuzzy membership degree scheduling approach (called "Approach 2" in Fig.6), control variable strategies in each layer are depicted to compare the proposed technique with the traditional regional scheduling approach (called "Approach 1" in Fig.6).

In Fig.6 (b), it is observed that when flight conditions change, the output N_f can be restored to the original expected setpoint in a very short time with minor deviations, which indicates that the adaptive predictive controller can deal with flight disturbances effectively during the steady state. It is obvious that different scheduling schemes result in different N_f response, and "Approach 2" is better than "Approach 1."

In Fig. 6(c)-(f), "$m1$" and "$m2$" denote control values at nominal power states $85\%N_f$ and $93\%N_f$ respectively with different scheduling methods. Taking "$m2$" for example, Fig. 6(a) shows that flight trajectories go across two sections of the control domain comparing with Fig. 3. According to the principle of the traditional regional scheduling approach, control value is determined by nominal controllers "1" or "2" with the changes of flight conditions, that is, there exists switching between nominal controller "1" and "2." As shown in Fig. 6(c), the control value is firstly dependent on controller "1," and at about 89s, then the control value jumps to be determined by controller "2," which results in a bigger deviation for N_f in Fig. 6(b). As for fuzzy membership degree scheduling approach, the control value is decided by a combination of three nominal controllers "1," "2," "3," control value changes continuously and smoothly without big jumps, which facilitates a better performance of N_f, as seen in Fig. 6(b).

When control variables for "$m1$" and "$m2$" are determined by the flight envelope scheduling layer, the final control variable can then be obtained by the power scheduling layer (linear interpolation method), as shown in Fig. 6(g) and 6(h).

This example shows that the adaptive LMPC controller with this two-layer scheduling scheme owns the ability to arrange these 9 nominal MPC controllers through parameters H, Ma, and N_f. Through the analysis of control variable trajectories in each layer, it is shown that the fuzzy membership degree scheduling approach together with the linear interpolation scheduling technique can make the control variable change in a continuous way, thus achieve the control objective with satisfactory dynamic effect. Similarly, for other expected constant power states objectives, it can be validated that the control effects under small disturbances are consistent with good disturbance rejection.

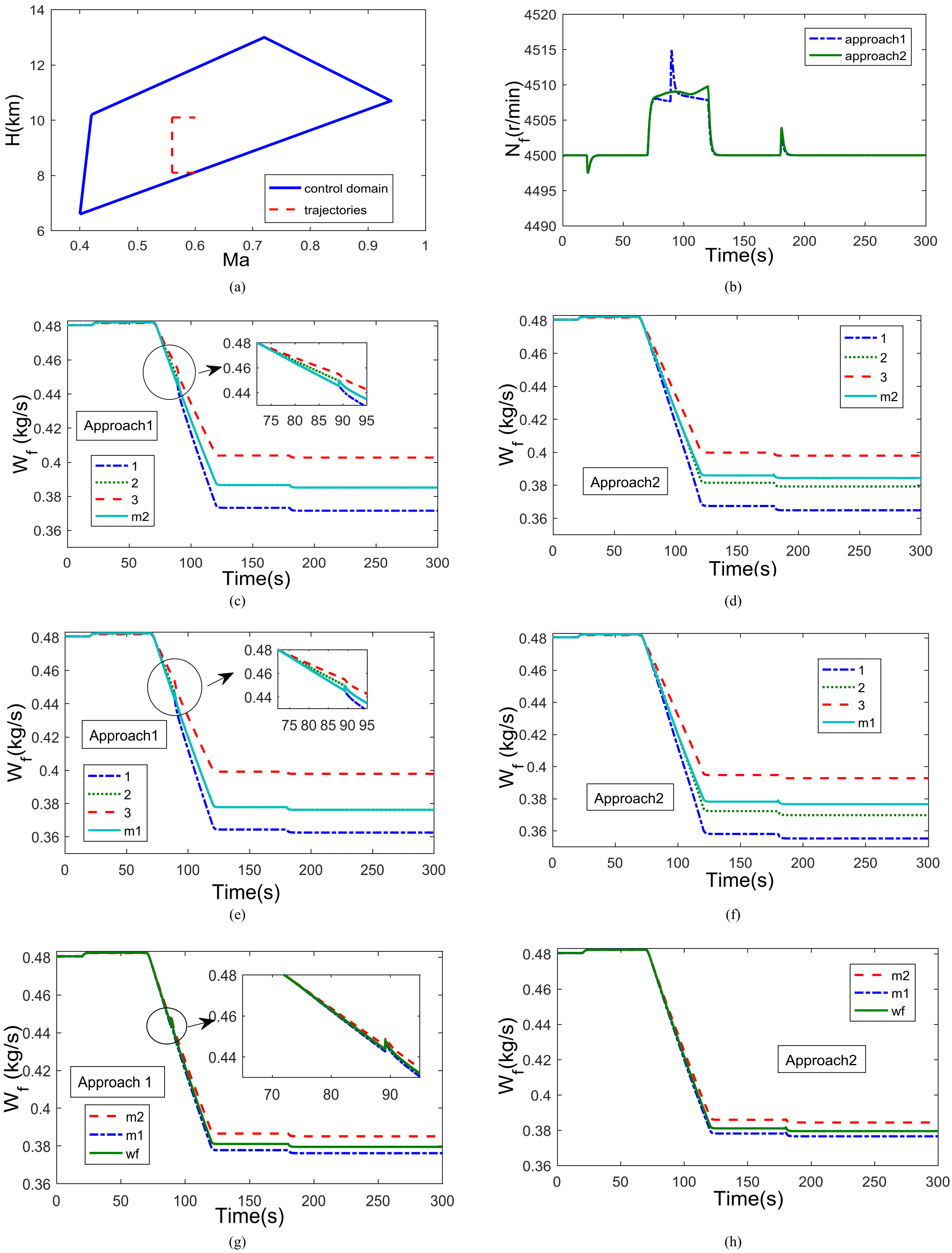

FIGURE 6. Input and output response trajectories with flight conditions changes. (a). Flight conditions trajectories. (b). Fan speed response with different approaches. (c). Fuel flow change trajectories of flight scheduling layer at 93%N_f power state under Approach 1. (d). Fuel flow trajectories of flight scheduling layer at 93%N_f power state with Approach 2. (e). Fuel flow trajectories of flight scheduling layer at 85%N_f power state with Approach 1. (f). Fuel flow trajectories of flight scheduling layer at 85%N_f power state with Approach 2. (g). Fuel flow trajectories of power scheduling layer with Approach 1. (h). Fuel flow trajectories of power scheduling layer with Approach 2.

B. CASE TWO

In this case, the control objective is to transfer power states for a large range from 82%N_f to 104%N_f (4200r/min-5200r/min), that is, the acceleration and deceleration transition process for engines. An idea of the acceleration and deceleration process arrangement is to limit the

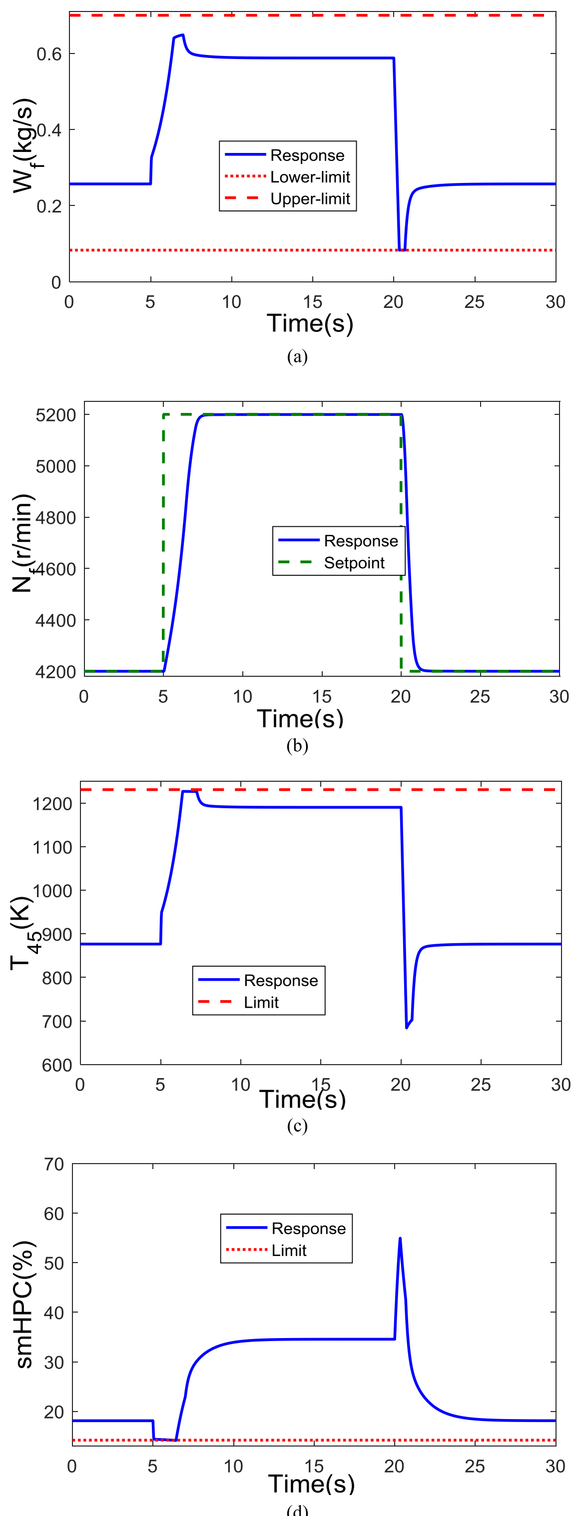

FIGURE 7. Response during transition state. (a). Fuel flow trajectories during transition state. (b). Fan speed response during transition state. (c). Temperature response during transition state. (d). Surge margin trajectories during transition state.

maximum/minimum change rate of the fuel flow W_f, as well as the maximum/minimum amplitude of the limited outputs T_{45} (temperature after high-pressure turbine), *smHPC* (surge margin).

Unlike the traditional transition controls (e.g. PID controller) where anti-windup (IWU) must be taken into account, MPC is well-known as a better way to deal with input and output constraints directly within the process of optimizations. However, such constraints are not included in the conventional control algorithms, which cannot produce a control input that breaks away from the constraints to overcome the "IWU" phenomenon. Therefore, for the adaptive MPC controller, there is no need to consider the "IWU" problem during the acceleration and deceleration transition state.

In this example, suppose the maximum limit of the W_f change rate is 0.03kg/s and the minimum limit is -0.04kg/s. In addition, for the output limits, $T_{45} \leq 1250K$ and $smHPC \geq 12\%$ are also taken into account during the transition state. These constraints are then added to the nominal MPC controller designed in Section II. Input and outputs trajectories are depicted accordingly in Fig.7.

As seen in Fig.7, N_f can track its setpoints with little overshoot and short settling time. Input and output constraints play an important role in the transition process. For the acceleration process, rate limits of control variable W_f are activated first, so W_f increases in an increment of 0.03kg/s, until *smHPC* reaches its limit and followed by the T_{45} limit. The deceleration process operates in a similar way. It is also observed that limited outputs T_{45} and *smHPC* are within their ranges during the transition state.

The simulation results show that the designed adaptive MPC controller with the two-layer scheduling scheme meets the performance requirements of both steady state and transition state processes. Therefore, it is feasible for the adaptive MPC controller to be applied to turbofan engines.

VI. CONCLUSIONS

An adaptive model predictive controller based on a two-layer scheduling scheme was designed and tested with a nonlinear component level turbofan engine model, which can drive the engine to operate randomly under the power states from 80%N_f to 104%N_f in the entire control domain. Acceleration and deceleration transition processes are realized by adding input and output constraints to the control system. The fuzzy membership degree scheduling approach proposed in the flight scheduling layer makes the control variable change smoothly with the flight conditions. Although the control domain considered in this paper is just a section of the full flight envelope, the proposed method to divide the entire envelope is the same, thus it is easy to extend the controller to realize the control in the whole flight envelope. Therefore, the two-layer scheduling method proposed in this paper gives instructions for adaptive controller design involving the whole power states and the entire flight envelope.

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