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A Low-Complexity Massive MIMO Precoding Algorithm Based on Chebyshev Iteration

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ABSTRACT Precoding algorithm is used to transmit signals effectively and to reduce the interferences from other user terminals in the massive multiple-input–multiple-output (MIMO) systems. In order to decrease the computational complexity of the precoding matrix, this paper proposes a new precoding algorithm. We use Chebyshev iteration to estimate the matrix inversion in the regularized zero-forcing precoding (RZF) algorithm. It does not need to compute the matrix inversion directly but uses iterations to estimate the matrix inversion. Therefore, the computational complexity can be decreased in this way. Furthermore, Chebyshev iteration has lower convergence rate, and it can gain precoding matrix quickly. This paper analyzes the performance of the Chebyshev-RZF precoding algorithm using average achievable rate and computes the complexity of the algorithm. Then, this paper optimizes initial values of the Chebyshev iteration algorithm on the basis of the feature of massive MIMO systems and makes initial values easier to be obtained. Simulation results show that after two iterations, the Chebyshev-RZF precoding algorithm can get similar average achievable rate as the RZF precoding algorithm does. An optimized Chebyshev-RZF precoding algorithm gets similar performance to the Chebyshev-RZF precoding algorithm after one iteration.

INDEX TERMS RZF precoding, Chebyshev iteration, Newton iteration, massive MIMO.

I. INTRODUCTION

Mobile communication has developed fast after several generations. The increase of data traffic and the popularity of intelligent terminals lead to high requirement of network. However, the 4G cannot satisfy users in aspects of capacity, speed and spectrum. The massive MIMO technology is one of important technologies of the 5G [1] and it provides high transmission rate, spectral efficiency and power efficiency [2]. In the massive MIMO systems, the base station (BS) is equipped with a large number of antennas and serves many users. However, a large number of antennas may result in pilot pollution, cell interference and multiuser interference at the user terminal (UT). In order to avoid this interference and enhance the accuracy of signal transmission, the transmitting signals are precoded at the BS using precoding algorithms.

Precoding algorithms need to get the channel state information (CSI) first. UTs transmit mutually orthogonal pilot signals to the BS and then the BS obtains CSI from pilot signals. Then Precoding algorithms use CSI to create a related precoding matrix. Precoding algorithms can be divided into linear precoding algorithms and non-linear precoding algorithms. Linear precoding algorithms contain zero-forcing (ZF) precoding algorithm, minimum mean square error (MMSE) precoding algorithm, matched filter (MF) precoding, regularized zero-forcing (RZF) precoding and so on. Non-linear precoding algorithms include Constant Enveloper (CE) algorithm, dirty paper coding (DPC) precoding algorithm, vector perturbation (VP) precoding algorithm and Tomlinson-Harashima precoding (THP) algorithm. In the massive MIMO systems, linear precoding algorithms are used because of their low complexity [3]. Particularly, RZF precoding algorithm is improved from ZF precoding algorithm and has better performance. Therefore, this study chooses RZF precoding algorithm.

There is a matrix inversion in the RZF precoding algorithm. If it is computed directly, it leads to a large computational complexity with the increase of the number of antennas as exponential form. Therefore, it is quite important to find a way to reduce the complexity of precoding algorithms [4].

There are two research directions in recent years to reduce the complexity of matrix inversion. The first method is to express the matrix inversion as the polynomial expansion and then truncate it [5]. Polynomial expansion applies into the precoding algorithm includes Taylor series, Neumann series [6] and Kapteyn series [7]. The Kapteyn series has better performance than Neumann series and Taylor series gets the worst performance under the same conditions. The second method is to use iteration algorithms to get the estimation of matrix inversion. Many iteration algorithms have been used in precoding algorithms. In [8], the successive over-relaxation (SOR) iteration is used to evaluate the matrix inversion in RZF precoding. Symmetric successive over relaxation (SSOR) is improved from SOR and estimates the matrix inversion in ZF precoding [9]. Recently, some researchers try to use Newton iteration algorithm to approximate the matrix inversion [10]–[12].

The main contributions of this paper are as follows. Firstly, the paper applies Chebyshev iteration into RZF precoding and reduces complexity of RZF precoding. Secondly, we can get the inversion matrix faster. Thirdly, the initial value of improved Chebyshev-RZF precoding is easier to be gotten and the performance of Chebyshev-RZF precoding is better.

The rest of this paper is organized as follows. In Section II, system model, RZF precoding matrix and the average achievable rate are described. In Section III, Newton iteration algorithm, Chebyshev iteration algorithm and improved Chebyshev iteration algorithm are used in RZF precoding algorithm. Then the complexity of Newton iteration and Chebyshev iteration are calculated. In Section IV, the average achievable rate of Newton-RZF precoding algorithm, Chebyshev-RZF precoding algorithm and RZF precoding algorithm are simulated. Section V concludes the paper.

II. SYSTEM MODEL

This section defines the massive MIMO system with flat-fading channel. We consider the downlink channel of a massive MIMO system. The system includes an M-antenna BS and K single-antenna UTs and it is shown in Fig. 1. In the uplink channel, UTs transmit pilot signals to the BS under TDD mode and then the BS gets the CSI using reciprocity between uplink channel and downlink channel. The estimated channel between the BS and UTs is modeled as $\hat{\mathbf{H}} \in \mathbb{C}^{M*K}$ and it can be expressed as $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, \ldots, \hat{\mathbf{h}}_K]$ where $\hat{\mathbf{h}}_k \in \mathbb{C}^{M*1}$ is the channel between the BS and the k-th UT. $\hat{\mathbf{h}}_k$ follows Gaussian distribution $CN(\mathbf{0}_{M*1}, \mathbf{\Phi})$ where $\mathbf{\Phi} \in \mathbb{C}^{M*M}$ is the channel covariance matrix. The imperfect channel estimate [5]

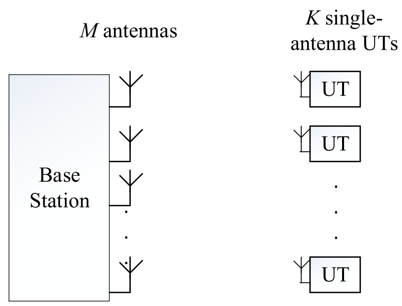

FIGURE 1. System model.

$$\hat{\mathbf{h}}_k = \sqrt{1-\tau^2}\mathbf{h}_k + \tau \mathbf{n}_k \tag{1}$$

where $\mathbf{h}_k$ is the real channel matrix. $\tau \in [0, 1]$ is the scalar parameter affecting the quality of channel estimation. $\mathbf{n}_k$ is the estimated channel noise which follows the Gaussian distribution $CN(0, \sigma^2)$ and it is independent and identically distributed with $\mathbf{h}_k$. When $\tau = 0$, the estimated channel matrix equals to the real channel.

The received signal $\mathbf{y}_k$ at the k-th UT is

$$\mathbf{y}_k = \mathbf{h}_k^{\mathrm{H}}\mathbf{x} + \mathbf{z}_k \tag{2}$$

where $\mathbf{x}$ is the transmitting signal. $\mathbf{z}_k$ is the additive white Gaussian noise following the Gaussian distribution $CN(0, \sigma^2)$.

In order to reduce multiuser interference, the transmitting signal is precoded at the BS. The data signal is set as $\mathbf{s} = [s_1, \ldots, s_K]^{\mathrm{T}}$ where s_k is the signal which is transmitted to the k-th UT. Based on this assumption, the transmitting signal can be expressed as

$$\mathbf{x} = \mathbf{G}\mathbf{s} \tag{3}$$

where $\mathbf{G}$ is a precoding matrix. In this study, we use RZF precoding algorithm and its precoding matrix is defined as

$$\mathbf{G} = \beta\, (\hat{\mathbf{H}}\hat{\mathbf{H}}^{\mathrm{H}} + \xi \mathbf{I}_M)^{-1}\hat{\mathbf{H}} \tag{4}$$

where β ensures $\mathbf{G}$ to satisfy the equation $\mathrm{tr}(\mathbf{G}\mathbf{G}^{\mathrm{H}}) = P$. P is the transmitting power at the BS. ξ is a regularization coefficient. When $\xi \to 0$, the RZF precoding matrix is equal to ZF precoding matrix. When $\xi \to \infty$, the RZF precoding matrix is similar to MF precoding matrix.

The signal to interference and noise ratio (SINR) at the k-th UT is [5]

$$\mathrm{SINR}_k = \frac{\mathbf{h}_k^{\mathrm{H}}\mathbf{g}_k\mathbf{g}_k^{\mathrm{H}}\mathbf{h}_k}{\mathbf{h}_k^{\mathrm{H}}\mathbf{G}\mathbf{G}^{\mathrm{H}}\mathbf{h}_k\text{-}\mathbf{h}_k^{\mathrm{H}}\mathbf{g}_k\mathbf{g}_k^{\mathrm{H}}\mathbf{h}_k+\sigma^2} \tag{5}$$

where $\mathbf{g}_k$ is the k-th column of the precoding matrix. Its corresponding achievable rate is shown as

$$\mathrm{r} = \log_2(1 + \mathrm{SINR}_k) \tag{6}$$

We use average achievable rate to measure the performance of precoding algorithms.

III. CHEBYSHEV ITERATION

In this section, we use Newton iteration algorithm and Chebyshev iteration algorithm to estimate the matrix inversion of RZF precoding algorithm and reduce its computational complexity. Then we optimize the initial value of Chebyshev iteration algorithm. Compared with Newton iteration algorithm, Chebyshev iteration algorithm has higher convergence rate.

A. NEWTON ITERATION

As for the question $f(x) = x^{-1} - A$, we can use Newton iteration algorithm to get the answer x. The traditional Newton iteration algorithm is represented as [13]

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{7}$$

where $n \in (0, +\infty)$ is the iteration time. Then we put equation (7) into $f(x) = x^{-1} - A$, and we can obtain the Newton iteration process.

$$\mathbf{X}_{n+1} = \mathbf{X}_n(2\mathbf{I} - \mathbf{A}\mathbf{X}_n) \tag{8}$$

When $n = 0$, $\mathbf{X}_0$ is the initial value of iteration. With the increase of iteration, $\mathbf{X}_n$ gets closer to $\mathbf{A}^{-1}$. When $n \to \infty$, $\lim\limits_{n\to\infty} \mathbf{X}_n = \mathbf{A}^{-1}$.

Newton iteration process can be converged when $\mathbf{X}_0 = \alpha\mathbf{A}^{\mathbf{T}}$. α can be gained as follows.

When $\mathbf{A}$ is a Hermite matrix, we can get the diagonalization based on eigenvalues of matrix $\mathbf{A}$ as

$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{U}^{\mathrm{T}} \tag{9}$$

where $\mathbf{U}$ is unitary matrix. $\mathbf{\Sigma} = diag(\delta_1, \delta_2, \ldots \delta_n)$ and δ_n is eigenvalue of $\mathbf{A}$.

According to $\mathbf{X}_0 = \alpha\mathbf{A}^{\mathbf{T}}$, if $\mathbf{A}$ is a Hermite matrix, $\mathbf{X}_0$ is also a Hermite matrix. Therefore, we can get the diagonalization based on eigenvalues of matrix $\mathbf{X}_0$ as

$$\mathbf{X}_0 = \mathbf{U}\Psi\mathbf{U}^{\mathrm{T}} \tag{10}$$

where $\Psi = \alpha diag(\delta_1, \delta_2, \ldots \delta_n)$.

Similarly, we can get the diagonalization of $\mathbf{A}\mathbf{X}_n$ as

$$\mathbf{A}\mathbf{X}_n = \mathbf{U}\mathbf{R}_n\mathbf{U}^{\mathrm{T}} \tag{11}$$

where $\mathbf{R}_n = diag(p_1^n, p_2^n, \ldots, p_k^n)$ is the eigenvalue of $\mathbf{A}\mathbf{X}_n$. $p_i^n = \alpha\delta_i^2$.

Then we multiply $\mathbf{A}$ on the left side of equation (8)

$$\mathbf{A}\mathbf{X}_{n+1} = \mathbf{A}\mathbf{X}_n(2\mathbf{I} - \mathbf{A}\mathbf{X}_n) \tag{12}$$

and put equation (11) into it. We can get

$$\mathbf{U}\mathbf{R}_{n+1}\mathbf{U}^{\mathrm{T}} = \mathbf{U}\mathbf{R}_n\mathbf{U}^{\mathrm{T}}\left(2\mathbf{I} - \mathbf{U}\mathbf{R}_n\mathbf{U}^{\mathrm{T}}\right) \tag{13}$$

Then equation (13) is reduced to

$$\mathbf{U}\mathbf{R}_{n+1}\mathbf{U}^{\mathrm{T}} = 2\mathbf{U}\mathbf{R}_n\mathbf{U}^{\mathrm{T}} - \mathbf{U}\mathbf{R}_n^2\mathbf{U}^{\mathrm{T}} \tag{14}$$

We can get iteration about eigenvalue

$$p_i^{n+1} = 2p_i^n - \left(p_i^n\right)^2 = 1 - \left(1 - p_i^n\right)^2 \tag{15}$$

According to equation (15), we can simply it as

$$1 - p_i^{n+1} = \left(1 - p_i^n\right)^2 \tag{16}$$

In order to make equation (8) converge, $\left|1 - p_1^0\right| < 1$. According to $p_i^n = \alpha\delta_i^2$, $p_1^0 = \alpha\delta_1^2$. Therefore, the condition of convergence is $0 < \alpha < \frac{2}{\delta_1^2}$.

According to [14], we can set $\alpha = \frac{2}{\delta_1^2 + \delta_r^2}$ where δ_1 is the minimum characteristic value of $\mathbf{A}$ and δ_r is the maximum characteristic value of $\mathbf{A}$.

As for RZF precoding algorithm, $\mathbf{A} = \hat{\mathbf{H}}\hat{\mathbf{H}}^{\mathrm{H}} + \xi\mathbf{I}_M$. After some iterations, $\mathbf{X}_n$ gets closer to the matrix inversion of $\hat{\mathbf{H}}\hat{\mathbf{H}}^{\mathrm{H}} + \xi\mathbf{I}_M$. In the massive MIMO system, we can treat $\delta_1^2 = \delta_r^2 = M^2$. Therefore, the initial value $\mathbf{X}_0$ is

$$\mathbf{X}_0 = \frac{1}{M^2}\mathbf{A}^{\mathrm{T}} \tag{17}$$

The Newton-RZF precoding algorithm is shown in detail as Algorithm 1 where n is the iteration times.

Algorithm 1 Newton-RZF Precoding (Input $\mathbf{H}$; Output $\mathbf{G}$)

1. $\mathbf{A} = (\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}} + \xi\mathbf{I}_k)$
2. $\sigma_1 = \lambda_{\max}(\mathbf{A})$, $\sigma_r = \lambda_{\min}(\mathbf{A})$
3. $\mathbf{X} = \frac{2}{\sigma_1 + \sigma_r}\mathbf{A}^{\mathrm{T}}$
4. times = 0
5. while times<n
6. $\quad\mathbf{T} = \mathbf{A}\mathbf{X}$
7. $\quad\mathbf{X} = \mathbf{X}(2\mathbf{I} - \mathbf{T})$
8. $\quad$times = times +1;
9. End
10. $\mathbf{G} = \beta\mathbf{N}\mathbf{H}$

B. CHEBYSHEV ITERATION

Newton iteration algorithm make the matrix inversion easier to be gotten. However, the Newton iteration algorithm is two-order convergence and its convergence rate is slow. In consideration of three-order convergence, Newton iteration algorithm can be changed into [15]

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f''(x_n)}{2f'(x_n)}\left(\frac{f(x_n)}{f'(x_n)}\right)^2 \tag{18}$$

The equation (18) is Chebyshev iteration algorithm. Similarly, we put it into $f(x) = x^{-1} - A$ and get Chebyshev iteration process

$$\mathbf{X}_{n+1} = \mathbf{X}_n(3\mathbf{I} - \mathbf{A}\mathbf{X}_n(3\mathbf{I} - \mathbf{A}\mathbf{X}_n)) \tag{19}$$

According to Newton iteration algorithm, the initial value of iteration is also $\mathbf{X}_0 = \alpha\mathbf{A}^{\mathbf{T}}$.

In terms of RZF precoding, $(\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}} + \xi\mathbf{I}_k)$ is Hermite matrix, so it replaces the matrix $\mathbf{A}$ and put it into Chebyshev iteration process. According to Newton iteration, we can first suppose the initial value is $\mathbf{X}_0 = \frac{2\mathbf{A}^{\mathrm{T}}}{\delta_1^2 + \delta_r^2}$ [16]. Then we put $\mathbf{X}_0$ into the condition of convergence, we can have

$$||\mathbf{I} - \mathbf{A}\mathbf{X}_0|| = \frac{\delta_1^2 - \delta_r^2}{\delta_1^2 + \delta_r^2} = \frac{\kappa^2 - 1}{\kappa^2 + 1} \tag{20}$$

where $\kappa = \frac{\delta_1}{\delta_r}$. According to (20), when $\mathbf{X}_0 = \frac{2\mathbf{A}^T}{\delta_1^2+\delta_r^2}$, $||\mathbf{I}-\mathbf{A}\mathbf{X}_0|| < 1$ [17].

The Chebyshev-RZF precoding algorithm is shown in detail as Algorithm 2 where n is the iteration time.

Algorithm 2 Chebyshev-RZF Precoding (Input $\mathbf{H}$; Output $\mathbf{G}$)

1. $\mathbf{A} = (\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \xi\mathbf{I}_k)$
2. $\sigma_1 = \lambda_{max}(\mathbf{A})$, $\sigma_r = \lambda_{min}(\mathbf{A})$
3. $\mathbf{X} = \frac{2}{\sigma_1+\sigma_r}\mathbf{A}^T$
4. times $= 0$
5. while times<n
6. $\quad\mathbf{T} = \mathbf{A}\mathbf{X}$
7. $\quad\mathbf{X} = \mathbf{X}(3\mathbf{I} - \mathbf{T}(3\mathbf{I} - \mathbf{T}))$
8. $\quad$times $=$ times $+1$;
9. End
10. $\mathbf{G} = \beta\mathbf{N}\mathbf{H}$

C. OPTIMIZATION OF INITIAL VALUE

The initial value decides the convergence rate. Therefore, a good initial value can make the convergence rate quicker. We find that $\mathbf{A} = (\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \xi\mathbf{I}_k)$ is a positive definite symmetric matrix, so we set the initial value as

$$\mathbf{X}_0 = \beta\mathbf{I} + \alpha\mathbf{A} \tag{21}$$

Then the diagonalization of $\mathbf{A}\mathbf{X}_0$ is

$$\mathbf{A}\mathbf{X}_0 = \mathbf{U}\left(\beta\boldsymbol{\Sigma} + \alpha\boldsymbol{\Sigma}^2\right)\mathbf{U}^T \tag{22}$$

Corresponding eigenvalue is

$$f(\delta) = \beta\delta + \alpha\delta^2 \tag{23}$$

α and β need to make $||\mathbf{I}-\mathbf{A}\mathbf{X}_0||_2$ minimum, and we can get

$$\beta = \frac{4\delta_{mid}}{2\delta_{mid}^2 - \theta^2} \tag{24}$$

$$\alpha = \frac{-2}{2\delta_{mid}^2 - \theta^2} \tag{25}$$

where $\delta_{mid} = \frac{\delta_1+\delta_n}{2}$ and $\theta = \delta_1 - \delta_{mid}$. Then we can get $\mathbf{X}_0 = \frac{4\delta_{mid}}{2\delta_{mid}^2-\theta^2}\mathbf{I} - \frac{2}{2\delta_{mid}^2-\theta^2}\mathbf{A}$.

In the massive MIMO system, we use approximate value of $\alpha = -\frac{1}{M^2}$ and $\beta = \frac{2}{M}$. In this way, we avoid finding eigenvalues of $\mathbf{A}$. Finally, the initial value is

$$\mathbf{X}_0 = \frac{2}{M}\mathbf{I} - \frac{1}{M^2}\mathbf{A} \tag{26}$$

D. ANALYSES OF COMPLEXITY

First of all, we define the complexity as the number of matrix additions and matrix multiplications.

We calculate the complexity of Newton iteration algorithm and Chebyshev iteration algorithm. According to equation (8), one Newton iteration needs one matrix addition and two matrix multiplications. According to equation (18), one Chebyshev iteration needs two matrix additions and three matrix multiplications. It can be seen clearly in the Table 1.

TABLE 1. Complexity of Newton iteraion and Chebyshev iteration.

Algorithm	Multiplication times	Addition times
Newton iteration	2	1
Chebyshev iteration	3	2

When Newton iteration algorithm and Chebyshev iteration algorithm are used in RZF precoding algorithm, we suppose that $\hat{\mathbf{H}}$, $\mathbf{s}$, $2\mathbf{I}$, $3\mathbf{I}$ and ξ, $\frac{\beta}{M^2}$ are known matrices and constants and we do not need to calculate them.

In the process of Newton-RZF precoding algorithm, the transmitting signal is $\mathbf{x} = \frac{\beta}{M^2}\hat{\mathbf{H}}(\hat{\mathbf{H}}^H\hat{\mathbf{H}}\mathbf{s} + \xi\mathbf{s})$ after one iteration. Steps of the algorithm are 1) one matrix and vector multiplication $\hat{\mathbf{H}}\mathbf{s}$; 2) one matrix and vector multiplication $\hat{\mathbf{H}}^H\hat{\mathbf{H}}\mathbf{s}$; 3) one constant and vector multiplication and one matrix addition $\hat{\mathbf{H}}^H\hat{\mathbf{H}}\mathbf{s} + \xi\mathbf{s}$; 4) one matrix and vector multiplication $\hat{\mathbf{H}}(\hat{\mathbf{H}}^H\hat{\mathbf{H}}\mathbf{s} + \xi\mathbf{s})$; 5) one constant and vector multiplication $\frac{\beta}{M^2}\hat{\mathbf{H}}(\hat{\mathbf{H}}^H\hat{\mathbf{H}}\mathbf{s} + \xi\mathbf{s})$. According to these steps, Newton-RZF precoding algorithm needs $3KM + M + K$ multiplications and $3KM$-$2M$ additions after one iteration.

When we get transmitting signal after two Newton iterations, the transmitting signal is

$$\mathbf{x} = \frac{\beta}{M^2}\hat{\mathbf{H}}\left(2\mathbf{I} - \frac{1}{M^2}\left(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \xi\mathbf{I}\right)\left(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \xi\mathbf{I}\right)\right) \times (\hat{\mathbf{H}}^H\hat{\mathbf{H}}\mathbf{s} + \xi\mathbf{s}) \tag{27}$$

In this process, $\mathbf{V}_1 = (\hat{\mathbf{H}}^H\hat{\mathbf{H}}\mathbf{s} + \xi\mathbf{s})$ has been calculated and it is a $K*1$ vector. Steps of the process are 1) One matrix and vector multiplication $2\mathbf{I}\mathbf{V}_1$; 2) one matrix and vector multiplication $\mathbf{V}_2 = \left(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \xi\mathbf{I}\right)\mathbf{V}_1$; 3) one matrix and vector multiplication $\mathbf{V}_3 = \left(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \xi\mathbf{I}\right)\mathbf{V}_2$; 4) one constant and vector multiplication $\frac{1}{M^2}\mathbf{V}_3$; 5) one vector addition $2\mathbf{I}\mathbf{V}_1 - \frac{1}{M^2}\mathbf{V}_3$; 6) one matrix and vector multiplication and one constant and vector multiplication $\frac{\beta}{M^2}\hat{\mathbf{H}}(2\mathbf{I}\mathbf{V}_1 - \frac{1}{M^2}\mathbf{V}_3)$. Newton-RZF precoding algorithm after two iterations needs $7KM + M + K$ multiplications and $7KM$-$4M + K$ additions.

Similarly, we can calculate Newton-RZF precoding algorithm after three iterations and it needs $15KM + 13K + M$ multiplications and $15kM$-$8M + 3K$ additions.

Analyses above are shown in Table 2.

TABLE 2. Complexity of Newton-RZF precoding algorithm.

Newton iteration	Multiplication times	Addition times
One iteration	$3KM$+M+K	$3KM$-$2M$
Two iterations	$7KM$+M+K	$7KM$-$4M$+K
Three iterations	$15KM$+$13K$+M	$15kM$-$8M$+$3K$

In the process of Chebyshev-RZF precoding algorithm after one iteration, the transmitting signal is also

$\mathbf{x} = \frac{\beta}{M^2}\hat{\mathbf{H}}(\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}}\mathbf{s} + \xi\mathbf{s})$ and its complexity is the same as Newton-RZF precoding algorithm.

When we gain the transmitting signal after two Chebyshev iterations, the transmitting signal is

$$\mathbf{x} = \frac{\beta}{M^2}\hat{\mathbf{H}}\left(3\mathbf{V}_3 - \frac{1}{M^2}\left(\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}} + \xi\mathbf{I}\right)\left(\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}} + \xi\mathbf{I}\right) \times \left(3\mathbf{V}_3 - \frac{1}{M^2}\mathbf{V}_3\right)\right) \tag{28}$$

where $\mathbf{V}_3$ is the vector given above. Steps of the process are 1) one matrix and vector multiplication and one constant and vector multiplication $3\mathbf{V}_3$ and $\frac{1}{M^2}\mathbf{V}_3$; 2) one vector and vector addition $\mathbf{V}_4 = \left(3\mathbf{V}_3 - \frac{1}{M^2}\mathbf{V}_3\right)$; 3) two matrix and vector multiplications $\mathbf{V}_5 = \left(\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}} + \xi\mathbf{I}\right)\left(\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}} + \xi\mathbf{I}\right)\mathbf{V}_4$; 4) one constant and vector multiplication $\frac{1}{M^2}\mathbf{V}_5$; 5) one vector and vector addition $3\mathbf{V}_3 - \frac{1}{M^2}\mathbf{V}_5$; 5) one matrix and vector multiplication and one constant and vector multiplication $\frac{\beta}{M^2}\hat{\mathbf{H}}\ (3\mathbf{V}_3 - \frac{1}{M^2}\mathbf{V}_5)$. Chebyshev-RZF precoding algorithm after two iterations needs $11KM + 9K + M$ multiplications and $11KM\text{-}6M + 2K$ additions.

Analyses above are clearly shown in Table 3.

TABLE 3. Complexity of Chebyshev-RZF precoding algorithm.

Chebyshev iteration	Multiplication times	Addition times
One iteration	$3KM$+M+K	$3KM$-$2M$
Two iterations	$11KM$+$9K$+M	$11KM$-$6M$+$2K$

According to analyses above, it can be seen that Chebyshev iteration is more complex after one iteration than Newton iteration. However, when two algorithms get the same performance with the same initial value, Chebyshev iteration algorithm needs less iterations and it is less complex than Newton iteration algorithm. The relationship between the number of iterations of Chebyshev iteration algorithm and Newton iteration algorithm is

$$\|E_0\|^{2^m} = \|E_0\|^{3^n} \tag{29}$$

where $E_0 = \mathbf{I} - \mathbf{A}\mathbf{X}_0$, m is the number of Newton iteration and n is the number of Chebyshev iteration. Then we can get

$$\frac{m}{n} = \frac{\ln 3}{\ln 2} \approx 1.585 \tag{30}$$

IV. SIMULATION RESULTS

In this section, we use Chebyshev iteration algorithm and some other algorithms to evaluate the matrix inversion in RZF precoding, and compare their average achievable rate. We assume the number of transmitting antennas is 256, the number of single-antenna users is 32. The channel covariance matrix uses the exponential model $[\Phi]_{i,j} = a^{j-i}$, $a = 0.1$.

Fig. 2 compares the average achievable rate of Chebyshev-RZF precoding algorithm with other precoding algorithms,

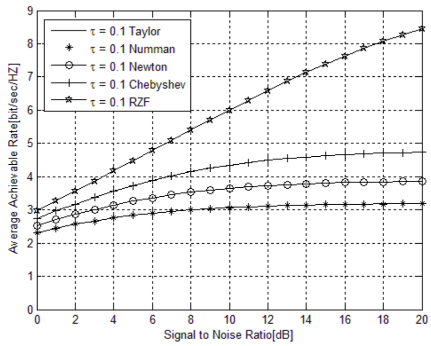

FIGURE 2. Average achievable rate ($\tau = 0.1, n = 1$).

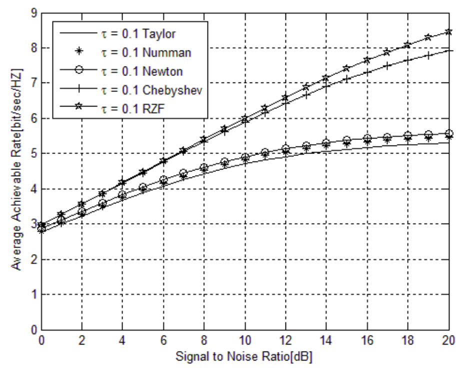

FIGURE 3. Average achievable rate ($\tau = 0.1, n = 2$).

including Newton-RZF precoding, Neumann-RZF precoding and Taylor-RZF precoding. The estimated channel is imperfect, and τ is 0.1. RZF precoding algorithm compute the matrix inversion directly, so it has best performance. Its performance is a standard. After one iteration, Chebyshev-RZF precoding algorithm has better performance. With the increase of SNR, the advantage of RZF precoding algorithm also enhances. Moreover, Chebyshev-RZF precoding algorithm is always better than other precoding algorithms.

Fig. 3 shows the performance of precoding algorithms after two iterations. The average achievable rate of Chebyshev-RZF precoding algorithm gets close to that of RZF precoding algorithm, and it is higher than other precoding algorithms. With the increase of SNR, the advantage is more obvious. Chebyshev-RZF precoding algorithm and Newton-RZF precoding algorithm are better than Taylor-RZF precoding algorithm and Neumann-RZF precoding algorithm in Fig. 2 and Fig. 3. Therefore, we compares Chebyshev-RZF precoding and Newton-RZF precoding in the next simulation.

In Fig. 4, we simulate average achievable rate of Chebyshev-RZF precoding and Newton-RZF precoding algorithm under different channel estimation parameter. We choose that Chebyshev-RZF precoding iterates two times and Newton-RZF precoding iterates three times. It can be found that Newton-RZF precoding algorithm's performance is similar to RZF precoding algorithm's performance, but Chebyshev-RZF precoding algorithm still better than

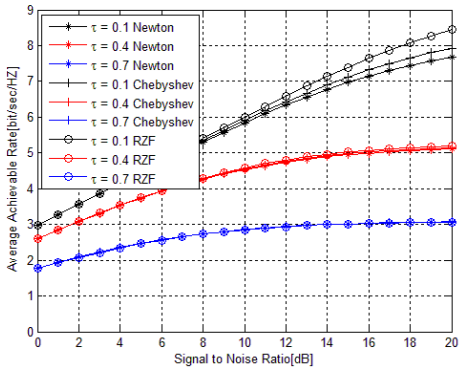

FIGURE 4. Average achievable rate (Chebyshev-RZF: $n = 2$; Newton-RZF: $n = 3$).

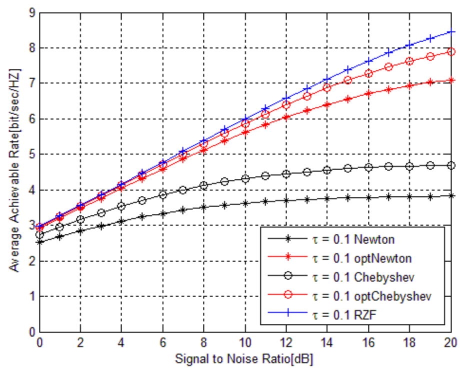

FIGURE 5. Average achievable rate after optimizing initial value ($\tau = 0.1, n = 1$).

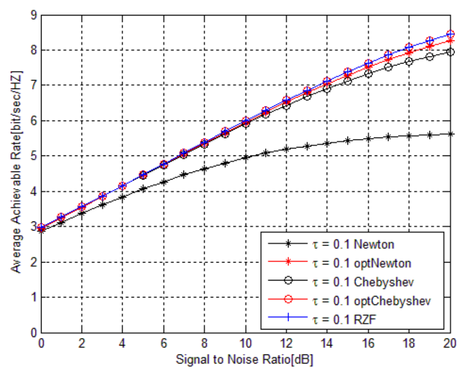

FIGURE 6. Average achievable rate after optimizing initial value ($\tau = 0.1, n = 2$).

Newton-RZF precoding algorithm. Besides, the complexity of Chebyshev-RZF precoding algorithm after two iterations is lower than Newton-RZF precoding algorithm after three iterations from analyses of complexity in Section III. Therefore, Chebyshev-RZF precoding gets better performance with lower complexity than Newton-RZF precoding.

In Fig. 5, red lines are the performances of Newton-RZF precoding algorithm and Chebyshev-RZF precoding algorithm after optimizing their initial values. It can be seen that their performance is much better than previous ones.

In Fig. 6, it can be seen that the average achievable rates of Newton-RZF precoding algorithm and Chebyshev-RZF precoding algorithm after optimizing initial values get much closer to RZF precoding algorithm after two iterations and the complexity of Newton-RZF precoding algorithm is lower than Chebyshev-RZF precoding algorithm when the number of iteration is the same. Therefore, we can use optimized Newton-RZF precoding algorithm after two iterations.

V. CONCLUSION

This paper introduces Newton iteration algorithm and Chebyshev iteration algorithm and uses them to estimate the matrix inversion in RZF precoding. Compared with computing matrix inversion directly, Chebyshev-RZF precoding reduces computational complexity. Moreover, we compare the Chebyshev iteration with Newton iteration in aspects of convergence rate and complexity. When they use the same initial value, Chebyshev-RZF can get good performance faster with lower complexity. Furthermore, this paper optimizes the initial value of Chebyshev iteration algorithm so as to make it easier to be gotten. According to simulation results, the performance of Chebyshev-RZF precoding algorithm is better than that of Newton-RZF precoding algorithm under the same iterations. By optimizing the initial values of Chebyshev algorithm, the improved Chebyshev-RZF precoding algorithm can get better performance and lower complexity compared with Chebyshev-RZF precoding. Furthermore, the improved Chebyshev-RZF precoding algorithm can get similar average user arrival rate to RZF precoding after one iterations.

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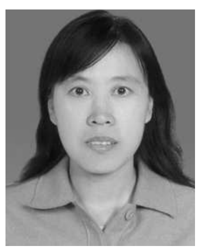

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