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Stability Analysis for GE T700 Turboshaft Distributed Engine Control Systems

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ABSTRACT Future gas turbine engine control systems will be based on a distributed architecture in which the sensors and actuators will be connected to the controllers via a communication network. The performance of the distributed engine control (DEC) system is dependent on the network performance. The network-induced time delay may degrade the performance of the closed-loop systems and even destabilize the systems if the controllers are designed without considering the effects of the delay. This paper introduces a new method to estimate the maximum tolerance of the time delay for analysis of the stability of the GE T700 turboshaft engine DEC system. The sufficient conditions for stability are derived and dynamic output feedback controllers for the turboshaft engine are applied. Hardware-in-the-loop simulation illustrates the effectiveness of the presented method.

INDEX TERMS Distributed engine control, networked control system, time delay, stability, gas turbine.

I. INTRODUCTION

With the increasing development of sophisticated electronics in gas turbine engine control systems, the increased performance, more convenient operation, and reduction of design and maintenance costs require a more effective architecture for the control systems; hence, the development of the distributed engine control (DEC) architecture [1]. The sensors and controllers are connected through communication networks [2]. For example, the GE T700 turboshaft engine is a two-spool engine consisting of a gas generator and a free power turbine [3]–[5], and the whole turboshaft engine system, combined with the control systems, can be viewed as a cascade control system (CCS) [6], [7]. In the DCS control structure, the GE T700 turboshaft engine DEC architecture can be viewed as a networked cascade control system (NCCS), and NCCS is a special case of a networked control system (NCS).

The main advantages of a NCS control structure are its modularity, simplified wiring, low cost, reduced weight, decentralization of control, integrated diagnosis, simple installation, quick and easy maintenance [8].

However, compared with the traditional point-to-point feedback closed-loop control system, the NCS may have a series of problems that are network-induced time delays and packet dropouts, network bandwidth, and security that will degrade the performance of the whole system or even destabilize the closed-loop system. Therefore, the stability of the NCS is the first and most important issue in this research field.

During the past few decades, extensive studies on network-induced imperfections have been carried out by both the control and the communication communities assuming different scenarios, and various methodologies have been proposed on how to deal with the imperfections [9]–[17].

Liu provided a method to guarantee the stability of time delay in NCSs by estimating the stability bound of the delay decay rate α and upper bound delay time τ, and based on the Lyapunov–Krasovskii method and linear matrix inequality (LMI), derived exponential stability [18]. Based on a switched linear systems approach, Donkers et al. proposed a new procedure to obtain a convex over-approximation in the form of a polytopic system with norm-bounded additive uncertainty [19]. To obtain far fewer conservative stability conditions, for the uncertain networked NCS, Wei et al. analyzed α-stability constraints subject to disturbance inputs. The sufficient condition for feasibility was presented in term of Lyapunov stability theory and a set of LMIs [20]. Moreover, most results in [21] were derived for the constant-delay case.

Despite the many methods to analyze the stability of a NCS presented in the literature, the less-conservative methods are still a problem in stability analysis for NCCSs [22], [23]. In this paper, a new stability analysis approach is presented and applied to a typical turboshaft DEC system. The method can be used as an approach for stability analysis and as DEC design guide. The paper gives a description of the proposed method for estimating the maximum time delay for a NCCS. The proposed method takes both time delays from sensor to controller and from controller to the plant input into consideration. Hardware-in-the-loop (HIL) simulation is illustrated and the results are compared with those proposed in the literature published previously. The rest of the paper is organized as follows: the architecture of the DEC system is thoroughly described in the next section. A NCCS model of a GE T700 turboshaft engine is established and the new stability analysis tool is provided based on Lyapunov stability theory in the following section. HIL simulation example is presented to illustrate the effectiveness of the approach in simulation results section. The conclusion is drawn in the final section.

II. DISTRIBUTED ARCHITECTURE OF THE GE T700 TURBOSHAFT ENGINE

A. ENGINE DESCRIPTION

This study utilized a GE T700 turboshaft engine. Figs 1 and 2 show simplified diagrams and Table 1 presents the abbreviations of the engine parameters.

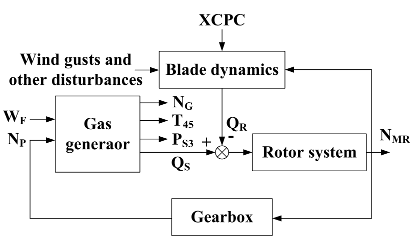

FIGURE 1. Block diagram of open-loop gas generator and rotor system.

B. MODEL DESCRIPTION

The controller design process begins with a linearized, state-space model of the system. Fig. 2 shows the simplified model in this case.

1) PRIMARY PLANT

The state-space representation of the rotor system is provided by the following equation:

$$\begin{cases} \dot{x}_1(t) = A_1 x_1(t) + B_1 y_2(t) \\ y_1(t) = C_1 x_1(t) \end{cases} \tag{1}$$

where $x_1 = \left[N_P \; N_{MR} \; Q_{MR}\right]^T$, and $y_1 = N_P$ are the state vector and the output of the rotor system, respectively.

TABLE 1. Symbols of the GE T700 turboshaft engine.

Symbols	Meaning
PLA	Power lever angle (throttle)
N_G	Gas generator speed
N_P	Power turbine speed
N_{MR}	Main rotor blade velocity
Q_{MR}	Rotor torque state
Q_S	Engine shaft torque
$XCPC$	Collective pitch
P_1	Inlet pressure
P_{S3}	Static pressure at Station 3
T_1	Inlet temperature
T_{45}	Inter-turbine gas temperature
W_F	Fuel flow
J_G	Power turbine inertia
J_T	Lumped power turbine/dynamometer inertia
J_{MR}	Main rotor blade inertia
KMR	Stiffness of the centrifugal restoring springs
DMR	Lag hinge damping
DAM	Aero damping
r	Reference input
x	Model state vector
y	Model output vector
u	Model input vector

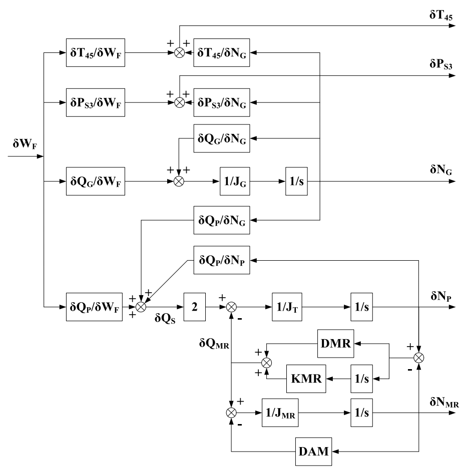

FIGURE 2. Block diagram of the simplified linearized gas generator and rotor system [7].

Here $y_2 = Q_S$ is the gas generator output. The matrices A_1, B_1, and C_1 are provided as follows:

$$A_1 = \begin{bmatrix} 0 & 0 & -\frac{1}{J_T} \\ 0 & -\dfrac{DAM}{J_{MR}} & \frac{1}{J_{MR}} \\ KMR & \dfrac{DMR \cdot DAM}{J_{MR}} - KMR & -\dfrac{DMR}{J_T} - \dfrac{DMR}{J_{MR}} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} \frac{2}{J_T} \\ 0 \\ \dfrac{2 \cdot DMR}{J_T} \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

2) SECONDARY PLANT

The continuous-time linear model of the gas generator is shown as follows:

$$\begin{cases} \dot{x}_2(t) = A_2 x_2(t) + B_2 u(t) \\ y_2(t) = C_2 x_2(t) \end{cases} \tag{2}$$

where $x_2 = \left[N_G\ Q_S\ T_{45}\ P_{S3}\ N_P\right]^T$, $y_2 = Q_S$ are the state and output vectors, and $u(t) = W_F$ is the gas generator input. The matrices A_2, B_2, and C_2 are presented as follows:

$$A_2 = \begin{bmatrix} \frac{1}{J_G}\cdot\frac{\delta Q_G}{\delta N_G} & 0 & 0 & 0 & 0 \\ \frac{2\cdot DMR}{J_T}\cdot\frac{\delta Q_P}{\delta N_G} & 0 & 0 & 0 & \frac{2\cdot DMR}{J_T}\cdot\frac{\delta Q_P}{\delta N_P} \\ \frac{\delta T_{45}}{\delta N_G} & 0 & 0 & 0 & 0 \\ \frac{\delta P_{S3}}{\delta N_G} & 0 & 0 & 0 & 0 \\ \frac{2}{J_T}\cdot\frac{\delta Q_P}{\delta N_G} & -\frac{1}{J_T} & 0 & 0 & \frac{2}{J_T}\cdot\frac{\delta Q_P}{\delta N_P} \end{bmatrix},$$

$$B_2 = \begin{bmatrix} \frac{1}{J_G}\cdot\frac{\delta Q_G}{\delta W_F} \\ \frac{2\cdot DMR}{J_T}\cdot\frac{\delta Q_P}{\delta W_F} \\ \frac{\delta T_{45}}{\delta W_F} \\ \frac{\delta P_{S3}}{\delta W_F} \\ \frac{2}{J_T}\cdot\frac{\delta Q_P}{\delta W_F} \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

III. STABILITY ANALYSIS

Fig. 3 shows the NCCS architecture.

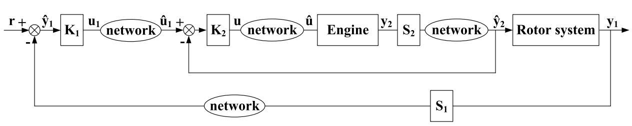

FIGURE 3. Block diagram of the engine NCCS model.

As the figure shows, the networked-induced system parameters are

$$\begin{cases} \dot{x}_1 = A_1 x_1 + B_1 \hat{y}_2 \\ y_1 = C_1 x_1 \end{cases} \tag{3}$$

$$\begin{cases} \dot{x}_2 = A_2 x_2 + B_2 \hat{u} \\ y_2 = C_2 x_2 \end{cases} \tag{4}$$

The dynamic controllers are given by

$$\begin{cases} \dot{x}_{C_1} = A_{C_1} x_{C_1} + B_{C_1} \hat{y}_1 \\ u_1 = C_{C_1} x_{C_1} + D_{C_1} \hat{y}_1 \end{cases} \tag{5}$$

$$\begin{cases} \dot{x}_{C_2} = A_{C_2} x_{C_2} + B_{C_2} \hat{u} \\ u = C_{C_2} x_{C_2} + D_{C_2} \hat{u} \end{cases} \tag{6}$$

where $\hat{u} = \hat{u}_1 - \hat{y}_2$, and x_{C_1}, x_{C_2}, u_1, and u are the states and outputs of the controllers respectively. Here $(A_{C_1}, B_{C_1}, C_{C_1}, D_{C_1})$ and $(A_{C_2}, B_{C_2}, C_{C_2}, D_{C_2})$ are real constant matrices with corresponding dimensions.

It is assumed that the non-networked closed feedback system presented by (7), as shown at the top of the next page, is globally exponentially stable.

Thus, there exists a P_{11} such that

$$A_{11}^T P_{11} + P_{11} A_{11} = -I \tag{8}$$

Walsh *et al.* [8] provided two scheduling methods, try-once-discard (TOD) and token-ring-type static scheduling, and under the maximum allowable transfer interval (MATI) [21] constraint, the result is given in the following theorem for both methods that preserve the stability of the closed-loop system.

Theorem 1 (Walsh et al. [8]): Given a NCCS (as shown in Eqs (1) and (2)) with p sensor nodes under a static scheduling, define $\lambda_1 = \lambda_{min}(P_{11})$, $\lambda_2 = \lambda_{max}(P_{11})$, and a MATI, τ_m, which satisfies (9), as shown at the top of the next page, then the NCCS is globally exponentially stable, where A_{cl} is showed in (10), as shown at the top of the next page, and

$$\begin{cases} e_{y_1} = y_1 - \hat{y}_1 \\ e_{y_2} = y_2 - \hat{y}_2 \\ e_{u_1} = u_1 - \hat{u}_1 \\ e_u = u - \hat{u} \end{cases} \tag{11}$$

However, numerical examples show that by using Theorem 1, the bound of the MATI is very conservative to guarantee the stable behavior [17].

To reduce this conservation and meanwhile guarantee the stable behavior, this study proposes a method for estimating the less-conservative bound of MATI. The proposed method takes both time delays from sensor to controller and from controller to the plant input into consideration. Suppose that the control signals are connected to the control plant through a kind of network, so the time delay from the controller output to the plant (τ_{cp}) is inevitably involved in the feedback loop. In addition, there is time delay between the measured feedback signals to the controller input (τ_{sc}). Fig. 4 shows a high-level description for a network modeled as time delay.

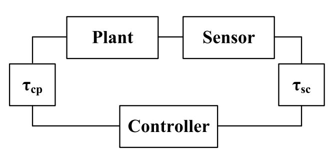

FIGURE 4. The network effect modeled as pure time delay [17].

Therefore, the total time delay of the closed-loop system can be calculated using

$$\tau = \tau_{cp} + \tau_{sc} \tag{12}$$

and making the following assumptions that are partially taken from [22].

- Sensors are time driven and controllers are event trigged.
- The data are transmitted as a single packet.
- All the states are available for measurements and for transmission.

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_{C_1} \\ \dot{x}_{C_2} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} A_1 & B_1C_2 & 0 & 0 \\ B_2D_{C_2}D_{C_1}C_1 & A_2 - B_2D_{C_2}C_2 & B_2D_{C_2}C_{C_1} & B_2C_{C_2} \\ B_{C_1}C_1 & 0 & A_{C_1} & 0 \\ B_{C_2}D_{C_1}C_1 & -B_{C_2}C_2 & B_{C_2}C_{C_1} & A_{C_2} \end{bmatrix}}_{A_{11}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_{C_1} \\ x_{C_2} \end{bmatrix}}_{x} \tag{7}$$

$$\tau_m < \min \left\{ \frac{\ln(2)}{p\|A_{cl}\|}, \frac{1}{8\|A_{cl}\|(\sqrt{\frac{\lambda_2}{\lambda_1}}+1)\sum_{i=1}^{p} i}, \frac{1}{16\lambda_2\sqrt{\frac{\lambda_2}{\lambda_1}}\|A_{cl}\|^2(\sqrt{\frac{\lambda_2}{\lambda_1}}+1)\sum_{i=1}^{p} i} \right\} \tag{9}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_{C_1} \\ \dot{x}_{C_2} \\ \dot{e}_{y_1} \\ \dot{e}_{y_2} \\ \dot{e}_{u_1} \\ \dot{e}_{u} \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & B_1C_2 & 0 & 0 & 0 & -B_1 & 0 & 0 \\ B_2D_{C_2}D_{C_1}C_1 & A_2 - B_2D_{C_2}C_2 & B_2D_{C_2}C_{C_1} & B_2C_{C_2} & -B_2D_{C_2}D_{C_1} & B_2D_{C_2} & -B_2D_{C_2} & 0 \\ B_{C_1}C_1 & 0 & A_{C_1} & 0 & -B_{C_1} & 0 & 0 & 0 \\ B_{C_2}D_{C_1}C_1 & -B_{C_2}C_2 & B_{C_2}C_{C_1} & A_{C_2} & -B_{C_2}D_{C_1} & B_{C_2} & -B_{C_2} & 0 \\ C_1A_1 & C_1B_1C_2 & 0 & 0 & 0 & -C_1B_1 & 0 & 0 \\ C_2B_2D_{C_2}D_{C_1}C_1 & C_2A_2 - C_2B_2C_2 & C_2B_2D_{C_2}C_{C_1} & C_2B_2C_{C_2} & -C_2B_2D_{C_1} & C_2B_2 & -C_2B_2D_{C_2} & -C_2B_2 \\ C_{C_1}B_{C_1}C_1 & 0 & C_{C_1}A_{C_1} & 0 & -C_{C_1}B_{C_1} & 0 & 0 & 0 \\ D_{C_2}D_{C_1}C_1 & -D_{C_2}C_2 & D_{C_2}C_{C_1} & C_{C_2} & -D_{C_2}D_{C_1} & D_{C_2} & -D_{C_2} & 0 \end{bmatrix}}_{A_{cl}}$$

$$\times \begin{bmatrix} x_1 \\ x_2 \\ x_{C_1} \\ x_{C_2} \\ e_{y_1} \\ e_{y_2} \\ e_{u_1} \\ e_{u} \end{bmatrix} \tag{10}$$

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_{C_1} \\ \dot{x}_{C_2} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & B_2D_{C_2}C_{C_1} & B_2C_{C_2} \\ 0 & 0 & A_{C_1} & 0 \\ 0 & 0 & B_{C_2}C_{C_1} & A_{C_2} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_{C_1} \\ x_{C_2} \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 & D_1C_2 & 0 & 0 \\ B_2D_{C_2}D_{C_1}C_1 & B_2D_{C_2}C_2 - B_2C_2 & 0 & 0 \\ B_{C_1}C_1 & 0 & 0 & 0 \\ B_{C_2}D_{C_1} & -B_{C_2}C_2 & 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} x_1(t-\tau_{sc}) \\ x_2(t-\tau_{sc}) \\ x_{C_1}(t-\tau_{sc}) \\ x_{C_2}(t-\tau_{sc}) \end{bmatrix}}_{x(t-\tau_{sc})} \tag{13}$$

First, a simplified analysis is discussed here by considering only the sensors-to-controllers time delay, τ_{sc}. In this model, all the delays and dropouts are lumped between the sensors and controllers. Applying the dynamic controllers proposed in (5) and (6) to the systems (3) and (4), and rewriting the systems showed in (13), as shown at the top of this page.

where $\hat{y}_1 = C_1x_1(t-\tau_{sc})$, $\hat{y}_2 = C_2x_2(t-\tau_{sc})$, $\hat{u}_1 = C_{C_1}x_{C_1} + D_{C_1}C_1x_1(t-\tau_{sc})$, and $\hat{u} = C_{C_2}x_{C_2} + D_{C_2}\hat{u}_1 - D_{C_2}C_2x_2(t-\tau_{sc})$.

Theorem 2: For both systems (3) and (4) with the dynamic controllers proposed in (5) and (6), the closed-loop system is globally asymptotically stable if $\lambda_i(\Phi) \in \mathbb{C}^-$, for $i = 1, 2, \ldots, n$ and all the state variables' second-order reminders are small enough for the given value of τ_{sc}, where Φ is showed in (14), as shown at the top of the next page.

Proof: The Taylor expression for $x(t-\tau_{sc})$ is

$$x(t-\tau_{sc}) = x(t) - \tau_{sc}\dot{x}(t) + \mathbf{R}_2(x, \tau_{sc}) \tag{15}$$

where $\mathbf{R}_2(x, \tau_{sc})$ is all the state variables' second-order reminders, and depends on the time delay, τ_{sc}, and the higher-order derivatives of $x(t)$. If τ_{sc} is small enough, then $\mathbf{R}_2(x, \tau_{sc})$ is small enough to be ignored. Then (15) can be rewritten as

$$x(t-\tau_{sc}) - x(t) \cong -\tau_{sc}\dot{x}(t) \tag{16}$$

Substituting (16) into (13), the closed-loop system can be derived as

$$\dot{x}(t) \cong (A+B)x(t) - \tau_{sc}B\dot{x}(t) \tag{17}$$

where (17) can be rewitten in (18), as shown at the top of the next page.

The closed-loop system will be globally asymptotically stable if

$$\lambda_i(\Phi) \in \mathbb{C}^-, \quad \text{for } i = 1, 2, \ldots, n \tag{19}$$

Then, the controllers to the plant time delay, τ_{cp}, is taken into consideration. Applying the dynamic controllers

$$\Phi = \frac{\begin{bmatrix} A_1 & D_1C_2 & 0 & 0 \\ B_2D_{C_2}D_{C_1}C_1 & A_2+B_2D_{C_2}C_2-B_2C_2 & B_2D_{C_2}C_{C_1} & B_2C_{C_2} \\ B_{C_1}C_1 & 0 & A_{C_1} & 0 \\ B_{C_2}D_{C_1} & -B_{C_2}C_2 & B_{C_2}C_{C_1} & A_{C_2} \end{bmatrix}}{I+\tau_{sc}\begin{bmatrix} 0 & D_1C_2 & 0 & 0 \\ B_2D_{C_2}D_{C_1}C_1 & B_2D_{C_2}C_2-B_2C_2 & 0 & 0 \\ B_{C_1}C_1 & 0 & 0 & 0 \\ B_{C_2}D_{C_1} & -B_{C_2}C_2 & 0 & 0 \end{bmatrix}} \tag{14}$$

$$\begin{aligned}\dot{x}(t) &\cong \frac{A+B}{I+\tau_{sc}B}x(t) \\ &= \underbrace{\frac{\begin{bmatrix} A_1 & D_1C_2 & 0 & 0 \\ B_2D_{C_2}D_{C_1}C_1 & A_2+B_2D_{C_2}C_2-B_2C_2 & B_2D_{C_2}C_{C_1} & B_2C_{C_2} \\ B_{C_1}C_1 & 0 & A_{C_1} & 0 \\ B_{C_2}D_{C_1} & -B_{C_2}C_2 & B_{C_2}C_{C_1} & A_{C_2} \end{bmatrix}}{I+\tau_{sc}\begin{bmatrix} 0 & D_1C_2 & 0 & 0 \\ B_2D_{C_2}D_{C_1}C_1 & B_2D_{C_2}C_2-B_2C_2 & 0 & 0 \\ B_{C_1}C_1 & 0 & 0 & 0 \\ B_{C_2}D_{C_1} & -B_{C_2}C_2 & 0 & 0 \end{bmatrix}}}_{\Phi} x(t)\end{aligned} \tag{18}$$

$$\begin{cases} \dot{x}_1 = A_1x_1 + D_1C_2x_2(t-\tau_{sc}-\tau_{cp}) \\ \dot{x}_2 = A_2x_2 + B_2C_2x_{C_2}(t-\tau_{cp}) + B_2D_{C_2}C_{C_1}x_{C_1}(t-\tau_{cp}) \\ \qquad +B_2D_{C_2}D_{C_1}C_1x_1(t-\tau_{sc}-\tau_{cp}) + B_2D_{C_2}C_2x_2(t-\tau_{sc}-\tau_{cp}) \\ \qquad -B_2C_2x_2(t-\tau_{sc}-\tau_{cp}) \\ \dot{x}_{C_1} = A_{C_1}x_{C_1} + B_{C_1}C_1x_1(t-\tau_{sc}) \\ \dot{x}_{C_2} = A_{C_2}x_{C_2} + B_{C_2}C_{C_1}x_{C_1} + B_{C_2}D_{C_1}x_1(t-\tau_{sc}) - B_{C_2}C_2x_2(t-\tau_{sc}) \end{cases} \tag{20}$$

$$\Psi = \frac{\begin{bmatrix} A_1 & D_1C_2 & 0 & 0 \\ B_2D_{C_2}D_{C_1}C_1 & A_2+B_2D_{C_2}C_2-B_2C_2 & B_2D_{C_2}C_{C_1} & B_2C_{C_2} \\ B_{C_1}C_1 & 0 & A_{C_1} & 0 \\ B_{C_2}D_{C_1} & -B_{C_2}C_2 & B_{C_2}C_{C_1} & A_{C_2} \end{bmatrix}}{I+\begin{bmatrix} 0 & (\tau_{sc}+\tau_{cp})D_1C_2 & 0 & 0 \\ (\tau_{sc}+\tau_{cp})B_2D_{C_2}D_{C_1}C_1 & (\tau_{sc}+\tau_{cp})B_2D_{C_2}C_2-\tau_{cp}B_2C_2 & 0 & 0 \\ \tau_{sc}B_{C_1}C_1 & 0 & 0 & 0 \\ \tau_{sc}B_{C_2}D_{C_1} & -\tau_{sc}B_{C_2}C_2 & 0 & 0 \end{bmatrix}} \tag{21}$$

proposed in (5) and (6) to the systems (3) and (4), and rewriting the systems as in (20), as shown at the top this page.

Theorem 3: For the systems (3) and (4) with the dynamic controllers proposed in (5) and (6), the closed-loop system is globally asymptotically stable if $\lambda_i(\Psi) \in \mathbb{C}^-$, for $i = 1, 2, \ldots, n$ and all the state variables' second-order reminders are small enough for the given values of τ_{sc} and τ_{cp}, where Ψ is shown in (21), as shown at the top this page.

Proof: Straightforward as in Theorem 2.

IV. SIMULATION RESULTS

This section presents the effectiveness evaluation of the proposed analysis method under hardware-in the-loop simulation in the GE T700 turboshaft gas turbine engine DEC control systems, and the coefficient matrices of rotor system are provided as follows:

$$A_1 = \begin{bmatrix} 0 & 0 & -285.7143 \\ 0 & -0.4533 & 9.0662 \\ 5.2650 & -5.2131 & -42.5958 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 571.4286 \\ 0 \\ 82.5714 \end{bmatrix}.$$

and the corresponding dynamic output feedback controller is:

$$A_{C_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_{C_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_{C_1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \quad D_{C_1} = -2.3760 \times 10^{-13}.$$

FIGURE 5. Hardware-in the-loop system: monitors, gas generator, and rotor system models.

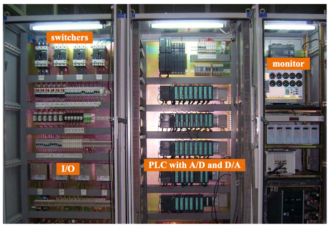

FIGURE 6. Hardware-in the-loop system: DEC system.

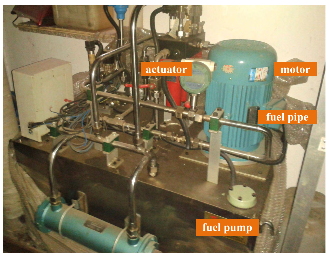

FIGURE 7. Hardware-in the-loop system: actuator and fuel supply system.

The coefficient matrices of gas generator model are given as:

$$A_2 = \begin{bmatrix} -126.8 & 27.04 & 12.36 & 22.17 & 16.72 \\ 54.67 & 57.21 & -77.02 & -76.21 & 50.81 \\ -336.6 & 223.3 & -130.7 & -83.32 & 172.1 \\ 161.2 & 2.459 & -21.8 & -63.09 & 1.799 \\ 62.42 & -73.55 & -104.2 & -91.44 & -102.3 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -11.7 \\ 44.24 \\ 53.56 \\ 17.45 \\ 59.35 \end{bmatrix}.$$

and the corresponding dynamic output feedback controller is:

$$A_{C_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_{C_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_{C_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_{C_2} = -6.4524 \times 10^{-9}.$$

The DEC system in this simulation was tested by using a HIL simulation testbed in Figs 5, 6, and 7 [7].

First, from several simulations, the unstable MATI of the closed-loop system is $\tau_m > 0.0461$ s, as shown in Figs. 8 and 9.

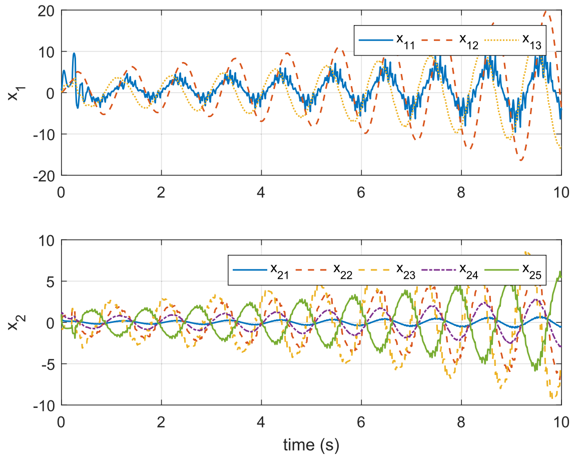

FIGURE 8. Unstable behavior of the closed-loop system for $\tau_m > 0.0461$ s (x).

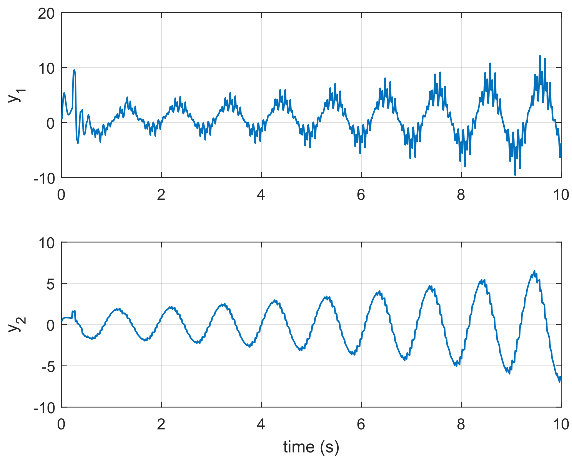

FIGURE 9. Unstable behavior of the closed-loop system for $\tau_m > 0.0461$ s (y).

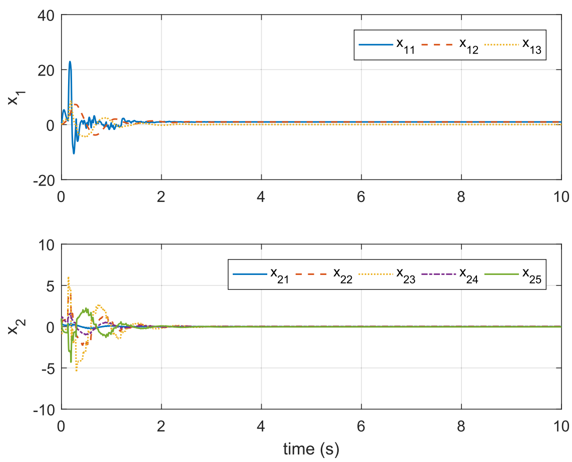

FIGURE 10. Stable behavior of the closed-loop system for $\tau_m < 0.0454$ s (x).

When using Theorem 1, the bound of MATI can be calculated as $\tau_m = 0.3614 \times 10^{-5}$ s, which is very conservative to guarantee the stable behavior of the closed-loop system. However, when using Theorem 3, $\tau_m = 0.0454$ s, which is

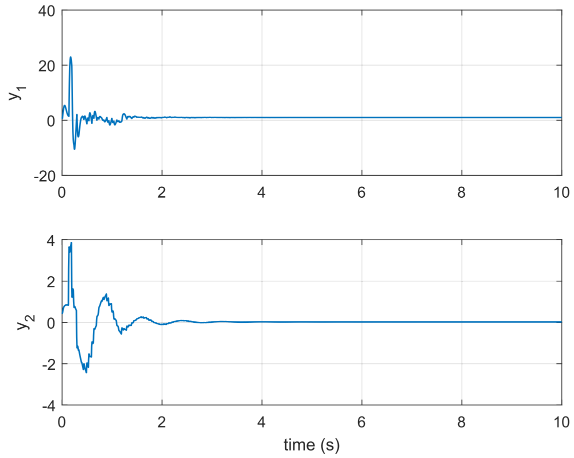

FIGURE 11. **Stable behavior of the closed-loop system for $\tau_m < 0.0454$ s (y).**

very close to the actual bound of stability ($\tau_m < 0.0461$ s). The stable behaviors of the closed-loop system are shown in Figs. 10 and 11.

As Figs. 10 and 11 show, when MATI is less than 0.00454 s, the closed-loop system can be globally asymptotically stable. That is, the method provided in this study gives less-conservative results than those in the literature published previously.

V. CONCLUSION

In this study, a new method has been derived for estimating the maximum time delay in a DEC system for an output feedback control architecture. A partially DCS architecture of a typical turboshaft engine has also been described. This distributed architecture can be transformed into a NCCS. The proposed method is simple for analyzing the stability of the networked control structure, and its advantages include using less computation time and producing less-conservative results. Hardware-in-the-loop simulation examples have been provided to show the effectiveness of the approach.

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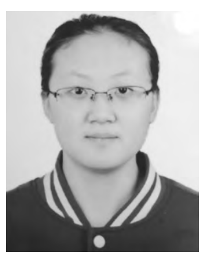

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