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Optimal Design of the *k*-Out-of-*n*: G (F) Majority Voter

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ABSTRACT The k-out-of-n: G (F) majority voter usually consists of n components (modules), and such a system is critical to ensure the correct operation of various computing systems for numerous critical applications. For a k-out-of-n: G (F) majority voter, a specific number of the components are required to operate correctly for the overall system to function. To deal efficiently with the reliability evaluation of a general majority voter, a stochastic architecture can be adopted. The corresponding system reliability can be obtained through analyzing the output sequence. Usually, the system reliability is improved if more components or redundancies are used. Nevertheless, the consumed cost or required space also increases accordingly. In this paper, a tradeoff between the cost and reliability value was made to pursue the most desirable design. The relationship between the cost and corresponding component parameters is also discussed thoroughly in this paper. Then, to find the most cost-effective design, a new evaluation standard was proposed, referred to as the *R_per_Cost*. Furthermore, the optimal designs under different standards are presented for the investigated example. The results are also pursued with respect to an analysis of several case studies.

INDEX TERMS Reliability evaluation, optimization, k-out-of-n: G (F) majority voter, cost-effective design.

I. INTRODUCTION

Today, the investigation of a k-out-of-n: G (F) majority voter is even more popular as it is very important to ensure the correct operation of various computing systems for numerous critical applications. Furthermore, the majority voter is widely applied in both industrial and military systems such as the multiengine system in an airplane, and the multi-pump system in a hydraulic control system [1]. Usually, the output performance distribution (OPD) of a k-out-of-n: G (F) majority voter is of great interest, and the prediction of the OPD has been extensively investigated in the technical literature [2]–[5]; where in particular, the reliability value is calculated to reflect the OPD. For the k-out-of-n: G majority voter, this consists of n components (modules or units), and the system operates correctly if the minimal total weight of all the good (fault-free, or correct) components is not less than the pre-provided threshold value k. As long as the reliability of the k-out-of-n: G system is found, then the unreliability of a $(w-k+1)$-out-of-n: F system can be easily determined where w denotes the total system weight; hence, the performance evaluation of a k-out-of-n: F majority voter is able to transform into the analysis of a $(w-k+1)$-out-of-n: G majority voter. For example, it may be possible to drive a car with a V8 engine if only four cylinders are firing. Thus, the functioning of the engine is indicated by a 4-out-of-8: G system. Nevertheless, the automobile cannot be driven if less than four cylinders fire (i.e., at least five cylinders are disabled). Then, the system can also be described as a 5-out-of-8: F system [6]. In this paper, without loss of generality, the reliability of a k-out-of-n: G system was mainly investigated.

Redundancy is usually added into a system for the purpose of achieving a higher reliability value, for instance, spare gate and majority voter [7]–[9]. For the spare gate, standby modules are critical for tolerating the failures, and fault tolerance is achieved by removing the faulty module from operation and replacing it with a spare unit [8]. Nevertheless, for a k-out-of-n: G system, module copies are critical for tolerating hardware failures or software errors; fault tolerance is achieved as long as the minimal total weight of all the good components is not less than the pre-provided threshold value k. Various approaches have been used to predict the corresponding reliability of a k-out-of-n: G system such as inclusion-exclusion [10], Markov analysis [11], Recursive algorithm [3], and Monte Carlo (MC) [12]. Recently, stochastic computational approaches using random binary bit streams have been proposed for the reliability analysis of logic circuits and dynamic fault tree (DFT) analysis [13]–[16]. As shown in [13], signal correlations are inherently preserved in stochastic sequences; hence, repeated components are readily accounted. Furthermore, the general case of non-exponential distributions was addressed efficiently with the adoption of a stochastic approach [13]. In [15], a stochastic analysis was performed to investigate the reliabilities of a 2-out-of-3 binary-state majority and 3-out-of-5 majority voters. In [17], stochastic architecture was proposed for the efficient analysis of a weighted binary-state majority voter. It has also been shown that the use of stochastic sequences led to an efficient, accurate evaluation with the adoption of the presented stochastic architecture. The stochastic approach has also been shown to be able to analyze systems consisting of non-exponentially distributed components.

Traditionally, it has been thought that the system achieves higher reliability with the increase of spare components or redundancies. The process of adding spare components proceeds for as long as the quantity of anticipated spares is not reached (for instance, space limit, module number limit, etc.). Nevertheless, with the increase of the number of spare modules, the corresponding consumed cost and space also grows accordingly. Furthermore, the amount of increased reliability is negligible as long as the number of modules reaches a certain amount. Thus, it is not cost effective to keep adding on modules. Hence, we are more interested in designing a system with a desirable reliability value and acceptable cost; therefore, a trade-off between the cost and reliability value needs to be made [18], and in [19], the optimal design of a k-out-of-n binary state majority voter was pursued.

This paper focuses on pursuing the optimal design of a general k-out-of-n majority voter (either binary or multi-state) with the incorporation of cost, including the operating cost factor to determine the optimal design. To evaluate the various designs, a new evaluation standard is also presented to find the most cost-effective design. Stochastic models are provided for the reliability calculation of a general majority voter. To improve the efficiency of determining the reliability value, stochastic analysis can be performed. Hence, in this paper, the stochastic model for a multi-state majority voter is also presented. Then, both exponential and non-exponential (e.g., Weibull) distributions can be efficiently addressed. As the optimal design is largely affected by the value of corresponding parameters, the relationships of different parameters and the obtained optimal design are discussed thoroughly in this paper.

II. ASSUMPTIONS

Some assumptions were made for the components of the investigated system in this paper.

1) For a k-out-of-n binary state majority gate, the subsystem was operational if and only if the total number of working component was at least k. For a multi-state majority voter, the sum of the states for the working components should be no less than k, which is a predefined specified threshold.

2) All components were operational at the beginning of the mission time due to the reason that all failed components of a critical system are definitely repaired or replaced prior to each mission.

3) All states of n components were mutually s-independent.

4) During the specified mission time, the components were assumed to be non-repairable [20]. This indicated that once a component failed during the mission time, the failure state of the component remained until the end of the mission time.

III. A GENERAL COMPUTATIONAL MODEL FOR K-OUT-OF-N MAJORITY VOTER

Stochastic computation was initially proposed in the 1960s for reliable circuit design [21]. For a binary-state majority voter, each component has two exclusive states: working and failure, indicated by 0 and 1, respectively. Signal probabilities are encoded into random binary bit streams by setting a proportional number of bits to a specific value, i.e., one or zero. The non-Bernoulli sequence is defined in [13] for performing stochastic analysis. Through stochastic logic, Boolean logic operations are transformed into probabilistic computations in the real domain. The stochastic computational approach has various attractive properties such as hardware simplicity, fault tolerance, high accuracy, high efficiency, and the ability to deal with repeated components. The typically emerged repeated/dependent relationships are inherently dealt with through the adoption of a stochastic approach as stochastic computing techniques efficiently handle the problem of signal re-convergence.

For a certain signal with a discretization level of m, the probability vector is usually described as $P = [p_{\_m-1} \cdots p_{\_0}]$, with $\sum_{h=0}^{m-1} p_{\_h} = 1$. A probability vector of a binary signal can be indicated by stochastic sequence through non-Bernoulli encoding as illustrated in Fig. 1 [14].

$$\text{" 0100010101"} \quad \text{for} \quad \begin{cases} p_{\_0} = 0.6 \\ p_{\_1} = 0.4 \end{cases}$$

FIGURE 1. An illustration of the encoding process of a binary probability vector given a sequence length of 10 bits.

As shown in [17], a multi-valued equal or larger (MVEL) operator performs the functions:

$$MVEL(A \geq k) = \begin{cases} 1, & A \geq k \\ 0, & A < k. \end{cases} \tag{1}$$

An illustration example is presented in Fig. 2 for a multi-valued equal or larger (MVEL) operator with a provided threshold k ($k = 2$).

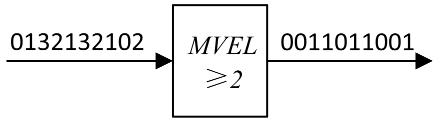


FIGURE 2. The stochastic example for a multi-valued equal or larger (MVEL) operator.

As analyzed in [15], a 2-out-of-3 majority voter can be implemented through the application of stochastic logic gates. The corresponding relationship among the components in the system is indicated by the combinations of the logic gates. Here, the stochastic architecture for a k-out-of-n majority voter is presented in Fig. 3.

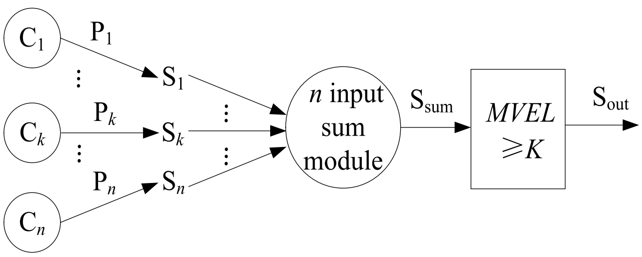


FIGURE 3. A stochastic model for a general k-out-of-n binary-state majority voter, $1 < k < n$.

For a binary state majority voter, the input signal probabilities P_k of components C_k are encoded as non-Bernoulli sequences according to the encoding process illustrated in Fig. 1. Next, the output sequence can be obtained by propagating the stochastic sequences in the stochastic model in Fig. 3. Then, an output sequence can be obtained at a corresponding output. Thus, the output signal probability can be predicted by analyzing the output sequence.

As indicated by the analysis in [15], the presented stochastic model is capable of predicting the reliability of a majority voter approximately. The corresponding stochastic fluctuation of the stochastic approach decreases with an increase in sequence length. By utilizing an appropriate sequence length, the reliability of a general k-out-of-n majority voter is able to be effectively and accurately predicted. Furthermore, the reliability evaluation of a system consisting of components with any failure distributions can be efficiently addressed through stochastic analysis. Thus, if a reliability evaluation is to be performed for a general majority voter, then a stochastic computational approach can be adopted.

Moreover, in practice, the components of a system might suffer from common cause failures (CCFs) that might be incurred by earthquakes, suddenly changes in the environment, design errors, and incorrect operations [22]. The existence of common cause failure is likely to affect parts or all of the components in the system. Here, by adopting the stochastic model provided, the effects of common cause failure are also easily addressed through stochastic analysis.

TABLE 1. Parameters for different scenarios for the purpose of performing cost analysis (here, 1 unit = 1000 dollars).

Parameters	Scenario 1	Scenario 2
c_c	1 unit	2 unit
c_s	100 units	200 units
λ	0.0001/h	0.0005/h
t	1000 hours	2000 hours

IV. SYSTEM OPTIMIZATION BY CONSIDERING COST

As discussed in Section I, though system reliability is supposed to improve with the adoption of more spare components or redundancies, the corresponding consumed cost and space also grows accordingly. Furthermore, the amount of increased reliability is negligible if the number of modules reaches a certain amount. Then, a trade-off between the cost and reliability value should be made. In this paper, three parameters were primarily investigated, i.e., the cost of components, the system failure cost, and the total cost.

For a k-out-of-n majority voter, the system reliability is calculated as

$$R = \sum\nolimits_{i=k}^{n} C_n^i (R_c)^i (1 - R_c)^{n-i} \tag{2}$$

where R and R_c denote the reliabilities of the majority and corresponding component, respectively. Here, n represents the number of components in the investigated system (this is also applicable to (3)).

For each component with any failure distribution such as exponential and Weibull distributions, R_c can be easily determined through various analysis approaches. For instance, if the failure of a component is exponentially distributed, then the corresponding reliability is obtained as $R_c = e^{-\lambda t}$ according to the corresponding *cdf*, where λ and t indicate the failure rate of each component and the investigated mission time, respectively. If the component failures are non-exponentially distributed, then corresponding reliabilities can be determined by adopting the corresponding *cdfs*. Aside from accurate analysis, the stochastic architecture presented in Section II can also be adopted for performing reliability evaluation.

Once the system reliability has been obtained, the calculation processes of the other interested parameters are presented as follows [19]:

$$\begin{cases} C = c_c * n \\ S = c_s * (1 - R) \\ T = S + C \end{cases} \tag{3}$$

where C means the cost of components; S denotes the system failure cost; and T represents the total cost. Here, c_c and c_s

TABLE 2. System reliability and cost for scenario 1.

Number of components (n)	3	4	**5**	6	7	8	9
Reliability	0.7408	0.9523	**0.9926**	0.9989	0.99986	0.99998	0.999998
Cost of components	3	4	**5**	6	7	8	9
System failure cost	25.92	4.769	**0.743**	0.105	0.014	0.00175	0.00021
Total cost	28.92	8.769	**5.743**	6.105	7.014	8.00175	9.00021

TABLE 3. System reliability and cost for scenario 2.

Number of components (n)	3	4	5	15	**16**	17	18
Reliability	0.0498	0.1442	0.2636	0.9534	**0.9669**	0.9766	0.9836
Cost of components	6	8	10	30	**32**	34	36
System failure cost	190.04	171.16	147.29	9.3123	**6.6224**	4.6792	3.2870
Total cost	196.04	179.16	157.29	39.3123	**38.6224**	38.6792	39.2870

denote the cost of each component and the cost of a system failure, respectively.

The parameters for cost analysis of the investigated k-out-of-n majority voter are presented in Table 1. Two different scenarios with different values for the parameters (i.e., Scenario 1 and Scenario 2) are provided for the analysis in this paper. By applying (3), the target values of interest such as system failure cost and total cost can be determined accordingly. The obtained values for different scenarios are shown in Tables 2 and 3, respectively.

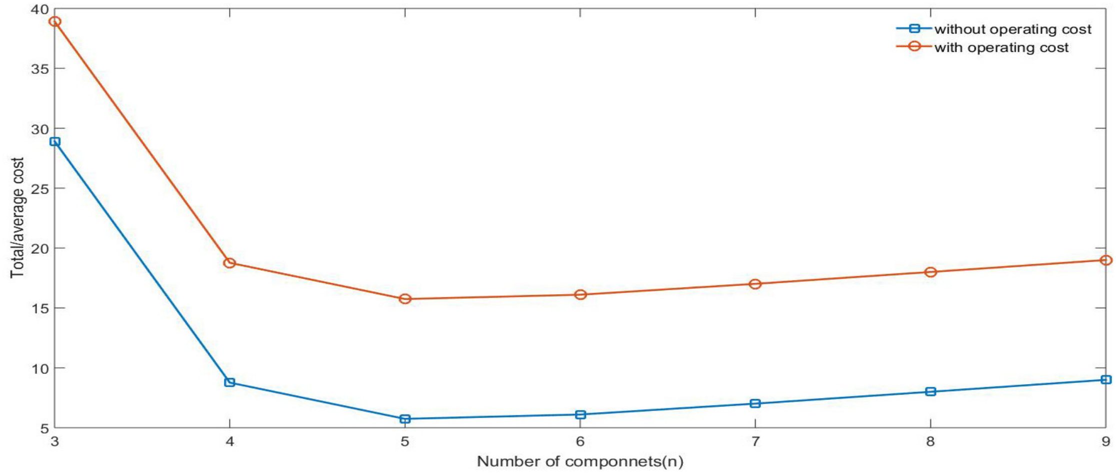
As indicated by the results in Tables 2 and 3, system reliability was improved with the increase of the number of components for the investigated system. The value of the system cost for the investigated voter also increased accordingly as more components were incorporated in the system. This is due to the fact that system cost is proportional to the number of components. As shown in Tables 2 and 3, the cost of system failure decreased with an increase in the number of components. This is due to the reason that as more components are incorporated in the system, system reliability increases. As the summation of the reliability and system failure probability always equals 1, the probability of system failure decreases with an increase in components. Then, by multiplying the factor of c_s, the system failure cost always decreases if more components are utilized. For the total cost of a system, first, the corresponding value decreases, then increases after reaching the lowest point; thus, there exists an optimal design which is a tradeoff between reliability and consumed cost, as is shown in Tables 2 and 3.

Hence, we can conclude that, in practice, more components is definitely not the more desirable design. For Scenario 1 of the investigated system, the case with five components was likely to incur the lowest total cost, while for Scenario 2, the design with a total number of 16 components was the most desirable. Thus, for the investigated system with the provided parameters, we could keep adding components into the system as long as the total cost decreased, and the adding process should stop if the total cost begins to increase. This then is regarded as the design with a desirable cost.

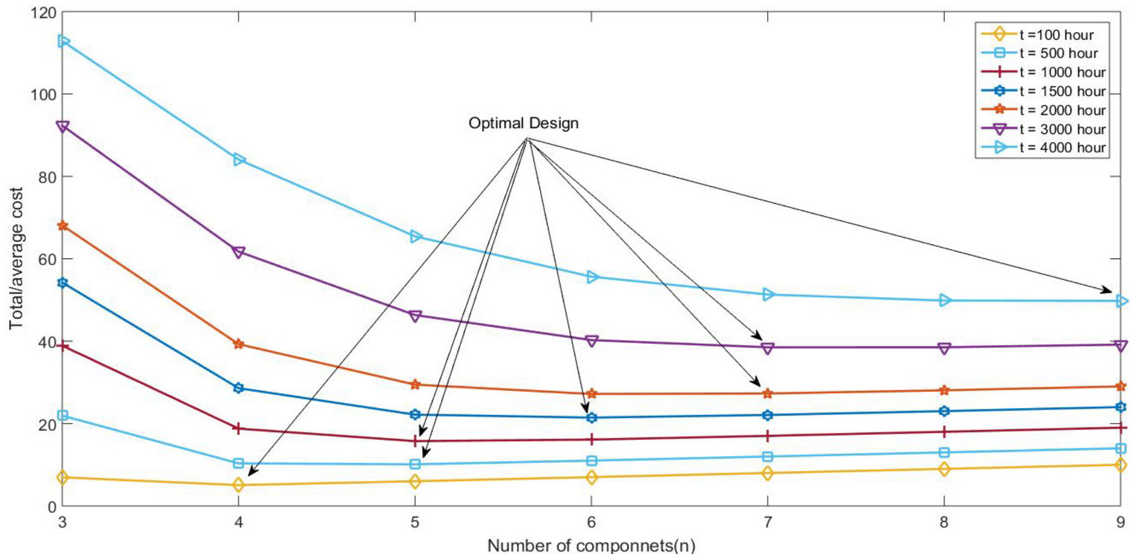
For a certain system, other than the investigated component cost, certain costs are usually required to operate the component such as power supply, etc. In this paper, this type of cost was referred to as c_o in a unit of mission time, i.e., an hour. Then, (3) was modified as

$$\begin{cases} C = c_c * n \\ S = c_s * (1 - R) \\ T = S + C + c_o * t \end{cases} \tag{4}$$

For the meaning of the corresponding variable, please refer to the illustration under (3). In this paper, c_o is 0.01. Then, the obtained total cost values for different cases (with and without the incorporation of operating cost, as described by (3) and (4), respectively) are presented in Fig. 4(a). As indicated by Fig. 4(a), we concluded that if the



(a)



(b)

FIGURE 4. **(a) An illustration of the obtained total cost under the evaluation standard described by (4). (b) Optimal design obtained by varying the mission time.**

operating cost was incorporated, then the total cost increased accordingly, and the increased amount was affected by the parameters of mission time and operating cost spent in a unit of mission time. Nevertheless, the optimal design seemed to remain if the evaluation standard of (4) was utilized.

Furthermore, to investigate the relationship between the mission time and the obtained optimal design, the analysis of various benchmarks was performed with the mission time varying from 500 hours to 4000 hours. The other parameters remained the same as those in Scenario 1. The optimal designs obtained for different cases are illustrated in Fig. 4(b) under the evaluation standard described by (4). Here, we concluded that the optimal design was also affected by the investigated mission time.

With an increase in mission time, more components tended to incur lower total costs. Nevertheless, for fixed parameters, it was definitely not the more components, the better design. Here, the total cost was calculated as $T = c_s * (1 - R) + c_c * n + c_o * t$; for a certain design, if only c_s or c_c varied, then the changing amount of the total cost as obtained as $\Delta T = \Delta c_s * (1 - R)$ or $\Delta T = \Delta c_c * n$. Then the value of the total cost varied for the different design and the optimal design was likely to be affected. The simulation results for cases where only the value of c_s or c_c varied for Scenario 1 are illustrated in Fig. 5(a) and Fig. 5(b), respectively. The value of different designs tended to become the same, due to the reason that the reliability of the designs with given parameters became close to 0.

If only c_o varied, then the changing amount of the total cost was obtained as $\Delta T = \Delta c_o * t$; as for the different designs, the mission time as pre-specified. Thus, the changing amount of total cost varied proportionally and the obtained optimal design was not affected by the variations of c_o.

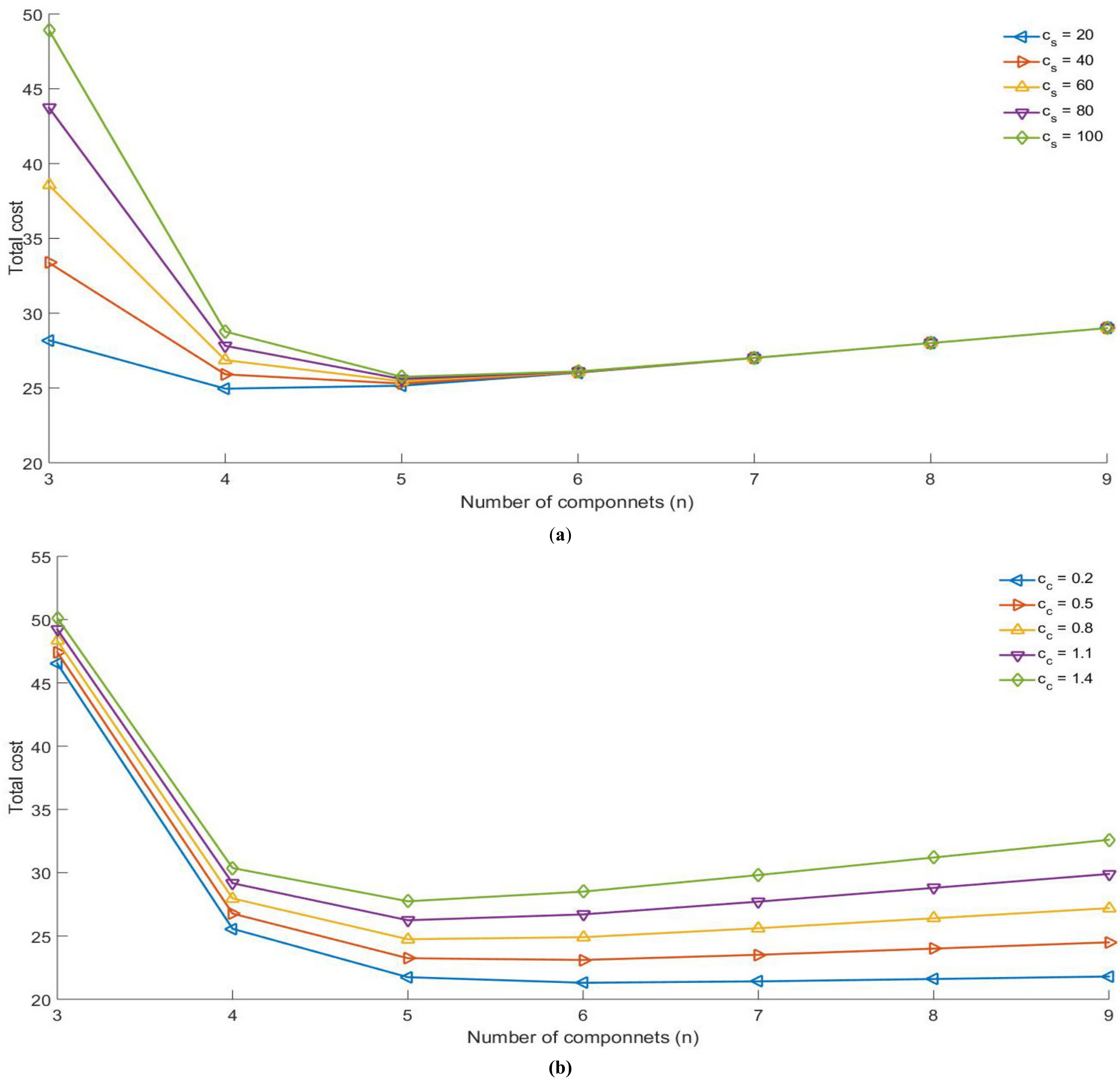


FIGURE 5. An illustration of total cost (described by Equation (4)) for Scenario 1. (a) the value of c_s varies; and (b) the value of c_c varies.

Moreover, the failure parameter also had an indirect impact on the total cost by affecting the reliability value. In this work, if the failure parameter increased, then the corresponding reliability value reduced; hence, the system failure value increased accordingly. Thus, the corresponding total cost also increased.

V. SYSTEM OPTIMIZATION BY *R_PER_COST*

By adopting the evaluation standard described by (3) or (4), the optimal system design can be obtained accordingly. This design is able to provide the lowest total cost with the anticipated number of components. Aside from the total cost value, we were also interested in finding the most cost-effective design if cost was limited. Nowadays, people are becoming more interested in pursuing the effect of the most cost-effective strategy, i.e., the highest feedback they achieved per unit of money spent. Thus, in this paper, a new evaluation standard, referred to as R_per_Cost, is also presented for the purpose of measuring various system designs.

The calculation formula of R_per_Cost is provided as follows

$$R\_per\_Cost = R/T \tag{5}$$

where R_per_Cost indicates the reliability value obtained for one unit of total cost, and is utilized to reflect the corresponding economic benefits. Here, the value of total cost is provided as $T = S + C + c_o * t$ (for the calculation of S and C, please refer to (4)). The meanings of corresponding parameters are the same as those presented previously.

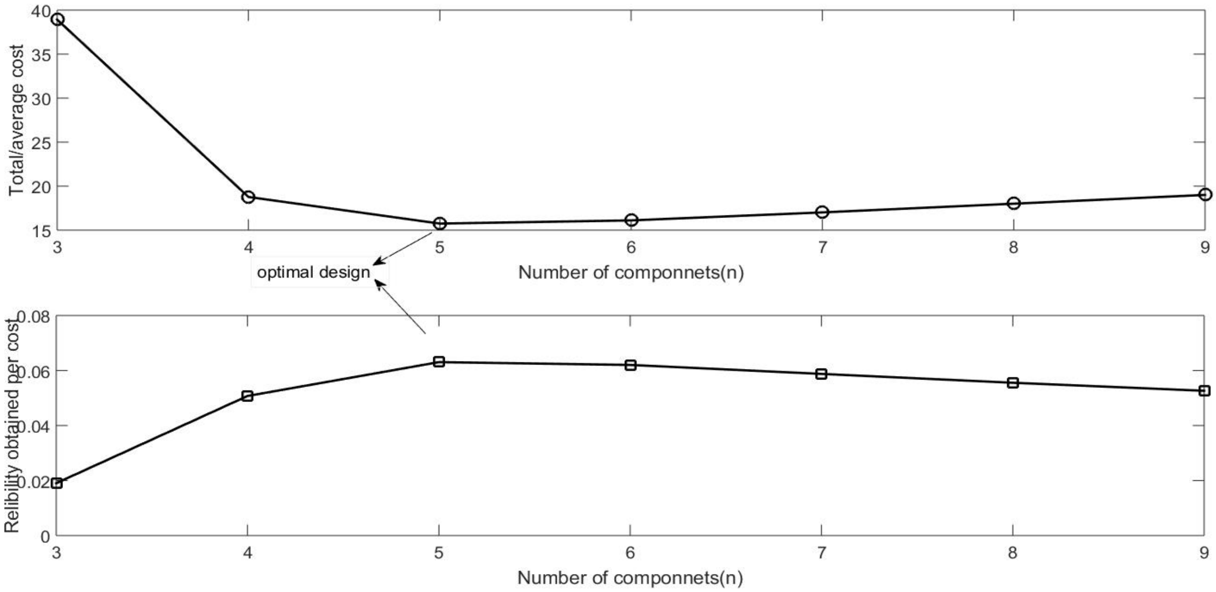


FIGURE 6. An illustration of total cost (described by (4)) and the reliability obtained per unit of cost (described by (5)) for Scenario 1 where the value varies with the number of components.

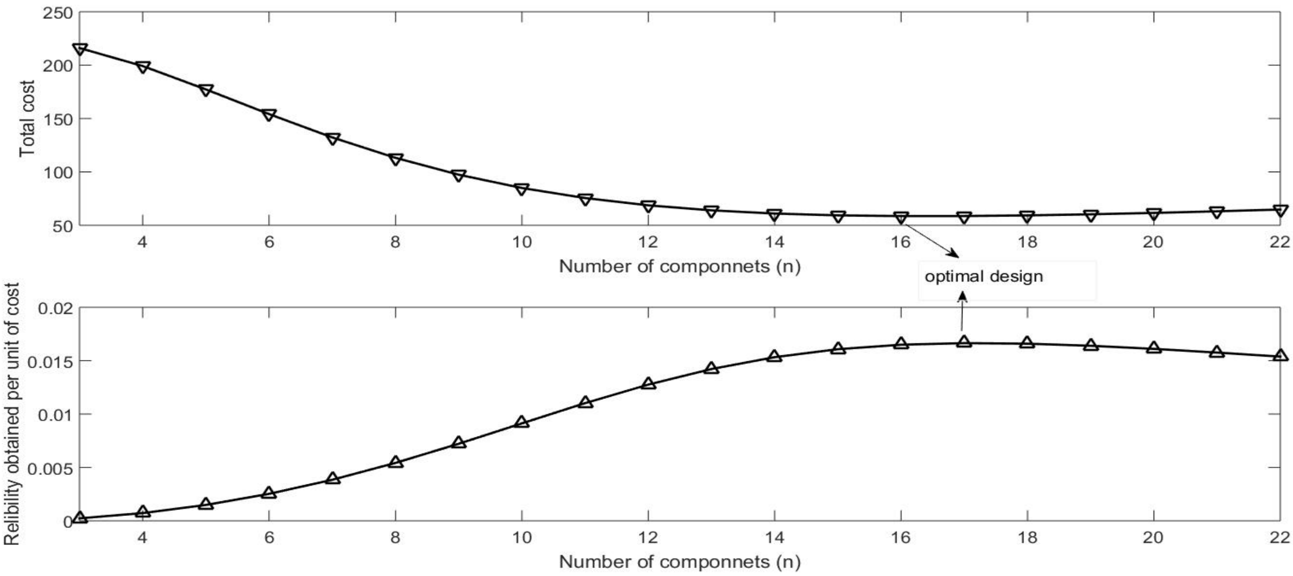


FIGURE 7. An illustration of the total cost (described by (4)) and the reliability obtained per unit of cost (described by (5)) for Scenario 2, where the corresponding value varies with the number of components.

Here, the corresponding values obtained for R_per_Cost using the evaluation standard of (5) are provided in Table 4 for Scenario 1.

Next, the distributions of total cost and reliability obtained per unit of cost spent (i.e., R_per_Cost) are presented in Fig. 6. The corresponding value varies with the total number of components. The parameters are the same as those for Scenario 1.

As shown in Fig. 6, the lowest value of total cost was obtained if the number of components equaled five; furthermore, under this condition, the highest value of R_per_Cost was also obtained. This indicated that per unit of total cost spent, the highest reliability was obtained if the total number of components equaled five. For Scenario 1, the obtained optimal designs under the evaluation standard described by (4) and (5) were the same. For $n = 6$, the reliability value obtained was the second highest, but was not the most effective economically. Hence, for different kinds of applications, different designs can be utilized. With the new proposed evaluation standard, the most cost effective design can be determined.

Furthermore, two evaluation standards (i.e., (4) and (5)) were performed for the analysis of a majority voter under Scenario 2. The corresponding results obtained

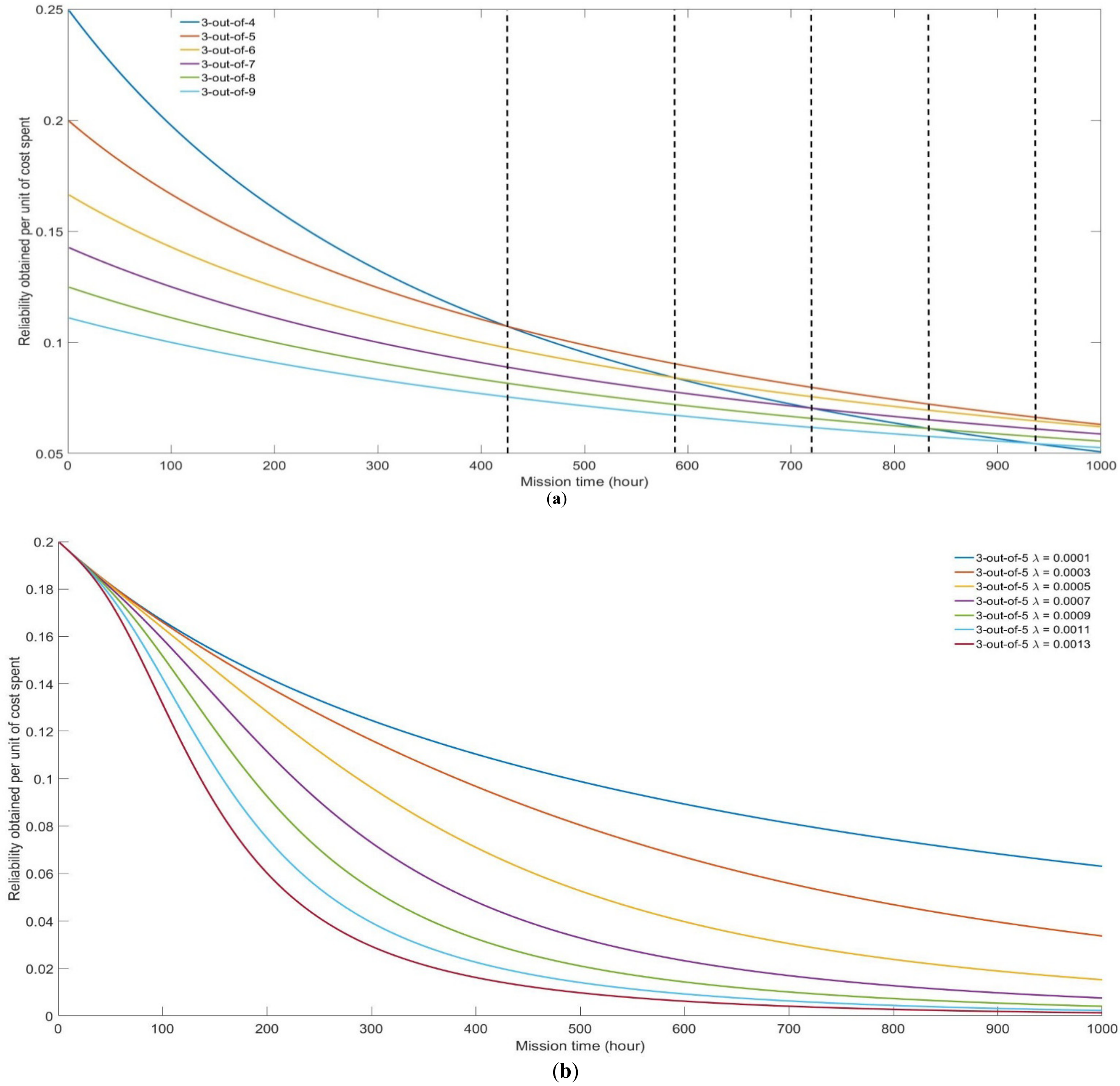


FIGURE 8. (a) An illustration of the reliability per unit of total cost for a mission time of 1000 hours and different designs (n varie); and (b) An illustration of the reliability per unit of total cost for a mission time of 1000 hours and different designs (λ varies). The other parameters are the same as Scenario 1 in Table 1.

for different evaluation standards are presented in Fig. 7.

As indicated in Fig. 7, if the number of components n equaled 16, then the design obtained under the evaluation standard given by (4) was the most desirable where the obtained minimal total cost was 58.6224. Furthermore, if the various designs were measured with the newly presented standard, a system design consisting of 17 components was supposed to be the most desirable. The obtained maximal reliability obtained per unit of cost spent was 0.01664. For Scenario 1, the most desirable design was obtained with a design consisting of five components under both evaluation standards. Nevertheless, for Scenario 2, the optimal design under different evaluation standard varied.

If the operating cost per unit of time was negligible compared with the fixed cost, then the total cost value can be obtained as per (3). Then, only the total cost value or *R_per_Cost* was affected. The obtained minimal total cost was calculated as 38.6224 when the investigated system was composed of 16 components, while the maximal

TABLE 4. Reliability obtained per unit of cost spent for a mission time of 1000 hours for scenario 1.

Number of components (n)	3	4	**5**	6	7	8	9
Reliability	0.7408	0.9523	**0.9926**	0.9989	0.99986	0.99998	0.999998
Cost of components	3	4	**5**	6	7	8	9
System failure cost	25.92	4.769	**0.743**	0.105	0.014	0.00175	0.00021
Total cost	38.92	18.769	**15.743**	16.105	17.014	18.00175	19.00021
R_per_Cost	0.019	0.0507	**0.0630**	0.0621	0.0587	0.0555	0.0526

R_per_Cost obtained equaled 0.02525 if $n = 17$. Therefore, the previously obtained optimal design remained the most desirable.

Furthermore, the relationship between the investigated evaluation standard R_per_Cost and the mission time is also presented in this paper. For convenient analysis, the value of threshold k was fixed to three, while the number of components in the system increased from four to nine. The corresponding obtained R_per_Cost is illustrated in Fig. 7. As indicated by the results, the most desirable designs for different mission times were totally different. As seen with the variation of failure rate, the most desirable design changed. As indicated by Fig. 8(b), with a larger failure rate, the amount of obtained value reliability for per unit of cost spent was always larger if components with smaller failure rates were adopted. For larger failure rates, the corresponding value decreased rapidly and then became placid. Similarly, if the other parameters varied such as c_c, c_s, or c_o, the process was performed accordingly and the most desirable design was likely to change.

Moreover, if common cause failure was incorporated, or the investigated system was composed of majority voter and other types of gates, then the optimal design obtained might also vary. Nevertheless, given the corresponding parameters and system topology (where the majority functions as a subsystem), then the optimal design can be obtained accordingly.

VI. CONCLUSION

To improve system reliability, redundancies are usually incorporated into the investigated system such as a majority voter and spare gates. The system is supposed to achieve a higher reliability with the increase of the number of spare components or redundancy. However, in practice, it is unrealistic to increase the number of redundancies without limit as the consumed cost or required space is proportional with the number of components. Hence, we were more interested in designing a system with a desirable reliability and acceptable cost. Thus, in this paper, a trade-off analysis between the cost and reliability for the spare gate was performed. Three factors were mainly investigated, i.e., the cost of components, the system failure cost, and the total cost. Furthermore, to determine the reliability value efficiently and accurately, a stochastic architecture was also provided. Through stochastic analysis, the reliability value can be easily obtained for further analysis in this paper. The effects of common cause failure are also easily addressed through stochastic analysis.

To incorporate the operating cost, the calculating formula for total cost was also revised. Furthermore, through analysis, the optimal design can also be determined accordingly. In this paper, the relationships between the total cost and system parameters (such as mission time, failure parameter, the cost of certain component, the cost of a system failure, and the operating cost required in a unit of mission time) were thoroughly discussed. The most desirable design varied if the corresponding parameters changed. For certain scenarios, if only the mission time was changed, then the optimal design was also likely to be affected.

Furthermore, an evaluation standard of R_per_Cost, which indicates the reliability value obtained for one unit of total cost was also proposed in this paper. This evaluation standard is more realistic if cost effectiveness was the main focus. A number of benchmarks were also presented for the cost-effective analysis. As indicated by the analysis in this paper for an investigated system with provided parameters, the most optimal design (incorporating total cost and cost-effective design) can be readily presented through corresponding analysis.

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