Received August 9, 2017, accepted September 28, 2017, date of publication October 3, 2017, date of current version October 25, 2017.

Digital Object Identifier 10.1109/ACCESS.2017.2759062

Sliding Mode Regulator for the Perturbations Attenuation in Two Tank Plants

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ABSTRACT The perturbations are the disturbances of motion presented in the plant inputs which affect the motion of the output and states. In this paper, a sliding mode regulator is recommended for the perturbations attenuation in the plant output and states. The novelty of the advised regulator is that it does not employ an observer for the perturbations estimation; consequently, it is more compact. The asymptotic stability of the regulator is ensured via the Lyapunov approach. The mentioned regulator is employed in two tank plants prototypes.

INDEX TERMS Regulator, sliding mode, perturbations, tank plants prototypes.

I. INTRODUCTION

The perturbations attenuation in the plant output and states has been recently studied. The perturbations attenuation is required since the perturbations can affect the sensors, actuators, plants, or regulators, causing accidents or unnecessary costs. Since a regulator is employed to get that all the plant states reach constant behaviors, a regulator for the perturbations attenuation is an important and current topic, in the theory and applications.

There are some works about the stable regulation for the perturbations attenuation. In [1]–[4], methods for the perturbations detection in plants are addressed. The geometric approach for the perturbations attenuation in perturbed plants is studied in [5] and [6]. In [7]–[10], active perturbation rejection regulators of plants with perturbations are advised. Sliding mode regulators for plants with perturbations are recommended in [11]–[13]. In [14]–[17], stable regulators for the perturbations attenuation are focused. H-infinity regulators of perturbed plants are addressed in [18]–[22]. In [23]–[26], several kind of regulators for perturbed plants are discussed. The aforementioned development shows that a regulator for the perturbations attenuation is a novel topic.

From the above research, in [1], [2], [4], [7], [8], [10], [11], [13], [14], [16], [22], and [24], several kind of observers are employed for the perturbations estimation, and these estimated perturbations are taken into account in the mentioned regulators as the perturbations compensation. It must be desired to design a regulator for the perturbations attenuation in which an observer is not required due to it must makes the development more compact.

In this development, a method is explained for the perturbations attenuation in the plant output and states. For this purpose, the next three improvements in the recommended regulator are taken into account:

1) Contrary with the interesting and mentioned works, the advised regulator does not employ an observer for the perturbation estimation. In this regulator, the sliding mode behavior is employed as the perturbations compensation. The improvement to employ a regulator and not an observer based regulator, is that it makes the advised method more compact.
2) The sliding mode regulator has an issue in the sliding mode surface due to it sometimes requires many derivatives of the plant [27], [28]. The difference of this study with respect to the previous methods is that the recommended regulator does not employ the sliding mode surface. It also makes the advised method more compact.
3) The Lyapunov technique is employed to ensure the stability of the sliding mode regulator for the perturbations attenuation. The recommended approach only requires the boundedness of perturbations.

This document is organized as follows. Section 2 explains the perturbed plants, they will be used for the regulator design. The sliding mode regulator for the perturbations attenuation in the plant output and states is advised in Section 3. In Sections 4, and 5, the sliding mode regulator is employed in two tank plants prototypes. Section 6 describes conclusions and suggests future development directions.

II. THE PERTURBED PLANT

Take into account the next perturbed plant:

$$\begin{aligned} \dot{x} &= Ax + Bu + Bp \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in \Re^n$ are the states, $u \in \Re^m$ are the regulator inputs, $y \in \Re$ is the output, $p \in \Re^m$ are the perturbations. $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, and $C \in \Re^{1\times n}$ are matrices representing the maps $\mathbf{A} : \Re^n \to \Re^n$, $\mathbf{B} : \Re^m \to \Re^n$, and $\mathbf{C} : \Re^n \to \Re$, respectively.

The reference model is:

$$\begin{aligned} \dot{x}_r &= Ax_r \\ y_r &= Cx_r \end{aligned} \tag{2}$$

where $x_r \in \Re^n$ are the reference states and $y_r \in \Re$ is the reference output.

The output error is:

$$\begin{aligned} \widetilde{y} &= y - y_r \\ \widetilde{y} &= C\widetilde{x} = C\left(x - x_r\right) \end{aligned} \tag{3}$$

where $\widetilde{y} \in \Re$ is the output error and $\widetilde{x} \in \Re^n$ are the states errors.

III. THE SLIDING MODE REGULATOR

A. THE REGULATOR DEVELOPMENT

In this section, a regulator will be developed based on the assumption that the output y is measured, but the perturbations p are not measured.

The next sliding mode regulator is recommended:

$$u = -B^* L\widetilde{y} - B^* K sgn(\widetilde{y}) \tag{4}$$

where $u \in \Re^m$ are the regulator inputs, $\widetilde{y} \in \Re$ is the output error, $K \in \Re^n$ is a constant vector which is chosen after, $L \in \Re^n$ is a constant vector, $sgn(\cdot) \in \Re$ is the signum map, $B^* \in \Re^{m\times n}$ is the pseudo-inverse of $B \in \Re^{n\times m}$.

The purpose of the regulator (4) is that the output y of the perturbed plant (1) should converge to the reference output y_r of the reference model (2) employing the output y. See the Fig. 1.

Remark 1: Please note that the behaviors of the perturbations p are not measured in the described regulator due to it only requires the measure of their upper bound.

B. THE STABILITY ANALYSIS

In this section, states of the sliding mode regulator applied to the perturbed plants are ensured to be stable by using the solution of the Lyapunov strategy.

Subtracting the reference model (2) to the model (1) and employing the states errors (3):

$$\begin{aligned} \dot{x} - \dot{x}_r &= A\left(x - x_r\right) + Bu + Bp \\ \dot{\widetilde{x}} &= A\widetilde{x} + Bu + Bp \end{aligned} \tag{5}$$

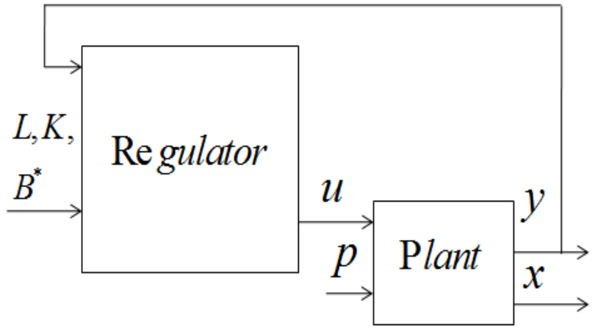

FIGURE 1. The sliding mode regulator.

Substituting the sliding mode regulator (4) into (5) and employing the output error (3) forms the closed loop regulator:

$$\begin{aligned} \dot{\widetilde{x}} &= A\widetilde{x} + Bp - L\widetilde{y} - Ksgn(\widetilde{y}) \\ \dot{\widetilde{x}} &= [A - LC]\widetilde{x} + Bp - Ksgn(\widetilde{y}) \\ \dot{\widetilde{x}} &= A_{smr}\widetilde{x} + Bp - Ksgn(\widetilde{y}) \end{aligned} \tag{6}$$

where $A_{smr} = A - LC$, p is described in (1).

The next Theorem shows the stability of the advised sliding mode regulator.

Theorem 1: States of the sliding mode regulator (4), used for the perturbations attenuation in the plant (1) are asymptotic stable; then, states errors $\widetilde{x}$ comply:

$$\|\widetilde{x}\|^2 \leq \rho e^{-\sigma t} \|\widetilde{x}_i\|^2 \tag{7}$$

where $\widetilde{x}_i$ are the initial conditions of $\widetilde{x}$, $\rho = \frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}$, $\sigma = \lambda_{\min}(WT^{-1})$, $|Bp| \leq B\overline{p}$ and $\frac{B\overline{p}}{sgn(C)} \leq K$, p are the perturbations described in (1), $\|\cdot\|$ is the Euclidean norm in $\Re^n$ and $|\cdot|$ is the absolute value, $T \in \Re^{n\times n}$ and $W \in \Re^{n\times n}$ are positive matrices which comply the next equality:

$$A_{smr}^T T + TA_{smr} = -W \tag{8}$$

where A_{smr} is described in (6).

Proof: Take into account the next candidate Lyapunov element:

$$V_{smr} = \widetilde{x}^T T\widetilde{x} \tag{9}$$

Taking into account (6), the derivation along the solution of (9) is:

$$\begin{aligned} \dot{V}_{smr} &= \dot{\widetilde{x}}^T T\widetilde{x} + \widetilde{x}^T T\dot{\widetilde{x}} \\ \dot{V}_{smr} &= \widetilde{x}^T \left(A_{smr}^T T + TA_{smr}\right)\widetilde{x} + 2\widetilde{x}^T TBp - 2\widetilde{x}^T TKsgn(\widetilde{y}) \end{aligned} \tag{10}$$

Taking into account $A_{smr}^T T + TA_{smr} = -W$ of (8) in (10), it is:

$$\dot{V}_{smr} = -\widetilde{x}^T W\widetilde{x} + 2\widetilde{x}^T TBp - 2\widetilde{x}^T TKsgn(\widetilde{y}) \tag{11}$$

Since (3), $sgn(\widetilde{y}) = sgn(C\widetilde{x}) = sgn(C)sgn(\widetilde{x})$, and taking into account that $\widetilde{x}^T sgn(\widetilde{x}) = |\widetilde{x}|^T$, it is:

$$\begin{aligned}\dot{V}_{smr} &= -\widetilde{x}^T W\widetilde{x} + 2\widetilde{x}^T TBp - 2\widetilde{x}^T TKsgn(C)sgn(\widetilde{x})\\ \dot{V}_{smr} &= -\widetilde{x}^T W\widetilde{x} + 2\widetilde{x}^T TBp - 2\,|\widetilde{x}|^T\, TKsgn(C)\end{aligned} \tag{12}$$

Taking into account $|Bp| \leq B\overline{p}$ and $\frac{B\overline{p}}{sgn(C)} \leq K$ of (7), it is:

$$\begin{aligned}\dot{V}_{smr} &\leq -\widetilde{x}^T W\widetilde{x} - 2\,|\widetilde{x}|^T\, T\left[Ksgn(C) - B\overline{p}\right]\\ \dot{V}_{smr} &\leq -\widetilde{x}^T W\widetilde{x}\end{aligned} \tag{13}$$

(13) can be expressed as:

$$\dot{V}_{smr} \leq -\sigma V_{smr} \tag{14}$$

where $\sigma = \lambda_{\min}(WT^{-1})$. From (14), it is recognized that the states of the regulator employed in perturbed plants are asymptotic stable. Taking into account (14), its solution is gotten as:

$$\begin{aligned}e^{\sigma t}\dot{V}_{smr} &\leq -e^{\sigma t}\sigma V_{smr}\\ e^{\sigma t}\dot{V}_{smr} + e^{\sigma t}\sigma V_{smr} &\leq 0\\ \frac{d}{dt}\left(e^{\sigma t}V_{smr}\right) &\leq 0\\ \int_0^t \frac{d}{d\tau}\left(e^{\sigma\tau}V_{smr}\right)d\tau &\leq 0\\ e^{\sigma\tau}V_{smr}\Big|_0^t &\leq 0\\ e^{\sigma t}V_{smr} - V_{smri} &\leq 0\\ e^{\sigma t}V_{smr} &\leq V_{smri}\\ V_{smr} &\leq e^{-\sigma t}V_{smri}\end{aligned} \tag{15}$$

where V_{smri} are the initial conditions of V_{smr}. Taking into account the definition of (9) in the last equality of (15) it is:

$$\begin{aligned}&\lambda_{\min}(T)\,\|\widetilde{x}\|^2 \leq \widetilde{x}^T T\widetilde{x} = V_{smr}\\ &\leq e^{-\sigma t}V_{smri} = e_i^{-\sigma t}\widetilde{x}_i^T T\widetilde{x}_i \leq \lambda_{\max}(T)e^{-\sigma t}\,\|\widetilde{x}_i\|^2\\ &\Longrightarrow \|\widetilde{x}\|^2 \leq \frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}e^{-\sigma t}\,\|\widetilde{x}_i\|^2\end{aligned} \tag{16}$$

where $\widetilde{x}_i$ are the initial conditions of $\widetilde{x}$. By using $\rho = \frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}$ of (7), the equality (7) is proved. ∎

Remark 2: The solvability of the mentioned regulator is that the solution of the equation (8) gives the gain L which is employed in the regulator (4) to reach the perturbations attenuation in the perturbed plant (1).

In the next two sections, the sliding mode regulator of (4) called SMR will be compared with the geometric regulator of [6] called GR in two experiments. The root mean square error (MSE) for the output and states will be exploited, it is:

$$MSE = \left(\frac{1}{T}\int_0^T x^2 d\tau\right)^{\frac{1}{2}} \tag{17}$$

where $x^2 = \sum_{j=1}^{n} \widetilde{x}_j^2$ for the states errors, or $x^2 = \widetilde{y}^2$ for the output error.

IV. THE THERMAL PLANT

This plant consists of three interconnected tanks [6]. It is understood that the liquid enters to the tank with a fixed temperature, it sends the tank liquid to a determined temperature. The valves allow a discharge of liquid outside the plant. The purpose is to keep the liquid temperature in the tanks in a desired value.

Fig. 2 shows the thermal plant prototype which is employed to get the real data of perturbations and to validate a model. Angular keys with barrel type of 1/2 inch are adapted as valves. Servo motors of 0.1 revolutions per minute are employed to move the valves. An aluminium resistance of 200 W and 120 V is employed to hot the water. Temperature sensors LM35 are employed to get the temperature measures. An Arduino microcontroller is employed to send the temperature measures to a personal computer. The personal computer is employed to receive and to save the temperature measures.

FIGURE 2. **The thermal plant prototype.**

Fig. 3 shows the thermal plant diagram where t_{02}, t_{03} are the temperatures of the water which fall in the tanks 1, 3 ($°C$), t_1, t_2, t_3 are the temperatures of the water for the tanks 1, 2, 3 ($°C$), f_1, f_2, f_3 are the flows of the water for the tanks 2, 3 (m^3/s), C_1, C_2, C_3 are the calorific capacities of the water for the tanks 1, 2, 3 (kcal/m^{3o}C), and w_1, w_2 are the heating energy employed in the tanks 2, 3 (kcal/m^3).

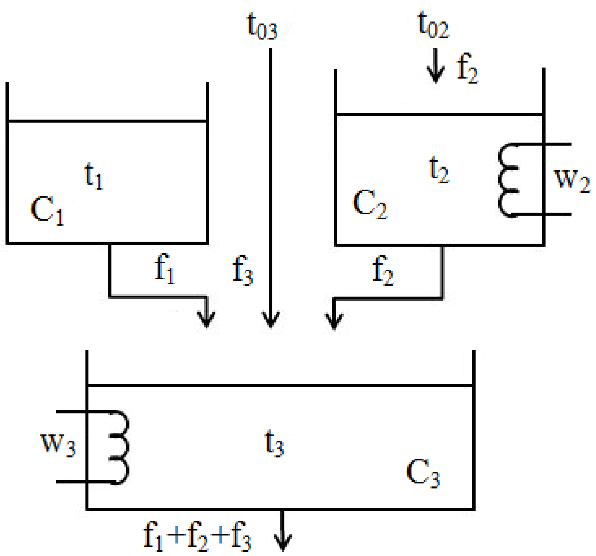

FIGURE 3. **The thermal plant diagram.**

The model of the thermal plant in states space is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + B_1 \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + B_2 \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$
$$y = C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{18}$$

where:

$$A = \begin{bmatrix} -\frac{f_1}{C_1} & 0 & 0 \\ 0 & -\frac{f_2}{C_2} & 0 \\ \frac{f_1}{C_3} & \frac{f_2}{C_3} & -\frac{f_1+f_2+f_3}{C_3} \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0 & 0 \\ \frac{1}{C_2} & 0 \\ 0 & \frac{1}{C_3} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ \frac{f_2}{C_2} & 0 \\ 0 & \frac{f_3}{C_3} \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$p_1 = t_{02}, p_2 = t_{03}$ are the plant perturbations, $u_1 = w_2, u_2 = w_3$ are the plant inputs, $x_1 = t_1, x_2 = t_2, x_3 = t_3$ are the plant states, and y is the plant output. If $f_1 = f_2 = f_3 = 1$ m^3/s and $C_1 = C_2 = C_3 = 1000$ kcal/m^{3o}C, then the plant matrices are:

$$A = \begin{bmatrix} -\frac{1}{1000} & 0 & 0 \\ 0 & -\frac{1}{1000} & 0 \\ \frac{1}{1000} & \frac{1}{1000} & -\frac{3}{1000} \end{bmatrix}$$
$$B = B_1 = B_2 = \begin{bmatrix} 0 & 0 \\ \frac{1}{1000} & 0 \\ 0 & \frac{1}{1000} \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \tag{19}$$

The regulability of the plant (18), (19) is analyzed as:

$$\mathbb{R} = \begin{bmatrix} B \\ AB \\ A^2B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{1000} & 0 & -\frac{1}{1000000} \\ 0 & \frac{1}{1000} & \frac{1}{1000000} \\ 0 & 0 & 0 \\ 0 & \frac{1}{1000000000} & 0 \\ -\frac{3}{1000000} & -\frac{1}{250000000} & \frac{9}{1000000000} \end{bmatrix} \tag{20}$$

Since the rank of $\mathbb{R}$ in (20) is 2, 2 states of the plant are able to be regulated and 1 state of the plant is not able to be regulated, it will be seen in the experiments.

In this experiment, the dynamic model of the thermal plant is given by the equality (1), (18), 39.15 °C, 39.15 °C, 39.15 °C are chosen as the initial conditions for the plant states x_1, x_2, x_3. The perturbations are p_1, p_2 are real data taken from 0 s to 10 s. The reference output is $y_r = 39.25$ °C.

The purpose of the GR is that the plant output y of (1), (18) should converge to the reference output y_r of (2) with the regulator of [6] using the output $y = x_3$. The GR is given with the factor $L = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$.

The purpose of the SMR is that the plant output y of (1), (18) should converge to the reference output y_r of (2) with the regulator of (4) using the output $y = x_3$. The SMR is given by the equalities (4) with the factors $B^* = \begin{bmatrix} 0 & 1000 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$, $L = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $K = \begin{bmatrix} 0.04 \\ 0.04 \\ 0.04 \end{bmatrix}$. The Lyapunov equality (8) is satisfied with $T = \begin{bmatrix} 333.67 & -166.33 & -166.33 \\ -166.33 & 333.67 & -166.33 \\ -166.33 & -166.33 & 332.17 \end{bmatrix}$, $W = \begin{bmatrix} 1.0 & 0.0 & 0.01251 \\ 0.0 & 1.0 & 0.01251 \\ 0.01251 & 0.01251 & 1.013 \end{bmatrix}$. Taking into account the Theorem 1, the states of the regulator are asymptotic stable with factors $\rho = 982.32$ and $\sigma = 1.9822 \times 10^{-3}$.

Fig. 4, 5, and 6 show the perturbations, states, and output for the regulators where P are the perturbations, DR are the desired references, GR is the geometric regulator, and SMR is the sliding mode regulator. Table 1 shows the MSE for the regulators using (17).

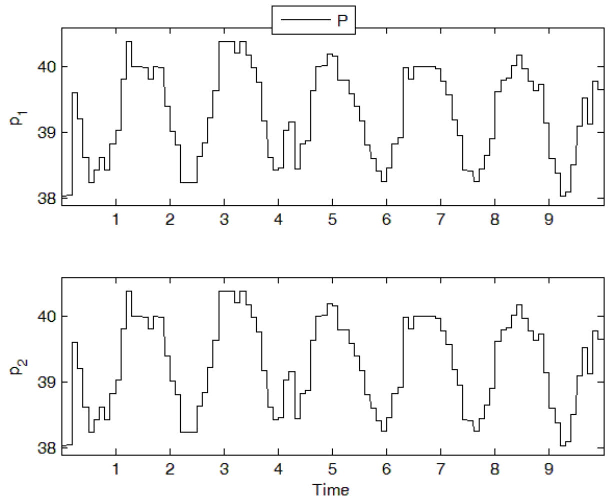

FIGURE 4. Perturbations for the experiment 1.

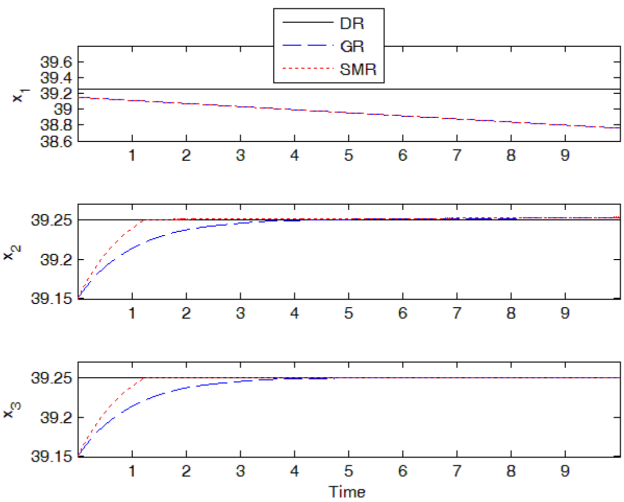

FIGURE 5. States for the experiment 1.

TABLE 1. The MSE for the regulators.

Strategies	MSE for $\widetilde{y}$	MSE for $\widetilde{x}$
GR	0.0159	0.2244
SMR	0.0123	0.2239

From Fig. 4, 5, and 6, it can be seen that the SMR improves the GR due to the output and states in the first get better perturbations attenuation than in the second, also it can be seen that states 2 and 3 are regulated while the state 1 is not regulated, it is due to the analysis (20) which proved that the plant is not completely able to be regulated. From Table 1, it can be shown that the SMR achieves better accuracy when compared with the GR due to the MSE for the first is smaller than for the second.

V. THE HYDRAULIC PLANT

This plant consists of three interconnected tanks [5]. It is understood that a fixed amount of liquid enters to the tank, it sends the tank liquid to a determined liquid level. The valves allow a discharge of liquid outside the plant. The purpose is to keep the liquid levels in the tanks in a desired value.

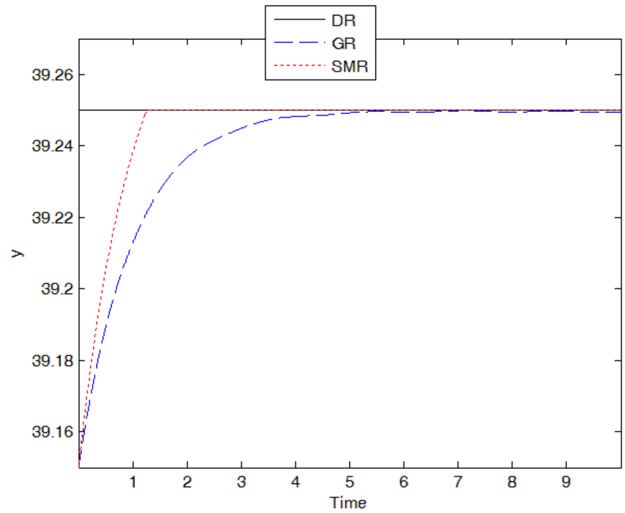

FIGURE 6. The output for the experiment 1.

Fig. 7 shows the liquid-level plant prototype which is employed to get the real data of perturbations and to validate a model. Angular keys with barrel type of 1/2 inch are adapted as valves. Servo motors of 0.1 revolutions per minute are employed to move the valves. Ultrasonic sensors HC-SR04 are employed to get the level measures. An Arduino microcontroller is employed to send the level measures to a personal computer. The personal computer is employed to receive and to save the level measures.

Fig. 8 shows the liquid-level plant diagram, where R_1, R_2, R_3 are the resistances of the valves 1, 2, 3 (m^2/s), C_1, C_2, C_3 are the capacitances of the tanks 1, 2, 3 (m^2), h_1, h_2, h_3 are the liquid levels in the tanks 1, 2, 3, and q_1, q_2 are the liquid inputs to the tanks 1, 2.

The model of the hydraulic plant in states space is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + B \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + B \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$
$$y = C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{21}$$

where:

$$A = \begin{bmatrix} -\frac{1}{R_1 C_1} & 0 & 0 \\ 0 & -\frac{1}{R_2 C_2} & 0 \\ \frac{1}{R_1 C_3} & \frac{1}{R_2 C_3} & -\frac{1}{R_3 C_3} \end{bmatrix}$$
$$B = \begin{bmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \\ 0 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \tag{22}$$

$x_1 = h_1$, $x_2 = h_2$, $x_3 = h_3$ are the plant states, $u_1 = q_1$, $u_2 = q_2$ are the plant inputs. If $R_1 = R_2 = R_3 = 0.5$ m^2/s

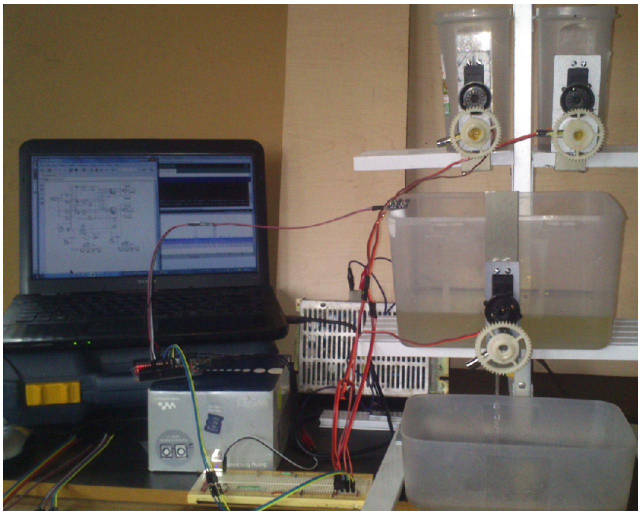

FIGURE 7. The liquid-level plant prototype.

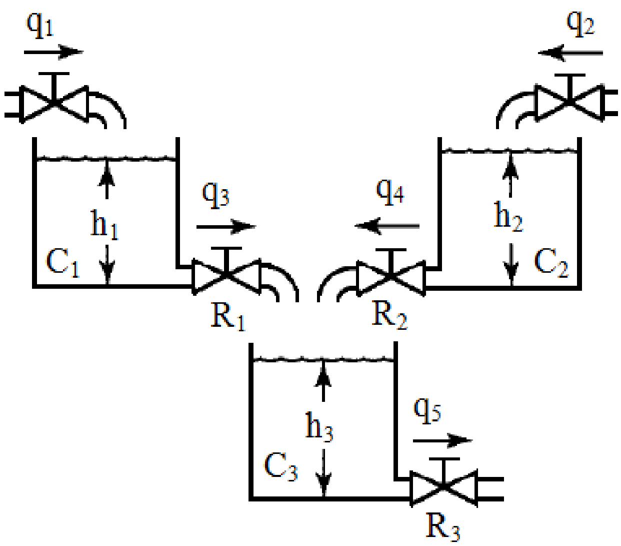

FIGURE 8. The liquid-level plant diagram.

and $C_1 = C_2 = C_3 = 0.5 \text{ m}^2$, then the plant matrices are:

$$A = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 4 & 4 & -4 \end{bmatrix}$$
$$B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

The regulability of the plant (21), (22) is analyzed as:

$$\mathbb{R} = \begin{bmatrix} B \\ AB \\ A^2B \end{bmatrix} = \begin{bmatrix} \frac{1}{0.5} & 0 & -8.0 & 0 & 32.0 & 0 \\ 0 & \frac{1}{0.5} & 0 & -8.0 & 0 & 32.0 \\ 0 & 0 & 8.0 & 8.0 & -64.0 & -64.0 \end{bmatrix} \quad (23)$$

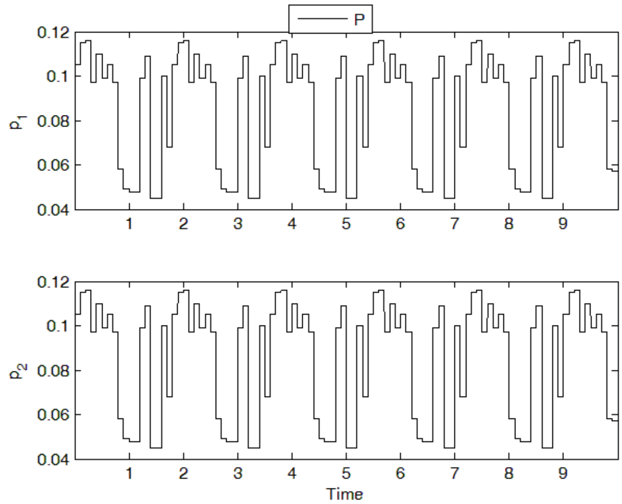

FIGURE 9. Perturbations for the experiment 2.

Since the rank of $\mathbb{R}$ in (23) is 3, the 3 states of the plant are able to be regulated, it will be seen in the experiments.

In this experiment, the dynamic model of the hydraulic plant is given by the equality (1), (21), 0 m, 0 m, 0 m are chosen as the initial conditions for the plant states x_1, x_2, x_3. The perturbations are p_1, p_2 are real data taken from 0 s to 10 s. The reference output is $y_r = 0.08$ m.

The purpose of the GR is that the plant output y of (1), (21) should converge to the reference output y_r of (2) with the regulator of [6] using the output $y = x_3$. The GR is given with the factor $L = \begin{bmatrix} 10 & 10 & 10 \end{bmatrix}^T$.

The purpose of the SMR is that the plant output y of (1), (21) should converge to the reference output y_r of (2) with the regulator of (4) using the output $y = x_3$. The SMR is given by the equality (4) with the factors $B^* = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$, $L = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$, $K = \begin{bmatrix} 0.08 \\ 0.08 \\ 0.08 \end{bmatrix}$. The Lyapunov equality (8) is satisfied with $T = \begin{bmatrix} 9.9673 \times 10^{-2} & -2.5327 \times 10^{-2} & -2.5327 \times 10^{-2} \\ -2.5327 \times 10^{-2} & 9.9673 \times 10^{-2} & -2.5327 \times 10^{-2} \\ -2.5327 \times 10^{-2} & -2.5327 \times 10^{-2} & 7.1895 \times 10^{-2} \end{bmatrix}$, $W = \begin{bmatrix} 1.0 & 0.0 & -6.0 \times 10^{-6} \\ 0.0 & 1.0 & -6.0 \times 10^{-6} \\ -6.0 \times 10^{-6} & -6.0 \times 10^{-6} & 0.99998 \end{bmatrix}$. Taking into account the Theorem 1, the states of the regulator are asymptotic stable with factors $\rho = 3.3528$ and $\sigma = 7.9997$.

Fig. 9, 10 and 11 show the perturbations, states, and output for the regulators where P are the perturbations, DR are the desired references, GR is the geometric regulator, and SMR is the sliding mode regulator. Table 2 shows the MSE for the regulators using (17).

From Fig. 9, 10, and 11, it can be seen that the SMR improves the GR due to the output and states in the first get

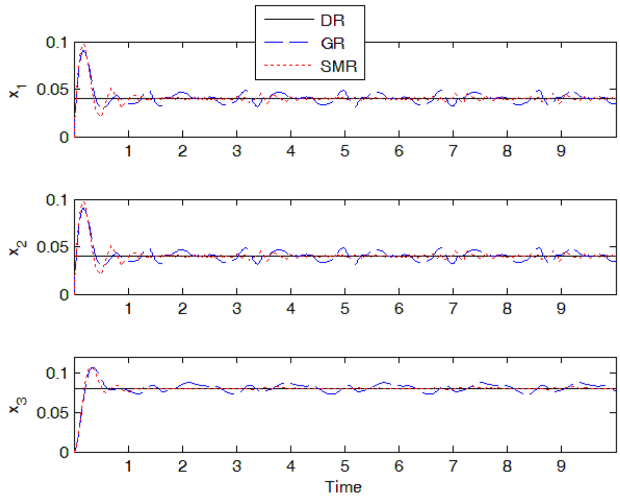

FIGURE 10. States for the experiment 2.

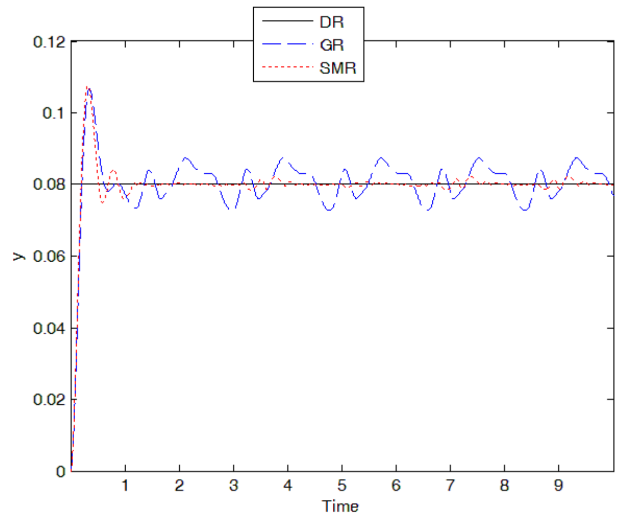

FIGURE 11. The output for the experiment 2.

TABLE 2. The MSE for the regulators.

Strategies	MSE for $\tilde{y}$	MSE for $\tilde{x}$
GR	0.0066	0.0104
SMR	0.0058	0.0097

better perturbations attenuation than in the second, also it can be seen that states 1, 2, and 3 are regulated, it is due to the analysis (23) which proved that the plant is completely able to be regulated. From Table 2, it can be shown that the SMR achieves better accuracy when compared with the GR due to the MSE for the first is smaller than for the second.

VI. CONCLUSIONS

In this work, the usefulness of the sliding mode approach is shown for solving the perturbations attenuation issue. The results showed that the sliding mode regulator achieves better perturbation attenuation when is compared with the geometric regulator. It is important to remark that the perturbations measurement is not required. This work represents the theoretical and application basis for developing the perturbations attenuation issues, for being able afterwards to perform an implementation in functioning plants. In the future, other kind of regulators will be designed for the perturbations attenuation, the adaptive learning will be combined with the advised regulator, or the trajectory tracking issue will be studied [29]–[32].

ACKNOWLEDGEMENTS

The authors are grateful with the editor and with the reviewers for their valuable comments and insightful suggestions, which can help to improves this development significantly. They thank the Instituto Politécnico Nacional, the Secretaría de Investigación y Posgrado, Comisión de Operación y Fomento de Actividades Académicas, and Consejo Nacional de Ciencia y Tecnología for their help in this study.

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