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An Improved S^2 Control Chart for Cost and Efficiency Optimization

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ABSTRACT Virtually, detection of shifts in the dispersion parameter of the process is more valuable before monitoring the location parameter of the process. For the monitoring of the dispersion parameter, the S^2 chart is a common choice in the literature. In this paper, we proposed a modified S^2 chart based on modified successive sampling, which is cost effective relative to simple random sampling. The run length properties are used as comparative measure and the findings depict that all proposed schemes outperform the classical S^2 chart. Finally, the application of the proposed scheme is demonstrated using a real-life engineering process.

INDEX TERMS Average run length, control chart, cost optimization, modified successive sampling, process dispersion, statistical process control.

I. INTRODUCTION

The world is a global village, where the supermarkets are filled with the variety of products or services. In our days customers not only purchase a product to fulfill their needs, but also consider the quality and cost efficiency of the product/service. Quality, in manufacturing perspective, is a measure of excellence or a state of being free from defects, deficiencies and significant variation. Generally, there are two causes of variation that affect the performance of the process. One is the chance cause or natural cause of variation that cannot be completely eliminated unless there is a major change in the equipment or material used in the process. The other is the special or the assignable cause of variation that can be divided further into two categories (transient and persistent). These causes of variation can precisely be identified, eliminated or reduced by investigating the problem and hence finding the cause results in the process improvement.

Statistical process control (SPC), a set of the well-known tool kits, is used to monitor the performance of any process. Control chart, one of the major tools of SPC, is commonly applied to monitor the performance of the process with respect to time. In control chart, there are two decision lines named as lower control limit (LCL) and an upper control limit (UCL), which are used to decide whether the process is working under in-control (IC) or out-of-control (OoC) situation. If the control chart identifies that the process is out-of-control, there is a need to diagnose the cause behind the curtain.

Control charts are designed to monitor the single process parameter such as location or dispersion. Dispersion charts are used to monitor with in samples variability while location charts are used to monitor between samples variability. So, it is preferable to monitor the process dispersion before location of the process. The Shewhart type charts such as S, R and S^2 charts are widely used to monitor the dispersion in many manufacturing processes. Many of the researchers are still engaged to improve these control charts. The dispersion charts under different sampling plans are discussed in Khoo [1], Zhang *et al.* [2], Lee [3], Lee *et al.* [4] and Guo and Wang [5] while other type of modifications are studied by Chen [6], David [7], Khoo [8], Huang and Chen [9], Riaz [10], Riaz and Saghir [11], Mahmoud *et al.* [12], Schoonhoven *et al.* [13], Rakitzis and Antzoulakos [14], Ahmad *et al.* [15], Kuo and Lee [16], Zafar *et al.* [17], Zhang [18], Aldosari *et al.* [19], Ahmad *et al.* [20], and Aslam *et al.* [21]. Recently, Yaqub *et al.* [22] used a cost-efficient sampling strategy (Modified successive sampling (MSS)) in the Shewhart chart to monitor the shifts in location parameter. The findings of their study depict that the proposed scheme out performs the existing schemes.

This study is designed to improve the existing S^2 chart by implementing successive sampling technique (cf. Jessen [23]). The rest of the study is as follows: Section II provides the basic structure of the classical Shewhart S^2 chart and the proposed S^2 chart. Section III induces design structure of the proposed S^2 chart and the performance of said charts under out-of-control situation of the process. The illustrative example will describe in section IV and conclusion of the stated study will report in section V.

II. STRUCTURE OF EXISTING AND PROPOSED CONTROL CHART

In this section, we will discuss the structure of classical Shewhart S^2 chart under simple random sampling (SRS) and the newly proposed chart based on modified successive sampling (MSS).

A. THE CLASSICAL SHEWHART S^2 CHART

Let $X_{1,j}$, $X_{2,j}$, $X_{3,j}, \ldots, X_{i,j} \ldots$ are the samples of size n (where $j = 1, 2, 3, \ldots\ldots, n$ and $i = 1, 2, 3, \ldots\ldots$) from normally distributed quality characteristic of interest with known mean (μ_0) and variance (σ_0^2). The plotting statistic and control limits (UCL, LCL) of the classical Shewhart S^2 chart are defined as

$$S_i^2 = \frac{\sum_{j=1}^{n} (X_{i,j} - \bar{X}_i)^2}{n-1},$$

where

$$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{i,j}$$

$$LCL_{SRS} = \mu_{S^2} - L_{SRS}\sigma_{S^2}$$

$$LCL_{SRS} = \sigma_0^2 - L_{SRS}\sqrt{\frac{2}{n-1}\sigma_0^4}$$

$$LCL_{SRS} = \sigma_0^2(1 - L_{SRS}\sqrt{\frac{2}{n-1}})$$

$$UCL_{SRS} = \mu_{S^2} + L_{SRS}\sigma_{S^2}$$

$$UCL_{SRS} = \sigma_0^2 + L_{SRS}\sqrt{\frac{2}{n-1}\sigma_0^4}$$

$$UCL_{SRS} = \sigma_0^2(1 + L_{SRS}\sqrt{\frac{2}{n-1}})$$

where L_{SRS} is the charting constant on the specific IC average run length (ARL_0).

B. THE SHEWHART S^2 CHART UNDER MSS

Earlier, we discussed that customers are not only willing to purchase a product to fulfill their needs and wants, but also consider the quality and cost optimization of the product/service. In manufacturing process, timely detection of defective product may cause gain in cost efficiency. For the single occasion inventory problem, simple random sampling (SRS) is referred in most surveys while Jessen [23] suggested the successive sampling for various occasions. In terms of industrial practice, the quality of a product is being assessed regularly from one-time period to the next. In such repetitive assessment, successive sampling plays a key role and provides reliable estimates. The design of successive sampling considers the first sample taken at first occasion and second sample (included some points from first sample) taken at the next occasion. Some modifications in the successive sampling can be seen in Patterson [24], Rao and Graham [25], Das [26] and Choudhary *et al.* [27].

Yaqub *et al.* [22] discussed the modified form of successive sampling for the quality characteristic variable which is defined in following steps;

Step 1: Take first sample $(X_{1,1}, X_{1,2}, X_{1,3}, \ldots\ldots, X_{1,n})$ of size n by using the SRS.

Step 2: Take second sample $(X_{2,1}, X_{2,2}, X_{2,3}, \ldots\ldots, X_{2,n-c})$ of size $n-c$ by using the SRS and the remaining c observations are picked as quantiles points of first sample in the following way: $X_{2,n-c+1} = Q_1(X_{1,1}, X_{1,2}, X_{1,3}, \ldots\ldots, X_{1,n})$, $X_{2,n-c+2} = Q_2(X_{1,1}, X_{1,2}, X_{1,3}, \ldots\ldots, X_{1,n})$ and so on, up to $X_{2,n} = Q_c(X_{1,1}, X_{1,2}, X_{1,3}, \ldots\ldots, X_{1,n})$.

Step 3: Similarly, third sample consist of $n-c$ new observations by using the SRS and remaining c observations from the quantile points of second sample, and this procedure is repeated for the specific run of production.

Generally, modified successive sampling symbolized as $MSS_{n,c,Q_1,Q_2,\ldots\ldots,Q_c}$. where n represents sample size, number of observations from previous sample is represented by c and $Q_p \forall p = 1, 2, 3, \ldots\ldots, c$. is the quantile point picked from the previous sample. In the stated study, we consider several cases which are given as follows:

i. $MSS_{n,2,Q_1,Q_2}$, where $n-2$ observations are generated by using SRS and the remaining two observations are taken from the specific quantile pairs (Q_1, Q_2) of the previous sample. In this study, the choice of quantile pairs (Q_1, Q_2) are $(Q_{0.25}, Q_{0.75})$, $(Q_{0.30}, Q_{0.70})$, $(Q_{0.35}, Q_{0.65})$, $(Q_{0.40}, Q_{0.60})$ and $(Q_{0.45}, Q_{0.55})$.

ii. $MSS_{n,3,Q_1,Q_2,Q_3}$, where $n-3$ observations are generated by using SRS and the remaining three observations are taken from the specific quantile pairs (Q_1, Q_2, Q_3) of the previous sample. In this study, the choice of quantile pairs (Q_1, Q_2, Q_3) are $(Q_{0.25}, Q_{0.50}, Q_{0.75})$, $(Q_{0.30}, Q_{0.50}, Q_{0.70})$, $(Q_{0.35}, Q_{0.50}, Q_{0.65})$, $(Q_{0.40}, Q_{0.50}, Q_{0.60})$ and $(Q_{0.45}, Q_{0.50}, Q_{0.55})$.

Finally, the plotting statistic and control limits (UCL, LCL) of the Shewhart S^2 chart under MSS are defined as:

$$S_i^2 = \frac{\sum_{j=1}^{n} (X_{i,j} - \bar{Y}_i)^2}{n-1}$$

$$LCL_{MSS} = \hat{\mu}_{S^2} - L_{MSS}\hat{\sigma}_{S^2} = \bar{S}_{S^2} - L_{MSS}MSE_{S^2}$$

$$UCL_{MSS} = \hat{\mu}_{S^2} + L_{MSS}\hat{\sigma}_{S^2} = \bar{S}_{S^2} + L_{MSS}MSE_{S^2}$$

where L_{MSS} is the charting constant on the specific IC average run length (ARL_0), $\bar{S}_{S^2}$ and MSE_{S^2} are the mean and mean square error of S^2 under MSS (cf. Table 1).

TABLE 1. Properties of S^2 under MSS.

c	Scheme	$n=5$			$n=7$		
		$\bar{S}_{S^2}$	MSE_{S^2}	L_{MSS}	$\bar{S}_{S^2}$	MSE_{S^2}	L_{MSS}
2	$MSS_{Q_{0.25},Q_{0.75}}$	1.0840	0.3678	4.290	0.9731	0.2580	4.130
	$MSS_{Q_{0.30},Q_{0.70}}$	0.8552	0.3415	4.390	0.8608	0.2604	4.070
	$MSS_{Q_{0.35},Q_{0.65}}$	0.7294	0.3758	4.160	0.7972	0.2775	3.920
	$MSS_{Q_{0.40},Q_{0.60}}$	0.6689	0.4004	4.006	0.7601	0.2922	3.830
	$MSS_{Q_{0.45},Q_{0.55}}$	0.6397	0.4159	3.920	0.7417	0.2992	3.780
3	$MSS_{Q_{0.25},Q_{0.50},Q_{0.75}}$	0.7565	0.2996	4.160	0.7768	0.2533	3.860
	$MSS_{Q_{0.30},Q_{0.50},Q_{0.70}}$	0.5514	0.4035	3.490	0.6827	0.2917	3.550
	$MSS_{Q_{0.35},Q_{0.50},Q_{0.65}}$	0.4600	0.4823	3.150	0.6277	0.3247	3.340
	$MSS_{Q_{0.40},Q_{0.50},Q_{0.60}}$	0.4291	0.5059	3.045	0.6016	0.3420	3.250
	$MSS_{Q_{0.45},Q_{0.50},Q_{0.55}}$	0.4126	0.5211	2.970	0.5901	0.3499	3.210

III. PERFORMANCE EVALUATION AND COMPARISONS

In this section, we discuss the design of control charting constant and OoC study of proposed chart. Moreover, we will provide a comprehensive comparison between the proposed chart and the existing charts.

A. DESIGN STRUCTURE OF CONTROL CHARTING CONSTANT

Construction and design of proposed control chart depends on the sample size (n), number of observations from previous sample (c) and quantile function ($Q_p \forall p = 1, 2, 3, \ldots\ldots, c$) which are used to pick observations from previous sample. For finding the control charting constant say L_{MSS} a simulation study is designed where 100,000 replicates are generated in R 3.1.1. This study is evaluated by the different performance measures such as average run length (ARL), standard deviation of run length ($SDRL$) and different quantiles (25^{th}, 75^{th} and 95^{th}) of run length distribution. The ARL is defined as the average number of items that are declared IC before the first OoC item detected while SDRL shows the dispersion of IC items in several iterations. Practically, the objective is to maximize the IC ARL (ARL_0) and minimize the OoC ARL (ARL_1) which is not possible for a fixed sample size. Therefore, we fix the ARL_0 and evaluate the ARL_1 values. A chart with the smaller ARL_1 is considered better and vice versa. The prefixed $ARL_0 = 370$ is used to search the appropriate L_{MSS} values which are given in Table 1.

B. OUT-OF-CONTROL PERFORMANCE OF THE INVESTIGATED CONTROL CHARTS

The dispersion charts such as S^2 chart are important and applicable to detect the degree of change in the variation of process. Along with explaining the IC properties of the charts, it is useful to examine the OoC performance of the charts.

The OoC average run length (ARL_1), standard deviation of run length ($SDRL$) and different quantiles (25^{th}, 75^{th} and 95^{th}) of run length distribution are given in Tables 2–4 for S^2 chart under SRS and MSS, respectively. To check the OoC performance of the proposed charts, shifts of different amount are introduced in dispersion parameter. That shift parameter is denoted by θ and is equal to $\theta = \frac{\sigma_1}{\sigma_0}$ where σ_1 is the OoC process standard deviation.

The run length study for S^2 chart under SRS is reported in Table 2. The findings indicate that an upward shift (20%) in dispersion parameter from the in-control situation resulted about 55.04% and 60.36% decrease in ARL_1 of S^2 chart under SRS for both cases i.e. $n = 5$ and 7 respectively.

The results for S^2 chart under MSS at fixed $c = 2$ are given in Table 3. If the choice of quantiles pair is ($Q_{0.25}$, $Q_{0.75}$) then (30%) upward shift in dispersion parameter, may decrease 80.47% and 83.83% ARL_1 of said S^2 chart for both cases i.e. $n = 5$ and 7 respectively. Moreover, when choice of quantiles pair is ($Q_{0.45}$, $Q_{0.55}$) then (50%) upward shift in dispersion parameter, may decrease 88.42% and 91.73% ARL_1 of said S^2 chart for both cases i.e. $n = 5$ and 7 respectively.

Finally, the run length study for S^2 chart under MSS at fixed $c = 3$ is reported in Table 4. If the choice of quantiles is ($Q_{0.30}$, $Q_{0.50}$, $Q_{0.70}$) then an upward 40% shift in dispersion parameter decreases the ARL_1 of said S^2 chart for both cases ($n = 5$ and 7) to 61.64 and 45.49 respectively. Further, when choice of quantiles is ($Q_{0.40}$, $Q_{0.50}$, $Q_{0.60}$) then an upward 60% shift in dispersion parameter may decrease up to 36.65 and 24.51 ARL_1 of the said S^2 chart for both cases i.e. $n = 5$ and 7 respectively. Considering the different sample sizes (i.e. $n = 5$ and 7), number of observations from previous sample (i.e. $c = 2$ and 3), shifts in dispersion parameter θ (on horizontal axis) and log average run length (ln (ARL)) (on vertical axis), we have portrayed the display in Figure 1. The results depict that the performance of charts increase with the increase of shift in dispersion parameter $\theta = 1.00$ up to 2.00. It is also examined that S^2 chart under MSS with choice of quantile pairs ($Q_{0.25}$, $Q_{0.75}$) and ($Q_{0.25}$, $Q_{0.50}$, $Q_{0.75}$), outperforms all other S^2 charts under different schemes. Finally, in terms of industrial practice, S^2 control chart based on SRS is based on the variations at single time point while proposed S^2 based on MSS is based on the shifts at various points in time which enables the practitioners to check the continuous performance of any process more efficiently.

IV. ILLUSTRATIVE EXAMPLE

Now a days electrical engineers take interest in the Z-source inverter for a grid connected PV system instead of conventional voltage source inverter (VSI) and conventional current source inverter (CSI). The Z-source inverter is preferable due to a significant property i.e. buck-boost inverter which eliminates the need of buck-boost converter and also overcomes various problems associated with conventional inverters. The physical structure of Z-source inverter consists of two capacitors, two inductors and 3-φ bridge inverter connected. The switches used in bridge circuit can have either series or anti-parallel diodes as shown in Figure 2.

The grid-connected PV system (shown in Figure 2) works in several steps given as; (i) To get the maximum available power from PV system, output of PV arrays is connected to DC link capacitor through maximum power point tracker (MPPT). (ii) To maintain the constant voltage at

TABLE 2. Run length Properties of S^2 under SRS.

θ	$n = 5$					$n = 7$				
	ARL	$SDRL$	$Q_{0.25}$	$Q_{0.75}$	$Q_{0.95}$	ARL	$SDRL$	$Q_{0.25}$	$Q_{0.75}$	$Q_{0.95}$
1.00	369.90	370.87	106.00	513.00	1110.00	373.15	374.60	108.00	516.00	1119.00
1.10	253.75	253.87	73.00	353.00	758.05	245.81	244.15	71.00	342.00	732.00
1.20	166.31	166.58	48.00	231.00	497.00	147.92	147.50	43.00	205.00	443.00
1.30	109.23	108.91	32.00	151.00	325.00	90.80	90.53	26.00	125.00	271.00
1.40	74.33	73.69	22.00	103.00	222.00	59.09	58.74	17.00	82.00	176.00
1.50	52.68	52.23	15.00	73.00	157.00	40.15	39.65	12.00	56.00	119.00
1.60	38.86	38.56	11.00	54.00	115.00	28.57	28.14	8.00	39.00	85.00
1.70	29.79	29.36	9.00	41.00	89.00	21.22	20.72	6.00	29.00	62.00
1.80	23.43	22.87	7.00	32.00	70.00	16.61	16.11	5.00	23.00	49.00
1.90	18.93	18.46	6.00	26.00	56.00	13.21	12.76	4.00	18.00	39.00
2.00	15.58	15.00	5.00	21.00	46.00	10.78	10.32	3.00	15.00	31.00

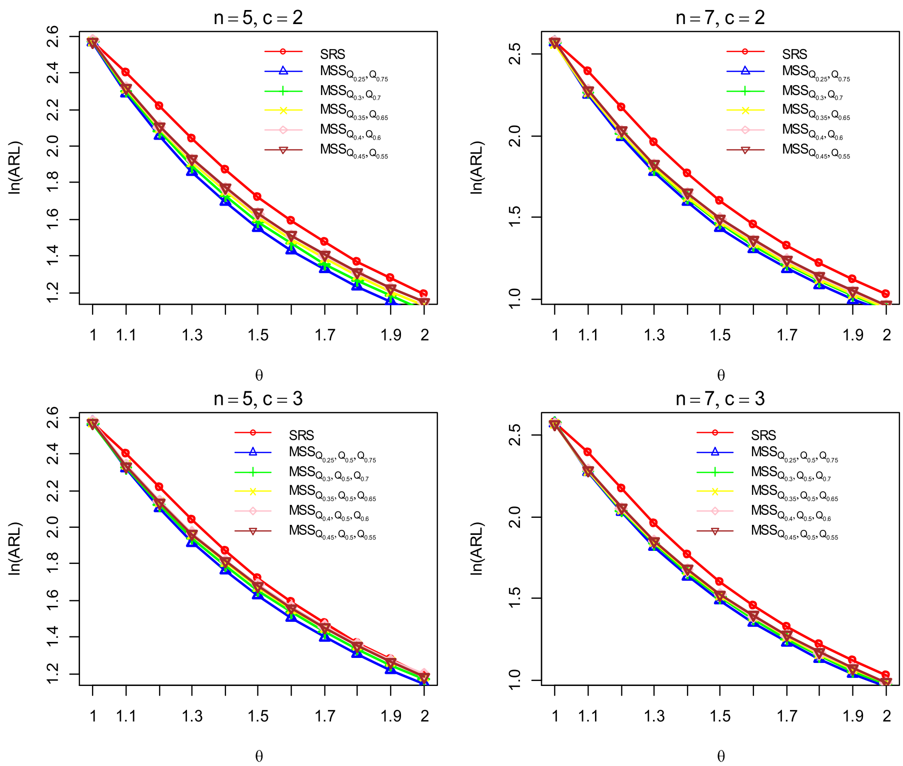

FIGURE 1. Comparative analysis of S^2charts under different schemes.

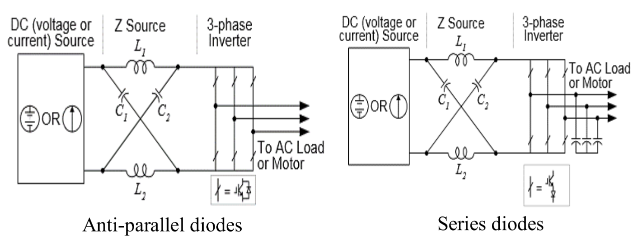

FIGURE 2. The Z-source inverter with respect to different switches.

DC link, the MPPT control is connected to DC-DC boost converter by adjusting the duty cycle of boost converter. (iii) Finally, DC voltages are converted to AC through inverter and connected to local grid. A diagram of grid-connected PV system is shown in Figure 3.

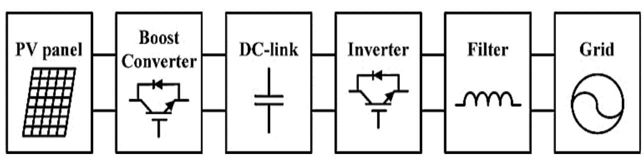

FIGURE 3. Diagram of PV system connected with Grid station.

Usually, parallel plate capacitors are used as a DC link which consists of two conductive plates separated by a dielectric material. In a parallel plate capacitor, capacitance is directly proportional to the surface area of the conductive

TABLE 3. Run length properties of S^2 chart under MSS at fixed $c = 2$.

Schemes	θ	$n = 5$					$n = 7$				
		ARL	$SDRL$	$Q_{0.25}$	$Q_{0.75}$	$Q_{0.95}$	ARL	$SDRL$	$Q_{0.25}$	$Q_{0.75}$	$Q_{0.95}$
$MSS_{Q_{0.25},Q_{0.75}}$	1.00	369.56	369.08	106.00	513.00	1109.00	369.21	371.56	105.00	511.25	1110.00
	1.10	193.53	194.28	55.00	269.00	579.00	177.70	177.88	51.00	247.00	536.00
	1.20	113.28	113.41	32.00	157.00	340.00	97.62	97.61	28.00	136.00	292.00
	1.30	72.18	72.09	21.00	100.00	217.00	59.70	60.09	17.00	83.00	179.00
	1.40	49.45	49.48	14.00	69.00	148.00	39.16	39.31	11.00	54.00	118.00
	1.50	35.53	35.46	10.00	49.00	106.00	27.24	27.26	8.00	38.00	81.00
	1.60	26.97	26.90	8.00	37.00	81.00	20.18	20.37	6.00	28.00	61.00
	1.70	21.23	21.13	6.00	29.00	63.00	15.36	15.33	4.00	21.00	46.00
	1.80	17.04	16.96	5.00	24.00	51.00	12.23	12.15	3.00	17.00	37.00
	1.90	14.09	14.00	4.00	19.00	42.00	9.94	9.81	3.00	14.00	30.00
	2.00	11.88	11.78	3.00	16.00	35.00	8.32	8.22	2.00	11.00	25.00
$MSS_{Q_{0.30},Q_{0.70}}$	1.00	371.31	371.98	106.00	514.00	1110.00	371.72	375.07	105.00	515.25	1120.00
	1.10	200.19	202.15	56.00	278.00	607.00	183.68	185.50	52.00	255.00	551.00
	1.20	119.88	121.58	33.00	166.00	362.00	102.47	103.13	29.00	143.00	309.00
	1.30	77.33	78.68	21.00	108.00	234.00	62.62	63.46	17.00	87.00	189.00
	1.40	53.30	54.19	14.00	74.00	161.00	41.59	42.56	11.00	58.00	126.00
	1.50	38.63	39.71	10.00	54.00	118.00	29.24	29.94	8.00	41.00	89.00
	1.60	29.29	30.11	8.00	41.00	89.00	21.33	21.85	6.00	30.00	65.00
	1.70	22.69	23.32	6.00	32.00	70.00	16.33	16.61	4.00	23.00	50.00
	1.80	18.49	19.18	5.00	26.00	57.00	12.93	13.27	3.00	18.00	39.00
	1.90	15.15	15.69	4.00	21.00	46.00	10.51	10.77	3.00	15.00	32.00
	2.00	12.70	13.19	3.00	18.00	39.00	8.65	8.82	2.00	12.00	26.00
$MSS_{Q_{0.35},Q_{0.65}}$	1.00	374.12	375.77	106.00	521.00	1122.00	365.67	367.15	104.00	509.00	1096.00
	1.10	206.36	208.55	57.00	288.00	623.00	184.49	185.84	52.00	256.00	558.00
	1.20	124.14	126.43	34.00	173.00	376.00	103.32	105.21	29.00	143.00	313.00
	1.30	81.44	83.01	22.00	114.00	246.00	63.94	65.25	17.00	89.00	194.00
	1.40	56.46	58.17	15.00	79.00	173.00	42.17	43.11	11.00	59.00	128.00
	1.50	41.10	42.65	11.00	58.00	126.00	29.71	30.59	8.00	42.00	91.00
	1.60	31.10	32.49	8.00	44.00	96.00	22.06	22.76	6.00	31.00	68.00
	1.70	24.32	25.58	6.00	34.00	76.00	16.70	17.38	4.00	23.00	52.00
	1.80	19.49	20.61	5.00	27.00	61.00	13.20	13.84	3.00	18.00	41.00
	1.90	16.00	16.99	4.00	23.00	50.00	10.63	11.12	3.00	15.00	33.00
	2.00	13.40	14.31	3.00	19.00	42.00	8.89	9.28	2.00	12.00	27.00
$MSS_{Q_{0.40},Q_{0.60}}$	1.00	376.03	379.63	106.00	521.00	1134.00	375.33	378.75	105.00	522.00	1129.00
	1.10	207.91	211.00	57.00	290.00	628.00	189.03	191.70	53.00	262.00	573.00
	1.20	127.41	129.93	35.00	178.00	386.00	107.02	108.71	29.00	149.00	324.00
	1.30	84.13	86.42	22.00	118.00	257.00	66.26	67.76	18.00	92.00	202.00
	1.40	58.41	60.58	15.00	82.00	179.05	44.02	45.16	12.00	61.00	134.00
	1.50	42.51	44.10	11.00	60.00	131.00	30.86	31.93	8.00	43.00	95.00
	1.60	32.41	34.13	8.00	45.00	101.00	22.66	23.57	6.00	32.00	70.00
	1.70	25.32	26.93	6.00	36.00	79.00	17.35	18.33	4.00	24.00	54.00
	1.80	20.21	21.49	5.00	28.00	63.00	13.68	14.42	3.00	19.00	42.00
	1.90	16.72	17.94	4.00	24.00	53.00	11.10	11.72	3.00	16.00	35.00
	2.00	14.02	15.26	3.00	20.00	45.00	9.09	9.62	2.00	13.00	29.00
$MSS_{Q_{0.45},Q_{0.55}}$	1.00	371.87	373.74	105.00	518.00	1122.00	376.04	377.20	107.00	525.00	1120.00
	1.10	208.89	211.68	58.00	291.00	637.00	188.64	190.19	53.00	262.00	570.00
	1.20	128.23	131.36	35.00	179.00	390.00	107.90	109.52	30.00	150.00	327.00
	1.30	85.32	88.56	22.00	120.00	261.00	66.78	68.19	18.00	93.00	201.00
	1.40	59.26	61.36	15.00	83.00	182.00	44.49	46.04	12.00	62.00	136.00
	1.50	43.07	45.14	11.00	60.00	134.00	31.10	32.17	8.00	44.00	96.00
	1.60	32.60	34.48	8.00	46.00	102.00	23.10	24.01	6.00	32.00	71.00
	1.70	25.61	27.48	6.00	36.00	81.00	17.55	18.42	4.00	25.00	55.00
	1.80	20.54	22.18	4.00	29.00	65.00	13.87	14.59	3.00	19.00	43.00
	1.90	16.79	18.27	3.00	24.00	54.00	11.22	11.90	3.00	16.00	35.00
	2.00	14.06	15.44	3.00	20.00	45.00	9.22	9.81	2.00	13.00	29.00

plates and inversely proportional to the distance between them. If the charge on the plates are $+q$ and $-q$, and V is the potential difference between the plates, then the capacitance C is given by

$$C \propto \frac{Q}{V}$$

For the illustrative example, we get 75456 sample values of Voltage (V) against each level of Capacitance (C) given in Mukhtar [28]. There exist 7 different capacitance levels such as, $50\mu F$, $100\mu F$, $150\mu F$, $200\mu F$, $250\mu F$, $300\mu F$ and $350\mu F$. In the stated study, we consider 75455 values of Voltage (V) against $150\mu F$, $250\mu F$ and $350\mu F$ capacitance level which are further divided into 15091 subgroups each of size 5.

For the classical Shewhart S^2 chart, we estimate sample variance of each subgroup belongs to $350\mu F$ capacitance level and through it we calculate the lower

TABLE 4. Run length properties of S^2 chart under MSS at fixed $c = 3$.

Scheme	θ	$n = 5$					$n = 7$				
		ARL	$SDRL$	$Q_{0.25}$	$Q_{0.75}$	$Q_{0.95}$	ARL	$SDRL$	$Q_{0.25}$	$Q_{0.75}$	$Q_{0.95}$
$MSS_{Q_{0.25},Q_{0.50},Q_{0.75}}$	**1.00**	373.83	379.94	102.00	520.00	1134.00	374.13	377.80	105.00	521.00	1129.00
	1.10	208.43	213.70	56.00	290.00	636.00	187.60	191.13	52.00	260.00	571.00
	1.20	126.35	130.14	33.00	177.00	386.00	106.39	108.87	29.00	149.00	322.00
	1.30	82.62	85.98	21.00	116.00	254.00	65.75	68.32	17.00	92.00	203.00
	1.40	58.02	61.31	14.00	81.00	182.00	43.29	45.25	11.00	61.00	134.00
	1.50	42.32	45.48	10.00	60.00	133.00	30.70	32.42	7.00	43.00	95.00
	1.60	31.92	34.57	7.00	45.00	101.00	22.47	23.80	5.00	32.00	71.00
	1.70	25.01	27.42	5.00	35.00	80.00	17.15	18.35	4.00	24.00	54.00
	1.80	20.22	22.22	4.00	29.00	65.00	13.51	14.64	3.00	19.00	43.00
	1.90	16.63	18.44	3.00	24.00	54.00	10.92	11.96	2.00	15.00	35.00
	2.00	13.88	15.63	2.00	20.00	45.00	9.09	9.88	2.00	13.00	29.00
$MSS_{Q_{0.30},Q_{0.50},Q_{0.70}}$	**1.00**	368.52	376.50	99.00	515.00	1121.00	374.10	379.64	104.00	522.00	1129.00
	1.10	211.02	219.56	55.00	296.00	650.00	190.71	195.74	51.00	265.00	580.00
	1.20	131.91	139.26	32.00	186.00	409.00	108.75	112.75	28.00	152.00	335.00
	1.30	87.46	93.80	20.00	123.00	276.00	68.32	71.99	17.00	96.00	212.00
	1.40	61.64	66.89	13.00	87.00	196.00	45.49	48.13	11.00	64.00	141.00
	1.50	45.13	50.20	9.00	64.00	145.00	32.17	34.72	7.00	46.00	102.00
	1.60	34.21	38.58	6.00	49.00	112.00	23.62	25.91	5.00	33.00	75.00
	1.70	26.68	30.66	4.00	38.00	88.00	17.98	19.77	3.00	26.00	57.00
	1.80	21.61	25.21	3.00	31.00	73.00	14.12	15.83	2.00	20.00	46.00
	1.90	17.65	20.85	2.00	25.00	60.00	11.36	12.81	2.00	16.00	37.00
	2.00	14.78	17.64	2.00	21.00	50.00	9.40	10.69	1.00	13.00	31.00
$MSS_{Q_{0.35},Q_{0.50},Q_{0.65}}$	**1.00**	375.11	385.43	99.00	524.00	1151.00	367.13	372.65	102.00	511.00	1111.00
	1.10	217.49	228.07	55.00	306.00	673.00	190.47	195.58	51.00	265.00	582.00
	1.20	135.84	145.65	32.00	191.00	428.00	110.47	114.50	29.00	155.00	340.00
	1.30	91.23	99.84	19.00	129.00	292.00	69.39	73.47	17.00	97.00	216.00
	1.40	64.46	71.81	12.00	92.00	209.00	46.38	49.53	11.00	66.00	146.00
	1.50	47.28	53.58	8.00	67.00	155.00	32.68	35.63	7.00	47.00	105.00
	1.60	35.71	41.39	5.00	51.00	119.00	24.33	26.87	5.00	34.00	78.00
	1.70	28.14	33.37	4.00	40.00	95.00	18.45	20.73	3.00	26.00	60.00
	1.80	22.69	27.22	3.00	32.00	78.00	14.44	16.40	2.00	21.00	47.00
	1.90	18.46	22.64	2.00	26.00	65.00	11.61	13.32	2.00	16.00	39.00
	2.00	15.32	19.01	1.00	22.00	54.00	9.58	11.14	1.00	14.00	32.00
$MSS_{Q_{0.40},Q_{0.50},Q_{0.60}}$	**1.00**	376.46	388.17	98.00	529.00	1151.00	368.15	375.18	101.00	512.00	1116.00
	1.10	219.78	232.80	53.00	308.00	685.00	193.73	198.72	52.00	269.25	592.00
	1.20	138.21	149.28	31.00	195.00	438.00	112.05	116.81	28.00	157.00	345.05
	1.30	92.89	101.98	19.00	132.00	297.00	70.82	74.91	17.00	100.00	219.00
	1.40	65.66	73.55	12.00	93.00	213.00	47.19	50.86	10.00	67.00	149.00
	1.50	48.34	55.43	8.00	69.00	159.00	33.43	36.61	7.00	48.00	107.00
	1.60	36.65	43.04	5.00	53.00	123.00	24.51	27.41	4.00	35.00	79.00
	1.70	28.73	34.63	3.00	41.00	98.00	18.72	21.27	3.00	27.00	62.00
	1.80	22.88	28.14	2.00	33.00	80.00	14.62	16.80	2.00	21.00	48.00
	1.90	18.65	23.12	2.00	27.00	65.00	11.83	13.71	2.00	17.00	40.00
	2.00	15.66	19.72	1.00	22.00	55.00	9.68	11.36	1.00	14.00	33.00
$MSS_{Q_{0.45},Q_{0.50},Q_{0.55}}$	**1.00**	370.20	386.05	94.00	519.00	1144.00	369.79	376.85	101.00	517.00	1123.00
	1.10	215.30	229.16	51.00	304.00	675.00	193.10	197.78	51.00	271.00	588.00
	1.20	136.48	148.08	29.00	194.00	432.00	113.43	118.53	29.00	159.00	350.00
	1.30	91.67	102.09	17.00	131.00	297.00	71.11	75.48	17.00	101.00	222.00
	1.40	65.05	73.81	11.00	93.00	214.00	47.94	51.99	10.00	68.00	152.00
	1.50	47.89	55.95	7.00	69.00	161.00	33.56	36.85	7.00	48.00	107.00
	1.60	36.22	43.49	5.00	52.00	124.00	24.99	27.98	5.00	35.00	81.00
	1.70	28.32	34.58	3.00	40.00	99.00	18.88	21.49	3.00	27.00	62.00
	1.80	22.61	28.12	2.00	32.00	80.00	14.85	17.18	2.00	21.00	49.00
	1.90	18.39	23.37	1.00	26.00	66.00	11.85	13.94	1.00	17.00	40.00
	2.00	15.15	19.54	1.00	21.00	55.00	9.73	11.51	1.00	14.00	33.00

control limit $LCL_{SRS} = 0.01442126$ and upper control limit $UCL_{SRS} = 2.46252780$. For the diagnosis purpose, we select first 100 subgroups from $350\mu F$, second 100 subgroups from $250\mu F$ and finally last 100 subgroups from $150\mu F$. We calculate sample variances (S^2_{SRS}) of selected 300 subgroups which are plotted against the control limits in Figure 4.

The classical Shewhart S^2 chart depicts that there exists no S^2_{SRS} out of control point in first 100 subgroups, in next 100 subgroups 10 S^2_{SRS} are declared out of control and in last 100 subgroups 70 S^2_{SRS} are declared out of control.

On the other hand, for the Shewhart S^2 chart under MSS, we estimate sample variances of subgroups after implemented the modified successive sampling on the existing subgroups and calculate the control limits (i.e. $LCL_{MSS} = 0.01833551$ and $UCL_{MSS} = 1.98684658$). For the diagnosis purpose, we select first 100 subgroups from $350\mu F$,

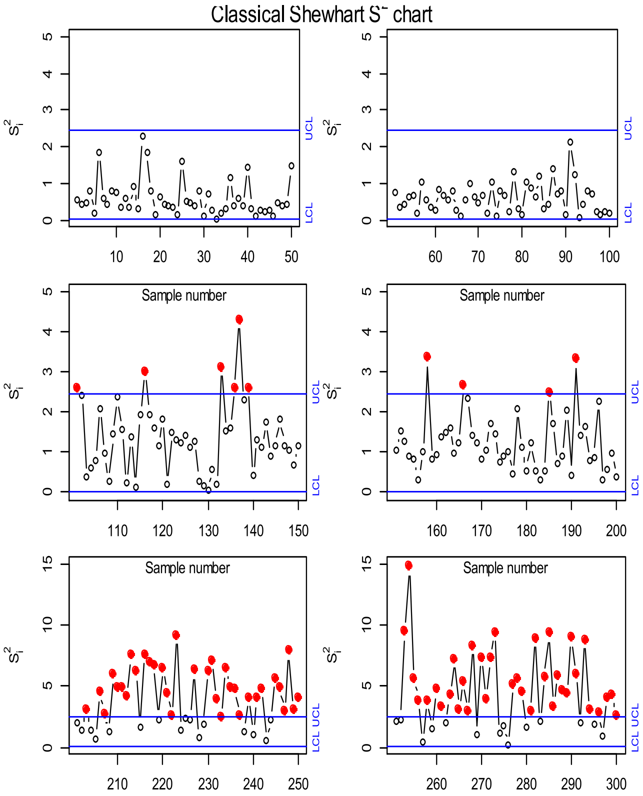

FIGURE 4. Portrayed of Illustrative example under SRS.

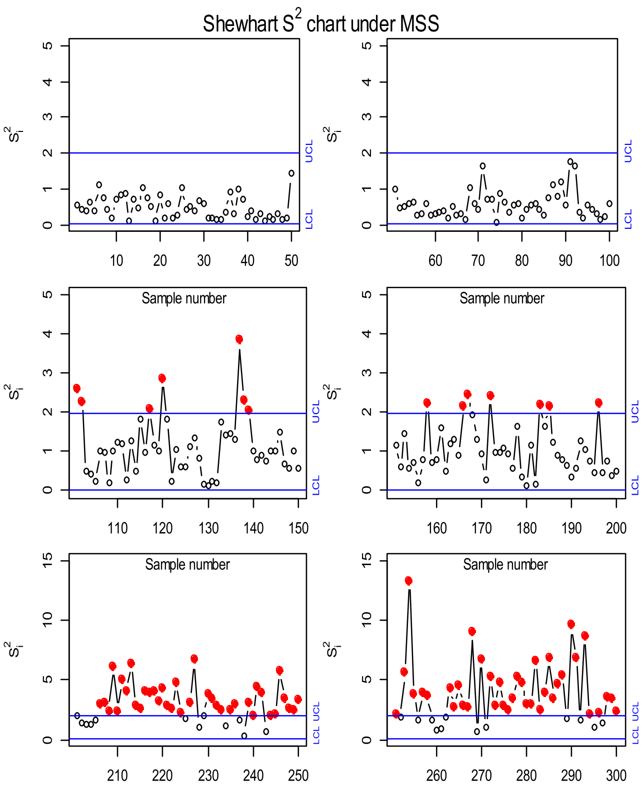

FIGURE 5. Portrayed of Illustrative example under MSS.

second 100 subgroups from $250\mu F$ and finally last 100 subgroups from $150\mu F$. We calculate sample variances (S^2_{MSS}) of selected 300 subgroups which are plotted against the control limits in Figure 5. The chart reveals no S^2_{MSS} out of control point in first 100 subgroups, in next 100 subgroups 14 S^2_{MSS} are declared out of control and in last 100 subgroups 76 S^2_{MSS} are declared out of control.

V. SUMMARY, CONCLUSION AND RECOMMENDATIONS

Generally, process is declared out of control due to a shift in the dispersion or location parameter of the process. Practically, detection of dispersion shift is important before the detection of location shift in the process. The classical S^2 chart is the best choice from the literature for the monitoring of the dispersion parameter. This study proposes a S^2 chart improved by using modified successive sampling, which has an advantage of cost optimization as compare to simple random sampling.

The run length properties are selected as performance measures which depict that all proposed schemes of S^2 chart under modified successive sampling outperforms the classical S^2 chart. Moreover, we can also use several run rules to get more efficiency of the proposed control chart under different schemes.

VI. LIMITATIONS OF THE STUDY

The performance of the proposed chart is evaluated for the industrial processes that operate under the ideal assumption of normality e.g. hard bake of wafers, voltage from z-source inverter and inner diameter measurements of engine piston rings etc. One may extend this study by assessing the performance of non-normal industrial processes such as insulation resistance, surface finishing, roundness, mold dimensions, customer waiting times and nuclear reactions.

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