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# Data-Driven Robust Non-Fragile Filtering for Cyber-Physical Systems

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**ABSTRACT** Filtering or state estimation plays an important role in the cyber-physical systems (CPSs). This paper aims to solve the data-driven non-fragile filtering problem for the cyber-physical system. Randomly occurring gain variations are considered so as to account for the parameter fluctuations occurring during the filter implementation. The data-driven communication mechanism is utilized to reduce the measurement transmission frequency and save energy for the CPSs. Therefore, a unified $H_\infty$ filtering framework that combines the data-driven communication mechanism and the non-fragility of filters is constructed. Based on this unified framework, the influence of the simultaneous presence of networked-induced packet dropouts, quantization, randomly occurring nonlinearities and randomly occurring parameter uncertainties in the CPS is investigated. A modified dropouts model is proposed under the data-driven communication mechanism. By utilizing stochastic analysis and Lyapunov functional theory, sufficient conditions guaranteeing the filtering performance are derived. The $H_\infty$ filter is obtained through the proposed algorithm. Last, a simulation is given to demonstrate the filtering method for CPS in this paper.

**INDEX TERMS** Cyber-physical systems (CPS), data-driven communication mechanism, ROGVs, data-driven non-fragile filter, quantization effects, packet dropouts.

## I. INTRODUCTION

In recent years, the cyber-physical systems which consist of not only computing and communication technologies, but also physical processes, have become an attractive research area [1], [2]. Moreover, the filtering or state estimation problem for cyber-physical systems has attracted considerable attention due to its importance in theory and comprehensive existence in a variety of cyber-physical systems, such as signal processing, smart grid, control engineering and target tracking [3], [4]. As such, many filtering methods have been proposed, for example, Kalman filtering and $H_\infty$ filtering. Kalman filter has been widely applied in practical engineering since it was proposed in 1960. However, it is necessary to point out that, the Kalman filtering is not useful for systems with parameter uncertainties or noises whose statistics are unknown or only partially known. Fortunately, the $H_\infty$ filtering that doesn't need statistical information of the noise signal can be well applied for systems with parameter uncertainties [5]. Moreover, the $H_\infty$ filter is more robust than the Kalman filer [6]. Therefore, the $H_\infty$ filtering technique has attracted persistent research attention. For example, $H_\infty$ filtering problems have been solved in [7] and [8] for time-delay systems, in [9] for linear uncertain systems, in [10] and [11] for stochastic systems, in [12] for fuzzy systems, and in [13] and [14] for nonlinear systems, etc. Also, $H_\infty$ performance index has been studied for fault diagnosis problem for two-dimensional systems in [15]. As such, we will design the $H_\infty$ filter for cyber-physical system in this paper.

As is known to all, CPSs contain colossal but redundant system information which is exchanged between different units through the communication network. It is of great significance to design effective strategies capable of reducing the data transmission frequency. In recent years, the data-driven strategy has attracted a lot of attention. Compared with the conventional time-driven communication mechanism that has been implicitly adopted by most filter algorithms, the data-driven strategy could reduce the signal communication frequency while maintaining guaranteed filtering performance. Thus, when the limited channel bandwidth is concerned, the data-driven communication mechanism is particularly significant for cyber-physical systems due to its advantage of reducing the communication burden and saving energy. So far, the data-driven strategies have been successfully applied in various cyber-physical objects, for example, network-based control systems [16], [17], sensor systems [19], neural networks [20] and multi-agent systems [21], [22]. However, in the field of filtering problem, the corresponding data-driven filter design for cyber-physical systems is relatively few.

It has been well known that, in practical cyber-physical systems, limited channel bandwidth and complex network circumstance inevitably cause random network-induced phenomena, for example, data missing or packet dropouts [23]–[25], signal quantization [8], [26], [27], RONs [28], [29], ROUs [30], [31], etc. It is worth pointing out that, if not properly coped with, these network-induced phenomena will seriously degrade the performance of CPSs. As such, significant research about filtering problem for cyber-physical systems with various network-induced phenomena has been carried out. Unfortunately, when the data-driven communication mechanism is adopted, the corresponding data-based filter design for cyber-physical systems with RONs, packet dropouts, quantization effect and ROUs hasn't been fully investigated. Therefore, we desire to examine how the simultaneous presence of these four phenomena affects the data-based filtering performance for CPSs.

Note that all the above filter algorithms implicitly assume that the devised filter could be precisely executed. However, fluctuations and uncertainties may happen during filter working, which are resulted from the finite word length of digital systems, numerical roundoff errors, and so on. In [32], Keel has proved that a tiny perturbation in the implementation of controller may have a significant impact on system performance. Therefore, the filter should be designed to tolerate some level of perturbation in its gain, that is to say, the filter is resilient or non-fragile. Also, some attention has been attracted to filter gain variations issue and relevant results have appeared (see, e.g. [33]–[38]). Furthermore, in networked circumstance, the filter gain may vary randomly. For example, filter parameters transmitted by network channel may be randomly varied because of limited bandwidth and complex network environmental circumstances. As such, ROGVs will be considered in the filter design problem. Very recently, in [39], the ROGVs phenomenon has begun to be investigated for $H_\infty$ filter problem. Unfortunately, up to now, relative filtering study that considers non-fragility of filter and data-driven transmission mechanism in a unified framework has seldom been done. Therefore, researching the data-driven non-fragile finite-horizon filtering for cyber-physical systems is an attractive question.

Motivated by the above discussions, we will study the data-driven non-fragile $H_\infty$ filtering problem for a class of cyber-physical systems which will be described in Section II. Both the data-driven communication mechanism and the non-fragility of filters are considered in a unified framework. Under constructed unified framework, several common network-induced phenomena, i.e., packet dropouts, ROUs, RONs and quantization effects are investigated about their influence on filtering performance. Sufficient conditions are derived for filtering issue of proposed cyber-physical systems. The main contributions are listed below:

1) The data-driven communication mechanism is introduced into the non-fragile filtering problem for cyber-physical systems. Thus, a unified $H_\infty$ filtering framework combining the data-driven communication mechanism and the non-fragility of filters is constructed, which could reduce the communication burden and decrease conservatism of filter design.

2) The considered cyber-physical system is fairly comprehensive, which is subject to ROUs, packet dropouts, quantization effects and ROGVs. The corresponding data-driven non-fragile filter algorithm is properly addressed.

3) The developed data-driven non-fragile filter design algorithm is suitable for online application due to its recursive characteristic.

The cyber-physical system is introduced and the problem is formulated in Section II. In Section III, sufficient condition for the data-driven filter is derived. The filter design algorithm is proposed in Section IV and an example is provided in Section V. Finally, we summarize in Section VI.

## II. PROBLEM FORMULATION AND PRELIMINARIES

Here, the physical process defined on $\kappa \in [0, N]$ will be studied:

$$\begin{cases} \chi(\kappa+1) = (\mathcal{G}(\kappa) + \rho(\kappa)\Delta\mathcal{G}(\kappa))\chi(\kappa) \\ \qquad\qquad + \varrho(\kappa)\mathcal{P}(\kappa, \chi(\kappa)) \\ \qquad\qquad + (1-\varrho(\kappa))\mathcal{Q}(\kappa, \chi(\kappa)) \\ \qquad\qquad + \mathcal{W}_1(\kappa)\nu(\kappa) \\ \tilde{y}(\kappa) = \mathcal{H}(\kappa)\chi(\kappa) + \mathcal{W}_2(\kappa)\vartheta(\kappa) \\ z(\kappa) = \mathcal{Z}(\kappa)\chi(\kappa) \end{cases} \tag{1}$$

where $\chi(\kappa) \in \mathbb{R}^n$ is the state vector, $\tilde{y}(\kappa) \in \mathbb{R}^r$ is the process output, $z(\kappa) \in \mathbb{R}^m$ is the signal to be estimated, $\nu(\kappa) \in \mathbb{R}^p$ and $\vartheta(\kappa) \in \mathbb{R}^q$ are the external disturbance signals that belong to $l_2[0, N-1]$, $\mathcal{P}(.,.) : R^+ \times R^n \to R^n$ and $\mathcal{Q}(.,.) : R^+ \times R^n \to R^n$ are nonlinear functions that represent the nonlinearity of the system. $\mathcal{G}(\kappa)$, $\mathcal{H}(\kappa)$, $\mathcal{W}_1(\kappa)$, $\mathcal{W}_2(\kappa) and \mathcal{Z}(\kappa)$ are known matrices.

$\Delta\mathcal{G}(\kappa)$ represents the norm-bounded parameter uncertainty described as

$$\Delta\mathcal{G}(\kappa) = H_{\mathcal{G}}(\kappa)F_{\mathcal{G}}(\kappa)E_{\mathcal{G}}(\kappa) \tag{2}$$

where $H_{\mathcal{G}}(\kappa)$ and $E_{\mathcal{G}}(\kappa)$ are known time-varying matrices and $F_{\mathcal{G}}(\kappa)$ is an unknown matrix satisfying

$$F_{\mathcal{G}}^T(\kappa)F_{\mathcal{G}}(\kappa) \leq I \tag{3}$$

The mutually independent Bernoulli distributed stochastic variables $\rho(\kappa)$ and $\varrho(\kappa)$ in (1) are used to describe ROUs and RONs, which take values on 0 or 1 with

$$\begin{aligned} \mathrm{Prob}\{\rho(\kappa)=1\} &= \mathbb{E}\{\rho(\kappa)\} = \bar{\rho} \\ \mathrm{Prob}\{\rho(\kappa)=0\} &= 1-\bar{\rho} \\ \mathrm{Prob}\{\varrho(\kappa)=1\} &= \mathbb{E}\{\varrho(\kappa)\} = \bar{\varrho} \\ \mathrm{Prob}\{\varrho(\kappa)=0\} &= 1-\bar{\varrho} \end{aligned} \tag{4}$$

respectively, where $\bar{\rho} \in [0, 1]$ and $\bar{\varrho} \in [0, 1]$ are known constants.

$\mathcal{P}(\kappa, \chi(\kappa))$ and $\mathcal{Q}(\kappa, \chi(\kappa))$ are nonlinear functions, assumed to satisfy $\mathcal{P}(\kappa, 0) = 0$, $\mathcal{Q}(\kappa, 0) = 0$ and

$$\begin{aligned} \left\|\mathcal{P}(\kappa, \chi(\kappa)+\delta(\kappa)) - \mathcal{P}(\kappa, \chi(\kappa))\right\| &\leq \left\|\mathcal{E}_1(\kappa)\delta(\kappa)\right\| \\ \left\|\mathcal{Q}(\kappa, \chi(\kappa)+\delta(\kappa)) - \mathcal{Q}(\kappa, \chi(\kappa))\right\| &\leq \left\|\mathcal{E}_2(\kappa)\delta(\kappa)\right\| \end{aligned} \quad (5)$$

where $\delta(\kappa)$ is a vector, and $\mathcal{E}_1(\kappa)$ and $\mathcal{E}_2(\kappa)$ are known matrices.

*Remark 1:* It is well known that nonlinearity is a common phenomenon for most practical plant and it has significant influence on system. In the networked systems, nonlinearity may experience abrupt changes and usually occurs randomly arising from the networked-induced problem, such as random changes and failures in the environmental circumstance. In this paper, $\mathcal{P}(.,.)$, $\mathcal{Q}(.,.)$ are used to depict the phenomenon of RONs. Random variable $\varrho(\kappa)$ is employed to govern their random nature and switch between each other.

In a cyber-physical system, the measurement signals are often quantized during the transmission process. Here, quantization is considered. Choose the similar logarithmic quantizer in [41] denoted as

$$\mathcal{L}(\tilde{y}(\kappa)) = \left[\, \mathcal{L}_1(\tilde{y}^{(1)}(\kappa)) \;\; \mathcal{L}_2(\tilde{y}^{(2)}(\kappa)) \;\; \cdots \;\; \mathcal{L}_r(\tilde{y}^{(r)}(\kappa)) \,\right]^T$$

Define $\Delta(\kappa) = \mathrm{diag}\{\Delta^{(1)}(\kappa), \Delta^{(2)}(\kappa) \cdots, \Delta^{(r)}(\kappa)\}$ and $\bar{\Delta} = \mathrm{diag}\{\delta_1, \delta_2, \cdots, \delta_r\}$ ($\delta_j = \frac{1-\chi_j}{1+\chi_j}$, $\chi_j (j = 1, 2, \cdots, r)$ is called the quantization density, $|\Delta^{(j)}(\kappa)| \leq \delta_j$). Similar to the analysis in [41], we can obtain an unknown real-valued time-varying matrix $F(\kappa) = \Delta(\kappa)\bar{\Delta}^{-1}$ satisfying $F(\kappa)F^T(\kappa) \leq I$ [41].

Then the quantized signals are described as

$$\begin{aligned} \bar{y}(\kappa) &= \mathcal{L}(\tilde{y}(\kappa)) = (I + \Delta(\kappa))\tilde{y}(\kappa) \\ &= (I + \Delta(\kappa))(\mathcal{H}(\kappa)\chi(\kappa) + \mathcal{W}_2(\kappa)\vartheta(\kappa)) \end{aligned} \quad (6)$$

Considering the limited bandwidth of network in cyber-physical systems, it has great significance to reduce data communication frequency. In this paper, the data-driven communication mechanism is adopted. For this purpose, such an data-driven function $e(\cdot, \cdot)$ is defined as:

$$e(r(\kappa), \delta) = r^T(\kappa)\Omega r(\kappa) - \delta\bar{y}^T(\kappa)\Omega\bar{y}(\kappa) \quad (7)$$

Here, $r(\kappa) = \bar{y}(\kappa_i) - \bar{y}(\kappa)$, where $\bar{y}(\kappa_i)$ is quantized sensor output at the latest transition time $\kappa_i$ and $\bar{y}(\kappa)$ is the current measurement. $\Omega$ is a symmetric positive-definite matrix and $\delta \in [0, 1)$ is the threshold [18].

The execution (i.e., $\bar{y}(\kappa)$ is transmitted to the filter) condition is

$$e(r(\kappa), \delta) > 0 \quad (8)$$

When the phenomenon of packet dropouts is considered, the information received by the filter is

$$y(\kappa_i) = \varsigma(\kappa)\bar{y}(\kappa_i) + (1 - \varsigma(\kappa))y(\kappa_{i-1}) \quad (9)$$

The stochastic variable $\varsigma(\kappa)$ in (9) is introduced to account for network-induced phenomenon of packet dropouts in cyber-physical systems. It is Bernoulli distributed white sequence uncorrelated to $\rho(\kappa)$ and $\varrho(\kappa)$ taking values on 0 or 1 with

$$\begin{aligned} \mathrm{Prob}\{\varsigma(\kappa) = 1\} &= \mathbb{E}\{\varsigma(\kappa)\} = \bar{\varsigma} \\ \mathrm{Prob}\{\varsigma(\kappa) = 0\} &= 1 - \bar{\varsigma} \end{aligned} \quad (10)$$

where $\bar{\varsigma} \in [0, 1]$ is known constants.

*Remark 2:* The dropout model (9) is a modification of the model in [40] due to the adoption of the data-driven communication mechanism. When data-driven strategy replaces the conventional time-triggered strategy, the modification could well describe the measurement missing. For instance, when $\varsigma(\kappa) = 1$, it means that the signal which satisfies the data-driven condition at time point k is successfully transmitted to the filter. If $\varsigma(\kappa) = 0$, it means that the measured output which satisfies the data-driven condition is missing. Compared with the traditional dropout model, such a dropout model is certainly more reasonable and easier to realize for cyber-physical systems. This is due to that, when data-driven strategy is adopted for signal transmission, if the filter doesn't receive any measurement at time point k, there is no way to determine whether a packet loss occurs or the data-driven condition is not satisfied. Thus, the modified dropout model is easier to implement in cyber-physical engineering applications and is less conservative.

Many filtering results usually assume that the filter could be implemented exactly. However, in cyber-physical systems, the filter is realized through a communication network. For instance, when filter parameters are transferred to filter through a channel, the networked circumstance may induce the change of filter gain parameters in a random way. Considering the phenomenon of ROGVs, the filter is constructed as:

$$\begin{cases} \hat{\chi}(\kappa+1) = \mathcal{G}(\kappa)\hat{\chi}(\kappa) + \bar{\varrho}\mathcal{P}(\kappa, \hat{\chi}(\kappa)) \\ \qquad\qquad + (1-\bar{\varrho})\mathcal{Q}(\kappa, \hat{\chi}(\kappa)) \\ \qquad\qquad + (\mathcal{F}(\kappa) + \psi(\kappa)\Delta\mathcal{F}(\kappa)) \\ [y(\kappa_i) - \bar{\varsigma}\mathcal{H}(\kappa)\hat{\chi}(\kappa)] \\ \hat{z}(\kappa) = \mathcal{Z}(\kappa)\hat{\chi}(\kappa) \end{cases} \quad (11)$$

where $\hat{\chi}(\kappa) \in \mathbb{R}^n$ represents the estimate of $\chi(\kappa)$, $\hat{z}(\kappa) \in \mathbb{R}^m$ is the estimate of $z(\kappa)$, and $\mathcal{F}(\kappa)$ is an filter gain matrix. $\Delta\mathcal{F}(\kappa)$ representing the phenomenon of ROGVs is defined as: $\Delta\mathcal{F}(\kappa) = H_k k(\kappa)F_k(\kappa)E_k(\kappa)$. In this case, $H_k(\kappa)$ and $E_k(\kappa)$ are known matrices, and $F_k(\kappa)$ is an unknown matrix satisfying $F_k^T(\kappa)F_k(\kappa) \leq I$.

The Bernoulli sequence $\psi(\kappa)$ uncorrelated to $\rho(\kappa)$, $\varrho(\kappa)$ and $\varsigma(\kappa)$ is defined as:

$$\begin{aligned} \mathrm{Prob}\{\psi(\kappa) = 1\} &= \mathbb{E}\{\psi(\kappa)\} = \bar{\psi} \\ \mathrm{Prob}\{\psi(\kappa) = 0\} &= 1 - \bar{\psi} \end{aligned} \quad (12)$$

where $\bar{\psi} \in [0, 1]$ is known constants.

*Remark 3:* Here, a unified $H_\infty$ filtering framework combining the data-driven communication mechanism and the non-fragility of filters is constructed and investigated. As far as I know, in the field of filtering, the non-fragile filtering problem that adopts data-driven transmission mechanism is still an open one. However, it is often the case that, with the development of network technologies, the components are located at different places in cyber-physical systems. Due to the limited communication ability of network, frequent signal transmission will increase the traffic burden of network, causing network congestion and reducing the stability of the system. Thus, the non-fragile filtering problem that adopts data-driven transmission mechanism has more advantage for cyber-physical systems. On one hand, the design of filter takes ROGVs into consideration, better revealing the actual situation of the network filtering. On the other hand, in data-driven transmission mechanism, signal transmission frequency will decrease for cyber-physical systems.

By letting $\eta(\kappa) = [\chi^T(\kappa) \quad \hat{\chi}^T(\kappa) \quad y(\kappa_{i-1})]^T$, $y(\kappa_{i-1})$ being the measurement received by the filter at event time $\kappa_{i-1}$, $\tilde{z}(\kappa) = z(\kappa) - \hat{z}(\kappa)$, $\bar{v}(\kappa) = [v^T(\kappa) \quad \vartheta^T(\kappa)]^T$, $\tilde{\rho}(\kappa) = \rho(\kappa) - \bar{\rho}$, $\tilde{\varrho}(\kappa) = \varrho(\kappa) - \bar{\varrho}$, $\tilde{\varsigma}(\kappa) = \varsigma(\kappa) - \bar{\varsigma}$, $\tilde{\psi}(\kappa) = \psi(\kappa) - \bar{\psi}$, we obtain the augmented system:

$$\begin{cases} \eta(\kappa+1) = \bar{\mathcal{G}}(\kappa)\eta(\kappa) + \Lambda_1 \mathcal{R}(\kappa, \eta(\kappa)) \\ \qquad + \bar{F}(\kappa) r(\kappa) + \bar{\mathcal{W}}(\kappa)\bar{v}(\kappa) \\ \qquad + \tilde{\rho}(\kappa)\tilde{\mathcal{G}}_1(\kappa)\eta(\kappa) \\ \qquad + \tilde{\varsigma}(\kappa)\tilde{\mathcal{G}}_2(\kappa)\eta(\kappa) \\ \qquad + \tilde{\psi}(\kappa)\tilde{\mathcal{G}}_3(\kappa)\eta(\kappa) \\ \qquad + \tilde{\psi}(\kappa)\varsigma(\kappa)\tilde{\mathcal{G}}_4(\kappa)\eta(\kappa) \\ \qquad + \tilde{\varrho}(\kappa)\Lambda_2 \mathcal{R}(\kappa, \eta(\kappa)) \\ \qquad + \tilde{\varsigma}(\kappa)\tilde{F}_1(\kappa) r(\kappa) \\ \qquad + \tilde{\psi}(\kappa)\varsigma(\kappa)\tilde{F}_2(\kappa) r(\kappa) \\ \qquad + \tilde{\varsigma}(\kappa)\tilde{\mathcal{W}}_1(\kappa)\bar{v}(\kappa) \\ \qquad + \tilde{\psi}(\kappa)\varsigma(\kappa)\tilde{\mathcal{W}}_2(\kappa)\bar{v}(\kappa) \\ \tilde{z}(\kappa) = \bar{\mathcal{Z}}(\kappa)\eta(\kappa) \end{cases} \tag{13}$$

where

$$\tilde{\mathcal{G}}_1(\kappa) = \begin{bmatrix} \Delta\mathcal{G}(\kappa) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\mathcal{G}}(\kappa) = \begin{bmatrix} \mathcal{G}(\kappa) + \bar{\rho}\Delta\mathcal{G}(\kappa) & 0 & 0 \\ \begin{gathered} \bar{\varsigma} F(\kappa)\mathcal{H}(\kappa) \\ + \bar{\varsigma} F(\kappa)\Delta(\kappa)\mathcal{H}(\kappa) \\ + \bar{\psi}\bar{\varsigma}\Delta F(\kappa)\mathcal{H}(\kappa) \\ + \bar{\psi}\bar{\varsigma}\Delta F(\kappa)\Delta(\kappa)\mathcal{H}(\kappa) \end{gathered} & \begin{gathered} \mathcal{G}(\kappa) - \bar{\varsigma} F(\kappa)\mathcal{H}(\kappa) \\ - \bar{\psi}\bar{\varsigma}\Delta F(\kappa)\mathcal{H}(\kappa) \end{gathered} & \begin{gathered} (1-\bar{\varsigma})F(\kappa) \\ + \bar{\psi}\Delta F(\kappa) \\ - \bar{\psi}\bar{\varsigma}\Delta F(\kappa) \end{gathered} \\ \bar{\varsigma}\mathcal{H}(\kappa) + \bar{\varsigma}\Delta(\kappa)\mathcal{H}(\kappa) & 0 & (1-\bar{\varsigma})I \end{bmatrix},$$

$$\tilde{\mathcal{G}}_3(\kappa) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\bar{\varsigma}\Delta F(\kappa)\mathcal{H}(\kappa) & \Delta F(\kappa) \\ 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\mathcal{G}}_2(\kappa) = \begin{bmatrix} 0 & 0 & 0 \\ \begin{gathered} F(\kappa)\mathcal{H}(\kappa) + F(\kappa)\Delta(\kappa)\mathcal{H}(\kappa) \\ + \bar{\psi}\Delta F(\kappa)\Delta(\kappa)\mathcal{H}(\kappa) + \bar{\psi}\Delta F(\kappa)\mathcal{H}(\kappa) \end{gathered} & 0 & -F(\kappa) - \bar{\psi}\Delta F(\kappa) \\ \mathcal{H}(\kappa) + \Delta(\kappa)\mathcal{H}(\kappa) & 0 & -I \end{bmatrix},$$

$$\tilde{\mathcal{G}}_4(\kappa) = \begin{bmatrix} 0 & 0 & 0 \\ \Delta F(\kappa)\mathcal{H}(\kappa) + \Delta F(\kappa)\Delta(\kappa)\mathcal{H}(\kappa) & 0 & -\Delta F(\kappa) \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Lambda_1 = \begin{bmatrix} \bar{\varrho}I & (1-\bar{\varrho})I & 0 & 0 \\ 0 & 0 & \bar{\varrho}I & (1-\bar{\varrho})I \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Lambda_2 = \begin{bmatrix} I & -I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{R}(\kappa, \eta(\kappa)) = [\mathcal{P}^T(\kappa, \chi(\kappa)) \quad \mathcal{Q}^T(\kappa, \chi(\kappa)) \quad \mathcal{P}^T(\kappa, \hat{\chi}(\kappa)) \quad \mathcal{Q}^T(\kappa, \hat{\chi}(\kappa))]^T,$$

$$\bar{F}(\kappa) = \begin{bmatrix} 0 \\ \bar{\varsigma}F(\kappa) + \bar{\psi}\bar{\varsigma}\Delta F(\kappa) \\ \bar{\varsigma}I \end{bmatrix},$$

$$\tilde{F}_1(\kappa) = \begin{bmatrix} 0 \\ F(\kappa) + \bar{\psi}\Delta F(\kappa) \\ I \end{bmatrix},$$

$$\tilde{F}_2(\kappa) = \begin{bmatrix} 0 \\ \Delta F(\kappa) \\ 0 \end{bmatrix},$$

$$\bar{\mathcal{W}}(\kappa) = \begin{bmatrix} \mathcal{W}_1(\kappa) & 0 \\ 0 & \begin{gathered} \bar{\varsigma}F(\kappa)\mathcal{W}_2(\kappa) + \bar{\varsigma}F(\kappa)\Delta(\kappa)\mathcal{W}_2(\kappa) \\ + \bar{\psi}\bar{\varsigma}\Delta F(\kappa)\mathcal{W}_2(\kappa) \\ + \bar{\psi}\bar{\varsigma}\Delta F(\kappa)\Delta(\kappa)\mathcal{W}_2(\kappa) \end{gathered} \\ 0 & \bar{\varsigma}\mathcal{W}_2(\kappa) + \bar{\varsigma}\Delta(\kappa)\mathcal{W}_2(\kappa) \end{bmatrix},$$

$$\tilde{\mathcal{W}}_1(\kappa) = \begin{bmatrix} 0 & 0 \\ 0 & \begin{gathered} F(\kappa)\mathcal{W}_2(\kappa) \\ + F(\kappa)\Delta(\kappa)\mathcal{W}_2(\kappa) \\ + \bar{\psi}\Delta F(\kappa)\mathcal{W}_2(\kappa) \\ + \bar{\psi}\Delta F(\kappa)\Delta(\kappa)\mathcal{W}_2(\kappa) \end{gathered} \\ 0 & \mathcal{W}_2(\kappa) + \Delta(\kappa)\mathcal{W}_2(\kappa) \end{bmatrix},$$

$$\tilde{\mathcal{W}}_2(\kappa) = \begin{bmatrix} 0 & 0 \\ 0 & \begin{gathered} \Delta F(\kappa)\mathcal{W}_2(\kappa) \\ + \Delta F(\kappa)\Delta(\kappa)\mathcal{W}_2(\kappa) \end{gathered} \\ 0 & 0 \end{bmatrix},$$

$$\bar{\mathcal{Z}}(\kappa) = \begin{bmatrix} \mathcal{Z}(\kappa) & -\mathcal{Z}(\kappa) & 0 \end{bmatrix}.$$

Our aim is to design a filter for proposed cyber-physical systems satisfying the following $H_\infty$ filtering performance:

$$J := \mathbb{E}\left\{\|\tilde{z}(\kappa)\|_2^2 - \gamma^2 \|\bar{v}(\kappa)\|_2^2\right\} - \gamma^2\eta^T(0)U\eta(0) < 0, \quad \forall(\{\bar{v}(\kappa)\}, \eta(0)) \neq 0 \tag{14}$$

where

$$\|\tilde{z}(\kappa)\|_2^2 = \sum_{k=0}^{N-1} |\tilde{z}(\kappa)|^2, \quad \|\bar{v}(\kappa)\|_2^2 = \sum_{k=0}^{N-1} |\bar{v}(\kappa)|^2$$

## III. ANALYSIS OF $H_\infty$ PERFORMANCES

First of all, several useful lemmas are introduced.

*Lemma 1 (Schur Complement, [42]):* Given constant matrices $P, Q$ and $U$, where $P = P^T$ and $0 < Q = Q^T$, then $P + U^TQ^{-1}U < 0$ if and only if

$$\begin{bmatrix} P & U^T \\ U & -Q \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -Q & U \\ U^T & P \end{bmatrix} < 0 \tag{15}$$

*Lemma 2 (S-Procedure, [42]):* Let $M = M^T$, $P$ and $Q$ be real matrics of appropriate dimensions, and $X^TX \leq I$. Then inequality $M + PXQ + (PXQ)^T < 0$ if and only if there exists a positive scalar $\alpha$ such that $M + \alpha PP^T + \alpha^{-1}Q^TQ < 0$ or, equivalently,

$$\begin{bmatrix} M & \alpha P & Q^T \\ \alpha P^T & -\alpha I & 0 \\ Q & 0 & -\alpha I \end{bmatrix} < 0 \tag{16}$$

*Theorem 1:* Consider the cyber-physical systems described in Section II. Let the disturbance attenuation level $\gamma > 0$, the data weighted matrix $\Omega > 0$, the scalar $\delta \in [0, 1)$, a positive definite matrix $U > 0$, and the filter gain matrix $\mathcal{F}(\kappa)_{\kappa\in[0,N-1]}$ be given. For the cyber-physical systems, the $H_\infty$ performance requirement defined in (14) is achieved for all nonzero $\bar{v}(\kappa)$ if there exists a sequence of positive definite matrices $\{\Pi(\kappa)\}_{\kappa\in[0,N]}$ and positive scalars $\{\tau_1(\kappa)\}_{\kappa\in[0,N-1]}$ and $\{\tau_2(\kappa)\}_{\kappa\in[0,N-1]}$ satisfying the following recursive matrix inequalities:

$$\Phi(\kappa) = \begin{bmatrix} \Phi_{11}(\kappa) & * & * & * \\ \Phi_{21}(\kappa) & \Phi_{22}(\kappa) & * & * \\ \Phi_{31}(\kappa) & \Lambda_1^T\Pi(\kappa+1)\bar{\mathcal{W}}(\kappa) & \Phi_{33}(\kappa) & * \\ \Phi_{41}(\kappa) & \Phi_{42}(\kappa) & \bar{\mathcal{F}}^T(\kappa)\Pi(\kappa+1)\Lambda_1 & \Phi_{44}(\kappa) \end{bmatrix} < 0 \tag{17}$$

with the initial condition

$$\Pi(0) - \gamma^2U < 0 \tag{18}$$

where

$$S = \begin{bmatrix} I & -I & 0 \end{bmatrix},$$
$$\bar{G}_2 = \begin{bmatrix} 0 \\ \mathcal{W}_2^T(\kappa)(I + \Delta^T(\kappa)) \end{bmatrix},$$
$$\mathcal{E}(\kappa) = \begin{bmatrix} \mathcal{E}_1(\kappa) & 0 & 0 \\ \mathcal{E}_2(\kappa) & 0 & 0 \\ 0 & \mathcal{E}_1(\kappa) & 0 \\ 0 & \mathcal{E}_2(\kappa) & 0 \end{bmatrix},$$
$$\bar{G}_1 = \begin{bmatrix} C^T(\kappa)(I + \Delta^T(\kappa)) \\ 0 \\ 0 \end{bmatrix},$$

$$\begin{aligned}
\tilde{\Phi}_{11}(\kappa) &= \bar{\mathcal{G}}^T(\kappa)\Pi(\kappa+1)\bar{\mathcal{G}}(\kappa) \\
&\quad + \bar{\rho}(1-\bar{\rho})\tilde{\mathcal{G}}_1^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{G}}_1(\kappa) \\
&\quad + \bar{\varsigma}(1-\bar{\varsigma})\tilde{\mathcal{G}}_2^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{G}}_2(\kappa) \\
&\quad + \bar{\psi}(1-\bar{\psi})(1+\bar{\varsigma})\tilde{\mathcal{G}}_3^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{G}}_3(\kappa) \\
&\quad + 3\bar{\psi}(1-\bar{\psi})\varsigma\tilde{\mathcal{G}}_4^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{G}}_4(\kappa) - \Pi(\kappa), \\
\tilde{\Phi}_{22}(\kappa) &= \bar{\mathcal{W}}^T(\kappa)\Pi(\kappa+1)\bar{\mathcal{W}}(\kappa) \\
&\quad + \bar{\varsigma}(1-\bar{\varsigma})\tilde{\mathcal{W}}_1^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{W}}_1(\kappa) \\
&\quad + 2\bar{\psi}(1-\bar{\psi})\bar{\varsigma}\tilde{\mathcal{W}}_2^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{W}}_2(\kappa), \\
\tilde{\Phi}_{44}(\kappa) &= \bar{\mathcal{F}}^T\Pi(\kappa+1)\bar{\mathcal{F}}(\mathcal{F}) \\
&\quad + \bar{\varsigma}(1-\bar{\varsigma})\tilde{\mathcal{F}}_1^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{F}}_1(\kappa) \\
&\quad + \bar{\psi}(1-\bar{\psi})\bar{\varsigma}\tilde{\mathcal{F}}_2^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{F}}_2(\kappa), \\
\Phi_{11}(\kappa) &= \tilde{\Phi}_{11}(\kappa) + S^T\mathcal{Z}^T(\kappa)\mathcal{Z}(\kappa)S \\
&\quad + \tau_1(\kappa)\mathcal{E}^T(\kappa)\mathcal{E}(\kappa) + 2\tau_2(\kappa)\delta\bar{G}_1\Omega\bar{G}_1^T, \\
\Phi_{21}(\kappa) &= \tilde{\Phi}_{21}(\kappa) = \bar{\mathcal{W}}^T(\kappa)\Pi(\kappa+1)\bar{\mathcal{G}}(\kappa) \\
&\quad + \bar{\varsigma}(1-\bar{\varsigma})\tilde{\mathcal{W}}_1^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{G}}_2(\kappa) \\
&\quad + \bar{\psi}(1-\bar{\psi})\bar{\varsigma}\tilde{\mathcal{W}}_2^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{G}}_3(\kappa), \\
\Phi_{22}(\kappa) &= \tilde{\Phi}_{22}(\kappa) - \gamma^2I + 2\tau_2(\kappa)\delta\bar{G}_2\Omega\bar{G}_2^T, \\
\Phi_{31}(\kappa) &= \Lambda_1^T\Pi(\kappa+1)\bar{\mathcal{G}}(\kappa), \\
\Phi_{33}(\kappa) &= \Lambda_1^T\Pi(\kappa+1)\Lambda_1 \\
&\quad + \bar{\varrho}(1-\bar{\varrho})\Lambda_2^T\Pi(\kappa+1)\Lambda_2 - \tau_1(\kappa)I, \\
\Phi_{41}(\kappa) &= \tilde{\Phi}_{41}(\kappa) = \bar{\mathcal{F}}^T\Pi(\kappa+1)\bar{\mathcal{G}}(\mathcal{F}) \\
&\quad + \bar{\varsigma}(1-\bar{\varsigma})\tilde{\mathcal{F}}_1^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{G}}_2(\kappa) \\
&\quad + \bar{\psi}(1-\bar{\psi})\bar{\varsigma}\tilde{\mathcal{F}}_2^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{G}}_3(\kappa) \\
&\quad + \bar{\psi}(1-\bar{\psi})\bar{\varsigma}\tilde{\mathcal{F}}_2^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{G}}_4(\kappa), \\
\Phi_{42}(\kappa) &= \tilde{\Phi}_{42}(\kappa) = \bar{\mathcal{F}}^T\Pi(\kappa+1)\bar{\mathcal{W}}(\kappa) \\
&\quad + \bar{\varsigma}(1-\bar{\varsigma})\tilde{\mathcal{F}}_1^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{W}}_1(\kappa) \\
&\quad + \bar{\psi}(1-\bar{\psi})\bar{\varsigma}\tilde{\mathcal{F}}_2^T(\kappa)\Pi(\kappa+1)\tilde{\mathcal{W}}_2(\kappa), \\
\Phi_{44}(\kappa) &= \tilde{\Phi}_{44}(\kappa) - \tau_2(\kappa)\Omega,
\end{aligned}$$

*Proof:* Construct the following Lyapunov function

$$V(\eta(\kappa)) := \eta^T(\kappa)\Pi(\kappa)\eta(\kappa) \tag{19}$$

The difference of $V(\eta(\kappa))$ is:

$$\Delta V(\eta(\kappa)) := V(\eta(\kappa+1)) - V(\eta(\kappa)). \tag{20}$$

Calculating the difference of $V(\eta(\kappa))$,

$$\begin{aligned}
&\mathbb{E}\{\Delta V(\eta(\kappa))\} \\
&= \mathbb{E}\left\{\eta^T(\kappa+1)\Pi(\kappa+1)\eta(\kappa+1) - \eta^T(\kappa)\Pi(\kappa)\eta(\kappa)\right\} \\
&\leq \mathbb{E}\left\{\xi^T(\kappa)\tilde{\Phi}(\kappa)\xi(\kappa)\right\}
\end{aligned} \tag{21}$$

where

$$\xi(\kappa) = \begin{bmatrix} \eta^T(\kappa) & \bar{v}^T(\kappa) & \mathcal{R}^T(\kappa, \eta(\kappa)) & r^T(\kappa) \end{bmatrix}^T,$$

$$\tilde{\Phi}(\kappa) = \begin{bmatrix} \tilde{\Phi}_{11}(\kappa) & * & * & * \\ \tilde{\Phi}_{21}(\kappa) & \tilde{\Phi}_{22}(\kappa) & * & * \\ \Lambda_1^T \Pi(\kappa+1)\bar{\mathcal{G}}(\kappa) & \Lambda_1^T \Pi(\kappa+1)\bar{\mathcal{W}}(\kappa) & \begin{array}{c}\Lambda_1^T \Pi(\kappa+1)\Lambda_1 \\ +\bar{\varrho}(1-\bar{\varrho})\Lambda_2^T \Pi(\kappa+1)\Lambda_2\end{array} & * \\ \tilde{\Phi}_{41}(\kappa) & \tilde{\Phi}_{42}(\kappa) & \bar{F}^T(\kappa)\Pi(\kappa+1)\Lambda_1 & \tilde{\Phi}_{44}(\kappa) \end{bmatrix}.$$

According to nonlinear constraint (5), we have

$$\left\|\mathcal{R}(\kappa, \eta(\kappa))\right\| \leq \left\|\mathcal{E}(\kappa)\eta(\kappa)\right\| \tag{22}$$

Then, substituting (22) into (21) results in

$$\begin{aligned} &\mathbb{E}\{\Delta V(\kappa)\} \\ &\leq \mathbb{E}\Big\{\xi^T(\kappa)\tilde{\Phi}(\kappa)\xi(\kappa) \\ &\qquad - \tau_1(\kappa)[\mathcal{R}^T(\kappa, \eta(\kappa))\mathcal{R}(\kappa, \eta(\kappa)) \\ &\qquad -\eta^T(\kappa)\mathcal{E}^T(\kappa)\mathcal{E}(\kappa)\eta(\kappa)]\Big\} \end{aligned} \tag{23}$$

Adopting the data-driven condition (8), we obtain

$$\begin{aligned} \mathbb{E}\{\Delta V(\kappa)\} \leq \mathbb{E}\Big\{&\xi^T(\kappa)\tilde{\Phi}(\kappa)\xi(\kappa) \\ &-\tau_2(\kappa)[r^T(\kappa)\Omega r(\kappa) - \delta\bar{y}^T(\kappa)\Omega\bar{y}(\kappa)] \\ &-\tau_1(\kappa)[\mathcal{R}^T(\kappa, \eta(\kappa))\mathcal{R}(\kappa, \eta(\kappa)) \\ &-\eta^T(\kappa)\mathcal{E}^T(\kappa)\mathcal{E}(\kappa)\eta(\kappa)]\Big\} \end{aligned} \tag{24}$$

where

$$\begin{aligned} \bar{y}^T(\kappa)\Omega\bar{y}(\kappa) = {} & [(I+\Delta(\kappa))\mathcal{H}(\kappa)\chi(\kappa) \\ & \quad + (I+\Delta(\kappa))\mathcal{W}_2(\kappa)\vartheta(\kappa)]^T\Omega \\ & [(I+\Delta(\kappa))\mathcal{H}(\kappa)\chi(\kappa) \\ & \quad +(I+\Delta(\kappa))\mathcal{W}_2(\kappa)\vartheta(\kappa)] \\ \leq {} & 2\eta^T(\kappa)\bar{G}_1\Omega\bar{G}_1^T\eta(\kappa) \\ & + 2\bar{v}^T(\kappa)\bar{G}_2\Omega\bar{G}_2^T\bar{v}(\kappa) \end{aligned} \tag{25}$$

Adding $\tilde{z}^T(\kappa)\tilde{z}(\kappa) - \gamma^2\bar{v}^T(\kappa)\bar{v}(\kappa) - \tilde{z}^T(\kappa)\tilde{z}(\kappa) + \gamma^2\bar{v}^T(\kappa)\bar{v}(\kappa)$ to $\mathbb{E}\{\Delta V(\kappa)\}$ results in

$$\begin{aligned} &\mathbb{E}\{\Delta V(\kappa)\} \\ &\leq \mathbb{E}\{\xi^T(\kappa)\Phi(\kappa)\xi(\kappa) - \tilde{z}^T(\kappa)\tilde{z}(\kappa) + \gamma^2\bar{v}^T(\kappa)\bar{v}(\kappa)\} \end{aligned} \tag{26}$$

Summing up (26) on both sides from 0 to $N-1$ with respect to $\kappa$, we obtain

$$\begin{aligned} &\mathbb{E}\{V(\eta(N))\} - V(\eta(0)) \\ &\quad = \mathbb{E}\left\{\eta^T(N)\Pi(N)\eta(N)\right\} - \eta^T(0)\Pi(0)\eta(0) \\ &\quad \leq \mathbb{E}\left\{\sum_{\kappa=0}^{N-1}\xi^T(\kappa)\Phi(\kappa)\xi(\kappa)\right\} \\ &\qquad -\mathbb{E}\{\sum_{\kappa=0}^{N-1}(\tilde{z}^T(\kappa)\tilde{z}(\kappa) - \gamma^2\bar{v}^T(\kappa)\bar{v}(\kappa))t\} \end{aligned} \tag{27}$$

According to the above inequality, one has

$$\begin{aligned} J \leq {} & \mathbb{E}\{\sum_{\kappa=0}^{N-1}\xi^T(\kappa)\Phi(\kappa)\xi(\kappa)\} \\ & - \mathbb{E}\{\eta^T(N)\Pi(N)\eta(N)\} \\ & + \eta^T(0)(\Pi(0) - \gamma^2 U)\eta(0) \end{aligned} \tag{28}$$

Noting that $\Phi(\kappa) < 0$, $\Pi(N) > 0$ and $\Pi(0) \leqslant \gamma^2 U$, there is $J < 0$. End of proof. ∎

## IV. DATA-DRIVEN ROBUST NON-FRAGILE FILTER DESIGN

*Theorem 2:* Consider the cyber-physical systems in Section II with non-fragile filter (11). For given $\gamma > 0$, $\Omega > 0$, the scalar $\delta \in [0, 1)$ and $U > 0$, the filter error $\tilde{z}(\kappa)$ satisfies the $H_\infty$ performance criterion (14) if there exist families of positive definite matrices $\{\Pi(\kappa)\}_{\kappa\in[0,N]}$, positive scalars $\{\tau_1(\kappa)\}_{\kappa\in[0,N-1]}$, $\{\tau_2(\kappa)\}_{\kappa\in[0,N-1]}$, $\{\varepsilon_1(\kappa)\}_{\kappa\in[0,N-1]}$, $\{\varepsilon_2(\kappa)\}_{\kappa\in[0,N-1]}$, $\{\varepsilon_3(\kappa)\}_{\kappa\in[0,N-1]}$, $\{\varepsilon_4(\kappa)\}_{\kappa\in[0,N-1]}$, and real-valued matrices $\{X(\kappa)\}_{\kappa\in[0,N-1]}$ satisfying the following recursive linear matrix inequalities (RLMIs):

$$\begin{bmatrix} \Theta_{11} & * \\ \Theta_{21} & \Theta_{22} \end{bmatrix} < 0 \tag{29}$$

with the initial condition

$$\Pi(0) - \gamma^2 U < 0 \tag{30}$$

where

$$\Theta_{11} = \begin{bmatrix} \Theta_{11}^{(1,1)} & * & * \\ \Theta_{11}^{(2,1)} & \Theta_{11}^{(2,2)} & * \\ \Theta_{11}^{(3,1)} & 0 & \Theta_{11}^{(3,3)} \end{bmatrix},$$

$$\Theta_{21} = \begin{bmatrix} \Theta_{21}^{(1,1)} \\ \Theta_{21}^{(2,1)} \\ \Theta_{21}^{(3,1)} \\ \Theta_{21}^{(4,1)} \end{bmatrix},$$

$$\Theta_{22} = \mathrm{diag}\{-\varepsilon_1(\kappa)I, G(\kappa), -\varepsilon_3(\kappa), -\varepsilon_4(\kappa)\},$$

$$\Theta_{11}^{(1,1)} = \begin{bmatrix} \vec{\phi}_{11} & * & * & * \\ 0 & \begin{array}{c}-\gamma^2 I + \varepsilon_2(\kappa)\check{\mathcal{W}}^T(\kappa)\bar{\Delta}^T\bar{\Delta}\check{\mathcal{W}}(\kappa) \\ +\varepsilon_3(\kappa)\check{\mathcal{W}}^T(\kappa)E^T(\kappa)E(\kappa)\check{\mathcal{W}}(\kappa)\end{array} & * & * \\ 0 & 0 & \phi_{33} & * \\ 0 & 0 & 0 & -\Omega + \varepsilon_3(\kappa)E_k^T(\kappa)E_k(\kappa) \end{bmatrix}$$

$$\begin{aligned} \vec{\phi}_{11} &= \phi_{11} + \varepsilon_1(\kappa)\bar{E}_A^T(\kappa)\bar{E}_G(\kappa) + \varepsilon_2(\kappa)\check{\mathcal{H}}^T(\kappa)\check{\mathcal{H}}(\kappa) \\ &\quad + \varepsilon_3(\kappa)\bar{E}_{c1}^T(\kappa)\bar{E}_{c1}(\kappa) + \varepsilon_4(\kappa)\bar{E}_{c2}^T(\kappa)\bar{E}_{c2}(\kappa) \\ \phi_{11} &= -\Pi(\kappa) + \tau_1(\kappa)\mathcal{E}^T(\kappa)\mathcal{E}(\kappa) + S^T\mathcal{Z}^T(\kappa)\mathcal{Z}(\kappa)S, \\ \vec{\Gamma}_{11} &= \Theta_{11}^{(1,1)} \\ \phi_{33} &= \bar{\varrho}(1-\bar{\varrho})\Lambda_2^T\Pi(\kappa+1)\Lambda_2 - \tau_1(\kappa)I \end{aligned}$$

$$\Theta_{11}^{(2,1)} = \hat{\Gamma}_{21} = \begin{bmatrix} \hat{\Lambda}_{11} & \hat{\Lambda}_{12} & \Lambda_{13} & \hat{\Lambda}_{14} \\ \hat{\Lambda}_{21} & \hat{\Lambda}_{22} & 0 & \hat{\Lambda}_{24} \\ \hat{\Lambda}_{31} & \hat{\Lambda}_{32} & 0 & \hat{\Lambda}_{34} \end{bmatrix},$$

$$\hat{\Lambda}_{11} = \begin{bmatrix} \Pi(\kappa+1)\bar{\mathcal{G}}_0(\kappa) + \Pi(\kappa+1)R_1F(\kappa)\hat{\mathcal{H}}_1(\kappa) \\ 0 \end{bmatrix}$$

$$\hat{\Lambda}_{21} = \begin{bmatrix} \sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)(\tilde{\mathcal{G}}_{20}(\kappa)+R_1F(\kappa)\hat{\mathcal{H}}_2(\kappa)) \\ 0 \end{bmatrix}$$

$$\hat{\Lambda}_{12} = \begin{bmatrix} \Pi(\kappa+1)\bar{\mathcal{W}}_0(\kappa) + \bar{\varsigma}\Pi(\kappa+1)R_1F(\kappa)\check{\mathcal{W}}(\kappa) \\ 0 \end{bmatrix}$$

$$\hat{\Lambda}_{22} = \begin{bmatrix} \sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)(\tilde{\mathcal{W}}_{10}(\kappa)+R_1F(\kappa)\check{\mathcal{W}}(\kappa)) \\ 0 \end{bmatrix},$$

$$\hat{\Lambda}_{14} = \begin{bmatrix} \bar{\varsigma}\Pi(\kappa+1)R_2 + \bar{\varsigma}\Pi(\kappa+1)R_1F(\kappa) \\ 0 \end{bmatrix}$$

$$\hat{\Lambda}_{24} = \begin{bmatrix} \sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)(R_2 + R_1F(\kappa)) \\ 0 \end{bmatrix},$$

$$\hat{\Lambda}_{31} = \hat{\Lambda}_{32} = \hat{\Lambda}_{34} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\Lambda_{13} = \begin{bmatrix} \Pi(\kappa+1)\Lambda_1 \\ 0 \end{bmatrix},$$

$$\Theta_{11}^{(3,3)} = \Gamma_{33} = \begin{bmatrix} -\tau_2(\kappa)\Omega^{-1} & 0 \\ 0 & -\tau_2(\kappa)\Omega^{-1} \end{bmatrix}$$

$$\Theta_{11}^{(2,2)} = \Gamma_{22} = \mathrm{diag}\left\{ \underbrace{-\Pi(\kappa+1), \cdots, -\Pi(\kappa+1)}_{8} \right\}$$

$$\Theta_{11}^{(3,1)} = \tilde{\Gamma}_{31} = \begin{bmatrix} \tau_2(\kappa)\sqrt{2\delta}\bar{G}_{11}^T & 0 & 0 & 0 \\ 0 & \tau_2(\kappa)\sqrt{2\delta}\check{\mathcal{W}}(\kappa) & 0 & 0 \end{bmatrix},$$

$$\vec{H}_A^T(\kappa) = \Theta_{21}^{(1,1)}$$

$$\Theta_{21}^{(1,1)} = \big[ 0 \quad 0 \quad 0 \quad 0 \quad (\bar{\rho}\Pi(\kappa+1)\bar{H}_{\mathcal{G}}(\kappa))^T \quad (\sqrt{\bar{\rho}(1-\bar{\rho})}\Pi(\kappa+1)\bar{H}_{\mathcal{G}}(\kappa))^T \quad \underbrace{0 \cdots 0}_{8} \big]$$

$$\Theta_{21}^{(2,1)} = \hat{H}^T(\kappa) = \big[ 0 \quad 0 \quad 0 \quad 0 \quad \hat{H}_1^T(\kappa) \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \hat{H}_2^T(\kappa) \big],$$

$$\hat{H}_1(\kappa) = \begin{bmatrix} \bar{\varsigma}\Pi(\kappa+1)R_1F(\kappa) & \bar{\varsigma}\Pi(\kappa+1)R_1F(\kappa) + \bar{\varsigma}\Pi(\kappa+1)R_2 \\ 0 & 0 \\ \sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)(R_1F(\kappa)+R_2) & \sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)(R_1F(\kappa)+R_2) \end{bmatrix}$$

$$\hat{H}_2(\kappa) = \begin{bmatrix} \tau_2(\kappa)\sqrt{2\delta}I & 0 \\ 0 & \tau_2(\kappa)\sqrt{2\delta}I \end{bmatrix}$$

$$\Theta_{21}^{(3,1)} = \vec{H}_{k1}^T(\kappa) = \big[ 0 \quad 0 \quad 0 \quad 0 \quad \vec{H}_{k11}^T(\kappa) \quad 0 \quad \vec{H}_{k12}^T(\kappa) \quad 0 \quad 0 \big]^T$$

$$\vec{H}_{k11}(\kappa) = \begin{bmatrix} \bar{\psi}\bar{\varsigma}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ 0 \\ \bar{\psi}\sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ 0 \\ \bar{\psi}\bar{\varsigma}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ 0 \\ \bar{\psi}\sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ \sqrt{\bar{\varsigma}\bar{\psi}(1-\bar{\psi})}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ \bar{\psi}\bar{\varsigma}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ 0 \\ \bar{\psi}\sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ \sqrt{\bar{\varsigma}\bar{\psi}(1-\bar{\psi})}\Pi(\kappa+1)\bar{H}_k(\kappa) \end{bmatrix}$$

$$\vec{H}_{k12}(\kappa) = \begin{bmatrix} \sqrt{\bar{\varsigma}\bar{\psi}(1-\bar{\psi})}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ \sqrt{2\bar{\varsigma}\bar{\psi}(1-\bar{\psi})}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ 0 \\ 0 \\ 0 \\ \sqrt{\bar{\varsigma}\bar{\psi}(1-\bar{\psi})}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ \sqrt{\bar{\varsigma}\bar{\psi}(1-\bar{\psi})}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ 0 \\ 0 \end{bmatrix}$$

$$\Theta_{21}^{(4,1)} = \vec{H}_{k2}^T(\kappa) = \big[ 0 \quad 0 \quad 0 \quad 0 \quad \vec{H}_{k21}^T(\kappa) \quad 0 \quad 0 \quad \vec{H}_{k22}^T(\kappa) \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \big]^T$$

$$\vec{H}_{k21}(\kappa) = \bar{\psi}\bar{\varsigma}\Pi(\kappa+1)\bar{H}_k(\kappa),$$

$$\vec{H}_{k22}(\kappa) = \begin{bmatrix} \sqrt{\bar{\psi}(1-\bar{\psi})}\bar{\varsigma}\Pi(\kappa+1)\bar{H}_k(\kappa) \\ \sqrt{\bar{\psi}(1-\bar{\psi})}\Pi(\kappa+1)\bar{H}_k(\kappa) \end{bmatrix}$$

$$G(\kappa) = \begin{bmatrix} -\varepsilon_2(\kappa)I + \varepsilon_3(\kappa)E_k^T(\kappa)E_k(\kappa) & 0 \\ 0 & -\varepsilon_2(\kappa)I + \varepsilon_3(\kappa)E_k^T(\kappa)E_k(\kappa) \end{bmatrix}$$

$$\bar{\mathcal{G}}_0(\kappa) = \begin{bmatrix} \mathcal{G}(\kappa) & 0 & 0 \\ 0 & \mathcal{G}(\kappa) & 0 \\ \bar{\varsigma}\mathcal{H}(\kappa) & 0 & (1-\bar{\varsigma})I \end{bmatrix},$$

$$\bar{H}_{\mathcal{G}}(\kappa) = \begin{bmatrix} H_{\mathcal{G}}(\kappa) \\ 0 \\ 0 \end{bmatrix},$$

$$R1 = \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, \quad R2 = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix},$$

$$\bar{E}_{c1}(\kappa) = \big[ E_k(\kappa)\mathcal{H}(\kappa) \quad 0 \quad -E_k(\kappa) \big],$$
$$\bar{E}_{\mathcal{G}}(\kappa) = \big[ E_{\mathcal{G}}(\kappa) \quad 0 \quad 0 \big],$$
$$\check{\mathcal{H}}(\kappa) = \big[ \bar{\Delta}\mathcal{H}(\kappa) \quad 0 \quad 0 \big],$$
$$\bar{E}_{c2}(\kappa) = \big[ 0 \quad -\bar{\varsigma}E_k(\kappa)\mathcal{H}(\kappa) \quad E_k(\kappa) \big],$$
$$\hat{\mathcal{H}}_1(\kappa) = \big[ \bar{\varsigma}\mathcal{H}(\kappa) \quad -\bar{\varsigma}\mathcal{H}(\kappa) \quad (1-\bar{\varsigma})I \big],$$
$$\hat{\mathcal{H}}_2(\kappa) = \big[ \mathcal{H}(\kappa) \quad 0 \quad -I \big],$$
$$\check{\mathcal{W}}(\kappa) = \big[ 0 \quad \mathcal{W}_2(\kappa) \big], \quad \bar{G}_{11}^T = \big[ \mathcal{H}(\kappa) \quad 0 \quad 0 \big]$$

Furthermore, if ( $\{\Pi(\kappa)\}$, $\{\tau_1(\kappa)\}$,$\{\tau_2(\kappa)\}$, $\{\varepsilon_1(\kappa)\}$, $\{\varepsilon_2(\kappa)\}$, $\{\varepsilon_3(\kappa)\}$, $\{\varepsilon_4(\kappa)\}$, $\{X(\kappa)\}$ ) is a feasible solution of (29) and (30), then filter matrices can be obtained as follows

$$F(\kappa) = (R_1^T\Pi(\kappa+1)R_1)^{-1}R_1^TX(\kappa) \tag{31}$$

*Proof:* From Theorem 1, we know that the filter in the form of (11) achieves the guaranteed $H_\infty$ performance if LMIs (17) and (18) are feasible. By the Schur complement,

(17) is equivalent to

$$\begin{bmatrix} \Gamma_{11} & * & * \\ \Gamma_{21} & \Gamma_{22} & * \\ \Gamma_{31} & 0 & \Gamma_{33} \end{bmatrix} < 0 \tag{32}$$

where

$$\Gamma_{11} = \mathrm{diag}\{\phi_{11}, -\gamma^2 I, \phi_{33}, -\Omega\},$$
$$\Gamma_{21} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ \Lambda_{21} & \Lambda_{22} & 0 & \Lambda_{24} \\ \Lambda_{31} & \Lambda_{32} & 0 & \Lambda_{34} \end{bmatrix},$$
$$\Gamma_{31} = \begin{bmatrix} \tau_2(\kappa)\sqrt{2\delta}\bar{G}_1^T & 0 & 0 & 0 \\ 0 & \tau_2(\kappa)\sqrt{2\delta}\bar{G}_2^T & 0 & 0 \end{bmatrix}$$
$$\Lambda_{11} = \begin{bmatrix} \Pi(\kappa+1)\bar{\mathcal{G}}(\kappa) \\ \sqrt{\bar{\rho}(1-\bar{\rho})}\Pi(\kappa+1)\tilde{\mathcal{G}}_1(\kappa) \end{bmatrix},$$
$$\Lambda_{12} = \begin{bmatrix} \Pi(\kappa+1)\bar{\mathcal{W}}(\kappa) \\ 0 \end{bmatrix}$$
$$\Lambda_{13} = \begin{bmatrix} \Pi(\kappa+1)\Lambda_1 \\ 0 \end{bmatrix}, \quad \Lambda_{14} = \begin{bmatrix} \Pi(\kappa+1)\bar{F}(\kappa) \\ 0 \end{bmatrix}$$
$$\Lambda_{21} = \begin{bmatrix} \sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)\tilde{\mathcal{G}}_2(\kappa) \\ \sqrt{\bar{\psi}(1-\bar{\psi})}\bar{\varsigma}\Pi(\kappa+1)\tilde{\mathcal{G}}_3(\kappa) \end{bmatrix},$$
$$\Lambda_{22} = \begin{bmatrix} \sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)\tilde{\mathcal{W}}_1(\kappa) \\ \sqrt{\bar{\psi}(1-\bar{\psi})}\bar{\varsigma}\Pi(\kappa+1)\tilde{\mathcal{W}}_2(\kappa) \end{bmatrix}$$
$$\Lambda_{24} = \begin{bmatrix} \sqrt{\bar{\varsigma}(1-\bar{\varsigma})}\Pi(\kappa+1)\tilde{F}_1(\kappa) \\ \sqrt{\bar{\psi}(1-\bar{\psi})}\bar{\varsigma}\Pi(\kappa+1)\tilde{F}_2(\kappa) \end{bmatrix},$$
$$\Lambda_{31} = \begin{bmatrix} \sqrt{\bar{\psi}(1-\bar{\psi})}\Pi(\kappa+1)\tilde{\mathcal{G}}_3(\kappa) \\ \sqrt{\bar{\psi}(1-\bar{\psi})\bar{\varsigma}}\Pi(\kappa+1)\tilde{\mathcal{G}}_4(\kappa) \\ \sqrt{2\bar{\psi}(1-\bar{\psi})\bar{\varsigma}}\Pi(\kappa+1)\tilde{\mathcal{G}}_4(\kappa) \\ 0 \end{bmatrix}$$
$$\Lambda_{32} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sqrt{\bar{\psi}(1-\bar{\psi})\bar{\varsigma}}\Pi(\kappa+1)\tilde{\mathcal{W}}_2(\kappa) \end{bmatrix},$$
$$\Lambda_{34} = \begin{bmatrix} 0 \\ \sqrt{\bar{\psi}(1-\bar{\psi})\bar{\varsigma}}\Pi(\kappa+1)\tilde{F}_2(\kappa) \\ 0 \\ 0 \end{bmatrix}$$

Next, the parameters in Theorem 1 are rewritten as follows:

$$\begin{aligned}
\bar{\mathcal{G}}(\kappa) &= \bar{\mathcal{G}}_0(\kappa) + \bar{\rho}\bar{H}_{\mathcal{G}}(\kappa)F_{\mathcal{G}}(\kappa)\bar{E}_{\mathcal{G}}(\kappa) \\
&\quad + \bar{\varsigma}R_1\digamma(\kappa)F(\kappa)\check{\mathcal{H}}(\kappa) + R_1\digamma(\kappa)\hat{\mathcal{H}}_1(\kappa) \\
&\quad + \bar{\psi}\bar{\varsigma}\bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)F(\kappa)\check{\mathcal{H}}(\kappa) + \bar{\varsigma}R_2F(\kappa)\check{\mathcal{H}}(\kappa) \\
&\quad + \bar{\psi}\bar{\varsigma}\bar{H}_k(\kappa)F_k(\kappa)\bar{E}_{c1}(\kappa) + \bar{\psi}\bar{H}_k(\kappa)F_k(\kappa)\bar{E}_{c2}(\kappa) \\
\tilde{\mathcal{G}}_1(\kappa) &= \bar{H}_{\mathcal{G}}(\kappa)F_{\mathcal{G}}(\kappa)\bar{E}_{\mathcal{G}}(\kappa) \\
\tilde{\mathcal{G}}_2(\kappa) &= \tilde{\mathcal{G}}_{20}(\kappa) + R_1\digamma(\kappa)F(\kappa)\check{\mathcal{H}}(\kappa) \\
&\quad + \bar{\psi}\bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)F(\kappa)\check{\mathcal{H}}(\kappa) + R_1\digamma(\kappa)\hat{\mathcal{H}}_2(\kappa) \\
&\quad + R_2F(\kappa)\check{\mathcal{H}}(\kappa) + \bar{\psi}\bar{H}_k(\kappa)F_k(\kappa)\bar{E}_{c1}(\kappa) \\
\tilde{\mathcal{G}}_3(\kappa) &= \bar{H}_k(\kappa)F_k(\kappa)\bar{E}_{c2}(\kappa) \\
\tilde{\mathcal{G}}_4(\kappa) &= \bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)F(\kappa)\check{\mathcal{H}}(\kappa) \\
&\quad + \bar{H}_k(\kappa)F_k(\kappa)\bar{E}_{c1}(\kappa)
\end{aligned}$$

$$\begin{aligned}
\bar{F}(\kappa) &= \bar{\varsigma}R_2 + \bar{\psi}\bar{\varsigma}\bar{H}_k(\kappa)F_k(\kappa)E_K(\kappa) + \bar{\varsigma}R_1\digamma(\kappa) \\
\tilde{F}_1(\kappa) &= R_2 + \bar{\varsigma}\bar{H}_k(\kappa)F_F(\kappa)E_k(\kappa) + R_1\digamma(\kappa), \\
\tilde{F}_2(\kappa) &= \bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa) \\
\bar{\mathcal{W}}(\kappa) &= \bar{\mathcal{W}}_0(\kappa) + \bar{\varsigma}R_1\digamma(\kappa)F(\kappa)\bar{\Delta}\check{\mathcal{W}}(\kappa) \\
&\quad + \bar{\varsigma}R_2F(\kappa)\Delta\check{\mathcal{W}}(\kappa) + \bar{\varsigma}R_1\digamma(\kappa)\check{\mathcal{W}}(\kappa) \\
&\quad + \bar{\psi}\bar{\varsigma}\bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)F(\kappa)\bar{\Delta}\check{\mathcal{W}}(\kappa) \\
&\quad + \bar{\psi}\bar{\varsigma}\bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)\check{\mathcal{W}}(\kappa) \\
\tilde{\mathcal{W}}_1(\kappa) &= \tilde{\mathcal{W}}_10(\kappa) + R_1\digamma(\kappa)F(\kappa)\bar{\Delta}\check{\mathcal{W}}(\kappa) \\
&\quad + R_2F(\kappa)\Delta\check{\mathcal{W}}(\kappa) + R_1\digamma(\kappa)\check{\mathcal{W}}(\kappa) \\
&\quad + \bar{\psi}\bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)F(\kappa)\bar{\Delta}\check{\mathcal{W}}(\kappa) \\
&\quad + \bar{\psi}\bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)\check{\mathcal{W}}(\kappa) \\
\tilde{\mathcal{W}}_2(\kappa) &= \bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)F(\kappa)\bar{\Delta}\check{\mathcal{W}}(\kappa) \\
&\quad + \bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)\check{\mathcal{W}}(\kappa) \\
\bar{G}_1^T &= \bar{G}_{11}^T + F(\kappa)\check{\mathcal{H}}(\kappa), \quad \bar{G}_2^T = \check{\mathcal{W}}(\kappa) + F(\kappa)\Delta\check{\mathcal{W}}(\kappa)
\end{aligned}$$

Rewriting (32) to eliminate $\Delta\mathcal{G}(\kappa)$:

$$N(\kappa) + \vec{H}_{\mathcal{G}}(\kappa)F_{\mathcal{G}}(\kappa)\vec{E}_{\mathcal{G}}(\kappa) + (\vec{H}_{\mathcal{G}}(\kappa)F_{\mathcal{G}}(\kappa)\vec{E}_{\mathcal{G}}(\kappa))^T < 0 \tag{33}$$

where

$$N(\kappa) = \begin{bmatrix} \Gamma_{11} & * & * \\ \bar{\Gamma}_{21} & \Gamma_{22} & * \\ \Gamma_{31} & 0 & \Gamma_{33} \end{bmatrix},$$
$$\vec{E}_{\mathcal{G}}(\kappa) = \begin{bmatrix} \bar{E}_{\mathcal{G}}(\kappa) & \underbrace{0\cdots 0}_{13} \end{bmatrix},$$
$$\bar{\Gamma}_{21} = \begin{bmatrix} \bar{\Lambda}_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ \Lambda_{21} & \Lambda_{22} & 0 & \Lambda_{24} \\ \Lambda_{31} & \Lambda_{32} & 0 & \Lambda_{34} \end{bmatrix}$$
$$\bar{\Lambda}_{11} = \begin{bmatrix} \Pi(\kappa+1)\bar{\mathcal{G}}_0(\kappa) \\ +\bar{\varsigma}\Pi(\kappa+1)R_1\digamma(\kappa)F(\kappa)\check{\mathcal{H}}(\kappa) \\ +\bar{\psi}\bar{\varsigma}\Pi(\kappa+1)\bar{H}_k(\kappa)F_k(\kappa)E_k(\kappa)F(F)\check{\mathcal{H}}(\kappa) \\ +\bar{\varsigma}\Pi(\kappa+1)R_2F(\kappa)\check{\mathcal{H}}(\kappa) \\ +\bar{\psi}\bar{\varsigma}\Pi(\kappa+1)\bar{H}_k(\kappa)F_k(\kappa)\bar{E}_{c1}(\kappa) \\ +\bar{\psi}\Pi(\kappa+1)\bar{H}_k(\kappa)F_k(\kappa)\bar{E}_{c2}(\kappa) \\ +\Pi(\kappa+1)R_1\digamma(\kappa)\hat{\mathcal{H}}_1(\kappa) \\ 0 \end{bmatrix}$$

Then, from S-procedure, we can obtain

$$\begin{bmatrix} N(\kappa) & * & * \\ \vec{H}_A^T(\kappa) & -\varepsilon_1(\kappa)I & * \\ \varepsilon_1(\kappa)\vec{E}_{\mathcal{G}}(\kappa) & 0 & -\varepsilon_1(\kappa)I \end{bmatrix} < 0 \tag{34}$$

by using the Schur Complement, (34) is equivalent to

$$\begin{aligned}
&\begin{bmatrix} N(\kappa) & * \\ \vec{H}_A^T(\kappa) & -\varepsilon_1(\kappa)I \end{bmatrix} \\
&\quad +\varepsilon_1(\kappa)\begin{bmatrix} \vec{E}_A^T(\kappa) \\ 0 \end{bmatrix}\begin{bmatrix} \vec{E}_{\mathcal{G}}(\kappa) & 0 \end{bmatrix} \\
&= \begin{bmatrix} N(\kappa) + \varepsilon_1(\kappa)\vec{E}_A^T(\kappa)\vec{E}_{\mathcal{G}}(\kappa) & * \\ \vec{H}_A^T(\kappa) & -\varepsilon_1(\kappa)I \end{bmatrix} < 0
\end{aligned} \tag{35}$$

Taking a similar way, we can easily eliminate $\Delta(\kappa)$ and $\Delta\mathcal{F}(\kappa)$ in (35) and obtain

$$\begin{bmatrix} \vec{N}(\kappa) & * & * & * & * \\ \vec{H}_A^T(\kappa) & -\varepsilon_1(\kappa)I & * & * & * \\ \hat{H}^T(\kappa) & 0 & G(\kappa) & * & * \\ \vec{H}_{k1}^T(\kappa) & 0 & 0 & -\varepsilon_3(\kappa)I & * \\ \vec{H}_{k2}^T(\kappa) & 0 & 0 & 0 & -\varepsilon_4(\kappa)I \end{bmatrix} < 0 \tag{36}$$

where

$$\vec{N}(\kappa) = \begin{bmatrix} \vec{\Gamma}_{11} & * & * \\ \hat{\Gamma}_{21} & \Gamma_{22} & * \\ \vec{\Gamma}_{31} & 0 & \Gamma_{33} \end{bmatrix},$$

$$G(\kappa) = \begin{bmatrix} -\varepsilon_2(\kappa)I + \varepsilon_3(\kappa)E_k^T(\kappa)E_k(\kappa) & 0 \\ 0 & -\varepsilon_2(\kappa)I + \varepsilon_3(\kappa)E_k^T(\kappa)E_k(\kappa) \end{bmatrix}$$

Then, it is easy to see that (36) is equivalent to (29). Thus, according to Theorem 1, the $H_\infty$ performance requirement for the cyber-physical systems is satisfied with initial conditions (30). The proof is complete. ■

*Remark 4:* For the considered cyber-physical systems, the sufficient conditions guaranteeing the designed data-driven robust non-fragile filter are derived in Theorem 1 such that the $H_\infty$ performance is satisfied. Besides, the data-driven robust non-fragile filter design method is addressed in Theorem 2. It can be observed from Theorem 2 that all information of the considered cyber-physical systems is contained in RLMIs, including the systems parameters, network-induced phenomena, data-driven parameters and ROGVs.

By means of Theorem 2, we can summarize the data-driven $H_\infty$ non-fragile finite-horizon filter design algorithm (NFHFD) as follows.

**Algorithm 1** NFHFD

Step 1. Given the disturbance attenuation level $\gamma > 0$, the event weighted matrix $\Omega > 0$, the scalar $\delta \in [0, 1)$ and the positive definite matrix $U > 0$, choose the initial value for matrice $\Pi(0)$ to satisfy the condition (30) and set $\kappa = 0$.

Step 2. Obtain the values of matrices $\{\Pi(\kappa+1), X(\kappa)\}$ and the desired filter parameters $\mathcal{F}(\kappa)$ for the sampling instant $\kappa$ by solving the LMIs (29), (31).

Step 3. Set $\kappa = \kappa + 1$ and then $\Pi(\kappa) = \Pi(\kappa+1)$.

Step 4. If $\kappa < N$,then go to Step 2, else go to Step 5.

Step 5. Stop.

## V. NUMERICAL SIMULATIONS

Next, a simulation is given to demonstrate the filtering method for cyber-physical systems. The considered cyber-physical system is given as follows:

$$\begin{cases} \chi(\kappa+1) = \left( \left( \begin{bmatrix} 0.5+0.1\sin(3\kappa) & 0 & 0.15+0.1\sin(0.2\kappa) \\ 0.1 & 0.45 & 0 \\ 0 & 0.2+0.1\sin(\kappa) & 0.4 \end{bmatrix} + \rho(\kappa)\Delta\mathcal{G}(\kappa) \right) \chi(\kappa) + \varrho(\kappa)\mathcal{P}(\kappa, \chi(\kappa)) \right. \\ \qquad + (1-\varrho(\kappa))\mathcal{Q}(\kappa, \chi(\kappa)) \\ \qquad + \begin{bmatrix} 0.3 - 0.1\sin(0.3\kappa) & 0.35 \\ -0.25 & 0.2 \\ 0.15 & -0.2+0.05\cos(\kappa) \end{bmatrix} \nu(\kappa) \\ \tilde{y}(\kappa) = \begin{bmatrix} 0.5 & 0.3+0.15\cos(0.5\kappa) & 0.1 \\ 0.2 & 0.2-0.1\sin(\kappa) & -0.4 \end{bmatrix} \chi(\kappa) \\ \qquad + \begin{bmatrix} 0.25 & -0.4+0.2\sin(2\kappa) \\ 0.3 & 0.2 \end{bmatrix} \vartheta(\kappa) \\ z(\kappa) = \begin{bmatrix} 0.15 & 0.2 & 0.1+0.05\sin(\kappa) \end{bmatrix} \chi(\kappa) \end{cases}$$

**TABLE 1.** Rescusive process.

| $\kappa$ | 0 | 1 | $\cdots$ |
|---|---|---|---|
| $\mathcal{F}(\kappa)$ | $\begin{bmatrix} 0.1145 & 0.0124 \\ 0.1101 & 0.0419 \\ 0.0654 & -0.0514 \end{bmatrix}$ | $\begin{bmatrix} 0.4295 & 0.0509 \\ 0.4018 & 0.0635 \\ 0.3182 & -0.1892 \end{bmatrix}$ | $\cdots$ |
| $\kappa$ | 50 | 51 | $\cdots$ |
| $\mathcal{F}(\kappa)$ | $\begin{bmatrix} 0.5447 & -0.1021 \\ 0.5464 & 0.1193 \\ 0.5986 & -0.8986 \end{bmatrix}$ | $\begin{bmatrix} 0.6231 & 0.0762 \\ 0.5769 & 0.0403 \\ 0.6196 & -0.8713 \end{bmatrix}$ | $\cdots$ |

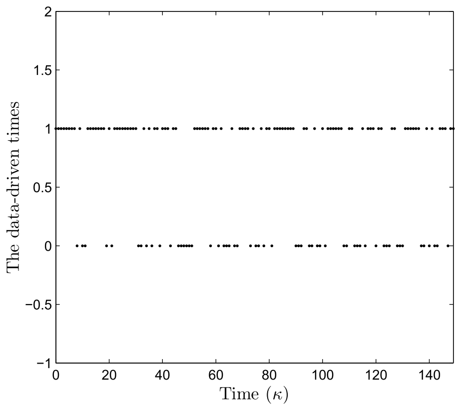


**FIGURE 1.** The communication times.

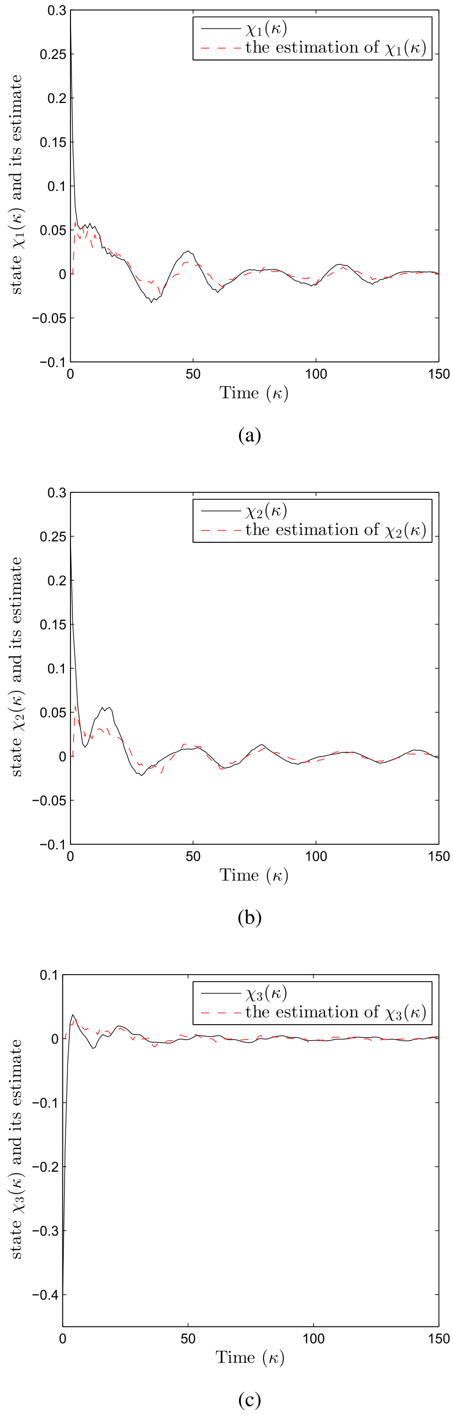


**FIGURE 2.** **State vector $x(\kappa)$ and its estimate. (a) State $\chi_1(\kappa)$ and its estimate. (b) State $\chi_2(\kappa)$ and its estimate. (c) State $\chi_3(\kappa)$ and its estimate.**

The nonlinear functions $\mathcal{P}(.,.)$, $\mathcal{Q}(.,.)$ and disturbance signals $\nu(\kappa)$, $\vartheta(\kappa)$ are chosen as

$$\mathcal{P}(\kappa,\chi(\kappa)) = \begin{bmatrix} \dfrac{(0.3\chi_1(\kappa))}{1+2\chi_3^2(\kappa)} \\ \dfrac{0.2\sin(\chi_2(\kappa))}{\sqrt{\chi_1^2(\kappa)+2}} \\ 0.2\chi_3(\kappa) \end{bmatrix},$$

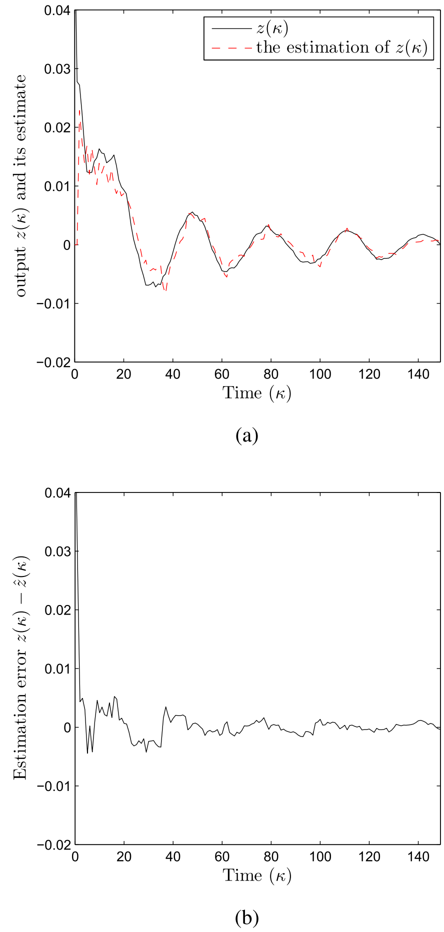


**FIGURE 3.** **Output $z(\kappa)$, its estimate and estimation error $z(\kappa)-\hat{z}(\kappa)$. (a) Output $z(\kappa)$ and its estimate. (b) Estimation error $z(\kappa)-\hat{z}(\kappa)$.**

$$\mathcal{Q}(\kappa,\chi(\kappa)) = \begin{bmatrix} \dfrac{\chi_1(\kappa)}{5+3\chi_1^2(\kappa)} \\ \dfrac{0.3\chi_2(\kappa)}{\chi_1^2(\kappa)+\chi_2^2(\kappa)+1} \\ 0.2\chi_3(\kappa) \end{bmatrix}$$

$$\nu(\kappa) = \begin{bmatrix} \dfrac{1}{2\kappa+14}\cos(0.3\kappa-1) \\ \dfrac{1}{\kappa+9}\sin(0.2\kappa-1) \end{bmatrix},$$

$$\vartheta(\kappa) = \begin{bmatrix} 0.1\exp(-0.5\kappa)\sin(\kappa) \\ 0.2\exp(-0.4\kappa)\cos(2\kappa) \end{bmatrix}$$

where $\chi_i(\kappa)(i=1,2,3)$ denotes the i-th component of $\chi(\kappa)$. Then, $\mathcal{E}_1(\kappa)$ and $\mathcal{E}_2(\kappa)$ are set as:

$$\mathcal{E}_1(\kappa) = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad \mathcal{E}_2(\kappa) = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

The probabilities of stochastic variable $\rho(\kappa)$, $\varrho(\kappa)$, $\varsigma(\kappa)$ and $\psi(\kappa)$ are taken as $\bar{\rho}=0.5$, $\bar{\varrho}=0.5$, $\bar{\varsigma}=0.9$, $\bar{\psi}=0.6$, respectively. The parameters in $\mathcal{L}(\cdot)$ are set as $\delta_1=0.4$, $\delta_2=0.3$. The parameter uncertainties $H_{\mathcal{G}}(\kappa)$, $E_{\mathcal{G}}(\kappa)$ and gain variations $H_k(\kappa)$, $E_k(\kappa)$ are set as follows:

$$H_{\mathcal{G}}(\kappa) = \begin{bmatrix} 0.15 \\ 0.2 \\ 0.1 \end{bmatrix}, \quad E_{\mathcal{G}}(\kappa) = \begin{bmatrix} 0.1 & 0.2 & 0.05 \end{bmatrix}$$

$$H_k(\kappa) = \begin{bmatrix} 0.2 & 0 \\ 0.2 + 0.1 \sin\kappa & 0.1 \\ 0 & 0.15 \end{bmatrix},$$

$$E_k(\kappa) = \begin{bmatrix} 0.2 & 0.15 + 0.05 \sin\kappa \\ 0.15 & 0.1 \end{bmatrix}$$

and the uncertain parameters $F_k(\kappa)$, $F_{\mathcal{G}}(\kappa)$ satisfy $F_k(\kappa)^T F_k(\kappa) \leq I$, $F_{\mathcal{G}}(\kappa)^T F_{\mathcal{G}}(\kappa) \leq I$.

Set $\chi(0) = [0.3 \quad 0.25 \quad -0.4]^T$ and $\hat{\chi}(0) = [0 \quad 0 \quad 0]^T$. Choose $\Omega = I$ and threshold $\delta = 0.1$. For given $\gamma = 0.95$ and $U = diag\{1, 1, 1, 1, 1, 1, 1, 1\}$, choose $\Pi(0) = 0.9\gamma^2 U$ to satisfy the initial condition (35).

Table 1 lists the desired parameters of filter $\mathcal{F}(\kappa)$ obtained by applying the algorithm in this paper.

The simulation results are shown in Figs 1-3. State variables $\chi_1(\kappa) - \chi_3(\kappa)$ and their estimate $\hat{\chi}_(\kappa) - \hat{\chi}_3(\kappa)$ are depicted in Figs 2, respectively, and Fig 3.$(a)$ plots $z(\kappa)$ and estimation, whereas $z(\kappa) - \hat{z}(\kappa)$ is shown in Fig 3.$(b)$. The transition times are plotted in Fig 1, whereas one represents the times that data-driven condition is satisfied and signal of sensors is transmitted, while zero represents the times that data-driven condition is not satisfied. The $H_\infty$ performance index is $J_1 = -0.3440$. From Fig 1, we could find that the data-driven communication mechanism may largely reduce the signal transmission frequency while maintaining the guaranteed filtering index. Results confirm the proposed filtering method which could well achieve the desired filtering requirement for the proposed cyber-physical systems.

## VI. CONCLUSIONS

The data-driven robust non-fragile $H_\infty$ filtering problem for the proposed cyber-physical systems has been researched in this paper. The data-driven communication mechanism has been adopted to reduce the signal transmission frequency while maintaining filtering performance requirement. The phenomenon of ROGVs in the implementation of filtering has been considered. Thus, a data-driven non-fragile filtering framework is constructed. Based on this unified framework, the influence of simultaneous presence of networked-induced packet dropouts, quantization, RONs and ROUs in cyber-physical systems is investigated. By employing stochastic analysis and Lyapunov functional approach, sufficient conditions guaranteeing the filter for proposed cyber-physical system have been derived. Moreover, $H_\infty$ filters parameters have been obtained in Theorem 2. Finally, the proposed data-driven robust non-fragile filtering method for cyber-physical systems is confirmed by a simulation.

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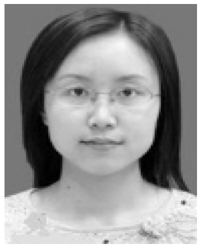

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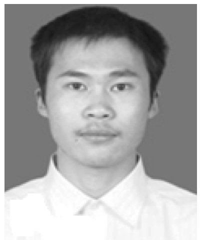

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