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Solvability of Output Regulation for Cascade Switched Nonlinear Systems

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ABSTRACT The problem of output regulation for a class of cascade switched nonlinear systems is investigated in this paper. Sufficient conditions for the problem to be solvable are given using the average dwell time method and the multiple Lyapunov function method. The problem for each subsystem to be solvable and may not be solvable are discussed, respectively. The main results are obtained based on full information feedback and error feedback. In addition, the method of designing the switched law is also presented in this paper. Finally, the simulation examples show that the results are very effective.

INDEX TERMS Output regulation, switched cascade nonlinear systems, average dwell time, multiple Lyapunov function.

I. INTRODUCTION

Output regulation is an important problem in the control theory. This problem aims to asymptotically track a class of reference trajectories and/or disturbances. It is generated by an autonomous finite-dimensional dynamical system. The problem of output regulation has been widely used in practical engineering. Thus, many researchers investigated this problem. For non-switched linear and nonlinear systems, this problem has been generally studied in references [1]–[5].

The switched system is a special kind of hybrid systems. It consists of a family of continuous time or discrete-time subsystems and a rule orchestrating the switching between these subsystems. Switched systems are used widely in the engineering applications, for instance, control of industrial processes and air traffic control [6], [7]. Generally, stability analysis and stabilization problems are fundamental issue of switched systems. The average dwell time method is an effective tool to solve these problems. The multiple Lyapunov function method is the other effective tools [8]–[13].

The output regulation problem is more challenging than stabilization, because the hybrid nature of the switched systems makes the problem more complicated. Actually, even if under the arbitrary switching, the solvability of the output regulation problem for switched systems could not be obtained from the solvability of the problem for each subsystem.

Up to now, there are few results about the output regulation problem of the switched systems. For linear switched systems, the solvability of the problem was solved using a convex combination method [14]. In order to reduce the conservativeness, reference [15] studied the output regulation problem for switched discrete-time linear systems. The multiple Lyapunov function method was used in this reference. References [16], [17] discussed the optimal output regulation problem for switched discrete-time linear systems. In [18], the problem was considered for non-switched systems with a switched exosystem. Reference [19] has solved the problem based on an error feedback according to an output error-defendant method. Reference [20] presented the sufficient condition for the problem to be solvable. The sufficient condition is based on the multiple quadratic Lyapunov convex null. At the same time, reference [21] discussed the output regulation problem for a class of switched linear multi-agent systems with a distributed observer approach. In [22], the problem was solved by a geometric approach. For nonlinear switched systems, reference [23] considered the solvability of the problem. The average dwell time method was used in this reference. Reference [24] applied the concept of geometric steady to research the solvability of the problem. References [25]–[27] presented the sufficient conditions for the problem based on the concept of the switched internal model. However, to the authors' best knowledge, the output regulation problem has not been investigated for cascade switched nonlinear systems. In the present work, this problem will be investigated.

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In this paper, we investigated the output regulation problem for cascade switched nonlinear systems. The average dwell time method and the multiple Lyapunov function method are used to solve this problem. Firstly, the regulation equations are derived for the solvability of the problem for the cascade switched nonlinear systems. Secondly, switching laws and controllers are designed to ensure the output regulation problem of switched systems is solvable. The problems for each subsystem with the assumption of the solvability are considered, while the problems with no assumption of the solvability are also considered. In addition, the above results are presented based on the full information feedback and the error feedback.

II. PROBLEM STATEMENT

Consider a cascade switched nonlinear system described by the equations of the form

$$\begin{cases}\dot{x}_1 = A_{1\sigma(t)}x_1(t) + A_{2\sigma(t)}x_2(t) + B_{\sigma(t)}u(t) + P_{\sigma(t)}\omega \\ \dot{x}_2 = f_{2\sigma(t)}(x_2(t)) \\ e(t) = C_{\sigma(t)}x_1(t) + Q_{\sigma(t)}\omega \end{cases} \tag{1}$$

where $x_1 \in R^{n-d}$ and $x_2 \in R^d$ are state vectors, $u \in R^m$ and $e \in R^p$ are control input and error variable respectively. The switching signal $\sigma : [0,\infty) \to I_N [1,\ldots,N]$ is a piecewise constant function of time, $A_{1i}, A_{2i}, B_i, P_i, C_i$ and Q_i are known constant matrices of appropriate dimensions, $f_{2i}(x_2)$ are known smooth functions. The exogenous input variable $\omega \in R^r$ includes disturbances and/or reference input and satisfies the following neutrally stable exosystem

$$\dot{\omega} = S\omega \tag{2}$$

where S has only simple eigenvalues on the imaginary axis.

In this paper, we consider the structure of controllers with two forms. When $x(t)$ and $\omega(t)$ are available, we adopt the full information controller, such as

$$u = K_{1\sigma}x_1 + K_{2\sigma}x_2 + L_\sigma\omega \tag{3}$$

and when the output error e is available for measurement, we apply the error feedback, that is,

$$\begin{aligned}\dot{\xi} &= F_\sigma\xi + G_\sigma e \\ u &= H_\sigma\xi\end{aligned} \tag{4}$$

Definition 1 (see [23]): For any switching signal $\sigma(t)$ and any $t > t_0 \geq 0$, let $N_\sigma(t_0, t)$ denote the number of switching of $\sigma(t)$ on the interval (t_0, t). If $N_\sigma(t_0, t) \leq N_0 + t - t_0/\tau_a$ holds, then the positive constant τ_a is called the average dwell time. In general, we assume $N_0 = 0$.

We now state the output regulation problem of the switched system (1).

A. OUTPUT REGULATION VIA FULL INFORMATION

Find, if possible, feedback laws (3) and a switching law $\sigma(t)$ such that:

i) the system (1) with the controllers (3) is asymptotically stable under the switching law $\sigma(t)$ without the disturbance input.
ii) for each $(x(0), \omega(0))$, the solution $(x(t), \omega(t))$ of

$$\begin{aligned}\dot{x}_1 &= (A_{1\sigma} + B_\sigma K_{1\sigma})x_1 + (A_{2\sigma} + B_\sigma K_{2\sigma})x_2 \\ &\quad + (B_\sigma L_\sigma + P_\sigma)\omega \\ \dot{x}_2 &= f_{2\sigma}(x_2) \\ \dot{\omega} &= S\omega\end{aligned} \tag{5}$$

satisfies

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}\left(C_{\sigma(t)}x + Q_{\sigma(t)}\omega\right) = 0.$$

B. OUTPUT REGULATION VIA ERROR FEEDBACK

Find, if possible, feedback laws (4) and a switching law $\sigma(t)$ such that:

i) the system (1) with the controllers (4) is asymptotically stable under the switching law $\sigma(t)$ without the disturbance input.
ii) for each $(x(0), \xi(0), \omega(0))$, the solution $(x(t), \xi(t), \omega(t))$ of

$$\begin{aligned}\dot{x}_1 &= A_{1\sigma}x_1 + A_{2\sigma}x_2 + B_\sigma H_\sigma\xi + P_\sigma\omega \\ \dot{x}_2 &= f_{2\sigma}(x_2) \\ \dot{\xi} &= G_\sigma C_\sigma x_1 + F_\sigma\xi + G_\sigma Q_\sigma\omega \\ \dot{\omega} &= S\omega\end{aligned} \tag{6}$$

satisfies

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}\left(C_{\sigma(t)}x + Q_{\sigma(t)}\omega\right) = 0.$$

III. OUTPUT REGULATION UNDER THE GIVEN SWITCHING LAW

In this section, we will give sufficient conditions for the solvability of the output regulation problem, based on the average dwell time method.

A. OUTPUT REGULATION VIA FULL INFORMATION FEEDBACK

Now, we will give a sufficient condition for the output regulation problem is solvable via full information feedback.

Theorem 1: Suppose there exist constants $\lambda_0 > 0, \delta > 0, \chi_1 > 0, \chi_2 > 0, \gamma > 0, \mu > 0$, positive definite matrices $\tilde{P}_i$, and matrices $K_{1i}, i = 1, 2\cdots N$, and a smooth positive definite function $G(x_2)$, such that

$$\begin{aligned}A_{1i}^T\tilde{P}_i + \tilde{P}_iA_{1i} + K_{1i}^TB_i^T\tilde{P}_i + \tilde{P}_iB_iK_{1i} + \gamma I + \lambda_0\tilde{P}_i < 0 \\ \tilde{P}_i \leq \mu\tilde{P}_j \quad i, j \in I_N\end{aligned} \tag{7}$$

$$\begin{aligned}&\chi_1\left\|x_2\right\|^2 \leq G(x_2) \leq \chi_2\left\|x_2\right\|^2 \\ &\frac{\partial G(x_2)}{\partial x_2}f_i(x_2) \leq -\delta\left\|x_2\right\|^2\end{aligned} \tag{8}$$

If there exist Π, Γ_i, for $\forall i \in I_N$, satisfying the following equations

$$\begin{aligned}\Pi S &= A_{1i}\Pi + B_i\Gamma_i + P_i \\ 0 &= C_i\Pi + Q_i\end{aligned} \tag{9}$$

then, under the switching law satisfying the following average dwell time

$$\tau_a \geq \tau_a^* = \frac{\ln \mu}{\lambda} \quad \lambda \in (0, \lambda_0) \tag{10}$$

the full information feedback controllers (3) solve the problem of output regulation for the switched system (1), where $\tilde{x}_1 = x_1 - \Pi\omega$.

Proof: Set $L_i = \Gamma_i - K_{1i}\Pi$, and consider the coordinate transformation $\tilde{x}_1 = x_1 - \Pi\omega$. According to (9), we have

$$\begin{aligned} \dot{\tilde{x}}_1 &= \dot{x}_1 - \Pi\dot{\omega} \\ &= A_{1\sigma}(x_1 - \Pi\omega) + B_\sigma K_{1\sigma}(x_1 - \Pi\omega) \\ &\quad + (A_{2\sigma} + B_\sigma K_{2\sigma}) x_2 \\ &= (A_{1\sigma} + B_\sigma K_{1\sigma})\tilde{x}_1 + (A_{2\sigma} + B_\sigma K_{2\sigma}) x_2 \\ \dot{x}_2 &= f_{2\sigma}(x_2) \\ e &= C_\sigma x_1 + Q_\sigma \omega \\ &= C_\sigma (x_1 - \Pi\omega) + (C_\sigma \Pi + Q_\sigma)\omega \\ &= C_\sigma \tilde{x}_1 \end{aligned} \tag{11}$$

We define the following Lyapunov function candidate $V(t) = V_\sigma(t) = \tilde{x}_1^T \tilde{P}_\sigma \tilde{x}_1 + kG(x_2)$ and for the i-th activated subsystem, the derivative of $V(t)$ along the trajectory of the corresponding subsystem is

$$\begin{aligned} \dot{V}(t) &= \dot{\tilde{x}}_1^T \tilde{P}_i \tilde{x}_1 + \tilde{x}_1^T \tilde{P}_i \dot{\tilde{x}}_1 + k\frac{\partial G(x_2)}{\partial x_2}\dot{x}_2 \\ &= \tilde{x}_1^T (A_{1i} + B_i K_{1i})^T \tilde{P}_i \tilde{x}_1 + \tilde{x}_1^T \tilde{P}_i (A_{1i} + B_i K_{1i})\tilde{x}_1 \\ &\quad + x_2^T (A_{2i} + B_i K_{2i})^T \tilde{P}_i \tilde{x}_1 + \tilde{x}_1^T \tilde{P}_i (A_{2i} + B_i K_{2i}) x_2 \\ &\quad + k\frac{\partial G(x_2)}{\partial x_2} f_{2i}(x_2) \\ &= \tilde{x}_1^T \left(A_{1i}^T \tilde{P}_i + \tilde{P}_i A_{1i} + K_{1i}^T B_i^T \tilde{P}_i + \tilde{P}_i B_i K_{1i}\right)\tilde{x}_1 \\ &\quad + 2\tilde{x}_1^T \tilde{P}_i A_{2i} x_2 + 2\tilde{x}_1^T \tilde{P}_i B_i K_{2i} x_2 \\ &\quad + k\frac{\partial G(x_2)}{\partial x_2} f_{2i}(x_2) \end{aligned}$$

there exist constants $c_i > 0$, $q_i > 0$ and $d_i > 0$, such that

$$\|A_{2i}x_2\| \leq c_i\|x_2\|, \quad \left\|\tilde{x}_1^T \tilde{P}_i\right\| \leq q_i\|\tilde{x}_1\|, \ \|B_i K_{2i} x_2\| \leq d_i\|x_2\|,$$

we set $m = \max\{c_i q_i, d_i q_i | i \in I_N\}$, according to (7) and (8), we get

$$\begin{aligned} \dot{V}(t) &\leq -\gamma\|\tilde{x}_1\|^2 - \lambda_0 \tilde{x}_1^T \tilde{P}_i \tilde{x}_1 + 4m\|\tilde{x}_1\|\|x_2\| - k\delta\|x_2\|^2 \\ &\leq -\lambda_0 \tilde{x}_1^T \tilde{P}_i \tilde{x}_1 - k\lambda_0 G(x_2) + k\lambda_0 G(x_2) - \gamma\|\tilde{x}_1\|^2 \\ &\quad + 4m\|\tilde{x}_1\|\|x_2\| - k\delta\|x_2\|^2 \\ &\leq -\lambda_0 V(t) + k\lambda_0 \chi_2\|x_2\|^2 - \gamma\|\tilde{x}_1\|^2 \\ &\quad + 4m\|\tilde{x}_1\|\|x_2\| - k\delta\|x_2\|^2 \\ &\leq -\lambda_0 V(t) - \gamma\left(\|\tilde{x}_1\| - \frac{2m}{\gamma}\|x_2\|\right)^2 \\ &\quad - \left(-\frac{4m^2}{\gamma} + k\delta - k\lambda_0\chi_2\right)\|x_2\|^2 \end{aligned}$$

where $k > \frac{4m^2}{\gamma(\delta - \lambda_0\chi_2)}$, thus $\dot{V}(t) < -\lambda_0 V(t)$. By virtue of (7), we have

$$V_i \leq \mu V_j \quad i, j \in I_N$$

For any $t > 0$, denote $t_1, t_2, \cdots, t_{k-1}, t_k, \cdots, t_{N_\sigma(0,T)}$ as the switching instants on the interval $(0, t)$, we get

$$\begin{aligned} V(t) &\leq e^{-\lambda_0(t - t_{N_\sigma(0,t)})} V(t_{N_\sigma(0,t)}) \\ &\leq \mu e^{-\lambda_0(t - t_{N_\sigma(0,t)})} V(t^-_{N_\sigma(0,t)}) \\ &\leq \cdots \\ &\leq \mu^{N_\sigma(0,t)} e^{-\lambda_0 t} V(0) \end{aligned}$$

According to (8), there exist constants φ_1, φ_2, such that

$$\varphi_1\left(\|\tilde{x}_1\|^2 + \|x_2\|^2\right) \leq V(t) \leq \varphi_2\left(\|\tilde{x}_1\|^2 + \|x_2\|^2\right)$$

Based on the above inequalities, we have

$$\|\tilde{x}(t)\| \leq \sqrt{\frac{\varphi_2}{\varphi_1}} e^{\frac{N_\sigma(0,t)\ln\mu - \lambda_0 t}{2}} \|\tilde{x}(0)\|$$

where $\tilde{x}(t) = \left[\tilde{x}_1^T, x_2^T\right]^T$. Because of the formula (10) and Definition 1, we have

$$\|\tilde{x}(t)\| \leq \sqrt{\frac{\varphi_2}{\varphi_1}} e^{\frac{(\lambda - \lambda_0)t}{2}} \|\tilde{x}(0)\|$$

Therefore, under the average dwell time (10), the system (11) is asymptotically stable and $\lim_{t\to\infty} e(t) = 0$.

Thus, the problem is solved via the full information feedback.

B. OUTPUT REGULATION VIA ERROR FEEDBACK

In this part, we consider the exosystem state $\omega(t)$ is not available, and the output error is available for measurement. Under these conditions, we will give a solvability condition.

Theorem 2: Assume that there exist constants $\bar{\lambda}_0 > 0$, $\bar{\delta} > 0$, $\bar{\chi}_1 > 0$, $\bar{\chi}_2 > 0$, $\bar{\gamma} > 0$, $\bar{\mu} > 0$ and positive definite matrices $\bar{P}_i$ and matrices G_i, $i = 1, 2 \cdots N$, and a smooth positive definite function $Z(x_2)$, satisfying the following inequalities

$$\begin{aligned} \bar{A}_{1i}^T \bar{P}_i + \bar{P}_i \bar{A}_{1i} + \bar{\gamma} I + \bar{\lambda}_0 \bar{P}_i &< 0 \\ \bar{P}_i &\leq \bar{\mu}\bar{P}_j \quad i, j \in I_N \end{aligned} \tag{12}$$

$$\begin{aligned} \bar{\chi}_1\|x_2\|^2 \leq Z(x_2) &\leq \bar{\chi}_2\|x_2\|^2 \\ \frac{\partial Z(x_2)}{\partial x_2} f_i(x_2) &\leq -\bar{\delta}\|x_2\|^2 \end{aligned} \tag{13}$$

If there exist Π, Σ, such that

$$\begin{aligned} \Pi S &= A_{1i}\Pi + B_i H_i \Sigma + P_i \\ \Sigma S &= F_i \Sigma \\ 0 &= C_i \Pi + Q_i \end{aligned} \tag{14}$$

then, under the switching law satisfying the following average dwell time

$$\tau_a \geq \tau_a^* = \frac{\ln\bar{\mu}}{\bar{\lambda}} \quad \bar{\lambda} \in \left(0, \bar{\lambda}_0\right) \tag{15}$$

the output regulation problem is solved by output feedback controllers (4), where $\bar{A}_{1i} = \begin{bmatrix} A_{1i} & B_iH_i \\ G_iC_i & F_i \end{bmatrix}$.

Proof: Define $\begin{pmatrix} \tilde{x}_1 \\ \tilde{\xi} \end{pmatrix} = \begin{pmatrix} x - \Pi\omega \\ \xi - \Sigma\omega \end{pmatrix}$, according to (14), we have

$$\begin{aligned}
\dot{\tilde{x}}_1 &= \dot{x}_1 - \prod \dot{\omega} \\
&= A_{1\sigma}(x_1 - \prod \omega) + B_\sigma H_\sigma\,(\xi - \Sigma\omega) \\
&\quad + (A_{1\sigma} \prod + B_\sigma H_\sigma \Sigma + P_\sigma - \prod S)\omega + A_{2\sigma}x_2 \\
&= A_{1\sigma}\tilde{x}_1 + B_\sigma H_\sigma \tilde{\xi} + A_{2\sigma}x_2 \\
\dot{\tilde{\xi}} &= F_\sigma \tilde{\xi} + G_\sigma C_\sigma \tilde{x}_1 \\
\dot{x}_2 &= f_{2\sigma}(x_2) \\
e &= C_\sigma \tilde{x}_1
\end{aligned} \tag{16}$$

Set $X_1 = \left(\tilde{x}_1^T \ \tilde{\xi}^T\right)^T$, then

$$\begin{aligned}
\dot{X}_1 &= \bar{A}_{1\sigma}\dot{X}_1 + \bar{A}_{2\sigma}x_2 \\
\dot{x}_2 &= f_{2\sigma}(x_2) \\
e &= \bar{C}_\sigma X_1
\end{aligned} \tag{17}$$

where $\bar{A}_{2i} = \begin{bmatrix} A_{2i} \\ 0 \end{bmatrix}$, $\bar{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}$.

We choose $V(t) = V_\sigma(t) = X_1^T\bar{P}_\sigma X_1 + \bar{k}Z(x_2)$ and for the i-th activated subsystem, the derivative of $V(t)$ along the trajectory of the corresponding subsystem is

$$\begin{aligned}
\dot{V}(t) &= \dot{X}_1^T\bar{P}_iX_1 + X_1^T\bar{P}_i\dot{X}_1 + \bar{k}\frac{\partial Z(x_2)}{\partial x_2}f_{2i}(x_2) \\
&= X_1^T\left(\bar{A}_{1i}^T\bar{P}_i + \bar{P}_i\bar{A}_{1i}\right)X_1 + 2X_1^T\bar{P}_i\bar{A}_{2i}x_2 \\
&\quad + \bar{k}\frac{\partial Z(x_2)}{\partial x_2}f_{2i}(x_2)
\end{aligned} \tag{18}$$

There exist constants $\bar{c}_i > 0$ and $\bar{q}_i > 0$, such that

$$\|\bar{A}_{2i}x_2\| \le \bar{c}_i\|x_2\|, \qquad \left\|X_1^T\bar{P}_i\right\| \le \bar{q}_i\,\|X_1\|,$$

we set $\bar{m} = \max\{\bar{c}_i\bar{q}_i | i \in I_N\}$, according to (12) and (13), we have

$$\begin{aligned}
\dot{V}(t) &\le -\bar{\gamma}\,\|X_1\|^2 - \bar{\lambda}_0X_1^T\bar{P}_iX_1 + 2\bar{m}\,\|X_1\|\,\|x_2\| - \bar{k}\bar{\delta}\,\|x_2\|^2 \\
&\le -\bar{\lambda}_0X_1^T\bar{P}_iX_1 - \bar{k}\bar{\lambda}_0Z(x_2) + \bar{k}\bar{\lambda}_0Z(x_2) - \bar{\gamma}\,\|X_1\|^2 \\
&\quad + 2\bar{m}\,\|X_1\|\,\|x_2\| - \bar{k}\bar{\delta}\,\|x_2\|^2 \\
&\le -\bar{\lambda}_0V(t) + \bar{k}\bar{\lambda}_0\bar{\chi}_2\,\|x_2\|^2 - \bar{\gamma}\,\|X_1\|^2 \\
&\quad + 2\bar{m}\,\|X_1\|\,\|x_2\| - \bar{k}\bar{\delta}\,\|x_2\|^2 \\
&\le -\bar{\lambda}_0V(t) - \gamma\left(\|X_1\| - \frac{\bar{m}}{\gamma}\,\|x_2\|\right)^2 \\
&\quad - \left(-\frac{m^2}{\gamma} + \bar{k}\bar{\delta} - \bar{k}\bar{\lambda}_0\bar{\chi}_2\right)\|x_2\|^2
\end{aligned}$$

where $\bar{k} > \frac{\bar{m}^2}{\bar{\gamma}(\bar{\delta} - \bar{\lambda}_0\bar{\chi}_2)}$, then $\dot{V}(t) < -\bar{\lambda}_0V(t)$.

Therefore, based on the proof of Theorem 1, the system (16) is asymptotically stable and $\lim\limits_{t\to\infty} e(t) = 0$.

Thus, the problem is solved via the error feedback.

IV. OUTPUT REGULATION UNDER THE CONSTRAINED SWITCHING LAW

In this part, solvability conditions for the problem will be presented based on the multiple Lyapunov function method, where the problem for each subsystem may not be solvable.

A. OUTPUT REGULATION VIA THE FULL INFORMATION FEEDBACK

Now, we will give a solvability condition under the constrained switching law via the full information feedback.

Theorem 3: Suppose that there exist constants $\beta_{ij} > 0$, $\beta > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, $\gamma > 0$, and positive definite matrices $\tilde{P}_i$ and matrices K_{1i}, $i, j = 1, 2 \cdots N$, and a smooth positive definite function $W(x_2)$, satisfying the following inequalities

$$A_{1i}^T\tilde{P}_i + \tilde{P}_iA_{1i} + K_{1i}^TB_i^T\tilde{P}_i + \tilde{P}_iB_iK_{1i} + \gamma I + \sum_{j=1, i\ne j}^{N}\beta_{ij}\left(\tilde{P}_j - \tilde{P}_i\right) < 0 \quad \forall i \in I_N \tag{19}$$

$$\begin{aligned}
&\alpha_1\,\|x_2\|^2 \le W(x_2) \le \alpha_2\,\|x_2\|^2 \\
&\frac{\partial W(x_2)}{\partial x_2}f_i(x_2) \le -\beta\,\|x_2\|^2
\end{aligned} \tag{20}$$

If there exist Π, Γ_i, for $\forall i \in I_N$, such that the formula (9) is satisfied, then, under the switching law

$$\sigma(t) = \arg\min_{i\in I_N}\left\{\tilde{x}_1^T\tilde{P}_i\tilde{x}_1 + W(x_2)\right\} \tag{21}$$

the problem is solved by controllers (3), where $\tilde{x}_1 = x_1 - \Pi\omega$.

Proof: Set $L_i = \Gamma_i - K_{1i}\Pi$, and consider the coordinate transformation $\tilde{x}_1 = x_1 - \Pi\omega$. According to (9), we have

$$\begin{aligned}
\dot{\tilde{x}}_1 &= \dot{x}_1 - \Pi\dot{\omega} \\
&= A_{1\sigma}\,(x_1 - \Pi\omega) + B_\sigma K_{1\sigma}\,(x_1 - \Pi\omega) \\
&\quad + (A_{2\sigma} + B_\sigma K_{2\sigma})\,x_2 \\
&= (A_{1\sigma} + B_\sigma K_{1\sigma})\,\tilde{x}_1 + (A_{2\sigma} + B_\sigma K_{2\sigma})\,x_2 \\
\dot{x}_2 &= f_{2\sigma}(x_2) \\
e &= C_\sigma x_1 + Q_\sigma\omega = C_\sigma\,(x_1 - \Pi\omega) + (C_\sigma\Pi + Q_\sigma)\,\omega \\
&= C_\sigma\tilde{x}_1
\end{aligned}$$

Define the following Lyapunov function $V(t) = V_\sigma(t) = \tilde{x}_1^T\tilde{P}_\sigma\tilde{x}_1 + lW(x_2)$ and for the i-th activated subsystem, the derivative of $V(t)$ along the trajectory of the corresponding subsystem is

$$\begin{aligned}
\dot{V}(t) &= \dot{\tilde{x}}_1^T\tilde{P}_i\tilde{x}_1 + \tilde{x}_1^T\tilde{P}_i\dot{\tilde{x}}_1 + l\frac{\partial W(x_2)}{\partial x_2}\dot{x}_2 \\
&= \tilde{x}_1^T\,(A_{1i} + B_iK_{1i})^T\,\tilde{P}_i\tilde{x}_1 + \tilde{x}_1^T\tilde{P}_i\,(A_{1i} + B_iK_{1i})\,\tilde{x}_1 \\
&\quad + x_2^T\,(A_{2i} + B_iK_{2i})^T\,\tilde{P}_i\tilde{x}_1 + \tilde{x}_1^T\tilde{P}_i\,(A_{2i} + B_iK_{2i})\,x_2 \\
&\quad + l\frac{\partial W(x_2)}{\partial x_2}f_{2i}(x_2) \\
&= \tilde{x}_1^T\left(A_{1i}^T\tilde{P}_i + \tilde{P}_iA_{1i} + K_{1i}^TB_i^T\tilde{P}_i + \tilde{P}_iB_iK_{1i}\right)\tilde{x}_1 \\
&\quad + 2\tilde{x}_1^T\tilde{P}_iA_{2i}x_2 + 2\tilde{x}_1^T\tilde{P}_iB_iK_{2i}x_2 + l\frac{\partial W(x_2)}{\partial x_2}f_{2i}(x_2)
\end{aligned}$$

there exist constants $a_i > 0, p_i > 0$, and $b_i > 0$, such that

$$\|A_{2i}x_2\| \leq \alpha_i \|x_2\|, \quad \left\|\tilde{x}_1^T \tilde{P}_i\right\| \leq p_i \|\tilde{x}_1\|,$$
$$\|B_iK_{2i}x_2\| \leq b_i \|x_2\|,$$

we set $k = \max\{a_ip_i, b_ip_i | i \in I_N\}$, according to (19) and (20), we get

$$\begin{aligned}\dot{V}(t) &\leq -\gamma \|\tilde{x}_1\|^2 + 4k \|\tilde{x}_1\| \|x_2\| - l\beta \|x_2\|^2 \\ &= -\gamma \left(\|\tilde{x}_1\| + \frac{2k}{\gamma} \|x_2\|\right)^2 + \frac{4k^2}{\gamma} \|x_2\|^2 - l\beta \|x_2\|^2\end{aligned}$$

where $l > \frac{4k^2}{\beta\gamma}$. Thus $\dot{V}(t) < 0$. Therefore, under the switching law (21), the system (1) is asymptotically stable without disturbances $\omega(t)$ and $\lim_{t\to\infty} e(t) = 0$.

Thus, the problem is solved by the dual design of controllers and the switching law.

B. OUTPUT REGULATION VIA THE ERROR FEEDBACK

The same as the part 3.2, we still consider the exosystem state $\omega(t)$ is not available, and the output error is available for measurement. Under these conditions, we will give a solvability condition.

Theorem 4: Assume that there exist constants $\bar{\beta}_{ij} > 0$, $\bar{\beta} > 0, \bar{\alpha}_1 > 0, \bar{\alpha}_2 > 0, \bar{\gamma} > 0$, and positive definite matrices $\bar{P}_i$ and matrices $G_i, i, j = 1, 2 \cdots N$, and a smooth positive definite function $U(x_2)$, satisfying the following inequalities

$$\bar{A}_{1i}^T\bar{P}_i + \bar{P}_i\bar{A}_{1i} + \bar{\gamma}I + \sum_{j=1,i\neq j}^{N} \bar{\beta}_{ij}\left(\bar{P}_j - \bar{P}_i\right) < 0, \quad \forall i \in I_N \tag{22}$$

$$\bar{\alpha}_1 \|x_2\|^2 \leq U(x_2) \leq \bar{\alpha}_2 \|x_2\|^2$$
$$\frac{\partial U(x_2)}{\partial x_2} f_i(x_2) \leq -\bar{\beta} \|x_2\|^2 \tag{23}$$

If there exist Π, Σ, such that the formula (14) is satisfied, then, under the switching law

$$\sigma(t) = \arg\min_{i\in I_N} \left\{e^T \bar{P}_{11i} e\right\} \tag{24}$$

the output regulation problem is solved by output feedback controllers (4), where

$$\bar{A}_{1i} = \begin{bmatrix} A_{1i} & B_iH_i \\ G_iC_i & F_i \end{bmatrix}, \quad \bar{P}_i = \begin{bmatrix} C_i^T\bar{P}_{11i}C_i & 0 \\ 0 & \bar{P}_{22} \end{bmatrix}.$$

Proof: Define $\begin{pmatrix} \tilde{x}_1 \\ \tilde{\xi} \end{pmatrix} = \begin{pmatrix} x - \Pi\omega \\ \xi - \Sigma\omega \end{pmatrix}$, according to (14), we have

$$\begin{aligned}\dot{\tilde{x}}_1 &= \dot{x}_1 - \prod \dot{\omega} \\ &= A_{1\sigma}(x_1 - \prod \omega) + B_\sigma H_\sigma (\xi - \Sigma\omega) \\ &\quad + (A_{1\sigma} \prod + B_\sigma H_\sigma \Sigma + P_\sigma - \prod S)\omega + A_{2\sigma}x_2 \\ &= A_{1\sigma}\tilde{x}_1 + B_\sigma H_\sigma \tilde{\xi} + A_{2\sigma}x_2 \\ \dot{\tilde{\xi}} &= F_\sigma\tilde{\xi} + G_\sigma C_\sigma \tilde{x}_1 \\ \dot{x}_2 &= f_{2\sigma}(x_2) \\ e &= C_\sigma \tilde{x}_1\end{aligned}$$

Set $X_1 = \left(\tilde{x}_1^T \ \tilde{\xi}^T\right)^T$, then

$$\begin{aligned}\dot{X}_1 &= \bar{A}_{1\sigma}X_1 + \bar{A}_{2\sigma}x_2 \\ \dot{x}_2 &= f_{2\sigma}(x_2) \\ e &= \bar{C}_\sigma X_1\end{aligned}$$

where $\bar{A}_{2i} = \begin{bmatrix} A_{2i} \\ 0 \end{bmatrix}, \bar{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}$.

We choose $V(t) = V_\sigma(t) = X_1^T\bar{P}_\sigma X_1 + lU(x_2)$ and for the i-th activated subsystem, the derivative of $V(t)$ along the trajectory of the corresponding subsystem is

$$\begin{aligned}\dot{V}(t) &= \dot{X}_1^T\bar{P}_iX_1 + X_1^T\bar{P}_i\dot{X}_1 + l\frac{\partial U(x_2)}{\partial x_2} f_{2i}(x_2) \\ &= X_1^T\left(\bar{A}_{1i}^T\bar{P}_i + \bar{P}_i\bar{A}_{1i}\right)X_1 + 2X_1^T\bar{P}_i\bar{A}_{2i}x_2 \\ &\quad + l\frac{\partial U(x_2)}{\partial x_2} f_{2i}(x_2)\end{aligned} \tag{25}$$

Because of

$$\begin{aligned}V_i(t) &= X_1^T\bar{P}_iX_1 + lU(x_2) \\ &= \left(\tilde{x}_1^T \quad \tilde{\xi}^T\right)\begin{pmatrix} C_i^T\bar{P}_{11i}C_i & 0 \\ 0 & \bar{P}_{22} \end{pmatrix}\begin{pmatrix} \tilde{x}_1 \\ \tilde{\xi} \end{pmatrix} + lU(x_2) \\ &= \tilde{x}_1^T C_i^T \bar{P}_{11i} C_i \tilde{x}_1 + \tilde{\xi}^T\bar{P}_{22}\tilde{\xi} + lU(x_2) \\ &= e^T\bar{P}_{11i}e + \tilde{\xi}^T\bar{P}_{22}\tilde{\xi} + lU(x_2)\end{aligned}$$

and

$$\sigma(t) = \arg\min_{i\in I_N}\left\{e^T\bar{P}_{11i}e\right\} = \arg\min_{i\in I_N}\left\{X_1^T\bar{P}_iX_1 + lU(x_2)\right\}$$

There exist constants $\bar{a}_i > 0$, and $\bar{p}_i > 0$ such that $\left\|\bar{A}_{2i}x_2\right\| \leq \bar{a}_i \|x_2\|$, $\left\|X_1^T\bar{P}_i\right\| \leq \bar{p}_i\|X_1\|$, we set $\bar{k} = \max\{\bar{a}_i\bar{p}_i | i \in I_N\}$, according to (22)-(25), we have

$$\begin{aligned}\dot{V}(t) &< -\bar{\gamma} \|X_1\|^2 + 2\bar{k} \|X_1\| \|x_2\| - l\bar{\beta} \|x_2\|^2 \\ &= -\bar{\gamma}\left(\|X_1\| - \frac{\bar{k}}{\bar{\gamma}} \|x_2\|\right)^2 + \frac{\bar{k}^2}{\bar{\gamma}} \|x_2\|^2 - l\bar{\beta} \|x_2\|^2\end{aligned}$$

We choose $l > \frac{\bar{k}^2}{\bar{\beta}\bar{\gamma}}$, then $\dot{V}(t) < 0$. Therefore, under the switching law (24), the system (1) is asymptotically stable without disturbances $\omega(t)$ and $\lim_{t\to\infty} e(t) = 0$.

Thus, the problem is solved by the dual design of controllers and the switching law via the error feedback.

V. SIMULATION

In this part, we will adopt the main method presented in this paper to solve two numerical examples and a practical example.

Example 1: Now, we will solve the output regulation problem via the full information feedback by the average dwell time method.

Consider the switched system (1) consisting of the following two subsystems.

The first subsystem with the parameters as follows:

$$A_{11} = \begin{bmatrix} -1.2 & 0.2 \\ 0.2 & -0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \quad P_1 = \begin{bmatrix} 1.6 & 0.9 \\ -0.45 & -0.95 \end{bmatrix}, f_{21} = -x_2 - x_2^3,$$
$$C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} -1.5 & 0 \end{bmatrix}.$$

The second subsystem with the parameters as follows:

$$A_{21} = \begin{bmatrix} -0.5 & 0.1 \\ 0.1 & -1.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$A_{22} = \begin{bmatrix} 2.5 \\ 0.5 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.25 & 0.4 \\ 0.15 & -1.65 \end{bmatrix},$$
$$f_{22} = -2x_2^3 - x_2 \cos^2 x_2, \quad C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} -1.5 & 0 \end{bmatrix}.$$

In (9),

$$S = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix}, \quad \Gamma_1 = \Gamma_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\Pi = \begin{bmatrix} 1 & 1 \\ 0.5 & -1 \end{bmatrix}.$$

Set $\gamma = 1$, $\lambda_0 = 0.5$, $\mu = 1.2$, we have $\tau_a \geq 0.364$.

Based on the above parameters, we have

$$\tilde{P}_1 = \begin{bmatrix} 1.3652 & 0.0449 \\ 0.0449 & 1.7917 \end{bmatrix}, \quad \tilde{P}_2 = \begin{bmatrix} 1.7904 & 0.0173 \\ 0.0173 & 1.4273 \end{bmatrix},$$
$$K_{11} = \begin{bmatrix} -0.4140 & -1.2570 \end{bmatrix},$$
$$K_{12} = \begin{bmatrix} -0.9513 & -0.2044 \end{bmatrix},$$
$$K_{21} = -0.25, \quad K_{22} = -0.75.$$

Therefore, the conditions of Theorem 1 are satisfied, and the output regulation problem is solved via the full information feedback. Figs. 1-3 depict the simulation results. Fig 1 gives the state response of the switched system; Fig 2 illustrates the output of the switched system. Fig 3 demonstrates the switching law.

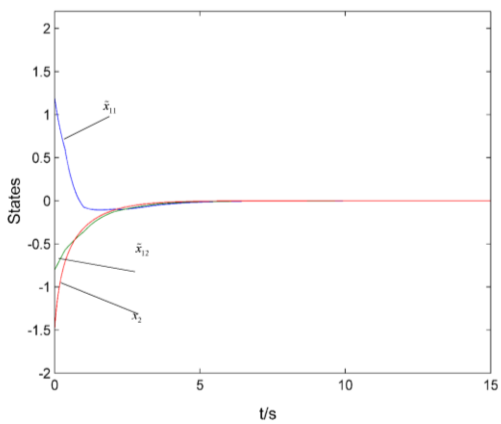

FIGURE 1. State responses of the switched systesm.

Example 2: Now, we will solve the output regulation problem via the error feedback by the multiple Lyapunov function method.

Consider the switched system (1) consisting of the following two subsystems.

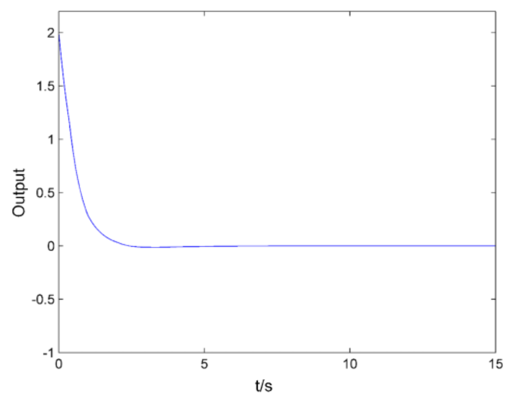

FIGURE 2. Output of the switched system.

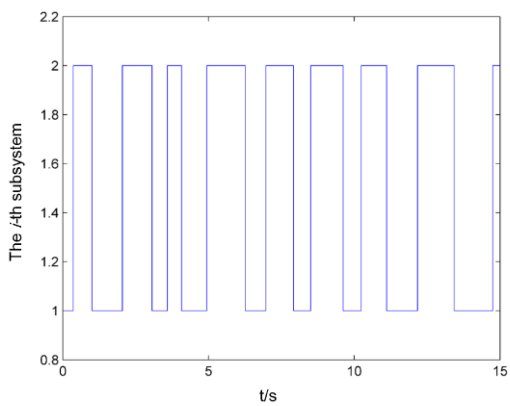

FIGURE 3. The switching law.

The first subsystem with the parameters as follows:

$$A_{11} = \begin{bmatrix} -2.5 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & -2.2 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad P_1 = \begin{bmatrix} -2 & 2.25 \\ 0.8 & -1.2 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 1 & -0.05 \\ -1 & -1 \end{bmatrix}, \quad f_{21}(x_2) = -x_2 - x_2^3.$$

The second subsystem with the parameters as follows:

$$A_{21} = \begin{bmatrix} 0.8 \\ 2 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 2.3 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 0.7 & 0.9 \\ 3.2 & 2.2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 1 & -0.5 \\ -1 & -1 \end{bmatrix},$$
$$f_{22}(x_2) = -2x_2^3 - x_2 \cos^2 x_2, \quad S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Based on the above parameters, there exist following matrices satisfying (14)

$$\Pi = \begin{bmatrix} 1 & -0.5 \\ 1 & 1 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$H_1 = [0.5 \quad -0.5], \quad H_2 = [-0.5 \quad 0.5],$$
$$F_1 = \begin{bmatrix} -1.5 & 0 \\ 0 & -1.1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix}.$$

Let $\bar{\beta}_{12} = 0.2$, $\bar{\beta}_{21} = 2$ and $\bar{\gamma} = 0.5$, according to Theorem 4, we have

$$\bar{P}_{111} = \begin{bmatrix} 9.7336 & 0 \\ 0 & 11.1797 \end{bmatrix},$$
$$\bar{P}_{112} = \begin{bmatrix} 35.3093 & 0 \\ 0 & 35.3543 \end{bmatrix},$$
$$\bar{P}_{22} = \begin{bmatrix} 14.8794 & 0.7878 \\ 0.7878 & 19.6276 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 2.7409 & -35.0432 \\ -47.8321 & 49.7168 \end{bmatrix},$$
$$G_2 = \begin{bmatrix} -3.1334 & 35.4921 \\ 48.1333 & -50.0612 \end{bmatrix}.$$

Therefore, the conditions of Theorem 4 are satisfied, and the output regulation problem is solved via the error feedback. Figs. 4-9 depict the simulation results. Fig.4 and Fig.5 give the state response and the output of the subsystem 1, Fig.6 and Fig.7 illustrate the state response and the output of the subsystem 2, Fig.8 and Fig.9 give the state response and the output of the switched system.

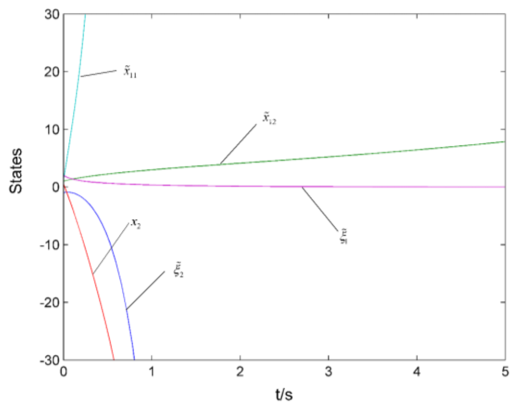

FIGURE 4. The state responses of the subsystem 1.

Figure 4-9 show that the output regulation problem is not solvable for the subsystem 1 and the subsystem 2, but the problem is solvable for the switched system under the dual design of the controller and the switching law.

Example 3: Consider a switched RLC circuit in Figure 10, the capacitor C_1 and C_2 could be switched. The switched RLC circuit is described by the following equations

$$\begin{cases} \dot{x}_1 = \begin{bmatrix} 0 & 1/L \\ 1/C_i & -R_1/L \end{bmatrix} x_1(t) + A_{2i}x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + P_i\omega \\ \dot{x}_2 = -\dfrac{1}{R_2C_3}x_2, \end{cases}$$
$$i = 1, 2$$

where $x_1 = \begin{bmatrix} q_c & \phi_L \end{bmatrix}$, x_2 is the voltage of the capacitor C_3, ω is the disturbance of the circuit, which satisfies the dynamic equation $\dot{\omega} = S\omega$, our control objective is the output $e = Cx_1 + Q\omega$ tends to zero. We choose the system parameters are $L = 0.5H$, $C_1 = 50\mu F$, $C_2 = 100\mu F$, $C_3 = 50\mu F$, $R_1 = 1\Omega$ and $R_2 = 0.1\Omega$.

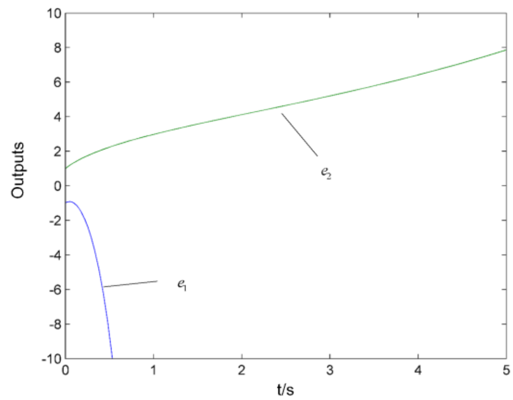

FIGURE 5. The outputs of the subsystem 1.

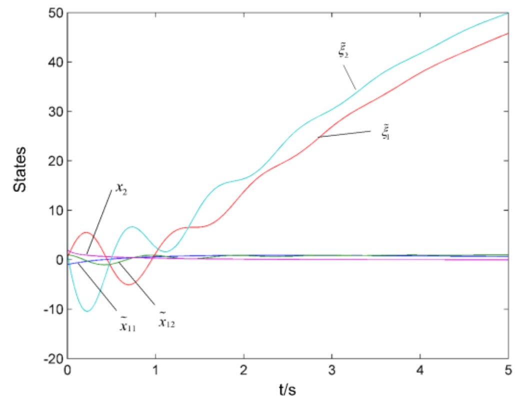

FIGURE 6. The state responses of the subsystem 2.

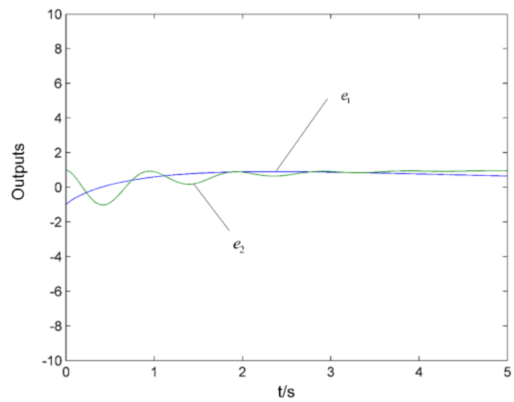

FIGURE 7. The outputs of the subsystem 2.

Therefore, we have

$$A_{11} = \begin{bmatrix} 0 & 2 \\ -0.02 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad P_1 = \begin{bmatrix} -1.8 & 1.6 \\ 1.84 & 2.18 \end{bmatrix}, f_{21} = -0.2x_2,$$
$$C_1 = \begin{bmatrix} 2 & -2 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} -2 & -4 \end{bmatrix}.$$
$$A_{21} = \begin{bmatrix} 0 & 2 \\ -0.01 & -2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

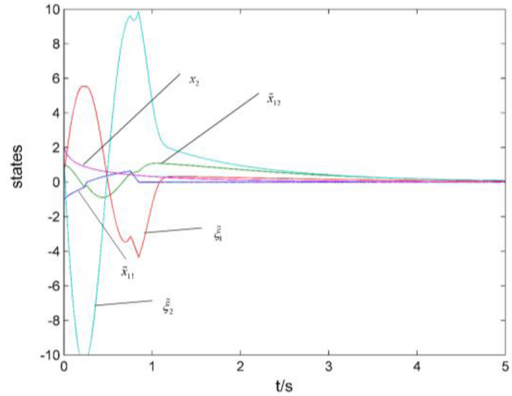

FIGURE 8. The state responses of switched systesms.

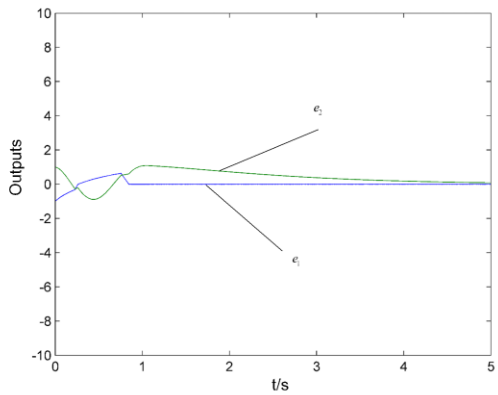

FIGURE 9. The outputs of switched systems.

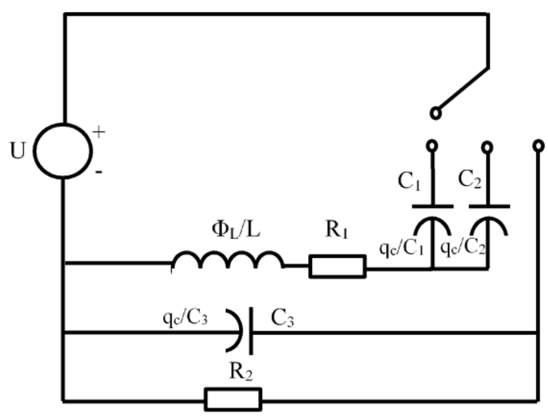

FIGURE 10. A switched RLC circuit.

$$A_{22} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad P_1 = \begin{bmatrix} -1.8 & 1.6 \\ 1.82 & 2.18 \end{bmatrix},$$

$$f_{22} = -0.2x_2, \quad C_2 = \begin{bmatrix} 2 & -2 \end{bmatrix}, \; Q_2 = \begin{bmatrix} -2 & -4 \end{bmatrix}.$$

In (9),

$$S = \begin{bmatrix} 0 & -0.2 \\ 0.2 & 0 \end{bmatrix}, \quad \Gamma_1 = \Gamma_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}.$$

Set $\gamma = 1$, $\lambda_0 = 0.3$, $\mu = 1.1$, we have $\tau_a \geq 0.333$.

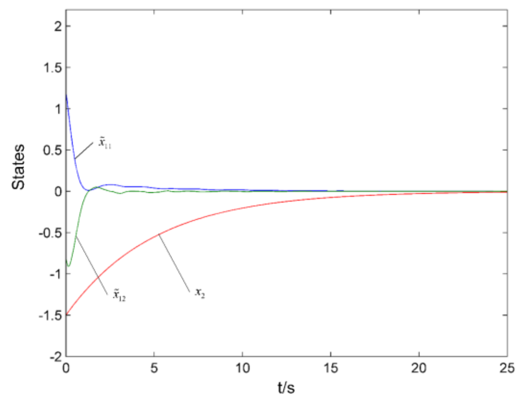

FIGURE 11. State responses of the switched systesm.

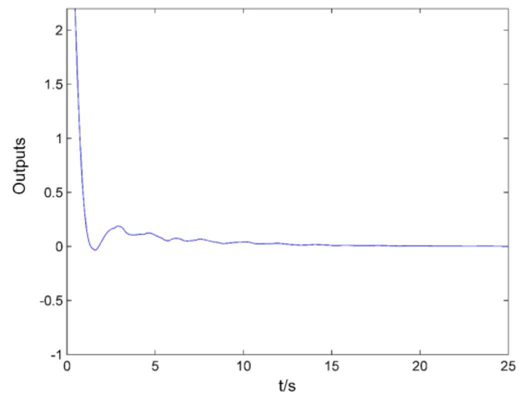

FIGURE 12. Output of the switched system.

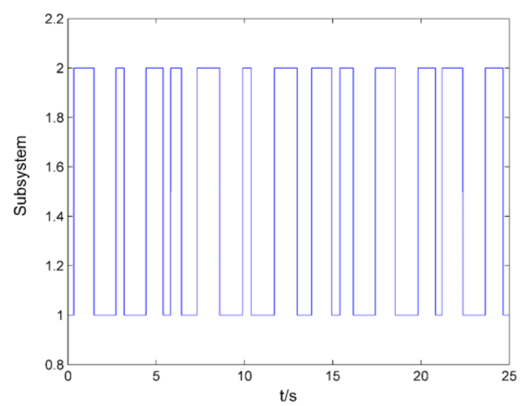

FIGURE 13. The switching law.

Based on the above parameters, we have

$$\tilde{P}_1 = \tilde{P}_2 = \begin{bmatrix} 4.3061 & 2.4527 \\ 2.4527 & 3.9382 \end{bmatrix},$$

$$K_{11} = \begin{bmatrix} -4.6326 & -2.2592 \end{bmatrix},$$

$$K_{12} = \begin{bmatrix} -4.6426 & -2.2592 \end{bmatrix},$$

$$K_{21} = -0.3675, \quad K_{22} = -0.2525.$$

Therefore, the conditions of Theorem 1 are satisfied, and the output regulation problem is solved via the full information feedback.

Figs. 11-13 depict the simulation results. Fig 11 gives the state response of the switched system; Fig 12 illustrates the output of the switched system. Fig 13 demonstrates the switching law.

VI. CONCLUSION

In this article, we considered the output regulation problem for cascade switched nonlinear systems. The problem for each subsystem to be solvable and may not be solvable are considered respectively. The average dwell time method and the multiple Lyapunov function method are employed to insure the problem is solvable for switched systems. The developed results are an extension to the output regulation problem for non-switched cascade nonlinear system.

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