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Stability Analysis for Discrete-Time Switched Nonlinear System Under MDADT Switching

LIJIE LI[1], LEI LIU[2], AND YUNFEI YIN[3]
[1]College of Computer Science and Technology, Harbin Engineering University, Harbin 150001, China
[2]School of Automation, Harbin Engineering University, Harbin 150001, China
[3]Research Institute of Intelligent Control and Systems, School of Astronautics, Harbin Institute of Technology, Harbin 150001, China

Corresponding author: Lei Liu (liuleiyyf@gmail.com)

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ABSTRACT The problem of stability analysis for discrete-time switched nonlinear system is investigated with mode-dependent average dwell time (MDADT) method in this paper. A slow switching strategy is adopted in the discrete-time nonlinear stable subsystems and unstable subsystems are handled by a fast switching strategy. Takagi–Sugeno (T–S) fuzzy model is utilized to approximate the switched nonlinear system. By constructing a multiple discontinuous Lyapunov function approach, the stability condition of switched T–S fuzzy system is built to get tighter bound on MDADT, which shows that our proposed method outperforms the classical one. Finally, through a numerical example, the effectiveness of the presented control approach is illustrated by comparison with result from classical one.

INDEX TERMS Mode-dependent average dwell time, switched system, stability analysis.

I. INTRODUCTION

Switched system [1], [2] is a typical hybrid dynamical system, which is composed of continuous states or discrete states and switching rules. During the last few decades, the switched systems have drawn much attention in complex dynamic control and application. This is partially due to its increasing practical potential applications, such as congestion control algorithms in packet switching computer networks [3], multi-autonomous underwater vehicle systems control [4], three-phase two-level grid-connected power converters [5], robot modeling control [6] and parallel control and management for intelligent transportation systems [7] and so on. Recent challenges may be related stability for switched system, and it primarily includes: the stability under arbitrary switching (UAS) [8] and the stability under constrained switching (UCS) [9]. For the UAS, all the subsystems must share common Lyapunov function, while it is difficult to find it in practice. On the other hand, multiple Lyapunov functions (MLFs) [10] are not only an valid method to solve above constraints but also easy to implement in practical engineering, which can guarantee system stability UCS. For now, the development UCS has been mainly focussed on time-switching signal [9], [11], [12]. In [13], the problems of stability and l_2-gain for discrete-time switched systems were investigated under average dwell time (ADT) switching by the extended Lyapunov functions ensuring to increase during the running time of subsystems. Based on ADT switching method, the authors extended this switching signal to the mode-dependent average dwell time (MDADT) signal in [9], in which the stability condition for switched systems was firstly proposed by MDADT switching in nonlinear plant. In practical applications, the MDADT switching signal is more flexible than ADT. The main reason is that under MDADT switching, each mode not only has its own ADT but also has its own control strategy. In this paper, the problem of stability analysis for discrete-time switched nonlinear system comprising unstable subsystems is considered by MDADT switching scheme shown its advantages in dealing with these problems, in which a multiple discontinuous Lyapunov function (MDLF) is adopted to obtain the tighter bound on MDADT.

On the other hand, some complex nonlinear systems are usually modeled by switched nonlinear systems. As we all know, Takagi-Sugeno (T-S) fuzzy model [14]–[21] has been a valid way to approximate the nonlinear systems in any arbitrary accuracy especially for switched nonlinear systems. Switched fuzzy systems [22]–[25] have been widely discussed in the latest academic research. To list a few, aiming at T-S fuzzy systems, a new switched dynamic parallel distributed compensation controllers had been proposed in [26]. By piecewise Lyapunov function and a switching fuzzy model, authors of [27] proposed the relaxed stabilization criteria for discrete-time T-S fuzzy control systems. But in many cases, the sample of conservatism is relatively high. Thus, the relaxed stabilization criteria for T-S fuzzy control systems was discussed in [28] by minimum-type piecewise Lyapunov function method. In this paper, based on IF-THEN rules of T-S fuzzy model, the switched nonlinear system is expressed as a linear combination of a series of local linear switched systems.

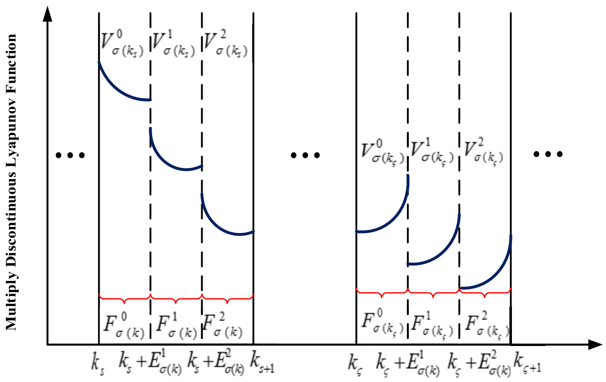

FIGURE 1. Multiply discontinuous Lyapunov function.

In this paper, the problem of stability analysis for discrete-time switched nonlinear unstable subsystems will be investigated. The contributions of the paper are stated as follow. The MDLF approach is adopted to obtain the tighter bound MDADT achieving the stability and desired system performances. The advantage of this method is that each Lyapunov function of the activated system mode is only piecewise continues during the dwell time (as shown in Fig. 1). Then, a slow switching strategy is applied to stable subsystems and unstable subsystems are handled by a fast switching strategy. Finally, T-S fuzzy model is developed to approximate the switched nonlinear system, and the stability condition for switched T-S fuzzy system is obtained formulating in terms of linear matrix inequalities.

The contents of this paper are organized as follows. Section II establishes the switched T-S fuzzy model. Section III presents stability analysis for switched nonlinear system by using a novel MDLF method. Section IV shows the proposed method through a numerical example. Section V is the conclusion of this paper.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ stand for the n-dimensional Euclidean space and set of $n \times m$ real matrices, respectively. Notation $\|\cdot\|$ refers to the Euclidean norm. Function $\alpha : [0,\infty) \to [0,\infty)$ is regarded of class $\mathcal{K}$, which is continuous and increasing strictly with $\alpha(0) = 0$. Class $\mathcal{K}_\infty$ is consisted of the subset of $\mathcal{K}$ with all unbounded functions. Function $\beta : [0,\infty) \times [0,\infty) \to [0,\infty)$ is considered of class $\mathcal{KL}$, which is of class $\mathcal{K}$, that is $t > 0, \beta(\cdot, t)$ and $r \geq 0, t \to \infty, \beta(r,t) \to 0$. A^T stands for the transpose of matrix A, and $P > 0$ (≥ 0) denotes that P is a real symmetric matrix of positive definite (semi-positive definite).

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following class of discrete-time switched nonlinear system:

$$x(k+1) = f_{\sigma(k)}(x(k), k), \quad x(k_0) = x_0, \tag{1}$$

where $x(k) \in \mathbb{R}^n$ denotes the state vector, and x_0 and k_0 denote the initial state and initial time, respectively; $f_{\sigma(k)}(\cdot)$, $\sigma(k) = \mathfrak{p} \in Q = \{1, \ldots, m\}$ are nonlinear functions from $\mathbb{R}^n \to \mathbb{R}^n$; $\sigma(k)$ is a switched signal; $m > 1$ is switching subsystems; $Q = S \cup U$, where S and U are represented stable modes and unstable modes, respectively.

The discrete-time switched T-S fuzzy system is represented by :

Model Rule $R^j_{\mathfrak{p}}$: If $\digamma_1(x(k))$ is $\widetilde{M}^j_{\mathfrak{p}1}$ and, $\cdots$, and $\digamma_\psi(x(k))$ is $\widetilde{M}^j_{\mathfrak{p}\psi}$,
then $x(k+1) = A_{\mathfrak{p}j}x(k)$,

where $\widetilde{M}^j_{\mathfrak{p}\gamma}$ denotes the j^{th} switched T-S fuzzy set corresponding to the function $\digamma_\gamma(x(k))$, $\alpha = 1, 2, \ldots, \psi$, $j = 1, 2, \ldots, r$; ψ is a positive integer; $A_{\mathfrak{p}j} \in \mathbb{R}^{n\times n}$ is known constant system matrices; $x(k) \in \mathbb{R}^n$ denotes the system state vector. The $\mathfrak{p}^{th}$ subsystem is obtained as following:

$$x(k+1) = \sum_{j=1}^{r} \widetilde{w}_{\mathfrak{p}j}(x(k))A_{\mathfrak{p}j}x(k), \tag{2}$$

where

$$\widetilde{w}_{\mathfrak{p}j}(x(k)) = \frac{\prod_{i=1}^{m} w_{\mathfrak{p}j}(x(k))}{\sum_{j=1}^{r}\prod_{i=1}^{m} w_{\mathfrak{p}j}(x(k))},$$

$$\sum_{j=1}^{r} \widetilde{w}_{\mathfrak{p}j}(x(k)) = 1.$$

The following definitions are given to facilitate the analysis below.

Definition 1 [1]: The equilibrium $x = 0$ of a switched system is global uniformly exponentially stable (GUES) under certain switching signal $\sigma(k)$ if there exists constants $\alpha > 0, 0 < \beta < 1$ satisfies the inequality $\|x(k)\| \leq \alpha\beta^{k-k_0}\|x(k_0)\|$, $\forall k \geq k_0$, with $u(k) = 0$ and any initial conditions $x(k_0)$.

Definition 2 [9]: For any time interval $[k_1, k_2]$, denote $N_{\sigma\mathfrak{p}}(k_1, k_2)$ as the numbers of the $\mathfrak{p}^{th}$ subsystem being activated, and $K_{\mathfrak{p}}(k_1, k_2)$ as the overall running time of the $\mathfrak{p}^{th}$ subsystem, $\mathfrak{p} \in S$. We can find two constants $N_{0\mathfrak{p}}$ and τ_{ap} satisfying

$$N_{\sigma\mathfrak{p}}(k_1, k_2) \leq N_{0\mathfrak{p}} + \frac{K_{\mathfrak{p}}(k_1, k_2)}{\tau_{a\mathfrak{p}}}, \quad 0 \leq k_1 \leq k_2, \tag{3}$$

where τ_{ap} is called the mode-dependent average dwell time of the switching signal $\sigma(k)$.

Definition 3 [22]: For any time interval $[k_1, k_2]$, denote $N_{\sigma\varrho}(k_1, k_2)$ as the numbers of the ϱ^{th} subsystem being activated, and $K_\varrho(k_1, k_2)$ as the overall running time of the

ϱ^{th} subsystem, $\varrho \in U$. We can find two constants $N_{0\varrho}$ and $\tau_{a\varrho}$ satisfying

$$N_{\sigma\varrho}(k_2, k_1) \geq N_{0\varrho} + \frac{K_\varrho(k_2, k_1)}{\tau_{a\varrho}}, \quad 0 \leq k_1 \leq k_2, \tag{4}$$

where $\tau_{a\varrho}$ is called the mode-dependent average dwell time of the switching signal $\sigma(k)$.

III. MAIN RESULTS

In practice, a large number of switched nonlinear systems are inevitably contained some unstable subsystems. Therefore, it is necessary to investigate the problem of stability analysis for discrete-time switched nonlinear comprising unstable subsystem. In this section, the multiple discontinuous Lyapunov function (MDLF) will be applied to the stability analysis for the switched systems to obtain the tighter bound on MDADT. The interval $[k_s, k_{s+1})$ is divided into $C_{\sigma(k_s)}$ parts. It is defined that the corresponding length of each part as $D^{\varrho}_{\sigma(k_s)}$, $n \in \{0, 1, 2, \ldots, C_{\sigma(k_s)}\}$. Let the $E^n_{\sigma(k_s)} = \sum_{\varrho=1}^{n} D^{\varrho}_{\sigma(k_s)}$, where $E^0_{\sigma(k_s)} = 0$, $\forall n \in \{0, 1, 2, \ldots, C_{\sigma(k_s)}\}$. $F^n_{\sigma(k_s)} = [k_s + E^n_{\sigma(k_s)}, k_s + E^{n+1}_{\sigma(k_s)})$, $n \in \mathcal{R}_{\sigma(k_s)} = \{0, 1, 2, \ldots, C_{\sigma(k_s)} - 1\}$. Then, the Fig. 1 (where $C_{\sigma(k_s)} = 3$) shows $[k_s, k_{s+1}) = \bigcup_n F^n_{\sigma(k_s)}$, $n \in \mathcal{R}_{\sigma(k_s)}$ in the time interval $[k_s, k_{s+1})$. We can describe the MDLF as follows:

$$V(k) = V^{\varepsilon(k)}_{\sigma(k)}(x(k)),$$

where $\varepsilon(k) = n, \forall k \in F^n_{\sigma(k)}$, $n \in \mathcal{R}_{\sigma(k)} = \{0, 1, 2, \ldots, C_{\sigma(k)} - 1\}$, and $V^n_{\mathfrak{p}}(x(k)) \in \mathcal{C}^1$, $\mathfrak{p} \in \mathcal{Q}, n \in \mathcal{R}_{\mathfrak{p}}$. Employing the above mentioned definitions and sets, the main results are presented as follows.

Theorem 1: Consider switched nonlinear system (1). For given scalars $-1 < \lambda_{\mathfrak{p}} < 0$, $0 < \theta_{\mathfrak{p}} \leq 1$, $\mu_{\mathfrak{p}} > 1$, $\mathfrak{p} \in S$ satisfying $(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}\mu_{\mathfrak{p}} > 1$, and $\lambda_{\mathfrak{p}} > 0$, $0 < \theta_{\mathfrak{p}} \leq 1$, $0 < \mu_{\mathfrak{p}} < 1$, $\mathfrak{p} \in U$, if there are a set of $\mathcal{C}^1$ non-negative functions $V^n_{\mathfrak{p}}(x(k)) : \mathbb{R}^n \to \mathbb{R}$, $\mathfrak{p} \in \mathcal{Q}, n \in \mathcal{R}_{\mathfrak{p}}$, and two class $\mathcal{K}_\infty$ functions κ_1 and κ_2, such that, $\forall n \in \mathcal{R}_{\sigma(k)}$

$$\kappa_1(\|x(k)\|) \leq V^n_{\mathfrak{p}}(x(k)) \leq \kappa_2(\|x(k)\|), \quad \mathfrak{p} \in \mathcal{Q}, \tag{5}$$

$$\Delta V^n_{\mathfrak{p}}(x(k)) \leq \lambda_{\mathfrak{p}} V^n_{\mathfrak{p}}(x(k)), \ \mathfrak{p} \in \mathcal{Q}, \tag{6}$$

$$V^n_{\mathfrak{p}}(x(k_s + E^n_{\mathfrak{p}})) \leq \theta_{\mathfrak{p}} V^{n-1}_{\mathfrak{p}}(x(k_s + E^n_{\mathfrak{p}})), \quad \mathfrak{p} \in \mathcal{Q}, n \neq 0, \tag{7}$$

$$V^0_{\mathfrak{p}}(x(k_s)) \leq \mu_{\mathfrak{p}} V^{C_\varrho - 1}_{\varrho}(x(k_s^-)), \quad (\mathfrak{p}, \varrho) \in S \times \mathcal{Q}, \mathfrak{p} \neq \varrho, \tag{8}$$

$$V^0_{\varrho}(x(k_s)) \leq \mu_\varrho V^{C_{\mathfrak{p}} - 1}_{\mathfrak{p}}(x(k_s^-)), \quad (\mathfrak{p}, \varrho) \in S \times U, \tag{9}$$

any MDADT switching signals satisfy:

$$\begin{cases} \tau_{a\mathfrak{p}} \geq \tau^*_{a\mathfrak{p}} = -\dfrac{ln\mu_{\mathfrak{p}} + (C_{\mathfrak{p}} - 1)ln\theta_{\mathfrak{p}}}{ln(1 + \lambda_{\mathfrak{p}})}, & \mathfrak{p} \in S, \\ \tau_{a\varrho} \leq \tau^*_{a\varrho} = -\dfrac{ln\mu_{\mathfrak{p}} + (C_{\mathfrak{p}} - 1)ln\theta_{\mathfrak{p}}}{ln(1 + \lambda_{\mathfrak{p}})}, & \mathfrak{p} \in U, \end{cases} \tag{10}$$

then the switched nonlinear system (1) is GUES.

Proof: In interval $[0, K]$, $k_1, k_2 \ldots k_s, k_{s+1} \ldots k_{N_{\sigma(K,0)}}$ are represented as the switching times, where $\sum_{\mathfrak{p} \in \mathcal{Q}} N_{\sigma\mathfrak{p}}(K, 0) = N_{\sigma(K,0)}$. According to (6) for $k \in F^n_{\sigma(k_s)}$, it can seen that

$$V^n_{\sigma(k_s)}(k) \leq (1 + \lambda_{\sigma(k_s)})^{(k - (k_s + E^n_{\sigma(k_s)}))} \times V^n_{\sigma(k_s)}(k_s + E^n_{\sigma(k_s)}). \tag{11}$$

Then, it follows from (7) and (11) that

$$\begin{aligned} V^{C_{\sigma(k_s)}-1}_{\sigma(k_s)}(k^-_{s+1}) &\leq (1 + \lambda_{\sigma(k_s)})^{(k_{s+1} - (k_s + E^{C_{\sigma(k_s)}-1}_{\sigma(k_s)}))} \\ &\quad \times V^{C_{\sigma(k_s)}-1}_{\sigma(k_s)}(k_s + E^{C_{\sigma(k_s)}-1}_{\sigma(k_s)}) \\ &\leq \theta_{\sigma(k_s)}(1 + \lambda_{\sigma(k_s)})^{(k_{s+1} - (k_s + E^{C_{\sigma(k_s)}-1}_{\sigma(k_s)}))} \\ &\quad \times V^{C_{\sigma(k_s)}-2}_{\sigma(k_s)}(k_s + E^{C_{\sigma(k_s)}-1}_{\sigma(k_s)}) \\ &\leq \theta_{\sigma(k_s)}(1 + \lambda_{\sigma(k_s)})^{(k_{s+1} - (k_s + E^{C_{\sigma(k_s)}-2}_{\sigma(k_s)}))} \\ &\quad \times V^{C_{\sigma(k_s)}-3}_{\sigma(k_s)}(k_s + E^{C_{\sigma(k_s)}-2}_{\sigma(k_s)}) \\ &\leq \ldots \leq (\theta_{\sigma(k_s)})^{C_{\sigma(k_s)}-1} \\ &\quad \times (1 + \lambda_{\sigma(k_s)})^{(k_{s+1} - (k_s + E^0_{\sigma(k_s)}))} \\ &\quad \times V^0_{\sigma(k_s)}(k_s + E^0_{\sigma(k_s)}). \end{aligned} \tag{12}$$

By integrating (8), (9) and (12), one can get that

$$\begin{aligned} &V^{C_{\sigma(k_{N_\sigma})}-1}_{\sigma(k_{N_\sigma})}(K) \\ &\leq (\theta_{\sigma(k_{N_\sigma})})^{C_{\sigma(k_{N_\sigma})}-1}(1 + \lambda_{\sigma(k_{N_\sigma})})^{(K - k_{N_\sigma})} \\ &\quad \times V^0_{\sigma(k_{N_\sigma})}(k_{N_\sigma}) \\ &\leq \mu_{\sigma(k_{N_\sigma})}(\theta_{\sigma(k_{N_\sigma})})^{C_{\sigma(k_{N_\sigma})}-1} \\ &\quad \times (1 + \lambda_{\sigma(k_{N_\sigma})})^{(K - k_{N_\sigma})} \\ &\quad \times V^{C_{\sigma(k_{N_\sigma}-1)}-1}_{\sigma(k_{N_\sigma}-1)}(k_{N_\sigma}) \\ &\leq \mu_{\sigma(k_{N_\sigma})}(\theta_{\sigma(k_{N_\sigma})})^{C_{\sigma(k_{N_\sigma})}-1} \\ &\quad \times (\theta_{\sigma(k_{N_\sigma-1})})^{C_{\sigma(k_{N_\sigma-1})}-1} \\ &\quad \times (1 + \lambda_{\sigma(k_{N_\sigma})})^{(K - k_{N_\sigma})} \\ &\quad \times (1 + \lambda_{\sigma(k_{N_\sigma-1})})^{(k_{N_\sigma} - k_{N_\sigma-1})} \\ &\quad \times V^0_{\sigma(k_{N_\sigma-1})}(k_{N_\sigma-1}) \\ &\leq \mu_{\sigma(k_{N_\sigma})}\mu_{\sigma(k_{N_\sigma-1})}(\theta_{\sigma(k_{N_\sigma})})^{C_{\sigma(k_{N_\sigma})}-1} \\ &\quad \times (\theta_{\sigma(k_{N_\sigma-1})})^{C_{\sigma(k_{N_\sigma-1})}-1} \\ &\quad \times (\theta_{\sigma(k_{N_\sigma-2})})^{C_{\sigma(k_{N_\sigma-2})}-1} \\ &\quad \times (1 + \lambda_{\sigma(k_{N_\sigma})})^{(K - k_{N_\sigma})} \\ &\quad \times (1 + \lambda_{\sigma(k_{N_\sigma-1})})^{(k_{N_\sigma} - k_{N_\sigma-1})} \\ &\quad \times (1 + \lambda_{\sigma(k_{N_\sigma-2})})^{(k_{N_\sigma-1} - k_{N_\sigma-2})} \\ &\quad \times V^0_{\sigma(k_{N_\sigma-2})}(k_{N_\sigma-2}) \\ &\leq \ldots \leq \prod_{\varrho=1}^{N_\sigma} \mu_{\sigma(k_\varrho)}(\theta_{\sigma(k_\varrho)})^{C_{\sigma(k_\varrho)}-1}(\theta_{\sigma(k_0)})^{C_{\sigma(k_\varrho)}-1} \\ &\quad \times (1 + \lambda_{\sigma(k_{N_\sigma})})^{(k - k_{N_\sigma})} \\ &\quad \times (1 + \lambda_{\sigma(k_{N_\sigma}-1)})^{(k_{N_\sigma} - k_{N_\sigma-1})} \end{aligned}$$

$$
\begin{aligned}
&\times \cdots \times \ (1+\lambda_{\sigma\ (k_{N_1})})^{(k_2-k_1)} \\
&\times \lambda_{\sigma\ (k_{N_0})}{}^{(k_1-k_0)} V^0_{\sigma\ (0)}(x(0)) \\
\leq &\prod_{\varrho=1}^{N_\sigma} \mu_{\sigma\ (k_\varrho)}(\theta_{\sigma\ (k_\varrho)})^{C_{\sigma\ (k_\varrho)}-1}(\theta_{\sigma\ (k_0)})^{C_{\sigma\ (k_0)}-1} \\
&\times \prod_{\mathfrak{p}=1, s\in\varphi\ (\mathfrak{p})}^{m} (1+\lambda_{\mathfrak{p}})^{(k_{s+1}-k_s)} V^0_{\sigma(0)}(x(0)),
\end{aligned} \tag{13}
$$

where $\varphi\ (\mathfrak{p})$ stands for the set of s satisfying $\sigma\ (k_s) = \mathfrak{p}\ (k_s \in \{k_1, k_2 \ldots k_n, k_{n+1} \ldots k_{N_\sigma - 1}\})$.

In fact, (13) can be rewritten as

$$
\begin{aligned}
V_{\sigma\ (K^-)}(K^-) \leq &\prod_{\mathfrak{p}\in S} (\mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1})^{N_{\sigma\mathfrak{p}}} \\
&\times \prod_{\mathfrak{p}\in U} (\mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1})^{N_{\sigma\mathfrak{p}}}(\eta_{\sigma\ (0)})^{C_{\mathfrak{p}}-1} \\
&\times \prod_{\mathfrak{p}\in S} (1+\lambda_{\sigma\ (k_{N_\sigma})})^{K_{\mathfrak{p}}(K,0)} \\
&\times \prod_{\mathfrak{p}\in U} (1+\lambda_{\sigma\ (k_{N_\sigma})})^{K_{\mathfrak{p}}(K,0)} \\
&\times V^0_{\sigma(0)}(x(0)),
\end{aligned} \tag{14}
$$

where $(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}\mu_{\mathfrak{p}} > 1, \mathfrak{p} \in S$ and $0 < (\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}\mu_{\mathfrak{p}} < 1$, $\mathfrak{p} \in U$. Then according to (3) and (4), the (14) indicates that

$$
\begin{aligned}
V_{\sigma\ (K^-)}(K^-) \leq \exp\{&\sum_{\mathfrak{p}\in S}(N_{0\mathfrak{p}} + \frac{K_{\mathfrak{p}}(K,0)}{\tau_{ap}}) \\
&\times \ln \mu_{\mathfrak{p}}(\eta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1} \\
&+ \sum_{\mathfrak{p}\in U}(N_{0\mathfrak{p}} + \frac{K_{\mathfrak{p}}(K,0)}{\tau_{ap}}) \\
&\times \ln \mu_{\mathfrak{p}}(\eta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1} \\
&+ \sum_{\mathfrak{p}\in S} K_{\mathfrak{p}}(K,0) \ln\ (1+\lambda_{\mathfrak{p}}) \\
&+ \sum_{\mathfrak{p}\in U} K_{\mathfrak{p}}(K,0) \ln\ (1+\lambda_{\mathfrak{p}})\} \\
&\times (\theta_{\sigma\ (0)})^{C_{\mathfrak{p}}-1} V_{\sigma\ (0)}(x(0)) \\
\leq \exp\{&\sum_{\mathfrak{p}\in S} (N_{0\mathfrak{p}} \ln \mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}) \\
&+ \sum_{\mathfrak{p}\in U} (N_{0\mathfrak{p}} \ln \mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1})\} \\
&\times \exp\{\sum_{\mathfrak{p}\in S}(\frac{\ln \mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}}{\tau_{ap}} \\
&+ \ln\ (1+\lambda_{\mathfrak{p}}))K_{\mathfrak{p}}(K,0) \\
&+ \sum_{\mathfrak{p}\in U}(\frac{\ln \mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}}{\tau_{ap}} \\
&+ \ln\ (1+\lambda_{\mathfrak{p}}))K_{\mathfrak{p}}(K,0)\} \\
&\times (\eta_{\sigma(0)})^{C_{\sigma(0)}-1} V^0_{\sigma(0)}(x(0)).
\end{aligned} \tag{15}
$$

Moveover, if τ_{ap} satisfy (10), we have $\frac{\ln \mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}}{\tau_{ap}} + \ln\ (1+\lambda_{\mathfrak{p}}) < 0, \mathfrak{p} \in Q$.

Then, it can be written as

$$
\begin{aligned}
V_{\sigma\ (K^-)}(K^-) \leq \exp\{&\sum_{\mathfrak{p}\in S} (N_{0\mathfrak{p}} \ln \mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}) \\
&+ \sum_{\mathfrak{p}\in U} (N_{0\mathfrak{p}} \ln \mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1})\} \\
&\times e^{\max_{\mathfrak{p}\in Q}\{(\frac{\ln \mu_{\mathfrak{p}}(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}}{\tau_{ap}} + \ln\ (1+\lambda_{\mathfrak{p}}))K_{\mathfrak{p}}(K,0)\}} \\
&\times (\theta_{\sigma(0)})^{C_{\sigma(0)}-1} V^0_{\sigma(0)}(x(0)).
\end{aligned} \tag{16}
$$

If MDADT satisfies (10), the consequences of $V_{\sigma\ (K^-)}(x(K))$ converges to 0 as $T \rightarrow \infty$. Therefore, based on (5) and Definition 1, the system (1) is considered as GUES. ■

Remark 1: In Theorem 1, the MDLF is adopted to obtain the tighter bound on MDADT, which will assuredly improve the flexibility in practice application. MDLF can guarantee that each Lyapunov function of the activated system mode is only piecewise continues during the dwell time. Specific details of MDLF can be seen Fig. 1.

Remark 2: The switched strategy is that slow switching and fast switched switching are respectively used among stable subsystems and unstable subsystems in Theorem 1. In fact, from the above switching strategy, one can observe that if the stable subsystem is activated, any subsystem can be activated at the next switching instance. While the unstable subsystem is activated, the next activated subsystem must be the stable subsystem.

Remark 3: According to (10), we suppose MDADT for one of the stable subsystem is that $\tau_1 \geq \tau_1^* = -\frac{ln\mu_1+(C_1-1)ln\theta_1}{ln(1+\lambda_1)}$. When μ_1, C_1, θ_1 is smaller value and the λ_1 is larger value, we can obtain the τ_1^* is smaller value. Namely, there is larger the range of τ_1, lowered conservative to stable subsystems. We choose MDADT for one of the unstable subsystem is that $\tau_2 \leq \tau_2^* = -\frac{ln\mu_2+(C_2-1)ln\theta_2}{ln(1+\lambda_2)}$. When the μ_2, C_2, θ_2 are increasing, the λ_2 is decreasing, we can obtain τ_2^* is increasing. In that way, the lowered conservative would be able to use all of unstable subsystem if there was smaller range of τ_2.

From Theorem 1, we can derive the following Theorem for switched linear system(2).

Theorem 2: Consider switched T-S fuzzy system (2). For given scalars $-1 < \lambda_{\mathfrak{p}} < 0$, $0 < \theta_{\mathfrak{p}} \leq 1$, $\mu_{\mathfrak{p}} > 1$, $\mathfrak{p} \in S$, satisfying $(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}\mu_{\mathfrak{p}} > 1$, and $\lambda_{\mathfrak{p}} > 0$, $0 < \theta_{\mathfrak{p}} \leq 1$, $0 < \mu_{\mathfrak{p}} < 1$, $\mathfrak{p} \in U$, if there are a set of matrices $P^n_{\mathfrak{p}} > 0$, $\mathfrak{p} \in Q$, $n \in \mathcal{R}_{\mathfrak{p}}$ such that, $\forall n \in \mathcal{R}_{\mathfrak{p}}$,

$$
\begin{bmatrix} -(1+\lambda_{\mathfrak{p}})P^n_{\mathfrak{p}} & * \\ A_{pj}P^n_{\mathfrak{p}} & -P^n_{\mathfrak{p}} \end{bmatrix} \leq 0, \ \mathfrak{p} \in Q, \tag{17}
$$

$$
P^n_{\mathfrak{p}} \leq \theta_{\mathfrak{p}} P^{n-1}_{\mathfrak{p}}, \ \mathfrak{p} \in S, \ n \neq 0, \tag{18}
$$

$$
P^0_{\mathfrak{p}} \leq \mu_{\mathfrak{p}} P^{C_\varrho - 1}_{\varrho}, \quad (\mathfrak{p}, \varrho) \in S \times Q, \quad \mathfrak{p} \neq \varrho, \tag{19}
$$

$$
P^0_{\varrho} \leq \mu_{\varrho} P^{C_{\mathfrak{p}} - 1}_{\mathfrak{p}}, \quad (\mathfrak{p}, \varrho) \in S \times U, \tag{20}
$$

then the switched T-S fuzzy system (2) is GUES, if any MDADT switching signals satisfy (10).

Proof: Consider the following quadratic form with (2):

$$V(x(k)) = x^T(k)P_{\mathfrak{p}}^n x(k), \sigma(k) = \mathfrak{p} \in Q, k \in F_{\mathfrak{p}}^n, \quad (21)$$

where positive definite matrices $P_{\mathfrak{p}}^n(\mathfrak{p} \in Q, n \in \mathfrak{R}_{\mathfrak{p}})$ achieved (17)-(20). By (2), (6)-(9), we have, $\forall n \in \mathfrak{R}_{\mathfrak{p}}$,

$$\begin{aligned}
&\Delta V_{\mathfrak{p}}^n(x(k)) - \lambda_{\mathfrak{p}} V_{\mathfrak{p}}^n(x(k)) \\
&\quad = \sum_{j=1}^{r} \widetilde{w}_{\mathfrak{p}j}(x(k))x^T(k) \\
&\quad\quad \times (A_{pj}^T P_{\mathfrak{p}}^n A_{\mathfrak{p}j} - P_{\mathfrak{p}}^n - \lambda_{\mathfrak{p}} P_{\mathfrak{p}}^n)x(k) \\
&\quad \le 0, \quad \forall \mathfrak{p} \in S, \\
&V_{\mathfrak{p}}^i(x(k_s + E_{\mathfrak{p}}^n)) - \eta_{\mathfrak{p}} V_{\mathfrak{p}}^{n-1} \\
&\quad\quad \times (x(k_s + E_{\mathfrak{p}}^n)) \\
&\quad \le x^T(k)(P_{\mathfrak{p}}^{n-1} - \eta_{\mathfrak{p}} P_{\mathfrak{p}}^n)x(k) \\
&\quad \le 0, \quad \forall \mathfrak{p} \in Q, \ n \ne 0, \\
&V_{\mathfrak{p}}^0(x(k)) - \mu_{\mathfrak{p}} V_{\varrho}^{C_{\varrho}-1}(x(k)) \\
&\quad = x^T(k)(P_{\mathfrak{p}}^0 - \mu_{\mathfrak{p}} P_{\varrho}^{C_{\varrho}-1})x(k), \\
&\quad \le 0, \quad \forall (\mathfrak{p}, \varrho) \in S \times Q, \ \mathfrak{p} \ne \varrho, \\
&V_{\varrho}^0(x(k)) - \mu_{\varrho} V_{\mathfrak{p}}^{C_{\mathfrak{p}}-1}(x(k)) \\
&\quad = x^T(k)(P_{\varrho}^0 - \mu_{\varrho} P_{\mathfrak{p}}^{C_{\mathfrak{p}}-1})x(k), \\
&\quad \le 0, \quad \forall (\mathfrak{p}, \varrho) \in S \times U.
\end{aligned}$$

Finally, based on Theorem 1, switched T-S system (2) is considered as GUES. ∎

Remark 4: The tighter bound on MDADT by increasing $C_{\mathfrak{p}}$ and/or decreasing $\theta_{\mathfrak{p}}$ is obtained in Theorem 2. However, $C_{\mathfrak{p}} = 1$ and/or $\theta_{\mathfrak{p}} = 1$, Theorem 2 don't show superiority.

In the MDADT switching signal, provided all discrete-time switched nonlinear subsystems are stable. Then Corollary 1 can be obtained as follows.

Corollary 1: Consider switched T-S fuzzy system (2). For given scalars $-1 < \lambda_{\mathfrak{p}} < 0$, $0 < \theta_{\mathfrak{p}} \le 1$, $\mu_{\mathfrak{p}} > 1$, $\mathfrak{p} \in S$, fulfilling $(\theta_{\mathfrak{p}})^{C_{\mathfrak{p}}-1}\mu_{\mathfrak{p}} > 1$, if there are a set of matrices $P_{\mathfrak{p}}^n > 0$, $\mathfrak{p} \in S$, $n \in \mathcal{R}_{\mathfrak{p}}$ such that, $(\mathfrak{p}, \varrho) \in S \times Q$, $\mathfrak{p} \ne \varrho$, $\forall n \in \mathcal{R}_{\mathfrak{p}}$,

$$\begin{aligned}
\begin{bmatrix} -(1+\lambda_{\mathfrak{p}})P_{\mathfrak{p}}^n & * \\ A_{pj}P_{\mathfrak{p}}^n & -P_{\mathfrak{p}}^n \end{bmatrix} &\le 0, \\
P_{\mathfrak{p}}^n &\le \theta_{\mathfrak{p}} P_{\mathfrak{p}}^{n-1}, \quad n \ne 0, \\
P_{\mathfrak{p}}^0 &\le \mu_{\mathfrak{p}} P_{\varrho}^{C_{\varrho}-1},
\end{aligned}$$

any MDADT switching signals satisfy

$$\tau_{a\mathfrak{p}} \ge \tau_{a\mathfrak{p}}^* = -\frac{ln\mu_{\mathfrak{p}} + (C_{\mathfrak{p}} - 1)ln\theta_{\mathfrak{p}}}{ln(1+\lambda_{\mathfrak{p}})},$$

then the switched T-S fuzzy stable system (2) is GUES.

Due to the process of proof is similar to Theorem 2, we omit it.

Stability conditions of discrete-time switched T-S fuzzy system (2) is established by MLFs, the result is presented in Corollary 2 as follows.

Corollary 2: Consider unstable switched T-S subsystem (2). For given scalars $-1 < \lambda_{\mathfrak{p}} < 0$, $\mu_{\mathfrak{p}} > 1$, and $\lambda_{\mathfrak{p}} > 0$, $0 < \mu_{\mathfrak{p}} < 1$, if there are a set of matrices $P_{\mathfrak{p}}^n > 0, \mathfrak{p} \in S, P_{\varrho} > 0, \varrho \in U$,

$$\begin{aligned}
\begin{bmatrix} -(1+\lambda_{\mathfrak{p}})P_{\mathfrak{p}} & * \\ A_{\mathfrak{p}j}P_{\mathfrak{p}} & -P_{\mathfrak{p}} \end{bmatrix} &\le 0, \ \mathfrak{p} \in S, \\
\begin{bmatrix} -(1+\lambda_{\varrho})P_{\varrho} & * \\ A_{\varrho j}P_{\varrho} & -P_{\varrho} \end{bmatrix} &\le 0, \ \varrho \in U, \\
P_{\mathfrak{p}} &\le \mu_{\mathfrak{p}} P_s, \\
&\quad \forall \mathfrak{p} \in S, \quad \forall s \in Q, \ \mathfrak{p} \ne s \\
P_{\varrho} &\le \mu_{\varrho} P_{\mathfrak{p}}, \quad \forall \mathfrak{p} \in S, \ \forall \varrho \in U.
\end{aligned}$$

Then, the system is GUES for the unstable switched T-S fuzzy subsystem with MDADT satisfying

$$\begin{cases} \tau_{a\mathfrak{p}} \ge \tau_{a\mathfrak{p}}^* = -\dfrac{ln\mu_{\mathfrak{p}}}{ln(1+\lambda_{\mathfrak{p}})}, & \mathfrak{p} \in S, \\ \tau_{a\varrho} \le \tau_{a\varrho}^* = -\dfrac{ln\mu_{\varrho}}{ln(1+\lambda_{\varrho})}, & \varrho \in U. \end{cases}$$

Similar to Theorem 2, Corollary 2 can be obtained directly. Due to space limitation, the proof process is omitted.

Remark 5: Based on MDADT switching method, the stability analysis of discrete-time switched T-S fuzzy system is studied by MLFs in corollary 2. Obviously, by comparing the results with Theorem 2 and Corollary 2, it is shown that one can obtain tighter bound on MDADT in Theorem 2 in which MDLF is applied to the discrete-time switched T-S fuzzy system.

IV. ILLUSTRATIVE EXAMPLES

In this section, we use a numerical example to illustrate the effectiveness of the results developed in the above section.

Consider nonlinear switched system including two subsystems, one is stable and the other is unstable as follows,

$$\Omega_1 = \begin{cases} x_1(k+1) = -0.2692x_1(k) + 0.3692\sin^2(x_1(k)) \\ \times x_2(k) + 0.3692x_2(k) \\ -0.8692\sin^2(x_1(k))x_1(k), \\ x_2(k+1) = -0.7692x_1(k) + 0.2692\sin^2(x_1(k)) \\ \times x_2(k) + 0.8692x_2(k) \\ -0.7692\sin^2(x_1(k))x_1(k). \end{cases}$$

$$\Omega_2 = \begin{cases} x_1(k+1) = -4.84x_1(k) - 3.6\sin^2(x_1(k))x_2(k) \\ +3.54x_2(k) + 3.7\sin^2(x_1(k))x_1(k), \\ x_2(k+1) = -4.72x_1(k) - 4.7\sin^2(x_1(k))x_2(k) \\ +3.42x_2(k) + 4.8\sin^2(x_1(k))x_1(k). \end{cases}$$

Based the T-S fuzzy model, the following two switched T-S fuzzy subsystems are proposed:

$$\begin{aligned}
&R_1^1\text{: IF } f(x(k)) \text{ is } 0, \text{ THEN} \\
&\quad x(k+1) = A_{11}x(k), \\
&R_1^2\text{: IF } f(x(k)) \text{ is } 1, \text{ THEN} \\
&\quad x(k+1) = A_{12}x(k),
\end{aligned}$$

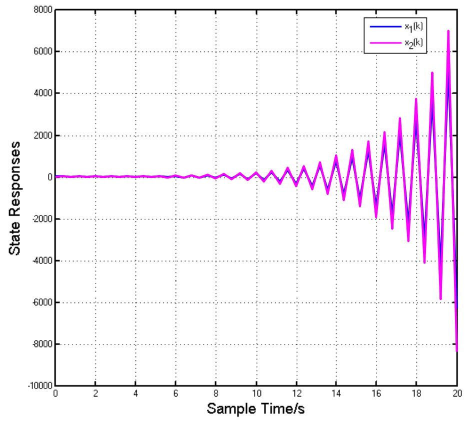

FIGURE 2. Stable responses for Ω_1 subsystem.

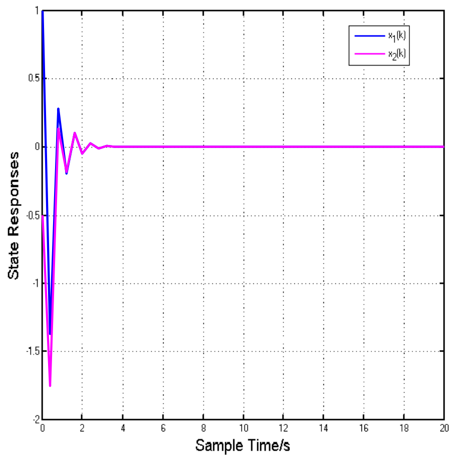

FIGURE 3. Stable responses for Ω_2 subsystem.

$$R_2^1\text{: IF } f(x(k)) \text{ is } 0\text{, THEN}$$
$$x(k+1) = A_{21}x(k),$$
$$R_2^2\text{: IF } f(x(k)) \text{ is } 1\text{, THEN}$$
$$x(k+1) = A_{22}x(k).$$

Setting normalized membership functions are calculated as follows:

$$\widetilde{w}_1(x(k)) = 1 - sin^2(x_1(k)), \widetilde{w}_2(x(k)) = sin^2(x_2(k)),$$

where

$$A_{11} = \begin{bmatrix} -0.2692 & 0.3692 \\ -0.7692 & 0.8692 \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} -0.8692 & 0.3692 \\ -0.7692 & 0.2692 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} -4.84 & 3.54 \\ -4.72 & 3.42 \end{bmatrix},$$
$$A_{22} = \begin{bmatrix} 3.7 & -3.6 \\ 4.8 & -4.7 \end{bmatrix}.$$

Firstly, the state trajectories of switched T-S fuzzy Ω_1 subsystem and Ω_2 subsystem are respectively shown in Figs. 2 and 3 under initial state condition $x(0) = [1, -0.5]$. From the Figs. 2 and 3 it can seen that the Ω_1 subsystem is unstable and Ω_2 subsystem is stable. Then, according to Theorem 2, if we choose $C_1 = 2$, $\lambda_1 = -0.55$, $\theta_1 = 0.72$, $\mu_1 = 2$, $C_2 = 2$, $\lambda_2 = 0.7$, $\theta_2 = 0.92$, $\mu_2 = 0.8$, the MDADT for stable subsystems and unstable subsystems are obtained

$$\tau_{a1} \geqslant \tau_{ap}^* = -\frac{ln\mu_p + (C_p - 1)ln\theta_p}{ln(1+\lambda_p)} = 0.457,$$

$$\tau_{a2} \leqslant \tau_{aq}^* = -\frac{ln\mu_p + (C_p - 1)ln\theta_p}{ln(1+\lambda_p)} = 0.577.$$

Therefore, based on MDADT switching signal property for stable subsystem and unstable subsystem, we choose $\tau_{a1} = 0.5$, $\tau_{a2} = 0.6$. Compared the result with τ_{ap}^* and τ_{aq}^*, we can see that unstable subsystems can stay longer than the stable ones, which is different traditional approach. The corresponding state trajectories of the switched T-S fuzzy system is shown in Fig. 4. In order to illustrate the proposed results in this example, the corresponding parameters of Theorem 2, Corollary 1 and Corollary 2 are obtained in the TABLE 1, TABLE 2 and TABLE 3, respectively. Furthermore, contrasting to the related data for Theorem 2 in TABLE 1 and Corollary 2 in TABLE 3, it is obvious that τ_{a1} and τ_{a2} of Theorem 2 is obtained the tighter bound on dwell time, when the $\lambda_1, \lambda_2, \mu_1$ and μ_2 are selected the same values. Therefore Theorem 2 in TABLE 1 is assuredly improved the application flexibility in practice application.

TABLE 1. On MDLF for switched T-S fuzzy unstable subsystem.

	Theorem 2
λ	$\lambda_1 = -0.55, \lambda_2 = 0.7$
θ	$\theta_1 = 0.72, \theta_2 = 0.92$
μ	$\mu_1 = 2, \mu_2 = 0.8$
C	$C_1 = 2, C_2 = 2$
τ_{aS}	$\tau_{a1} = 0.457$
τ_{aU}	$\tau_{a2} = 0.577$

TABLE 2. On MDLF for switched T-S fuzzy stable system.

	Corollary 1
λ	$\lambda_1 = 0.7$
μ	$\mu_1 = 0.8$
C	$C_1 = 2$
τ_{aS}	$\tau_{a1} = 0.868$

TABLE 3. On MLFs for switched T-S fuzzy unstable subsystem.

	Corollary 2
λ	$\lambda_1 = -0.55, \lambda_2 = 0.7$
μ	$\mu_1 = 2, \mu_2 = 0.8$
τ_{aS}	$\tau_{a1} = 0.868$
τ_{aU}	$\tau_{a2} = 0.421$

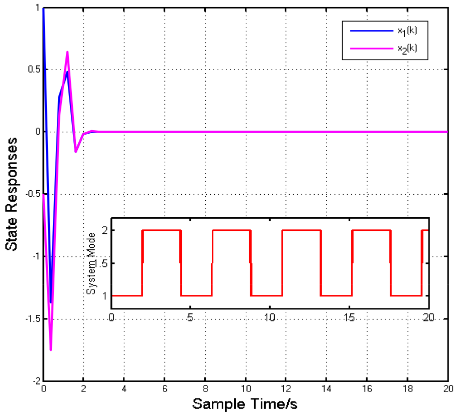

FIGURE 4. On MDLF approach for switched T-S fuzzy unstable subsystem with MDADT $\tau_{a1} = 0.5$, $\tau_{a2} = 0.6$.

V. CONCLUSIONS

The problem of stability analysis for discrete-time switched nonlinear system has been investigated in this paper under MDADT switching. Slow switching strategy and fast switching strategy are applied to stable subsystems and unstable subsystems, respectively. A sufficient stability condition for the switched nonlinear system has been obtained via a MDLF approach, which can get tighter bound on MDADT. By using the T-S fuzzy model to approximate the switched nonlinear system, the stability condition for switched T-S fuzzy system is also obtained. The merits of the proposed results in comparison with existing works are shown through a numerical example.

REFERENCES

[1] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Control Syst.*, vol. 19, no. 5, pp. 59–70, Oct. 1999.

[2] D. Liberzon, J. P. Hespanha, and A. S. Morse, "Stability of switched systems: A Lie-algebraic condition," *Syst. Control Lett.*, vol. 37, no. 3, pp. 117–122, 1999.

[3] C.-Q. Yang and A. V. S. Reddy, "A taxonomy for congestion control algorithms in packet switching networks," *IEEE Netw.*, vol. 9, no. 4, pp. 34–45, Jul. 1995.

[4] W. Xing, Y. Zhao, and H. R. Karimi, "Convergence analysis on multi-AUV systems with leader-follower architecture," *IEEE Access*, vol. 5, pp. 853–868, 2017.

[5] J. Liu, S. Vazquez, L. Wu, A. Marquez, H. Gao, and L. G. Franquelo, "Extended state observer-based sliding-mode control for three-phase power converters," *IEEE Trans. Ind. Electron.*, vol. 64, no. 1, pp. 22–31, Jan. 2017.

[6] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*, vol. 3. New York, NY, USA: Wiley, 2006.

[7] F.-Y. Wang, "Parallel control and management for intelligent transportation systems: Concepts, architectures, and applications," *IEEE Trans. Intell. Transp. Syst.*, vol. 11, no. 3, pp. 630–638, Sep. 2010.

[8] R. Ma and J. Zhao, "Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings," *Automatica*, vol. 46, no. 11, pp. 1819–1823, 2010.

[9] X. Zhao, L. Zhang, P. Shi, and M. Liu, "Stability of switched positive linear systems with average dwell time switching," *Automatica*, vol. 48, no. 6, pp. 1132–1137, 2012.

[10] M. S. Branicky, "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems," *IEEE Trans. Autom. Control*, vol. 43, no. 4, pp. 475–482, Apr. 1998.

[11] L. I. Allerhand and U. Shaked, "Robust stability and stabilization of linear switched systems with dwell time," *IEEE Trans. Autom. Control*, vol. 56, no. 2, pp. 381–386, Feb. 2011.

[12] L. Zhang and H. Gao, "Asynchronously switched control of switched linear systems with average dwell time," *Automatica*, vol. 46, no. 5, pp. 953–958, 2010.

[13] L. Zhang and P. Shi, "Stability, l_2-gain and asynchronous H_∞ control of discrete-time switched systems with average dwell time," *IEEE Trans. Autom. Control*, vol. 54, no. 9, pp. 2192–2199, Sep. 2009.

[14] G. Feng, "A survey on analysis and design of model-based fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 5, pp. 676–697, Oct. 2006.

[15] C.-S. Tseng, B.-S. Chen, and H.-J. Uang, "Fuzzy tracking control design for nonlinear dynamic systems via T-S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 3, pp. 381–392, Jun. 2001.

[16] C.-H. Fang, Y.-S. Liu, S.-W. Kau, L. Hong, and C.-H. Lee, "A new LMI-based approach to relaxed quadratic stabilization of T-S fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 3, pp. 386–397, Jun. 2006.

[17] Y. Zheng, H. Fang, and H. O. Wang, "Takagi–Sugeno fuzzy-model-based fault detection for networked control systems with Markov delays," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 36, no. 4, pp. 924–929, Aug. 2006.

[18] L. Liu, Y. Zhao, and H. Li, "Fuzzy control for discrete-time nonlinear switched systems with improved average dwell time," *Int. J. Mach. Intell. Sensory Signal Process.*, vol. 1, no. 4, pp. 321–340, 2016.

[19] J. Liu, C. Wu, Z. Wang, and L. Wu, "Reliable filter design for sensor networks using type-2 fuzzy framework," *IEEE Trans Ind. Informat.*, vol. 13, no. 4, pp. 1742–1752, Aug. 2017.

[20] H. K. Lam, "LMI-based stability analysis for fuzzy-model-based control systems using artificial T–S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 19, no. 3, pp. 505–513, Jun. 2011.

[21] C. Han, Y. Zeng, H. Zhang, and Y. Zhao, "Admissibility analysis for nonlinear singular system with time-delay via T–S fuzzy model," *Int. J. Fuzzy Syst.*, vol. 19, no. 1, pp. 207–214, 2017.

[22] L. Liu, Y. Yin, J. Wang, and Q. Wu, "Stability analysis of discrete-time switched nonlinear systems via T–S fuzzy model approach," *Neurocomputing*, vol. 173, pp. 1967–1971, 2016.

[23] X. Zhao, Y. Yin, B. Niu, and X. Zheng, "Stabilization for a class of switched nonlinear systems with novel average dwell time switching by T–S fuzzy modeling," *IEEE Trans. Cybern.*, vol. 46, no. 8, pp. 1952–1957, Aug. 2016.

[24] L. Liu, Q. Zhou, H. Liang, and L. Wang, "Stability and stabilization of nonlinear switched systems under average dwell time," *Appl. Math. Comput.*, vol. 298, pp. 77–94, Apr. 2017.

[25] X. Zhao, Y. Yin, L. Zhang, and H. Yang, "Control of switched nonlinear systems via T–S fuzzy modeling," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 1, pp. 235–241, Feb. 2016.

[26] J. Dong and G.-H. Yang, "Dynamic output feedback control synthesis for continuous-time T–S fuzzy systems via a switched fuzzy control scheme," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 38, no. 4, pp. 1166–1175, Aug. 2008.

[27] W.-J. Wang, Y.-J. Chen, and C.-H. Sun, "Relaxed stabilization criteria for discrete-time T–S fuzzy control systems based on a switching fuzzy model and piecewise Lyapunov function," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 37, no. 3, pp. 551–559, Jun. 2007.

[28] Y.-J. Chen, H. Ohtake, K. Tanaka, W.-J. Wang, and H. O. Wang, "Relaxed stabilization criterion for T–S fuzzy systems by minimum-type piecewise-Lyapunov-function-based switching fuzzy controller," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 6, pp. 1166–1173, Dec. 2012.

LIJIE LI received the M.S. degree from the College of Computer Science and Technology, Harbin Engineering University, China in 2006, the Ph.D. degree from the College of Automation, Harbin Engineering University, China, in 2012. She is currently a Lecturer with the College of Computer Science and Technology, Harbin Engineering University, China. Her research interests include privacy protection, social network and pattern recognition.

LEI LIU received the M.Eng. degree in control theory and control engineering from Bohai University, Jinzhou, China, in 2017. She is currently pursuing the Ph.D. degree in control science and engineering from Harbin Engineering University, Harbin, China. Her current research interests include switched control, fuzzy control and their applications.

YUNFEI YIN received the M.Eng. degree in control theory and control engineering from Bohai University, Jinzhou, China, in 2016. He is currently pursuing the Ph.D. degree in control science and engineering with the Harbin Institute of Technology, Harbin, China. His current research interests include switched control, sliding mode control, adaptive control and their applications to power electronic systems for renewable energy systems.