

Received July 31, 2017, accepted September 7, 2017, date of publication September 11, 2017, date of current version October 12, 2017. *Digital Object Identifier 10.1109/ACCESS.2017.2751105*

# DF Relaying Networks With Randomly Distributed Interferers

### XIAZHI LAI<sup>1,2</sup>, WANXIN ZOU<sup>2</sup>, DONGQING XIE<sup>1</sup>, XUTAO LI<sup>2</sup>, (Member, IEEE), AND LISHENG FAN<sup>1</sup>

<sup>1</sup> School of Computer Science and Educational Software, Guangzhou University, Guangzhou 510006, China <sup>2</sup>Department of Electronic Engineering, Shantou University, Shantou 515063, China

Corresponding author: Lisheng Fan (lsfan@gzhu.edu.cn)

This work was supported in part by the Science and Technology Planning Key Project of Guangdong under Grant 2016B010124014, in part by the National Natural Science Foundation of China under Grant 61471229, Grant 61573233, and Grant 61772007, in part by the Innovation Improvement Project for Universities under Grant 2015KTSCX040, in part by the Natural Science Foundation of Guangdong under Grant 2014A030306027 and Grant 2015A030311017, and in part by the Guangzhou University's 2017 Training Program for Young Top-Notch Personnels under Grant BJ201702.

**ABSTRACT** In this paper, we study a decode-and-forward (DF) relaying network with random interferers, in which the source transmits its message to the destination with the help of *N* DF relays. We consider the interference-limited environments, where the reception at the relays and the destination are corrupted by random interferers, which are distributed according to a homogeneous Poisson point process. To improve the system performance, relay selection based on received signal-to-interference ratio (SIR) has been employed to select the best relay among *N* ones. We examine the network performance by deriving the analytical outage probability under Rayleigh fading transmission channels and Nakagami-*m* fading interference channels. Moreover, we compute the asymptotic expressions of outage probability, and confirm that the system diversity order on SIR is equal to  $\frac{2}{\alpha}$ , where  $\alpha$  denotes the path loss factor. Furthermore, we see that the major limitation of system results from the second hop. Numerical and simulation results are demonstrated to validate the proposed analysis as well.

**INDEX TERMS** DF relaying networks, random interferers, Poisson point process, outage probability, diversity order.

#### **I. INTRODUCTION**

Cooperative relaying has been proved to be an efficient method to extend the network coverage, mitigate shadowing fading and increase transmission reliability without additional transmit power at the sources [1]–[12]. The major performance limitations on wireless networks are noise and interference. As [13] suggested, cooperative relaying networks might be more vulnerable to co-channel interference than noise under certain scenarios, such as high signalto-interference-plus-noise ratio (SINR) regime. Co-channel interference might weaken the transmission reliability and degrade the system performance, hence extensive literatures have been dedicated to study the statistical models of interference and their impact [14]–[19].

Taking into account the mobility and uncertainty of terminals in wireless networks, a practical communication scenario in which the number and locations of interferers are randomly distributed has been considered. With stochastic geometry, [19] suggested that interferers in an uncertain wireless network were distributed in a plane according to a homogeneous Poisson point process (PPP). By applying stochastic geometry, [20] characterized the random interference with the first two moments, and therefore derived an approximate expression of probability density function (PDF) for the interference modeled under the PPP assumption. And more importantly, [21] provided the Laplace transform function of random interference associated with PPP under independent fading channels.

In recent years, researchers extended the above work to relaying networks in a Poisson field of interferers, and analyzed the system transmission performance by examining the outage probability. For instance, [22] considered the interference at the destination solely, while [23]–[25] analyzed a three-node relaying system with interferers distributed according to PPP at both the relay and the destination. In [26]–[28], multiple-hop relaying communications were studied with the distributions of interferers following PPP. Moreover, [29] studied multiple decode-and-forward (DF)

relays networks with interference-limited relays, and it also considered direct link and noise at the destination. However, the results were untraceable and no diversity gain on the system was revealed analytically.

In this paper, we study a DF relaying network with randomly distributed interferers corrupting the reception at both the relays and the destination, where the number and positions of interferers are modeled according to independent PPP. To improve the performance of the proposed system, we aim to achieve the minimum system outage probability. To this end, relay selection based on signal-to-interference ratio (SIR) has been employed. System performance is analyzed by deriving the outage probability under Rayleigh fading transmission channels and Nakagami-*m* fading interference channels. In order to acquire some insights into the system, we also provide the asymptotic outage probability in the high SIR and the low interferer density regimes respectively. Numerical and simulation results are demonstrated to validate our analysis as well.

The main contributions of our work can be summarized as follows:

- We analyze a DF relaying network degraded by randomly distributed interferers at the relays and the destination, where the number and positions of interferers follow independent PPP. Close-form expressions of outage probability for the proposed system are given.
- We provide new asymptotic expressions of outage probability, which enable us to determine the crucial system parameters in the high SIR regime and evaluate their effects on the system performance.
- Attribute to the asymptotic expressions, we can observe that the system diversity order is equals to  $\frac{2}{\alpha}$ . Moreover, we can confirm that the system is unable to gain extra system diversity by increasing the number of relays, since the limitation of the second hop.

This paper is organized as follows. Following the introduction, Section II provides a detailed description of a two-hop cooperative relaying system model with multiple fixed DF relays and randomly distributed interferes. Then Section III presents the outage probability analysis, including the analytical outage probability and asymptotic outage probability, and a specific case will be introduced as well. In Section IV, numerical and simulation results are demonstrated to verify our analysis and provide desirable insights on the system performance. Conclusion of our work will be revealed in Section V.

*Notations:*  $X \sim \mathcal{CN}(0, \sigma^2)$  denotes a zero-mean circularly symmetric complex Gaussian random variable (RV) X with variance  $\sigma^2$ . *Y* ∼ Naka(*m*, 1) denotes a RV Y following Nakagami-*m* distribution. Pr[·] denotes the probability.

#### **II. SYSTEM MODEL**

As depicted in Fig. 1, we consider a DF relaying network, which comprises one source *S*, one destination *D* and multiple DF relays  ${R_n | n = 1, 2, ..., N}$ . There are multiple interferers degrading the transmission from the source to the relays



**FIGURE 1.** Selective DF relaying system model with random interferers.

as well as the transmission from the relays to the destination. We assume that all transmission channels experience independent flat Rayleigh fading. Moreover, we adopt the assumption that interference channels experience independent flat Nakagami-*m* fading, since the Nakagami-*m* distribution often offers the best fit to the land-mobile [30]–[32] and indoor-mobile [33] multipath propagation.

During the first time slot, *S* transmits the normalized signal *s*. In this work, we consider an interference-limited environment, where noise is negligible with respect to interference, hence we ignore the effect of noise on the proposed system. The signal received at relay  $R_n$  can be accordingly expressed by

$$
y_{R_n} = \sqrt{P_S} r_{SR_n}^{-\frac{\alpha}{2}} h_{SR_n} s + I_{R_n}, \qquad (1)
$$

where  $P_S$  is the transmit power at the source;  $r_{SR_n}$  is the distance from *S* to  $R_n$ ;  $\alpha \geq 2$  is the exponential path loss factor;  $h_{SR_n} \sim \mathcal{CN}(0, 1)$  is the channel fading coefficient for  $S \rightarrow R_n$  link and  $I_{R_n}$  is the aggregated interference at the relay  $R_n$ . We assume that the interferers around each relay are randomly distributed in a two-dimensional plane according to independent homogeneous Poisson point process  $\Phi_{R_n}$  with node density  $\lambda_R$ , and  $\Phi_{R_n}$  varies from different time slot.<sup>1</sup> Therefore,  $I_{R_n}$  can be expressed by

$$
I_{R_n} = \sum_{i \in \Phi_{R_n}} \sqrt{P_I} h_{I_i R_n} r_{I_i R_n}^{-\frac{\alpha}{2}} x_{I_i R_n}
$$
 (2)

where  $P_I$  is the transmit power at interferers;  $h_{I_iR_n} \sim \text{Naka}(m, 1)$  is the channel fading coefficient for  $I_i \rightarrow R_n$  link;  $r_{I_iR_n}$  is the distance between interferers  $I_i$  and relay  $R_n$  and  $x_{I_iR_n}$  is the transmit signal from interferer  $I_i$ .

Once the received SIR at  $R_n$  is above the given threshold  $\gamma_{th}$ ,  $R_n$  can successfully decode the information. Out of

 $1<sup>1</sup>$ As noticed in [29] and [34], relays might be interfered by the same source of randomness, and therefore, the correlation of interference degrades the reception at relays significantly. However, according to the analysis from [35], this correlation decreases as the distance between each relay increases. Thus, by independent homogeneous Poisson point process, we assume that the distance between each relay is far enough, hence the correlation of interference is negligible.

the successfully decoding relays, the optimal relay  $R_{n*}$  is selected to forward the information. The signal received at the destination can be accordingly expressed by

$$
y_D = \sqrt{P_R} r_{R_n^*D}^{-\frac{\alpha}{2}} h_{R_n^*D} s + I_D \tag{3}
$$

where  $P_R$  is the transmit power at relays;  $r_{R_n * D}$  is the distance from relay  $R_{n^*}$  to *D*;  $h_{R_{n^*}D} \sim \mathcal{CN}(0, 1)$  is the channel fading coefficient for  $R_{n^*} \rightarrow D$  link and  $I_D$  is the aggregated interference at *D*. We also assume that the interferers around the destination are randomly located according to a homogeneous Poisson point process  $\Phi_D$  with node density  $\lambda_D$ , and  $\Phi_D$  varies from different time slot. Therefore,  $I_D$  can be expressed by

$$
I_D = \sum_{i \in \Phi_D} \sqrt{P_I} h_{I_i D} r_{I_i D}^{-\frac{\alpha}{2}} x_{I_i D}
$$
 (4)

where  $h_{I_iD} \sim \text{Naka}(m, 1)$  is the channel fading coefficient for  $I_i \rightarrow D$  link;  $r_{I_iD}$  is the distance from interferer  $I_i$  to  $D$  and  $x_{I_i}$ *D* is the transmit signal from interferer  $I_i$ .

For the simplicity of notation, we use  $\omega_n = |h_{SR_n}|^2$  and  $\nu_n = |h_{R_nD}|^2$  to denote the channel gains of  $S \rightarrow R_n$  and  $R_n \rightarrow D$  links, respectively. In addition, we use  $\eta_{R_n} = \sum |h_{I,R_n}|^2 r_{I,R}^{-\alpha}$  and  $\eta_D = \sum |h_{I,D}|^2 r_{I,D}^{-\alpha}$  to represent the  $i \in \overline{\Phi}_{R_n}$  $|h_{I_iR_n}|^2r_{I_iR_i}^{-\alpha}$  $\sum_{I_i R_n}^{-\alpha}$  and  $\eta_D = \sum_{I \subset \mathcal{I}}$  $\sum_{i\in\Phi_D}$   $|h_{I_iD}|^2 r_{I_iD}^{-\alpha}$  $I_i D$ <sup> $\alpha$ </sup> to represent the summed power of interference at  $R_n$  and  $D$ , respectively. Moreover, we adopt notations that  $\rho_1 = \frac{P_S}{P_I}$  and  $\rho_2 = \frac{P_R}{P_I}$ . Therefore,  $\rho_1$  and  $\rho_2$  represent the source-to-interferer transmit power ratio and the relay-to-interferer transmit power ratio, respectively. Accordingly, the SIR at relay  $R_n$  can be obtained as

$$
SIR_{R_n} = \frac{\rho_1 r_{SR_n}^{-\alpha} \omega_n}{\eta_{R_n}}
$$
 (5)

and the SIR at the destination through  $S \rightarrow R_n \rightarrow D$  link is given by

$$
SIR_{R_nD} = \begin{cases} 0, & \text{If } \frac{\rho_1 r_{SR_n}^{-\alpha} \omega_n}{\eta_{R_n}} \le \gamma_{th} \\ \frac{\rho_2 r_{R_nD}^{-\alpha} \nu_n}{\eta_D}, & \text{If } \frac{\rho_1 r_{SR_n}^{-\alpha} \omega_n}{\eta_{R_n}} > \gamma_{th} \end{cases} . \tag{6}
$$

We denote the set composed of all successfully decoding relays by *C* and the set including all relays by  $\Omega$ . To maximizing the received SIR at the destination, the best relay *R<sup>n</sup>* ∗ is selected by

$$
n^* = \arg\max_{n \in C} \left( \frac{\rho_2 r_{R_n D}^{-\alpha} v_n}{\eta_D} \right). \tag{7}
$$

#### **III. PERFORMANCE ANALYSIS**

The aim of this section is to measure the quality of the proposed communication system in terms of outage performance, thus we derive the analytical and asymptotic expressions of outage probability. In addition, a reduced but practical case is discussed to demonstrate our analysis.

α  $*_{D}s + I_{D}$  (3) system can be written as

$$
P_{out} = \Pr\left[\text{SIR}_{R_n * D} \le \gamma_{th}\right]
$$
 (8)

The outage probability of this system represents the averaged probability that outage of transmission occurs over different positions of interferers with Poisson distribution. By considering that each relay is influenced by interference following independent PPP  $\Phi_{R_n}$ , and then applying total probability law, the outage probability can be rewritten as

According to the system model and the relay selection cri-

A. EXACT OUTAGE PROBABILITY ANALYSIS

$$
P_{out} = \sum_{l=0}^{N} \sum_{\substack{\forall c \subset \Omega, \ \forall n \in C}} \prod_{\forall n \in C} P_{suc, R_n} \times P_{out, C} \times \prod_{\forall m \in (\Omega \setminus C)} P_{out, R_m}
$$
\n(9)

where

$$
P_{suc,R_n} = \Pr\left[\frac{\rho_1 r_{SR_n}^{-\alpha} \omega_n}{\eta_{R_n}} > \gamma_{th}\right]
$$
 (10)

$$
P_{out,R_m} = \Pr\left[\frac{\rho_1 r_{SR_m}^{-\alpha} \omega_m}{\eta_{R_m}} \le \gamma_{th}\right]
$$
 (11)

$$
P_{out,C} = \Pr\left[\frac{\max_{\forall n \in C} \left(\rho_2 r_{R_n D}^{-\alpha} v_n\right)}{\eta_D} \le \gamma_{th}\right] \tag{12}
$$

and |*C*| denotes the cardinal of set *C*, which represents the number of successfully decoding relays. Here *Psuc*,*R<sup>n</sup>* denotes the success probability of  $S \rightarrow R_n$  link.  $P_{out,R_m}$  and  $P_{out,C}$ denote the outage probabilities of  $S \rightarrow R_n$  link and second hop respectively. The exact close-form expressions for the PDFs of  $\eta_{Rn}$  and  $\eta_D$  are difficult to obtain, except for some approximate approaches, such as [20]. However, by using the result from [36] and applying the PDF of  $\omega_n$ ,  $f_{\omega_n}(x) = e^{-x}$ ,  $P_{\textit{succ},R_n}$  and  $P_{\textit{out},R_m}$  can be computed as

$$
P_{suc,R_n} = \int_0^\infty \int_{\frac{\gamma_{th}y r_{SR_n}^{\alpha}}{\rho_1}}^{\infty} f_{\omega_n}(x) dx f_{\eta_{R_n}}(y) dy
$$
  
=  $\mathcal{L}_I \left\{ \frac{\gamma_{th} y r_{SR_n}^{\alpha}}{\rho_1} \right\}$   
=  $\exp \left\{ -\lambda_R \pi \mathbb{E} \left[ |h|^{\frac{4}{\alpha}} \right] \Gamma \left( 1 - \frac{2}{\alpha} \right) r_{SR_n}^2 \left( \frac{\gamma_{th}}{\rho_1} \right)^{\frac{2}{\alpha}} \right\}$  (13)

*Pout*,*R<sup>m</sup>*

$$
= \int_0^\infty \int_0^{\frac{\gamma_{th} yr_{SR_m}^{\alpha}}{\rho_1}} f_{\omega_m}(x) dx f_{\eta_{R_m}}(y) dy
$$
  
=  $1 - \mathcal{L}_I \left\{ \frac{\gamma_{th} yr_{SR_m}^{\alpha}}{\rho_1} \right\}$   
=  $1 - \exp \left\{ -\lambda_R \pi \mathbb{E} \left[ |h|^{\frac{4}{\alpha}} \right] \Gamma \left( 1 - \frac{2}{\alpha} \right) r_{SR_m}^2 \left( \frac{\gamma_{th}}{\rho_1} \right)^{\frac{2}{\alpha}} \right\}$  (14)

where  $\mathcal{L}_I(s)$  is the Laplace transform of the interference in an infinite network;  $\mathbb{E}\left[\left|h\right|^{\frac{4}{\alpha}}\right]$  can be calculated as

$$
\mathbb{E}\left[|h|^{\frac{4}{\alpha}}\right] = \int_0^\infty x^{\frac{2}{\alpha}} f_{|h|^2}(x) dx
$$

$$
= \frac{\Gamma\left(m + \frac{2}{\alpha}\right) m^{-\frac{2}{\alpha}}}{\Gamma\left(m\right)} \tag{15}
$$

and  $f_{|h|^2}(x) = \frac{m^m}{\Gamma(m)}$  $\frac{m^m}{\Gamma(m)} x^{m-1}$  exp ( $-mx$ ) is the PDF of interference channel gain  $|h_{I_iR_n}|^2$  and  $|h_{I_iD}|^2$ .

Similarly, by using the PDF of  $v_n$ ,  $f_{v_n}(x) = e^{-x}$ , we can calculate  $P_{out,C}$  as

$$
P_{out,C} = \int_0^\infty \left[ \prod_{\forall n \in C} \int_0^{\frac{\gamma_{th} y r_{Rn}^{\alpha} D}{\rho_2}} f_{v_n}(x) dx \right] f_{\eta_D}(y) dy.
$$
  
\n
$$
= \int_0^\infty \prod_{\forall n \in C} \left[ 1 - e^{-\frac{\gamma_{th} y r_{Rn}^{\alpha} D}{\rho_2}} \right] f_{\eta_D}(y) dy
$$
  
\n
$$
= \sum_{k=0}^{|C|} (-1)^k \sum_{\substack{\forall B \subset C, \\|B|=k}} \exp \left\{ -\lambda_D \pi \mathbb{E} \left[ |h|^{\frac{4}{\alpha}} \right] \Gamma \left( 1 - \frac{2}{\alpha} \right) \right\}
$$
  
\n
$$
\times \left( \frac{\gamma_{th}}{\rho_2} \sum_{\forall n \in B} r_{Rn}^{\alpha} D \right)^{\frac{2}{\alpha}} \right\}
$$
(16)

where *B* is a subset of *C*, and |*B*| denotes the cardinal of set *B*.

To conveniently analyze the proposed system and demonstrate the analytical results in a straightforward form, we consider a scenario in which all source-to-relay distances are identical with  $r_{SR_n} = r_{SR}$ , and all relay-to-destination distances are identical with  $r_{R_nD} = r_{RD}$ , since relays are placed symmetrically. Moreover, we set *m* to 1, which is associated to Rayleigh fading interference channels. Accordingly, the outage probability for identical *rSR* and *rRD* with Rayleigh fading interference channels is expressed as

$$
P_{out}^{identical} = \sum_{l=0}^{N} {N \choose l} \underbrace{\prod_{n=1}^{l} Pr\left[\frac{P_{S}r_{SR}^{-\alpha}\omega_{n}}{P_{I}\eta_{R_{n}}} > \gamma_{th}\right]}_{J_{1}}
$$
\n
$$
\times \underbrace{\prod_{m=1}^{N-l} Pr\left[\frac{P_{S}r_{SR}^{-\alpha}\omega_{m}}{P_{I}\eta_{R_{m}}} \le \gamma_{th}\right]}_{J_{2}}
$$
\n
$$
\times \underbrace{\prod_{n=1}^{l} Pr\left[\frac{P_{R}r_{RD}^{-\alpha}\nu_{n}}{P_{I}\eta_{D}} \le \gamma_{th}\right]}_{J_{3}}.
$$
\n(17)

By applying the results of eqs. (13), (14) and (16), we can calculate  $J_1$ ,  $J_2$  and  $J_3$  as follows

$$
J_1 = \exp\left[-\frac{l\lambda_R 2\pi^2 r_{SR}^2 \left(\frac{\gamma_{th}}{\rho_1}\right)^{\frac{2}{\alpha}}}{\alpha \sin \frac{2\pi}{\alpha}}\right]
$$
(18)

$$
J_3 = \sum_{n=0}^{l} {l \choose n} \left(-1\right)^n \exp\left[\frac{-\lambda_D 2\pi^2 r_{RD}^2 \left(\frac{n\gamma_{th}}{\rho_2}\right)^{\frac{2}{\alpha}}}{\alpha \sin \frac{2\pi}{\alpha}}\right].
$$
 (20)

#### B. ASYMPTOTIC OUTAGE PROBABILITY ANALYSIS

To acquire some meaningful insights with the crucial network parameters, we aim to investigate the asymptotic behavior of the outage probability for the proposed system in the high SIR and the low interferer density regimes. By using the Taylor expansion method of  $e^x \simeq 1 + x$  for small value of |x| and omitting the higher order terms, *Pout* can be approximated by

• If  $\rho_1 \rightarrow \infty$  or  $\lambda_R \rightarrow 0$ ,

$$
P_{out} \simeq \sum_{k=0}^{N} (-1)^{k} \sum_{\substack{\forall B \subset \Omega, \\ |B|=k}} \exp\left[-\frac{\lambda_{D} \pi \Gamma(1-\frac{2}{\alpha}) \Gamma(m+\frac{2}{\alpha}) m^{-\frac{2}{\alpha}}}{\Gamma(m)} + \frac{\lambda_{D} \pi \Gamma(1-\frac{2}{\alpha}) \Gamma(m+\frac{2}{\alpha}) m^{-\frac{2}{\alpha}}}{\Gamma(m)}\right]
$$
\n
$$
\times \left(\frac{\gamma_{th}}{\rho_{2}} \sum_{\forall n \in B} r_{R_{n}D}^{\alpha}\right)^{\frac{2}{\alpha}}\left.\right].
$$
 (21)

• If 
$$
\rho_2 \to \infty
$$
 or  $\lambda_D \to 0$ ,

$$
P_{out} \simeq \prod_{n=1}^{N} \left\{ 1 - \exp\left[ \frac{\lambda_R \pi \Gamma(1 - \frac{2}{\alpha}) \Gamma(m + \frac{2}{\alpha}) r_{SR_n}^2 m^{-\frac{2}{\alpha}}}{\Gamma(m)} \right. \right. \times \left. \left. \left( \frac{\gamma_{th}}{\rho_1} \right)^{\frac{2}{\alpha}} \right] \right\} . \tag{22}
$$

• If 
$$
\rho_1 = \kappa \rho_2
$$
,  $\rho_2 \to \infty$  or  $\lambda_R = \theta \lambda_D$ ,  $\lambda_D \to 0$ 

$$
P_{out} \simeq \begin{cases} \pi \Gamma \left( 1 - \frac{2}{\alpha} \right) \Gamma \left( m + \frac{2}{\alpha} \right) m^{-\frac{2}{\alpha}} \\ \times \left[ \lambda_R r_{SR}^2 \left( \frac{\gamma_{th}}{\rho_1} \right)^{\frac{2}{\alpha}} + \lambda_D r_{RD}^2 \left( \frac{\gamma_{th}}{\rho_2} \right)^{\frac{2}{\alpha}} \right] \\ \text{If } N = 1 \\ \sum_{n=0}^N (-1)^{n+1} \sum_{\substack{|B|=\kappa \\ |B|=\kappa}} \left( \sum_{\forall n \in B} r_{R_n D}^{\alpha} \right)^{\frac{2}{\alpha}} \\ \times \frac{\lambda_D \pi \Gamma \left( 1 - \frac{2}{\alpha} \right) \Gamma \left( m + \frac{2}{\alpha} \right) m^{-\frac{2}{\alpha}}}{\Gamma(m)} \\ \text{If } N \ge 2 \end{cases} \tag{23}
$$

For the identical *rSR* and *rRD* with Rayleigh fading interference channels scenario, we can further simplify the asymptotic expressions for  $P_{out}^{identical}$  as eq. (24), as shown at the bottom of the next page.

From the above approximate expressions of outage probability, the following meaningful insights can be achieved accordingly:

*Remark 1:* As  $\rho_1$  and  $\rho_2$  tend to infinity with  $\rho_1 = \kappa \rho_2$ , we use  $\rho_1$  as the SIR parameter, and define the system diversity order  $\Delta$  as

$$
\Delta = \lim_{\rho_1 \to \infty} \frac{-\log(P_{out})}{\log(\rho_1)}
$$
(25)

which is equal to  $\frac{2}{\alpha}$ . This indicates that the system performance mainly depends on channel condition. Specifically, the diversity gains of system decrease as the path loss factor increases.

*Remark 2:* According to eqs. (21), (22) and (24), we see that the improvement of either hop may confront the bottleneck results from the other hop. Moreover, the degradation of multiple relay networks mainly results from the interference at the destination, since the performance of the first hop can be significantly improved with the increasing number of relays by obtaining more diversity gains, yet the second hop fails. This is due to the fact that as interferers emerge nearby the destination, it is highly possible that outage of transmission occurs. However, the system can select other optional relays as outage is inevitable at one relay.

*Remark 3:* As the SIR at the first and second hops tend to infinity or the overall interferer density tends to 0, when  $N = 1$ , the outage probability is effected by both the first and second hops. However, when the number of relays  $N > 2$ , the second hop becomes the crucial link of the system, which indicates that the second hop is the dominant link of the whole system. This is due to the fact that the interference at the destination has made the second hop a weaker link regarding the whole system, as the first hop is able to gain full diversity while the second hop fails.

#### **IV. NUMERICAL AND SIMULATION RESULTS**

In this section, by several numerical and simulation results, we verify the aforementioned analysis results, and evaluate the effects of several important networks parameters on the system performance. We adopt the scenario in which the distance from source to destination is 400 meters and all transmission channels experience flat Rayleigh fading. Unless specified otherwise, the distances from source to relays and the distances from relays to destination are both identical with  $r_{SR} = r_{RD} = 200$  meters, and *m* is set to 1 for interference channels. Besides,  $\lambda_R$  and  $\lambda_D$  are set to 10<sup>-5</sup>. The simulation performed 10<sup>6</sup> iterations with different interferer position configurations and transmission channel realizations. For each interferer position configuration, the interference nodes are placed according to Poisson point process within a square plane with side length of  $10<sup>6</sup>$  meters, where the relay or destination is placed in the central. Normalized as benchmark, the transmit power at interferers is set to 1. The transmission rate is configured to 1 bit per second per hertz (bps/HZ) and the corresponding SIR threshold γ*th* is equal to 3.

Fig. 2(a) and 2(b) show how the outage probabilities vary with SIR for  $\rho_1 = \rho_2$ . Fig. 2(a) corresponds to the scenario in which relays are evenly placed on a straight line between the source and destination, and thus the sum of  $r_{SR_n}$  and  $r_{R_nD}$ is a constant and set to 400 meters. In addition, we set *m* to 2. Fig. 2(b) refers to the scenario with  $r_{SR_n} = r_{SR}, r_{R_nD} = r_{RD}$ and *m*=1. It can be observed that analytical results approximately match the related simulation results for different *N* and  $\alpha$ , due to the fact that we consider an infinite network in previous analysis. Moreover, the asymptotic results are consistent to the related simulation results in high SIR regime, which confirms our analysis. Furthermore, we see that the increase of the number of relays is able to improve the system performance yet slightly, and the diversity gains are inversely

$$
P_{out}^{identical} \simeq \begin{cases} \sum_{n=0}^{N} {N \choose n} (-1)^n \exp\left[\frac{-\lambda_D 2\pi^2 r_{RD}^2 (\frac{n\gamma_{th}}{\rho_2})^{\frac{2}{\alpha}}}{\alpha \sin \frac{2\pi}{\alpha}}\right], & \text{if } \rho_1 \to \infty, \text{ or } \lambda_R \to 0 \\ \begin{cases} 1 - \exp\left[-\frac{\lambda_R 2\pi^2 r_{SR}^2 (\frac{\gamma_{th}}{\rho_1})^{\frac{2}{\alpha}}}{\alpha \sin \frac{2\pi}{\alpha}}\right] \end{cases}, & \text{if } \rho_2 \to \infty, \text{ or } \lambda_D \to 0 \\ 2\pi^2 \left[\lambda_R r_{SR}^2 (\frac{\gamma_{th}}{\rho_1})^{\frac{2}{\alpha}} + \lambda_D r_{RD}^2 (\frac{\gamma_{th}}{\rho_2})^{\frac{2}{\alpha}}\right], & \text{if } \rho_1 = \kappa \rho_2, \rho_2 \to \infty \\ \text{or } \lambda_R = \theta \lambda_D, \lambda_D \to 0, & N = 1 \\ \frac{\lambda_D 2\pi^2 r_{RD}^2 (\frac{\gamma_{th}}{\rho_2})^{\frac{2}{\alpha}}}{\alpha \sin \frac{2\pi}{\alpha}} \sum_{n=1}^{N} {N \choose n} (-1)^{n+1} n^{\frac{2}{\alpha}}, & \text{if } \rho_1 = \kappa \rho_2, \rho_2 \to \infty \\ (\text{or } \lambda_R = \theta \lambda_D, \lambda_D \to 0), & N \ge 2 \end{cases} \tag{24}
$$





**FIGURE 2.** Outage probabilities versus SIR  $\rho_1 = \rho_2$  for  $\lambda_R = \lambda_D = 10^{-5}$ . (a) Relays with different distances. (b) Relays with identical distances.

proportional to the path loss factor α, resulting from the interference at the destination.

Fig. 3 plots the variation of throughput with relay number for  $\rho_1 = \rho_2 = 10 \text{ dB}, \lambda_R = \lambda_D =$  $10^{-5}$ ,  $m=1$  and  $r_{SR}$  =  $r_{RD}$  = 200 meters. For the unit target data rate, the system throughput is equal to  $(1 - P_{out})$ . We see that the network throughput increases as the relay number grows. In addition, for smaller value of  $\alpha$ , the enhancement is more obvious, since more diversity gain can be obtained, which is consistent to our analysis.

To evaluate each hop individually, Fig. 4 and Fig. 5 plot the variation of outage probabilities with source-to-interferer transmit power ratio  $\rho_1$  and relay-to-interferer transmit power ratio  $\rho_2$  respectively for  $\alpha = 3$ . As expected, for either  $\rho_1 \rightarrow \infty$  or  $\rho_2 \rightarrow \infty$ , their exact outage probabilities converge to corresponding asymptotic outage probabilities



**FIGURE 3.** Throughput versus relay number, for  $\rho_1 = \rho_2 = 10$  dB,  $\lambda_R = \lambda_D = 10^{-5}$ , m=1 and  $r_{SR} = r_{RD} = 200$  meters.



**FIGURE 4.** Outage probabilities versus source-to-interferer transmit power ratio  $\rho_1$ , for  $\rho_2 = 10$ ,  $\alpha = 3$ ,  $\lambda_R = \lambda_D = 10^{-5}$ , m=1 and  $r_{SR} = r_{RD} = 200$  m.



**FIGURE 5.** Outage probabilities versus relay-to-interferer transmit power ratio  $\rho_2$ , for  $\rho_1 = 10$ ,  $\alpha = 3$ ,  $\lambda_R = \lambda_D = 10^{-5}$ , m=1 and  $r_{SR} = r_{RD} = 200$  m.

for different values of *N*. Moreover, Fig. 6 and Fig. 7 show the outage probabilities versus interferer densities  $\lambda_R$  and  $\lambda_D$  respectively for  $\alpha = 3$ . We note that for either  $\lambda_R \to 0$ 



**FIGURE 6.** Outage probabilities versus interferer density at relays  $\lambda_{\boldsymbol{R}}$ , for  $\lambda_D = 10^{-5}$ ,  $\alpha = 3$ , m=1 and  $r_{SR} = r_{RD} = 200$  m.



**FIGURE 7.** Outage probabilities versus interferer density at the destination  $\lambda_D$ , for  $\lambda_R = 10^{-5}$ ,  $\alpha = 3$ , m=1 and  $r_{SR} = r_{RD} = 200$  m.

or  $\lambda_D \rightarrow 0$ , their exact outage probabilities converge to related asymptotic outage probabilities. From Fig. 4 to Fig. 7, we see that the enhancement of either hop, such as increasing  $\rho_1$  or loweringf down  $\lambda_R$ , may confront the bottleneck results from the other hop. In addition, for  $\rho_2 \rightarrow \infty$  or  $\lambda_D \rightarrow 0$ , augmenting the number of relays can significantly improve the system performance, yet the improvement for the scenario when  $\rho_1 \rightarrow \infty$  or  $\lambda_R \rightarrow 0$  is much less obvious. This is due to the fact that the interference at the destination degrades the transmission of the second hop, hence it reduces the influence of the number of relays, especially on relay-to-destination links.

Fig. 8 plots the variation of outage probabilities with interferer density  $\lambda_R = \lambda_D$  for  $\alpha = 3$ . As both approximate outage probabilities and numerical results suggested, increasing  $\rho_1$  is unable to fundamentally improve the system performance, which is evident in low interferer density regime. However, the enhancement of  $\rho_2$  can noticeably decrease the outage probability. This indicates that the second hop is



**FIGURE 8.** Outage probabilities versus interferer density  $\lambda_R = \lambda_D$ , for  $\alpha = 3$ , m=1 and  $r_{SR} = r_{RD} = 200$  m.

the dominant link of the whole system, which validates our analysis.

#### **V. CONCLUSION**

In this work, the performance of a DF relaying network in a Poisson field of interferers has been evaluated. Both analytical and approximate expressions of outage probability for the proposed communications system have been derived under Rayleigh fading transmission channels and Nakagami*m* fading interference channels. With the given asymptotic expressions, we can confirm that the system is unable to gain extra system diversity by increasing the number of relays, due to the interference at the destination. Moreover, the improvement of either hop may confront the bottleneck results from the other hop. In addition, in the high SIR or the low interferer density regime, the second hop becomes the dominant hop for the whole system. Numerical and simulation results illustrate the proposed analysis and bring some meaningful insights as well.

#### **REFERENCES**

- [1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior,'' *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [2] H. Nourizadeh, S. Nourizadeh, and R. Tafazolli, ''Performance evaluation of cellular networks with mobile and fixed relay station,'' in *Proc. IEEE Veh. Technol. Conf.*, Montreal, QC, Canada, Sep. 2006, pp. 1–5.
- [3] L. Fan, S. Zhang, T. Q. Duong, and G. K. Karagiannidis, ''Secure switchand-stay combining (SSSC) for cognitive relay networks,'' *IEEE Trans. Commun.*, vol. 64, no. 1, pp. 70–82, Jan. 2016.
- [4] J. Hu and N. C. Beaulieu, ''Performance analysis of decode-and-forward relaying with selection combining,'' *IEEE Commun. Lett.*, vol. 11, no. 6, pp. 489–491, Jun. 2007.
- [5] Y. Zhao, R. Adve, and T. J. Lim, ''Outage probability at arbitrary SNR with cooperative diversity,'' *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 700–702, Aug. 2005.
- [6] S. S. Ikki and M. H. Ahmed, ''Performance of multiple-relay cooperative diversity systems with best relay selection over Rayleigh fading channels,'' *EURASIP J. Adv. Signal Process.*, vol. 2008, no. 1, pp. 1–7, Mar. 2008.
- [7] Y. Jing and H. Jafarkhani, ''Single and multiple relay selection schemes and their achievable diversity orders,'' *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1414–1423, Mar. 2009.
- [8] L. Fan, R. Zhao, F.-K. Gong, N. Yang, and G. K. Karagiannidis, ''Secure multiple amplify-and-forward relaying over correlated fading channels,'' *IEEE Trans. Commun.*, vol. 65, no. 7, pp. 2811–2820, Jul. 2017.
- [9] L. Fan, X. Lei, N. Yang, T. Q. Duong, and G. K. Karagiannidis, ''Secure multiple amplify-and-forward relaying with cochannel interference,'' *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 8, pp. 1494–1505, Dec. 2016.
- [10] T. Zhang, W. Chen, and Z. Cao, ''Opportunistic DF-AF selection relaying with optimal relay selection in Nakagami-*m* fading environments,'' in *Proc. IEEE Int. Conf. Commun. China*, Beijing, China, Aug. 2012, pp. 619–624.
- [11] S. S. Ikki and M. H. Ahmed, "Performance analysis of cooperative diversity wireless networks over Nakagami-*m* fading channel,'' *IEEE Commun. Lett.*, vol. 11, no. 4, pp. 334–336, Apr. 2007.
- [12] X. Lei, L. Fan, D. S. Michalopoulos, P. Fan, and R. Q. Hu, ''Outage probability of TDBC protocol in multiuser two-way relay systems with Nakagami-*m* fading,'' *IEEE Commun. Lett.*, vol. 17, no. 3, pp. 487–490, Mar. 2013.
- [13] H. Yu, I. H. Lee, and G. L. Stuber, "Outage probability of decode-andforward cooperative relaying systems with co-channel interference,'' *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 266–274, Jan. 2012.
- [14] O. Dousse, F. Baccelli, and P. Thiran, "Impact of interferences on connectivity in ad hoc networks,'' *IEEE/ACM Trans. Netw.*, vol. 13, no. 2, pp. 425–436, Apr. 2003.
- [15] H.-S. Ryu, J.-S. Lee, and C. G. Kang, "Performance analysis of relay systems in an interference-limited environment,'' in *Proc. Veh. Technol. Conf.*, Budapest, Hungary, May 2011, pp. 1–5.
- [16] H. A. Suraweera, H. K. Garg, and A. Nallanathan, ''Performance analysis of two hop amplify-and-forward systems with interference at the relay,'' *IEEE Commun. Lett.*, vol. 14, no. 8, pp. 692–694, Aug. 2010.
- [17] M. Z. Win, P. C. Pinto, and L. A. Shepp, "A mathematical theory of network interference and its applications,'' *Proc. IEEE*, vol. 97, no. 2, pp. 205–230, Feb. 2009.
- [18] J. Ilow and D. Hatzinakos, "Analytic alpha-stable noise modeling in a Poisson field of interferers or scatterers,'' *IEEE Trans. Signal Process.*, vol. 46, no. 6, pp. 1601–1611, Jun. 1998.
- [19] T. Mattfeldt, *Stochastic Geometry and Its Applications*, 3rd ed. Hoboken, NJ, USA: Wiley, 1996.
- [20] R. W. Heath, M. Kountouris, and T. Bai, ''Modeling heterogeneous network interference using Poisson point processes,'' *IEEE Trans. Signal Process.*, vol. 61, no. 16, pp. 4114–4126, Aug. 2013.
- [21] M. Haenggi and R. K. Ganti, ''Interference in large wireless networks,'' *Found. Trends Netw.*, vol. 3, no. 2, pp. 127–248, 2009.
- [22] M. D. Renzo and W. Lu, "On the diversity order of selection combining dual-branch dual-hop AF relaying in a Poisson field of interferers at the destination,'' *IEEE Trans. Veh. Technol.*, vol. 64, no. 4, pp. 1620–1628, Apr. 2015.
- [23] H. S. Ryu and C. G. Kang, "Performance analysis of interference-limited relay systems over large-scale fading channels,'' in *Proc. 18th Asia–Pacific Conf. Commun. (APCC)*, Oct. 2012, pp. 997–1001.
- [24] A. Altieri, L. R. Vega, C. G. Galarza, and P. Piantanida, ''Cooperative strategies for interference-limited wireless networks,'' in *Proc. IEEE Int. Symp. Inf. Theory Proc.*, Saint Petersburg, Russia, Aug. 2011, pp. 1623–1627.
- [25] R. Tanbourgi, H. Jäkel, and F. K. Jondral, "Cooperative relaying in a Poisson field of interferers: A diversity order analysis,'' in *Proc. IEEE Int. Symp. Inf. Theory*, Istanbul, Turkey, Jul. 2013, pp. 3100–3104.
- [26] V. A. Aalo, G. P. Efthymoglou, T. Soithong, M. Alwakeel, and S. Alwakeel, ''Performance analysis of multi-hop amplify-and-forward relaying systems in Rayleigh fading channels with a Poisson interference field,'' *IEEE Trans. Wireless Commun.*, vol. 13, no. 1, pp. 24–35, Jan. 2014.
- [27] J. Lee, H. Shin, I. Lee, and J. Heo, ''Optimal linear multihop system for DF relaying in a Poisson field of interferers,'' *IEEE Commun. Lett.*, vol. 17, no. 11, pp. 2029–2032, Nov. 2013.
- [28] A. A. Abdelnabi, F. S. Al-Qahtani, M. Shaqfeh, S. S. Ikki, and H. M. Alnuweiri, ''Performance analysis of MIMO multi-hop system with TAS/MRC in Poisson field of interferers,'' *IEEE Trans. Commun.*, vol. 64, no. 2, pp. 525–540, Feb. 2016.
- [29] N. Suraweera and N. C. Beaulieu, "Optimum combining for cooperative relaying in a Poisson field of interferers,'' *IEEE Trans. Commun.*, vol. 63, no. 9, pp. 3132–3142, Jul. 2015.
- [30] H. Suzuki, "A statistical model for urban radio propagation," *IEEE Trans. Commun.*, vol. COM-25, no. 7, pp. 673–680, Jul. 1977.
- [31] T. Aulin, ''Characteristics of a digital mobile radio channel,'' *IEEE Trans. Veh. Technol.*, vol. VT-30, no. 2, pp. 45–53, May 1981.
- [32] W. Braun and U. Dersch, ''A physical mobile radio channel model,'' *IEEE Trans. Veh. Technol.*, vol. 40, no. 2, pp. 472–482, May 1991.
- [33] A. U. Sheikh, M. Abdi, and M. Handforth, "Indoor mobile radio channel at 946 MHz: Measurements and modeling,'' in *Proc. IEEE Veh. Technol. Conf.*, Secaucus, NJ, USA, May 1993, pp. 73–76.
- [34] M. Haenggi, ''Diversity loss due to interference correlation,'' *IEEE Commun. Lett.*, vol. 16, no. 10, pp. 1600–1603, Oct. 2012.
- [35] R. K. Ganti and M. Haenggi, ''Spatial and temporal correlation of the interference in ALOHA ad hoc networks,'' *IEEE Commun. Lett.*, vol. 13, no. 9, pp. 631–633, Sep. 2009.
- [36] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, ''Stochastic geometry and random graphs for the analysis and design of wireless networks,'' *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029–1046, Sep. 2009.



XIAZHI LAI received the B.S. degree in communication engineering from Shantou University in 2015, where he is currently pursuing the M.S. degree. His research interests include energy harvesting, physical-layer secure communications, and interference modeling.



WANXIN ZOU received the B.S. degree from the Department of Electronic Engineering, Shantou University, China, in 2015, where she is currently pursuing the M.S. degree in the Department of Electronic Engineering.



DONGQING XIE received the B.Sc. degree in applied mathematics and the M.Sc. degree in computer software from Xidian University, China, and the Ph.D. degree in applied mathematics from Hunan University, China. He is the Dean and a Professor with the School of Computer and Education Software, Guangzhou University. He has been a Visiting Scholar with Nipissing University, Canada. His research interests include information security and cryptography. He is a member of CCF.

## **IEEE** Access



XUTAO LI (M'12) received the Ph.D. degree in electronics engineering from the Huazhong University of Science and Technology in 2006. From 2006 to 2008, he was a Post-Doctoral Fellow with the South China University of Technology. Since 2013, he has been a Professor with the Department of Electronic Engineering, Shantou University. He has authored or co-authored over 20 papers published in refereed journals. His research interests include array signal processing, radar systems,

computer vision, and nonlinear presentation to signals.



LISHENG FAN received the B.S. degree from the Department of Electronic Engineering, Fudan University, in 2002, and the M.S. degree from the Department of Electronic Engineering, Tsinghua University, China, in 2005, and the Ph.D. degree from the Department of Communications and Integrated Systems, Tokyo Institute of Technology, Japan, in 2008. He is currently a Professor with Guangzhou University. He has authored or coauthored many papers in international journals,

such as the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, the IEEE TRANSACTIONS ON COMMUNICATIONS, the IEEE TRANSACTIONS ON INFORMATION THEORY, as well as papers in conferences, such as the IEEE ICC, the IEEE Globecom, and the IEEE WCNC. His research interests span in the areas of wireless cooperative communications, physical-layer secure communications, interference modeling, and system performance evaluation. He has also served as a member of Technical Program Committees for IEEE conferences, such as Globecom, ICC, WCNC, and VTC. He is a Guest Editor of *EURASIP Journal on Wireless Communications and Networking*, and served as the Chair of Wireless Communications and Networking Symposium for Chinacom 2014.

 $\bullet$   $\bullet$   $\bullet$