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Resource Allocation for Weighted Sum-Rate Maximization in Multi-User Full-Duplex Device-to-Device Communications: Approaches for Perfect and Statistical CSIs

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ABSTRACT In this paper, we investigate the resource allocation problem for multi-user full-duplex device-to-device (D2D) underlay communication, considering both perfect channel state information (CSI) and statistical CSI scenarios. In perfect CSI scenario, the weighted sum-rate maximization problem under cellular users' minimum rate constraints is formulated as a mixed integer programming problem. To solve the challenging problem, we decouple it into two subproblems as power allocation and channel assignment. Then we proposed a power allocation algorithm based on difference of two convex functions programming and a channel assignment algorithm based on Kuhn–Munkres algorithm, respectively. In statistical CSI scenario, we formulate the resource allocation problem as an outage probability constrained weighted ergodic sum-rate maximization problem. To solve the problem, the closed-form expressions of outage probability and weighted ergodic sum-rate are derived first. Then we decouple resource allocation problem into power allocation and channel assignment. An optimization solution that consists of a 2-D global searching and Kuhn–Munkres algorithm is then developed. Simulation results demonstrate that the proposed algorithms can improve the weighted sum-rate of full-duplex D2D communications significantly both in perfect CSI and statistical CSI scenarios and confirm the accuracy of our derived closed-form expressions.

INDEX TERMS Device-to-device (D2D), full-duplex, outage probability, resource allocation, weighted sum-rate.

I. INTRODUCTION

With the rapid development of mobile communications, the tremendous growth of smart phones and mobile applications bring severe challenges to current mobile wireless cellular system [1], [2]. Device-to-device (D2D) communication and full-duplex communication, as two promising technologies for the next generation mobile communication networks (5G), have received a great deal of attention [3]–[5].

D2D communication enables any two proximate D2D users to set up direct transmission under the control of base station. In traditional D2D communication, which is referred to as half-duplex D2D communication in this paper, each D2D pair includes a D2D transmitter and a D2D receiver. By reusing the same spectrum resources with cellular links, D2D transmitter sends signals to D2D receiver, as an underlay to cellular networks, which can potentially improve network capacity and spectrum efficiency. However, if the spectrum resources are not properly allocated, co-channel interference between cellular users and D2D users causes performance degradation. Thus resource allocation becomes one of the critical issues for D2D underlay communications [6]. To tackle this challenging problem, various resource allocation issues in half-duplex D2D communication networks has been extensively explored previously to enhance network performance, including spectrum allocation, power allocation and mode selection [7]–[12]. In [7], the resource allocation of D2D underlaying communication cellular system is investigated to maximize the overall throughput with QoS constraints of both cellular users and D2D users, and a three-step solution is proposed, including admission control, power control and maximum weighted matching. In [8], a hypergraph based channel allocation method is proposed to maximize the cell capacity, allowing multiple D2D pairs to share uplink channels with cellular users. In [9], both centralized resource allocation method based on convex optimization and decentralized method based on Stackelberg game are proposed for half-duplex D2D communications to maximize sum-rate of D2D links, allowing multiple D2D pairs reuse same cellular link. In [10], a distributed resource allocation scheme based on game theoretical approach is investigated to optimize energy efficiency of half-duplex D2D enabled cellular networks. Spectrum allocation between D2D links and cellular links using pairing methods or matching methods have also been exploited in many works. In [11], cellular links and D2D links are formulated as two disjoint sets and various pairing algorithms such as Hopcroft-Karp and Gale-Shapley algorithms, are used to optimize different performance metrics including sum-rate, number of active D2D links et al. The resource allocation problem for D2D communications is modeled as a many-to-one matching game and is further solved by matching and swap operation in [12]. A resource allocation approach based on the many-to-many matching game with externalities is proposed in [13], which is proved to converge the two-side exchange stability. Mode selection also provides another approach to improve system performance, by enabling mobile users to switch between cellular mode and D2D mode. In [14], the joint mode selection and channel assignment is investigated, allowing multiple D2D links to share the same channel. A dynamic programming based algorithm is proposed to tackle the problem. In [15], the joint mode selection and base station selection in D2D-enabled multi-BS cellular networks is investigated and a global optimal solution is proposed based on maximum weighted bipartite matching approach.

However, all the above mentioned papers focus on the traditional half-duplex D2D communication. To further improve the spectral efficiency, full-duplex communication which enables mobile devices to receive and transmit signal in identical spectral resource simultaneously, has emerged as another promising technology toward 5G. The performance of full-duplex D2D communication with single cellular link and single full-duplex D2D link is derived in [16], proving that full-duplex D2D outperforms traditional half-duplex D2D. The sum-rate of full-duplex underlay D2D network is analyzed and the closed-form approximation is derived in [18]. In [19], a power allocation scheme is developed for full-duplex D2D communication to maximize the ergodic capacity of D2D link. However, existing researches [17]–[19] only focus on full-duplex D2D communication with single D2D pair and single cellular user. To the best of our knowledge, the resource allocation problem for multi-user full-duplex D2D communication system has rarely been addressed yet.

In this paper, we investigate resource allocation for full-duplex D2D communication with multiple full-duplex D2D pairs and multiple cellular users. Specifically, we consider both perfect and statistical CSI scenarios. For both scenarios, we propose efficient resource allocation algorithms respectively, by solving power allocation and channel assignment separately. The main contributions of this paper are summarized as follows:

(1) For perfect CSI scenario, we formulate resource allocation problem as maximizing weighted sum-rate of both D2D pairs and cellular users under cellular users' rate constraints. Then we propose OPtimal Power Allocation and OPtimal Channel Assignment algorithm with Perfect CSI (OPPA-OPCA-P), in which the power allocation is solved by DC programming, while channel assignment is solved based on Kuhn–Munkres algorithm.

(2) For statistical CSI scenario, we firstly derive the closed-form expressions of ergodic weighted sum-rate and outage probability by assuming Rayleigh fading distribution for each channel. The resource allocation problem is formulated as maximizing ergodic weighted sum-rate of both D2D pairs and cellular users under cellular users' outage constraints. Similarly, we propose an efficient resource allocation algorithm referred to as OPtimal Power Allocation and OPtimal Channel Assignment with Statistical CSI (OPPA-OPCA-S), in which power allocation and channel assignment are solved by 2-dimensional global searching and Kuhn–Munkres algorithm respectively.

The rest of this paper is organized as follows. Section II describes the considered full-duplex D2D communication system model. In section III, the resource allocation scheme considering perfect CSI is addressed. In section IV, the resource allocation scheme with statistical CSI is proposed. Simulation results are presented in Section V. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

We consider a cellular system with one base station and multiple users, including M cellular uplink users and N pairs of D2D users, as depicted in Fig. 1 (a). The base station and cellular users are half-duplex and equipped with single antenna. D2D users have isolate transmit antenna and receive antenna to facilitate full-duplex transmission. The set of cellular users and D2D pairs are denoted as $\mathrm{C} = \{CU_1, \ldots, CU_j, \ldots CU_M\}$ and $\mathrm{D} = \{D2D_1, \ldots, D2D_i, \ldots, D2D_N\}$ respectively, where CU_j and $D2D_i$ denote the jth cellular user and the ith D2D pair respectively. The twins of D2D pair $D2D_i$ are denoted as D_i^1 and D_i^2, $i = 1, 2 \ldots N$.

The available bandwidth is divided into a set of channels $\{CH_1, \ldots, CH_j, \ldots, CH_M\}$. We assume that the channel

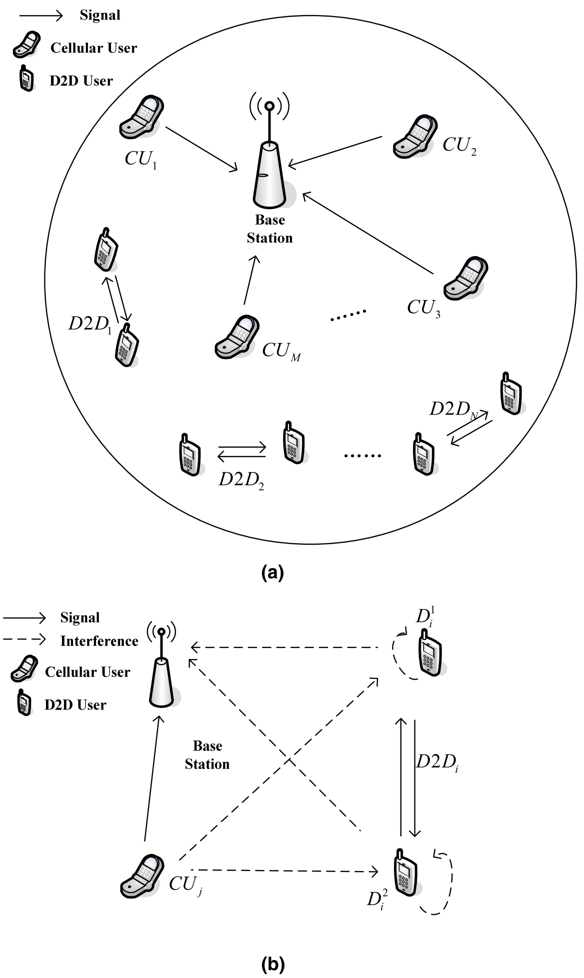

FIGURE 1. System model. (a) Cellular system with multiple D2D pairs and multiple cellular users. (b) Interference model between D2D pair and cellular user sharing same channel.

assignment for cellular users has been pre-determined and channel CH_j is assigned to CU_j. In this paper, we focus on the channel assignment for D2D pairs. The channel assignment constraints for D2D pairs are assumed as follows: one cellular user can only share its uplink channel with one D2D pair, to avoid interference between D2D links. Also, each D2D pair is allowed to access one uplink channel at most.

The channel gains contain the normalized small-scale fading and distance based large-scale pathloss. Assume that all channels experience block Rayleigh fading. We define the following symbols:

h_i^D: channel gain between D_i^1 and D_i^2.

h_j^{CB}: channel gain between CU_j and base station.

$h_{ji,k}^{CD}$: interference channel gain from CU_j to D_i^k, $k = 1, 2$.

$h_{ji,k}^{BD}$: interference channel gain from D_i^k to base station on channel i, $k = 1, 2$.

$h_{i,k}^D$: self-interference channel gain of D_i^k, $k = 1, 2$.

Two full-duplex D2D users in each D2D pair transmit and receive signal at the same time on the same frequency. Advanced analog and digital self-interference cancellation techniques are implemented to suppress amount of self-interference. Also there is co-channel interference between a D2D pair and a cellular user on the same channel, as illustrated in Fig. 1 (b). Compared with that in half-duplex D2D underlaying cellular system, the interference model in full-duplex D2D system is more complicated. The base station suffers the co-channel interferences from two users in one D2D pair. Meanwhile, each D2D user suffers co-channel interference from cellular user and self-interference from its transmit antenna.

Assuming $D2D_i$ shares the uplink channel of CU_j, the signals received by D_i^k include information signal from $D_i^{k'}$ ($k' \neq k$), self-interference caused by full-duplex transmission and interference caused by CU_j. Thus we can derive the received signal to interference and noise ratio (SINR) at D_i^1 and D_i^2 on channel CH_j as follows:

$$\gamma_{ij,1}^D = \frac{P_{i,2}^D \left|h_i^D\right|^2}{P_j^C \left|h_{ji,1}^{CD}\right|^2 + P_{i,1}^D \left|\beta h_{i,1}^D\right|^2 + N_0}, \tag{1}$$

$$\gamma_{ij,2}^D = \frac{P_{i,1}^D \left|h_i^D\right|^2}{P_j^C \left|h_{ji,2}^{CD}\right|^2 + P_{i,2}^D \left|\beta h_{i,2}^D\right|^2 + N_0}, \tag{2}$$

where $P_{i,k}^D$ denotes the transmit power of D_i^k, $k = 1, 2$, P_j^C denotes the transmit power of CU_j, β is self-interference cancellation factor and N_0 represents the power density of additive white Gaussian noise at D_i^k, $k = 1, 2$.

According to Shannon's theory, the achievable sum-rate of D2D pair $D2D_i$ on channel j can be calculated by

$$r_{ij}^D = \log_2(1 + \gamma_{ij,1}^D) + \log_2(1 + \gamma_{ij,2}^D), \tag{3}$$

where $\gamma_{ij,1}^D$ and $\gamma_{ij,2}^D$ are given in (1) and (2) respectively.

Meanwhile, base station will receive information signals from CU_j and the interference signals caused by D_i^1 and D_i^2. Thus, the received SINR at base station on CH_j is

$$\gamma_{ij}^C = \frac{P_j^C \left|h_j^{CB}\right|^2}{P_{i,1}^D \left|h_{ji,1}^{BD}\right|^2 + P_{i,2}^D \left|h_{ji,2}^{BD}\right|^2 + N_0}. \tag{4}$$

Also, the achievable rate of cellular link CU_j can be calculated by

$$r_{ij}^C = \log_2(1 + \gamma_{ij}^C). \tag{5}$$

III. RESOURCE ALLOCATION WITH PERFECT CSI

A. PROBLEM FORMULATION

Because D2D pairs reuse the same spectrum resource with cellular users, the co-channel interference can be generated between D2D pairs and cellular users sharing the same channel. Thus efficient resource allocation needs to be implemented to mitigate the co-channel interference. In this section, we solve the full-duplex D2D resource allocation problem assuming that the base station knows perfect CSI of all channels.

Denote $\mathbf{P}$ as the power allocation matrix for all cellular users and D2D users, $\mathbf{P} = [\boldsymbol{P}_{ij}]$, where $\boldsymbol{P}_{ij}$ is denoted as $(P_j^C, P_{i,1}^D, P_{i,2}^D)$. Define weighted sum-rate matrix $\mathbf{F} = [f_{ij}]$, where f_{ij} represents the weighted sum-rate of CU_j and D2D_i on channel CH_j.

$$f_{ij}(\boldsymbol{P}_{ij}) = a_j r_{ij}^C + b_i r_{ij}^D. \tag{6}$$

The coefficients a_j and b_i in (6) are dependent on the quality of service (QoS) and priority of cellular user CU_j and D2D pair $D2D_i$ respectively.

Denote the channel assignment matrix as $\mathbf{X} = [x_{ij}]_{N\times M}$, where $x_{ij} = 1$, if channel CH_j is assigned to D2D pair $D2D_i$; $x_{ij} = 0$, otherwise. The utilization function of multi-user full-duplex D2D communication system is defined as weighted sum-rate of all cellular users and D2D pairs.

$$U(\mathbf{X}, \mathbf{P}) = \sum_{i=1,\ldots,N} \sum_{j=1,\ldots,M} x_{ij} f_{ij}(\boldsymbol{P}_{ij}). \tag{7}$$

It is worth mentioning that the utilization function defined in (7) is a generalized performance metric, which can be transformed as sum-rate of D2D pairs or overall network throughput, by assigning different coefficients to each cellular user and D2D pair.

In order to maximize the weighted sum-rate for the overall system while guaranteeing each cellular user's minimum transmit rate, we formulate the following optimization problem:

$$P1: \quad \max_{\mathbf{X},\mathbf{P}} U(\mathbf{X}, \mathbf{P}) \tag{8a}$$

$$s.t. \quad r_j^C \geq R_j^C, \quad j = 1, \ldots, M \tag{8b}$$

$$P_j^C \leq P_{\max}^C, \quad j = 1, \ldots, M \tag{8c}$$

$$P_{i,1}^D + P_{i,2}^D \leq P_{\max}^D, \quad i = 1, \ldots, N \tag{8d}$$

$$\sum_{i=1}^{N} x_{ij} \leq 1, \quad j = 1, \ldots, M \tag{8e}$$

$$\sum_{j=1}^{M} x_{ij} = 1, \quad i = 1, \ldots, N \tag{8f}$$

$$x_{ij} \in \{0, 1\}. \tag{8g}$$

Constraint (8b) denotes the minimum rate requirements of cellular users. (8c) denotes the maximum power constraint of each cellular user and (8d) denotes the aggregate power constraint of each D2D pair. (8e) and (8f) represent that one D2D pair can only be allocated one uplink subcarrier, and one uplink subcarrier can only be allocated to one D2D pair.

Note that the optimization problem P1 is a mixed integer non-linear programming (MINLP), which contains both continuous and binary variables [20]. Although the optimal solution can be derived by joint channel and power allocation, the computational complexity is rather high. In order to reduce the computational complexity, we propose an efficient algorithm, named OPtimal Power Allocation and OPtimal Channel Assignment with Perfect CSI (OPPA-OPCA-P), by decomposing the problem into two subproblems: power allocation and channel assignment. In power allocation, optimal power on each node is derived under every channel assignment choice. In channel assignment, the channel assignment matrix is optimized, with optimal power allocation on each node. By decomposition, power allocation only involves continuous variables and channel assignment only involves binary variables.

B. POWER ALLOCATION

In this section, we investigate the power allocation problem for each possible pair $(CU_j, D2D_i)$. Assuming D2D pair $D2D_i$ shares channel CH_j with CU_j, the power allocation problem on CH_j can be formulated as follows:

$$P2: \quad \max_{\boldsymbol{P}_{ij}} f_{ij}(\boldsymbol{P}_{ij}) \tag{9a}$$

$$s.t. \text{ Constraints (8b) (8c) (8d).} \tag{9b}$$

To tackle the optimization problem P2, we first derive the feasible region of vector $\boldsymbol{P}_{ij}$ from cellular users' minimum rate constraint (8b) and power constraints (8c) (8d). Considering the QoS constraint (8b) and the expression of cellular link's capacity in equation (4), the minimum rate constraint of cellular user CU_j (8b) can be transformed as

$$\log_2\Big(1 + \frac{P_j^C \left|h_j^{CB}\right|^2}{P_{i,1}^D \left|h_{ji,1}^{BD}\right|^2 + P_{i,2}^D \left|h_{ji,2}^{BD}\right|^2 + N_0}\Big) \geq R_j^C. \tag{10}$$

Then we can derive the lower bound of transmit power of cellular user as follow:

$$P_j^C \geq A_j(P_{i,1}^D \left|h_{ji,1}^{BD}\right|^2 + P_{i,2}^D \left|h_{ji,2}^{BD}\right|^2 + N_0), \tag{11}$$

where $A_j = (2^{R_j^C} - 1)/\left|h_j^{CB}\right|^2$. The feasible region of transmit power of cellular user CU_j and D2D pair $D2D_i$ can be expressed as

$$\begin{aligned} \mathrm{F} = \{(P_j^C, P_{i,1}^D, P_{i,2}^D) | P_{i,1}^D + P_{i,2}^D \leq P_{\max}^D, \\ A_j(P_{i,1}^D \left|h_{ji,1}^{BD}\right|^2 + P_{i,2}^D \left|h_{ji,2}^{BD}\right|^2 + N_0) \leq P_j^C \leq P_{\max}^C\}. \end{aligned} \tag{12}$$

From (12), we can observe that the feasible region of transmit power is a polygon. According to [7], which focuses on power control for half-duplex D2D underlay cellular networks, the feasible region of transmit power can be derived as a polygon and the optimal transmit power resides on one of the corner points of the feasible region. However, this conclusion no long holds for full-duplex D2D underlay cellular networks due to the aggregate power constraint for D2D pairs (8d).

Due to the non-convexity of the objective function of problem P2, we cannot solve it directly. Fortunately, we can convert it into a Difference of Convex (DC) programming. First we rewrite problem P2 as follows

$$P3: \quad \max_{\boldsymbol{P}_{ij}} u_{ij}(\boldsymbol{P}_{ij}) - v_{ij}(\boldsymbol{P}_{ij}) \tag{13a}$$
$$s.t. \text{ Constraints (11) (8c) (8d),} \tag{13b}$$

where

$$\begin{aligned}
&u_{ij}(\boldsymbol{P}_{ij})\\
&= a_j \log_2(P_j^C \left|h_j^{CB}\right|^2 + P_{i,1}^D \left|h_{ji,1}^{BD}\right|^2 + P_{i,2}^D \left|h_{ji,2}^{BD}\right|^2 + N_0)\\
&\quad + b_i[\log_2(P_{i,2}^D \left|h_i^D\right|^2 + P_j^C \left|h_{ji,1}^{CD}\right|^2 + P_{i,1}^D \left|\beta h_{i,1}^D\right|^2 + N_0)\\
&\quad + \log_2(P_{i,1}^D \left|h_i^D\right|^2 + P_j^C \left|h_{ji,2}^{CD}\right|^2 + P_{i,2}^D \left|\beta h_{i,2}^D\right|^2 + N_0)]\\
&v_{ij}(\boldsymbol{P}_{ij})\\
&= a_j \log_2(P_{i,1}^D \left|h_{ji,1}^{BD}\right|^2 + P_{i,2}^D \left|h_{ji,2}^{BD}\right|^2 + N_0)\\
&\quad + b_i[\log_2(P_j^C \left|h_{ji,1}^{CD}\right|^2 + P_{i,1}^D \left|\beta h_{i,1}^D\right|^2 + N_0)\\
&\quad + \log_2(P_j^C \left|h_{ji,2}^{CD}\right|^2 + P_{i,2}^D \left|\beta h_{i,2}^D\right|^2 + N_0)].
\end{aligned}$$

We can observe that optimization problem P3 is equivalent to P2. Since both $u_{ij}(\boldsymbol{P}_{ij})$ and $v_{ij}(\boldsymbol{P}_{ij})$ are convex, the objective function of P3 is DC function. Moreover, the constraints (11) (8c) and (8d) are all linear constraints. Thus we can solve problem P3 based on DC programming by convexifying the non-convex objective function [21]. By approximating the discounted term by its first order Taylor series, the DC function can be converted into a convex function.

According to [21], $v_{ij}(\boldsymbol{P}_{ij})$ is approximated with its first order approximation as $v_{ij}(\boldsymbol{P}_{ij}^{(t)}) + < v_{ij}(\boldsymbol{P}_{ij}^{(t)}), (\boldsymbol{P}_{ij} - \boldsymbol{P}_{ij}^{(t)}) >$ at point $\boldsymbol{P}_{ij}^{(t)}$, where <X, Y> denotes the standard inner product on $R^{3\times 1}$. Then optimization problem P3 can be transformed as follows:

$$P4: \quad \max_{P_{ij}} u_{ij}(\boldsymbol{P}_{ij}) - v_{ij}(\boldsymbol{P}_{ij}^{(t)}) - < \nabla v_{ij}(\boldsymbol{P}_{ij}^{(t)}), (\boldsymbol{P}_{ij} - \boldsymbol{P}_{ij}^{(t)}) > \tag{14a}$$
$$s.t. \text{ Constraints (11) (8c) (8d).} \tag{14b}$$

The objective function of optimization problem P4 is convex and the constraints are linear constraints. Thus P4 is a convex optimization problem and can be solved by interior pointer method.

From analysis above, we propose an iterative algorithm for the optimization problem P3, referred to as OPtimal Power Allocation with Perfect CSI (OPPA-P). The details of OPPA-P are summarized in Algorithm 1.

In OPPA-P, a feasible point $P_{ij}^{(0)}$ is selected at the first iteration and $P_{ij}^{(t+1)}$ at t-th iteration is generated as the optimal solution of problem P4. Because $v_{ij}(\boldsymbol{P}_{ij})$ is a concave function, we have

$$v_{ij}(\boldsymbol{P}_{ij}) \leq v_{ij}(\boldsymbol{P}_{ij}^{(t)}) - < \nabla v_{ij}(\boldsymbol{P}_{ij}^{(t)}), (\boldsymbol{P}_{ij} - \boldsymbol{P}_{ij}^{(t)}) > . \tag{15}$$

Algorithm 1 OPPA-P

1: Initialization: $\boldsymbol{P}_{ij}^{(0)}$ is a feasible solution to problem P3, $t = 0$, $\varepsilon > 0$;
2: Repeat:
3: Construct optimization problem P4 with point $\boldsymbol{P}_{ij}^{(t)}$;
4: Solve convex optimization problem P4 using interior point method and denote the solution point as P_{ij}^{*};
5: Update t: $t = t + 1$
6: Update $\boldsymbol{P}_{ij}^{(t)}$: $\boldsymbol{P}_{ij}^{(t)} = \boldsymbol{P}_{ij}^{*}$;
7: until $\left\|\boldsymbol{P}_{ij}^{(t)} - \boldsymbol{P}_{ij}^{(t-1)}\right\| < \varepsilon$
8: return $\boldsymbol{P}_{ij,opt} = \boldsymbol{P}_{ij}^{(t)}$.

The solution of problem P3 after each iteration follows

$$\begin{aligned}
&u_{ij}(\boldsymbol{P}_{ij}^{(t+1)}) - v_{ij}(\boldsymbol{P}_{ij}^{(t+1)})\\
&\geq u_{ij}(\boldsymbol{P}_{ij}^{(t+1)}) - v_{ij}(\boldsymbol{P}_{ij}^{(t)}) - < \nabla v_{ij}(\boldsymbol{P}_{ij}^{(t)}), (\boldsymbol{P}_{ij}^{(t+1)} - \boldsymbol{P}_{ij}^{(t)}) >\\
&\geq u_{ij}(\boldsymbol{P}_{ij}^{(t)}) - v_{ij}(\boldsymbol{P}_{ij}^{(t)}).
\end{aligned} \tag{16}$$

Therefore, the objective function of optimization problem P3 is either increased or unchanged after each iteration. Considering the objective function of P3 has an upper bound due to the power limitation, we can conclude that the Algorithm 1 will converge after finite iterations.

C. KUHN-MUNKRES BASED CHANNEL ASSIGNMENT

In this section, we focus on the channel assignment with optimized power allocation. Denote $f_{ij,opt}$ as the optimal solution of power allocation problem in III.B.

$$f_{ij,opt} = f_{ij}(\boldsymbol{P}_{ij})|_{\boldsymbol{P}_{ij}=\boldsymbol{P}_{ij,opt}},$$

where $\boldsymbol{P}_{ij,opt}$ is solution of the problem P2.

After power allocation, the channel assignment problem can be formulated as follows:

$$P5: \quad \max_{X} \sum_{i=1,\ldots,M} \sum_{j=1,\ldots,N} x_{ij} f_{ij,opt} \tag{17a}$$
$$s.t. \text{ Constraints (8e) (8f) (8g).} \tag{17b}$$

The channel assignment problem P5 only includes binary variables x_{ij}. According to the definition of utilization matrix F and channel assignment constraints (8e) and (8f), channel assignment problem is equivalent to finding N elements with maximum sum from utilization matrix **F**, which belong to different rows and different columns. The optimization problem turns out to be maximum weighted matching problem of bipartite graph. Inspired by Kuhn-Munkres algorithm [15], [22], we propose OPtimal Channel Assignment algorithm (OPCA) to solve the channel assignment problem (17). The details of OPCA are presented in Algorithm 2.

Since the computational complexity of Kuhn-Munkres is $O(M^3)$, the computational complexity of OPCA is also $O(M^3)$.

Considering the solution of power allocation and channel assignment, we can obtain OPPA-OPCA-P algorithm for full-duplex D2D resource allocation problem with perfect CSI

Algorithm 2 OPCA

1: Initialization: expand utilization matrix $\mathbf{F}$ by adding $M-N$ columns with 0 and construct a square matrix $\underline{\mathbf{F}} = [\underline{f}_{ij}]$.
2: $s_i = \max \underline{f}_{ij}$, $i = 1, 2, \ldots, M$; $t_j = 0, j = 1, 2, \ldots, M$.
3: Calculate the excess matrix $\mathbf{C}$ as follows: $c_{ij} = s_i + t_j - \underline{f}_{ij}$.
4: Denote Z_{line} and Z_{column} as number of lines containing 0 and the number of columns containing 0, $E = \min(Z_{line}, Z_{column})$.
5: if E equals to M
6: Go to step 14.
7: else
8: Go to step 10.
9: end if
10: R $= \{D2D_i | c_{ij} = 0\}$, G $= \{CU_j | c_{ij} = 0\}$.
11: $\delta = \min\{c_{ij} | D2D_i \in \mathrm{D} \backslash \mathrm{R}, CU_j \in \mathrm{G}\}$.
12: Update s_i, t_j: $s_i = s_i - \delta$, $D2D_i \in \mathrm{D} \backslash \mathrm{R}$; $t_j = t_j + \delta$, $CU_j \in \mathrm{G}$.
13: Go to step 3.
14: $x_{ij} = \begin{cases} 1, & if\ c_{ij} = 0 \\ 0, & others. \end{cases}$

(optimization problem P1), as listed in Algorithm 3. In OPPA-OPCA-P, OPPA-P and OPCA are implemented sequentially. Then, after channel assignment, we further adjust the transmit power of cellular users. Specifically, the cellular users whose spectrum resources are not shared with any D2D pairs will transmit signals with their maximum power, because there are no co-channel interferences in this case.

Algorithm 3 OPPA-OPCA-P

1: C: The set of cellular users.
2: D: The set of D2D pairs.
3: Step1: Optimal power allocation.
4: for all $i \in$ D and $j \in$ C
5 Solve power allocation problem P2 using OPPA-P in Algorithm1 and obtain solution $\boldsymbol{P}_{ij,opt}$.
6: end for
7: Calculate $f_{ij,opt} = f_{ij}(\boldsymbol{P}_{ij,opt})$.
8: Step 2: Optimal channel assignment, input: $[f_{ij,opt}]$.
9: Obtain optimal $\mathbf{X}$ using OPCA in Algorithm 2.
10: for $j \in$ C and $\sum_{i=1}^{N} x_{ij} = 0$
11: $P_j^C = P_{\max}^C$.
12: end for

IV. RESOURCE ALLOCATION WITH STATISTICAL CSI

In the resource allocation problem with perfect CSI addressed in section III, all channel gain information between any two nodes is required. We note that, in a practical cellular system, channel gains between cellular users/D2D users and base station (h_j^{CB}, $h_{ji,1}^{BD}$ and $h_{ji,1}^{BD}$) can be obtained by base station via sending pilot signal before information transmission. However, channel gain between any two users (h_i^D, $h_{ji,1}^{CD}$, $h_{ji,2}^{CD}$) need to be measured by cellular user or D2D user and feedback to base station, which involves numerous computational complexity and then feedback overhead. Thus in this section we further consider resource allocation for full-duplex D2D communication with statistical CSI to facilitate practical application.

A. PROBLEM FORMULATION

Assume Rayleigh fading channel is adopted for all users and the channel power gains follow an exponential distribution. Denote Exp(λ) as an exponential distribution with the mean λ. The probability density function (p.d.f.) of the channel power gain x is

$$f(x) = \frac{1}{\lambda} e^{-x/\lambda} u(x), \tag{18}$$

where $u(x)$ is the unit step function.

Assume that the base station only has statistical information of all channels, $|h_j^{CB}|^2 \sim \mathrm{Exp}(\gamma_j^{CB})$, $|h_{ji,1}^{BD}|^2 \sim \mathrm{Exp}(\gamma_{ji,1}^{BD})$, $|h_{ji,2}^{BD}|^2 \sim \mathrm{Exp}(\gamma_{ji,2}^{BD})$, $|h_i^D|^2 \sim \mathrm{Exp}(\gamma_i^D)$, $|h_{ji,1}^{CD}|^2 \sim \mathrm{Exp}(\gamma_{ji,1}^{CD})$, $|h_{ji,2}^{CD}|^2 \sim \mathrm{Exp}(\gamma_{ji,2}^{CD})$. In this case, the instantaneous capacity of cellular links and D2D links cannot be obtained according to (3) and (5). Thus the QoS requirements of cellular users are guaranteed by outage probability constraints. Then we formulate the resource allocation problem with statistical CSI as maximizing ergodic weighted sum-rate of cellular users and D2D users, under cellular users' maximum outage constraints.

$$P6: \quad \max_{\mathbf{X},\mathbf{P}} E[U(\mathbf{X}, \mathbf{P})] \tag{19a}$$

$$s.t.\ \Pr\{r_j^C \le R_j^C\} \le P_j^{out} \quad j = 1, \ldots, M \tag{19b}$$

$$\text{Constraints (8c) (8d) (8e) (8f) (8g),} \tag{19c}$$

where E[$\bullet$] denotes the expectation of a random variable. The constraint (19b) represents outage constraints of each cellular link, the constraints (8c) (8d) (8e) (8f) and (8g) are the same with those in the problem P1. To solve the optimization problem P6, we analyze the outage probability and ergodic rate of cellular user and D2D user firstly.

B. OUTAGE ANALYSIS

In this subsection, we derive the expression of outage probability of cellular link in full-duplex D2D underlay communication. From the outage probability expression (19b) and equation (4), the outage probability can be formulated as

$$\Pr\{r_j^C \le R_j^C\} = \Pr\{\frac{P_j^C \left|h_j^{CB}\right|^2}{P_{i,1}^D \left|h_{ji,1}^{BD}\right|^2 + P_{i,2}^D \left|h_{ji,2}^{BD}\right|^2 + N_0} \le \Gamma_j^C\}, \tag{20}$$

where $\Gamma_j^C = 2^{R_j^C} - 1$.

Denote $X = P_j^C |h_j^{CB}|^2 / N_0$, $Y = P_{i,1}^D |h_{ji,1}^{BD}|^2 / N_0$ and $Z = P_{i,2}^D |h_{ji,2}^{BD}|^2 / N_0$. It is obvious that X, Y, Z also follow exponential distribution as follows $X \sim \text{Exp}(P_j^C \gamma_j^{CB}/N0)$, $Y \sim \text{Exp}(P_{i,1}^D \gamma_{ji,1}^{BD}/N_0)$, $Z \sim \text{Exp}(P_{i,2}^D \gamma_{ji,2}^{BD}/N_0)$. The expression of outage probability in equation (20) can be transferred as

$$\Pr\{r_j^C \le R_j^C\} = \Pr\{\frac{X}{Y+Z+1} \le \Gamma_j^C\}. \tag{21}$$

To analyze the outage probability of cellular user in (21), we first introduce Lemma 1.

Lemma 1: For random variables $X \sim \text{Exp}(\alpha_1)$, $Y \sim \text{Exp}(\alpha_2)$, $Z \sim \text{Exp}(\alpha_3)$, it holds that

$$\Pr\{\frac{X}{Y+Z+1} \le \tau\} = 1 - (\frac{\alpha_1}{\alpha_1 + \tau\alpha_2} \times \frac{\alpha_1}{\alpha_1 + \tau\alpha_3})e^{-\frac{\tau}{\alpha_1}}.$$

Proof: See Appendix A.

According to Lemma 1, we can derive the closed-form expression of cellular users' outage probability as follows:

$$\Pr\{r_j^C \le R_j^C\} = 1 - (\frac{P_j^C \gamma_j^{CB}}{P_j^C \gamma_j^{CB} + \Gamma_j^C P_{i,2}^D \gamma_{ji,2}^{BD}} \times \frac{P_j^C \gamma_j^{CB}}{P_j^C \gamma_j^{CB} + \Gamma_j^C P_{i,1}^D \gamma_{ji,1}^{BD}}) \exp(-\frac{\Gamma_j^C N_0}{P_j^C \gamma_j^{CB}}) \tag{22}$$

From equation (22), outage probability is an increasing function of ΓC^j, $P_{i,1}^D$ and $P_{i,2}^D$. In other words, more outages will occur if D2D user transmits signals with larger power or cellular user requires a higher transmit rate. After we obtain the expression of outage probability (22), the outage constraint (19b) can be rewritten as follows:

$$1 - (\frac{P_j^C \gamma_j^{CB}}{P_j^C \gamma_j^{CB} + \Gamma_j^C P_{i,2}^D \gamma_{ji,2}^{BD}} \times \frac{P_j^C \gamma_j^{CB}}{P_j^C \gamma_j^{CB} + \Gamma_j^C P_{i,1}^D \gamma_{ji,1}^{BD}}) \times \exp(-\frac{\Gamma_j^C N_0}{P_j^C \gamma_j^{CB}}) \le P_j^{out}. \tag{23}$$

Considering the outage constraint of cellular users (23) and power constraint (19c), we can derive the feasible region of transmit power F as

$$\begin{aligned} \text{F} = \{(P_j^C, P_{i,1}^D, P_{i,2}^D) | 1 - (&\frac{P_j^C \gamma_j^{CB}}{P_j^C \gamma_j^{CB} + \Gamma_j^C P_{i,2}^D \gamma_{ji,2}^{BD}} \\ &\times \frac{P_j^C \gamma_j^{CB}}{P_j^C \gamma_j^{CB} + \Gamma_j^C P_{i,1}^D \gamma_{ji,1}^{BD}}) \exp(-\frac{\Gamma_j^C N_0}{P_j^C \gamma_j^{CB}}) \\ &\le P_j^{out}, P_j^C \le P_{\max}^C, P_{i,1}^D + P_{i,2}^D \le P_{\max}^D\} \end{aligned} \tag{24}$$

By transforming equation (24), we can obtain another form of F as follows:

$$\begin{aligned} \text{F} = \{(P_j^C, P_{i,1}^D, P_{i,2}^D) | (1 + A_{i,2}^D P_{i,2}^D)(1 + B_{i,1}^D P_{i,1}^D) \\ \le C_j, P_j^C \le P_{\max}^C, P_{i,1}^D + P_{i,2}^D \le P_{\max}^D\} \end{aligned} \tag{25}$$

where $A_i^D = \Gamma_j^C \gamma_{ji,2}^{BD}/(P_j^C \gamma_j^{CB})$, $B_i^D = \Gamma_j^C \gamma_{ji,1}^{BD}/(P_j^C \gamma_j^{CB})$ and $C_j = \frac{1}{(1-P_j^{out})} \exp(-\frac{\Gamma_j^C N_0}{P_j^C \gamma_j^{CB}})$. Obviously, the feasible power region F is 3-dimensional. To illustrate the feasible region clearly, we consider the feasible region of $P_{i,1}^D$, $P_{i,2}^D$ with an arbitrary P_j^C, which is a 2-dimensional domain. The feasible region of $P_{i,1}^D$ and $P_{i,2}^D$ can be constrained by two bounds: $P_{i,1}^D + P_{i,2}^D \le P_{max}^D$ and $(1 + A_i^D P_{i,2}^D)(1 + B_i^D P_{i,1}^D) \le C_j$. Considering different relationship of two bounds, there are four cases about the feasible region of $P_{i,1}^D$ and $P_{i,2}^D$, as illustrated in Fig. 2.

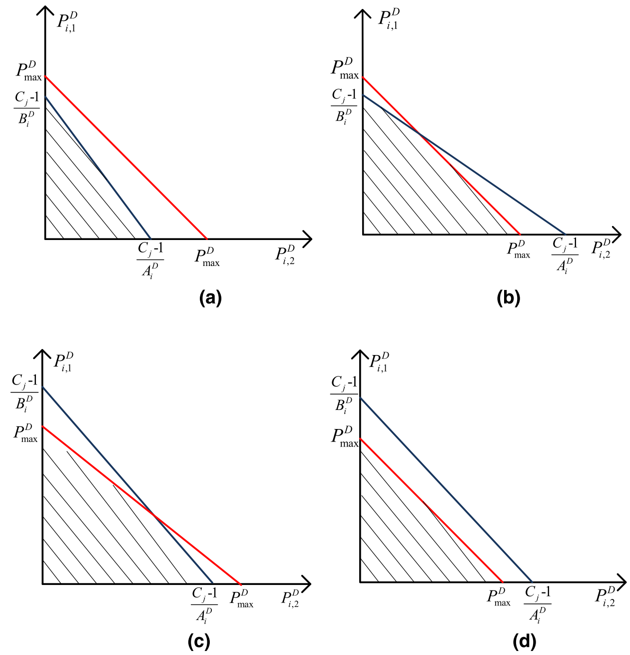

FIGURE 2. Feasible region of $P_{i,1}^D$ and $P_{i,2}^D$. (a) case I (b) case II. (c) case III (d) case IV.

The red line in Fig. 2 refers to the equation $P_{i,1}^D + P_{i,2}^D = P_{max}^D$, and the blue line corresponds to the equation $(1 + A_i^D P_{i,2}^D)(1+B_i^D P_{i,1}^D) = C_j$. we refer to the red line and the blue one as total power constraint line and outage constraint line respectively. The shadow part represents the feasible region of D2D users' transmit power $P_{i,1}^D$, $P_{i,2}^D$. Then we analyze the impact of several parameters on the feasible region.

Impact of P_j^{out} on F: If P_j^{out} increases, which means cellular user CU_j can tolerate worse outage performance, C_j will increase, causing the outage constraint line in Fig. 2 upward. We will get a larger feasible region. However, if P_j^{out} is larger than certain threshold, the outage constraint line lies upon the total power constraint line, as illustrated in Fig. 2 (d). In this case, the feasible domain is dependent on the total power constraint line and remains unchangeable with the increase of P_j^{out}.

Impact of R_j^C on F: If R_j^C increases, which means the cellular user CU_j must be guaranteed a better performance, Γ_j^C will increase at the same time, causing a reduction in C_j, increase in A_i^D and B_i^D. In this case, the outage constraint line in Fig. 2 will move downward, and the feasible region will get smaller.

Impact of P_j^C on F: If P_j^C increases, which means cellular user transmits signals to the base station with a larger power,

C_j will increase, causing reduction in A_i^D and B_i^D. In this case, the outage constraint line in Fig. 2 will move upward, and the feasible region will get larger.

Impact of γ_j^{CB} on F: If γ_j^{CB} increases, which means the cellular link has a better channel, the result is the same with increase of P_j^C. C_j will increase, causing reduction in A_i^D and B_i^D. The outage constraint line in Fig. 2 will move upward, and the feasible region will get larger.

Impact of $\gamma_{ji,1}^{BD}$ on F: If $\gamma_{ji,1}^{BD}$ increases, which means the D2D user D_i^1 performs worse on self-interference cancellation, $B_{i,1}^D$ will increase. The intersection of the outage constraint line and y-axis will move downward, which means D2D user D_i^1 needs to transmit signal with less power.

Impact of $\gamma_{ji,2}^{BD}$ on F: If $\gamma_{ji,2}^{BD}$ increases, A_i^D will increase. The intersection of the outage constraint line and x-axis will move downward, which means D2D user D_i^2 needs to transmit signal with less power.

C. ERGODIC WEIGHTED SUM-RATE

In this subsection, we derive the expression of ergodic weighted sum-rate of cellular link and D2D link. We consider the following lemma of exponential distributed random variables firstly.

Lemma 2: For random variables $X \sim \mathrm{Exp}(\alpha_1)$, $Y \sim \mathrm{Exp}(\alpha_2)$ and $Z \sim \mathrm{Exp}(\alpha_3)$, it holds that

$$E[\ln(1+\frac{X}{1+Y+Z})] = \alpha_1 F_c(\alpha_1, \alpha_2, \alpha_3),$$

where

$$F_c(x,y,z) = \frac{x\phi(x)}{(x-y)(x-z)} + \frac{y\phi(y)}{(y-x)(y-z)} + \frac{z\phi(z)}{(z-y)(z-x)},$$
$$\phi(x) = e^{1/x}E_1(1/x)$$

and

$$E_1(x) = \int_x^{+\infty} \frac{1}{t} e^{-t} dt.$$

Proof: see Appendix B.

From equations (3) and (5), we can observe that the achievable rate of cellular user and D2D pair (r_j^C and r_i^D) have the same form as $\ln(1+\frac{X}{1+Y+Z})$. Considering the distribution of channel gains and equations (3) (5), the closed-form expressions of ergodic capacity of cellular user CU_j and ergodic sum-rate of D2D pair $D2D_i$ can be derived as

$$E[r_j^C] = P_j^C \gamma_j^{CB} F_c(P_j^C \gamma_j^{CB}, P_{i,1}^D \gamma_{ji,1}^{BD}, P_i^R \gamma_{ji,2}^{BD}), \quad (26)$$

$$E[r_i^D] = P_{i,1}^D \gamma_i^D F_c(P_{i,1}^D \gamma_i^D, P_j^C \gamma_{ji,2}^{CD}, P_{i,2}^D \gamma_{i,2}^D) + P_{i,2}^D \gamma_i^D F_c(P_{i,2}^D \gamma_i^D, P_j^C \gamma_{ji,1}^{CD}, P_{i,1}^D \gamma_{i,1}^D). \quad (27)$$

Substituting equations (26) and (27) into (6), we can obtain the expectation of utilization function f_{ij} as follows:

$$\begin{aligned} E[f_{ij}] = {} & a_j P_j^C \gamma_j^{CB} F_c(P_j^C \gamma_j^{CB}, P_{i,1}^D \gamma_{ji,1}^{BD}, P_i^R \gamma_{ji,2}^{BD}) \\ & + b_i[P_{i,1}^D \gamma_i^D F_c(P_{i,1}^D \gamma_i^D, P_j^C \gamma_{ji,2}^{CD}, P_{i,2}^D \gamma_{i,2}^D) \\ & + P_{i,2}^D \gamma_i^D F_c(P_{i,2}^D \gamma_i^D, P_j^C \gamma_{ji,1}^{CD}, P_{i,1}^D \gamma_{i,1}^D)]. \end{aligned} \quad (28)$$

D. OPPA-OPCA WITH STATISTICAL CSI

After the analysis on outage probability and weighted ergodic sum-rate, we consider the solution to the resource allocation problem P6. Since P6 includes continuous and binary variables, we decouple the resource allocation problem into power allocation and channel assignment to reduce the number of variables.

During the power allocation step, we aim to derive optimal power allocation for each (CU_j, $D2D_i$) pair. We consider the weighted sum-rate maximization problem of $D2D_i$ and CU_j as follows:

$$P7: \quad \max_{\boldsymbol{P}_{ij}} E[f_{ij}(\boldsymbol{P}_{\mathbf{ij}})] \quad (29a)$$

$$s.t.\ (P_j^C, P_{i,1}^D, P_{i,2}^D) \in \mathrm{F}. \quad (29b)$$

Since the objective function in optimization function P7 is in a semi-closed form as shown in equation (28), the optimization problem can be solved by global searching. Note that $\rho P_1 X/(1+\rho P_2 Y + \rho P_3 Z) > P_1 X/(1+P_2 Y+P_3 Z)$, when $\rho > 1$. We can hold that the optimal point should appear at the boundary of feasible power region, which means at least one of maximization constraints $P_j^C \leq P_{max}^C$, $P_{i,1}^D + P_{i,2}^D \leq P_{max}^D$ should reach its boundary. So a 2-dimensional global searching can be used to solve power allocation problem P7, instead of 3-dimensional global searching. We refer to this global searching based power allocation as OPtimal Power Allocation with Statistical CSI (OPPA-S).

After power allocation, the channel assignment problem is the same with that in perfect CSI scenario, as shown in section III.C. Thus we solve channel assignment problem using OPCA algorithm proposed in Algorithm 2, which is based on Kuhn-Munkres algorithm.

Hence, to solve the optimization problem P6, we propose the following OPPA-OPCA-S algorithms, as described in Algorithm 4. In OPPA-OPCA-S, OPPA-S and OPCA are implemented sequentially. Similar to the OPPA-OPCA-P algorithm in section III.C, for the cellular users whose spectrum resources are not shared with any D2D pairs, they will transmit using their maximum power.

V. SIMULATION RESULTS

In this section, we present the simulation results of our proposed resource allocation algorithms for full-duplex D2D communication, OPPA-OPCA-P and OPPA-OPCA-S. The performance of OPPA-OPCA-P and OPPA-OPCA-S is compared with three benchmarks: Exhaustive Searching (ES), EQual Power Allocation and OPtimal Channel Assignment (EQPA-OPCA), EQual Power Allocation and RAndom Channel Assignment (EQPA-RACA).

The upper bound of the system utilization can be achieved by ES, in which the power and channel is joint optimized using global searching. In EQPA-OPCA, channels are assigned based on Kuhn Munkres algorithm [8]. Cellular users transmit signals with maximum power and two full-duplex D2D users in one D2D pair transmit signals with

Algorithm 4 OPPA-OPCA-S

C: The set of cellular users.
D: The set of D2D pairs.
Step1: Optimal Power Allocation.
for all $i \in$ D and $j \in$ C
    Calculate the feasible region F with outage probability constraint.
    Solve power allocation problem P7 by global searching and obtain $f_{ij,opt}$.
end for
Step 2: Optimal Channel assignment, input: $[f_{ij,opt}]$.
Get optimal $\mathbf{X}$ according to OPCA in Algorithm 2.
for $j \in$ C and $\sum_{i=1}^{N} x_{ij} = 0$
    $P_j^C = P_{\max}^C$.
end for

TABLE 1. Simulation parameters.

Parameters	Value
Pathloss factor	4
Cellular diameter	100m
The largest distance between D2D users	25m
The largest transmit power of D2D user	20dBm
The largest transmit power of cellular user	25dBm
Power density of noise	-94dBm

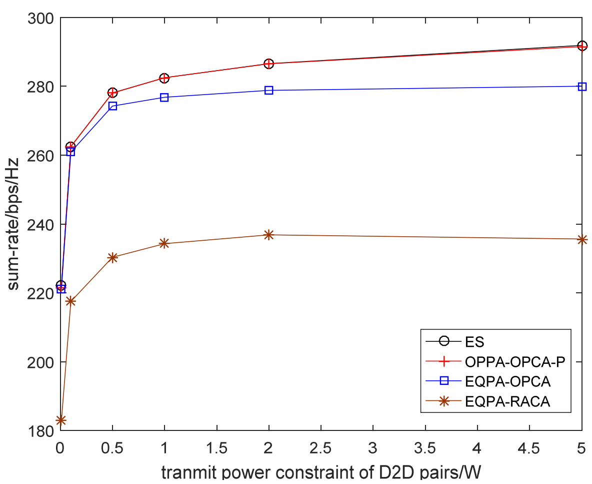

FIGURE 3. Sum-rate vs transmit power constraint of D2D pairs.

equal power while guaranteeing cellular users' transmission rate constraints. In EQPA-RACA, the channels are assigned randomly and the power allocation algorithm is the same with EQPA-OPCA. The parameters in simulation are presented in Table 1.

To indicate that our proposed algorithm OPPA-OPCA-P has better performance than two benchmark algorithms, Fig. 3 plots the weighted sum-rate versus total transmit power for the proposed OPPA-OPCA-A algorithms and other three benchmark algorithms, with $M = 15$, $N = 15$, $a_j = 1$ and $b_i = 1$. As shown in Fig.3, OPPA-OPCA-P has almost the same performance with ES and outperforms the other two benchmarks. Comparing with EQPA-OPCA and EQPA-RACA, the performance gain achieved by channel assignment is almost the same with the increase of power, while the performance gain achieved by power allocation becomes much higher with the increase of power.

The theoretical results and simulation results of ergodic cellular rate E$[r^C]$, ergodic D2D sum-rate E$[r^D]$ and ergodic sum-rate of both cellular user and D2D pair are illustrated in Fig. 4, with $M = 1$, $N = 1$, $a_j = 1$ and $b_i = 1$. The sum transmit power of D2D pair increase from 0 to 5 and the two D2D users in one D2D pair transmit with equal power. As illustrated in Fig. 4, the theoretical results of cellular rate and D2D rate match the simulation results very well, which validate the accuracy of the derived expressions (26) (27) and (28). With the increase of transmit power of D2D users, the cellular data rate decreases due to the increase of interference caused by D2D users, while both the D2D rate and sum-rate increase.

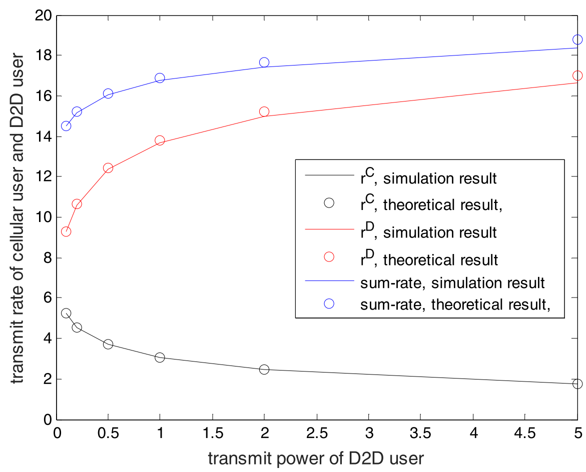

FIGURE 4. Rate of cellular user and D2D users with different transmit power of D2D users.

The theoretical and simulation results of outage probability versus total transmit power of D2D pair are illustrated in Fig. 5 with $M = 1$ and $N = 1$, considering three different values of cellular user's minimum rate R_{min}. As shown in Fig.5, the theoretical result matches the simulation result perfectly, which confirm the accuracy of the derived closed-form expression (22). The outage probability increases with the increase of D2D transmit power. This is due to the reason that when D2D users transmit signal with more power, the cellular user will get a larger interference. Also, more outages will occur if cellular user requires a higher rate constraint R_{min}, which confirms our theoretical analysis of equation (22).

To analyze the impact of cellular users' minimum rate R_{min} and outage probability P^{out} on full-duplex D2D

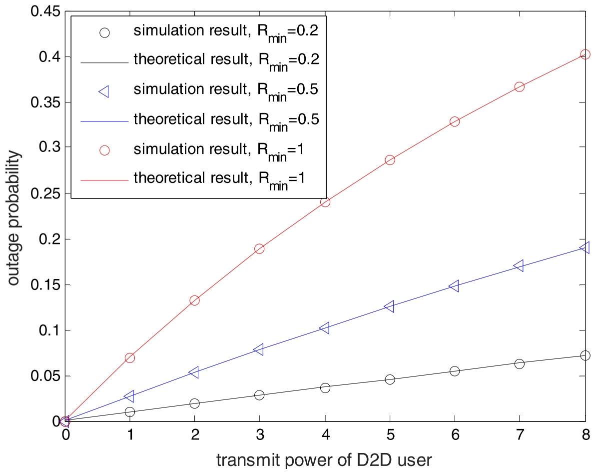

FIGURE 5. Outage probability with different transmit power of D2D user.

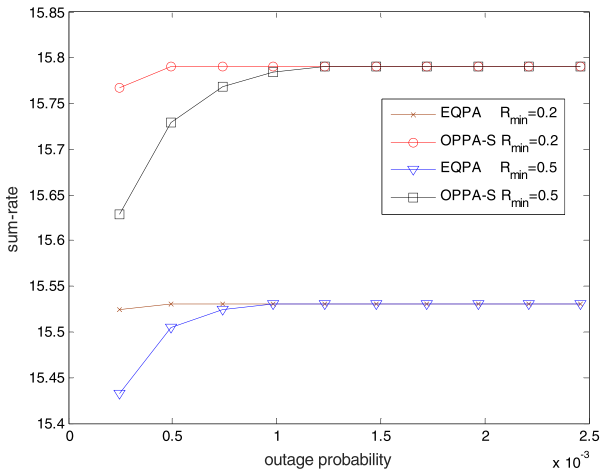

FIGURE 6. Sum-rate with different outage probability constraints (M = 1 and N = 1).

communication, Fig. 6 plots the results of ergodic sum-rate versus outage probability, for OPPA-S and EQPA algorithms considering two different values of R_{min}, with $M = 1$ and $N = 1$. The optimal power allocation (OPPA-S) is achieved by using the 2-dimensional global searching, while according to equal power allocation (EQPA), two DUs in each D2D pair transmits with equal power. As shown in Fig. 6, for each power allocation algorithm, with the increase of outage probability the sum-rate increases until a maximum sum-rate achieved. Also, the OPPA-S always outperforms EQPA algorithm. A better sum-rate performance can be achieved when cellular user requires a smaller minimum rate constraint. This is due to the fact that the system has a larger feasible domain of transmit power with a smaller R_{min}, as analyzed in IV.B. When the outage probability is larger than 1.2×10^{-3}, the sum-rate remains constant. This is due to that the outage probability constraint can always be achieved even with the largest D2D transmit power in this condition and the achievable sum-rate is dependent on total power constraint. In other words, the feasible domain of transmit power is

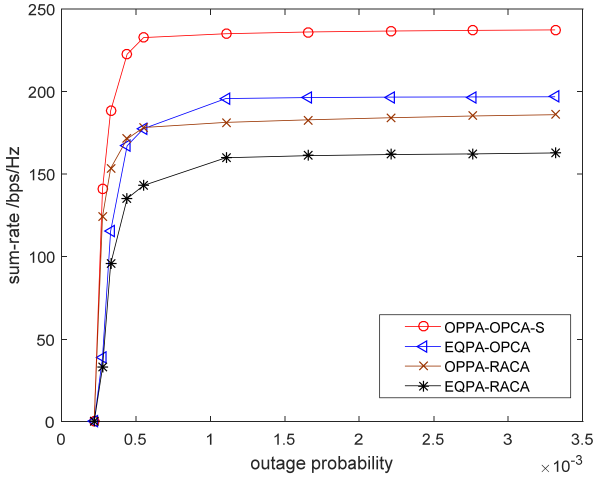

FIGURE 7. Sum-rate with different outage probability constraints (M = 15 and N = 15).

constrained by total power constraint in this case as illustrated in Fig. 2(d). This confirms our analysis of the feasible domain of transmit power Φ in section IV.B.

To indicate that the proposed OPPA-OPCA-S algorithm outperforms other state-of-art algorithms, Fig. 7 plots the ergodic sum-rate versus outage probability for OPPA-OPCA-S, EQPA-OPCA, OPPA-RACA and EQPA-RACA algorithms, with $N = 15, M = 15, R_{min} = 0.5, a_j = 1$ and $b_i = 1$. From Fig.7, the performance of random channel assignment (OPPA-RACA and EQPA-RACA) is similar to the performance in the system with one cellular user one D2D pair as shown in Fig.6. For optimal channel assignment (OPPA-OPCA-S and EQPA-OPCA), the sum-rate is almost constant when outage probability is larger than 0.5×10^{-3}. This is due to that when optimal channel assignment is performed, the D2D pair will be allocated on the CU with small interference and the CUs can always achieve outage probability constraint. Thus the sum-rate performance is only limited by the transmit power constraint of cellular users and D2D pairs as illustrated in Fig. 2(d), which again confirms the accuracy of our theoretical analysis of feasible domain of transmit power Φ in IV.B.

VI. CONCLUSION

In this paper, we investigated the resource allocation problem for full-duplex D2D communication considering both perfect CSI and statistical CSI. For perfect CSI scenario, the power allocation problem was solved by DC programming, while channel assignment problem was solved based on Kuhn-Munkre algorithm. For statistical CSI scenario, the closed-form expressions of ergodic sum-rate and outage probability were derived firstly. The power allocation and channel assignment were solved by global searching and Kuhn-Munkre algorithm respectively. The performance of the proposed algorithms OPPA-OPCA-P and OPPA-OPCA-S was verified by simulation results.

APPENDIX A
PROOF OF LEMMA 1

Proof: Considering the p.d.f. of random variable X, Y and Z, the probability can be written as

$$\Pr\{\frac{X}{Y+Z+1} \le \tau\} = \int_0^{+\infty} dz \int_0^{+\infty} dy \int_0^{+\tau(y+z+1)} f(z)f(y)f(x)dx \quad (30)$$

Since X, Y and Z follow exponential distribution, equation (30) can be transformed as

$$\Pr\{\frac{X}{Y+Z+1} \le \tau\} = \frac{1}{\alpha_1\alpha_2\alpha_3}\int_0^{+\infty} dz \int_0^{+\infty} dy \int_0^{+\tau(y+z+1)} e^{-\frac{x}{\alpha_1}-\frac{y}{\alpha_2}-\frac{z}{\alpha_3}}dx. \quad (31)$$

By calculating the integral in the right part of (31), we have

$$\Pr\{\frac{X}{Y+Z+1} \le \tau\} = 1-(\frac{\alpha_1}{\alpha_1+\tau\alpha_2}\times\frac{\alpha_1}{\alpha_1+\tau\alpha_3})e^{-\frac{\tau}{\alpha_1}}. \quad (32)$$

APPENDIX B
PROOF OF LEMMA 2

Proof: Firstly, we expand the equation of $E[\ln(1+\frac{X}{1+Y+Z})]$ as

$$E[\ln(1+\frac{X}{1+Y+Z})] = E[\ln(1+X+Y+Z)] - E[\ln(1+Y+Z)]. \quad (33)$$

According to [23], the expectation of ln(1+$Y+Z$) can be derived as

$$E[\ln(1+Y+Z)] = \frac{\alpha_2\phi(\alpha_2)-\alpha_3\phi(\alpha_3)}{\alpha_2-\alpha_3}, \quad (34)$$

where $\phi(x) = e^{1/x}E_1(1/x)$, $E_1(x) = \int_x^{+\infty}\frac{1}{t}e^{-t}dt$.

The expectation of $\ln(1+X+Y+Z)$ can be derived as follows. We expand $E[\ln(1+X+Y+Z)]$ as follows

$$\begin{aligned}
&E[\ln(1+X+Y+Z)] \\
&= \int_0^{+\infty}\frac{1}{\alpha_3}e^{-\frac{z}{\alpha_3}}dz\int_0^{+\infty}\frac{1}{\alpha_2}e^{-\frac{y}{\alpha_2}}dy \\
&\quad\times\int_0^{+\infty}\frac{1}{\alpha_1}e^{-\frac{x}{\alpha_1}}\ln(1+x+y+z)dx \\
&= \underbrace{\int_0^{+\infty}\frac{1}{\alpha_3}e^{-\frac{z}{\alpha_3}}dz\int_0^{+\infty}\frac{1}{\alpha_2}e^{-\frac{y}{\alpha_2}}\ln(1+y+z)dy}_{E(\ln(1+Y+Z))} \\
&\quad+\underbrace{\int_0^{+\infty}\frac{1}{\alpha_3}e^{-\frac{z}{\alpha_3}}dz\int_0^{+\infty}\frac{1}{\alpha_2}e^{-\frac{y}{\alpha_2}}dy\int_0^{+\infty}e^{-\frac{x}{\alpha_1}}\frac{1}{1+x+y+z}dx}_{d_1}.
\end{aligned} \quad (35)$$

The first term of equation (35) equals $E[\ln(1+Y+Z)]$ and follows equation (34). The second term can be derived as follows

$$\begin{aligned}
d_1 &= \int_0^{+\infty}\frac{1}{\alpha_3}e^{-\frac{z}{\alpha_3}}dz\int_0^{+\infty}\frac{1}{\alpha_2}e^{-\frac{y}{\alpha_2}}dy\int_0^{+\infty}e^{-\frac{x}{\alpha_1}}\frac{1}{1+x+y+z}dx \\
&= \frac{\alpha_1}{\alpha_2}e^{\frac{1}{\alpha_1}}\int_0^{+\infty}\frac{1}{\alpha_3}e^{-(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})z}e^{\frac{1}{\alpha_2}-\frac{1}{\alpha_1}}dz(\int_0^{+\infty}e^{-(\frac{\alpha_1}{\alpha_2}-1)t}E_1(t)dt \\
&\quad-\int_0^{\frac{1+z}{\alpha_1}}e^{-(\frac{\alpha_1}{\alpha_2}-1)t}E_1(t)dt)
\end{aligned} \quad (36)$$

Using equation (3) on Page 197 in [24], we have

$$\begin{aligned}
d_1 &= \frac{\alpha_1}{\alpha_2}e^{\frac{1}{\alpha_1}}[\frac{\alpha_2}{(\alpha_1-\alpha_2)}\frac{\alpha_2}{(\alpha_2-\alpha_3)}\ln(\frac{\alpha_1}{\alpha_2})e^{(\frac{1}{\alpha_2}-\frac{1}{\alpha_1})} \\
&\quad-\underbrace{\frac{1}{\alpha_3}\int_0^{+\infty}e^{-(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})z+(\frac{1}{\alpha_2}-\frac{1}{\alpha_1})}dz\int_0^{\frac{1+z}{\alpha_1}}e^{-(\frac{\alpha_1}{\alpha_2}-1)t}E_1(t)dt}_{d_2}],
\end{aligned} \quad (37)$$

where

$$\begin{aligned}
d_2 &= \frac{1}{\alpha_3}\int_0^{+\infty}e^{-(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})z+(\frac{1}{\alpha_2}-\frac{1}{\alpha_1})}dz\int_0^{\frac{1+z}{\alpha_1}}e^{-(\frac{\alpha_1}{\alpha_2}-1)t}E_1(t)dt \\
&= \frac{1}{\alpha_3}e^{(\frac{1}{\alpha_2}-\frac{1}{\alpha_1})}[\int_0^{\frac{1}{\alpha_1}}e^{-(\frac{\alpha_1}{\alpha_2}-1)t}E_1(t)dt\underbrace{\int_0^{+\infty}e^{-(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})z}dz}_{d_3} \\
&\quad+\int_{\frac{1}{\alpha_1}}^{+\infty}e^{-(\frac{\alpha_1}{\alpha_2}-1)t}E_1(t)dt\underbrace{\int_{\alpha_1 t-1}^{+\infty}e^{-(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})z}dz}_{d_4}].
\end{aligned} \quad (38)$$

d_3 and d_4 in (38) can be obtained as follows

$$d_3 = \int_0^{+\infty}e^{-(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})z}dz = \frac{1}{\frac{1}{\alpha_3}-\frac{1}{\alpha_2}} \quad (39)$$

$$d_4 = \int_{\alpha_1 t-1}^{+\infty}e^{-(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})z}dz = \frac{1}{\frac{1}{\alpha_3}-\frac{1}{\alpha_2}}e^{-(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})(\alpha_1 t-1)}. \quad (40)$$

Then, substituting (39) and (40) into (38), we have

$$\begin{aligned}
d_2 &= \frac{1}{\alpha_3}e^{(\frac{1}{\alpha_2}-\frac{1}{\alpha_1})}\frac{1}{\frac{1}{\alpha_3}-\frac{1}{\alpha_2}}[\int_0^{\frac{1}{\alpha_1}}e^{-(\frac{\alpha_1}{\alpha_2}-1)t}E_1(t)dx \\
&\quad+\int_{\frac{1}{\alpha_1}}^{+\infty}E_1(t)e^{-(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})(\alpha_1 t-1)-(\frac{\alpha_1}{\alpha_2}-1)t}dt]
\end{aligned}$$

$$= \frac{\alpha_2}{\alpha_2-\alpha_3} e^{(\frac{1}{\alpha_2}-\frac{1}{\alpha_1})} [\underbrace{\int_0^{\frac{1}{\alpha_1}} e^{(1-\frac{\alpha_1}{\alpha_2})t} E_1(t)dt}_{d_5} + e^{(\frac{1}{\alpha_3}-\frac{1}{\alpha_2})} \times \underbrace{\int_{\frac{1}{\alpha_1}}^{+\infty} E_1(t) e^{(1-\frac{\alpha_1}{\alpha_3})t} dt}_{d_6}] \quad (41)$$

Define $\alpha_1/\alpha_2 = \xi_1$, $\alpha_1/\alpha_3 = \xi_2$. We have

$$d_5 = \int_0^{\frac{1}{P_1\alpha_1}} e^{(1-\xi_1)t} E_1(t)dt = \frac{\alpha_2}{\alpha_1-\alpha_2}[\ln(\frac{\alpha_1}{\alpha_2}) + e^{\frac{1}{\alpha_1}-\frac{1}{\alpha_2}} E_1(\frac{1}{\alpha_1}) + E_1(\frac{1}{\alpha_2})] \quad (42)$$

and

$$d_6 = \int_{\frac{1}{\alpha_1}}^{+\infty} e^{(1-\xi_2)t} E_1(t)dt = \frac{1}{\xi_2-1}(e^{\frac{1-\xi_2}{\alpha_1}} E_1(\frac{1}{\alpha_1}) - E_1(\frac{\xi_2}{\alpha_1})) = \frac{\alpha_3}{\alpha_1-\alpha_3}(e^{\frac{1}{\alpha_1}-\frac{1}{\alpha_3}} E_1(\frac{1}{\alpha_1}) - E_1(\frac{1}{\alpha_3})). \quad (43)$$

Then substituting (42) and (43) into (41), we have

$$d_2 = \frac{\alpha_2}{\alpha_2-\alpha_3}\{\frac{1}{\frac{\alpha_1}{\alpha_2}-1}\ln(\frac{\alpha_1}{\alpha_2}) e^{\frac{1}{\alpha_2}-\frac{1}{\alpha_1}} + (\frac{1}{\frac{\alpha_1}{\alpha_2}-1} + \frac{1}{\frac{\alpha_1}{\alpha_3}-1}) E_1(\frac{1}{\alpha_1}) + \frac{1}{\frac{\alpha_1}{\alpha_2}-1} E_1(\frac{1}{\alpha_2}) e^{\frac{1}{\alpha_2}-\frac{1}{\alpha_1}} - \frac{1}{\frac{\alpha_1}{\alpha_3}-1} e^{\frac{1}{\alpha_3}-\frac{1}{\alpha_1}} E_1(\frac{1}{\alpha_3})\}. \quad (44)$$

Substituting (44) into (36), we have

$$d_1 = \frac{\alpha_1}{(\alpha_3-\alpha_1)}\frac{\alpha_1}{(\alpha_1-\alpha_2)}\phi(\alpha_1) + \frac{\alpha_1}{\alpha_3-\alpha_2}\frac{\alpha_2}{\alpha_1-\alpha_2}\phi(\alpha_2) + \frac{\alpha_1}{\alpha_2-\alpha_3}\frac{\alpha_3}{\alpha_1-\alpha_3}\phi(\alpha_3) \quad (45)$$

Substituting (45) into (35), we have

$$E[\ln(1+X+Y+Z)] = -\frac{\alpha_1}{\alpha_1-\alpha_2}\frac{\alpha_1}{\alpha_1-\alpha_3}\phi(\alpha_1) + \frac{\alpha_2}{\alpha_2-\alpha_1}\frac{\alpha_2}{\alpha_2-\alpha_3}\phi(\alpha_2) + \frac{\alpha_3}{\alpha_3-\alpha_2}\frac{\alpha_3}{\alpha_3-\alpha_1}\phi(\alpha_3) \quad (46)$$

Then we derive the expression of $E[\ln(1+X+Y+Z)]$ and $E[\ln(1+Y+Z)]$. By substituting equation (34) and (46) into (33), we have

$$E[\ln(1+\frac{X}{1+Y+Z})] = \alpha_1 F_c(\alpha_1, \alpha_2, \alpha_3), \quad (47)$$

where

$$F_c(x, y, z) = \frac{x\phi(x)}{(x-y)(x-z)} + \frac{y\phi(y)}{(y-x)(y-z)} + \frac{z\phi(z)}{(z-y)(z-x)}.$$

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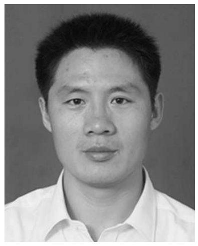

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