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On Computation Reduction of Liveness-Enforcing Supervisors

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ABSTRACT It is well recognized that the computation of an optimal liveness-enforcing supervisor (optimal supervisor for short) for a sequential resource allocation system is intractable due to the nature of the problem itself, since an integer linear programming model has to be formulated to find its solution. In this paper, a novel vector covering approach is developed to reduce the computational cost of designing maximally permissive supervisors for flexible manufacturing systems (FMSs), by reducing the number of operation places and legal markings (LMs) that need to be considered. A vector covering approach is used to find the minimal covered set of first-met bad markings (FBMs). Then, a novel vector covering approach is proposed to reduce the set of LMs to a smaller covering set. The proposed method can reduce the number of both variables and constraints in the integer linear programming problems (ILPPs). It raises the efficiency of designing optimal supervisors by solving ILPPs.

INDEX TERMS Petri net, flexible manufacturing system, deadlock prevention, optimal liveness-enforcing supervisor.

I. INTRODUCTION

Manufacturing is the source of the progress of human society. In developed or developing countries, manufacturing is usually a pillar industry. FMSs are a novel production mode, operated in an automatic way, with limited and shared resources. Every process of the system is running in a sequence that is previously established and competing with others for the limited shared resources when the FMS is working [20], [30], [34]. Deadlocks occur because of the unreasonable allocation of shared resources [19]. Deadlocks must be prevented since their occurrence often reduces the efficiency of a system and even might result in some devastating consequences [11], [23], [25], [27]. Therefore, many deadlock control policies have been put forward to prevent the occurrence of deadlocks [8], [9], [12]–[16], [20], [26].

Petri nets [16], [26], [42], [45]–[47], automata [41], and graph theory [23], [24] can solve the deadlock problems in resource allocation systems (RASs) where resource-sharing is a distinguished feature; therefore, the competition for resources by a number of concurrently executing processes becomes the main cause of blocking. In recent years, Petri nets have become a popular tool to manage deadlocks in FMSs [3], [5]–[7], [33], [35]–[37], [43], [44], as well as the pertinent control problems in discrete event systems (DESs) [17], [18]. In general, there are mainly three ways to solve the deadlocks: prevention, detection-and-recovery, and avoidance. The first is achieved by designing an off-line decision mechanism, whose goal is to prevent the reachability of the deadlock states by enforcing some constraints on a system [10]–[12], [20], [25], [29]. The role of Petri nets in an RAS is essential and obvious. Petri nets can naturally represent system static structure and dynamic behavior. Specifically, a net structure can describe the structure of a system under consideration, while its initial resource configuration, i.e., initial marking, provides the dynamics of system evolution. The distinction of places and transitions can be thought of as the convenient elements to well express a system from both static and dynamic aspects. Petri nets are extensively adopted as a tool to investigate DESs, such as resource allocation systems that are a generalization of many computer-integrated complexes including automated flexible manufacturing systems.

A deadlock control strategy is often evaluated from a number of aspects. If all LMs are included in a controlled system, the policy is said to be optimal with respect to the behavior permission. The structure of a supervisor depends on the number of control places restricting the firings of enabled events, and computational complexity means the computational cost when a supervisor is computed. The maximal permissiveness usually means the full use of the system resources. A supervisor that is structurally concise means low software and hardware overheads to implement the control policy. If the computational cost of a deadlock control method is not high, it is possibly applied to real-world systems. These years, many strategies of deadlock prevention are developed to achieve the aspects mentioned above [21], [28], [31], [32], [37]–[39].

Uzam and Zhou [8] propose a deadlock prevention method for FMSs. In their research, a reachability graph (RG) is dichotomized, i.e., one region that consists of LMs and the other that consists of illegal markings (states), which are sometimes called the live-zone (LZ) and deadlock-zone (DZ), respectively. The DZ is composed of dead and bad markings, from which there is not a feasible transition sequence to LMs, and the LZ includes all the LMs.

Deadlocks are prevented by first selecting an FBM, and then the selected FBM can be prohibited by designing a control place. Although the method presented in [8] is easy to use, it cannot guarantee the optimality of the obtained supervisor. Chen *et al.* [1] develop a recipe to establish a maximally permissive supervisor (MPS) for an FMS if it exists. In their work, an FBM is selected out at each iteration and a control place is designed to forbid the selected FBM and ensure that all of the LMs are reachable. To decrease the computational cost, a vector covering approach is presented to reduce the cardinalities of LMs and FBMs that are involved in computing. Then, the two reduced sets are considered for the design of optimal supervisors only. Chen and Li [2] develop a method that can obtain an optimal controlled FMS with the minimal number of control places. However the scale of the ILPP designed by this method is too large and cannot be solved within a short period of time in some large-size Petri nets models. Chen *et al.* [4] formulate a technique to compute an MPS that is composed of a compact structure. However, there are still too many variables and constraints in an ILPP since the number of LMs and FBMs that need to be considered is large.

This research reports a novel vector covering technique to further reduce the computational cost in [2]–[4], without affecting the existence of the MPSs only. We use the vector covering approach to work out the minimal set of LMs (SLM) and FBMs. In general, since there exist operation places that are not marked (have no token) at all markings in the minimal set of FBMs (MSF), the operation places that we need to consider can be further reduced. Thus, a vector covering approach can be used again to reduce the cardinality of the minimal SLM. In this case, the numbers of operation places and markings in the minimal SLM to be considered, which determine the number of constraints and variables in an ILPP, can be reduced in general. Thus, the proposed method can reduce the computational cost for the design of optimal supervisors.

The paper is structured as follows. Section II reviews an optimal control place design method [1]. Section III proposes a novel vector covering approach to further reduce the places to be dealt with and the minimal covering SLMs. We also introduce the applications of this method. Section IV provides some widely studied examples. Finally, Section V concludes this paper.

II. CONTROL PLACE SYNTHESIS

A. ANALYSIS OF REACHABILITY GRAPH

This paper uses the standard notations and concepts of Petri nets. We assume that a reader has the knowledge of the preliminaries of Petri nets. For details, the reader is referred to [40].

Markings in a reachability graph can be categorized into four classes: good, dangerous, bad, and deadlock markings. A reachability graph can be partitioned into a deadlock-zone and a live-zone [8]. The DZ contains deadlock and bad markings that unavoidably reach deadlocks. The LZ consists of all the good and dangerous markings, i.e., the LMs. The SLM is denoted as $\mathcal{M}_L$, from any of which the initial marking M_0 can be reached. Generally, the SLM of a net (N, M_0) can be defined as

$$\mathcal{M}_L = \{M | M \in R(N, M_0) \wedge M_0 \in R(N, M)\}. \tag{1}$$

An FBM is a marking in DZ, and it can be reached from LZ by firing a single transition only. The set of FBMs can be defined as follows:

$$\mathcal{M}_{\text{FBM}} = \{M \in DZ | \exists M' \in LZ, t \in T,\ s.t.\ M'[t\rangle M\}. \tag{2}$$

For a Petri net, if control places forbid all the FBMs of a system, i.e., the controlled system always works in the LZ and cannot enter the DZ, we say that the resulting net model is live.

B. GENERALIZED MUTUAL EXCLUSION CONSTRAINTS (GMECs) AND CONTROL PLACES

The work in [22] develops a method to enforce a GMEC via a control place whose computation involves algebraic operations only. Specifically, let $[N_p]$ be the incidence matrix of a Petri net model with n places and m transitions and $[N_c]$ be the incidence matrix with respect to control places. A control place can enforce a constraint taking the form

$$[L] \cdot \mu_p \leq b \tag{3}$$

where μ_p is the marking vector of the Petri net model, $[L]$ is an $n_c \times n$ nonnegative integer matrix, b is an $n_c \times 1$ nonnegative integer vector, and n_c represents the number of constraints. By introducing a non-negative slack variable vector μ_c, Eq. (3) can be transformed into equalities:

$$[L] \cdot \mu_p + \mu_c = b \tag{4}$$

where μ_c is an $n_c \times 1$ vector that represents the marking of the control places. $[N_c]$ can be computed as follows:

$$[N_c] = -[L] \cdot [N_p]. \tag{5}$$

The initial marking μ_{c0} of the supervisor can be calculated as follows:

$$\mu_{c_0} = b - [L] \cdot \mu_0. \tag{6}$$

where μ_0 is the initial marking of the net.

C. OPTIMAL CONTROL PLACE SYNTHESIS

Deadlocks in a system can be prohibited via enforcing a set of GMECs that govern the evolution of the system. The deadlock-free requirements can be represented by a set of GMECs. To enforce the GMECs, we need to find a set of control places, i.e., control place synthesis. If a control place does not exclude any LM, it said to be maximally permissive [1]. There are three kinds of places in a Petri net model of an FMS. In this paper, P^0, P_A, and P_R are used to represent the sets of idle, activity (operation), and resource places, respectively. The place partition strategy is natural since they can respectively represent three different kinds of components in a manufacturing system. For example, a system is usually composed of a number of processes. Each process can process one or more types of products using some resources according to the predefined processing stages on the raw parts. The availability of raw parts is modeled by process idle places; the operations performed on raw parts corresponding to the processing stages are modeled with activity places, and the manufacturing resources are modeled with resource places. In this sense, Petri nets are a natural representation of an FMS in which there are three kinds of distinctions as mentioned. To prevent an FBM, we just need to consider the operation places to construct a PI (place invariant) [8]. Let $\mathbb{N}_A$ stand for $\mathbb{N}_A = \{i | p_i \in P_A\}$. An FBM $M \in \mathcal{M}_{\text{FBM}}$ can be forbidden by the constraint below:

$$\sum_{i \in \mathbb{N}_A} l_i \cdot u_i \leq \beta. \tag{7}$$

where

$$\beta = \sum_{i \in \mathbb{N}_A} l_i \cdot M(p_i) - 1. \tag{8}$$

Eq. (7) represents the forbidding condition, implying that its satisfaction prohibits the reachability of the FBM M. Thus, a marking $M \in \mathcal{M}_{\text{FBM}}$ is prohibited due to a control place if

$$\sum_{i \in \mathbb{N}_A} l_i \cdot M(p_i) \geq \beta + 1. \tag{9}$$

To ensure the reachability of every marking $M' \in \mathcal{M}_L$, the coefficients $l_i (i \in \mathbb{N}_A)$ in Eq. (9) should satisfy the following constraint:

$$\sum_{i \in \mathbb{N}_A} l_i \cdot M'(p_i) \leq \beta, \quad \forall M' \in \mathcal{M}_L. \tag{10}$$

Eq. (10) is the reachability condition, implying that any marking that satisfies Eq. (10) can be reachable after the enforcement of the constraint. By combining Eqs. (8) and (10), the reachability conditions of LMs can be converted into:

$$\sum_{i \in \mathbb{N}_A} l_i \cdot (M'(p_i) - M(p_i)) \leq -1, \quad \forall M' \in \mathcal{M}_L. \tag{11}$$

Eq. (11) means that the prohibition of an FBM M does not forbid any LM. Then, with the method proposed in Section II-B, we can design an optimal control place.

Definition 1: Let $M, M' \in R(N, M_0)$. M A-covers M' if $\forall p \in P_A, M(p) \geq M'(p)$, which is denoted as $M \geq_A M'$.

Let M and M' be two markings in $R(N, M_0)$ with $M \geq_A M'$. If M' is forbidden by a PI, M is forbidden. If M is not forbidden by a PI, M' is neither forbidden.

Definition 2: Let $\mathcal{M}^{\star}_{\text{FBM}}$ be a subset of $\mathcal{M}_{\text{FBM}}$. $\mathcal{M}^{\star}_{\text{FBM}}$ is called a minimal covered set of FBMs if it meets the following requirements:

1): $\forall M \in \mathcal{M}_{\text{FBM}}, \exists M' \in \mathcal{M}^{\star}_{\text{FBM}}$, s.t. $M \geq_A M'$;

2): $\forall M \in \mathcal{M}^{\star}_{\text{FBM}}, \nexists M'' \in \mathcal{M}^{\star}_{\text{FBM}}$, s.t. $M \geq_A M''$ and $M \neq M''$.

Corollary 1: If all markings in $\mathcal{M}^{\star}_{\text{FBM}}$ are forbidden by PIs, all the FBMs are forbidden.

Definition 3: Let $\mathcal{M}^{\star}_L$ be a subset of $\mathcal{M}_L$. $\mathcal{M}^{\star}_L$ is called a minimal covering SLM if the following conditions hold:

1): $\forall M \in \mathcal{M}_L, \exists M' \in \mathcal{M}^{\star}_L$, s.t. $M' \geq_A M$;

2): $\forall M \in \mathcal{M}^{\star}_L, \nexists M'' \in \mathcal{M}^{\star}_L$, s.t. $M'' \geq_A M$ and $M \neq M''$.

Corollary 2: If no markings in $\mathcal{M}^{\star}_L$ are forbidden by PIs, all the LMs in $\mathcal{M}_L$ are reachable.

Corollaries 1 and 2 mean that we just need to consider $\mathcal{M}^{\star}_{\text{FBM}}$ and $\mathcal{M}^{\star}_L$ for designing MPSs. Thus, for an FBM M, Eq. (11) can be reduced as follows:

$$\sum_{i \in \mathbb{N}_A} l_i (M'(p_i) - M(p_i)) \leq -1, \quad \forall M' \in \mathcal{M}^{\star}_L. \tag{12}$$

In general, $\mathcal{M}^{\star}_{\text{FBM}}$ and $\mathcal{M}^{\star}_L$ are much smaller than $\mathcal{M}_{\text{FBM}}$ and $\mathcal{M}_L$, respectively. This method can improve the computation efficiency for designing MPSs.

The set of FBMs that are forbidden by a PI I is defined as $F_I = \{M \in \mathcal{M}^{\star}_{\text{FBM}} | \sum_{i \in \mathbb{N}_A} l_i \cdot M(p_i) \geq \beta + 1\}$.

III. A NOVEL VECTOR COVERING APPROACH

This section presents a novel vector covering approach to reduce the number of operation places and LMs involved in optimal supervisor design and introduces some applications of the proposed method.

A. THE REDUCTION OF CONSIDERING OPERATION PLACES AND LEGAL MARKINGS

By considering the operation places only, the SLM can be reduced to be a much smaller one, i.e., $\mathcal{M}_L^\star$. In the following, a vector covering approach further reducing $\mathcal{M}_L^\star$ is formulated.

Definition 4: Given $\mathcal{M}_{\text{FBM}}^\star$, for a place $p_i \in P_A$ $(i \in \mathbb{N}_A)$, $\mathcal{M}_{\text{FBM}}^\star(p_i) = 0$, if $\forall M_k \in \mathcal{M}_{\text{FBM}}^\star$, $M_k(p_i) = 0$; else $\mathcal{M}_{\text{FBM}}^\star(p_i) \neq 0$.

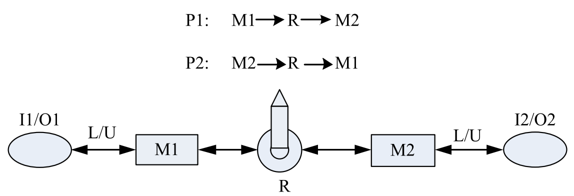

FIGURE 1. An FMS.

Considering the FMS as shown in Fig. 1, it includes a robot, two machines, two input buffers and two output buffers. The robot R can hold two parts at a time and each machine can hold only one part. The parts enter the processing sequence through the input buffers I1 and I2, and leave the sequence through the output buffers O1 and O2. There are two processing sequences P1 and P2 in this FMS, as shown below:

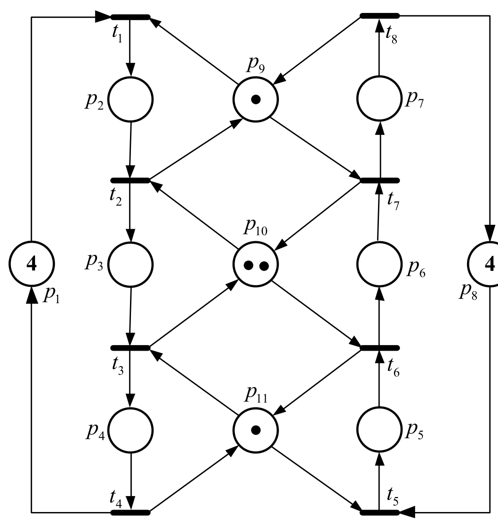

FIGURE 2. Petri net model of the FMS in Fig. 1.

Fig. 2 shows the Petri net model of Fig. 1. Places p_9 and p_{11} represent the machines M1 and M2, respectively. p_6 represents the robot, and $M_0(p_6) = 2$ means that the robot can hold two parts at a time. For processing sequence P1, p_2, p_3, and p_4 represent the operations of M1, R, and M2, respectively. p_1 represents the input and output buffers, and the tokens in p_1 mean the raw parts in the input buffer I1. Similarly, for P2, p_5, p_6, and p_7 represent the operations of M1, R, and M2, respectively. The tokens in p_8 mean the raw parts in the input buffer I2.

Let us consider the Petri net model shown in Fig. 2. The system contains 47 reachable markings, 42 of them are legal, and three are FBMs. The idle places, operation places, and resource places are $\{p_1, p_8\}$, $\{p_2 - p_7\}$, and $\{p_9 - p_{11}\}$, respectively. We can work out its MSF via the proposed vector covering approach with $\mathcal{M}_{\text{FBM}}^\star = \{p_2 + 2p_6, 2p_3 + p_5, p_2 + p_3 + p_5 + p_6\}$. It can be seen that operation places p_4 and p_7 have no token in $\mathcal{M}_{\text{FBM}}^\star$. Then we denote $\mathcal{M}_{\text{FBM}}^\star(p_4) = 0$ and $\mathcal{M}_{\text{FBM}}^\star(p_7) = 0$.

Given $\mathcal{M}_{\text{FBM}}^\star$ and $\mathcal{M}_L^\star$, it is not necessary to consider all the operation places in P_A for the design of optimal control places. In the next, the operation places that we need to consider can be reduced into a smaller set and it does not affect the existence of the optimal control places.

Theorem 1: $\forall M \in \mathcal{M}_{\text{FBM}}^\star$, if there exist coefficients $l_i (i \in \mathbb{N}_A)$ that satisfy constraint $\sum_{i\in\mathbb{N}_A} l_i \cdot (M'(p_i) - M(p_i)) \leq -1$, then $l_i^\star (i \in \mathbb{N}_A)$ can also satisfy the constraint if

$$l_i^\star = \begin{cases} l_i, & \text{if } \mathcal{M}_{\text{FBM}}^\star(p_i) \neq 0 \\ 0, & \text{if } \mathcal{M}_{\text{FBM}}^\star(p_i) = 0 \end{cases}$$

Proof: $\forall M \in \mathcal{M}_{\text{FBM}}^\star$, $\forall M' \in \mathcal{M}_L^\star$, we have

$$\begin{aligned}
&\sum_{i\in\mathbb{N}_A} l_i^\star \cdot (M'(p_i) - M(p_i)) \\
&= \sum_{i\in\mathbb{N}_A, \mathcal{M}_{\text{FBM}}^\star(p_i)\neq 0} l_i^\star \cdot (M'(p_i) - M(p_i)) \\
&\quad + \sum_{i\in\mathbb{N}_A, \mathcal{M}_{\text{FBM}}^\star(p_i)=0} l_i^\star \cdot (M'(p_i) - M(p_i)) \\
&= \sum_{i\in\mathbb{N}_A, \mathcal{M}_{\text{FBM}}^\star(p_i)\neq 0} l_i \cdot (M'(p_i) - M(p_i)) \\
&\leq \sum_{i\in\mathbb{N}_A, \mathcal{M}_{\text{FBM}}^\star(p_i)\neq 0} l_i \cdot (M'(p_i) - M(p_i)) \\
&\quad + \sum_{i\in\mathbb{N}_A, \mathcal{M}_{\text{FBM}}^\star(p_i)=0} l_i \cdot M'(p_i) \\
&= \sum_{i\in\mathbb{N}_A, \mathcal{M}_{\text{FBM}}^\star(p_i)\neq 0} l_i \cdot (M'(p_i) - M(p_i)) \\
&\quad + \sum_{i\in\mathbb{N}_A, \mathcal{M}_{\text{FBM}}^\star(p_i)=0} l_i \cdot (M'(p_i) - M(p_i)) \\
&= \sum_{i\in\mathbb{N}_A} l_i \cdot (M'(p_i) - M(p_i))
\end{aligned}$$

Then, $\sum_{i\in\mathbb{N}_A} l_i^\star \cdot (M'(p_i) - M(p_i)) \leq \sum_{i\in\mathbb{N}_A} l_i \cdot (M'(p_i) - M(p_i)) \leq -1$ can be derived. It means that coefficients $l_i^\star$ $(i \in \mathbb{N}_A)$ can also meet Eq. (12). And then all markings in $\mathcal{M}_L^\star$ are reachable. ■

From Theorem 1, we can conclude that not all the operation places need to be considered. It only needs to consider all of the places p_i with $\mathcal{M}_{\text{FBM}}^\star(p_i) \neq 0, p_i \in P_A, \forall i \in \mathbb{N}_A$. Let $P_A^\star = \{p_i | \mathcal{M}_{\text{FBM}}^\star(p_i) \neq 0, p_i \in P_A, i \in \mathbb{N}_A\}$. It is obvious that $|P_A^\star| \leq |P_A|$. $\mathbb{N}_A^\star$ can be defined as $\mathbb{N}_A^\star = \{i | p_i \in P_A, \mathcal{M}_{\text{FBM}}^\star(p_i) \neq 0\}$.

From what is mentioned above, to design optimal control places, we just need to consider operation places in $P_A^\star$.

There will be some new vector cover relationships in $\mathcal{M}_L^\star$. Then, the vector covering approach is needed to perform again for the set $\mathcal{M}_L^\star$.

Definition 5: Let $M_1, M_2 \in M_L^\star$. M_1 F-covers M_2 if $\forall p \in P_A^\star$, $M_1(p) \geq M_2(p)$ holds, which is denoted as $M_1 \geq_F M_2$ (or $M_2 \leq_F M_1$).

Theorem 2: Let $M_1 \geq_F M_2$. If M_1 is reachable, M_2 is also reachable.

Proof: Suppose that there exists a PI that satisfies constraint $\sum_{i\in\mathbb{N}_A} l_i \cdot M_k(p_i) \geq \beta + 1$ to forbid any FBM $M_k \in \mathcal{M}_{\text{FBM}}^\star$, where $\beta = \sum_{i\in\mathbb{N}_A} l_i \cdot M_k(p_i) - 1$. The two constraints can be reduced as $\sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_k(p_i) \geq \beta + 1$ and $\beta = \sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_k(p_i) - 1$ by Theorem 1. To ensure the reachability of M_1, the PI should also satisfy the constraint $\sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_1(p_i) \leq \beta$. Since $M_1 \geq_F M_2$, then $\sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_2(p_i) \leq \sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_1(p_i) \leq \beta$, implying that M_2 is also reachable. ■

Theorem 2 means that if a control place can ensure the reachability of marking $M_1 \in \mathcal{M}_L^\star$, then any marking M_2, satisfying $M_1 \geq_F M_2$, can also be reached. Based on this theorem, $\mathcal{M}_L^\star$ can be further reduced into a smaller one. The set $\mathcal{M}_L^\star$ of the net model in Fig. 2 is shown in Table 1. Let us consider markings $M_a, M_b \in \mathcal{M}_L^\star$ with $M_a = p_2 + 2p_3 + p_4$ and $M_b = p_3 + p_4 + p_7$. Since $\mathcal{M}_{\text{FBM}}^\star(p_4) = 0$ and $\mathcal{M}_{\text{FBM}}^\star(p_7) = 0$, the two operation places p_4 and p_7 can be discarded. Then we have $M_a \geq_F M_b$. By Theorem 2, if M_a is reachable, M_b is also reachable.

TABLE 1. The novel vector covering approach in Fig. 2.

$\mathcal{M}_{\text{FBM}}^\star$	$\mathcal{M}_L^\star$	$\mathcal{M}_L^{\star\star}$
	$p_5 + 2p_6 + p_7$	
	$p_2 + 2p_3 + p_4$	$p_2 + 2p_3$
$p_2 + 2p_6$	$p_2 + p_3 + p_4 + p_6$	$p_2 + p_3 + p_6$
$2p_3 + p_5$	$p_3 + p_4 + p_7$	$p_2 + p_3 + p_5$
$p_2 + p_3 + p_5 + p_6$	$p_2 + p_3 + p_5$	$p_3 + p_5 + p_6$
	$p_4 + p_6 + p_7$	$p_2 + p_5 + p_6$
	$p_3 + p_5 + p_6 + p_7$	$p_5 + 2p_6$
	$p_2 + p_5 + p_6$	

Definition 6: Let $\mathcal{M}_L^{\star\star}$ be a subset of $\mathcal{M}_L^\star$. $\mathcal{M}_L^{\star\star}$ is said to be the minimal covering SLM related to $\mathcal{M}_{\text{FBM}}^\star$ if the following conditions hold:

1): $\forall M \in \mathcal{M}_L^\star, \exists M' \in \mathcal{M}_L^{\star\star}$, s.t. $M' \geq_F M$;

2): $\forall M \in \mathcal{M}_L^{\star\star}, \nexists M'' \in \mathcal{M}_L^{\star\star}$, s.t. $M'' \geq_F M$ and $M \neq M''$.

Corollary 3: If every FBM in $\mathcal{M}_{\text{FBM}}^\star$ is forbidden by a PI and every marking in $\mathcal{M}_L^{\star\star}$ is reachable, then all of LMs in $\mathcal{M}_L$ are reachable.

Proof: By Definition 6, Theorem 2, and Corollary 2, the result holds. ■

For the model in Fig. 2, there are eight markings in $\mathcal{M}_L^\star$, and three markings in $\mathcal{M}_{\text{FBM}}^*$, as shown in Table 1. Since $\mathcal{M}_{\text{FBM}}^\star(p_4) = \mathcal{M}_{\text{FBM}}^\star(p_7) = 0$, both p_4 and p_7 can be discarded, and the further reduction method can be performed to $\mathcal{M}_L^\star$. The resulting $\mathcal{M}_L^{\star\star}$, including six markings, has less elements than $\mathcal{M}_L^\star$.

B. APPLICATIONS OF THE REDUCTION METHOD

The further vector covering approach can be applied to the study that needs $\mathcal{M}_L^\star$ such as in [1]–[5]. In this section we select several examples from them to illustrate the reduction problem.

According to Corollary 3, given $\mathcal{M}_{\text{FBM}}^\star$ and $\mathcal{M}_L^{\star\star}$, if PIs can forbid every element in $\mathcal{M}_{\text{FBM}}^\star$ and ensure that $\forall M \in \mathcal{M}_L^{\star\star}$, M is reachable, all the markings in $\mathcal{M}_L$ are reachable. To find an optimal supervisor, the maximal number of forbidding FBM problem 1 (MFFP1) proposed in [4] can be modified as follows, denoted as MFFP1*.

MFFP1*:

$$max\ f = \sum_{k\in\mathbb{N}_{\text{FBM}}^\star} f_k \tag{13}$$

$$\text{subject to: } \sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_l(p_i) \leq \beta, \quad \forall M_l \in \mathcal{M}_L^{\star\star} \tag{14}$$

$$\sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_k(p_i) \geq \beta + 1 - Q \cdot (1 - f_k),$$

$$\forall M_k \in \mathcal{M}_{\text{FBM}}^\star$$

$$l_i \in \{0, 1, 2, \ldots\}, \quad \forall i \in \mathbb{N}_A^\star$$

$$\beta \in \{1, 2, \ldots\}$$

$$f_k \in \{0, 1\}, \quad \forall k \in \mathbb{N}_{\text{FBM}}^\star \tag{15}$$

where f_k=1 indicates that M_k is forbidden by a PI, and otherwise $f_k = 0$. Q is a positive integer constant which is big enough. $\mathbb{N}_{\text{FBM}}^\star$ is the set $\{i | M_i \in \mathcal{M}_{\text{FBM}}^\star\}$. The objective function f means that the designed PI can forbid as many FBMs as possible, and f^* denotes the optimal value. A PI is said to be maximally permissive if its enforcement does not exclude any LM from the controlled system. Such a PI is abbreviated as a maximally permissive PI (MPPI).

Theorem 3: If $f^\star = 0$, there does not exist an MPPI that can forbid any FBM in $\mathcal{M}_{\text{FBM}}^\star$.

Proof: Suppose that $f^\star = 0$ and there exists an MPPI that can forbid an FBM M_k in $M_{\text{FBM}}^\star$. It means that a set of parameters $l_i (i \in \mathbb{N}_A^\star)$ and β satisfy Eq. (14), $\sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_k(p_i) \geq \beta + 1$, and f_k=1. Then we have $f^* = \sum_{k\in\mathbb{N}_{\text{FBM}}^\star} f_k \geq 1$, which comes into conflict with the assumption. Thus, the conclusion holds. ■

Theorem 4: If there is a feasible solution that satisfies MFFP1, then the solution also satisfies MFFP1*.

Proof: Suppose that there is a feasible solution l_i ($i \in \mathbb{N}_A$), β and f_k ($k \in \mathbb{N}_{\text{FBM}}^\star$) that satisfies the constraints in MFFP1. We have $\sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_l(p_i) \leq \sum_{i\in\mathbb{N}_A} l_i \cdot M_l(p_i) \leq \beta$ and $\sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_k(p_i) = \sum_{i\in\mathbb{N}_A^\star} l_i \cdot M_k(p_i) + \sum_{i\in\mathbb{N}_A\setminus\mathbb{N}_A^\star} l_i \cdot M_k(p_i) = \sum_{i\in\mathbb{N}_A} l_i \cdot M_k(p_i) \geq \beta + 1 - Q(1 - f_k)$. It means the conclusion holds. ■

Theorem 5: If there exists a feasible solution satisfying MFFP1*, then it satisfies MFFP1.

Proof: Suppose that there exists a feasible solution l_i ($i \in \mathbb{N}_A^\star$), β and f_k ($k \in \mathbb{N}_{\text{FBM}}^\star$) that satisfies the

TABLE 2. The number of Constraints and Variables in an MFFP1, MFFP2, and MCPP [4].

Parameters	MFFP1	MFFP2	MCPP
No. constraints	$\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert + \lvert\mathcal{M}^{\star}_{L}\rvert$	$\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert + \lvert\mathcal{M}^{\star}_{L}\rvert - 1$	$\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert \cdot (\lvert\mathcal{M}^{\star}_{L}\rvert + 2\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert - 1)$
No. variables	$\lvert P_A\rvert + \lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert + 1$	$\lvert P_A\rvert + \lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert - 1$	$\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert \cdot (\lvert P_A\rvert + \lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert)$

TABLE 3. The number of Constraints and Variables in an MFFP1⋆, MFFP2⋆, and MCPP⋆.

Parameters	MFFP1⋆	MFFP2⋆	MCPP⋆
No. constraints	$\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert + \lvert\mathcal{M}^{\star\star}_{L}\rvert$	$\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert + \lvert\mathcal{M}^{\star\star}_{L}\rvert - 1$	$\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert \cdot (\lvert\mathcal{M}^{\star\star}_{L}\rvert + 2\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert - 1)$
No. variables	$\lvert P^{\star}_A\rvert + \lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert + 1$	$\lvert P^{\star}_A\rvert + \lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert - 1$	$\lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert \cdot (\lvert P^{\star}_A\rvert + \lvert\mathcal{M}^{\star}_{\mathrm{FBM}}\rvert)$

constraints in MFFP1⋆. Then we have the solution with $l_i = 0$ ($i \in \mathbb{N}_A \setminus \mathbb{N}^{\star}_A$), l_i ($i \in \mathbb{N}^{\star}_A$), β and f_k such that $\sum_{i\in\mathbb{N}^{\star}_A} l_i \cdot M_l(p_i) = \sum_{i\in\mathbb{N}^{\star}_A} l_i \cdot M_l(p_i) + \sum_{i\in\mathbb{N}_A\setminus\mathbb{N}^{\star}_A} l_i \cdot M_l(p_i) \leq \beta$ and $\sum_{i\in\mathbb{N}^{\star}_A} l_i\cdot M_k(p_i) = \sum_{i\in\mathbb{N}^{\star}_A} l_i\cdot M_k(p_i)+\sum_{i\in\mathbb{N}_A\setminus\mathbb{N}^{\star}_A} l_i\cdot M_k(p_i) = \sum_{i\in\mathbb{N}_A} l_i\cdot M_k(p_i) \geq \beta+1-Q\cdot(1-f_k)$. It means the conclusion holds. ■

From Theorems 4 and 5, we can conclude that all the solutions of MFFP1 can satisfy MFFP1⋆, and all the solutions of MFFP1⋆ can also satisfy MFFP1, i.e., MFFP1 and MFFP1⋆ have the same feasible region, and MFFP1⋆ does not affect the existence of the solution of MFFP1. Similarly, the maximal number of forbidding FBM problem 2 (MFFP2) proposed in [4] can be modified to MFFP2⋆ by replacing $M^{\star}_L$ and P_A by $M^{\star\star}_L$ and $P^{\star}_A$, respectively. Moreover, the minimal number of control places problem (MCPP) proposed in [2] can be modified as follows, named as MCPP⋆.

MCPP⋆:

$$\min \sum_{M_j\in\mathcal{M}^{\star}_{\mathrm{FBM}}} pm_j$$

$$\begin{aligned}
\text{subject to: } & \sum_{i\in\mathbb{N}^{\star}_A} l_{j,i}\cdot(M_l(p_i)-M_j(p_i)) \leq -1,\\
& \forall M_j \in \mathcal{M}^{\star}_{\mathrm{FBM}} \text{ and } \forall M_l \in \mathcal{M}^{\star\star}_L\\
& \sum_{i\in\mathbb{N}^{\star}_A} l_{j,i}\cdot(M_k(p_i)-M_j(p_i)) \geq -Q\cdot(1-f_{j,k}),\\
& \forall M_j, M_k \in \mathcal{M}^{\star}_{\mathrm{FBM}} \text{ and } j\neq k\\
& f_{j,k} \leq pm_j, \quad \forall j,k \in \mathbb{N}^{\star}_{\mathrm{FBM}} \text{ and } j\neq k\\
& pm_j + \sum_{k\in\mathbb{N}^{\star}_{\mathrm{FBM}},k\neq j} f_{k,j} \geq 1, \quad \forall j\in\mathbb{N}^{\star}_{\mathrm{FBM}}\\
& l_{j,i}\in\{0,1,2,\ldots\}, \quad \forall i\in\mathbb{N}^{\star}_A \text{ and } \forall j\in\mathbb{N}^{\star}_{\mathrm{FBM}}\\
& f_{j,k}\in\{0,1\}, \quad \forall j,k\in\mathbb{N}^{\star}_{\mathrm{FBM}} \text{ and } j\neq k\\
& pm_j\in\{0,1\}, \quad \forall j\in\mathbb{N}^{\star}_{\mathrm{FBM}}
\end{aligned}$$

where $\mathbb{N}^{\star}_{\mathrm{FBM}}$ denotes $\{i|M_i \in M^{\star}_{\mathrm{FBM}}\}$. $f_{j,k}$=1 indicates that M_k is forbidden by PI$_j$ and $f_{j,k}$=0 means that M_k cannot be forbidden by PI$_j$. pm_j=1 indicates that PI$_j$ is selected to compute a control place and pm_j=0 indicates that it is unnecessary to add a control place.

Theorem 6: If there is a feasible solution satisfying MCPP, then it satisfies MCPP⋆.

Proof: Similar to Theorem 4. ■

Algorithm 1 Deadlock Prevention Policy by Using MFFP1⋆

Input: Petri net model (N, M_0) of an FMS.
Output: An optimally controlled system (N_1, M_1).

1: Compute FBMs and $\mathcal{M}_L$ for (N, M_0).
2: Compute $\mathcal{M}^{\star}_{\mathrm{FBM}}$ and $\mathcal{M}^{\star}_L$ for (N, M_0).
3: $V_M := \emptyset$. /⋆ V_M denotes the set of control places. ⋆/
4: **while** $\mathcal{M}^{\star}_{\mathrm{FBM}} \neq \emptyset$ **do**
5: Compute $P^{\star}_A$ and $\mathcal{M}^{\star\star}_L$ with respect to the remaining markings in $\mathcal{M}^{\star}_{\mathrm{FBM}}$.
6: Design MFFP1⋆ as proposed in Section 3.2.
7: Solve the MFFP1⋆. If $f^{\star} \neq 0$, $l_i (i \in \mathbb{N}^{\star}_A)$ and β are the solution. Otherwise, exit, as there is no maximally permissive Petri net supervisor.
8: Design a control place p_c.
9: $V_M := V_M \cup \{p_c\}$ and $\mathcal{M}^{\star}_{\mathrm{FBM}} := \mathcal{M}^{\star}_{\mathrm{FBM}} - F_I$.
10: **end while**
11: Add V_M to (N, M_0) and the controlled net is denoted as (N_1, M_1).
12: Output (N_1, M_1).
13: End.

Theorem 7: If there exists a feasible solution satisfying MCPP⋆, then it satisfies MCPP.

Proof: Similar to Theorem 5. ■

From Theorems 6 and 7, we can conclude that all the solutions of MCPP can satisfy MCPP⋆, and all the solutions of MCPP⋆ can also satisfy MCPP, i.e., MCPP and MCPP⋆ have the same feasible region, and MCPP⋆ does not affect the existence of the solution of MCPP.

From what is mentioned above, $\mathcal{M}^{\star}_L$ can be further reduced into $\mathcal{M}^{\star\star}_L$ and P_A, the places to be considered, can be reduced into $P^{\star}_A$. It is well known that an ILPP is NP-hard, and the time cost to solve it is mainly subject to its constraints and variables. From the comparison of Tables 2 and 3, it is clear that the number of variables and constraints in the three ILPPs for each method can be reduced due to $|P^{\star}_A| \leq |P_A|$ and $|\mathcal{M}^{\star\star}_L| \leq |\mathcal{M}^{\star}_L|$. Meanwhile, the existence of the optimal solution is still guaranteed. For example, the total number of constraints is reduced from $|\mathcal{M}^{\star}_{\mathrm{FBM}}| + |\mathcal{M}^{\star}_L|$ in MFFP1 to $|\mathcal{M}^{\star}_{\mathrm{FBM}}| + |\mathcal{M}^{\star\star}_L|$ in MFFP1⋆, and the number of variables is reduced from $|P_A| + |\mathcal{M}^{\star}_{\mathrm{FBM}}| - 1$ to $|P^{\star}_A| + |\mathcal{M}^{\star}_{\mathrm{FBM}}| - 1$. Both of them make it more efficient to solve the ILPP.

Algorithm 2 Deadlock Prevention Policy by Using MCPP*

Input: Petri net model (N, M_0) of an FMS.
Output: An optimally controlled system (N_1, M_1).

Compute FBMs and $\mathcal{M}_L$ for (N, M_0).
Compute $\mathcal{M}^\star_{\text{FBM}}$ and $\mathcal{M}^\star_L$ for (N, M_0).
Compute $P^\star_A$ and $\mathcal{M}^{\star\star}_L$.
$V_M := \emptyset$. /$\star$ V_M denotes the set of control places to be computed. $\star$/
Solve the MCPP*. If there is no solution, exit, as there does not exist a linear optimal supervisor for this Petri net model.
foreach $pm_j = 1$ **do**
  Use $l_{j,i}$ in the solution as the coefficients of a PI and design a place C_j to forbid $M_j \in \mathcal{M}^\star_{\text{FBM}}$.
  $V_M := V_M \cup C_j$.
Add V_M to (N, M_0) and the controlled net is denoted as (N_1, M_1).
Output (N_1, M_1).
End.

C. DEADLOCK PREVENTION POLICY BY THE REDUCTION METHOD

This section improves two deadlock prevention algorithms originally developed in [2] and [4], respectively, using MFFP1* and MCPP*.

Algorithm 1 using MFFP1* can find an optimal supervisor if such a supervisor exists. Compared with the original Algorithm proposed in [4], at each iteration, we perform the further reduction method and obtain $\mathcal{M}^{\star\star}_L$ and $P^\star_A$ related to the remaining markings in $\mathcal{M}^\star_{\text{FBM}}$. The sizes of the two sets are generally less than $|\mathcal{M}^\star_L|$ and $|P_A|$, respectively, in the original Algorithm. The MFFP1* has less constraints and variables.

Algorithm 2 using MCPP* can also find an optimal supervisor with the minimal number of control places. The numbers of operation places and legal markings to be considered are small, compared with those in the original Algorithm proposed in [2]. We can perform the further reduction only once since Algorithm 2 is a non-iterative method and all control places can be obtained by solving an ILPP. The numbers of constraints and variables in the ILPP by MCPP* are reduced and the optimal solution is still guaranteed.

IV. EXAMPLES

This section exposes the application of the developed methodology to four examples from manufacturing, and the fourth has been studied as a benchmark. The computation is carried out by a desktop computer under Windows 7 operating system with an Intel Core 2.4-GHz CPU and 4-GB memory. The Petri net in Fig. 3 has 132 reachable markings. The number of LMs is 120 while that of FMBs is 12. By the technique reported in [1], we have $\mathcal{M}^\star_L = \{2p_2+p_9+2p_{10}, p_2+p_3+p_4+2p_7, p_2+p_3+p_4+p_7+p_{10}, p_2+p_3+p_4+2p_{10}, 2p_4+p_6+2p_7, 2p_4+p_6+p_7+p_{10}, 2p_4+p_7+p_9+p_{10}, 2p_4+p_9+2p_{10}, p_2+p_4+p_6+2p_7, p_2+p_4+p_6+p_7+p_{10}, p_2+p_4+p_7+p_9+p_{10}, p_2+p_4+p_9+2p_{10},$

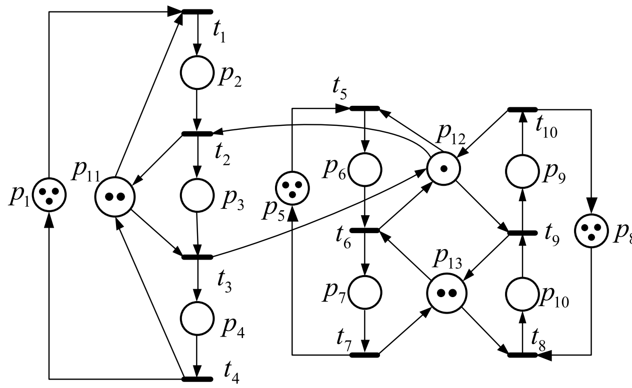

FIGURE 3. Petri net model of an FMS.

$2p_2+p_6+2p_7, 2p_2+p_6+p_7+p_{10}, 2p_2+p_7+p_9+p_{10}\}$ and $\mathcal{M}^\star_{\text{FBM}} = \{2p_2+p_3, p_6+2p_{10}\}$, i.e., the number of elements in $\mathcal{M}^\star_{\text{FBM}}$ and $\mathcal{M}^\star_L$ are 2 and 15, respectively, and the number of operation places is 7. We can illustrate the reduction effects by the comparison between MFFP1 proposed in [4] and MFFP1*.

First we use the MFFP1 proposed in [4]. At the first iteration, let I_1 be a PI, ensuring the reachability of $\mathcal{M}^\star_L$, such that as many FBMs as possible are forbidden. MFFP1 is hence presented as follows:

MFFP1:

$$
\begin{aligned}
& max\ f = f_1 + f_2 \\
& \text{subject to } 2l_2 + l_9 + 2l_{10} \le \beta \\
& \qquad l_2 + l_3 + l_4 + 2l_7 \le \beta \\
& \qquad l_2 + l_3 + l_4 + l_7 + l_{10} \le \beta \\
& \qquad l_2 + l_3 + l_4 + 2l_{10} \le \beta \\
& \qquad 2l_4 + l_6 + 2l_7 \le \beta \\
& \qquad 2l_4 + l_6 + l_7 + l_{10} \le \beta \\
& \qquad 2l_4 + l_7 + l_9 + l_{10} \le \beta \\
& \qquad 2l_4 + l_9 + 2l_{10} \le \beta \\
& \qquad l_2 + l_4 + l_6 + 2l_7 \le \beta \\
& \qquad l_2 + l_4 + l_6 + l_7 + l_{10} \le \beta \\
& \qquad l_2 + l_4 + l_7 + l_9 + l_{10} \le \beta \\
& \qquad l_2 + l_4 + l_9 + 2l_{10} \le \beta \\
& \qquad 2l_2 + l_6 + 2l_7 \le \beta \\
& \qquad 2l_2 + l_6 + l_7 + l_{10} \le \beta \\
& \qquad 2l_2 + l_7 + l_9 + l_{10} \le \beta \\
& \qquad 2l_2 + l_3 \ge \beta + 1 - Q \cdot (1 - f_1) \\
& \qquad l_6 + 2l_{10} \ge \beta + 1 - Q \cdot (1 - f_2) \\
& \qquad l_i \in \{0, 1, 2, \ldots\}, \quad \forall i \in \{2, 3, 4, 6, 7, 9, 10\} \\
& \qquad \beta \in \{1, 2, \ldots\} \\
& \qquad f_k \in \{0, 1\}, \quad \forall k \in \{1, 2\}.
\end{aligned}
$$

Solving the ILPP, an optimal solution can be obtained with $l_2 = 1, l_3 = 1, \beta = 2, f_1 = 1$, and all other variables being

TABLE 4. The comparison of MFFP1 and MFFP1⋆ in Fig. 3.

	MFFP1						MFFP1⋆						
iter.	$\lvert P_A\rvert$	$\lvert\mathcal{M}_L^\star\rvert$	$\lvert\mathcal{M}_{\text{FBM}}^\star\rvert$	N_{LP}	N_{var}	τ_{LP}(S)	$\lvert P_A^\star\rvert$	$\lvert\mathcal{M}_L^{\star\star}\rvert$	$N_{LP}^\star$	$N_{var}^\star$	$\tau_{LP}^\star$(S)	$r_V\%$	$r_C\%$
1	7	15	2	17	10	0.3276	4	3	5	7	0.3120	70.0	29.4
2	7	15	1	16	9	0.0468	2	2	3	4	0.0312	44.4	18.8

zero. Then a supervisor p_{c1} is designed by I_1: $u_2+u_3+u_{p_{c1}} = 2$ to forbid FBM$_1$. By removing the FBM from $\mathcal{M}_{\text{FBM}}^\star$, we have $\mathcal{M}_{\text{FBM}}^\star$=$\{p_6 + 2p_{10}\}$.

At the second iteration, let I_2 be a PI to be computed. Then the new MFFP1 can be constructed as follows:

MFFP1:

$$
\begin{aligned}
&max\, f = f_2\\
&\text{subject to } 2l_2 + l_9 + 2l_{10} \le \beta\\
&\quad l_2 + l_3 + l_4 + 2l_7 \le \beta\\
&\quad l_2 + l_3 + l_4 + l_7 + l_{10} \le \beta\\
&\quad l_2 + l_3 + l_4 + 2l_{10} \le \beta\\
&\quad 2l_4 + l_6 + 2l_7 \le \beta\\
&\quad 2l_4 + l_6 + l_7 + l_{10} \le \beta\\
&\quad 2l_4 + l_7 + l_9 + l_{10} \le \beta\\
&\quad 2l_4 + l_9 + 2l_{10} \le \beta\\
&\quad l_2 + l_4 + l_6 + 2l_7 \le \beta\\
&\quad l_2 + l_4 + l_6 + l_7 + l_{10} \le \beta\\
&\quad l_2 + l_4 + l_7 + l_9 + l_{10} \le \beta\\
&\quad l_2 + l_4 + l_9 + 2l_{10} \le \beta\\
&\quad 2l_2 + l_6 + 2l_7 \le \beta\\
&\quad 2l_2 + l_6 + l_7 + l_{10} \le \beta\\
&\quad 2l_2 + l_7 + l_9 + l_{10} \le \beta\\
&\quad l_6 + 2l_{10} \ge \beta + 1 - Q \cdot (1 - f_2)\\
&\quad l_i \in \{0, 1, 2, \ldots\}, \quad \forall i \in \{2, 3, 4, 6, 7, 9, 10\}\\
&\quad \beta \in \{1, 2, \ldots\}\\
&\quad f_2 \in \{0, 1\}.
\end{aligned}
$$

Solving the ILPP leads to an optimal solution with $l_6 = 1, l_{10} = 1, \beta = 2, f_2 = 1$, and all other variables being zero. Then we can have I_2: $u_6 + u_{10} + u_{p_{c2}} = 2$. The iteration terminates since $\mathcal{M}_{\text{FBM}^\star} = \emptyset$.

Next, we consider MFFP1⋆ proposed in this paper. At the first iteration, we perform the further reduction method, and the set of places to consider $\mathbb{N}_A^\star$ is $\{p_2, p_3, p_6, p_{10}\}$. We have $\mathcal{M}_L^{\star\star} = \{2p_2 + 2p_{10}, p_2 + p_3 + 2p_{10}, 2p_2 + p_6 + p_{10}\}$. Then, the following constraints can be obtained:

MFFP1⋆:

$$
\begin{aligned}
&max\, f = f_1 + f_2\\
&\text{subject to } 2l_2 + 2l_{10} \le \beta\\
&\quad l_2 + l_3 + 2l_{10} \le \beta\\
&\quad 2l_2 + l_6 + l_{10} \le \beta\\
&\quad 2l_2 + l_3 \ge \beta + 1 - Q \cdot (1 - f_1)\\
&\quad l_6 + 2l_{10} \ge \beta + 1 - Q \cdot (1 - f_2)\\
&\quad l_i \in \{0, 1, 2 \ldots\}, \quad \forall i \in \{2, 3, 6, 10\}\\
&\quad \beta \in \{1, 2, \ldots\}\\
&\quad f_k \in \{0, 1\}, \quad \forall k \in \{1, 2\}.
\end{aligned}
$$

An optimal solution, $l_2 = 1, l_3 = 1, \beta = 2$, and $f_1 = 1$, can be obtained. i.e., the same solution as the first iteration of MFFP1. Then we have $\mathcal{M}_{\text{FBM}}^\star = \{p_6 + 2p_{10}\}$.

In the next iteration, using the further reduction method, the set of places to consider, i.e., $\mathbb{N}_A^\star$, is $\{p_6, p_{10}\}$. Then the following constraints are constructed:

MFFP1⋆:

$$
\begin{aligned}
&max\, f = f_2\\
&\text{subject to } l_6 + l_{10} \le \beta\\
&\quad 2l_{10} \le \beta\\
&\quad l_6 + 2l_{10} \ge \beta + 1 - Q \cdot (1 - f_2)\\
&\quad l_i \in \{0, 1, 2 \ldots\}, \quad \forall i \in \{6, 10\}\\
&\quad \beta \in \{1, 2, \ldots\}\\
&\quad f_2 \in \{0, 1\}.
\end{aligned}
$$

We solve the above ILPP and obtain an optimal solution with $l_6 = 1, l_{10} = 1, \beta = 2$ and $f_2 = 1$, i.e., the same solution as the second iteration of MFFP1. The iteration terminates since $\mathcal{M}_{\text{FBM}}^\star = \emptyset$.

From the comparison of MFFP1 and MFFP1⋆, we can see that both MFFP1 and MFFP1⋆ can find the same supervisor since the optimal solutions of the two ILPPs are the same at each iteration. However, the number of variables and constraints in MFFP1⋆ is much smaller than that in MFFP1, as shown in Table 4. The numbers of variables and constraints in MFFP1 are denoted as N_{var} and N_{LP}, respectively. $N_{var}^\star$ and $N_{LP}^\star$ denote the numbers of variables and constraints in MFFP1⋆, respectively. r_V=$N_{var}^\star/N_{var}$ and $r_C = N_{LP}^\star/N_{LP}$ show the comparison between the two ILPPs. τ_{LP} and $\tau_{LP}^\star$ denote the computational time of solving the MFFP1 and MFFP1⋆ at each iteration, respectively.

The Petri net model in Fig. 4 is an S^{4}PR with nine places and seven transitions. It has the following place set partition: $P^0 = \{p_1, p_7\}$, $P_A = \{p_2 - p_6\}$, and $P_R = \{p_8, p_9\}$. The net model has 13 reachable markings. There are 11 LMs and one FBM. Using the vector covering approach, we have $|\mathcal{M}_{\text{FBM}}^\star| = 3$ and $|\mathcal{M}_L^\star| = 1$. Table 5 shows the reduction of constraints, variables and computational time by comparing MFFP1 and MFFP1⋆.

The third example, a model that has 17 places and 13 transitions, is considered as visualized in Fig. 5.

TABLE 5. The comparison of MFFP1 and MFFP1* in Fig. 4.

	MFFP1						MFFP1*						
iter.	$\|P_A\|$	$\|\mathcal{M}_L^\star\|$	$\|\mathcal{M}_{\mathrm{FBM}}^\star\|$	N_{LP}	N_{var}	τ_{LP}(S)	$\|P_A^\star\|$	$\|\mathcal{M}_L^{\star\star}\|$	$N_{LP}^\star$	$N_{var}^\star$	$\tau_{LP}^\star$(S)	$r_V\%$	$r_C\%$
1	5	3	1	4	7	0.2815	2	2	3	4	0.2541	57.1	75

TABLE 6. The comparison of MFFP1 and MFFP1* in Fig. 5.

	MFFP1						MFFP1*						
iter.	$\|P_A\|$	$\|\mathcal{M}_L^\star\|$	$\|\mathcal{M}_{\mathrm{FBM}}^\star\|$	N_{LP}	N_{var}	τ_{LP}(S)	$\|P_A^\star\|$	$\|\mathcal{M}_L^{\star\star}\|$	$N_{LP}^\star$	$N_{var}^\star$	$\tau_{LP}^\star$(S)	$r_V\%$	$r_C\%$
1	10	290	13	303	24	0.3900	7	48	61	21	0.3276	87.5	20.1
2	10	290	4	294	15	0.1248	6	24	28	11	0.0468	73.3	9.5
3	10	290	2	292	13	0.0468	5	8	10	8	0.0312	61.5	3.4
4	10	290	1	291	12	0.0468	3	3	4	5	0.0312	41.7	1.4

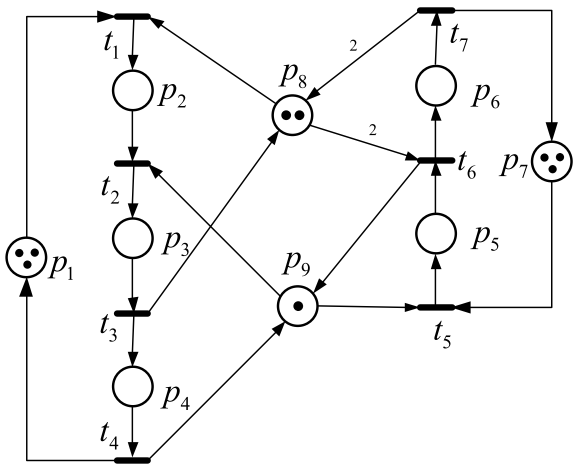

FIGURE 4. S^{4}PR model of an FMS.

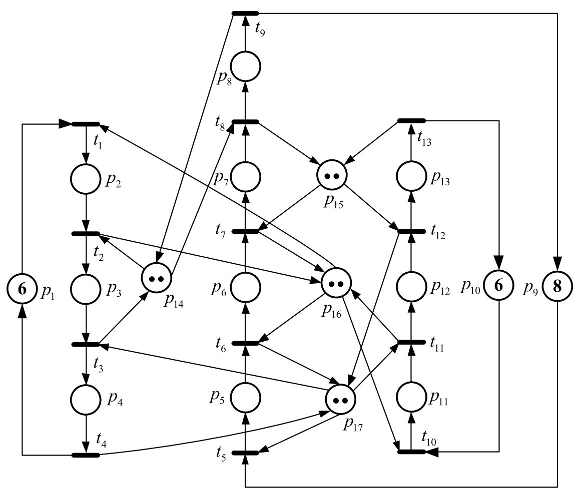

FIGURE 5. Petri net model of an FMS.

The places can be partitioned into $P^0 = \{p_1, p_9, p_{10}\}$, $P_R = \{p_{14} - p_{17}\}$, and $P_A = \{p_2 - p_8, p_{11} - p_{13}\}$. The total number of reachable markings is 3531. There are 61 FBMs and 3465 LMs. $\mathcal{M}_L^\star$ has 290 markings and $\mathcal{M}_{\mathrm{FBM}}^\star$ has 13 markings. There are 10 operation places. Table 6 shows the reduction of constraints, variables and computational time by comparing MFFP1 and MFFP1*. Both the variables and constraints can be simplified by using the proposed vector covering approach. For MCPP*, the constraints and variables decrease from 4095 and 299 to 949 and 260, respectively. Moreover, the computational time of solving MCPP* is 17s, which is less than the time cost for solving MCPP, i.e., 48s.

To further illustrate the advantages of this approach, we put different tokens in idle places and resource places at the initial marking. Table 7 shows some parameters of the net, where the first column represents the tokens of places $p_1, p_9, p_{10}, p_{14}, p_{15}, p_{16}$, and p_{17} at initial marking M_0, $|M_R|$, $|P_A|$ and $|P_A^\star|$ indicate the numbers of reachable markings, operation places, and the operation places to be considered, respectively. The last column is $r_L = |M_L^{\star\star}|/|M_L^\star|$.

From Table 7, it can be seen that r_L decreases with the increase of initial markings, which actually implies that the proposed methodology in this paper can be efficient for large-size real-world systems since the number of markings that are taken into account decreases significantly.

Table 8 shows the comparison of MFFP1 and MFFP1* in Fig. 5 with $M_0(p_1) = M_0(p_{10}) = 11$, $M_0(p_9) = 14$, $M_0(p_{14}) = M_0(p_{15}) = 3$, and $M_0(p_{16}) = M_0(p_{17}) = 4$. It is clear that solving the MFFP1* is more efficient.

Let us consider the fourth example in Fig. 6. It is a widely used net for the deadlock control problem in the recent literature [20], [26]. This flexible manufacturing cell has four machine tools and three robots. Machine tools are used to perform processing stages of different part types. Robots are in charge of part movement among the machine tools. A machine tool can process two parts at a time while a robot can hold one part type at a time. Three part types can be produced, implying that the system has three concurrent processes. We use seven places to model the four machine tools and three robots and 16 activity places to model the processing stages of the three part types. Three process idle places model the maximal number of raw parts that can be concurrently processed in the system.

The considered system can be modeled as a Petri net with 26 places and 20 transitions. The net model has 26750 reachable markings. The number of LMs is 21581 and that

TABLE 7. Parameters in the model depicted in Fig. 5 with different markings.

$M_0(p_i)(i = 1, 9, 10, 14, 15, 16, 17)$	$\|\mathcal{M}_R\|$	$\|\mathcal{M}_L\|$	$\|\mathcal{M}_{\text{FBM}}\|$	$\|\mathcal{M}_L^\star\|$	$\|\mathcal{M}_{\text{FBM}}^\star\|$	$\|\mathcal{M}_L^{\star\star}\|$	$\|P_A\|$	$\|P_A^\star\|$	$r_L\%$
3, 4, 3, 1, 1, 1, 1	127	111	14	22	4	11	10	7	50
5, 6, 5, 1, 1, 2, 2	872	839	28	118	13	48	10	7	40.7
6, 8, 6, 2, 2, 2, 2	3531	3465	61	290	13	48	10	7	16.6
8, 10, 8, 2, 2, 3, 3	14309	14210	90	851	31	121	10	7	14.2
9, 12, 9, 3, 3, 3, 3	39815	39640	166	1546	31	127	10	7	8.2
11, 14, 11, 3, 3, 4, 4	122278	122049	215	3530	61	242	10	7	6.9

TABLE 8. The comparison of MFFP1 and MFFP1* in Fig. 5 with $M_0(p_1) = M_0(p_{10}) = 11$, $M_0(p_9) = 14$, $M_0(p_{14}) = M_0(p_{15}) = 3$, $M_0(p_{16}) = M_0(p_{17}) = 4$.

	MFFP1						MFFP1*						
iter.	$\|P_A\|$	$\|\mathcal{M}_L^\star\|$	$\|\mathcal{M}_{\text{FBM}}^\star\|$	N_{LP}	N_{var}	τ_{LP}(S)	$\|P_A^\star\|$	$\|\mathcal{M}_L^{\star\star}\|$	$N_{LP}^\star$	$N_{var}^\star$	$\tau_{LP}^\star$(S)	$r_V\%$	$r_C\%$
1	10	3530	61	3591	72	0.9048	7	242	303	69	0.4368	95.8	8.4
2	10	3530	6	3536	17	0.4524	6	80	86	13	0.1092	76.5	2.4
3	10	3530	2	3532	13	0.3900	5	16	18	8	0.1404	61.5	0.5
4	10	3530	1	3531	12	0.2028	3	3	4	5	0.0780	41.7	0.1

TABLE 9. The comparison of MFFP1 and MFFP1* in Fig. 6.

	MFFP1						MFFP1*						
iter.	$\|P_A\|$	$\|\mathcal{M}_L^\star\|$	$\|\mathcal{M}_{\text{FBM}}^\star\|$	N_{LP}	N_{var}	τ_{LP}(S)	$\|P_A^\star\|$	$\|\mathcal{M}_L^{\star\star}\|$	$N_{LP}^\star$	$N_{var}^\star$	$\tau_{LP}^\star$(S)	$r_V\%$	$r_C\%$
1	16	393	34	427	51	2.4960	13	151	185	48	1.2012	94.1	43.3
2	16	393	15	408	32	0.2496	13	151	166	29	0.1248	90.6	40.7
3	16	393	7	400	24	0.1529	13	151	158	21	0.0952	87.5	39.5
4	16	393	4	397	21	0.1092	7	12	16	12	0.0624	57.1	4.0
5	16	393	2	395	19	0.0936	4	6	8	7	0.0468	36.8	2.0
6	16	393	1	394	18	0.0624	2	2	3	4	0.0312	22.2	0.8

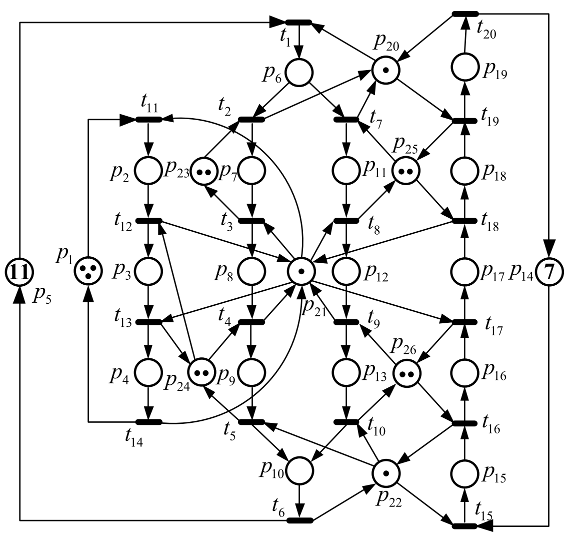

FIGURE 6. A Petri net model in [20].

of FBMs is 4211. We further have $|\mathcal{M}_{\text{FBM}}^\star| = 34$ and $|\mathcal{M}_L^\star| = 393$. There are 16 places in P_A. Table 9 shows that the constraints are reduced significantly at each iteration step by the comparison between MFFP1 and MFFP1*, and the number of variables is reduced too. It can also be seen that MFFP1* is more efficient than MFFP1. Comparing with MCPP, the constraints and variables in MCPP* decrease from 15640 and 1700 to 7412 and 1598, respectively. The number of constraints is reduced more than 50%.

V. CONCLUSION

The method presented in this particular research significantly lowers the computational overheads in designing optimal Petri net supervisors. As is known, an ILP problem is in theory NP-hard. This effective method is reported to decrease the places to be considered and LMs, as well as the set of FBMs such that the computational cost, when solving an ILPP related to $\mathcal{M}_L^\star$ and $\mathcal{M}_{\text{FBM}}^\star$, is reduced. The experimental results show that the constraints remarkably decrease, especially in large-size models. Hence, the computational overhead to solve an ILPP decreases accordingly. However, the method still needs the enumeration of all reachable markings. In future work, structural analysis techniques can be combined into a deadlock prevention policy to efficiently identify the FBMs and the minimal set of covering LMs.

APPENDIX

BASICS OF PETRI NETS

A Petri net is a 4-tuple $N = (P, T, F, W)$ where P and T are finite and non-empty sets. P is a place set and T is a transition set with $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$. $F \subseteq (P \times T) \cup (T \times P)$ represents the arcs with arrows from places (transitions) to transitions (places). $W : (P \times T) \cup (T \times P) \to \mathbb{N}$ is a mapping that assigns a weight to an arc: $W(x, y) \geq 0$ iff $(x, y) \in F$, and $W(x, y) = 0$, otherwise, where $x, y \in P \cup T$ and $\mathbb{N}$ is the set of non-negative integers. Given a node $x \in P \cup T$, ${}^\bullet x = \{y \in P \cup T | (y, x) \in F\}$ is called the preset of x, while $x^\bullet = \{y \in P \cup T | (x, y) \in F\}$ is called the postset of x.

A marking is a mapping $M : P \to \mathbb{N}$. $M(p)$ denotes the number of tokens in place p. Usually $\sum_{p\in P} M(p)p$ is used to denote vector M. For instance, $M = (1, 0, 2, 4, 0, 3)^T$ is a marking of a net with six places. It can be written as $p_1+2p_3+4p_4+3p_6$. The pair (N, M_0) is called a Petri net system and M_0 is called the initial making. Incidence matrix $[N]$ of net N is a $|P| \times |T|$ integer matrix with $[N](p, t) = W(t, p) - W(p, t)$.

A P-vector is a column vector $I : P \to \mathbb{Z}$ indexed by P and a T-vector is a column vector $J : T \to \mathbb{Z}$ indexed by T, where $\mathbb{Z}$ is the set of integers. P-vector I is called a P-invariant (place invariant, PI) if $I \neq 0$ and $I^T[N] = 0^T$. Let I be a PI of (N, M_0) and M be a reachable marking from M_0. Then, $I^T M = I^T M_0$. The dynamic of a Petri net system depends on the transition enabling and firing. A transition is enabled if its input places has enough tokens. At a marking M, an enabling transition t can fire, leading to a new marking M', which is denoted by $M[t\rangle M'$. The concepts of the transition enabling, firing rules, initial marking (denoted by M_0), reachability graph (denoted by $G(N, M_0)$, and marking reachability set (denoted by $R(N, M_0)$ for a net system (N, M_0)) can be found in [40].

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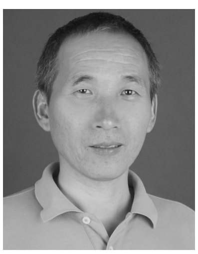

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