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Random, Persistent, and Adaptive Spectrum Sensing Strategies for Multiband Spectrum Sensing in Cognitive Radio Networks With Secondary User Hardware Limitation

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ABSTRACT In this paper, we consider hardware limitation at the secondary user, which makes multiband (wideband) spectrum sensing more challenging. Under secondary user (SU) hardware limitation, the SU can only sense a small portion of the multiband spectrum for a given time period, which introduces a design issue of selecting subchannels to sense at a given time. A random spectrum sensing strategy (RSSS) is presented to select the subchannels to sense in a totally random fashion. With the Markov assumption of the primary user (PU) behavior, a persistent spectrum sensing strategy (PSSS) is proposed to take advantage of the PU traffic patterns in determining the channels to sense. Theoretical and simulation results show that RSSS and PSSS display different performance in different ranges of PU traffic parameters. We finally propose an adaptive spectrum sensing strategy (ASSS), which determines whether to use RSSS or PSSS for spectrum sensing at a given time based on the estimated PU traffic parameters. Numerical results under various system parameters are presented to evaluate the performance of RSSS, PSSS, and ASSS. The ASSS is shown to gain the advantages of both RSSS and PSSS in different ranges of PU traffic parameters and provide more available subchannels for SU.

INDEX TERMS Adaptive spectrum sensing, cognitive radio, subchannel selection, wideband spectrum sensing, Markov model.

I. INTRODUCTION

The amount of wireless devices and services has grown explosively during past decade, which makes the limited frequency spectrum resources becomes increasingly scarce and valuable. The current spectrum allocation policies regulate the wireless network to operate in certain time frames, over certain frequency bands, and within certain geographical regions. While the entire radio spectrum from 6 kHz to 300 GHz has been allocated [1], the traditional spectrum policies has been shown to be highly inefficient that some radio bands are overcrowded while others are underutilized [2]–[4]. In fact, this statical spectrum allocation policy, in many frequency bands, is a more significant problem than the physical scarcity of spectrum itself [5], [6]. Cognitive radio (CR) proposed by Mitola in 1999 [7], which allows secondary users (SUs) to opportunistically access unused spectrum bands, has been widely considered as a promising solution to the above problem [6], [8], [9]. One of the fundamental functions of CR is spectrum sensing conducted by secondary user (SU) that involves monitoring the spectrum usage and locating the unused spectrum bands [10]–[14]. In the field of spectrum sensing, multiband spectrum sensing and access are regarded as a great promise for future CR networks for its potential of providing more access opportunities and higher aggregate throughput [15]–[18].

Multiband spectrum sensing techniques include serial sensing, parallel sensing and wideband sensing [15]. In serial sensing, SUs deal with only one channel at one time while in parallel sensing, SUs are assumed to detect all the channels at the same time by using filter banks. Wideband sensing captures a wideband spectrum using high-speed analog-to-digital converters and high-performance signal processing components. In this paper, we consider a practical scenario in multiband sensing in which the spectrum sensing conducted by SUs is constrained by hardware limitation. SU hardware limitation can be summarized as two categories [19]–[21]: (1) *Sensing capability limitation*: given a wide multiband spectrum, a SU can only sense a small portion of the whole range of spectrum bands during a given time period; (2) *Access capability limitation*: a SU can not access all of the available channels simultaneously. In this paper, we address the spectrum sensing problem under SU sensing capability limitation.

With limited sensing capability, a design issue for SU is to determine the specific subchannels to sense at a given time. The problem of channel selection for multiband sensing in cognitive radio networks has been studied in a multitude of literatures. In [20], a single user-single channel access problem is considered as a bandit problem and an asymptotically optimal strategy is presented. Wu *et al.* [21], [22] model spectrum sensing under sensing capability limitation as a Partial Observable Markov Decision Process (POMDP) problem which is known to be P-Space hard to solve or even to approximate [23]. Both [21] and [22] search the optimal sensing policy using dynamic programming which is computationally expensive. If the PU traffic parameters change, it may take a long time to find a new optimal sensing policy. The multiband spectrum sensing problem is formulated as an optimal stopping problem to determine the channel sensing order in advance in both [24] and [25]. The complexity of the dynamic programming method and backward induction method used in these two papers, respectively, is relatively high when the number of the channels grows large. The work in [26] aims at finding the most likely available channels to sense by employing an estimated channel availability probability.

In this paper, we study the multiband spectrum sensing problem with limited sensing capability, where SU can only sense a small portion of the given multiband spectrum during a given time period. Our research differs from previous work in the following aspects. First, unlike the aforementioned studies, most of which only consider the channel stationary availability rather than the PU state transition property, we model the PU traffic of a channel as a discrete time Markov chain (DTMC) model [27] and investigate the impact of both stationary availability of channels and PU state dynamic transition on the spectrum sensing performance. Second, while [21], [22], [24]–[26] treat the channels to be independent from each other, we consider the correlation of the channels and it is often seen in the presence of wideband PU signals, e.g., broadcast television or wireless local area networks systems [28]. By channel correlation, it means that several consecutive channels could be occupied by one PU. We define a performance metric in terms of normalized available channel quantity (NACQ) and present two spectrum sensing strategies referred to as random spectrum sensing strategy (RSSS) and persistent spectrum sensing strategy (PSSS). Third, based on theoretical and simulation results, we find that PSSS outperforms RSSS in a certain range of PU traffic parameters, while for other PU traffic parameter values, the contrary is true. Therefore, an adaptive spectrum sensing strategy (ASSS) is proposed to combine the advantages of both PSSS and RSSS, i.e., one of the two strategies is adaptively selected to perform spectrum sensing according to the PU traffic parameters, which can be estimated during the process of spectrum sensing.

The rest of this paper is organized as follows. Section II describes the system model of multiband spectrum sensing under SU hardware limitation. The random spectrum sensing strategy and persistent spectrum sensing strategy are presented in Section III. The adaptive spectrum sensing strategy is proposed in Section IV and the PU traffic parameter estimation is also studied in this section. Section V presents numerical results and discussions on the performance of the proposed algorithms. Conclusions are drawn in Section VI.

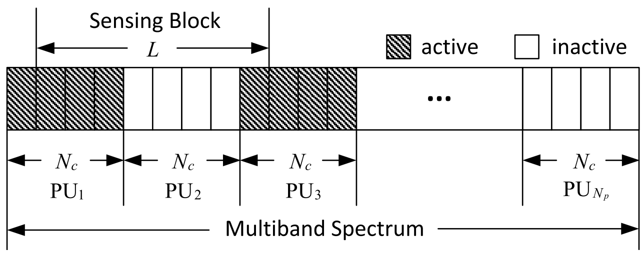


FIGURE 1. Multiband spectrum sensing under hardware limitation.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The problem of multiband spectrum sensing with SU hardware limitation is depicted in Fig. 1. A given wide frequency band is divided into K subchannels, which are assigned to N_p PUs. As is shown in Fig. 1, the correlation of subchannels is considered as N_c consecutive subchannels are assigned to one PU, which is called a PU block and thus we have $K = N_pN_c$. Both PU and SU are assumed to function in a time-slotted fashion. The N_p PUs are assumed to be independent from each other and the state of each PU follows a discrete time Markov chain (DTMC) with identical parameters as depicted in Fig. 2(a). The transition process of PU blocks is illustrated in Fig. 2(b). A PU block transits from an inactive state to an active state with probability P_{01} and from an active state to an inactive state with probability P_{10}. We assume that if a PU is in an active state, it occupies all the N_c subchannels assigned.

As a result, the stationary distribution of the DTMC can be computed as

$$P_0 = \frac{P_{10}}{P_{01} + P_{10}}, \tag{1a}$$

$$P_1 = \frac{P_{01}}{P_{01} + P_{10}}, \tag{1b}$$

where P_0 and P_1 are the stationary probabilities of PU inactive and active state respectively, which measure the availability of a given PU block.

An SU performs spectrum sensing in each time slot. We denote the sensing capability (i.e, the number of consecutive

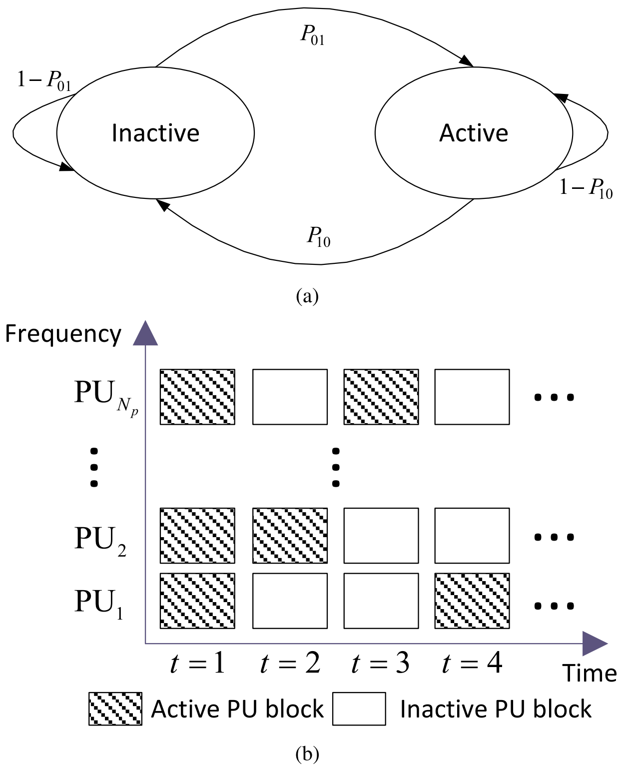


FIGURE 2. Statistic model of PU. (a) Markov model of PU; (b) PU state transition.

subchannels that can be sensed at a given time) of the SU as L as shown in Fig. 1. The L subchannels covered by SU at a given time slot is called a sensing block. Notice that the length of the sensing block L can be either $L < N_c$ or $L \geq N_c$, which will be treated as Case 1 and Case 2, respectively, in the following sections.

For convenience, a state vector $\mathbf{S} = (s_0, s_2, \ldots, s_{L-1})$ is used to represent the state of subchannels in a sensing block for a time slot, where $s_k = 1, k = 0, 1, 2, \ldots, L-1$, if the corresponding subchannel is detected in an inactive state; Otherwise $s_k = 0$ as the corresponding subchannel is detected in an active state. For a given limited sensing capability L, the objective of SU is to perform spectrum sensing to obtain a larger number of available subchannels under the system model described above. To be more specific, we construct a metric Y, which is referred to as normalized available channel quantity(NACQ), to evaluate any proposed spectrum sensing algorithms.

$$Y = \frac{1}{LP_0} \sum_{k=0}^{L-1} s_k. \tag{2}$$

Note that the summation term in (2) is the total number of available subchannels detected by SU in a time slot. However, the number of the available subchannels for SU is also greatly related to how many subchannels are checked and the number of available subchannels in the wideband range, which are represented by L and P_0, respectively. Therefore, it is more reasonable to normalize the absolute value of the available subchannels with L and P_0. Further, Eq. (2) measures a single realization of random variable Y. Therefore, we will investigate $\mathbb{E}[Y]$, instead of Y, in the following sections, where $\mathbb{E}[\cdot]$ denotes the mathematical expectation operator.

III. RANDOM AND PERSISTENT SPECTRUM SENSING STRATEGIES

As discussed above, given a wide frequency band with a total of K subchannels and limited SU sensing capability L, it is necessary to determine which subchannels (total L consecutive subchannels) to sense for each time slot. One straightforward approach is to select the subchannels randomly, which is referred to as random spectrum sensing strategy (RSSS). The other approach, which will be introduced in Section III.B, is called persistent spectrum sensing strategy (PSSS). In this section, we will present these two spectrum sensing strategies and derive the corresponding NACQ $\mathbb{E}[Y]$ under both Case 1 and Case 2 as mentioned in the last section.

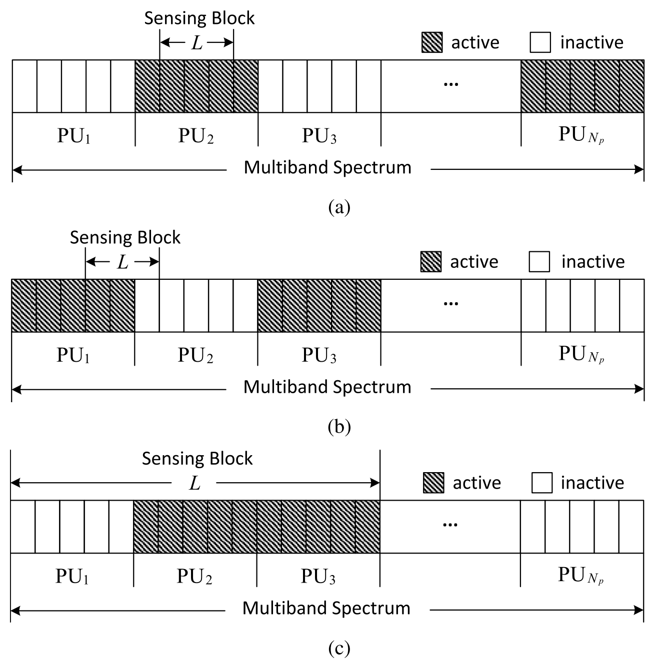


FIGURE 3. (a) Event A of Case 1; (b) Event B of Case 1; (c) Case 2.

A. RANDOM SPECTRUM SENSING STRATEGY

In the random spectrum sensing strategy (RSSS), for each time slot, SU randomly selects a sensing block (L consecutive subchannels) in the range of the wide frequency band. We will derive the performance of RSSS under two cases.

Case 1: $L < N_c$

In this case, the sensing block of SU for a particular time slot can be within the range of one PU block or crosses over two PU blocks (see Fig. 3(a) and Fig. 3(b)). Let's refer to these two events as Event A and Event B respectively. When a SU randomly selects the sensing block, the probabilities of these two events are given by

$$P_{\mathrm{A}} = \Pr(\mathrm{A}) = \frac{K\,(N_c - L + 1)}{N_c\,(K - L + 1)}, \tag{3a}$$

$$P_{\mathrm{B}} = \Pr(\mathrm{B}) = \frac{(L-1)\,(K/N_c - 1)}{(K - L + 1)}. \tag{3b}$$

For Event A, the elements s_k, $k = 0, 1, \ldots, L-1$, of the state vector $\mathbf{S}$ are either all 1 or all 0, following the Bernoulli distribution with parameter P_0. Therefore, the expectation of Y under Event A is given by

$$\begin{aligned}\mathbb{E}[Y|\,\mathrm{A}] &= \mathbb{E}\left[\frac{1}{LP_0}\sum_{k=0}^{L-1} s_k\right] \\ &= \frac{1}{LP_0}\mathbb{E}\left[\sum_{k=0}^{L-1} s_k\right] \\ &= 1. \end{aligned} \tag{4}$$

Event B consists of $L-1$ equiprobable subcases each corresponding to the $L-1$ possible locations of a sensing block considering two adjacent PU blocks. For the sensing block, the number of subchannels in the first PU block (n subchannels) can be $1, 2, \ldots, L-1$ and, accordingly, the number of subchannels in the second PU block is $L-n$. Thus the expectation of Y under Event B can be computed as

$$\begin{aligned}\mathbb{E}[Y|\,\mathrm{B}] &= \mathbb{E}\left[\frac{1}{LP_0}\sum_{k=0}^{L-1} s_k\right] = \frac{1}{LP_0}\mathbb{E}\left[\sum_{k=0}^{L-1} s_k\right] \\ &= \frac{1}{LP_0}\frac{1}{L-1}\sum_{n=1}^{L-1}\left(nP_0P_1 + (L-n)P_0P_1 + LP_0^2\right) \\ &= 1. \end{aligned} \tag{5}$$

Consequently, the expectation of Y, the NACQ metric, can be obtained as

$$\begin{aligned}\mathbb{E}[Y] &= P_{\mathrm{A}}\mathbb{E}[Y|\,\mathrm{A}] + P_{\mathrm{B}}\mathbb{E}[Y|\,\mathrm{B}] \\ &= 1. \end{aligned} \tag{6}$$

Case 2: $L = qN_c$, $q \in \mathbb{Z}^+$

Under Case 2, we assume that sensing block length L is an integral multiple of PU block length N_c and the sensing block aligns with the boundaries of PU blocks, which implies that the SU sensing block covers exactly q PU blocks (see Fig. 3(c)). The number of inactive PUs in the sensing block follows the Binomial distribution with parameters q and P_0. Therefore, the expectation of Y is given as

$$\begin{aligned}\mathbb{E}[Y] &= \mathbb{E}\left[\frac{1}{LP_0}\sum_{k=0}^{L-1} s_k\right] = \frac{1}{LP_0}\mathbb{E}\left[\sum_{k=0}^{L-1} s_k\right] \\ &= \frac{1}{LP_0}qP_0N_c, \end{aligned} \tag{7}$$

where $qN_c = L$. Therefore $\mathbb{E}[Y] = 1$ under Case 2.

Based on the derivations for Case 1 and Case 2, we conclude that RSSS ensures SU to achieve a spectrum sensing result at a level of the average number of available subchannels in a sensing block.

B. PERSISTENT SPECTRUM SENSING STRATEGY

Considering the PU state transition probability, it is possible to devise a spectrum sensing strategy to achieve improved NACQ compared to RSSS. Basically, we intend to move away from the total random approach in selecting a sensing block. Instead, if the SU finds the number of the available subchannels to be equal or larger than the average number of available subchannels in a sensing block, it stays in the current sensing block for the next time slot; Otherwise SU randomly selects a sensing block in the next time slot. This is the idea of proposed persistent spectrum sensing strategy (PSSS). Notice that the average number of available subchannels in a sensing block can be found as $\gamma = \lfloor LP_0 \rfloor$, where $\lfloor \cdot \rfloor$ denotes the floor operator. We thus have the policy of PSSS,

$$\text{Selection of next sensing block} = \begin{cases}\text{Stay in the current sensing block}, & x \geq \gamma \\ \text{Randomly select a sensing block}, & x < \gamma, \end{cases} \tag{8}$$

where x is a value of random variable X, which is the number of available subchannels found by SU for current time slot. Note that, the value of x for a time slot equals to $\sum_{k=0}^{L-1} s_k$, where s_k is the kth element of the state vector $\mathbf{S}$. In the following, we study the NACQ metric under both Case 1 and Case 2.

Case 1: $L < N_c$

Since the sensing capability of SU is L, the number of available subchannels found by SU at time slot t is a random variable denoted as X_t which draws from set $\mathbb{S}_1 = \{0, 1, 2, \ldots, L\}$. In other words, there are $|\mathbb{S}_1| = L+1$ states that X_t could possibly achieve. According to the PSSS policy in (8), SU will either stay in the current sensing block or randomly select another sensing block in time slot $t+1$, depending on the value of X_t in time slot t. Notice that, X_{t+1} only depends on X_t but not earlier states $\{X_{t'}, t' < t\}$. It is easy to see that the sequence of random variables $\{X_1, X_2, \ldots, X_t, \ldots\}$ follows a stationary Markov chain and the state space is $\mathbb{S}_1$, which implies that

$$\begin{aligned}&\Pr\{X_{t+1} = x_{t+1}|X_1 = x_1, X_2 = x_2, \ldots, X_t = x_t\} \\ &\quad = \Pr\{X_{t+1} = x_{t+1}|X_t = x_t\} \\ &\quad = \Pr\{X_{t+m+1} = x_{t+1}|X_{t+m} = x_t\}, \end{aligned} \tag{9}$$

where $x_t \in \mathbb{S}_1$, $t = 1, 2, \ldots$ and m is an arbitrary time shift. Thus the NACQ can be computed by

$$\mathbb{E}[Y] = \frac{1}{LP_0}\sum_{i=0}^{L} iP_i^{c_1}, \tag{10}$$

where $P_i^{c_1}$, $i \in \mathbb{S}_1$, is the ith entry of the stationary distribution of X defined as $\mathbf{P}^{c_1} \triangleq (P_0^{c_1}, P_1^{c_1}, \ldots, P_L^{c_1})$, with each entry $P_i^{c_1}$ being the stationary probability of i availabele subchannels that can be found by SU in a sensing block. The superscript c_1 represents Case 1 for short.

Eq. (10) shows that it requires the stationary distribution of X to compute $\mathbb{E}[Y]$. To this end, following the theorem of DTMC, we construct an $(L+1)\times(L+1)$ probability transition matrix $\mathbf{T}^{c_1}$ to characterize the transition from current state to the next state. The element $T_{ij}^{c_1}$ in the ith row and jth column, $i, j = 0, 1, 2, \ldots, L$, of $\mathbf{T}^{c_1}$ represents the transition probability from state i to state j, i.e., the probability that there are j available subchannels in the next time slot under the condition that there are i available subchannels in current time slot. To obtain the stationary distribution $\mathbf{P}^{c_1}$ of X, we first provide the following theorem.

Theorem 1: The transition matrix $\mathbf{T}^{c_1}$ for **Case** 1 is given as follows.

For $0 \le i < \gamma$,

$$T_{ij}^{c_1} = \begin{cases} P_{\mathrm{A}}P_1 + P_{\mathrm{B}}P_1^2, & j = 0 \\ 2\dfrac{(K/N_c - 1)}{(K-L+1)}P_1P_0, & 1 \le j \le L-1 \\ P_{\mathrm{A}}P_0 + P_{\mathrm{B}}P_0^2, & j = L. \end{cases} \tag{11}$$

For $\gamma \le i \le L-1$,

$$T_{ij}^{c_1} = \begin{cases} P_{01}(1-P_{10}), & j = 0 \\ (1-P_{01})(1-P_{10}), & j = i \\ P_{01}P_{10}, & j = L-i \\ (1-P_{01})P_{10}, & j = L \\ 0, & \text{others}. \end{cases} \tag{12}$$

For $i = L$,

$$T_{ij}^{c_1} = \begin{cases} P_{01} + \Pr(B|i=L)\,P_{01}^2, & j = 0 \\ 2\dfrac{\Pr(B|i=L)}{(L-1)}P_{01}(1-P_{01}), & 1 \le j \le L-1 \\ \Pr(A|i=L)(1-P_{01}) \\ \quad + \Pr(B|i=L)(1-P_{01})^2, & j = L, \end{cases} \tag{13}$$

where the posterior probabilities are given as

$$\Pr(\mathrm{A}|i=L) = \frac{K(N_c - L + 1)}{K(N_c - L + 1) + (L-1)(K-N_c)P_0}, \tag{14a}$$

$$\Pr(\mathrm{B}|i=L) = \frac{(L-1)(K-N_c)P_0}{K(N_c - L + 1) + (L-1)(K-N_c)P_0}. \tag{14b}$$

Proof: See Appendix A. ∎

With Theorem 1, we obtained the state transition matrix of the Markov chain constructed by the sequence of random variables $\{X_1, X_2, \ldots, X_t, \ldots\}$. It can be verified that $\mathbf{T}^{c_1}$ meets the property of a transition matrix that $\sum_{j=0}^{L} T_{ij}^{c_1} = 1$ for an arbitrary i. The process of computing $\mathbf{T}^{c_1}$ is summarized in Table 1.

To find the stationary distribution of X, we define the probability vector of the tth time slot as $\mathbf{P}_t^{c_1} = (P_{t,0}^{c_1}, P_{t,1}^{c_1}, \ldots, P_{t,L}^{c_1})$ with each element $P_{t,i}^{c_1}$, $i = 0, 1, 2, \ldots, L$, representing the probability that there are i available subchannels in the sensing block in the tth time slot. Therefore, the probability vector $\mathbf{P}_{t+1}^{c_1}$ of the $(t+1)$th time slot can be expressed as

$$\mathbf{P}_{t+1}^{c_1} = \mathbf{P}_t^{c_1}\mathbf{T}^{c_1}. \tag{15}$$

TABLE 1. Compute transition matrix for case 1.

A: **Initialization**
Initialize the transition matrix $\mathbf{T}^{c_1}$ as $\mathbf{0}_{(L+1)\times(L+1)}$. Input parameters P_{01}, P_0, K, L and N_c.
B: **Compute Transition Matrix $\mathbf{T}^{c_1}$**
$P_A = K(N_c - L + 1)/(N_c(K - L + 1))$
$P_B = (L-1)(K/N_c - 1)/(K - L + 1)$
for $i = 0$ to $\gamma - 1$ **do**
  for $j = 0$ to L **do**
    if $j = 0$ **do**
      $T_{ij}^{c_1} = P_A P_1 + P_B P_1^2$
    elseif $1 \le j \le L-1$ **do**
      $T_{ij}^{c_1} = 2P_0(1-P_0)(K/N_c - 1)/(K - L + 1)$
    else do
      $T_{ij}^{c_1} = P_A P_0 + P_B P_0^2$
  end for
end for
for $i = \gamma$ to $L-1$ **do**
  for $j = 0$ to L **do**
    if $j = 0$ **do** $T_{ij}^{c_1} = P_{01}(1-P_{10})$
    elseif $j = i$ **do** $T_{ij}^{c_1} = (1-P_{01})(1-P_{10})$
    elseif $j = L-i$ **do** $T_{ij}^{c_1} = P_{01}P_{10}$
    elseif $j = L$ **do** $T_{ij}^{c_1} = (1-P_{01})P_{10}$
    else **do** $T_{ij}^{c_1} = 0$
  end for
end for
$\Pr(A|i=L) = K(N_c-L+1)/(K(N_c-L+1)+P_0(L-1)(K-N_c))$
$\Pr(B|i=L) = P_0(L-1)(K-N_c)/(K(N_c-L+1)+P_0(L-1)(K-N_c))$
for $j = 0$ to L **do**
  if $j = 0$ **do**
    $T_{Lj}^{c_1} = \Pr(A|i=L)P_{01} + \Pr(B|i=L)P_{01}^2$
  elseif $1 \le j \le L-1$ **do**
    $T_{Lj}^{c_1} = P_{01}(1-P_{01})\Pr(B|i=L)/(L-1)$
  else do
    $T_{Lj}^{c_1} = \Pr(A|i=L)(1-P_{01}) + \Pr(B|i=L)(1-P_{01})^2$
end for

Applying (15) recursively yields

$$\mathbf{P}_{t+1}^{c_1} = \mathbf{P}_0^{c_1}(T^{c_1})^{t+1}. \tag{16}$$

where $\mathbf{P}_0^{c_1} = (P_{0,0}^{c_1}, P_{0,1}^{c_1}, \ldots, P_{0,L}^{c_1})$ is the initial probability vector for the first sensing time slot of SU. Particularly under the system model described earlier, SU has no prior knowledge to make decision in the first time slot and randomly selects a sensing block and then follows the policy of PSSS (eq.(8)) in the subsequent time slots. Therefore, $P_{0,i}^{c_1}$ can be computed as

$$P_{0,i}^{c_1} = \begin{cases} P_A P_1 + P_B P_1^2, & i = 0 \\ 2\dfrac{(K/N_c - 1)}{(K-L+1)}P_1P_0, & i = 1, 2, \ldots L-1 \\ P_A P_0 + P_B P_0^2, & i = L. \end{cases} \tag{17}$$

With (16) and (17), the probability vector $\mathbf{P}_t^{c_1}$ at any time slot t can be computed. The stationary distribution can be obtained by the following limit

$$\mathbf{P}^{c_1} = \lim_{t\to\infty} \mathbf{P}_t^{c_1} = \mathbf{P}_0^{c_1} \lim_{t\to\infty} (T^{c_1})^t. \tag{18}$$

And according to the stationary property of Markov chain, we have $\lim_{t\to\infty} (\mathbf{T}^{c_1})^t = \widetilde{\mathbf{T}}^{c_1}$, which implies that $(\mathbf{T}^{c_1})^t$ converges to a constant matrix $\widetilde{\mathbf{T}}^{c_1}$ with increasing t. Therefore, the stationary distribution $\mathbf{P}^{c_1}$ of X can be computed as

$$\mathbf{P}^{c_1} = \mathbf{P}_0^{c_1} \widetilde{\mathbf{T}}^{c_1}. \tag{19}$$

In practice, computing the limit in (18) can be realized using an iterative approach that taking $(\mathbf{T}^{c_1})^t$ as the true $\widetilde{\mathbf{T}}^{c_1}$ if the following condition has been met

$$\max_{i,j} \left| \left(\left(\mathbf{T}^{c_1}\right)^{t+1} - \left(\mathbf{T}^{c_1}\right)^{t} \right)_{ij} \right| \leq \varepsilon_1, \tag{20}$$

where the $(\cdot)_{ij}$ represents the element in the ith row and jth column of a matrix and ε_1 is a preset error precision factor. Finally, the NACQ under Case 1 can be obtained by (10) with the stationary distribution of X.

Case 2: $L = qN_c$, $q \in \mathbb{Z}^+$ Under this case, the assumption of L is the same as that in Section III.A. The sensing block is assumed to be aligned with the boundaries of PU blocks. For simplicity, we can consider the number of inactive PUs in the sensing block rather than the number of available subchannels.

As the sensing block covers q PU blocks, the number of inactive PUs denoted by Z in a particular time slot is a discrete random variable which draws value from the set $\mathbb{S}_2 = \{0, 1, 2, \ldots, q\}$. In other words, there are $|\mathbb{S}_2| = q + 1$ states for the value of Z. Similar to Case 1, the sequence of random variables $\{Z_1, Z_2, \ldots, Z_t, \ldots\}$ can be treated as a stationary Markov chain and the state space is $\mathbb{S}_2$, where Z_t represents the number of inactive PUs in the sensing block of SU in time slot t.

To obtain NACQ under Case 2, we proceed to find the stationary distribution of Z which is defined as $\mathbf{P}^{c_2} \triangleq (P_0^{c_2}, P_1^{c_2}, \ldots, P_L^{c_2})$ and compute $\mathbb{E}[Y]$ with

$$\mathbb{E}[Y] = \frac{1}{qP_0} \sum_{i=0}^{q} iP_i^{c_2}, \tag{21}$$

where $P_i^{c_2}$, $i \in \mathbb{S}_2$, is the ith element of $\mathbf{P}^{c_2}$.

To find the stationary distribution of Z, we construct a probability transition matrix $\mathbf{T}^{c_2}$ for Case 2 with dimension of $(q+1) \times (q+1)$, where superscript c_2 represents Case 2 for short. The element $T_{ij}^{c_2}$ in the ith row and jth column, $i, j = 0, 1, 2, \ldots, q$, represents the probability that there are j inactive PUs in the next time slot under the condition that there are i inactive PUs in the current time slot. The elements of $\mathbf{T}^{c_2}$ can be obtained using the following theorem.

Theorem 2: The transition matrix $\mathbf{T}^{c_2}$ for **Case 2** is given as follows.

TABLE 2. Compute transition matrix for case 2.

A: Initialization

Initialize the transition matrix $\mathbf{T}^{c_2}$ as $\mathbf{0}_{(q+1)\times(q+1)}$. Input parameters P_{01}, P_0 and q.

B: Compute Transition Matrix $\mathbf{T}^{c_2}$

for $i = 0$ to $\gamma' - 1$ **do**
  for $j = 0$ to q **do**
    $T_{ij}^{c_2} = \binom{q}{j} P_0^j (1 - P_0)^{q-j}$
  end for
end for
$P_{10} = P_{01} / (1 - P_0) - P_{01}$
for $i = \gamma'$ to q **do**
  for $j = 0$ to q **do**
    $u = \max(j, q - i)$
    $v = \max(0, i - j)$
    for $k = v$ to $q - u$ **do**
      $T_{ij}^{c_2} = T_{ij}^{c_2} + \binom{i}{k} P_{01}^k (1 - P_{01})^{i-k} \times \binom{q-i}{k-i+j} P_{10}^{k-i+j} (1 - P_{10})^{q-k-j}$
    end for
  end for
end for

For $i < \gamma'$,

$$T_{ij}^{c_2} = \binom{q}{j} P_0{}^j (1 - P_0)^{q-j}, \tag{22}$$

where $0 \leq i < \gamma'$ and $0 \leq j \leq q$.

For $\gamma' \leq i \leq q$,

$$T_{ij}^{c_2} = \sum_{l=v}^{q-u} \binom{i}{l} P_{01}^l (1 - P_{01})^{i-l} \binom{q-i}{l-i+j} \times P_{10}^{(l-i+j)} (1 - P_{10})^{(q-l-j)} \tag{23}$$

where $\gamma' = \left\lfloor \frac{\gamma}{N_c} \right\rfloor$, $0 \leq j \leq q$, $u = \max(j, q - i)$ and $v = \max(0, i - j)$.

Proof: See Appendix B. ■

Theorem 2 provides the state transition matrix of the Markov chain constructed by $\{Z_1, Z_2, \ldots, Z_t, \ldots\}$ in Case 2. The process of computing $\mathbf{T}^{c_2}$ is summarized in Table 2.

Furthermore, we define the probability vector $\mathbf{P}_t^{c_2} = (P_{t,0}^{c_2}, P_{t,1}^{c_2}, \ldots, P_{t,L}^{c_2})$ for Case 2, where $P_{t,i}^{c_2}$, $i = 0, 1, \ldots, L$, is the probability that there are i inactive PUs in the sensing block in the tth time slot. Then we have

$$\mathbf{P}_{t+1}^{c_2} = \mathbf{P}_t^{c_2} \mathbf{T}^{c_2}. \tag{24}$$

$\mathbf{P}_{t+1}^{c_2}$ can be computed recursively

$$\mathbf{P}_{t+1}^{c_2} = \mathbf{P}_0^{c_2} (T^{c_2})^{t+1}, \tag{25}$$

where $\mathbf{P}_0^{c_2} = (P_{0,0}^{c_2}, P_{0,1}^{c_2}, \ldots, P_{0,L}^{c_2})$ is the initial probability vector for Case 2. For the first time slot, SU randomly selects a sensing block and the number of inactive PUs in the sensing block follows the binomial distribution with parameter q and P_0. The elements of $P_0^{c_2}$ can be expressed as

$$P_{0,i}^{c_2} = \binom{q}{i} P_0^i (1 - P_0)^{q-i}, \tag{26}$$

where $0 \le i \le q$. With (26) and (25), the probability vector $\mathbf{P}_t^{c_2}$ of any time slot t can be calculated. Therefore, the stationary distribution of the Markov chain satisfies the following limits

$$\mathbf{P}^{c_2} = \lim_{t\to\infty} \mathbf{P}_t^{c_2} = \mathbf{P}_0^{c_2} \lim_{t\to\infty} (T^{c_2})^t, \tag{27}$$

where $\lim_{t\to\infty} (T^{c_2})^t = \widetilde{\mathbf{T}}^{c_2}$. Thus the stationary distribution of Z is finally obtained as

$$\mathbf{P}^{c_2} = \mathbf{P}_0^{c_2} \widetilde{\mathbf{T}}^{c_2}. \tag{28}$$

Substituting P^{c_2} in (21) yields the NACQ for Case 2. Notice that the computation of the limit in (27) can be implemented using an iterative approach as for Case 1.

In summary, for a given PU statistic model and sensing capability of SU, the NACQ of RSSS and PSSS can be obtained using (6), (7), (10) and (21) for both Case 1 and Case 2.

IV. ADAPTIVE SPECTRUM SENSING STRATEGY

A. RSSS VERSUS PSSS

RSSS and PSSS have been described in Section III. Their performance results will be presented in Section V. It is clear that RSSS NACQ is a constant value (see (6) and (7)), which is independent of the PU traffic statistics, while PSSS performance results depend on PU traffic statistics (e.g., parameters of the PU traffic model). Considering PSSS, there are two interesting traffic scenarios, a stable PU traffic scenario and a highly dynamic PU traffic scenario.

In the stable PU traffic scenario, parameters P_{01} and P_{10} are relatively small and the state of PU (active or inactive) changes slowly. There is a high probability that PU will keep the same state in the next time slot. Consequently, if SU stays in the current sensing block in the next time slot, following the policy of PSSS given by (8), there will be a high probability for SU to obtain/find a similar level of available subchannels in the next time slot, which is above the average level (i.e., RSSS performance). Therefore, on average, SU will obtain a higher NACQ in PSSS than that in RSSS. In dynamic PU traffic scenario, parameters P_{01} and P_{10} are relatively large and PU tends to change its state more frequently between adjacent time slots. Following the policy in PSSS, SU will stay in the same sensing block in the next time slot if meeting the stated condition. However, PU will likely transit to its opposite traffic state with a high probability, which means that SU will obtain/find the number of available subchannels below the average level in the next time slot. Thus, on average, SU will obtain a NACQ lower than than in RSSS.

B. ADAPTIVE SPECTRUM SENSING STRATEGY

Based on the considerations in Section IV.A, we conclude that the proposed PSSS outperforms RSSS in a certain range of the PU traffic parameters, while for other PU traffic parameter values, the contrary is true. Therefore, we are able to develop an adaptive spectrum sensing strategy (ASSS) based on PSSS and RSSS using PU traffic parameters for adaptation. Basically, in ASSS, PSSS or RSSS is selected adaptively to conduct spectrum sensing at a given time according to PU traffic parameter values. Thus, the challenge is to determine the PU traffic parameter range for selecting PSSS operation and that for RSSS operation.

To this end, it is helpful to find the values of the PU traffic parameters at the intersection point of the PSSS and RSSS performance. We first let the NACQ of PSSS ((10) and (21)) equal to 1 (i.e., the RSSS NACQ) for both Case 1 and Case 2 respectively,

$$\frac{1}{LP_0} \sum_{i=0}^{L} i P_i^{c_1} = 1. \tag{29}$$

$$\frac{1}{LP_0} \sum_{i=0}^{q} i P_i^{c_2} = 1. \tag{30}$$

Notice that the left hand side of (29) and (30) is related to K, N_c and L besides the PU traffic parameters. For given K, N_c and L, finding the values of PU traffic parameters at the intersection point of the performance of RSSS and PSSS for both Case 1 and Case 2 is equivalent to solving (29) and (30) respectively. In (29) and (30), there are four PU traffic parameters (i.e., P_{01}, P_{10}, P_0 and P_1). According to (1), we only need P_{01} and P_0 to fully describe the PU model.

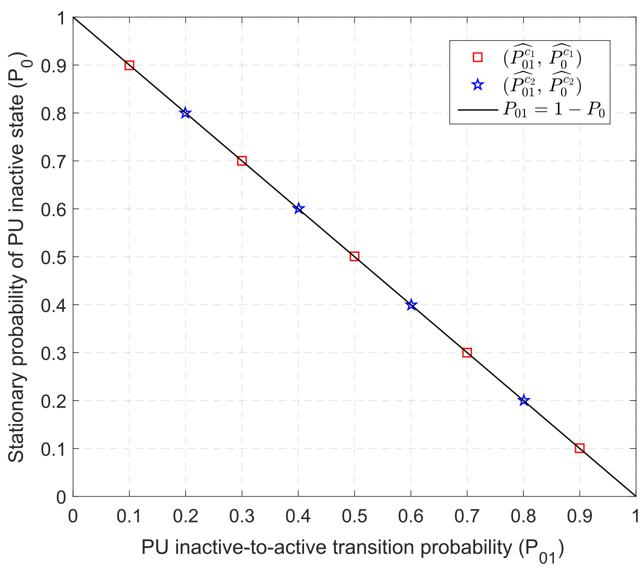


FIGURE 4. Numerical results of intersection point for Case 1 and Case 2.

As shown in Section III, P_{01} and P_0 are contained in the transition matrix ($\mathbf{T}^{c_1}$ and $\mathbf{T}^{c_2}$), which are on the left side of (29) and (30). Both (29) and (30) are difficult to derive analytically in closed forms. Instead, we use numerical methods to search P_{01} and P_0 values that satisfy (29) and (30). Based on numerical solutions (see results in Fig. 4), we observe that, the values of PU traffic parameters, $(\widehat{P_{01}^{c_1}}, \widehat{P_0^{c_1}})$ and $(\widehat{P_{01}^{c_2}}, \widehat{P_0^{c_2}})$, for Case 1 and Case 2 at the

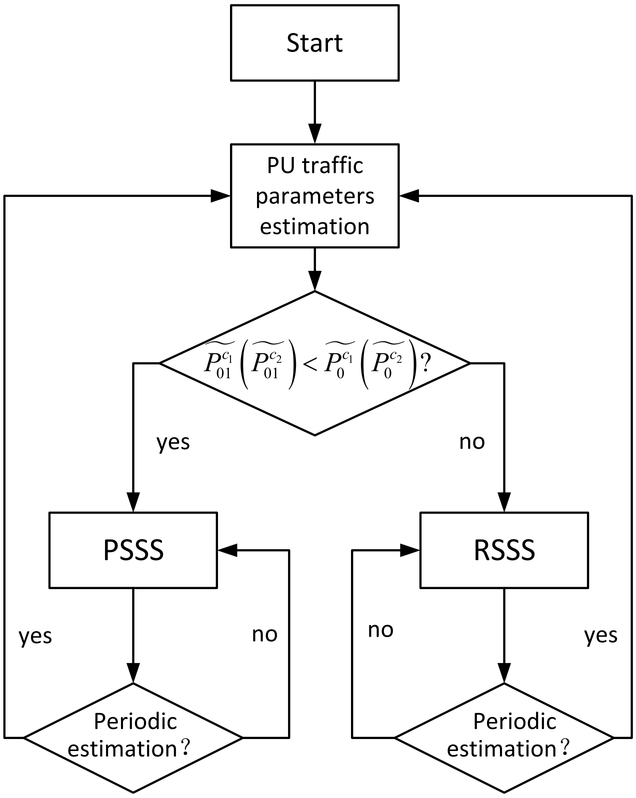


FIGURE 5. The flow chart of ASSS algorithm.

intersection point of the NACQ of PSSS and RSSS match the function $P_{01} = 1 - P_0$ very well, i.e.,

$$\widehat{P_{01}^{c_1}} = 1 - \widehat{P_0^{c_1}}, \tag{31}$$

$$\widehat{P_{01}^{c_2}} = 1 - \widehat{P_0^{c_2}}. \tag{32}$$

Eq. (31) and (32) imply that, for both Case 1 and Case 2, if P_{01} is smaller than $1-P_0$, PSSS outperforms RSSS and PSSS is selected for spectrum sensing; otherwise, RSSS is selected.

C. ESTIMATION OF PU TRAFFIC PARAMETERS

From the descriptions above, it requires the knowledge of real-time PU traffic statistics (i.e., P_{01} and P_0) to implement ASSS. Therefore, we present a simple and effective method for estimating P_{01} and P_0. The estimates of (P_{01}, P_0) under Case 1 and Case 2, denoted as $(\widetilde{P_{01}^{c_1}}, \widetilde{P_0^{c_1}})$ and $(\widetilde{P_{01}^{c_2}}, \widetilde{P_0^{c_2}})$, can be obtained by monitoring the state transition of PUs. In Case 1, SU is able to sense the state transition of one PU and, in Case 2, SU is able to sense the state transition of q PUs. As a result, the estimates of PU traffic parameters for Case 1 and Case 2 can be expressed as follows.

For Case 1, $\widetilde{P_{01}^{c_1}}$ and $\widetilde{P_0^{c_1}}$ are given by

$$\widetilde{P_{01}^{c_1}} = \frac{N_{01}^{c_1}}{N_{01}^{c_1} + N_{00}^{c_1}}, \tag{33a}$$

$$\widetilde{P_0^{c_1}} = \frac{N_0^{c_1}}{N^{c_1}}, \tag{33b}$$

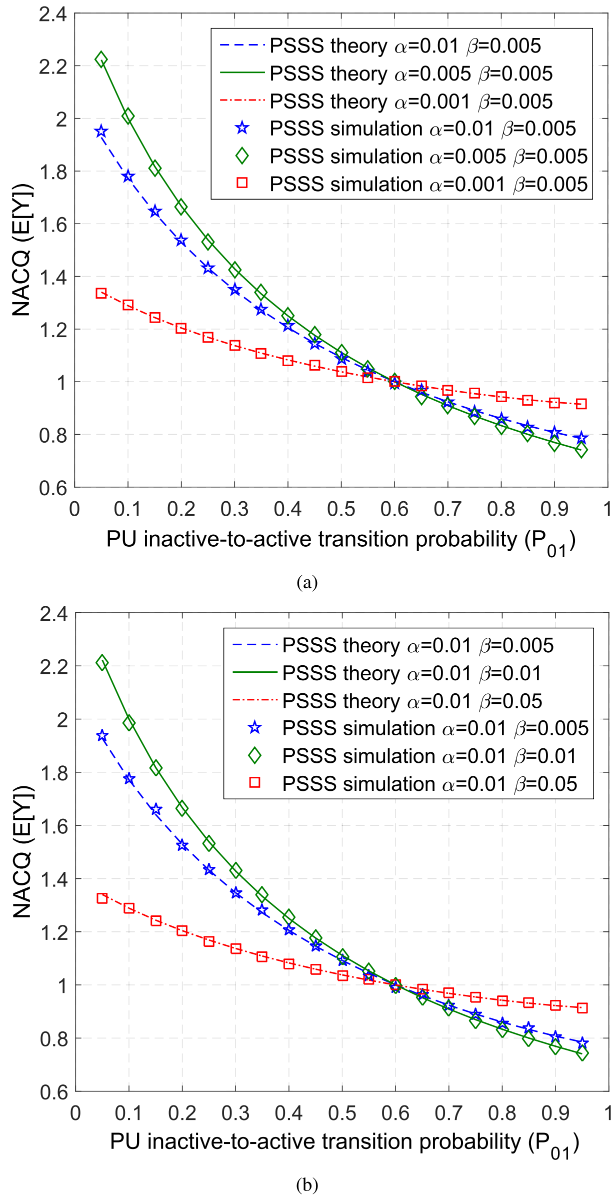


FIGURE 6. Spectrum sensing efficiency of PSSS with $P_0 = 0.4$. (a) Fixed β with different α; (b) Fixed α with different β.

where $N_{01}^{c_1}$ and $N_{00}^{c_1}$ are the number of PU inactive-to-active state transitions and inactive-to-inactive state transitions between adjacent time slots, respectively, and $N_0^{c_1}$ is the total number of time slots that PU is in an inactive state and N^{c_1} is total number of monitored time slots.

For Case 2, $\widetilde{P_{01}^{c_2}}$ and $\widetilde{P_0^{c_2}}$ are given by

$$\widetilde{P_{01}^{c_2}} = \frac{\sum_{i=1}^{q} N_{01}^{c_2}(i)}{\sum_{i=1}^{q} N_{01}^{c_2}(i) + \sum_{i=1}^{q} N_{00}^{c_2}(i)}, \tag{34a}$$

$$\widetilde{P_0^{c_2}} = \frac{\sum_{i=1}^{q} N_0^{c_2}(i)}{qN^{c_2}}, \tag{34b}$$

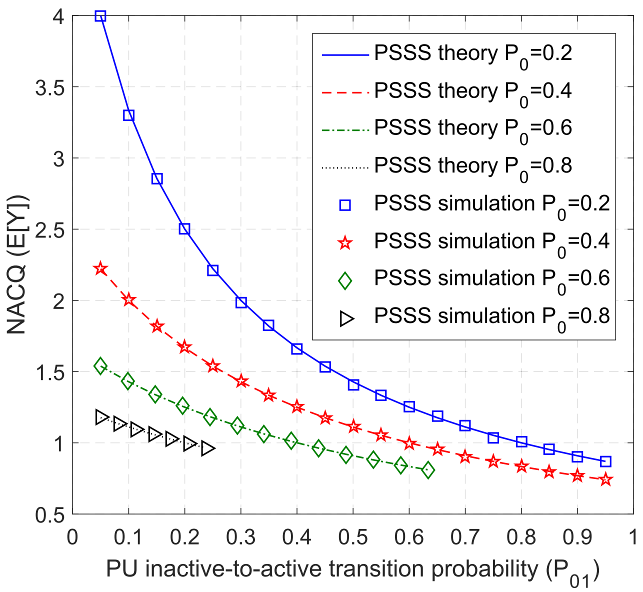


FIGURE 7. **Spectrum sensing efficiency of PSSS under different P_0 with $\alpha = 0.005$ and $\beta = 0.005$.**

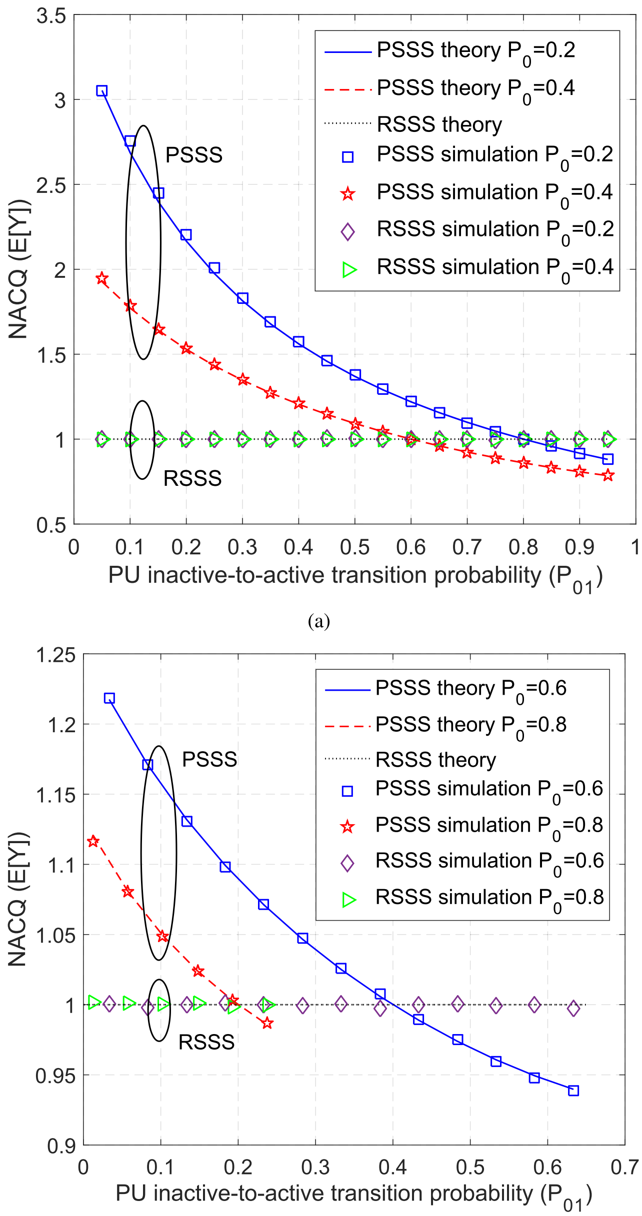
where $N_{01}^{c2}(i)$ and $N_{00}^{c2}(i)$ are the number of inactive-to-active state transitions and inactive-to-inactive state transitions between adjacent time slots, respectively, of the ith PU in the sensing block, $N_0^{c2}(i)$ is the number of time slots that the ith PU is in an inactive state, and N^{c2} is the total number of monitored slots. With (33) and (34), the PU traffic parameters are estimated and the impact of the estimation accuracy on ASSS performance will be discussed in the next section.

With the estimates of P_{01} and P_0, we can follow the process depicted in Fig. 5 to perform ASSS. To consider the potential changes of the PU traffic parameters, periodic estimations of the PU traffic parameters are included in Fig. 5

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, theoretical and simulation results are presented to evaluate the performance of RSSS, PSSS and ASSS. We introduce two factors, $\alpha = N_c/K$, PU block length normalized to the total number of subchannels, and $\beta = L/K$, SU sensing block length normalized to the total number of subchannels. During the following simulations, K is set to be 1000 and P_{01} is set as 0.05 : 0.05 : 0.95. Also, in the following simulations, we assume perfect spectrum sensing which means the sensing error is ignored in order to focus on designing subchannel selection strategies.

Fig. 6 illustrates the NACQ of PSSS versus P_{01} under different parameter settings, and the stationary probability of PU inactive state P_0 is fixed as 0.4. In Fig. 6(a), the factor β is fixed while in Fig. 6(b) the factor α is fixed. Based on the values of α and β in Fig. 6(a) and Fig. 6(b), both performance results for Case 1 and Case 2 are presented. As shown in Fig. 6, both theoretical and simulation are presented and they match very well. It is seen that, in PSSS, the NACQ drops with increased P_{01}, which agrees with the discussions in Section IV.A.

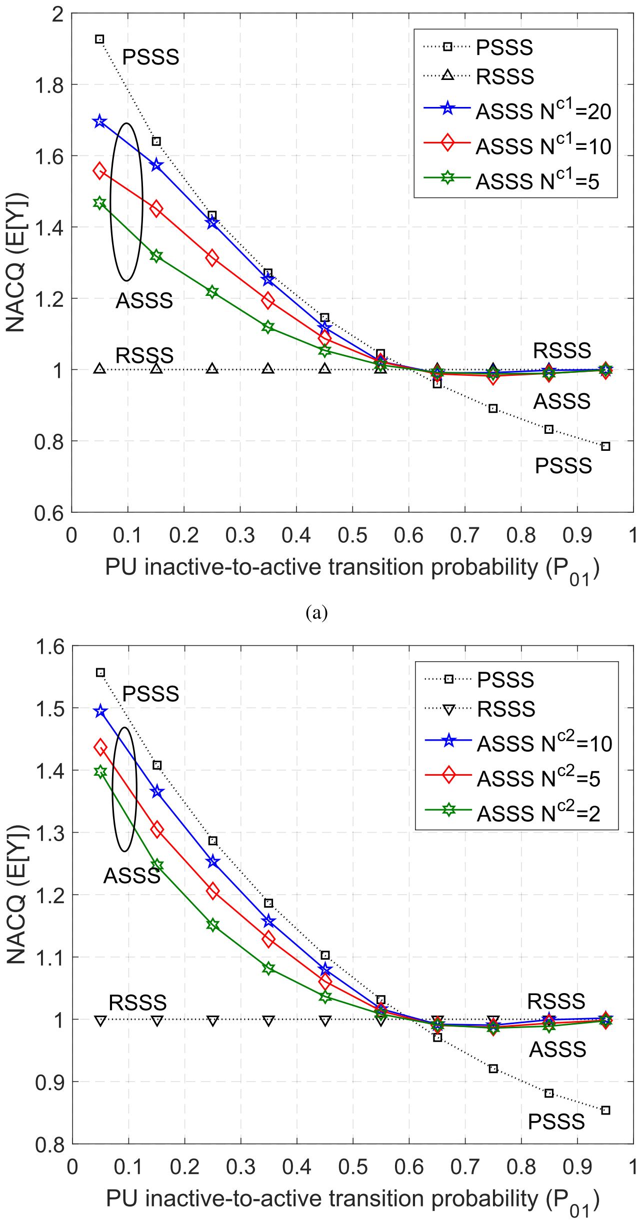


(a)

(b)

FIGURE 8. **Performance comparison between RSSS and PSSS. (a) Case 1, $\alpha = 0.01$ and $\beta = 0.005$; (b) Case 2, $\alpha = 0.005$ and $\beta = 0.025$.**

The performance of PSSS under various values of P_0 are illustrated in Fig. 7 with both α and β being fixed as 0.005. Notice that the parameters in the PU traffic model P_{01}, P_{10}, P_0 and P_1 follow the relationship in (1). The maximum value that P_{01} can achieve is $\max(1, (1 - P_0)/P_0)$. Therefore, for $P_0 <= 0.5$, P_{01} can be set from 0 to 1. On the other hand, for $P_0 > 0.5$, the value of P_{01} is set from 0 to $(1 - P_0)/P_0$. Fig. 7 shows that the theoretical analysis matches well with the simulation results under various values of P_{01} and P_0. Further, the NACQ $\mathbb{E}[Y]$ is a value normalized by the average number of available subchannels that can be found in a sensing block. Therefore, as shown in Fig. 7, larger P_0 implies more available subchannels and it reduces the percentage of the available subchannels found by SU.



(a)

(b)

FIGURE 9. Performance of ASSS with different estimation length with $P_0 = 0.4$. (a) Case 1, $\alpha = 0.01$ and $\beta = 0.005$; (b) Case 2, $\alpha = 0.005$ and $\beta = 0.01$.

The performance comparisons of PSSS and RSSS under Case 1 and Case 2 are given by Fig. 8(a) and Fig. 8(b) respectively. In Case 1, the factors α and β are set as 0.01 and 0.005, respectively, and in Case 2, they are set as 0.005 and 0.01, respectively. As shown in Fig. 8(a) and Fig. 8(b) (and (6) and (7)), the NACQ of RSSS is 1. Additionally, PSSS outperforms RSSS when P_{01} is relatively small while the contrary is true if P_{01} is relatively large, which agrees with the discussions presented in Section IV.A. The location of the intersection point of PSSS and RSSS varies with different values of P_0, which is consistent with (31), (32) and Fig. 4.

In Fig. 9, simulation results are presented to show the performance of ASSS and its comparison with PSSS and RSSS for both Case 1 and Case 2. As described in Section IV.C, the PU traffic parameter estimation method ((33) and (34)) is utilized in implementing ASSS. With the estimated values of $\widetilde{P_{01}^{c_1}}$, $\widetilde{P_0^{c_1}}$, $\widetilde{P_{01}^{c_2}}$ and $\widetilde{P_0^{c_2}}$, ASSS adaptively selects RSSS or PSSS to perform spectrum sensing according to the process shown in Fig. 5. It is observed that ASSS performance approaches PSSS performance when P_{01} is relatively small and it approaches RSSS performance when P_{01} is relatively large. Further, it is seen that the performance of ASSS is influenced by the PU traffic parameter estimation duration (N^{c_1} and N^{c_2}), which determines the parameter estimation accuracy. According to the ASSS policy described in Section IV.B, ASSS uses the estimated parameters to decide which spectrum sensing strategy to select. If the parameter estimation is not accurate enough, there exists a chance that the ASSS algorithm selects a wrong spectrum sensing strategy in a particular range of PU traffic parameters, leading to the performance loss as compared to the ideal situation (PSSS in lower P_{01} region or RSSS in higher P_{01} region). However, as long as N^{c_1} and N^{c_2} are sufficiently large, the performance of ASSS approaches the ideal results. From Fig. 9, we also show that with a stronger SU sensing capability (Case 2), ASSS requires fewer estimation slots to achieve the ideal results.

VI. CONCLUSION

In this paper, we have considered the multiband spectrum sensing problem under SU hardware limitation, where an SU can only sense a small portion of the total frequency band for a given time period. We defined a performance metric in terms of NACQ and two spectrum sensing strategies, RSSS and PSSS, are presented. The expressions of the NACQ of RSSS and PSSS are derived. Theoretical and simulation results reveal the performance difference between RSSS and PSSS in various ranges of PU traffic parameters. An ASSS algorithm is then proposed to adaptively select a spectrum sensing strategy (RSSS or PSSS) according to the values of PU traffic parameters. An effective and simple method is provided to estimate the PU traffic parameters. The proposed ASSS algorithm is shown to take the advantages of both RSSS and PSSS and it achieves a good NACQ performance under a wide range of PU traffic scenarios.

APPENDIX A
THE PROOF OF THEOREM 1

Notice that SU will take one of the two actions in (8) depending on the value of i which represents the available subchannels in the current time slot. Corresponding to different values of i, the following three subcases are considered.

Subcase 1-1: $0 \le i < \gamma$

For $0 \le i < \gamma$, i.e., $x < \gamma$, SU will randomly select a sensing block in the next time slot and the random variable X will reach to state j, $j \in \mathbb{S}_1$. Since the sensing block of SU in the next time slot can be within the range of one PU block (i.e., Event A) or crosses over two PU blocks (i.e., Event B), $T_{ij}^{c_1}$ can be computed by considering different values of j.

If $j = 0$, which implies no available subchannels in the sensing block in the next time slot, the PU block (or the two PU blocks) involved in the sensing block is (are) in an active state. Thus the transition probability $T_{ij}^{c_1}, j = 0$, is

$$T_{ij}^{c_1} = P_{\rm A}P_1 + P_{\rm B}P_1^2, \tag{35}$$

where $P_{\rm A}$ and $P_{\rm B}$ are given in (3a) and (3b), respectively.

For a specific value of $j \in \{1, 2, \ldots, L-1\}$, which only occurs in Event B with the probability of $P_{\rm B}/(L-1)$, the sensing block crosses over two PU blocks and the two PUs are in opposite states. Therefore, the transition probability $T_{ij}^{c_1}$ is computed as

$$T_{ij}^{c_1} = 2\frac{(K/N_c - 1)}{(K - L + 1)}P_1P_0. \tag{36}$$

For $j = L$, the computation is similar to that for $j = 0$ except that the PU block (or the two PU blocks) involved in the sensing block is (are) in an inactive state. Thus the transition probability $T_{ij}^{c_1}, j = L$, is

$$T_{ij}^{c_1} = P_{\rm A}P_0 + P_{\rm B}P_0^2 \tag{37}$$

With (35), (36) and (37), $T_{ij}^{c_1}$ for $0 \le i < \gamma$ in (11) is obtained.

Subcase 1-2: $\gamma \le i \le L-1$

For $\gamma \le i < L-1$, SU will stay in the same sensing block for the next time slot. Notice that, under this subcase, the sensing block in current time slot must cross over two PU blocks (Event B only). For a specific value of i, the sensing block covers i subchannels of one PU block, which is in an inactive state, and covers $L-i$ subchannels of an adjacent PU block, which is in an active state. Thus, there will be four possible values of j, $j \in \{0, i, L-i, L\}$, the available subchannels in the next time slot, depending on the state transitions of the two PU blocks involved. The conditional probabilities of these four possible values of j can be calculated as

$$\Pr(j = 0|i) = P_{01}(1 - P_{10}), \tag{38a}$$
$$\Pr(j = i|i) = (1 - P_{01})(1 - P_{10}), \tag{38b}$$
$$\Pr(j = L - i|i) = P_{01}P_{10}, \tag{38c}$$
$$\Pr(j = L|i) = (1 - P_{01})P_{10}. \tag{38d}$$

Therefore, the transition probability $T_{ij}^{c_1}$ for $\gamma \le i < L-1$ can be obtained as (12) .

Subcase 1-3: $i = L$

When $i = L$, both Event A and Event B need to be considered. Clearly, SU will stay in the same sensing block and the number of available subchannels, j, in the next time slot depends on whether the sensing block involves one PU block or two PU blocks in the current time slot. Therefore, we first study the posterior probabilities $\Pr(\mathrm{A}|i = L)$ and $\Pr(\mathrm{B}|i = L)$, which can be obtained by using the Bayes theorem

$$\Pr(\mathrm{A}|i = L) = \frac{\Pr(i = L|\mathrm{A})\Pr(\mathrm{A})}{\Pr(i = L)}, \tag{39a}$$
$$\Pr(\mathrm{B}|i = L) = \frac{\Pr(i = L|\mathrm{B})\Pr(\mathrm{B})}{\Pr(i = L)}. \tag{39b}$$

For (39a), Pr(A) is given by (3a). The conditional probability $\Pr(i = L|\mathrm{A})$ is P_0 and the probability $\Pr(i = L)$ can be expressed as

$$\Pr(i = L) = P_{\rm A}P_0 + P_{\rm B}{P_0}^2. \tag{40}$$

Similarly, $\Pr(\mathrm{B}|i = L)$ can be found. As a result, the final expressions of $\Pr(\mathrm{A}|i = L)$ and $\Pr(\mathrm{B}|i = L)$ given in (14a) and (14b) can be derived.

Afterwards, the transition probability $T_{ij}^{c_1}$ can be computed by considering different values of j. For $j = 0$, the involved PU should transit from inactive state to active state under Event A and the two PUs should transit from inactive state to active state under Event B. Therefore, the transition probability $T_{ij}^{c_1}, j = 0$, is

$$T_{ij}^{c_1} = \Pr(A|i = L)P_{01} + \Pr(B|i = L)P_{01}^2. \tag{41}$$

For a specific value of $j \in \{1, 2, \ldots, L-1\}$, which only occurs in Event B with probability $\Pr(\mathrm{B}|i = L)/(L-1)$, the two PU blocks involved in the sensing block are in opposite states. The transition probability $T_{ij}^{c_1}, j = 1, 2, \ldots, L-1$, is computed as

$$T_{ij}^{c_1} = 2\frac{\Pr(B|i = L)}{(L-1)}P_{01}(1 - P_{01}). \tag{42}$$

For $j = L$, the analysis is similar to that of $j = 0$ except that the involved PU (or PU blocks) stays inactive state in the next time slot. Thus the transition probability $T_{ij}^{c_1}, j = L$, is given by

$$T_{ij}^{c_1} = \Pr(A|i = L)(1 - P_{01}) + \Pr(B|i = L)(1 - P_{01})^2. \tag{43}$$

With (41), (42) and (43), the expression of transition probability for $i = L$ in (13) is obtained.

APPENDIX B
THE PROOF OF THEOREM 2

To find $\mathbf{T}^{c_2}$, we follow the decision rule in (8) while the threshold used in Case 2 can be equivalently replaced as $\gamma' = \lfloor \gamma/N_c \rfloor$ as we only consider the number of inactive PUs in the sensing block. Considering the two possible actions taken by SU, we split the process of computing $\mathbf{T}^{c_2}$ into two subcases.

Subcase 2-1: $i < \gamma'$

In this subcase, SU will randomly select another sensing block in the next time slot. It is easy to show that the number of an inactive PUs j in a new sensing block follows the binomial distribution with parameter q and P_0, i.e., $j \sim B(q, P_0)$, where P_0 is the stationary probability of inactive state for a given PU block. Therefore, transition probability $T_{ij}^{c_2}$ is calculated as

$$T_{ij}^{c_2} = \binom{q}{j}{P_0}^j(1 - P_0)^{q-j}, \tag{44}$$

where $0 \le i < \gamma'$ and $0 \le j \le q$.

Subcase 2-2: $\gamma' \le i \le q$

In this subcase, SU stays in the current sensing block in the next time slot and there are i inactive PUs and $q-i$ active PUs in current time slot. Notice that the number of inactive PUs, j, in the next time slot is a result of various combinations of transition patterns of all the PUs in current time slot. For convenience, we denote the number of PUs that transit from an inactive state to an active state as l. The number of PUs that transit from an active state to an inactive state is l', which equals $j+l-i$. Also notice that $l \sim B(i, P_{01})$ and $l' \sim B(q-i, P_{10})$. In computing $T_{ij}^{c_2}, j = 0, 1, 2, \ldots, q$, for a particular i, we consider two separate parts of the value of j, $0 \le j \le i$ and $i < j \le q$.

• Part 1: $0 \le j \le i$

If $j \le q-i$, to obtain the number of inactive PUs, j, in the next time slot, the possible value of l is from $\{i-j, i-j+1, \ldots, i\}$. Correspondingly, the possible value of l' is from $\{0, 1, \ldots, j\}$. The total probability that these combinations of transition patterns for the next time slot is the transition probability $T_{ij}^{c_2}$ which is given by

$$T_{ij}^{c_2} = \sum_{l=i-j}^{i} \binom{i}{l} P_{01}^{l}(1-P_{01})^{i-l} \binom{q-i}{l-i+j} \times P_{10}^{(l-i+j)}(1-P_{10})^{(q-l-j)}. \quad (45)$$

If $j > q-i$, $l \in \{i-j, i-j+1, \ldots, q-j\}$ and $l' \in \{0, 1, \ldots, q-i\}$, the transition probability $T_{ij}^{c_2}$ is computed as

$$T_{ij}^{c_2} = \sum_{l=i-j}^{q-j} \binom{i}{l} P_{01}^{l}(1-P_{01})^{i-l} \binom{q-i}{l-i+j} \times P_{10}^{(l-i+j)}(1-P_{10})^{(q-l-j)}. \quad (46)$$

• Part 2: $i < j \le q$

If $j \le q-i$, $l \in \{0, 1, \ldots, i\}$ and $l' \in \{j-i, j-i+1, \ldots, j\}$, the transition probability $T_{ij}^{c_2}$ is derived as

$$T_{ij}^{c_2} = \sum_{l=0}^{i} \binom{i}{l} P_{01}^{l}(1-P_{01})^{i-l} \binom{q-i}{l-i+j} \times P_{10}^{(l-i+j)}(1-P_{10})^{(q-l-j)}. \quad (47)$$

If $j > q-i$, $l \in \{0, 1, \ldots, q-j\}$ and $l' \in \{j-i, j-i+1, \ldots, q-i\}$, $T_{ij}^{c_2}$ is given by

$$T_{ij}^{c_2} = \sum_{l=0}^{q-j} \binom{i}{l} P_{01}^{l}(1-P_{01})^{i-l} \binom{q-i}{l-i+j} \times P_{10}^{(l-i+j)}(1-P_{10})^{(q-l-j)}. \quad (48)$$

Combining (45)∼(48), the state transition probability $T_{ij}^{c_2}$ for Subcase 2-2 can be written in a more compact form,

$$T_{ij}^{c_2} = \sum_{l=v}^{q-u} \binom{i}{l} P_{01}^{l}(1-P_{01})^{i-l} \binom{q-i}{l-i+j} \times P_{10}^{(l-i+j)}(1-P_{10})^{(q-l-j)}, \quad (49)$$

where $\gamma' \le i \le q$, $0 \le j \le q$, $u = \max(j, q-i)$ and $v = \max(0, i-j)$.

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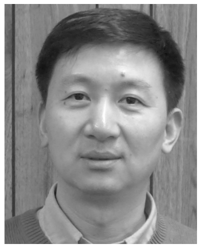

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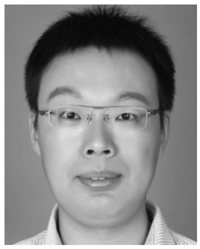

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