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New Cumulative Sum Control Chart for Monitoring Poisson Processes

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ABSTRACT The cumulative sum (CUSUM) control charts are widely used for measurement control of continuous processes. However, the quality characteristics of interest in many production processes, follows a sequence of discrete counts for non-conformities often modeled using a Poisson distribution. This paper introduces new CUSUM control chart design structure to monitor the location of a Poisson parameter. The proposed two-sided scheme is based on ranked set sampling, a more well-structured data collection method than the traditional random sampling. Extensive simulations were used to compute the average, standard deviation and percentiles of the run-length distribution for the new Poisson CUSUM charts. Relative run-length performances achieved were compared with the classical schemes for monitoring improvements or deteriorations in a Poisson process. Consequently, it turns out that the new scheme has greatly enhanced the sensitivity of the classical chart in detecting changes in Poisson processes. The practical application of the new Poisson CUSUM chart is illustrated through a numerical example.

INDEX TERMS Average run length, cumulative sum, poisson processes, ranked set sampling, statistical process control.

I. INTRODUCTION

Cumulative sum (CUSUM) control charts are effective tools widely used to monitor various aspects of production processes. The scheme was introduced by Page [1] and has proven to be more efficient in detecting small and moderate changes in quality characteristics of interest than the classical control charts, such as the Shewhart control chart. The CUSUM statistics are based on information from the past and present data points which gives the scheme a tighter process control that allows early detection of changes in a normal process. Besides monitoring of continuous data, the scheme has found applications in monitoring sequence of discrete count data often modeled with a Poisson distribution. The count data CUSUM, popularly known as the Poisson CUSUM chart, was first suggested by Brook and Evans [2] to monitor the location of a Poisson parameter. The scheme is a superior alternative to the traditional Shewhart *c*-chart or *u*-chart for monitoring quality characteristics that take on attribute data [3]. The *c*-chart is based on single observation while the *u*-chart is applicable to multiple observations [4].

Various approaches on the enhancements of Poisson CUSUM charts to monitor count data have been proposed. Lucas [5] gave the necessary design structure and implementation procedures for the development of Poisson CUSUM chart. Using run-length properties, White *et al.* [6] gave a comprehensive comparison between Poisson CUSUM chart and Shewhart *c*-chart for monitoring the mean of nonconforming units. The theoretical foundation of Poisson CUSUM chart was provided by Hawkins and Olwell [7] and showed that departures from assumed Poisson distribution have adverse effects on performance of the Poisson CUSUM control chart. Chen *et al.* [8] studied the situation where the count data follows a compound Poisson distribution by developing the geometric Poisson CUSUM control scheme. For more recent studies on the Poisson CUSUM chart, see Han *et al.* [9], Ryan and Woodall [10], Jiang *et al.* [11], Saghir and Lin [12], and He *et al.* [13]. The exponentially weighted moving average (EWMA) control chart introduced by Roberts [14] and commonly used to monitor continuous data is an alternative to the CUSUM control chart that may also be used for controlling count data. See, for example, Gan [15], Borror *et al.* [16], Zhang *et al.* [17], Sparks *et al.* [18], Shu *et al.* [19] and Abujiya, *et al.* [20].

Recently, ranked set sampling (RSS) technique has found applications in statistical quality control to enhance the efficiency of a production process. Traditionally, the theories of control charts are based on simple random sampling (SRS) which often yields observations with highly skewed distribution that affect their performance as a result of wide control limits. To improve the effectiveness of several control chart procedures, a more well-structured sampling schemes, such as RSS [21]–[26] and repetitive sampling [27], [28], that give tighter control limits have been employed to enhance measurement control and reduce manufacturing cost. The RSS protocol involves the collection of n random observations, each of size n units, from the target population. Rank the n units within each set with respect to a quality characteristic of interest by visual inspection or some negligible cost method. The n measurements are obtained by selecting the smallest ranked sample $X_{(1:n)}$ in the first set, the second smallest ranked sample $X_{(2:n)}$ in the second set, and this process of quantification continues until the largest sample $X_{(n:n)}$ is selected in the last set. These $X_{(1:n)}, X_{(2:n)}, \ldots, X_{(n:n)}$ represents a single cycle of RSS data for subgroup size n. The cycle may be repeated m times for a total of n^2m samples collected from the population, where only nm observations have actually been measured [29], [30].

The RSS method introduced by McIntyre [29] is an alternative to the traditional SRS technique that offers improvements in parameter estimates in practical applications where the actual measurements of quality characteristics of interest may be costly, time-consuming or even destructive but could be ranked by visual inspection or some economical method without actual measurements [29], [30]. The scheme is at least as efficient as SRS with the same number of quantification when there are ranking errors [31]. Although, the concept of RSS first found its application in estimating mean pasture and forage yields [29], the scheme has since been applied to several other areas. For example, the scheme has found applications in environmental and ecological studies [31], [32], reliability theories [33], medical studies [34], marketing and economics [35], and statistical quality control [21], [23], [24], [36], [37]. Recent theoretical developments and applications of RSS can be found in Wolfe [38], and Al-Omari and Bouza [39]. In this article, we propose a new Poisson CUSUM control chart using RSS methods for efficient monitoring of changes in a mean of a Poisson process.

The rest of the article is organized as follows. In the next section, we present an overview of the classical Poisson CUSUM chart using SRS. Next is the design structure of the proposed Poisson CUSUM control charts using RSS scheme to monitor Poisson process location parameter. Section IV presents, a detail simulation study and the corresponding statistical performance of the proposed new Poisson CUSUM chart based on RSS and its variants in terms of the run-length properties. Furthermore, the numerical comparison of the proposed Poisson control charts and the classical schemes for monitoring changes in a Poisson process location parameter is also presented in Section IV. The effect of imperfectness in ranking of units on performance of the proposed schemes is studied in Section V. Numerical examples are provided to demonstrate the practical application of the proposed Poisson CUSUM charts in Section VI. Finally, some concluding remarks are given in Section VII.

II. OVERVIEW OF CLASSICAL POISSON CUSUM CHART

In classical Poisson CUSUM chart, the quality characteristic of interest is the mean parameter μ of a Poisson process obtained via SRS data [5]. To detect changes in μ, the in-control process mean value μ_0 is used to design and operate a Poisson CUSUM chart either with individual or subgroup observations. Let $x_{1j}, x_{2j}, x_{3j}, \ldots, x_{nj}; j = 1, 2, \ldots, m$ be independent Poisson random samples of subgroup size n with mean μ. The Poisson process is deemed out-of-control if there is a shift from the in-control mean of $\mu = \mu_0$ to an unknown mean value $\mu = \mu_1$ where $\mu_1 \neq \mu_0$. The mean $\hat{\mu}_{SRS\,j} = (1/n)\sum_{i=1}^{n} x_{ij}$, of the j^{th} sample, is an unbiased estimator of the process parameter μ with variance μ/n.

The classical Poisson CUSUM control statistics for monitoring increases ($\mu_1 > \mu_0$) and decreases ($\mu_1 < \mu_0$) in the process mean are respectively given by [5]

$$S^{+}_{SRS\,j} = \max\left[0, \hat{\mu}_{SRS\,j} - k + S^{+}_{SRS\,j-1}\right]$$
$$S^{-}_{SRS\,j} = \max\left[0, k - \hat{\mu}_{SRS\,j} + S^{-}_{SRS\,j-1}\right], \tag{1}$$

where max (a, b) is the maximum of a and b; the constant k is called the reference value. While $\hat{\mu}_{SRS\,j}$ represents the current information, the statistics $S^{+}_{SRS\,j-1}$ and $S^{-}_{SRS\,j-1}$ are the accumulated differences between $\hat{\mu}_{SRS\,j}$ and k representing the past information. A standard CUSUM chart has a starting value of $S^{+}_{SRS0} = S^{-}_{SRS0} = 0$. Here, the Poisson CUSUM control chart gives an out-of-control signal when either $S^{+}_{SRS\,j}$ or $S^{-}_{SRS\,j}$ exceeds the predetermined decision interval $h > 0$, called the control limit.

III. DESIGN OF THE NEW POISSON CUSUM CHART

This section presents the design procedure for the construction of a Poisson CUSUM control chart to monitor the mean of a Poisson process based on RSS technique. The new chart is designed with the primary aim of increasing the sensitivity and overall effectiveness of the basic Poisson CUSUM chart to detect all kinds of fluctuations in a Poisson process. RSS has played a significant role in improving the performance of various control chart schemes at detecting wide range of shifts in a process, particularly the CUSUM control charts [22], [25], [26], [40]. Abujiya *et al.* [41] also, used RSS to enhance the performance of the classical Poisson EWMA control chart to monitor the mean of count data.

Define $X_{(i:n)j}$, $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$ to be the i^{th} order statistic for the i^{th} observation from the Poisson distribution of subgroup size n in j^{th} cycle. The unbiased estimator for the Poisson process location parameter based on RSS technique, is given by Barnett and Barreto [42]

$$\hat{\mu}_{RSS\,j} = \frac{\gamma_n}{n}\sum\nolimits_{i=1}^{n} X_{(i:n)j}, \tag{2}$$

where $\gamma_n = (1/n)\sum_{i=1}^{n}\gamma_{(i:n)}$ is the correction constant chosen to minimize

$$\mathrm{Var}\left(\hat{\mu}_{RSS}\right) = \frac{\lambda\left(\sum_{i=1}^{n}\alpha_{(i:n)}^2/\beta_{(i:n)}\right)}{\left(\sum_{i=1}^{n}1/\beta_{(i:n)}\right)\left(\sum_{i=1}^{n}\alpha_{(i:n)}^2/\beta_{(i:n)}\right)-\left(\sum_{i=1}^{n}\alpha_{(i:n)}/\beta_{(i:n)}\right)^2} \tag{3}$$

In this study, γ_n is found to be approximately unity, while $\alpha_{(i:n)}$ is the mean and $\beta_{(i:n)}$ is the variance of reduced ordered variables $\eta_{(i:n)j} = \left(X_{(i:n)j} - \mu_0\right)/\sqrt{\mu}_0$. In practice, $\eta_{(i:n)j}$ is often used to transform the Poisson distribution to approximately normal.

The problem here is that the transformation $\eta_{(i:n)j}$ only changes the values of the location and dispersion parameters to zero and one, respectively, leaving the shape of the Poisson distribution intact. To achieve better normal approximation, Anscombe [43] suggested the square root transformation

$$Y_{(i:n)j} = 2\sqrt{X_{(i:n)j} + c}, \tag{4}$$

where c is a constant. $Y_{(i:n)j}$ is approximately normal with mean $\mu_{Y_{(i:n)j}} = 2\sqrt{\mu + c} - 1/4\sqrt{\mu}$ and variance $\sigma^2_{Y_{(i:n)j}} = 1 + (3 - 8c)/8\mu$ (cf. Shu *et al.* [19]). Using the suggested $c = 3/8$, we found the square root transformation

$$Z_{RSS\,j} = \frac{2\sqrt{\hat{\mu}_{RSS} + 3/8} - \mu_0}{\sqrt{\mathrm{Var}\left(\hat{\mu}_{RSS}\right)}} \tag{5}$$

to be approximately standard normal for an in-control Poisson process. This study assumes that μ_0 is either known or estimated while the variance is known and kept constant. Thus, the new Poisson CUSUM scheme under RSS structure for monitoring process location parameter is defined by

$$\begin{aligned} S^+_{RSS\,j} &= \max\left[0, Z_{RSS\,j} - k + S^+_{RSS\,j-1}\right] \\ S^-_{RSS\,j} &= \max\left[0, k - Z_{RSS\,j} + S^-_{RSS\,j-1}\right], \end{aligned} \tag{6}$$

where $S^+_{RSS\,0} = S^-_{RSS\,0} = 0$. The Poisson CUSUM chart triggers an out-of-control signal when either $S^+_{RSS\,j} > h$ or $S^-_{RSS\,j} > h$, the predefined control limit.

IV. PERFORMANCE ANALYSIS

CUSUM control charts are usually evaluated by computing their average run length (ARL); that is, the average number of observations plotted on a control chart before the out-of-control signal. However, in situations where the run-length distribution is heavily skewed, other measures such as the standard deviation (SDRL) and percentile points of the run-length distribution have been suggested. The run-length properties, ARL, SDRL and percentile points, measures the response power of a control scheme to detect departures from the specified requirements. In contrast, such response may be a false alarm. That is an out-of-control signal when indeed the process is at a satisfactory level (in-control). The in-control ARL is denoted by ARL_0 and the out-of-control ARL by ARL_1, with the subscripts 0 and 1 corresponding to the null and alternative hypothesis, respectively. A control chart with sufficiently large ARL_0 to avoid unnecessary false alarms and small ARL_1 to enable quick detection of process shifts is desirable.

A. PERFORMANCE EVALUATION

This study uses Monte Carlo simulation approach to numerically calculate the ARL, SDRL and percentile points of the new Poisson CUSUM control charts through an algorithm developed in R. The structure assumes that the process is initially at a satisfactory level with a known mean $\mu = \mu_0$, and changes to $\mu = \mu_1$ $(\mu_0 \neq \mu_1)$ after a certain point in time. The run-length properties for the proposed Poisson CUSUM chart for detecting both increases and decreases in the mean number of Poisson count data are calculated by simultaneous implementation of the two-sided control charts, equation (6). Using 10^5 independent seeded iterations, the ARL and SDRL values were estimated for the case of perfect ranking. Furthermore, the P_{25}, P_{50} and P_{75} values, which represents the 25th, 50th and 75th percentiles of the run-length distribution, respectively, are also computed. P_{50} is the median of run-length (MRL). The performances of the new Poisson CUSUM charts were investigated under sample sizes of $n = 3, 4, 5$ and 6 using different combinations of h and k.

The parameter k is the reference value of Poisson CUSUM chart, which is often chosen between the in-control and the out-of-control process mean of count data. It is defined by $k = (\mu_1 - \mu_0)/\ln(\mu_1/\mu_0)$ and lies closer to μ_0. Furthermore, for any fixed value of k, an appropriate control limit h that corresponds to the desired ARL_0 value can be determined, numerically, via trial procedure. In practice, the zero value of μ_0 is usually avoided because a Poisson CUSUM design structure with $\mu_0 = 0$ would have $k = 0$ and $h = 1$, triggering an alarm at every count [5]. In this study, we set the values of $\mu_0 = 7.0$ and for the quick detection of small changes in the mean of count data while maintaining relatively good detection performance of large shifts, a small value of $k = 0.5$ is used. Each of the Poisson CUSUM control charts used in this study is designed to have $\mathrm{ARL}_0 = 200$ for fair comparisons.

Outlined below is the stepwise procedure for the calculations of ARL, SDRL and percentile points of the run-length distribution. The simulation approach is based on Poisson CUSUM control chart statistics.

1. Generate pseudo random numbers x_{ij} of size n from Poisson with mean μ_0.
2. Apply the RSS procedures and compute the mean parameter $\hat{\mu}_{RSS\,j}$.
3. Calculate the variance $\mathrm{Var}\left(\hat{\mu}_{RSS\,j}\right)$ via simulations.
4. Perform the normal transformation to obtain $Z_{RSS\,j} \sim N(0, 1)$.
5. Initialize the CUSUM statistics, $S^+_{RSS\,0}$ and $S^-_{RSS\,0}$ equal to zero.
6. Specify parameter k and update the statistics $S^+_{RSS\,j}$ and $S^-_{RSS\,j}$.

TABLE 1. Run-length properties for two-sided Poisson CUSUM control charts ($n = 3$, $k = 0.5$, $ARL_0 = 200$).

	RSS ($h = 4.198$)					Classical ($h = 4.189$)				
μ	ARL	SDRL	P_{25}	MRL	P_{75}	ARL	SDRL	P_{25}	MRL	P_{75}
2	1.143	0.350	1	1	1	1.703	0.474	1	2	2
3	1.669	0.485	1	2	2	2.171	0.549	2	2	2
4	2.249	0.599	2	2	3	3.107	1.005	2	3	4
5	3.671	1.318	3	3	4	5.396	2.366	4	5	7
6	10.053	6.018	6	9	13	17.124	12.365	8	14	22
7	200.225	193.841	63	143	275	201.770	196.401	62	141	273
8	11.099	6.722	6	9	14	18.747	13.715	9	15	24
9	4.288	1.667	3	4	5	6.469	3.094	4	6	8
10	2.781	0.842	2	3	3	3.993	1.481	3	4	5
11	2.164	0.540	2	2	2	2.938	0.921	2	3	3
12	1.848	0.451	2	2	2	2.390	0.648	2	2	3

TABLE 2. Run-length properties for two-sided Poisson CUSUM control charts ($n = 4$, $k = 0.5$, $ARL_0 = 200$).

	RSS ($h = 4.169$)					Classical ($h = 4.153$)				
μ	ARL	SDRL	P_{25}	MRL	P_{75}	ARL	SDRL	P_{25}	MRL	P_{75}
2	1.002	0.044	1	1	1	1.437	0.496	1	1	2
3	1.214	0.410	1	1	1	1.922	0.461	2	2	2
4	1.820	0.468	2	2	2	2.639	0.776	2	3	3
5	2.765	0.835	2	3	3	4.462	1.786	3	4	5
6	6.909	3.441	4	6	9	13.300	8.918	7	11	17
7	197.572	194.962	60	136	272	199.682	193.704	63	139	274
8	7.541	3.896	5	7	9	14.618	9.907	8	12	19
9	3.225	1.063	2	3	4	5.298	2.315	4	5	6
10	2.191	0.545	2	2	2	3.337	1.116	3	3	4
11	1.754	0.473	1	2	2	2.515	0.722	2	2	3
12	1.401	0.490	1	1	2	2.100	0.516	2	2	2

7. Choose an experimental decision interval, $h > 0$.
8. Compare the statistics, $S^+_{RSS\,j}$ and $S^-_{RSS\,j}$, with h and record run-length.
9. Repeat steps 1 to 8 until the desired ARL_0 value of 200 is achieved.
10. Compute the ARL, SDRL, MRL, 1st and 3rd quartile points of the run-length distribution after each 10^5 iterations.

These outlined procedures are for the computation of ARL_0 when $\mu = \mu_0$. For shifts in the mean of count data from $\mu = \mu_0$ to $\mu = \mu_1$, the ARL_1 may be computed from the above steps 1 to 9, excluding step 7 since the value h is pre-determined from the computation of ARL_0. Tables 1 – 4 presents the results obtained based on the proposed schemes using perfect ranking of RSS as well as the classical Poisson CUSUM charts.

Following are the summary of our findings based on results in Tables 1 – 4, for the two-sided Poisson CUSUM control charts.

1. There is no significant difference between the nominal ARL_0 values and its corresponding in-control SDRL values for both the proposed charts.
2. The out-of-control ARL_1 and SDRL values of the newly developed Poisson CUSUM chart decrease rapidly with changes in mean level μ_1. The decreases are more evident for changes in downward shifts in mean rate of a Poisson process.
3. In all out-of-control cases, the run-length properties for the proposed charts under RSS are uniformly smaller as compared to the classical charts under SRS scheme. This is an indication that the new schemes are more efficient than the classical Poisson CUSUM chart in detecting all kinds of changes in a Poisson rate.
4. The strong right skewness of the run-length distributions is evident from the MRL values as well as the lower (25th percentile) and upper (75th percentile) quartile points of the sampling distribution for both the proposed and classical Poisson CUSUM control schemes.
5. As the sample size n increases, the sensitivities of the two schemes also increased with the new Poisson CUSUM chart having a better overall performance in terms of ARLs and SDRLs as compared to the corresponding classical chart.

TABLE 3. Run-length properties for two-sided Poisson CUSUM control charts ($n = 5, k = 0.5$, $ARL_0 = 200$).

	RSS ($h = 4.175$)					Classical ($h = 4.159$)				
μ	ARL	SDRL	P_{25}	MRL	P_{75}	ARL	SDRL	P_{25}	MRL	P_{75}
2	1.000	0.010	1	1	1	1.232	0.422	1	1	1
3	1.024	0.153	1	1	1	1.761	0.466	1	2	2
4	1.511	0.502	1	2	2	2.365	0.640	2	2	3
5	2.281	0.607	2	2	3	3.889	1.439	3	4	5
6	5.278	2.319	4	5	6	11.073	6.957	6	9	14
7	198.256	194.355	60	139.5	273	198.580	195.936	61	136	268
8	5.726	2.572	4	5	7	12.067	7.639	7	10	15
9	2.622	0.758	2	3	3	4.577	1.866	3	4	5
10	1.868	0.460	2	2	2	2.941	0.930	2	3	3
11	1.410	0.493	1	1	2	2.273	0.588	2	2	3
12	1.102	0.302	1	1	1	1.916	0.469	2	2	2

TABLE 4. Run-length properties for two-sided Poisson CUSUM control charts ($n = 6, k = 0.5$, $ARL_0 = 200$).

	RSS ($h = 4.170$)					Classical ($h = 4.160$)				
μ	ARL	SDRL	P_{25}	MRL	P_{75}	ARL	SDRL	P_{25}	MRL	P_{75}
2	1.000	0.000	1	1	1	1.095	0.293	1	1	1
3	1.001	0.030	1	1	1	1.607	0.495	1	2	2
4	1.218	0.413	1	1	1	2.176	0.555	2	2	2
5	1.989	0.487	2	2	2	3.483	1.197	3	3	4
6	4.356	1.713	3	4	5	9.539	5.588	6	8	12
7	197.606	192.748	59	140	272	198.776	193.564	60	136	277
8	4.653	1.870	3	4	6	10.225	6.107	6	9	13
9	2.255	0.585	2	2	3	4.109	1.562	3	4	5
10	1.614	0.499	1	2	2	2.686	0.803	2	3	3
11	1.147	0.354	1	1	1	2.100	0.519	2	2	2
12	1.011	0.105	1	1	1	1.771	0.474	1	2	2

6. The run-length distribution of the RSS based Poisson CUSUM control chart appeared to be more symmetric than the classical scheme, particularly, when $n > 3$.
7. The design structure of the new Poisson CUSUM chart is a special case of the RSS based Shewhart u-chart as rightly pointed out by White *et al.* [6]. As with the c-chart, this is achievable by setting $k = u + 3\sqrt{u}$ and $h = 0$ to detect large shifts in mean count rate.

B. PERFORMANCE COMPARISON WITH OTHER RSS VARIANTS

It is well-known in the literature that further gain in the efficiency of RSS can be achieved when an appropriate unequal allocation of units is used instead of equal allocation in RSS procedure. In this sub-section, we compare the performance of the design structures for Poisson CUSUM charts based on unequal allocation of units: median-RSS and extreme-RSS with the balanced RSS and classical charts to monitor the Poisson process mean of a quality characteristic. Using the same design parameters for the RSS in the above sub-section IV-A, the ARLs, SDRLs and MRLs of the new control charts were computed via simulations, and the results are displayed in Tables 5 – 8. The upper and lower quartile points of the run-length distribution of Poisson CUSUM charts based on median-RSS and extreme-RSS are also presented in Tables 5 – 8.

C. COMPARISON WITH THE MEDIAN-RSS

The median-RSS procedure [44] involves the collection of n random samples, each of size n from the target population. Rank the units within each set with respect to a variable of interest. Then the median observation is selected for measurement if the set size is odd, otherwise select the two middlemost observations for even set size. From the run-length properties summarized in Tables 5 - 8, it is observed that the Poisson CUSUM chart based on median-RSS encompasses all the findings in sub-section IV-A. In addition, the scheme can detect increases in the mean level of a Poisson process better than the decreases when $7 < \mu_1 < 9$. Comparison with the balanced RSS and classical schemes (cf. Tables 1 - 4) indicates that the median-RSS design

TABLE 5. Run-length properties for two-sided Poisson CUSUM control charts ($n = 3, k = 0.5, ARL_0 = 200$).

	Median-RSS ($h = 4.198$)					Extreme-RSS ($h = 4.178$)				
μ	ARL	SDRL	P_{25}	MRL	P_{75}	ARL	SDRL	P_{25}	MRL	P_{75}
2	1.049	0.215	1	1	1	1.139	0.346	1	1	1
3	1.522	0.502	1	2	2	1.665	0.486	1	2	2
4	2.105	0.525	2	2	2	2.239	0.589	2	2	3
5	3.388	1.155	3	3	4	3.635	1.288	3	3	4
6	9.458	5.410	6	8	12	10.025	5.950	6	9	13
7	198.888	191.418	62	140	278	202.452	199.672	61	141	277
8	9.066	5.174	5	8	11	11.055	6.646	6	9	14
9	3.792	1.380	3	4	4	4.250	1.668	3	4	5
10	2.506	0.703	2	2	3	2.779	0.851	2	3	3
11	2.008	0.470	2	2	2	2.161	0.532	2	2	2
12	1.688	0.483	1	2	2	1.832	0.466	2	2	2

TABLE 6. Run-length properties for two-sided Poisson CUSUM control charts ($n = 4, k = 0.5, ARL_0 = 200$).

	Median-RSS ($h = 4.233$)					Extreme-RSS ($h = 4.241$)				
μ	ARL	SDRL	P_{25}	MRL	P_{75}	ARL	SDRL	P_{25}	MRL	P_{75}
2	1.000	0.000	1	1	1	1.015	0.120	1	1	1
3	1.032	0.176	1	1	1	1.360	0.480	1	1	2
4	1.552	0.501	1	2	2	1.943	0.464	2	2	2
5	2.356	0.630	2	2	3	2.972	0.924	2	3	3
6	5.685	2.503	4	5	7	7.367	3.760	5	7	9
7	198.277	191.246	61	138	273	199.121	195.003	60	139	276
8	5.507	2.408	4	5	7	9.162	5.187	6	8	11
9	2.578	0.741	2	2	3	3.627	1.259	3	3	4
10	1.864	0.462	2	2	2	2.410	0.653	2	2	3
11	1.415	0.494	1	1	2	1.923	0.448	2	2	2
12	1.102	0.303	1	1	1	1.586	0.501	1	2	2

TABLE 7. Run-length properties for two-sided Poisson CUSUM control charts ($n = 5, k = 0.5, ARL_0 = 200$).

	Median-RSS ($h = 4.358$)					Extreme-RSS ($h = 4.270$)				
μ	ARL	SDRL	P_{25}	MRL	P_{75}	ARL	SDRL	P_{25}	MRL	P_{75}
2	1.000	0.000	1	1	1	1.000	0.000	1	1	1
3	1.006	0.078	1	1	1	1.078	0.268	1	1	1
4	1.382	0.486	1	1	2	1.669	0.482	1	2	2
5	2.167	0.532	2	2	2	2.455	0.673	2	2	3
6	5.030	2.057	4	5	6	5.642	2.497	4	5	7
7	198.761	194.901	61	138	271	202.641	198.709	62	141	275
8	4.723	1.857	3	4	6	6.899	3.367	4	6	8
9	2.330	0.591	2	2	3	2.929	0.896	2	3	3
10	1.719	0.473	1	2	2	2.037	0.478	2	2	2
11	1.240	0.427	1	1	1	1.620	0.494	1	2	2
12	1.027	0.163	1	1	1	1.237	0.425	1	1	1

structure has smaller ARL and SDRL values than the former two charts when monitoring both the upward and downward shifts in the Poisson parameter. Thus, the median-RSS based Poisson CUSUM chart can effectively handle all kinds of shifts in the mean level of a Poisson process better than its corresponding RSS and classical control charts.

TABLE 8. Run-length properties for two-sided Poisson CUSUM control charts ($n = 6$, $k = 0.5$, $ARL_0 = 200$).

	Median-RSS ($h = 4.482$)					Extreme-RSS ($h = 4.491$)				
μ	ARL	SDRL	P_{25}	MRL	P_{75}	ARL	SDRL	P_{25}	MRL	P_{75}
2	1.000	0.000	1	1	1	1.000	0.000	1	1	1
3	1.000	0.000	1	1	1	1.038	0.191	1	1	1
4	1.053	0.224	1	1	1	1.573	0.497	1	2	2
5	1.821	0.440	2	2	2	2.305	0.578	2	2	3
6	3.805	1.302	3	4	4	5.044	2.012	4	5	6
7	200.884	196.788	62	140	276	202.926	197.733	61	140	279
8	3.587	1.188	3	3	4	6.873	3.190	5	6	8
9	1.938	0.432	2	2	2	2.860	0.827	2	3	3
10	1.294	0.456	1	1	2	2.003	0.433	2	2	2
11	1.017	0.128	1	1	1	1.570	0.500	1	2	2
12	1.000	0.000	1	1	1	1.179	0.383	1	1	1

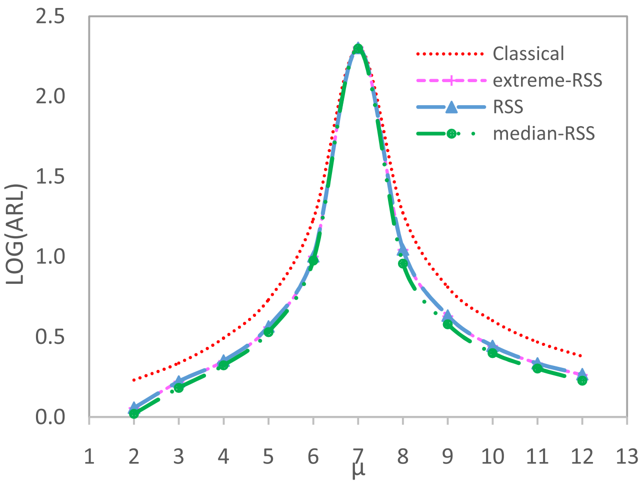

FIGURE 1. ARL curves for classical Poisson CUSUM chart versus variants of RSS schemes when $n = 3$, $k = 0.5$, $\mu_0 = 7$ and $ARL_0 = 200$.

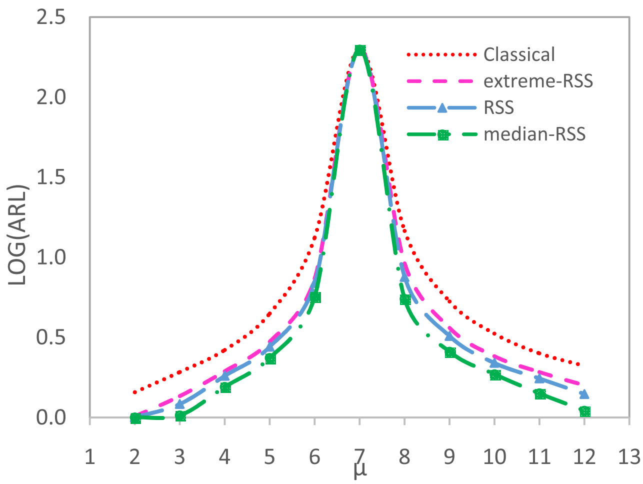

FIGURE 2. ARL curves for classical Poisson CUSUM chart versus variants of RSS schemes when $n = 4$, $k = 0.5$, $\mu_0 = 7$ and $ARL_0 = 200$.

D. COMPARISON WITH THE EXTREME-RSS

In extreme-RSS [45], n random samples each of size n is collected from the target population. Rank the units within each set with respect to the quality characteristics of interest. Then the lowest and highest rank units are selected for measurement if the set size is even. For odd subgroup size, the lowest, median and highest ranked observations are measured. Just like the median-RSS, this scheme also shares the findings in sub-section IV-A based on the extreme-RSS run-length results in Tables 5 – 8. The scheme also outperformed the classical Poisson CUSUM chart in detecting both the upward and downward shifts in the mean level of a Poisson process (cf. Tables 1 - 4). Further comparison shows that the balanced RSS scheme has smaller ARL_1 values than the extreme-RSS for all shift values and sample sizes.

E. COMPARISON AMONG ALL THE SCHEMES

To get a clearer picture on the performance of the four schemes in detecting the mean of count data Poisson process, we present the ARL curves for all the charts with design parameters $k = 0.5$, $ARL_0 = 200$ when $n = 3$ and $n = 4$ in Figures 1 and 2, respectively. Statistically, the lower the ARL curve of a control chart, the greater the probability of shorter ARL_1 and quick detection of smaller shifts. As expected, Figures 1 and 2 shows that the classical Poisson CUSUM chart has higher ARL_1 values than the proposed schemes based on ranked data. In other words, all the new schemes have a high tendency to quickly detect small shifts in the process mean. Furthermore, observe that the median-RSS has the highest tendency of faster detection of small mean shifts, closely followed by the balanced RSS for larger sample sizes (cf. Figure 2). There is no significant difference between the new charts when $n < 4$ (cf. Figure 1).

V. THE EFFECT OF IMPERFECT RANKING ON ARL PERFORMANCE

In practice, ranking of units in a subgroup with respect to the quality characteristic of interest may not always be perfect as it largely depends on the operator's judgment. For instance, the 2^{nd} smallest unit in a subgroup measured by an operator may not be the actual 2^{nd} smallest observation in the subgroup. Hence, in this section, the effects of errors in ranking

TABLE 9. Run length properties for two-sided Poisson CUSUM charts using IRSS ($n = 5, k = 0.5, \text{ARL}_0 = 200$).

	$\rho_{xy} = 0.25$ $h = 4.178$			$\rho_{xy} = 0.50$ $h = 4.177$			$\rho_{xy} = 0.75$ $h = 4.176$			$\rho_{xy} = 0.90$ $h = 4.176$		
μ	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
2	1.06	0.23	1	1.03	0.17	1	1.00	0.07	1	1.00	0.01	1
3	1.56	0.50	2	1.45	0.50	1	1.24	0.43	1	1.10	0.31	1
4	2.12	0.51	2	2.03	0.48	2	1.87	0.45	2	1.70	0.48	2
5	3.37	1.11	3	3.18	1.04	3	2.83	0.87	3	2.55	0.73	2
6	9.09	5.11	8	8.28	4.50	7	7.11	3.60	6	6.17	2.88	6
7	202.15	196.03	143	200.66	197.96	139	199.28	194.57	139	199.03	194.14	138
8	9.97	5.93	8	9.20	5.26	8	7.83	4.11	7	6.72	3.33	6
9	3.98	1.49	4	3.75	1.35	4	3.31	1.11	3	2.94	0.92	3
10	2.61	0.77	2	2.47	0.70	2	2.23	0.58	2	2.05	0.50	2
11	2.05	0.51	2	1.97	0.48	2	1.79	0.47	2	1.62	0.50	2
12	1.73	0.49	2	1.65	0.50	2	1.44	0.50	1	1.25	0.43	1

of quality characteristic of interest on the performance of the proposed Poisson CUSUM control charts, is investigated. In practice, the ranking of units of the main quality characteristic may not be that easy, hence it is often recommended to use the corresponding auxiliary information for ranking purposes. This is called ranking with auxiliary variable or imperfect ranking of units. The run-length performance of the proposed Poisson CUSUM charts under imperfect ranking largely depends on the linear relationship between the so-called auxiliary variable and the main quality characteristic of interest.

Denote a bivariate Poisson random variable with (X, Y), where X and Y represent the main quality characteristic of interest and its auxiliary variable, respectively. Also denote the i^{th} order statistic of X and Y in the i^{th} sample of size n in the j^{th} cycle by $X_{(i:n)j}$ and $Y_{(i:n)j}$, respectively, where $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$. Suppose that the regression of X on Y is linear, then the i^{th} judgment order statistic [46] is given by

$$X_{[i:n]j} = \mu_x + \rho_{xy}\frac{\sigma_x}{\sigma_y}\left[X_{(i:n)j} - \mu_y\right] + \varepsilon_{ij} \tag{7}$$

where ρ_{xy} is the correlation between X and Y; μ_x and μ_y are the means of X and Y; σ_x and σ_y represent the standard deviations of X and Y, respectively; and ε_{ij} is an error term with a mean of zero and variance $\sigma_x^2\left(1 - \rho_{xy}^2\right)$. Note that ε_{ij} which is independent of Y. The mean estimator of the Poisson imperfect RSS (IRSS) data $X_{[i:n]j}$ based on the judgment ranking of $Y_{(i:n)}$ can be defined as

$$\hat{\mu}_{IRSS\,j} = \frac{\zeta_n}{n}\sum\nolimits_{i=1}^{n} X_{[i:n]j} \tag{8}$$

where $\zeta_n \approx 1$. Also, the variance of $\hat{\mu}_{IRSS\,j}$ can be defined as

$$\text{Var}\left(\hat{\mu}_{IRSS}\right) = \frac{\sigma_x^2}{n}\left[\left(1 - \rho_{xy}^2\right) + \frac{\rho_{xy}^2}{n\sigma_y^2}\sum\nolimits_{i=1}^{n}\sigma_{y(i:n)}^2\right] \tag{9}$$

where $\sigma_{y(i:n)}^2$ is the variance of the i^{th} observation from a Poisson rate of subgroup size n. In an analogous to Section III, the standard normalizing transformation of $\hat{\mu}_{IRSS\,j}$ can be achieved via

$$Z_{IRSS\,j} = \frac{2\sqrt{\hat{\mu}_{IRSS} + 3/8} - \mu_0}{\sqrt{\text{Var}\left(\hat{\mu}_{IRSS}\right)}}. \tag{10}$$

Hence, the proposed Poisson CUSUM chart for monitoring location parameters under IRSS scheme triggers an out-of-control signal when either

$$\begin{aligned} S_{IRSS\,j}^{+} &= \max\left[0, Z_{IRSS\,j} - k + S_{IRSS\,j-1}^{+}\right] \quad \text{or} \\ S_{IRSS\,j}^{-} &= \max\left[0, k - Z_{IRSS\,j} + S_{IRSS\,j-1}^{-}\right], \end{aligned} \tag{11}$$

exceeds the decision interval h, with the initial values of $S_{IRSS\,0}^{+}$ and $S_{IRSS\,0}^{-}$ set equal to zero.

Statistically, SRS and RSS are special cases of IRSS with correlation coefficients of $\rho_{xy} = 0$ and $\rho_{xy} = 1$, respectively. Setting the subgroup size at $n = 5$, we follow the Monte Carlo simulation design procedure outlined in Section IV, using IRSS data generated from a bivariate Poisson distribution with a mean of seven and variance of one. We then investigate the effects of correlation coefficients ρ_{xy} on the run-length performance of the proposed Poisson CUSUM control charts by setting $\rho_{xy} = 0.25, 0.50, 0.75$ and 0.90. For a fixed value of ρ_{xy}, we also investigated the effect of reference value k on the proposed charts when the in-control ARL_0 is approximately 200. The performance measures used in this section include: ARL, SDRL and MRL. The results obtained for the IRSS based Poisson CUSUM charts are displayed in Tables 9 and 10. To gain more insight, a graphical display on the effects of ρ_{xy} is presented in Figure 3. Below is the summary of our findings.

A. EFFECT OF CORRELATION COEFFICIENT ρ_{xy}

For a fixed sample size $n = 5$ and reference value $k = 0.5$, Table 9 presents the run-length properties when $\rho_{xy} = 0.25, 0.50, 0.75$ and 0.90 to study the effect of ρ_{xy}

TABLE 10. Run length properties for two-sided Poisson CUSUM charts using IRSS ($n = 5$, $\rho_{xy} = 0.75$, $\mathrm{ARL}_0 = 200$).

	$k = 0.25$ $h = 5.743$			$k = 0.50$ $h = 4.176$			$k = 0.75$ $h = 2.404$			$k = 1.00$ $h = 1.809$		
μ	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
2	1.30	0.46	1	1.00	0.07	1	1.00	0.01	1	1.00	0.00	1
3	1.92	0.29	2	1.24	0.43	1	1.03	0.17	1	1.01	0.11	1
4	2.34	0.49	2	1.87	0.45	2	1.40	0.51	1	1.26	0.46	1
5	3.61	0.89	4	2.83	0.87	3	2.37	0.94	2	2.26	1.07	2
6	8.08	3.04	8	7.11	3.60	6	7.66	5.28	6	9.39	7.72	7
7	199.44	187.86	141	199.28	194.57	139	201.19	197.69	138	201.03	199.56	139
8	8.76	3.47	8	7.83	4.11	7	8.66	6.09	7	10.95	9.05	8
9	4.19	1.12	4	3.31	1.11	3	2.84	1.25	3	2.80	1.49	2
10	2.88	0.66	3	2.23	0.58	2	1.80	0.65	2	1.63	0.69	2
11	2.26	0.45	2	1.79	0.47	2	1.34	0.49	1	1.21	0.42	1
12	2.00	0.26	2	1.44	0.50	1	1.10	0.30	1	1.05	0.22	1

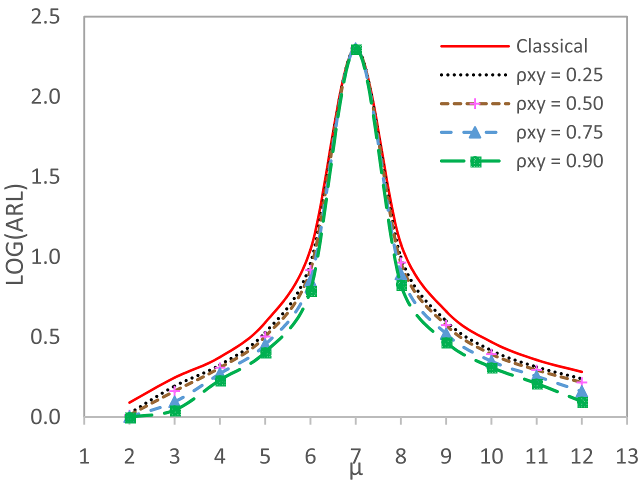

FIGURE 3. ARL curves for classical Poisson CUSUM chart versus IRSS schemes when $n = 5$, $k = 0.5$, $\mu_0 = 7$ and $\mathrm{ARL}_0 = 200$ for different values of ρ_{xy}.

on the performance of the new Poisson CUSUM control charts. It is evident from the results that imperfect ranking does have a negative impact on performance of the proposed Poisson CUSUM chart. However, the effect is negligible, particularly as ρ_{xy} increases. In other words, the proposed schemes are still more efficient than the classical charts even in presence of ranking errors. Furthermore, the IRSS based control charts maintained all the properties exhibited by the charts under perfect ranking. Comparison with the classical Poisson CUSUM chart (cf. Tables 3 and 9) shows that the new IRSS based charts dominate the former for all the correlation coefficient ρ_{xy} values. This point is equally supported by Figure 3. For example, if $\mu_1 = 6$, the ARL of the classical chart is 11.07 (cf. Table 3) while the IRSS based chart has 9.09, 8.28, 7.11, 6.17 for $\rho_{xy} = 0.25, 0.50, 0.75$ and 0.90, respectively (cf. Table 9).

B. EFFECT OF REFERENCE VALUE k

To study the effect of reference value k in presence of ranking errors, we fixed the sample size $n = 5$ and correlation coefficient $\rho_{xy} = 0.75$. Then using different values of $k = 0.25, 0.5, 0.75$ and 1.0, we studied the effect of k on the performance of the proposed Poisson CUSUM control charts based on IRSS and the results are displayed in Table 10. It has been observed that k is inversely proportional to h and directly proportional to the ARL_1 values at smaller shift levels. This means that better run-length performances are achievable for detecting small process shifts when smaller values of $k \le 0.75$ are used but reduce the effectiveness of the control chart in detecting moderate to large disturbances. Comparison with the classical charts (cf. Table 3) reveals that the IRSS based Poisson CUSUM charts are more sensitive to changes than the former, for all the shift values, in Poisson count rate (cf. Table 10, columns 5 and 6). It is also observed that even in the presence of ranking errors and changes in value of k, the distributions of the new charts are more symmetric about μ_0 than the classical Poisson CUSUM control chart when $k > 0.25$.

VI. ILLUSTRATIVE EXAMPLE

To demonstrate the practical application of the proposed Poisson CUSUM chart based on RSS method, we revisited the number of non-conformities in samples of 100 printed circuit board example in Zhang *et al.* [17] and Abujiya *et al.* [41]. The data set consist of thirty samples obtained from two sources. The first twenty data on the number of non-conformities per board is from Table 7.8 of Montgomery [3] with an in-control non-conformities rate of $\mu_0 = 19.67$. The last ten samples are simulated Poisson random variables with two different target out-of-control non-conformities rate of $\mu_1 = 16.0$ and $\mu_1 = 23.0$, signifying some improvements and deterioration in a process after the twentieth observation, respectively.

For illustration, we used the re-sampling approach of Takahasi and Wakimoto [30] to randomly collect thirty observations using RSS and classical random sampling schemes based on subgroup size of $n = 3$. The first twenty subgroup data points were from the above number of non-conformities

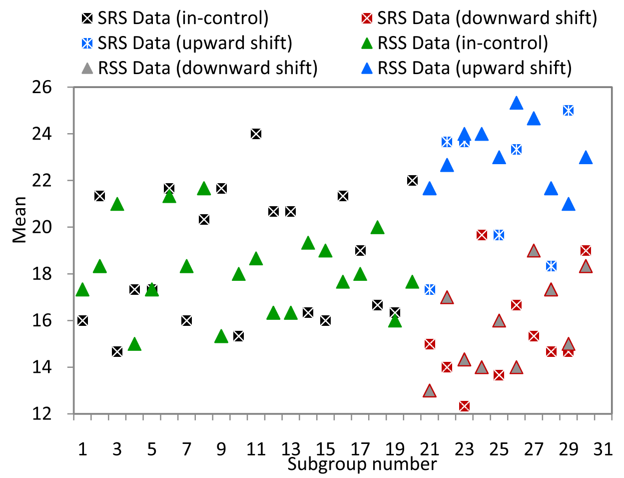

FIGURE 4. Means of the thirty re-sampled data points collected using SRS and RSS schemes.

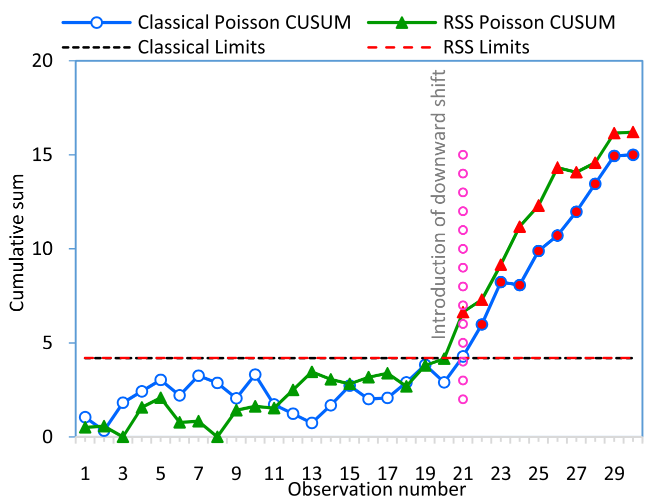

FIGURE 5. Means of the thirty re-sampled data points collected using SRS and RSS schemes.

per board [3] and the last ten were from the assumed out-of-control system whose averages are displayed in Figure 4. Using the parameters $\mu_0 = 19.67$, $k = 0.5$, $\mathrm{ARL}_0 = 200$, the Poisson CUSUM control chart statistics for all the classical and RSS based schemes were computed. The control limit for the classical and RSS Poisson CUSUM chart was found to be $h = 4.189$ and $h = 4.198$, respectively. The graphical representations of the plotting statistics for the detection of upward and downward shifts are displayed in Figures 5 and 6, respectively. Since CUSUM is well-known for its effectiveness with small shifts, the target here, is to identify the scheme that is more effective in the detection of moderate to large downward and upward shifts, the so-called quality improvements or deteriorations in the Poisson rate.

From Figure 5, we observed that both the classical and the proposed schemes are not doing badly in the detection of quality improvements. While the classical Poisson CUSUM chart signals at the twenty-second sample point, the proposed

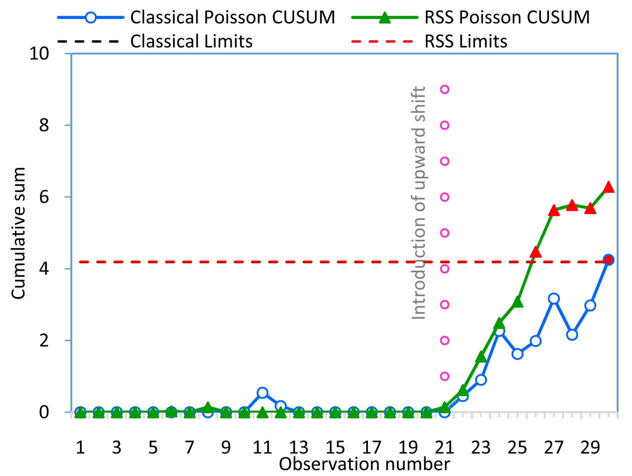

FIGURE 6. Classical versus RSS Poisson CUSUM control charts for monitoring upward shift.

scheme triggers right on the twenty-first sample point giving all the ten improvement signals. Interestingly, the classical case appeared not to be doing so well when there is a deterioration in the process (cf. Figure 6). Although the scheme shows some pattern, it only triggers an out-of-control signal on the thirtieth sample point. The proposed scheme, on the other hand, gives a signal on the twenty-sixth sample with a total of five out-of-control deterioration points. Hence, the superiority of the proposed Poisson CUSUM chart using RSS over the classical scheme is once again demonstrated.

VII. CONCLUDING REMARK

Three new Poisson CUSUM control charts based on RSS technique were developed for effective monitoring the Poisson process location parameter. The Poisson CUSUM charts based on RSS, and its variants are designed to detect both upward and downward shifts in the process mean of Poisson count data more effectively. The run-length properties of the proposed charts, namely ARL, SDRL and percentile points were computed via simulation approach. The two-sided Poisson CUSUM control charts based on RSS are more effective in detection of both the upward and downward shifts in a Poisson process than the existing classical Poisson CUSUM control charts. Moreover, the presence of ranking errors does not have much negative impact on performance of the proposed schemes as they have better symmetric distribution than the classical charts. Using real data, an application example is presented to demonstrate how the proposed scheme can be used in manufacturing industrial setting. The scope of this study may be extended to other non-symmetric distributions and variety of control charts design structures.

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