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Optimal Allocation of Distributed Generation Using Hybrid Grey Wolf Optimizer

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ABSTRACT Optimal allocation of distributed generation units is essential to ensure power loss minimization, while meeting the real and reactive power demands in a distribution network. This paper proposes a solution to this non-convex, discrete problem by using the hybrid grey wolf optimizer, a new metaheuristic algorithm. This algorithm is applied to IEEE 33-, IEEE 69-, and Indian 85-bus radial distribution systems to minimize the power loss. The results show that there is a considerable reduction in the power loss and an enhancement of the voltage profile of the buses across the network. Comparisons show that the proposed method outperforms all other metaheuristic methods, and matches the best results by other methods, including exhaustive search, suggesting that the solution obtained is a global optimum. Furthermore, unlike for most other metaheuristic methods, this is achieved with no tuning of the algorithm on the part of the user, except for the specification of the population size.

INDEX TERMS Distributed generation (DG), optimal DG location, optimal DG size, loss minimization, radial distribution system, metaheuristic algorithm.

I. INTRODUCTION

Recent developments such as the advances in power electronics, renewable energy sources, liberalization of electric power markets, and the need for environment protection have drastically necessitated and made possible the decentralization of power networks. This has led to new sets of problems and opportunities, and the proliferation of distributed generation systems.

Distributed generation (DG) units can be classified into different types, based on whether they generate or consume reactive power along with generation of real power: (a) P-type or Type-I DG units, which supply real power alone, such as photovoltaic cells (b) Q-type or Type-II DG units which supply reactive power alone, like capacitor banks (c) PQ^{+}-type or Type-III DG units which supply real power and can either generate or consume reactive power, like synchronous generators, and (d) PQ^{-}-type or Type-IV DG units which produce real power and consume reactive power, like induction generators for wind power.

The optimal DG allocation problem is to determine the optimal bus location, and the optimal size of the DG units, to minimize the total power loss in the system network. The importance of this problem has been highlighted by surveys such as [1] and [2], and a large number of papers, using a variety of approaches that can be broadly classified into four categories:(a) classical (b) analytical (c) metaheuristic and heuristic, and (d) hybrid. The latest works in each of these categories are reviewed next.

Classical approaches to solving the DG allocation problem include mixed integer nonlinear programming (MINLP) [3], and the use of bifurcation analysis and dynamic programming [4]. Considering the analytical approaches next, [5] presents an analytical method (AM) using loss sensitivity factor (LSF) that is claimed to be simpler and faster than other classical methods in it. Reference [6] proposes an analytical expression, [7] an improved analytical (IA) method, and [8] a dual index analytical approach. Other works that use the analytical approach include [9] which uses sensitivity approaches, [10] which uses sensitivity analysis, [11], [12], which uses efficient analytical (EA) method and EA with optimal power flow (EA-OPF).

The metaheuristic and heuristic methods used for solving the DG allocation problem outnumber the analytical and classical approaches. Some of these methods are: artificial bee colony (ABC) [13], heuristic curve-fitted technique [14], modified honey bee mating [15], improved particle swarm optimization (IPSO) [16], modified teaching learning-based optimization (MTLBO) algorithm [17], multi-objective harmony search [18], Pareto front differential evolution [19], particle swarm optimization (PSO) with constriction factor approach [20], backtracking search algorithm (BSA) [21], big bang big crunch [22], krill herd algorithm (KHA) [23], improved PSO [24].

Given the relative advantages and disadvantages of the four categories of approaches, some hybrid methods to eliminate the disadvantages and combine the advantages too have been tried out on the DG allocation problem. Hybridization of metaheuristic approaches with analytical approaches has been tried in [25] which uses LSF and simulated annealing (LSFSA), [26] which uses sensitivity analysis and PSO, and [27] which uses the analytical method and PSO (named as 'Hybrid' in [27]). Another kind of hybridization is combining operators of one metaheuristic algorithm with those of another. Examples of such methods are [28] which uses an imperialist competitive algorithm and genetic algorithm, [29] which uses ant colony optimization and artificial bee colony (HACO) and [30] which combines harmony search and particle ant bee colony (PABC).

The cost function in the DG allocation problem, the total power loss in the system, is subject to nonlinear equality constraints. This makes the problem non-convex. The bus numbers and the DG capacities can assume only discrete values, hence making the problem a discrete one. Classical methods have the advantage of small computation time but assume that the problem is a convex programming problem, while the optimality of analytical methods is an open issue [1].

The contribution of this paper is to find the globally optimal solution of the non-convex, discrete DG allocation problem, using the hybrid grey wolf optimizer (HGWO), a hybrid metaheuristic method. The HGWO outperforms or performs as well as all the other methods -including the other metaheuristic methods and exhaustive search - in terms of optimality of the solution, thereby suggesting that the solution is a globally optimal one. Further, unlike for most other metaheuristic methods, this is achieved with no tuning of the algorithm on the part of the user, except for the specification of the population size.

The remainder of this paper is constituted as follows. Section II explains the statement of the problem of loss optimization in a distribution network. Section III summarizes the grey wolf optimizer, and Section IV gives a brief review of the hybrid grey wolf algorithm (HGWO). Section V explains how the HGWO is applied to solve the DG allocation problem. Section VI contains the results and discussion, and Section VII concludes the paper.

II. PROBLEM FORMULATION

The problem involves identifying the location, the size and the type of distributed generators or distributed generation units (DG units) to be introduced at different nodes of the distribution network while ensuring the operational constraints are met to ensure system integrity.

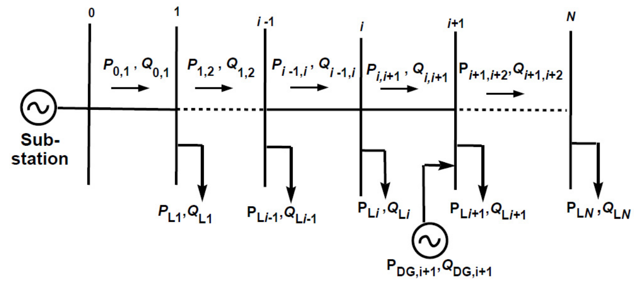

FIGURE 1. Single line diagram of an RDS.

A. OBJECTIVE FUNCTION

Fig. 1 shows the single line diagram of the main feeder of a radial distribution system (RDS) with N number of buses. The real power or I^2R loss in the line section between buses i and $i+1$ is given by

$$P_{i,i+1}^{\text{Loss}} = \frac{P_{i,i+1}^2 + Q_{i,i+1}^2}{|V_i|^2} R_{i,i+1}. \tag{1}$$

The total power loss of the feeder, which is the sum of losses in all the line sections of the feeder is given by

$$P^{\text{TLoss}} = \sum_{i=0}^{N-1} \frac{P_{i,i+1}^2 + Q_{i,i+1}^2}{|V_i|^2} R_{i,i+1} \tag{2}$$

where $P_{i,i+1}, Q_{i,i+1}$ are the real and reactive power flow between buses i and $i+1$, in kW and kVAr, $R_{i,i+1}$ is the resistance of the line between the buses i and $i+1$, and V_i is the voltage at bus i.

Similarly, the reactive power loss in the line section between buses i and $i+1$ is given by

$$Q_{i,i+1}^{\text{Loss}} = \frac{P_{i,i+1}^2 + Q_{i,i+1}^2}{|V_i|^2} X_{i,i+1} \tag{3}$$

where $X_{i,i+1}$ is the reactance of the line between buses i and $i+1$.

The objective is to minimize the total real power loss incurred by placing the DG units at optimal bus locations and choosing the optimal sizes of the DG units. With the addition of the DG units, (2) becomes

$$P_{\text{DG}}^{\text{TLoss}} = \sum_{i=0}^{N-1} \frac{\left(P_{i,i+1} - \alpha_{\text{pDG}} P_{\text{DG},i+1}\right)^2 + \left(Q_{i,i+1} - \alpha_{\text{qDG}} Q_{\text{DG},i+1}\right)^2}{|V_i|^2} \times R_{i,i+1} \tag{4}$$

where $\alpha_{\text{pDG}}, \alpha_{\text{qDG}}$ are the real and reactive power multiplier, set to 1 if DG unit is present, and 0 if DG unit is absent, $P_{\text{DG},i+1}, Q_{\text{DG},i+1}$ are the size of the DG unit or active and reactive power injections at bus $i+1$ in kW and kVAr, respectively.

B. CONSTRAINTS

The inequality constraints of the problem are given by

1) The voltage at each bus should be well within the permissible limits:

$$V_{i,\min} \le V_i \le V_{i,\max} \tag{5}$$

2) The size of individual DG units is to be maintained within the set limits [13]:

$$S_{\mathrm{DG},i,\min} \le S_{\mathrm{DG},i} \le S_{\mathrm{DG},i,\max} \tag{6}$$

3) The operating power factor of the DG unit has to be within the set limits [13]:

$$pf_{\mathrm{DG},i,\min} \le pf_{\mathrm{DG},i} \le pf_{\mathrm{DG},i,\max} \tag{7}$$

4) At every bus in the system, the following nonlinear equality constraints must be satisfied (see Fig. 1) [13]:

$$P_{i+1,i+2} = P_{i,i+1} - P^{\mathrm{Loss}}_{i,i+1} - P_{\mathrm{L},i+1} + \alpha_{\mathrm{pDG}} P_{\mathrm{DG},i+1} \tag{8}$$

$$Q_{i+1,i+2} = Q_{i,i+1} - Q^{\mathrm{Loss}}_{i,i+1} - Q_{\mathrm{L},i+1} + \alpha_{\mathrm{qDG}} Q_{\mathrm{DG},i+1} \tag{9}$$

$$|V_{i+1}|^2 = |V_i|^2 - 2\left(R_{i,i+1}P_{i,i+1} + X_{i,i+1}Q_{i,i+1}\right) + \left(R^2_{i,i+1} + X^2_{i,i+1}\right)\left(\frac{P^2_{i,i+1} + Q^2_{i,i+1}}{|V_i|^2}\right) \tag{10}$$

where $P_{\mathrm{L},i+1}$ and $Q_{\mathrm{L},i+1}$ are the real and reactive power loads at bus $i+1$.

C. BUS VOLTAGE DIFFERENCE WITH DG UNIT

For DG units of the P-type, Q-type and PQ^+-type considered in this paper, the real and reactive powers are to be injected such that the bus voltages are maintained within their lower and upper limits. The bus voltage difference after the addition of the DG units is approximately given by [26] (see Fig. 1)

$$V_i - V_{i+1} = \left(P_{\mathrm{L},i+1} - \alpha_{\mathrm{pDG}} P_{\mathrm{DG},i+1}\right) R_{i,i+1} + \left(Q_{\mathrm{L},i+1} \pm \alpha_{\mathrm{qDG}} Q_{\mathrm{DG},i+1}\right) X_{i,i+1} \tag{11}$$

where$R_{i,i+1}$, $X_{i,i+1}$ are the resistance and reactance of the line between the buses i and $i+1$.

III. THE GREY WOLF OPTIMIZER

The grey wolf optimizer (GWO) is a swarm intelligence algorithm introduced by Mirjalili, Mirjalili, and Lewis in 2014 [31], that does not require any tuning on the part of the user. It employs the two operators for its working: (i) encircling prey, and (ii) hunting. These are given by [32]:

A. ENCIRCLING PREY

The distance between any wolf and the prey is given by

$$\vec{D} = \left|\vec{C} \otimes \vec{X}^p(t) - \vec{X}(t)\right| \tag{12}$$

$$\vec{C} = 2\vec{r}_1 \tag{13}$$

where $\vec{X}^p$ is the position vector of the prey, $\vec{X}$ is the position vector of a wolf, and t indicates the iteration number. $\vec{r}_1$ is a vector of random numbers in the range [0, 1], of the same dimensions as $\vec{X}^p$ and $\vec{X}$. The $\otimes$ between $\vec{C}$ and $\vec{X}^p$ corresponds to component-wise multiplication.

B. HUNTING

Hunting involves moving closer to the prey using the information obtained in encircling, given by (12) and (13). This is given by

$$\vec{X}(t+1) = \vec{X}^p(t) - \vec{A} \otimes \vec{D} \tag{14}$$

$$\vec{A} = a(2\vec{r}_2 - 1) \tag{15}$$

where a is linearly decreased from 2 to 0 over the course of iterations, and $\vec{r}_2$ a vector of random numbers in the range[0, 1], and of the same dimensions as $\vec{X}^p$, $\vec{X}$ and $\vec{D}$. The $\otimes$ between $\vec{A}$ and $\vec{D}$ means corresponding component-wise multiplication, as in (12).

The position of the prey $\vec{X}^p$, or the optimizer being searched for in the solution landscape is unknown, it is assumed that the α, β and δ wolves [32] have the best knowledge of the prey. Hence their positions are used for updating the positions of all the other (omega) wolves. Using these three best solutions in the decreasing order of their fitness, the distances between any wolf $\vec{X}$ and these three best wolves are given by

$$\vec{D}_\alpha = \left|\vec{C}_1 \otimes \vec{X}^\alpha - \vec{X}\right|, \quad \vec{D}_\beta = \left|\vec{C}_2 \otimes \vec{X}^\beta - \vec{X}\right|, \quad \vec{D}_\delta = \left|\vec{C}_3 \otimes \vec{X}^\delta - \vec{X}\right| \tag{16}$$

These distances can be used to obtain the new position of the wolf $\vec{X}(t+1)$ using the following equations.

$$\vec{X}_1 = \vec{X}^\alpha - \vec{A}_1 \otimes \vec{D}_\alpha, \quad \vec{X}_2 = \vec{X}^\beta - \vec{A}_2 \otimes \vec{D}_\beta, \quad \vec{X}_3 = \vec{X}^\delta - \vec{A}_3 \otimes \vec{D}_\delta \tag{17}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{18}$$

Applying the two operators of encircling and hunting repeatedly, the prey or the best solution is located.

IV. THE HYBRID GREY WOLF OPTIMIZER (HGWO)

The DG allocation problem is inherently discrete owing to the discrete values of bus numbers and the DG unit capacities, and it is also non-convex due to the nonlinear power balance equality constraint. To enhance the search capability in this kind of non-convex solution spaces, [32] proposed hybridization of the GWO with operators from evolutionary algorithms. These are given next.

A. CROSSOVER

The uniform or binomial crossover proposed in [33] is used here. The jth component of the ith wolf applying the crossover operator is given by

$$X_j^i = \begin{cases} X_j^r & \text{if } \mathrm{rand}_j^i < C_r \\ X_j^i & \text{else} \end{cases} \quad j = 1, 2, \ldots, D, \quad i = 1, 2, \ldots, N_w \tag{19}$$

where $r \in [1, 2, \ldots, N_w]$, chosen randomly, $r \neq i$, and $rand_j^i \in [0, 1]$ is randomly generated $\forall j, i$. D is the dimensionality of each solution vector, and N_w is the number of solutions or wolves.

For the work reported in this paper, the crossover probability C_r is varied dynamically unlike in the classical differential evolution. This is given by [34]

$$C_r = 0.2 \times \hat{F}^{i,\mathrm{best}} \tag{20}$$

$$\hat{F}^{i,\mathrm{best}} = \frac{F^i - F^{\mathrm{best}}}{F^{\mathrm{worst}} - F^{\mathrm{best}}} i = 1, 2, \ldots, N_w \tag{21}$$

where F^i is the fitness value of the ith wolf. The best and the worst fitness values in the wolf pack for the current iteration are termed as F^{best} and F^{worst} respectively. (20) and (21) ensure that the best wolf vector remains unchanged, and the crossover probability is directly proportional to the relative fitness of the vector.

B. MUTATION

The mutation scheme used is as in [34]. After mutation, the jth component of the ith wolf is given by

$$x_j^i = \begin{cases} x^{gBest,j} + r_r \left(x_j^p - x_j^q\right) & \text{if } rand_j^i < \mu \\ x_j^i & \text{else} \end{cases}$$

$$j = 1, 2, \ldots, D, i = 1, 2, \ldots, N_w \tag{22}$$

$$\mu = 0.05 \times \hat{F}^{i,\mathrm{best}} \tag{23}$$

where $p, q \in [1, 2, \ldots, N_w]$, randomly generated, $p \neq q \neq i$, and $r_r, rand_j^i \in [0, 1]$ both randomly generated $\forall j, i$. μ is the mutation probability or mutation rate, $x^{\mathrm{gBest},j}$ is the jth component of the global best wolf in the whole of the iterative process so far, across the iterations, up to the current iteration. The best wolf in the pack in any given iteration is compared with this global best wolf. If the best wolf is better than the global best wolf, it becomes the new global best wolf. Equations (21) and (23) mean that the mutation probability or mutation rate μ is 0 for the best wolf and 0.05 for the worst wolf in the pack of the current iteration.

V. IMPLEMENTATION OF THE HGWO FOR OPTIMAL DG ALLOCATION

The allocation of DG units in appropriate locations reduces the losses and improves the voltage profile. The control variables of the problem are the (a) the locations, (b) capacity, and (c) operating power factor of DG units. A set of these control variables forms a grey wolf (or solution vector). The fitness of each solution is determined by substituting this in the fitness function given by (4) and executing a load flow by the direct approach proposed by Teng [35]. The grey wolves which give the best fitness values (the least or minimum losses) are chosen as the α, β and δ wolves respectively.

The step by step procedure of HGWO to solve the optimal DG allocation problem is given below.

Step 0: Choose the population size N_w, maximum number of iterations, total number of locations for DG units to be installed $N_{\mathrm{DG,loc}}$, capacity of DG unit S in kVA, and the operating power factor pf, set as

$$pf = \begin{cases} 1 & \text{for P-type} \\ 0 & \text{for Q-type} \\ [0.7, 1] & \text{for PQ}^{+}\text{-type} \end{cases}$$

Generate the initial population of N_w number of wolves or feasible solution vectors, that satisfy all the constraints listed in Section 2.2, (24), as shown at the bottom of this page, where $i = 1, 2, \ldots, N_{\mathrm{DG,loc}}$ is the location or the bus number. DG can take values of 1, 2 and 3 for 1 no DG, 2 no DG and 3 no DG locations respectively.

Step 1: Run the load flow for each grey wolf and find the power loss in the distribution system. Evaluate the fitness using the fitness or objective function (4). Identify the α, β and δ wolves and the global best solution P^{gBest}. In the very first iteration, $P^{\mathrm{gBest}} = P^{\alpha}$.

Step 2: Apply the encircling operator (12) to compute

$$\vec{D}_\alpha = \left|\vec{C}_1 \otimes \vec{P}^{\alpha} - \vec{P}^i\right|, \quad \vec{D}_\beta = \left|\vec{C}_2 \otimes \vec{P}^{\beta} - \vec{P}^i\right|, \quad \vec{D}_\delta = \left|\vec{C}_3 \otimes \vec{P}^{\delta} - \vec{P}^i\right| \tag{25}$$

Apply the hunting operator (14), to compute $\vec{P}_1$, $\vec{P}_2$ and $\vec{P}_3$. For this problem, using the distances calculated by (25), these are given by

$$\vec{P}_1 = \vec{P}^{\alpha} - \vec{A}_1 \otimes \vec{D}_\alpha, \quad \vec{P}_2 = \vec{P}^{\beta} - \vec{A}_2 \otimes \vec{D}_\beta, \quad \vec{P}_3 = \vec{P}^{\delta} - \vec{A}_1 \otimes \vec{D}_\delta \tag{26}$$

Compute the population of the next generation $\boldsymbol{P}(t+1)$, comprising N_w number of solutions or wolves each

$$\boldsymbol{P} = \begin{bmatrix} P^1 \\ P^2 \\ \vdots \\ P^{N_w} \end{bmatrix} = \begin{bmatrix} l_1^1 & \cdots & l_{N_{\mathrm{DG,loc}}}^1 & S_1^1 & \cdots & S_{N_{\mathrm{DG,loc}}}^1 & pf_1^1 & \cdots & pf_{N_{\mathrm{DG,loc}}}^1 \\ l_1^2 & \cdots & l_{N_{\mathrm{DG,loc}}}^2 & S_1^2 & \cdots & S_{N_c}^2 & pf_1^1 & \cdots & pf_{N_{\mathrm{DG,loc}}}^2 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ l_1^{N_w} & \cdots & l_{N_{\mathrm{DG,loc}}}^{N_w} & S_1^{N_w} & \cdots & S_{N_{\mathrm{DG,loc}}}^{N_w} & pf_1^{N_w} & \cdots & pf_{N_{\mathrm{DG,loc}}}^{N_w} \end{bmatrix} \tag{24}$$

TABLE 1. Comparison of simulation results of HGWO for P-type or type-I, and Q-type or type-II DG units for 33-bus RDS.

Method	1 DG unit		2 DG units				3 DG units				
	(Optimal bus no, Optimal DG unit size, in kVA)	Total power loss (kW)	(Optimal bus no, Optimal DG size, in kVA)	(Optimal bus no, Optimal DG unit size, in kVA)	Total DG (kVA)	Total power loss (kW)	(Optimal bus no, Optimal DG unit size, in kVA)	(Optimal bus no, Optimal DG unit size, in kVA)	(Optimal bus no, Optimal DG unit size, in kVA)	Total DG (kVA)	Total power loss (kW)
Base Case	-	210.98	-			210.98	-				210.98
P-type (Type-I)											
HGWO	**(6,2590)**	**111.018**	**(13,852)**	**(30,1158)**	**2010**	**87.164**	**(13,802)**	**(24,1090)**	**(30,1054)**	**2946**	**72.784**
MINLP[3]	(6,2590)	111.018	(13,850)	(30,1150)	2000	87.167	(13,800)	(24,1090)	(30,1050)	2940	72.784
Exhaustive OPF [12]	(6,2590)	111.02	(13,852)	(30,1158)	2010	87.17	(13, 802)	(24, 1091)	(30, 1054)	2947	72.79
EA-OPF[12]	(6,2590)	111.02	(13,852)	(30,1158)	2010	87.17	(13, 802)	(24, 1091)	(30, 1054)	2947	72.79
EA[12]	(6,2530)	111.07	(13, 844)	(30,1149)	1993	87.172	(13,798)	(24,1099)	(30, 1050)	2947	72.787
Hybrid[27]	(6,2490)	111.170	(13,830)	(30,1110)	1940	87.280	(13,790)	(24,1070)	(30,1010)	2870	72.890
PSO[27]	(6,2590)	111.030	(13,850)	(30,1160)	2010	87.170	(14,770)	(24,1090)	(30,1070)	2930	72.790
PABC [30]	(6,2598)	111.030	-	-	-	-	(14,755)	(24,1073)	(30,1068)	2896	72.810
KHA [23]	-	-	-	-	-	-	(13,810)	(25,836)	(30,841)	2487	75.412
IA[7]	(6,2601)	111.020	(12,1020)	(30,1020)	2040	87.550	(13,900)	(24,900)	(30,900)	2700	74.200
Q-type (Type-II)											
HGWO	**(30,1258)**	**151.36**	**(12,467)**	**(30,1054)**	**1521**	**141.83**	**(13,387)**	**(24 ,543)**	**(30,1037)**	**1967**	**138.25**
Hybrid[27]	(30,1230)	151.41	(12,430)	(30,1040)	1470	141.94	(13,360)	(24,510)	(30,1020)	1890	138.37

given by

$$\vec{P}^i(t+1) = \frac{\vec{P}_1 + \vec{P}_2 + \vec{P}_3}{3} \tag{27}$$

Step 3: Apply the crossover operator (19) and mutation operator (22), replacing X_j^i with P_j^i.

Step 4: Check if DG unit kVA limits and operating power factor limits are violated. Fix the generation at the limit(s) violated.

Step 5: Is the stopping criterion satisfied? If yes, stop. Else, repeat steps 1 to 4.

VI. RESULTS AND DISCUSSION

The problem considered in this paper, the optimal allocation of distributed generation using HGWO is applied and tested on three test cases comprising IEEE-33-, IEEE-69- and 85-bus Indian RDSs, to the three different types of DG units mentioned in Section I. The scope of this paper is restricted to P-type, Q-type and PQ$^+$-type DG units.

The population size of the herd or the number of wolves or trial solutions is fixed as 20. The stopping criterion used is the maximum number of iterations. For the work in this paper, this value is fixed as 200. The minimum and maximum bus voltage limits are set at 0.9 p.u. and 1.05 p.u. respectively, as in [7]. The lower and upper kVA limits of the DG unit are respectively set as 20 % and 100 % of the total load plus losses incurred in the system. The operating power factor is set as explained in Step 0 in Section V. All calculations are done in per unit (p.u.) system.

The HGWO approach and the load flow solution used are implemented in MATLAB® software on a personal computer with a 64-bit, 3.0 GHz, i7 processor and 6 GB RAM. The results of these test cases are presented and discussed next.

A. IEEE 33-BUS RDS

This RDS has 33 buses and 32 distribution lines. The total real and reactive power demands are 3,715 kW and 2,300 kVAr respectively [36]. The base values are 10 MVA and 12.66 kV. The voltage goes on decreasing as one proceeds from the source to the end, due to the presence of loads at the buses. The voltage profiles of the buses may be improved by connecting DG units to the buses to take up part of the load demand, thereby reducing the current flow and losses.

The uncompensated or base case power loss for this system is 210.98 kW. This RDS is solved for P-type, Q-type and PQ$^+$-type DG units, using the proposed HGWO algorithm. Each type is applied to the three Cases, of 1 no. DG unit, 2 no. DG unit, and 3 no. DG unit. The methods compared against include all the other four categories of classical, analytical, metaheuristic methods and hybrid approaches. Table 1 shows the comparison of simulation results for P-type DG units (real power injection only), and Q-type DG units (reactive power injection) for the all these three Cases.

For the 1 DG unit Case, the proposed HGWO algorithm determined the optimal location and size as bus no. 6, and 2,590 kVA. At 111.018 kW, it is seen that this is the best result, matched only by MINLP, EA-OPF, and Exhaustive OPF. It is to be noted that Exhaustive OPF method is an exhaustive search method, and hence, can be only be matched, and not outdone. An exhaustive search is naturally expected to produce the best results, given the nature of, and computational load of the method. It is to the credit of the

TABLE 2. Comparison of simulation results of HGWO for PQ+-type or type-III DG units, 33-bus RDS.

Method	1 DG unit		2 DG units				3DG units				
	(Optimal bus no, Optimal DG unit size (kVA), Optimal pf)	Total power loss (kW)	(Optimal bus no, Optimal DG unit size (kVA), Optimal pf)	(Optimal bus no, Optimal DG unit size (kVA), Optimal pf)	Total DG (kVA)	Total power loss (kW)	(Optimal bus no, Optimal DG unit size (kVA), Optimal pf)	(Optimal bus no, Optimal DG unit size (kVA), Optimal pf)	(Optimal bus no, Optimal DG unit size (kVA), Optimal pf)	Total DG (kVA)	Total power loss (kW)
Base Case	-	210.98	-			210.98	-				210.98
HGWO	**(6,3106,0.82)**	**67.855**	**(13,932,0.90)**	**(30,1558,0.72)**	**2490**	**28.50**	**(13,878,0.90)**	**(24,1182,0.90)**	**(30,1454.7,0.71)**	**3514**	**11.74**
MINLP[3]	(6,3105,0.82)	67.855	(13,926,0.88)	(30,1984,0.8)	2477	29.31	(13,869,0.87)	(24,1180,0.88)	(30,1432,0.8)	3481	12.74
EA-OPF[12]	(6, 3119, 0.82)	67.86	(13, 940, 0.90)	(30, 1558, 0.73)	2498	28.50	(13, 882, 0.90)	(24, 1189, 0.90)	(30, 1450, 0.71)	3521	11.74
EA[12]	(6, 3119, 0.82)	67.87	(13, 938, 0.90)	(30, 1573, 0.73)	2511	28.52	(13, 886, 0.90)	(24, 1189, 0.90)	(30, 1450, 0.71)	3525	11.80
LSFSA[25]	-	-	-	-	-	-	(6,1382,0.86)	(18,550,0.86)	(30,1062,0.86)	2994	26.72
Hybrid[27]	(6,3028,0.82)	67.937	(13,1039,0,91)	(30,1508,0.72)	2547	28.98	(13,873,0.9)	(24,1186,0.89)	(30,1439,0.71)	3498	11.76
PSO[27]	(6,3035,0.82)	67.928	(13,914,0.91)	(30,1535,0.73)	2448	28.56	(13,863,0.91)	(24,1188,0.9)	(30,1431,0.71)	3482	11.76
PABC [30]	(6,3011,0.85)	68.290	-	-	-	-	(12,1014,0.85)	(25,960,0.85)	(30,1363,0.85)	2880	15.91
AM [11]	(6,3070,0.85)	68.050	(6,3070,0.85)	(15,590,0.85)	3660	51.50	-	-	-	-	-
KHA [23]	-	-	-	-	-	-	(13,853,0.86)	(24,900,0.86)	(30,899,0.86)	2652	19.57
IA[7]	(6,3103,0.82)	67.860	(6,2195,0.82)	(30,1098,0.82)	3293	44.36	(6,1098,0.82)	(14,768,0.82)	(30,1098,0.82)	2964	22.26

other methods like MINLP, EA-OPF, and HGWO that they produce results that match those of exhaustive search, with a lower computational burden. Of these three, HGWO is the only metaheuristic method. The optimal location of the DG unit for minimal power loss is the same (bus no. 6) for all the methods.

For the 2 DG units Case, the optimal bus locations are the same at 13 and 30 for all the algorithms, except for IA method, for which it is 12 and 30. However, the HGWO solution of 87.164 kW of power loss, is the least of all the methods. The optimal DG unit sizes by HGWO differs from those by the other methods, except those by EA-OPF and Exhaustive OPF.

For the 3 DG units Case, the power loss by the HGWO is the least of all methods, with the exception of MINLP, which produces the same value for loss, with the same bus locations.

From Table 1, it is also seen that, as the number of DG units increases, the power loss reduces. Thus, for the HGWO, the power loss reduces to 72.784 kW for the 3 DG units Case, as compared to 87.164 kW for the 2 DG units Case, and 111.018 kW for the 1 DG unit Case.

For Q-type DG units, for the three Cases of 1, 2 and 3 DG units, the comparison here is with the Hybrid method of [27], which is the only work in recent literature for this particular problem data. The bus locations are the same for both the methods, but the DG unit sizes are different. However, as with P-type DG units, the results are marginally better than those of the Hybrid method, suggesting that the HGWO is superior, even if only marginally.

As with P-type DG units, the power loss reduces with the increase in the number of Q-type DG units, as can be seen in Table 1. The power loss for the HGWO reduces to 138.25 kW for the 3 DG units Case, as compared to 141.83 kW for the 2 DG units Case, and 151.36 kW for the 1 DG unit Case.

Table 2 shows the results for PQ^+-type (injection of both real and reactive power), for the three Cases of 1, 2 and 3 DG units. In this Type, the optimal location, size and power factor of the DG units are to be determined. For the Case 1, the HGWO produces the least power loss of all methods, with the exception of the MINLP. The bus location is the same for all methods.

In Case 2, the HGWO produces the least power loss with the exception of the EA-OPF method. This is better than the MINLP solution. The bus locations of 13 and 30 of the other methods that produce results comparable to the HGWO are all the same as those of the HGWO. For Case 3, as in Case 2, the HGWO produces the least power loss (11.74 kW) of all methods, with the exception of EA-OPF. Thus, it can be concluded that the HGWO matches the other best results in the literature.

As with the other Types of DG units, the power loss reduces with the increase in the number of PQ^+-type DG units, as can be seen in Table 2. The power loss for the HGWO reduces from 67.855 kW for 1 DG unit, to 28.50 kW for 2 DG units, to 11.74 kW for 3 DG units.

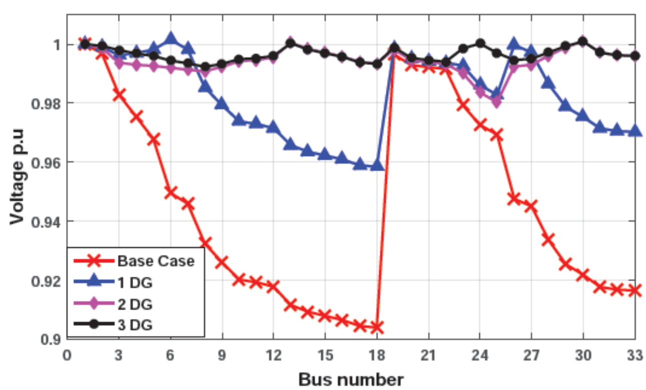

FIGURE 2. Voltage profile improvement of the IEEE 33-bus RDS with the addition of PQ+-type DG units.

The addition of DG units improves the voltage profile of the buses across the network, as given approximately by (11). Expectedly, this improvement is the most for PQ^+-type DG units. Fig. 2 shows the improvement in voltage profile, with

TABLE 3. Least and highest bus voltages for the 33-bus RDS after DG allocation.

	Case	Least		Highest	
		Bus No.	V (p.u.)	Bus No.	V (p.u.)
P-type or Type-I	1 DG unit	18	0.945517	1	1
	2 DG units	18	0.971479	1	1
	3 DG units	18	0.971487	1	1
Q-type or Type-II	1 DG unit	18	0.916321	1	1
	2 DG units	18	0.933805	1	1
	3 DG units	18	0.933389	1	1
PQ^{+}-type or Type-III	1 DG unit	18	0.958457	6	1.0015
	2 DG units	25	0.980238	30	1.009921
	3 DG units	8	0.99222	30	1.000918

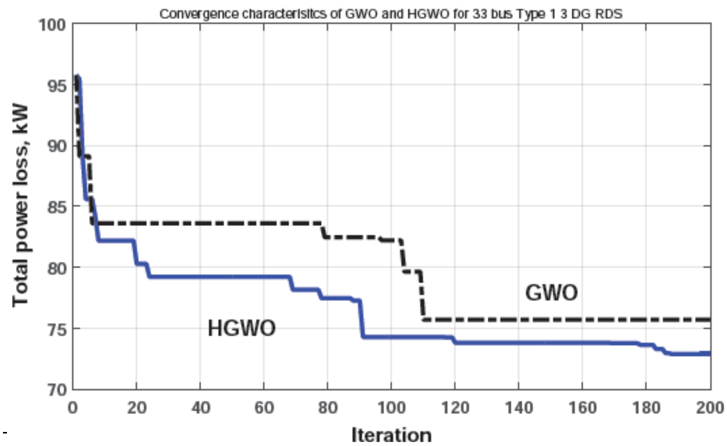

FIGURE 3. Convergence characteristics of GWO and HGWO for IEEE 33-bus RDS, with the addition of 3 DG units of P-type.

the addition of PQ^{+}-type DG units. It can be seen that the addition of more DG units results in more improvement of the voltage profile. Hence, the 3 DG units Case has the least drop in voltage and an almost flat voltage profile.

Table 3 shows the least and highest bus voltages in the RDS after DG allocation. It is seen that these are within the pre-specified lower and upper limits of 0.9 p.u. and 1.05 p.u. respectively. It is also seen that, for each type, as the number of DG units increase, there is an improvement in the least bus voltages (column 4). The effect of power factor as a variable in the problem formulation shows up as a marked improvement in both the least and highest voltages for PQ^{+}-type (Columns 4 and 6 of the last three rows, compared to the same columns of the first six rows).

Fig. 3 shows the convergence characteristics of the GWO and HGWO for the sample case of P-type, 3 DG units RDS for the 33-bus system. It can be seen that hybridization of the GWO, by the addition of the evolutionary operators of crossover and mutation, produces considerable improvement in both the speed of convergence and optimality of the solution.

B. IEEE 69-BUS RDS

This system is made up of 69 buses and 68 distribution lines. The base values are 10 MVA and 12.66 kV. The system has a total real power demand of 3,801.89 kW and a total reactive power demand of 2,694.1 kVAr [37]. Bus 3 has three branches and buses 4, 7, 9, 11 and 12 have two branches, while the other buses have only one branch connected to their next bus.

The uncompensated or base case power loss for this system is 224.99 kW. This RDS is solved for P-type, Q-type and PQ^{+}-type DG units, using the proposed HGWO algorithm. Each type is applied to the three Cases, of 1 no. DG unit, 2 no. DG units, and 3 no. DG units. The methods compared against include all the other four categories of classical, analytical, metaheuristic methods and hybrid approaches. Table 4 shows the comparison of simulation results for P-type and Q-type DG units, for the all these three Cases.

For the 1 DG unit Case, the optimal location of the DG unit for minimal power loss is the same (bus no. 61) for all the methods. The proposed HGWO algorithm determines the optimal size as 1,872 kVA, and the loss is 83.222 kW, that matches with the loss by other methods.

TABLE 4. Comparison of simulation results of HGWO for P-type or type-I and Q-type or type-II DG units for 69-bus RDS.

Method	1 DG unit		2 DG units				3 DG units				
	(Optimal bus no, Optimal size, kVA)	Total power loss (kW)	(Optimal bus no, Optimal size, kVA)	(Optimal bus no, Optimal size, kVA)	Total DG (kVA)	Total power loss (kW)	(Optimal bus no, Optimal size, kVA)	(Optimal bus no, Optimal size, kVA)	(Optimal bus no, Optimal size, kVA)	Total DG (kVA)	Total power loss (kW)
Base Case	-	224.99	-			224.99	-				224.99
P-type (Type-I)											
HGWO	**(61,1872)**	**83.222**	**(17, 531)**	**(61, 1781)**	**2312**	**71.674**	**(11, 527)**	**(17, 380)**	**(61, 1718)**	**2625**	**69.425**
MINLP[3]	(61,1870)	83.222	(17,510)	(61,1780)	2290	71.693	(11,530)	(17,380)	(61,1720)	2630	69.426
Exhaustive OPF [12]	(61, 1870)	83.23	(17, 531)	(61, 1781)	2312	71.68	(11, 527)	(18, 380)	(61, 1719)	2626	69.43
EA-OPF[12]	(61, 1870)	83.23	(17, 531)	(61, 1781)	2312	71.68	(11, 527)	(18, 380)	(61, 1719)	2626	69.43
EA[12]	(61, 1878)	83.23	(17, 534)	(61, 1795)	2329	71.68	(11, 467)	(18, 380)	(61, 1795)	2642	69.62
MTLBO[17]	(61,1819)	83.323	(17,519)	(61,1732)	2251	71.776	(11,493)	(18,378)	(61,1672)	2544	69.539
HACO[29]	(61,1872)	83.222	(18,530)	(61,1781)	2311	71.675	(11,559)	(21,346)	(61,1715)	2622	69.429
KHA [23]	-	-	-	-	-	-	(12,496)	(22,311)	(61,1735)	2542	69.56
Hybrid[27]	(61,1810)	83.372	(17,520)	(61,1720)	2240	71.82	(11,510)	(17,380)	(61,1670)	2560	69.52
PSO[27]	(61,1870)	83.222	(17,1780)	(61,530)	2310	71.68	(11,460)	(17,440)	(61,1700)	2600	69.541
Q-type (Type-II)											
HGWO	**(61,1330)**	**152.041**	**(61,1277)**	**(17,364)**	**1641**	**146.44**	**(11,412)**	**(21,230)**	**(61,1231)**	**1873**	**145.115**
Hybrid[27]	(61,1290)	152.104	(61,1240)	(18,350)	1590	146.49	(11,330)	(18,250)	(61,1190)	1770	145.283

For the 2 DG units Case, the optimal bus locations are the same at 17 and 61 for all the algorithms, except the HACO. The power loss of 71.674 kW by HGWO is the least of all methods, matched by the EA-OPF and HACO.

For the 3 DG units Case, the power loss of 69.425 kW by the HGWO is the least of all methods, with the exception of MINLP, which produces 69.426 kW. The bus locations of these two methods too are the same but differ from those by other methods.

In summary, MINLP, EA-OPF, Exhaustive OPF and HGWO produce comparable results. It is to be noted that Exhaustive OPF method is an exhaustive search method, and hence, can only be matched, and not outdone. An exhaustive search is naturally expected to produce the best results, given the nature of, and computational load of the method. It is to the credit of the other methods like MINLP, EA-OPF, and HGWO that they produce results that match those of exhaustive search, with a lower computational burden. Of these three, HGWO is the only metaheuristic method.

From Table 4, it is also seen that, as the number of DG units increases, the power loss reduces. Thus, for the HGWO, the power loss reduces to 69.425 kW for the 3 DG units Case, as compared to 71.674 kW for the 2 DG units Case, and 83.222 kW for the 1 DG unit Case. For Q-type DG units, for the three Cases of 1, 2 and 3 DG units, the comparison here is with the Hybrid method of [27], which is the only work in recent literature for this particular problem data. The results by HGWO are marginally better than those by the Hybrid method. For these almost comparable solutions, there is a variation in either the bus numbers or DG sizes, suggesting that there is more than one globally optimal solution, in finding which the HGWO is better at.

Table 5 shows the results for PQ^+-type, for the three Cases of 1, 2 and 3 DG units, for the IEEE 69-bus RDS. For the 1 DG unit Case, the bus location of 61 is the same for all the methods. However, the HGWO and the MINLP produce the least power loss of 23.16 kW.

For Case 2, the HGWO produces the least power loss of 7.20 kW, with the exception of the EA-OPF method, that matches the HGWO. For Case 3, as in the other Cases, the HGWO produces the least power loss of 4.26 kW, to be matched only by MINLP. As with the other Types of DG units, the power loss reduces with the increase in the number of PQ^+-type DG units, as can be seen in Table 5.

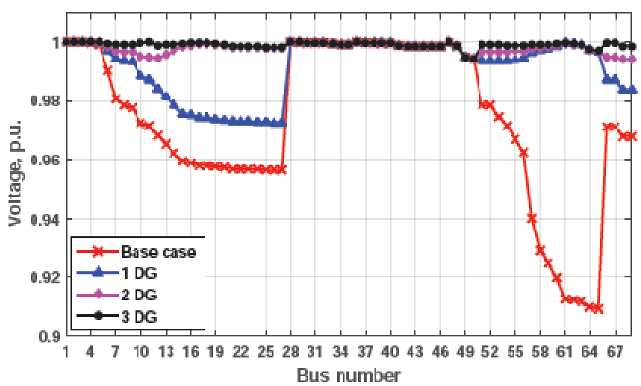

FIGURE 4. Voltage profile improvement of the IEEE 69-bus RDS with the addition of PQ^+-type DG units.

As for the previous 33-bus problem, the addition of DG units improves the voltage profile of the buses across the network. Expectedly, this improvement is the most for

TABLE 5. Comparison of simulation results of HGWO for PQ⁺-type or type-III DG units, 69-bus RDS.

Method	1 DG unit		2 DG units				3DG units				
	(Optimal bus no, Optimal size (kVA), Optimal pf)	Total power loss (kW)	(Optimal bus no, Optimal size (kVA), Optimal pf)	(Optimal bus no, Optimal size (kVA), Optimal pf)	Total DG (kVA)	Total power loss (kW)	(Optimal bus no, Optimal size (kVA), Optimal pf)	(Optimal bus no, Optimal size (kVA), Optimal pf)	(Optimal bus no, Optimal size (kVA), Optimal pf)	Total DG (kVA)	Total power loss (kW)
Base Case	-	224.99	-			224.99	-				224.99
HGWO	**(61, 2246, 0.81)**	**23.16**	**(17,628,0.82)**	**(61,2127,0.81)**	**2755**	**7.20**	**(11,614,0.81)**	**(18,452,0.83)**	**(61,2056,0.81)**	**3122**	**4.26**
MINLP[3]	(61, 2244, 0.81)	23.16	(17,658,0.82)	(61,2196,0.82)	2854	7.44	(11,607,0.813)	(50,1058,0.82)	(61,1058,0.82)	3123	4.26
EA-OPF[12]	(61,2229, 0.82)	23.17	(17, 629, 0.83)	(61, 2142, 0.81)	2771	7.20	(11, 611, 0.81)	(18, 456, 0.83)	(61, 2067, 0.81)	3134	4.27
EA[12]	(61,2290, 0.82)	23.26	(17,643,0.83)	(61, 2189, 0.82)	2832	7.35	(11, 668, 0.82)	(18, 458, 0.83)	(61, 2113, 0.82)	3239	4.48
LSFSA [25]	(61, 2200, 0.82)	23.92	-	-	-	-	(18,633,0.86)	(60,1389,0.86)	(65,362,0.86)	2384	16.26
Hybrid[27]	(61, 2200, 0.82)	23.92	(17,630,0.82)	(61,2120,0.81)	2750	7.21	(18,480,0.77)	(61,2060,0.83)	(66,530,0.82)	3070	4.30
PSO[27]	(61, 2240, 0.81)	23.16	(17,630,0.82)	(61,2130,0.81)	2760	7.20	(11,600,0.83)	(18,460,0.81)	(61,2060,0.81)	3120	4.28
AM [11]	(61, 2220, 0.81)	23.20	(17,610,0.82)	(61,2290,0.82)	2900	8.06	-	-		-	-
KHA [23]	(61, 2290, 0.82)	23.22	-	-	-	-	(11,560,0.86)	(22,357,0.86)	(61,1773,0.86)	2690	5.91
IPSO[16]	(61, 2235, 0.85)	23.90	(21,378,0.85)	(61,1861,0.85)	2567	13.600	(21,375,0.85)	(61,1515,0.85)	(64,353,0.85)	2243	12.80

TABLE 6. Least and highest bus voltages for 69-bus RDS after DG allocation.

69-bus		Least		Highest	
		Bus	V (p.u.)	Bus	V (p.u.)
P-type or Type-I	1DG unit	27	0.968272	1	1
	2DG units	65	0.979958	1	1
	3DG units	65	0.979956	1	1
Q-type or Type-II	1DG unit	65	0.931131	1	1
	2DG units	65	0.931556	1	1
	3DG units	65	0.931773	1	1
PQ⁺-type or Type-III	1DG unit	27	0.972454	62	1.000753
	2DG units	69	0.994158	62	1.000593
	3DG units	50	0.994271	19	1.003012

TABLE 7. Simulation results of HGWO for P-type and PQ⁺-type DG units, for 85-bus RDS.

Method	1 DG unit			2 DG units					3DG units					
	(Optimal bus no, Optimal size (kVA), Optimal pf)	Total power loss (kW)	Per cent reduction	(Optimal bus no, Optimal size (kVA), Optimal pf)	(Optimal bus no, Optimal size (kVA), Optimal pf)	Total DG (kVA)	Total power loss (kW)	Per cent reduction	(Optimal bus no, Optimal size (kVA), Optimal pf)	(Optimal bus no, Optimal size (kVA), Optimal pf)	(Optimal bus no, Optimal size (kVA), Optimal pf)	Total DG (kVA)	Total power loss (kW)	Per cent reduction
Base Case	-	316.1	-	-			316.1	-	-				316.1	-
P-type (Type-I)														
HGWO	(8,2368,1)	175.5	44.48	(34,676,1)	(9,1629,1)	2305	156.53	50.48	(48,416,1)	(70,198,1)	(9,1876,1)	2490	46.394	85.32
PQ⁺-type (Type-III)														
HGWO	(8,3192,0.708)	62.656	80.18	(34,952,0.7)	(9,2123,0.7)	3075	29.33	90.72	(34,967,0.7)	(10,1062,0.7)	(64,931,0.62)	2960	16.543	94.77

PQ⁺-type DG units. Fig.4 shows the improvement in the voltage profile, with the addition of PQ⁺-type DG units. It can be seen that addition of more DG units results in more improvement of the voltage profile. Hence, the 3 DG units Case has the least drop in voltage, as compared to the other Cases.

Table 6 shows the least and highest bus voltages in the RDS after DG allocation. It is seen that these are within the pre-specified lower and upper limits of 0.9 p.u. and 1.05 p.u. respectively. It is also seen that, for each type, as the number of DG units increase, there's an improvement in the least bus voltages (Column 4).

The effect of power factor as a variable in the problem formulation shows up as a marked improvement in both the least and highest voltages for PQ^+-type (Columns 4 and 6 of the last three rows, compared to the same columns of the first six rows). The voltage profile across the RDS is almost flat, for this Case.

C. INDIAN 85-BUS RDS

This system consists of 85 buses and 84 distribution lines. The total real power demand of the system is 2,570.28 kW and the reactive power demand is 2,621.936 kVAr. The base values are 100 MVA, 11 kV [38]. The uncompensated, or base case power loss is 316.1 kW. This RDS is solved for P-type, Q-type and PQ^+-type DG units, using the proposed HGWO algorithm. Since there are no other results in the literature for this problem, the only comparison is the base case.

Table 7 shows the simulation results for P-type and PQ^+-type DG units, for the all these three Cases. The percentage reduction in the power loss as compared to the base case is, 44.48 % for the 1 DG unit Case, 50.48 % for the 2 DG units Case, and 85.32 % for the 3 DG units Case.

The percentage reduction in power loss for the PQ^+-type DG units is 80.18 % for the 1 DG unit Case, 90.72 % for the 2 DG units Case, and 94.77 % for the 3 DG units Case. Obviously, among all the Types, it is the PQ+-type that leads to the maximum reduction of power loss.

VII. CONCLUSION

The work reported in this paper applied the HGWO, a new metaheuristic algorithm, to solve the non-convex, discrete, optimal DG allocation problem to the IEEE 33-, IEEE 69- and Indian 85-bus RDS. The HGWO compares favorably with other best results in the literature - including exhaustive search - in terms of optimality of the solution, thereby suggesting that the solution is a globally optimal one. A careful study of the tables also shows that, by and large, the HGWO outperforms all the other metaheuristic methods, and matches the results by exhaustive search.

Between P-type, Q-type and PQ^+-type DG units, the best results were produced by PQ^+-type DG units, for all the three test cases of IEEE 33-, IEEE 69- and Indian 85-bus RDSs, considered in this paper. This is easily explained, as the addition of one more variable (the power factor) in the solution vector leads to more freedom in the choice of other variables.

Classical, gradient-based methods are most suitable for solving convex programming problems. Metaheuristic algorithms are free from the convexity condition required by classical methods, and hence are inherently capable of finding the global optimum of non-convex problems like the optimal DG allocation problem. However, this capability is not automatic; it is dependent on factors like the operators the algorithm has, to perform an efficient search of the non-convex solution space. The results in this paper show that HGWO meets this requirement eminently, thereby outperforming all the other metaheuristic algorithms on this problem.

Metaheuristic algorithms being probabilistic in nature, the solution produced is often nearly the global optimum, but not the exact global optimum. HGWO performs so well that, the results produced by it matches those by exhaustive search, indicating that HGWO is an exceptional performer among metaheuristic algorithms.

Metaheuristic methods have the limitation that, they often require tuning on the part of the user, to work correctly. Given some practical optimization problem, this is can often make the difference between solving successfully and not solving the problem. The HGWO is an exception in this respect too: except for the specification of the population size by the user, no other tuning is needed for the HGWO.

Future works can consider applying this promising algorithm to other difficult optimization problems in power systems and other areas. Future refinements of the optimal DG allocation problem can include other real life objectives like reliability and robustness, under uncertainties of load and generation.

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