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A Stochastic Computational Approach for the Analysis of Fuzzy Systems

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ABSTRACT Fault tree analysis (FTA) has been widely utilized as a reliability evaluation technique for complex systems, such as nuclear power plants and aerospace systems. However, it is hard to obtain the crisp failure probabilities of basic events, owning to the insufficient information about some complex engineering systems. Hence, fuzzy set theory and fuzzy arithmetic operation (FAO) have been used as effective methods to analyze system reliability. However, it is cumbersome to evaluate complex systems based on FAO. To improve the evaluation efficiency, stochastic computational models are proposed in this paper to perform reliability analysis of a fuzzy system. Due to the features of Gaussian distribution in stochastic computation, a basic event's failure possibility given by a fuzzy number is transformed into the expected value of it. The standard deviation of stochastic computational results gives the spread of the fuzzy number. A fuzzy system is then converted into a deterministic system. The analysis of an illustrating example shows that the proposed stochastic approach can efficiently evaluate the failure probability of a system.

INDEX TERMS Fault tree analysis, fuzzy arithmetic operation, stochastic computation, Gaussian distribution.

NOTATION

$\boldsymbol{A}$	Fuzzy set.
$u(x)$	Membership function.
e_i	Expert *i*.
w_i	Weight of expert *i*.
L	Sequence length.
u	Gaussian distribution with expected value u.
σ	Gaussian distribution with the standard deviation σ.

I. INTRODUCTION

Reliability evaluation plays an important role in the design and development of engineering systems. Fault tree analysis (FTA) is a widely used technique for the reliability evaluation of systems [1]. In conventional FTA, the failure probabilities of basic events are assumed to be precisely determined for a system [2]. However, such precise parameters are difficult or even impossible to be obtained in practice. To address this issue, failure possibility has been proposed to substitute failure probability [3], [4]. The relative frequencies of basic events, referred to as fuzzy numbers, are used to represent the failure possibilities [5]. Then, the failure possibility of the top event can be obtained by FTA. The system failure possibility can be estimated using the fuzzy set theory [6]. The *L*-*R* fuzzy numbers are applied to define the failure possibilities of a system in [7]. A fuzzy-based algorithm is presented in [8] to rank the basic events according to their contributions to a system's failure possibility. Using these approaches, the top event's failure possibility can be obtained as a fuzzy number. The scope of the fuzzy number indicates the fluctuations incurred by uncertainty [9], [10]. These methods are effective in evaluating the reliability of a system, but with a rather complex computation process.

The expected value of a fuzzy number has been used to calculate the crisp system failure probability and to simplify a fuzzy computation; the notations of l*ower possibilistic* and *upper possibilistic* mean values have been used to obtain the bounds of the system failure probability [11]. However, the fuzzy expected value approach (FEVA) can only compute the interval bound given by l*ower* and *upper possibilistic* mean values of the system failure probability, whereas the probability distribution of the system failure probability cannot be obtained.

Monte Carlo (MC) simulation [12]–[14] has widely been used to evaluate fault trees. However, a long run time is usually required because of the slow convergence of the result in an MC approach [15]. A fuzzy Markov model [16], [17], [37] has been proposed to evaluate a fault tree system. However, the size of a state transition matrix required in the analysis increases exponentially with the number of components in a system [18].

In [19] and [20], a stochastic approach is used to model the dynamic behaviors in a fault tree. Stochastic computation has advantages such as computational simplicity, high speed, and fault-tolerance [21]. For stochastic analysis, the output is probabilistic; due to inevitable stochastic fluctuations, it follows approximately a Gaussian distribution with a reasonable sequence length [22]. This feature of randomness in stochastic computation provides a natural means to efficiently model a fuzzy network.

In this paper, a stochastic computational model is proposed for improving the evaluation efficiency of a fuzzy system. In this model, a fuzzy system is first converted into a conventional system. The features of Gaussian distribution in stochastic computation are exploited such that the failure possibilities of basic events with fuzzy numbers are converted into precise failure probabilities values. The failure probabilities of basic events are represented by non-Bernoulli sequences [22]. The output sequence of the proposed stochastic computation model denotes the top event's failure probability by performing stochastic computing, and the result follows Gaussian distribution. The proposed stochastic approach is more efficient compared to Monte Carlo (MC) method and fuzzy arithmetic operation.

The remainder of the paper is organized as follows. Section II reviews the fuzzy set theory and fuzzy expert systems. Section III presents the stochastic computational models. The validation of the stochastic computational approach is presented in Section IV. In Section V, the efficiency of the proposed approach is revealed by the analysis of several benchmarks. Finally, Section VI concludes the paper.

II. FUZZY SET THEORY AND FUZZY EXPERT SYSTEMS

Fuzzy set theory was first introduced in 1965 to cope with the ambiguity or uncertainty in a system [23]. For such a system, the subjective judgement or estimation of an individual plays a vital role. This technique provides an alternative method to process data by allowing partial set memberships rather than crisp set memberships. The failure possibilities of events replace the failure probabilities in the fuzzy set theory [24], [38]. A fuzzy expert system is used to transform the ambiguous knowledge of experts to the TFNs [25] to obtain the failure possibilities of the basic events.

A. FUZZY SET AND ITS OPERATION

1) FUZZY SET

A fuzzy set was introduced as an extension and generalization of the concept of crisp sets [7]. A fuzzy set $\boldsymbol{A}$ in the universal set U is characterized as

$$\boldsymbol{A} = \{(x, u(x))|x \in U \text{ and } 0 \leq u(x) \leq 1\}, \tag{1}$$

where $u(x)$ is the membership function of fuzzy set $\boldsymbol{A}$, the value of $u(x)$ is the grade (i.e., *degree or confidence level*) of membership x in A, indicating the degree that x belongs to $\boldsymbol{A}$. If x is not in A, $u(x) = 0$; if x is totally in A, $u(x) = 1$; if x is partly in A, $u(x) \in (0, 1)$.

The triangular fuzzy number (TFN) is widely used by experts to express the failure possibility of an event [8]. A TFN is defined in [4] as follows.

Definition 1: The membership function of a TFN $\tilde{A}$ parameterized by (α, m, β) is defined as:

$$u_{\tilde{A}}(x) = \begin{cases} \dfrac{x-\alpha}{m-\alpha} & \text{if } \alpha \leq x \leq m \\ \dfrac{\beta - x}{\beta - m} & \text{if } m \leq x \leq \beta \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where m is the center of $u_{\tilde{A}}(x)$, while α and β indicate the lower and upper bounds respectively.

2) L-R FUZZY NUMBER AND FUZZY OPERATIONS

The membership function $u(x)$ can be approximated by two functions $L(x)$ and $R(x)$. Functions $L(\cdot)$ and $R(\cdot)$ are referred to as reference functions $f(\cdot)$, with the following properties [26]:

(i) $f(x) = f(-x)$;

(ii) $f(0) = 1$;

(iii) $f(x)$ is a decreasing function in the interval $[0, +\infty]$.

This type of fuzzy number is referred to as an *L-R* fuzzy number. If a fuzzy number $\tilde{\theta}$ is *L-R* type, then the membership function is given by:

$$u_{\tilde{m}}(x) = \begin{cases} L(\dfrac{\theta - x}{\rho}) & \textit{if } x \leq \theta,\ \rho > 0 \\ R\left(\dfrac{x-\theta}{\zeta}\right) & \textit{if } x \geq \theta,\ \zeta > 0, \end{cases} \tag{3}$$

where θ represents the center of $u_{\tilde{\theta}}(x)$, ρ and ζ indicate the *left* and *right spreads* respectively. When the spreads are zero, $\tilde{\theta}$ is a non-fuzzy number by convention. As the spreads increase, $\tilde{\theta}$ will be fuzzier and fuzzier [26]. Similarly, an *L-R* fuzzy number can be denoted as $(\theta, \rho, \zeta)_{LR}$. The arithmetic operations of *L-R* fuzzy numbers are given in [5] and [7]:

Change of sign:

$$-(\theta, \rho, \zeta)_{LR} = (-\theta, \zeta, \rho)_{RL}. \tag{4}$$

Addition $\oplus$:

$$(\theta, \rho, \zeta)_{LR} \oplus (n, \gamma, \delta)_{LR} = (\theta + n, \rho + \gamma, \zeta + \delta)_{LR}. \tag{5}$$

Subtraction $\ominus$:

$$(\theta, \rho, \zeta)_{LR} \ominus (n, \gamma, \delta)_{RL} = (\theta - n, \rho + \delta, \zeta + \gamma)_{LR}. \tag{6}$$

Multiplication $\otimes$:

$$(\theta, \rho, \zeta)_{LR} \otimes (n, \gamma, \delta)_{LR} \cong (\theta n, \theta\gamma + n\rho, \theta\delta + n\zeta)_{LR}. \quad \text{if } \theta,\ n > 0 \quad (7)$$

Scalar multiplication $\odot$:

$$k \odot (\theta, \rho, \zeta)_{LR} = \begin{cases} (k\theta, k\rho, k\zeta)_{LR}, & \text{for } k \in R,\ k > 0 \\ (k\theta, -k\zeta, -k\rho)_{RL}, & \text{for } k \in R,\ k < 0. \end{cases} \quad (8)$$

B. FUZZY EXPERT SYSTEMS

Due to the lack of historical failure data for complex systems, such as nuclear power, the failure parameters for basic events are often determined according to the experience of experts. The fuzzy expert system includes an expert assessment unit, a transform unit and an expert weighting unit.

1) EXPERT ASSESSMENT UNIT

The selected experts must be familiar with the working environment and have considerable knowledge of the target system [25], [27]. The expert set (E) is described as $E = \{e_i | i = 1, 2, 3, \ldots, n\}$, where n indicates the number of experts involved. The basic event set (B) is denoted as $B = \{b_j | j = 1, 2, 3, \ldots, k\}$, where k is the total number of basic events of the target system.

The output of the expert assessment unit is a set of qualitative linguistic values (V) that reflect the basic event failure possibilities [25]. Usually, seven linguistic values (V) [8], [28] are used to assess the basic events, $V =$ {"very high," "high," "reasonably high," "moderate," "reasonable low," "low," "very low"}.

TABLE 1. Linguistic value of basic event failure possibility and corresponding triangular fuzzy numbers [8], [30].

Levels	Linguistic value	Triangular fuzzy number (TFN)
1	very high	(0.9, 1.0, 1.0)
2	High	(0.7, 0.9, 1.0)
3	reasonably high	(0.5, 0.7, 0.9)
4	moderate	(0.3, 0.5, 0.7)
5	reasonable low	(0.1, 0.3, 0.5)
6	low	(0, 0.1, 0.3)
7	very low	(0, 0, 0.1)

2) TRANSFORMATION UNIT

The transform unit converts the linguistic values generated by expert assessments to TFNs. The obtained fuzzy number represents qualitative failure possibility in [0, 1]. This means that the closer the fuzzy number is to 0, the less likely that the failure of a basic event occurs. Table I provides an example of the transformation from the linguistic values to the TFNs.

3) EXPERT WEIGHT UNIT

In practice, the involved experts have different levels of expertise, background and working experience. Hence, due to the discrepancy in experts' knowledge and experience, different perceptions about the same event may lead to different assessments [29], [30]. Let w_i indicate the different justification weight being assigned to expert e_i, $i = 1, 2, 3, \ldots, n$. Here n indicates the number of selected experts, $0 \le w_i \le 1$ and $\sum_{i=1}^{n} w_i = 1$. The final weighted-mean of TFNs indicated by a matrix Q_B can be obtained as:

$$Q_B = Q_{TFN} * Q_W = \begin{bmatrix} u^{e_1 b_1} & u^{e_2 b_1} & \ldots u^{e_n b_1} \\ \ldots & \ldots & \ldots\ldots \\ u^{e_1 b_k} & u^{e_2 b_k} & \ldots u^{e_n b_k} \end{bmatrix} * \begin{bmatrix} w_1 \\ \ldots \\ w_n \end{bmatrix} = \begin{bmatrix} (\alpha^{b_1}, m^{b_1}, \beta^{b_1}) \\ \ldots \\ (\alpha^{b_k}, m^{b_k}, \beta^{b_k}) \end{bmatrix}, \quad (9)$$

where Q_{TFN} and Q_W contain fuzzy failure possibilities of basic events and expert weights respectively, $u^{e_i b_j}$ is the TFN for b_j evaluated by expert e_i, while w_i represents the weight for the ith expert.

III. THE PROPOSED STOCHASTIC COMPUTATIONAL MODEL

The computational model consists of an expert evaluation module and a stochastic computational module, as shown in Fig. 1. This model is based on the assumptions below:

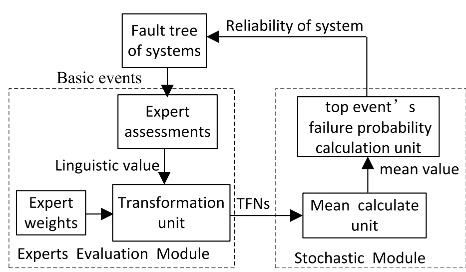

FIGURE 1. The Structure of the proposed computational model.

(i) The basic events operate independently, which means that the failure of one event does not affect other events.
(ii) The failure possibilities of the basic event are described by fuzzy numbers provided by experts.

In the stochastic computational model, the failure possibility of a basic event is described by linguistic values in expert assessments unit, and the linguistic values are converted to triangular fuzzy numbers by expert weight and transformation units. The mean values of the triangular fuzzy numbers can be obtained by the mean calculate unit, and mean values of basic events' failure probabilities are inputs of the top event's failure probability calculation unit. Then the reliability of the system can be obtained by the proposed computational model.

A. STOCHASTIC COMPUTATION

Stochastic computation was first introduced in 1960s for performing arithmetic operations with standard logic elements [31]. In stochastic computation, non-Bernoulli sequences are used to encode real numbers or probabilities [22]. In the binary streams, a proportional number of bits are set to "1" to indicate a probability. For instance, a probability of 0.4 is represented by a bit stream containing 40% 1s and 60% 0s. If a probability (e.g. 0.4) is the failure probability of a basic event, a '1' or '0' indicates one failure or one success of the basic event respectively.

Fig. 2(a-d) show several logic gates used for stochastic computation in this paper, including a buffer, an inverter (NOT), an AND, and an OR gate. In Fig. 2, the probabilities are encoded into bit streams of 10 bits. An output sequence encoding the output signal probability can be obtained by propagating the bit streams through the logic gates.

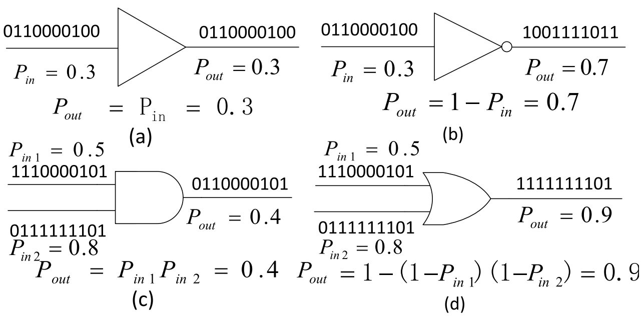

FIGURE 2. Logic gates for stochastic computation: (a) A buffer with a random binary bit stream as the input; (b) An inverter; (c) An AND gate with statistically independent inputs; (d) An OR gate with statistically independent inputs. (The percentage of '1' in a stochastic sequence indicates the signal probability).

Scaled addition can be implemented by a multiplexer (MUX), as shown in Fig. 3. Let A, B, and S indicate stochastic bit streams for p_A, p_B and p_S respectively. Then, p_C is given by $p_C = p_S \cdot p_A + (1-p_S) \cdot p_B$ and can be obtained by propagating the sequences through the MUX. If $p_S = 0.5$, $p_C = 0.5(p_A + p_B)$.

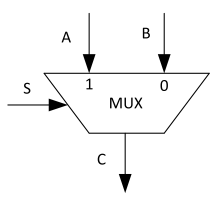

FIGURE 3. Scaled addition.

B. MEAN CALCULATION UNIT

The fuzzy operation can be implemented by stochastic computation. The fuzzy numbers can be transformed into the crisp number by calculating the expected value of the TFNs. The matrix Q_B can be obtained and the failure possibilities with expert weights in the matrix Q_B can be transformed into the exact failure probabilities using the stochastic computational model proposed for this unit.

The fuzzy scalar multiplication of triangular fuzzy number is characterized by

$$p_w \odot (p_u, p_v, p_q) = (p_w \cdot p_u, p_w \cdot p_v, p_w \cdot p_q) = (p_u, p_v, p_q), \tag{10}$$

which can be implemented with the application of AND gates.

Assume that u, v, q and w are stochastic bit streams to encode p_u, p_v, p_q and p_w respectively, the failure possibility $u^{e_i b_j} (i \in [1, n]$ and $j \in [1, k])$ in Q_{TFN} is described by the TFN (p_u, p_v, p_q) and the expert weights $w_i (i \in [1, n])$ in Q_W are represented by p_w. Then, $p_{u'}$, $p_{v'}$ and $p_{q'}$ can be obtained by analyzing the output sequences u', v' and q' in Fig. 4.

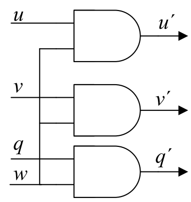

FIGURE 4. Fuzzy scalar multiplication.

As discussed previously, the scaled addition can be implemented by a MUX. The fuzzy addition of triangular fuzzy numbers is given by (11). Let γ_1, η_1, δ_1, γ_2, η_2, δ_2, S_1, S_2, and S_3 be stochastic bit streams indicating probabilities p_{γ_1}, p_{η_1}, p_{δ_1}, p_{γ_2}, p_{η_2}, p_{δ_2}, p_{S_1}, p_{S_2}, p_{S_3} respectively. Then, the fuzzy addition of TFNs $(p_{\gamma_1}, p_{\eta_1}, p_{\delta_1})$, $(p_{\gamma_2}, p_{\eta_2}, p_{\delta_2})$ can be performed by using the stochastic model in Fig. 5.

$$\begin{aligned}
&(p_{\gamma_1}, p_{\eta_1}, p_{\delta_1}) \oplus (p_{\gamma_2}, p_{\eta_2}, p_{\delta_2}) \\
&\quad = (p_{\gamma_1} + p_{\gamma_2}, p_{\eta_1} + p_{\eta_2}, p_{\delta_1} + p_{\delta_2}) \\
&\quad = (2(p_{S_1} \cdot p_{\gamma_1} + (1 - p_{S_1}) \cdot p_{\gamma_2}), \\
&\qquad 2(p_{S_2} \cdot p_{\eta_1} + (1 - p_{S_2}) \cdot p_{\eta_2}), \\
&\qquad 2(p_{S_3} \cdot p_{\delta_1} + (1 - p_{S_3}) \cdot p_{\delta_2}))
\end{aligned} \tag{11}$$

Let $p_{S_1} = p_{S_2} = p_{S_3} = 0.5$, then

$$(p_{\gamma_1}, p_{\eta_1}, p_{\delta_1}) \oplus (p_{\gamma_2}, p_{\eta_2}, p_{\delta_2}) = (2p_{\gamma'}, 2p_{\eta'}, 2p_{\delta'}).$$

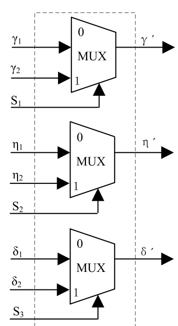

FIGURE 5. Fuzzy scaled addition.

The failure possibility of a basic event is then transformed to a precise failure probability, because the TFN can be described by the expected value of it. A method to calculate the expected value of a TFN $A\ (\tau, \varepsilon, \upsilon)$ is presented in [11] and the expected value is given by:

$$E_A = \varepsilon + \frac{\upsilon - \tau}{4}, \tag{12}$$

where ε is the center of A, τ and υ are left-width and right-width of A respectively ($\alpha > 0, \beta > 0$).

As per *Definition* 1, A' is a TFN parameterized by (α', m', β'), m' is the center of a membership function $u_{A'}(x)$, α' and β' are the lower and upper bounds. The TFN A' can be described by (m'-α', m', β'-m'), m'-α' and β'-m' are the left-width and right-width respectively. As per (12), the expected value of A' is obtained by:

$$E_{A'} = m' + \frac{(\beta' - m') - (m' - \alpha')}{4} = \frac{2 * m' + \beta' + \alpha'}{4}. \tag{13}$$

Hence, the matrix (Q_M) that contains the failure probabilities of basic events is given by

$$Q_M = \begin{bmatrix} \bar{m}^{b_1} \\ \cdots \\ \bar{m}^{b_k} \end{bmatrix} = \begin{bmatrix} \frac{2*m^{b_1} + \alpha^{b_1} + \beta^{b_1}}{4} \\ \cdots \\ \frac{2*m^{b_k} + \alpha^{b_k} + \beta^{b_k}}{4} \end{bmatrix} = \begin{bmatrix} \frac{\frac{\alpha^{b_1}+\beta^{b_1}}{2} + m^{b_1}}{2} \\ \cdots \\ \frac{\frac{\alpha^{b_k}+\beta^{b_k}}{2} + m^{b_k}}{2} \end{bmatrix}, \tag{14}$$

where $\bar{m}^{b_j}$ is the exact failure probability of the basic event, converted from the final weighted-mean TFN, and $(\alpha^{b_j}, m^{b_j}, \beta^{b_j})$ is the element in the final weighted-mean TFN matrix (Q_B) with $j \in (1, k)$ and k being the number of basic events.

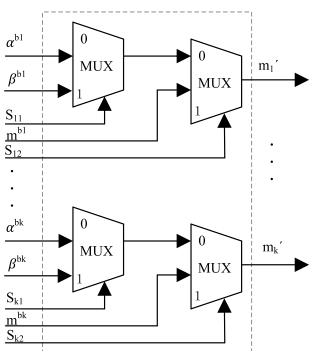

FIGURE 6. Triangular Fuzzy number mean.

Equation (14) can be implemented by the stochastic architecture shown in Fig. 6. Through the propagation of the stochastic sequences, α^{b_j}, m^{b_j}, β^{b_j}, S_{j1} and S_{j2}, the output probabilities can be obtained by analyzing the output sequences. Given $S_{j1} = S_{j2} = 0.5$, we have $m_j' = \frac{\alpha^{b_j}+2*m^{b_j}+\beta^{b_j}}{4}$.

C. FAILURE PROBABILITY CALCULATION UNIT

In stochastic computation, Bernoulli and non-Bernoulli sequences can be used to represent the initial input probabilities. As shown in [15], the use of non-Bernoulli sequences leads to a faster convergence of the output. Hence, non-Bernoulli sequences are adopted to represent the basic event's failure probability $\bar{m}^{b_j}$ ($j \in (1, k)$, k is the number of the basic events) in this paper.

Based on a system's topology, a fault tree can be derived according to the relationships among events and these relationships can be modeled by combinations of logic gates. Then, the output sequence S_{P_T} is obtained as

$$S_{P_T} = f(S_{\bar{m}^{b_1}}, S_{\bar{m}^{b_2}}, \ldots, S_{\bar{m}^{b_k}}), \tag{15}$$

where $f(\cdot)$ is the function of the constructed stochastic architecture due to the system topology; $S_{\bar{m}^{b_j}}(j \in (1, k))$ is the non-Bernoulli sequence representing the precise failure probability of b_j, i.e., $\bar{m}^{b_j}$, and k is number of basic events. Then, the failure probability of the top event, i.e., P_T, is obtained by analyzing the output sequence S_{P_T}, given by $P_T = \frac{\sum_{i=1}^{L} S_{P_T}}{L}$. The stochastic model to obtain failure probability of the top event is shown in Fig. 7.

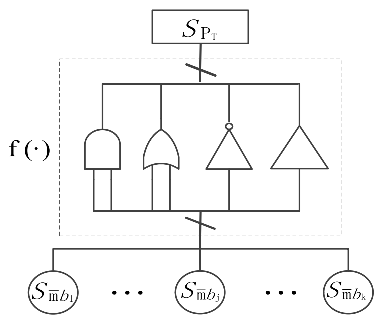

FIGURE 7. Stochastic model of failure probability of the top event calculation.

IV. STOCHASTIC MODEL VALIDATION

To validate the proposed stochastic model, the logic gates in Fig. 2 (b, c) are verified first. The theoretical proof is presented as follows:

Theorem 1: For an inverter with a triangular fuzzy number (TFN) as the input, using the stochastic approach with the expected value of the TFN encoded as the probability of 1's in a long non-Bernoulli sequence as the input, the probability of 1's in the output sequence of the inverter has the same expected value as the output of the inverter using a fuzzy arithmetic operation (FAO).

Proof: Assume that the input of the inverter is a triangular fuzzy number (TFN) $F_1\ (\alpha, m, \beta)$ and that Z_1 and Z_2 indicate the output of the inverter using the stochastic approach and FAO respectively. F_1 denotes a probability value occurred by an event. As per (13), the expected value of F_1 is given by

$$E(F_1) = \frac{2m+\alpha+\beta}{4}. \tag{16}$$

In the stochastic model for an inverter, assume that the input sequence has a probability of a to be "1", then the expected value of the probability of a 1 in the output sequence is given by:

$$u_1 = 1 - a. \tag{17}$$

If the fuzzy number $F_1(\alpha, m, \beta)$ is represented by the expected value $E(F_1)$, and $E(F_1) = \text{a}$, then the expected value of Z_1 is given by

$$E(Z_1) = 1 - \frac{2m+\alpha+\beta}{4}. \tag{18}$$

In the fuzzy arithmetic operation (FAO), *L-R* fuzzy numbers are used as the failure possibilities of the basic events. *L-R* fuzzy numbers can be defined by a triplet (n, γ, δ) and the value of n is considered as the expected value of a triangular fuzzy number from expert assessment [5], [7], [34].

According to (3) and (13), the TFN F_1 (α, m, β) is converted into *L-R* fuzzy number F_2 [5] as the input of the inverter gate:

$$\begin{aligned} F_2 &= (n, \gamma, \delta) \\ &= (E(F_1), \lambda E(F_1), \lambda E(F_1)) \\ &= (\frac{2m+\alpha+\beta}{4}, \lambda\frac{2m+\alpha+\beta}{4}, \lambda\frac{2m+\alpha+\beta}{4})_{LR}, \end{aligned}$$

where F_2 is defined by a triplet (n, γ, δ) and the value of n is obtained by the expected value of the triangular fuzzy number F_1, λ is a small number from the membership function of the *L-R* fuzzy number F_2.

According to (4) and (17), Z_2 is obtained as:

$$\begin{aligned} Z_2 &= 1 - F_2 \\ &= 1 + (-\frac{2m+\alpha+\beta}{4}, \lambda\frac{2m+\alpha+\beta}{4}, \lambda\frac{2m+\alpha+\beta}{4})_{RL} \\ &= \left(1 - \frac{2m+\alpha+\beta}{4}, \lambda\frac{2m+\alpha+\beta}{4}, \lambda\frac{2m+\alpha+\beta}{4}\right)_{RL}. \end{aligned}$$

Let $m_1 = 1 - \frac{2m+\alpha+\beta}{4}$, $\alpha_1 = \lambda\frac{2m+\alpha+\beta}{4}$, according to the method in [11], the expected value of Z_2 is given by:

$$E(Z_2) = \frac{1}{2}\left(m_1 - \alpha_1 \int_0^{+\infty} L(x)\,dx + m_1 + \alpha_1 \int_0^{+\infty} R(x)\,dx\right).$$

For a symmetric *L-R* fuzzy number, we have:

$$E(Z_2) = m_1 = 1 - \frac{2m+\alpha+\beta}{4}. \tag{19}$$

Then, we obtain $E(Z_1) = E(Z_2)$.

This proves *Theorem 1*.

Theorem 2: For a two-input AND gate with two triangular fuzzy numbers (TFNs) as the inputs, using the stochastic approach with the expected values of the two TFNs encoded as the probabilities of 1's in two long non-Bernoulli sequences as the inputs, the probability of 1's in the output sequences of two-input AND gate has the same expected value as the output of the two-input AND gate using a fuzzy arithmetic operation (FAO).

Proof: Assume that the two inputs of the AND gate are triangular fuzzy numbers (TFNs) F_3 (α, m, β) and F_4 (γ, n, δ), and that Z_3 and Z_4 are the outputs of the AND gate using the stochastic approach and FAO respectively. F_3 and F_4 denote the probabilities value occurred by events. In stochastic model, F_3 and F_4 are represented by the expected values $E(F_3)$ and $E(F_4)$ as the probabilities of a 1 in the input non-Bernoulli sequences. According to (13), the expected values of F_3 and G_1 are given as:

$$E(F_3) = \frac{2m+\alpha+\beta}{4}, \tag{20}$$

$$E(F_4) = \frac{2n+\gamma+\delta}{4}. \tag{21}$$

The [15, Lemma 1] shows that for an AND gate with two non-Bernoulli input sequences with the 1's probabilities a, b, if the sequence length is large, the probability of a 1 in the output sequence of the AND gate follows approximately a Gaussian distribution with the expected value $u_2 = ab$.

We can obtain:

$$E(Z_3) = E(F_3) \cdot E(F_4) = \frac{2m+\alpha+\beta}{4} \cdot \frac{2n+\gamma+\delta}{4}. \tag{22}$$

In fuzzy arithmetic operation (FAO) [5], [7], [32], the TFNs F_3 and F_4 are converted into *L-R* fuzzy numbers F_5 and F_6 as follows:

$$F_5 = \left(\frac{2m+\alpha+\beta}{4}, \lambda\frac{2m+\alpha+\beta}{4}, \lambda\frac{2m+\alpha+\beta}{4}\right)_{LR},$$

$$F_6 = \left(\frac{2n+\gamma+\delta}{4}, \lambda\frac{2n+\gamma+\delta}{4}, \lambda\frac{2n+\gamma+\delta}{4}\right)_{LR},$$

where $L(\cdot)$ and $R(\cdot)$ are called reference functions, λ is a small number from the membership function of the *L-R* fuzzy number F_5 and F_6.

Assume that F_5 and F_6 are the two inputs of the AND gate, according to (7), the output Z_4 is obtained as:

$$\begin{aligned} Z_4 &= F_5 \otimes F_6 \\ &= \left(\frac{2m+\alpha+\beta}{4} \cdot \frac{2n+\gamma+\delta}{4}, 2\lambda\frac{2m+\alpha+\beta}{4}\right. \\ &\qquad \left.\cdot\frac{2n+\gamma+\delta}{4}, 2\lambda\frac{2m+\alpha+\beta}{4} \cdot \frac{2n+\gamma+\delta}{4}\right)_{LR}. \end{aligned} \tag{23}$$

If we set $m_2 = \frac{2m+\alpha+\beta}{4} \cdot \frac{2n+\gamma+\delta}{4}$, $\alpha_2 = 2\lambda\frac{2m+\alpha+\beta}{4} \cdot \frac{2n+\gamma+\delta}{4}$, according to the method in [11], the expected value of Z_2 is given by:

$$E(Z_4) = \frac{1}{2}\left(m_2 - \alpha_2 \int_0^{+\infty} L(x)\,dx + m_2 + \alpha_2 \int_0^{+\infty} R(x)\,dx\right). \tag{24}$$

For a symmetric *L-R* fuzzy number, we obtain:

$$E(Z_4) = m_2 = \frac{2m+\alpha+\beta}{4} \cdot \frac{2n+\gamma+\delta}{4}.$$

Then, we have $E(Z_3) = E(Z_4)$, which completes the proof.

The OR gate and other logic functions can be implemented by the inverter and AND gate.

Theorem 3: For a two-input AND gate with two triangular fuzzy numbers (TFNs) as the inputs, the stochastic simulation results (i.e., the probability of a 1 in the output sequence), obtained by using the expected values of the TFNs encoded as the 1's probabilities in the long non-Bernoulli sequences as inputs, are close to the center of the output fuzzy number obtained by using a fuzzy arithmetic operation (FAO).

Proof: As a 1's probability in the output sequence of the AND gate by stochastic computation approximately follows a Gaussian distribution, the standard deviation σ reflect the confidence level of the output signal probability. In [15], for an AND gate with two L bits non-Bernoulli input sequences to represent the 1's probability a and b respectively, the probability of a 1 in the output sequence follows a Gaussian distribution with standard deviation σ:

$$\sigma = \sqrt{a(1-a)b(1-b)/L}.$$

Based on the assumptions of $F_3(\alpha, m, \beta)$ and $F_4(\gamma, n, \delta)$ and Z_3 in *Theorem 2*, the standard deviation of Z_3 is calculated as (25), as shown at the bottom of the next page.

The TFNs F_3 and F_4 are converted into *L-R* fuzzy numbers F_5 and F_6 as above to be the input of AND gate. By applying the fuzzy arithmetic operation (FAO), we can get the output Z_4 as in (23).

Due to the 3σ rule [33] of a Gaussian distribution, the area under the normal curve within the range $u \pm 3\sigma$ is 0.9973, where u and σ are the mean and the standard deviation respectively. Hence, the $3\sigma_{Z_3}$ in stochastic computation and the α_2 (the *left* or *right spread* of fuzzy number Z_4) in FAO are compared by (26), as shown at the bottom of the next page.

By (26), when $L > \frac{9(1-E_{F_3})(1-E_{F_4})}{4\lambda^2 E_{F_3} E_{F_4}}$, we obtain: $\Delta < 0$, which indicates that the stochastic computational results are located within the range of the fuzzy number calculated by FAO. This proves *Theorem 3*.

V. CASE STUDIES

A stochastic analysis is performed for a nuclear power plant to show the feasibility and the effectiveness of the proposed model. The results are compared with those obtained by a fuzzy arithmetic operation (FAO) [5], [7], [32], Monte Carlo simulation [12]–[14] and the fuzzy expected value approach (FEVA) [11]. In the nuclear power plant, the radiation release is a hazard, and assumed as the top event in fault tree analysis. The radiation release may be caused by the occurrence of some events. The descriptions of the nuclear power plant are presented in Table II and Fig. 8.

TABLE 2. The descriptions of the events in nuclear power plant [34].

events	Description
b_1	Safety signals fail
b_2	Power fails
b_3	Explosive damage to the core
b_4	Insufficient thermal dissipation
b_5	Reactor on
b_6	Logic of alarm system fails
b_7	Sensor fails
b_8	Overpressure
b_9	Room hot
b_{10}	Breach of physical boundary
Mid event 1	Release of fission product due to core damage
Mid event 2	Physical damage to the core
Mid event 3	Mechanical damage to the core
Mid event 4	Thermal damage to the core
Mid event 5	Unwanted radiation
Mid event 6	Alarm system fails
Mid event 7	Radiation present

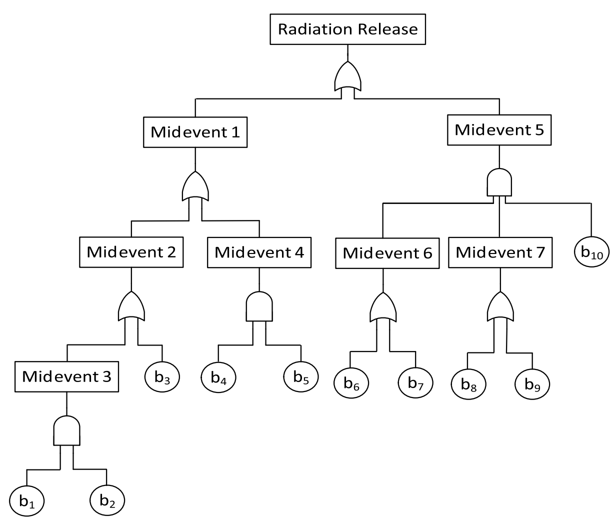

FIGURE 8. A fault tree for the nuclear power plant [34]. b_i indicates the basic event, $i \in [1, 10]$.

The linguistic values provided by the expert assessment unit and corresponding TFNs generated by the transform unit are presented in Table III.

A. ANALYSIS USING STOCHASTIC COMPUTATION

Table IV shows the weights of the involved five experts in the assessment of this nuclear power plant.

TABLE 3. Experts' TFNs of basic events ("R low" and "R high" mean "reasonably low" and "reasonably high" respectively).

Basic event	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5
b_1	high (0.7,0.9,1)	very high (0.9,1.0,1.0)	R high (0.5,0.7,0.9)	high (0.7,0.9,1)	high (0.7,0.9,1)
b_2	R low (0,0,0.1)	low (0,0.1,0.3)	R low (0.1,0.3,0.5)	low (0,0.1,0.3)	low (0,0.1,0.3)
b_3	very low (0,0,0.1)	very low (0,0,0.1)	low (0,0.1,0.3)	very low (0,0,0.1)	low (0,0.1,0.3)
b_4	R low (0,0,0.1)	very low (0,0,0.1)	low (0,0.1,0.3)	R low (0.1,0.3,0.5)	very low (0,0,0.1)
b_5	low (0,0.1,0.3)	R low (0.1,0.3,0.5)	low (0,0.1,0.3)	very low (0,0,0.1)	low (0,0.1,0.3)
b_6	low (0,0.1,0.3)	very low (0,0,0.1)	low (0,0.1,0.3)	very low (0,0,0.1)	ow (0,0.1,0.3)
b_7	moderate (0,0.1,0.3)	low (0,0.1,0.3)	low (0,0.1,0.3)	very low (0,0,0.1)	low (0,0.1,0.3)
b_8	R low (0.1,0.3,0.5)	very low (0,0,0.1)	low (0,0.1,0.3)	very low (0,0,0.1)	low (0,0.1,0.3)
b_9	moderate (0,0.1,0.3)	very low (0,0,0.1)	low (0,0.1,0.3)	very low (0,0,0.1)	low (0,0.1,0.3)
b_{10}	moderate (0,0.1,0.3)	low (0,0.1,0.3)	low (0,0.1,0.3)	very low (0,0,0.1)	very low (0,0,0.1)

TABLE 4. Experts and the corresponding weights [8].

Experts	Weight
e_1	0.3
e_2	0.2
e_3	0.2
e_4	0.2
e_5	0.1

The evaluation results of experts are synthesized in the mean calculation unit. According to (9), the final weighted-mean TFN of the basic event b_k, $k \in [1, 10]$, is shown in Table V, by propagating the stochastic sequences through the stochastic circuits Figs. 5 and 6. Here, non-Bernoulli sequences of 1k bits are used to encode the TFNs and the weights. According to (13), the failure probability of the basic event $\bar{m}^{b_j}$, $j \in [1, 10]$, can be obtained by using the non-Bernoulli sequences encoding the final weight-mean TFNs of the basic events; the obtained results are also shown in Table V.

TABLE 5. The weight mean TFNs and failure probability $\bar{m}^{b_j}$ of basic events ($L = 1$k bits).

Basic event	Weight- mean triangular fuzzy number (TFN)	Failure probability $\bar{m}^{b_j}$
b_1	(0.6989,0.8805,0.9793)	0.8596
b_2	(0.0493,0.2007,0.3992)	0.2125
b_3	(0,0.0311,0.1592)	0.0552
b_4	(0.0494,0.1713,0.3388)	0.1823
b_5	(0.0207,0.1203,0.2987)	0.1401
b_6	(0,0.0596,0.2204)	0.0850
b_7	(0.0905,0.1999,0.3807)	0.2174
b_8	(0.0312,0.1201,0.2788)	0.1374
b_9	(0.0894,0.1806,0.3384)	0.1973
b_{10}	(0.0901,0.1905,0.3592)	0.2074

For the nuclear power plant in Fig. 8, the stochastic computational results for 10000 simulations are illustrated in Fig. 9.

As revealed in the simulation results, P_T approximately follows a Gaussian distribution. The mean and standard deviation of P_T are calculated as 0.2614 and 0.0021 respectively, the values of P_T are within the interval [0.254, 0.268].

B. THE FUZZY ARITHMETIC OPERATION

The fuzzy arithmetic operation (FAO) was presented to solve the fuzzy system reliability analysis problems in [5], [7], and [32]. The *L-R* fuzzy numbers defined by (m, α, β) are suitable to represent the failure possibilities of hazardous events in industrial systems [24]. The value

$$\sigma_{Z_3} = \sqrt{\frac{2m+\alpha+\beta}{4}\cdot\frac{2n+\gamma+\delta}{4}\cdot\left(1-\frac{2m+\alpha+\beta}{4}\right)\cdot\left(1-\frac{2n+\gamma+\delta}{4}\right)/L}. \tag{25}$$

$$\begin{aligned}\Delta &= 3\sigma_{Z_3} - a_2 \\ &= 3\sqrt{\frac{2m+\alpha+\beta}{4}\cdot\frac{2n+\gamma+\delta}{4}\cdot\left(1-\frac{2m+\alpha+\beta}{4}\right)\cdot\left(1-\frac{2n+\gamma+\delta}{4}\right)/L} \\ &\quad - 2\lambda\frac{2m+\alpha+\beta}{4}\cdot\frac{2n+\gamma+\delta}{4}.\end{aligned} \tag{26}$$

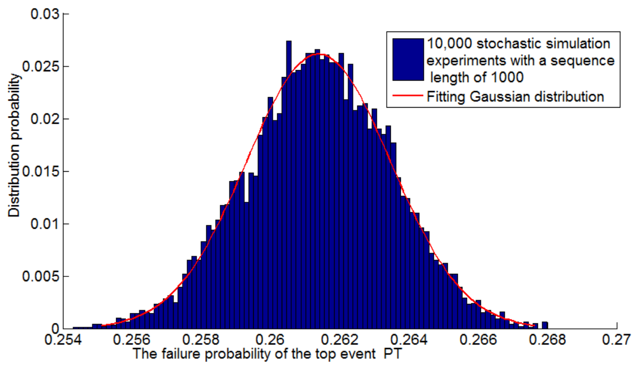

FIGURE 9. Distribution of P_T for 10,000 stochastic simulation runs with $L = 1$k bits.

of m is calculated from the expected value of the triangular fuzzy number from expert assessment [5], [7], [32]. To obtain the failure possibility distribution functions (or membership functions), it is assumed that the functions are approximated by a function of L-R type [5], [7] as

$$U_i = \begin{cases} L\left(\dfrac{\bar{m}_i - x}{\alpha_i}\right) = \dfrac{1}{1+|\frac{\bar{m}_i - x}{\alpha_i}|}, & for\ x \le \bar{m}_i,\ \alpha_i > 0 \\ R\left(\dfrac{x - \bar{m}_i}{\beta_i}\right) = \dfrac{1}{1+|\frac{x-\bar{m}_i}{\beta_i}|}, & for\ x \ge \bar{m}_i,\ \beta_i > 0, \end{cases} \tag{27}$$

where $\bar{m}_i$ is the mean value of the final weight-mean TFN from the expert assessment; α_i and β_i indicate the left and right spreads respectively: $\alpha_i = \beta_i$, $i \in (1, 10)$.

As in [5], [7], assume that α_i and β_i are considered as the fuzzy relative frequencies; all the values of u_{p_i} are equal or less than 0.1, if the deviation from the center value $\bar{m}_i$ is $x = \pm 0.5\bar{m}_i$. In other words, we assume that the relative frequency that a value differs from the middle value $\bar{m}_i$ by $\pm 50\%$ has a possibility value of only 0.1. According to (27), we obtain:

$$\frac{1}{1+|\dfrac{\bar{m}_i - x}{\alpha_i}|} = 0.1, \quad for\ x \le \bar{m}_i,\ \alpha_i > 0.$$

$$\frac{1}{1+|\dfrac{x-\bar{m}_i}{\beta_i}|} = 0.1, \quad for\ x \ge \bar{m}_i,\ \beta_i > 0.$$

Then, we have:

$$\alpha_i = \beta_i = 0.0556\bar{m}_i. \tag{28}$$

For the nuclear power plant, by the fuzzy arithmetic operation (FAO) the L-R fuzzy number P_T is obtained as $P_T = (m_T, \alpha_T, \beta_T) = (0.2611, 0.0251, 0.0251)$, where $m_T = 0.2611$ is the mean value of the failure possibility of the occurrence of the top event. The *left* and *right spreads* of the top event are given by $\alpha_T = 0.0251$, and $\beta_T = 0.0251$. The membership function of P_T is shown in Fig. 10, where the values of m_T, α_T, β_T are inserted into (27). As shown in Fig. 10, $x1$ and $x3$ are $m_T - \alpha_T$ and $m_T + \beta_T$ respectively when the membership degree is 0.5, the membership degree of $x2$ (m_T) is 1.

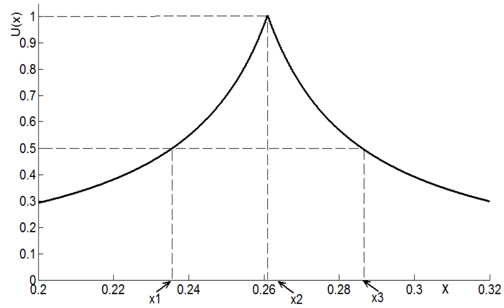

FIGURE 10. The membership function of P_T, x is the center of P_T and U(x) is the membership degree of P_T.

C. DISCUSSION

The top event failure probability obtained by the stochastic analysis is compared with that by the fuzzy arithmetic operation (FAO), Monte Carlo (MC) simulation and the fuzzy expected value approach (FEVA).

1) STOCHASTIC APPROACH VS. FAO

The non-fuzzy values of P_T obtained by 10,000 stochastic experiments are all within the interval [0.254, 0.268], and the L-R fuzzy number P_T is obtained by FAO as (0.2611, 0.0251, 0.0251). The membership function of P_T gives the possibility distribution of P_T [23] and is calculated by FAO, as shown in Fig. 10. The possibility distribution can be converted into a probability distribution [35], [36] via a simple normalization as:

$$P(x_j) = \frac{U(x_j)}{\sum_{i=1}^{n} U(x_i)}, \tag{29}$$

where x_j is in $[a, b]$, a and b are two values of x in (27) when the membership degree of P_T is 0.1 (i.e., the confidence level is 90%), $P(x_j)$ and $U(x_j)$ are the probability and membership degree of x_j respectively, and n is the number of samples in $[a, b]$. The probability distributions of P_T using the FAO and stochastic approach (with 10,000 simulation runs and 1000 bits as the sequence length) are shown in Fig. 11. The comparison of the probability distributions of P_T indicates that all the stochastic simulation results concentrate on the high probability region of P_T.

To obtain a wider spread of the stochastic computational results, a shorter sequence length of 100 bits is used to obtain the probability distribution of P_T by 10,000 stochastic simulations. The probability distributions of P_T using FAO and the stochastic approach are shown in Fig. 12. It can be seen that the distribution of the stochastic computational results resembles the results obtained by FAO.

According to the simulation results of P_T in Fig. 8 and the 3σ rule [33] of Gaussian distribution, we can obtain the function to calculate the u_{P_T} (i.e., the membership degree

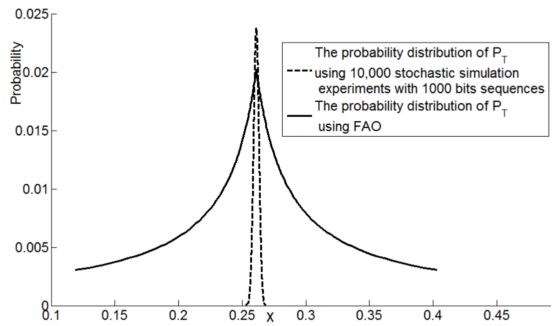

FIGURE 11. The probability distribution of P_T calculated by fuzzy arithmetic operation (FAO) and the stochastic approach (with 10,000 simulation runs using a sequences length of 1000 bits).

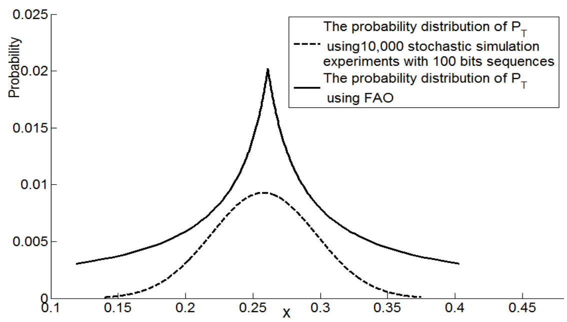

FIGURE 12. The probability distribution of P_T calculated by fuzzy arithmetic operation (FAO) and the stochastic approach (with 10,000 simulation runs using a sequences length of 100 bits).

of P_T), by inserting the value of $(u \pm n\sigma)$ into (27):

$$u_{P_T} = \begin{cases} \dfrac{1}{1+\left|\frac{m_T - x}{\alpha_T}\right|}, & \text{for } x \le m_T,\ \alpha_T > 0 \\ \dfrac{1}{1+\left|\frac{x - m_T}{\beta_T}\right|}, & \text{for } x \ge m_T,\ \beta_T > 0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{1+\left|\frac{m_T-(u-n\sigma)}{\alpha_T}\right|}, & \text{for } x \le m_T,\ \alpha_T > 0 \\ \dfrac{1}{1+\left|\frac{(u+n\sigma)-m_T}{\beta_T}\right|}, & \text{for } x \ge m_T,\ \beta_T > 0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{1+\left|\frac{n\sigma+(m_T-u)}{\alpha_T}\right|}, & \text{for } x \le m_T,\ \alpha_T > 0 \\ \dfrac{1}{1+\left|\frac{n\sigma+(u-m_T)}{\beta_T}\right|}, & \text{for } x \ge m_T,\ \beta_T > 0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{1+\frac{n\sigma}{\alpha_T}}, & \text{for } x \le m_T,\ \alpha_T > 0 \\ \dfrac{1}{1+\frac{n\sigma}{\beta_T}}, & \text{for } x \ge m_T,\ \beta_T > 0, \end{cases} \quad (30)$$

where m_T is the mean value of P_T calculated by FAO, α_T and β_T are the *left* and *right spreads* of P_T respectively, μ and σ are the expected value and the standard deviation of P_T from the stochastic simulation, n is the coefficient of σ in 3σ rule [33] of Gaussian distribution and $m_T = u$.

In this system, $\alpha_T = \beta_T = 0.0251$ and $\sigma = 0.0021$; according to (30) and the 3σ rule of Gaussian distribution, we obtain the relationship between P_T located in the range $(u - n\sigma, u + n\sigma)$ and the membership degree of the P_T using the stochastic approach and FAO, as shown in Table VI (the sequence length used in the stochastic approach is 1k bits, u and σ are the expected value and standard deviation of system failure probability P_T respectively).

As revealed in Table VI, when the value of P_T obtained by the stochastic analysis falls within the 1σ (68.2%) centerline (the expected value u of the distribution), the u_{P_T} (membership degree of the P_T) calculated by FAO is above 0.916. When n is 3, the minimum membership degree of a large percentage (99.7%) of the stochastic computational results is 0.792.

TABLE 6. The relationship between P_T and the membership degree of P_T using the stochastic approach and FAO respectively.

n	P_T $(u - n\sigma, u + n\sigma)$	u_{P_T} (membership degree of the P_T)
1	68.2%	0.916
2	95.4%	0.845
3	99.7%	0.792

2) STOCHASTIC APPROACH VS. MONTE CARLO SIMULATION

The results for P_T using 10,000 MC simulations with 1000 runs in each simulation are presented in Fig. 13.

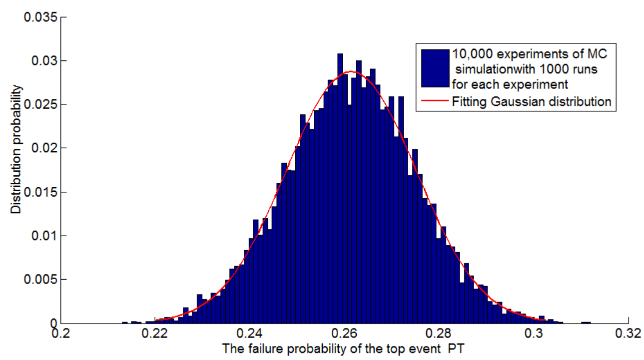

FIGURE 13. Distribution of P_T using 10,000 MC simulations with 1000 runs in each simulation.

As shown in the simulation results, P_T approximately follows a Gaussian distribution. The mean and standard deviation of P_T are 0.2616 and 0.0139 respectively; all values of P_T are within the interval [0.215, 0.3073]. The simulation results by the stochastic approach with non-Bernoulli sequences and the MC method (equivalent to the use of Bernoulli sequences as initial inputs) with different numbers of simulations are shown in Table VII. Fig. 9, Fig. 13 and Table VII indicate that the stochastic approach accomplishes a faster convergence

TABLE 7. The P_T distribution and run time by Monte Carlo simulation, FAO and the stochastic approach with different sequence length (L) (u and σ are the expected value and standard deviation of system failure probability P_T, m is the expected value of P_T calculated by FAO, N and L are respectively the Monte Carlo simulation runs and the stochastic sequences length).

	N/L	P_T distribution	$\|u-m\|$	σ	Average Runtime
Proposed stochastic approach	500	[0.2289, 0.288]	0.0007	0.0096	0. 0278s
	1k	[0.254, 0.268]	0.0003	0.0021	0.0526s
	10k	[0.257, 0.265]	0	0.0013	0.833s
Monte Carlo [12-14]	500	[0.202, 0.3264]	0.0015	0.0199	0.0611s
	1k	[0.215, 0.3073]	0.0005	0.0139	0.142s
	10k	[0.2438, 0.275]	0.0002	0.0045	3.167s
FAO [5, 7]	(0.2611,0.0251,0.0251)				0.259s

and requires a shorter runtime compared with MC simulation. The accuracy (in u) improves with the increase of the length of the stochastic sequences.

3) STOCHASTIC APPROACH VS. FUZZY EXPECTED VALUE APPROACH

The notations of l*ower possibilistic* and *upper possibilistic* mean values are used to obtain the bounds of the system failure probability in [11]. The l*ower possibilistic* and *upper possibilistic* mean values of a fuzzy number A (α, m, β) is given by:

$$M_A = \left[M_{A*}, \mathrm{M}^{A*}\right] = [\mathrm{m} - \frac{\alpha}{3}, \mathrm{m}+\frac{\beta}{3}]. \tag{31}$$

where m is the center of A, α and β are the left width and right width of A respectively ($\alpha > 0$, $\beta > 0$). As per *Definition* 1, the interval bound given by the *lower* and *upper possibilistic* mean values of A' is obtained by:

$$\begin{aligned} M_{A'} &= \left[M_{A*'}, \mathrm{M}^{A*'}\right] \\ &= [\mathrm{m}' - \frac{\mathrm{m}'-\alpha'}{3}, \mathrm{m}' + \frac{\beta'-\mathrm{m}'}{3} \\ &= [\frac{2*\mathrm{m}'+\alpha'}{3}, \frac{2*\mathrm{m}'+\beta'}{3}]. \end{aligned} \tag{32}$$

The expected values given in (13) and the interval bound by the *lower* and *upper possibilistic* mean values of P_{b_i} calculated by FEVA are shown in Table VIII.

Based on a system's fault tree in Fig. 8, the expected value of system failure probability is 0.2611. The closed interval bounded by *lower and upper possibilistic* mean values of system failure probability is [0.1562, 0.3485]. Compared with the stochastic approach, the probability distribution of the system failure probability (P_T) cannot be obtained by the fuzzy expected value approach (FEVA).

TABLE 8. The expected values and Interval bounded by lower/upper possibilistic mean values of basic events' failure probabilities calculated by fuzzy expected value approach (P_{b_i} is the failure probability of basic event b_i, $i \in [1, 10]$).

Basic event b_i	Weight- mean TFN of P_{b_i}	Interval bounded by lower and upper possibilistic mean values of P_{b_i}	Expected value of P_{b_i}
b_1	(0.7, 0.88, 0.98)	[0.82, 0.9133]	0.86
b_2	(0.05, 0.2, 0.4)	[0.15, 0.2667]	0.2125
b_3	(0, 0.03, 0.16)	[0.02, 0.0733]	0.055
b_4	(0.05, 0.17, 0.34)	[0.13, 0.2267]	0.1825
b_5	(0.02, 0.12, 0.3)	[0.0867, 0.18]	0.14
b_6	(0, 0.06, 0.22)	[0.04, 0.1133]	0.085
b_7	(0.09, 0.2, 0.38)	[0.1633, 0.26]	0.2175
b_8	(0.03, 0.12, 0.28)	[0.09, 0.1733]	0.1375
b_9	(0.09, 0.18, 0.34)	[0.15, 0.2333]	0.1975
b_{10}	(0.09, 0.19, 0.36)	[0.1567, 0.2467]	0.2075

VI. CONCLUSION

This paper proposes a model that consists of an expert evaluation module and a stochastic computational module to evaluate the reliability of complex fuzzy systems. In a fuzzy system, the probabilities of basic events cannot be accurately determined but can be approximated by linguistic values from assessment experts, and the linguistic values are converted to triangular fuzzy number by transformation unit. In this model, triangular fuzzy numbers are converted to mean value, the failure possibility of a basic event is transformed into a crisp failure probability. The proposed method transforms a fuzzy probability analysis into simple bit-wise operations on non-Bernoulli sequences of random permutations of 1's and 0's. The reliability of the fuzzy system can be obtained by the proposed model. Compared with the fuzzy arithmetic operation (FAO), the runtime to obtain the reliability parameters of a fuzzy system is reduced. The proposed approach results in a faster convergence than Monte Carlo simulation and can produce a system's failure probability distribution that cannot be easily obtained by the fuzzy expected value approach (FEVA).

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