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Robustness Analysis of Global Exponential Stability of Nonlinear Systems With Deviating Argument and Stochastic Disturbance

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ABSTRACT Robust performance of nonlinear systems has attracted phenomenal worldwide attention. It is well known that deviating argument and stochastic disturbance may derail the evolution properties of nonlinear systems. Then the following issue has become a major bottleneck: for a given globally exponentially stable nonlinear system, the perturbed nonlinear system can sustain how much the length of the interval of the deviating function and the noise intensity so that the perturbed nonlinear system in the presence of deviating argument and stochastic disturbance may remain to be exponentially stable. In this paper, theoretical investigation has been made on the robustness of global exponential stability of nonlinear systems with deviating argument and stochastic disturbance. The allowable upper bounds of the length of interval of deviating function and the noise intensity are derived for the perturbed nonlinear systems to remain exponentially stable. It is also proven that, if the length of interval of deviating function and the noise intensity of perturbed nonlinear systems are lower than the upper bounds derived herein, the nonlinear systems infected by deviating argument and stochastic disturbance are still exponentially stable. Finally, we give several simulation examples to demonstrate the efficacy of the proposed results.

INDEX TERMS Nonlinear systems, deviating argument, stochastic disturbance, robustness.

I. INTRODUCTION

Analysis and synthesis of nonlinear systems is a hot issue [1]–[16]. Many control methods have been developed for real nonlinear systems, for instance, right coprime factorization approach [1], fixed-time control [2], adaptive control [3], [6], [8], [9], [15], fault design scheme [4], [5], [11], output feedback control [7], [14], H_∞ control [12], event-trigger control [13], fuzzy control [16]. In practice, lots of nonlinear systems possess completely unknown nonlinearity, unmodeled dynamics, and arbitrary switchings. These high levels of uncertainty and complexity may seriously influence the system performance. Furthermore, control schemes for nonlinear systems still have a long way to go.

Deviating argument, which can have serious impacts in the running process of nonlinear systems, is one of the most significant nonsmooth nonlinearities arisen inactuators [17]–[19]. In the last few years, there has been rapidly growing interest in nonlinear systems with deviating argument. For example, to describe the stationary distribution of the temperature of length of wire that is bended, nonlinear dynamic model with deviating argument is often used. The right-hand side in nonlinear systems with deviating argument features a combination of continuous and discrete systems. Thus, nonlinear systems with deviating argument unify the advanced and retarded systems. Because this strange property, it is pretty hard to design the control strategies for nonlinear systems with deviating argument. From the perspective of system cybernetics, differential equations and difference equations are included in the analytical framework [19]. However, it is still obvious that many basic issues on nonlinear systems with deviating argument remain to be addressed, such as nonlinear dynamics, systems design and analysis.

Stochastic disturbance is recently examined using systems theory. Stochastic motion, which has a very rich and new structure, nontrivially generalizes the classical deterministic process [20]–[23]. Then nonlinear realized stochastic models come into play important roles in many real systems including control engineering, economy and finance. After the success of systematic control design for deterministic nonlinear systems, how to expand promptly and evaluate accurately this approach to stochastic nonlinear systems always is a challenging and meaningful issue.

Meanwhile, as mentioned in [24]–[26], it remains unclear on how to analyze the robustness of of nonlinear systems in the presence of external disturbance. There are a large number of tools available for analyzing the stability of control systems, including Lyapunov theory, Razumikhin-type method, comparison principle, monotone/mixed monotone operator and many others. However, analogous techniques for robustness analysis of nonlinear systems are still quite incipient.

Summarizing the above statements, some natural questions arise: It is generally known that deviating argument and stochastic disturbance in nonlinear systems can cause instability or destabilize stable nonlinear systems if the length of interval of the deviating function or the noise intensity exceeds a certain limit. The stability of nonlinear systems in the presence of deviating argument and stochastic disturbance often depends on their intensity. For a stable undisturbed nonlinear system, if deviating argument, stochastic disturbance, or both are low, the disturbed nonlinear system may still maintain stable. Therefore, it is interesting to investigate how much the length of interval of deviating function and the noise intensity nonlinear systems can withstand without losing stability. As a matter of fact, robustness of global exponential stability of nonlinear systems is rarely analyzed directly with respect to deviating argument and stochastic disturbance. Is it feasible to obtain the allowable upper bounds of the length of interval of deviating function and the noise intensity on the perturbed nonlinear systems to remain exponentially stable? This paper will devote to solving the matters. We reveal the robust performance of nonlinear systems subjected to deviating argument and stochastic disturbance. We address theoretically that, for a given exponentially stable nonlinear systems, if deviating argument, stochastic disturbance, or both are lower than the obtained upper bounds herein, then the perturbed nonlinear systems can keep exponentially stable.

II. EFFECT OF DEVIATING ARGUMENT

Let N and $\Re^+$ be the sets of natural numbers and nonnegative real numbers, respectively. Denote $\Re^n$ as the n-dimensional real space. The Euclidean norm in $\Re^n$ is recorded as $\|\cdot\|$. Fix two real-value sequences $\{\alpha_k\}$, $\{\eta_k\}$, $k \in N$, such that $\alpha_k < \alpha_{k+1}$, $\alpha_k \leq \eta_k \leq \alpha_{k+1}$ for all $k \in N$ with $\alpha_k \to +\infty$ as $k \to +\infty$. In this section, we will consider the effect of deviating argument on the exponential stability to the following nonlinear systems:

$$\begin{cases} \dot{y}(t) = f(y(t), y(\beta(t)), t), & t \geq t_0 \geq 0, \\ y(t_0) = y_0, \end{cases} \tag{1}$$

where the deviating function $\beta(t) = \eta_k$, if $t \in [\alpha_k, \alpha_{k+1})$, $k \in N$, $t \in \Re^+$, $f : \Re^n \times \Re^n \times \Re^+ \to \Re^n$.

Remark 1: System (1) is a kind of mixed system. Actually, consider (1) on the interval $[\alpha_k, \alpha_{k+1})$, $k \in N$, when $\alpha_k \leq t < \eta_k$, (1) is an advanced system, and when $\eta_k < t < \alpha_{k+1}$, (1) is a retarded system. That is, system (1) can be alternately advanced and retarded argument.

Throughout this section, suppose that $f(\cdot)$ is local Lispchitz, that is, there are constants $l_1 > 0$ and $l_2 > 0$, such that

$$\|f(\mathcal{X}_1, \mathcal{Y}_1, t) - f(\mathcal{X}_2, \mathcal{Y}_2, t)\| \leq l_1\|\mathcal{X}_1 - \mathcal{X}_2\| + l_2\|\mathcal{Y}_1 - \mathcal{Y}_2\|,$$

for all $\mathcal{X}_1 \in \Re^n$, $\mathcal{X}_2 \in \Re^n$, $\mathcal{Y}_1 \in \Re^n$, $\mathcal{Y}_2 \in \Re^n$, $t \in \Re^+$, and $f(0, 0, t) = 0$.

Then, it is clear that system (1) has a trivial state $y = 0$.

Without loss of generality, we assume that system (1) has an unique state $y(t; t_0, y_0)$ for any given initial data t_0 and y_0. Now, consider the undisturbed system of (1) as follows:

$$\begin{cases} \dot{x}(t) = f(x(t), x(t), t), & t \geq t_0 \geq 0, \\ x(t_0) = x_0 = y_0. \end{cases} \tag{2}$$

Obviously, system (2) has the trivial state $x = 0$ and exists an unique state $x(t; t_0, x_0)$ for any given initial data t_0 and x_0.

Next, the definition of global exponential stability for system (2) is given.

Definition 1: The state of system (2) is said to be globally exponentially stable if for any $t_0 \in \Re^+$, $x_0 \in \Re^n$, there exist constants $l > 0$ and $\kappa > 0$ such that

$$\|x(t; t_0, x_0)\| \leq l\|x_0\| \exp\{-\kappa(t - t_0)\}, \quad t \geq t_0, \tag{3}$$

where $x(t; t_0, x_0)$ is the state of system (2).

In the following, we give some presumptions:

(A1) There exists a constant $\alpha > 0$ such that $\alpha_{k+1} - \alpha_k \leq \alpha$, $k \in N$.

(A2) $\alpha[l_1(1 + l_2\alpha)\exp\{l_1\alpha\} + l_2] < 1$.

The lemma developed below unmasks the relationship of state of system (1) in current time t and deviating function $\beta(t)$.

Lemma 1: Let (A1) and (A2) hold, and $y(t)$ be a solution of system (1). Then the following inequality

$$\|y(\beta(t))\| \leq \gamma\|y(t)\| \tag{4}$$

holds for all $t \in \Re^+$, where $\gamma = \{1 - \alpha[l_1(1 + l_2\alpha)\exp\{l_1\alpha\} + l_2]\}^{-1}$.

Proof: Fix $k \in N$. Then, for any $t \in [\alpha_k, \alpha_{k+1})$, we have

$$\begin{aligned} \|y(t)\| &= \|y(\eta_k) + \int_{\eta_k}^t f(y(s), y(\eta_k), y)\mathrm{d}s\| \\ &\leq \|y(\eta_k)\| + \int_{\eta_k}^t \|f(y(s), y(\eta_k), y)\|\mathrm{d}s \\ &\leq \|y(\eta_k)\| + \int_{\eta_k}^t [l_1\|y(s)\| + l_2\|y(\eta_k)\|]\mathrm{d}s \\ &\leq (1 + l_2\alpha)\|y(\eta_k)\| + \int_{\eta_k}^t l_1\|y(s)\|\mathrm{d}s. \end{aligned} \tag{5}$$

Applied Gronwall-Bellman lemma to (5), we obtain

$$\|y(t)\| \leq (1 + l_1\alpha)\exp\{l_1\alpha\}\,\|y(\eta_k)\|. \tag{6}$$

Similarly, for $t \in [\alpha_k, \alpha_{k+1})$, it follows

$$\|y(\eta_k)\| \leq \|y(t)\| + l_1 \int_{\eta_k}^{t} \|y(t)\|\mathrm{d}s + l_2 \int_{\eta_k}^{t} \|y(\eta_k)\|\mathrm{d}s. \quad (7)$$

Together with (6) and (7),

$$\|y(\eta_k)\| \leq \|y(t)\| + \alpha[l_1(1 + l_2\alpha)\exp\{l_1\alpha\} + l_2]\|y(\eta_k)\|,$$

then

$$\begin{aligned}\|y(\eta_k)\| &\leq \{1 - \alpha[l_1(1 + l_2\alpha)\exp\{l_1\alpha\} + l_2]\}^{-1}\|y(t)\| \\ &= \gamma\|y(t)\|,\end{aligned}$$

for $t \in [\alpha_k, \alpha_{k+1})$. By the randomicities of t and k, (4) holds for all $t \in \Re^+$.

Now, we investigate the effect of deviating argument on the robustness of global exponential stability of system (1).

Theorem 1: Let (A1) and (A2) hold, and system (2) to be globally exponentially stable. System (1) still remains to be globally exponentially stable if $\alpha < \min(\frac{\eta}{2}, \bar{\alpha}, \bar{\bar{\alpha}})$, where $\bar{\alpha}$ is the unique positive solution $\hat{x}$ of the following equation (8)

$$\begin{aligned} & l\exp\{-\kappa(\eta - \hat{x})\} + \frac{l_2(1+\gamma)l}{\kappa}\exp\left\{2\eta\left[l_1 + 2l_2 \right.\right.\\ &\qquad \left.\left. + \left(1 - \hat{x}\left[l_1(1 + l_1\hat{x})\exp\left\{l_1\hat{x}\right\} + l_2\right]\right)^{-1} l_2\right]\right\} \\ &= 1, \end{aligned} \quad (8)$$

and $\bar{\bar{\alpha}}$ is the unique positive solution $\check{x}$ of the following equation (9)

$$\check{x}\left[l_1(1 + l_2\check{x})\exp\left\{l_1\check{x}\right\} + l_2\right] = 1, \quad (9)$$

and $\eta > \frac{\ln(l)}{\kappa} > 0$.

Proof: For convenience, we write $x(t; t_0, x_0) \equiv x(t)$ and $y(t; t_0, y_0) \equiv y(t)$. From (1) and (2), together with Lemma 1, for any $t \geq t_0 \geq 0$, we have

$$\begin{aligned} &\|x(t) - y(t)\| \\ &= \|\int_{t_0}^{t} [f(x(s), x(s), s) - f(y(s), y(\beta(s)), s)]\mathrm{d}s\| \\ &\leq l_1 \int_{t_0}^{t} \|x(s) - y(s)\|\mathrm{d}s + l_2 \int_{t_0}^{t} \|x(s) - y(\beta(s))\|\mathrm{d}s \\ &\leq (l_1 + l_2) \int_{t_0}^{t} \|x(s) - y(s)\|\mathrm{d}s + l_2 \int_{t_0}^{t} \|y(s) - y(\beta(s))\|\mathrm{d}s \\ &\leq (l_1+l_2) \int_{t_0}^{t} \|x(s)-y(s)\|\mathrm{d}s + l_2 \int_{t_0}^{t} [\|y(s)\| + \|y(\beta(s))\|]\mathrm{d}s \\ &\leq (l_1 + l_2) \int_{t_0}^{t} \|x(s) - y(s)\|\mathrm{d}s + l_2(1 + \gamma) \int_{t_0}^{t} \|y(s)\|\mathrm{d}s \\ &\leq (l_1 + 2l_2 + \gamma l_2) \int_{t_0}^{t} \|x(s) - y(s)\|\mathrm{d}s \\ &\quad + l_2(1 + \gamma) \int_{t_0}^{t} \|x(s)\|\mathrm{d}s. \end{aligned} \quad (10)$$

In addition, from the global exponential stability of system (2), then for any $t \geq t_0 \geq 0$,

$$\begin{aligned} &\|x(t) - y(t)\| \\ &\leq (l_1 + 2l_2 + \gamma l_2) \int_{t_0}^{t} \|x(s) - y(s)\|\mathrm{d}s \\ &\quad + l_2(1 + \gamma) \int_{t_0}^{t} l\|x_0\| \exp\{-\kappa(s - t_0)\}\,\mathrm{d}s \\ &\leq (l_1 + 2l_2 + \gamma l_2) \int_{t_0}^{t} \|x(s) - y(s)\|\mathrm{d}s + \frac{l_2(1 + \gamma)l}{\kappa}\|x_0\|. \end{aligned} \quad (11)$$

Applied Gronwall-Bellman lemma to (11), for $t_0 + \alpha \leq t \leq t_0 + 2\eta$,

$$\|x(t) - y(t)\| \leq \frac{l_2(1 + \gamma)l}{\kappa}\|x_0\| \exp\{2\eta\,(l_1 + 2l_2 + \gamma l_2)\}.$$

Then

$$\begin{aligned} \|y(t)\| &\leq \|x(t)\| + \|x(t) - y(t)\| \\ &\leq \|x(t)\| + \frac{l_2(1 + \gamma)l}{\kappa}\|x_0\| \exp\{2\eta(l_1 + 2l_2 + \gamma l_2)\}. \end{aligned} \quad (12)$$

Note that $\alpha < \min(\frac{\eta}{2}, \bar{\alpha})$, by (12) and the global exponential stability of system (2), for $t_0 - \alpha + \eta \leq t \leq t_0 - \alpha + 2\eta$, we can get

$$\begin{aligned} \|y(t)\| &\leq l\|x_0\| \exp\{-\kappa(\eta - \alpha)\} \\ &\quad + \frac{l_2(1 + \gamma)l}{\kappa}\|x_0\| \exp\{2\eta(l_1 + 2l_2 + \gamma l_2)\} \\ &\triangleq \delta\|y_0\|, \end{aligned}$$

where

$$\delta = l\exp\{-\kappa(\eta - \alpha)\} + \frac{l_2(1+\gamma)l}{\kappa}\exp\{2\eta(l_1 + 2l_2 + \gamma l_2)\}.$$

From (8), we see $\delta < 1$ when $\alpha < \min(\frac{\eta}{2}, \bar{\alpha}, \bar{\bar{\alpha}})$. Setting $\lambda = -\frac{\ln\,(\delta)}{\eta}$, we obtain

$$\|y(t)\| \leq \exp(-\eta\lambda)\|y_0\|. \quad (13)$$

From the uniqueness of solution of system (1),

$$y(t; t_0, y_0) = y\big(t; t_0 + (m - 1)\eta, y(t_0 + (m - 1)\eta; t_0, y_0)\big), \quad (14)$$

where m is positive integer. Therefore, from (13) and (14), for $t \geq t_0 - \alpha + m\eta$,

$$\begin{aligned} &\|y(t; t_0, y_0)\| \\ &= \|y\big(t; t_0 + (m - 1)\eta, y(t_0 + (m - 1)\eta; t_0, y_0)\big)\| \\ &\leq \exp\{-\eta\lambda\}\,\|y\big(t_0 + (m - 1)\eta; t_0, y_0\big)\| \\ &= \exp\{-\eta\lambda\}\,\|y\big(t; t_0+(m-2)\eta, y(t_0+(m-2)\eta; t_0, y_0)\big)\| \\ &\leq \exp\{-m\eta\lambda\}\,\|y_0\|. \end{aligned}$$

Thus, for any $t > t_0 - \alpha + \eta$, there is a positive integer m such that $t_0 - \alpha + (m-1)\eta \leq t \leq t_0 - \alpha + m\eta$,

$$\|y(t; t_0, y_0)\| \leq \exp(-\lambda(t-t_0)) \exp(\lambda(\eta - \alpha))\|y_0\|. \quad (15)$$

Clearly, (15) also holds for $t_0 \leq t \leq t_0 - \alpha + \eta$. So system (1) is globally exponentially stable.

Remark 2: In many published works, it is supposed that nonlinear systems are only related to the present or past state. However, in this paper, nonlinear systems with deviating argument are related to not only the present state but also the past and future ones.

Remark 3: Theorem 1 demonstates that if system (2) is globally exponentially stable, the disturbed system evoked by deviating argument in (1) may still be globally exponentially stable when the length of interval of deviating function $\beta(t)$ is lower than the obtained bound $\min(\frac{\eta}{2}, \bar{\alpha}, \bar{\bar{\alpha}})$.

Remark 4: The allowable length of interval of deviating function $\beta(t)$ in Theorem 1 can be given expediently. By solving the transcendental equations (8) and (9) via MATLAB, the allowable length α of interval of deviating function $\beta(t)$ can be effectively estimated.

Remark 5: Wu et al. [19] have investigated the stability of neurodynamic systems with deviating argument. Whereas, in this paper, the considered controlled systems are more general models, and we can see from the following examples that the obtained criteria here possess wider application scope.

III. EFFECTS OF DEVIATING ARGUMENT AND STOCHASTIC DISTURBANCE

Let N and $\Re^+$ be the sets of natural numbers and nonnegative real numbers, respectively. Denote $\Re^n$ as the n-dimensional real space. The Euclidean norm in $\Re^n$ is recorded as $\|\cdot\|$. $E(\cdot)$ stands for mathematical expectation. Fix two real-value sequences $\{\alpha_k\}$, $\{\eta_k\}$, $k \in N$, such that $\alpha_k < \alpha_{k+1}$, $\alpha_k \leq \eta_k \leq \alpha_{k+1}$ for all $k \in N$ with $\alpha_k \to +\infty$ as $k \to +\infty$. Let $(\Omega, \mathscr{F}, P)$ be a compete probability space with $\{\mathscr{F}_t\}_{t \geq t_0 \geq 0}$. $B(t)$ represents one-dimensional Brownian motion on the compete space. In this section, we will consider the effects of deviating argument and stochastic disturbance on the exponential stability to the following nonlinear systems:

$$\begin{cases} dy(t) = f(y(t), y(\beta(t)), t)dt + \sigma y(t)dB(t), & t \geq t_0 \geq 0, \\ y(t_0) = y_0, \end{cases} \quad (16)$$

where the deviating function $\beta(t) = \eta_k$, if $t \in [\alpha_k, \alpha_{k+1})$, $k \in N$, $t \in \Re^+$, $f : \Re^n \times \Re^n \times \Re^+ \to \Re^n$, σ is the noise intensity.

Remark 6: System (16) is also a kind of mixed system in stochastic environment. Actually, consider (16) on the interval $[\alpha_k, \alpha_{k+1})$, $k \in N$, when $\alpha_k \leq t < \eta_k$, (16) is an advanced system, and when $\eta_k < t < \alpha_{k+1}$, (16) is a retarded system. That is, system (16) can be alternately advanced and retarded argument in stochastic environment.

System (16) can be regarded as the perturbed system of

$$\begin{cases} dx(t) = f(x(t), x(t), t)dt, & t \geq t_0 \geq 0, \\ x(t_0) = x_0 = y_0. \end{cases} \quad (17)$$

Throughout this section, suppose that $f(\cdot)$ is local Lispchitz, that is, there are constants $l_1 > 0$ and $l_2 > 0$, such that

$$\|f(\mathcal{X}_1, \mathcal{Y}_1, t) - f(\mathcal{X}_2, \mathcal{Y}_2, t)\| \leq l_1\|\mathcal{X}_1 - \mathcal{X}_2\| + l_2\|\mathcal{Y}_1 - \mathcal{Y}_2\|,$$

for all $\mathcal{X}_1 \in \Re^n$, $\mathcal{X}_2 \in \Re^n$, $\mathcal{Y}_1 \in \Re^n$, $\mathcal{Y}_2 \in \Re^n$, $t \in \Re^+$, and $f(0, 0, t) = 0$.

It is clear that system (16) has a trivial state $y = 0$. Certainly, system (17) also has a trivial state $x = 0$. Without loss of generality, we assume that system (16) has an unique state $y(t; t_0, y_0)$ for any given initial data t_0 and y_0.

The exponential stability of system (17) is defined in Definition 1 and the exponential stability of system (16) is defined as shown below.

Definition 2: The state $y(t; t_0, y_0)$ of system (16) is said to be almost surely exponentially stable if for any $t_0 \in \Re^+$, $y_0 \in \Re^n$, there exist $\Delta > 0$ and $\Lambda > 0$ such that

$$\|y(t; t_0, y_0)\| \leq \Delta\|y_0\| \exp\{-\Lambda(t - t_0)\}, \quad t \geq t_0 \geq 0,$$

holds almost surely.

Definition 3: The state $y(t; t_0, y_0)$ of system (16) is said to be mean square exponentially stable if for any $t_0 \in \Re^+$, $y_0 \in \Re^n$, there exist $\Delta > 0$ and $\Lambda > 0$ such that

$$E\|y(t; t_0, y_0)\|^2 \leq \Delta\|y_0\|^2 \exp\{-\Lambda(t - t_0)\}, \quad t \geq t_0 \geq 0.$$

Remark 7: From Definitions 2 and 3, obviously, almost sure exponential stability implies mean square exponential stability, but not vice versa. However, under

$$\|f(\mathcal{X}_1, \mathcal{Y}_1, t) - f(\mathcal{X}_2, \mathcal{Y}_2, t)\| \leq l_1\|\mathcal{X}_1 - \mathcal{X}_2\| + l_2\|\mathcal{Y}_1 - \mathcal{Y}_2\|,$$

for all $\mathcal{X}_1 \in \Re^n$, $\mathcal{X}_2 \in \Re^n$, $\mathcal{Y}_1 \in \Re^n$, $\mathcal{Y}_2 \in \Re^n$, $t \in \Re^+$, we have the following claim: The mean square exponential stability of system (16) implies the almost sure exponential stability of system (16), see [27].

Next, we give a presumption:

$$\text{(A3)}\, 6\alpha^2 l_2^2 + 9\alpha(2\alpha l_1^2 + \sigma^2)(1 + 2\alpha^2 l_2^2) \exp\left\{3\alpha(2\alpha l_1^2 + \sigma^2)\right\} < 1.$$

The lemma developed below unmasks the relationship of state of system (16) in current time t and deviating function $\beta(t)$.

Lemma 2: Let (A1) and (A3) hold, and $y(t)$ be a solution of system (16). Then the following inequality

$$E\|y(\beta(t))\|^2 \leq \varpi E\|y(t)\|^2 \quad (18)$$

holds for all $t \in \Re^+$, where

$$\varpi = 3(1 - \hat{o})^{-1},$$
$$\hat{o} = 6\alpha^2 l_2^2 + 9\alpha(2\alpha l_1^2 + \sigma^2)(1 + 2\alpha^2 l_2^2)\exp\left\{3\alpha(2\alpha l_1^2 + \sigma^2)\right\}.$$

Proof: Fix $t \in \Re^+$, there exists $k \in N$, such that $t \in [\alpha_k, \alpha_{k+1})$, $\beta(t) = \eta_k$. Then it follows that

$$\begin{aligned}
&E\|y(t)\|^2 \\
&= E\|y(\eta_k) + \int_{\eta_k}^{t} f(y(s), y(\eta_k), s)\mathrm{d}s \\
&\quad + \int_{\eta_k}^{t} \sigma y(s)\mathrm{d}B(s)\|^2 \\
&\le 3\Big[E\|y(\eta_k)\|^2 + E\|\int_{\eta_k}^{t} f(y(s), y(\eta_k), s)\mathrm{d}s\|^2 \\
&\quad + E\|\int_{\eta_k}^{t} \sigma y(s)\mathrm{d}B(s)\|^2\Big] \\
&\le 3\Big[E\|y(\eta_k)\|^2 + 2\alpha E\|\int_{\eta_k}^{t}\Big(l_1^2\|y(s)\|^2 + l_2^2\|y(\eta_k)\|^2\Big)\mathrm{d}s \\
&\quad + E\int_{\eta_k}^{t}\sigma^2\|y(s)\|^2\mathrm{d}s\Big] \\
&\le 3(1 + 2\alpha^2\, l_2^2)E\|y(\eta_k)\|^2 \\
&\quad + 3(2\alpha l_1^2 + \sigma^2)\int_{\eta_k}^{t} E\|y(s)\|^2\mathrm{d}s. \qquad (19)
\end{aligned}$$

Applied Gronwall-Bellman lemma to (19), we obtain

$$E\|y(t)\|^2 \le 3(1 + 2\alpha^2 l_2^2)E\|y(\eta_k)\|^2 \exp\Big\{3\alpha(2\alpha l_1^2 + \sigma^2)\Big\}. \qquad (20)$$

Similarly, for $t \in [\alpha_k, \alpha_{k+1})$, from (20), we have

$$\begin{aligned}
&E\|y(\eta_k)\|^2 \\
&\le 3E\|y(t)\|^2 + 6\alpha^2\, l_2^2 E\|y(\eta_k)\|^2 \\
&\quad + 3(2\alpha l_1^2 + \sigma^2)\int_{\eta_k}^{t} E\|y(s)\|^2\mathrm{d}s \\
&\le 3E\|y(t)\|^2 + 6\alpha^2 l_2^2 E\|y(\eta_k)\|^2 \\
&\quad + 9\alpha(2\alpha l_1^2 + \sigma^2)(1 + 2\alpha^2 l_2^2) \\
&\quad \times \exp\Big\{3\alpha(2\alpha l_1^2 + \sigma^2)\Big\} E\|y(\eta_k)\|^2 \\
&= 3E\|y(t)\|^2 + \hat{o}E\|y(\eta_k)\|^2, \qquad (21)
\end{aligned}$$

where

$$\hat{o} = 6\alpha^2\, l_2^2 + 9\alpha(2\alpha l_1^2 + \sigma^2)(1 + 2\alpha^2\, l_2^2) \times \exp\Big\{3\alpha(2\alpha l_1^2 + \sigma^2)\Big\}.$$

Combined with (A3),

$$\begin{aligned}
E\|y(\eta_k)\|^2 &\le 3(1 - \hat{o})^{-1}E\|y(t)\|^2 \\
&= \varpi E\|y(t)\|^2,
\end{aligned}$$

where $\varpi = 3(1 - \hat{o})^{-1}$. Therefore, (18) holds for $t \in [\alpha_k, \alpha_{k+1})$. By the randomicities of t and k, (18) holds for all $t \in \Re^+$.

In the following, we investigate the effects of deviating argument and stochastic disturbance on the robustness of global exponential stability of system (16).

Theorem 2: Let (A1) and (A3) hold, and system (17) to be globally exponentially stable. System (16) is mean square exponentially stable and also almost surely exponentially stable if $|\sigma| < \frac{\bar{\sigma}}{\sqrt{2}}$ and $\alpha < \min(\frac{\rho}{2}, \bar{\alpha})$, where $\bar{\sigma}$ is the unique positive solution $\hat{w}$ of the following equation (22)

$$\begin{aligned}
&2l\exp\{-\kappa\rho\} + 8[128 l_2^2\rho + 2\hat{w}^2]\frac{\rho l}{\kappa} \\
&\quad \times \exp\Big\{2\rho\Big(8\rho[l_1^2 + 2l_2^2] + 2[128 l_2^2\rho + \hat{w}^2]\Big)\Big\} = 1, \qquad (22)
\end{aligned}$$

and $\bar{\alpha}$ is the unique positive solution $\check{w}$ of the following equation (23)

$$\begin{aligned}
&2l\exp\{-\kappa(\rho - \check{w})\} + 8[32 l_2^2\rho(1+\vartheta) + \bar{\sigma}^2]\frac{\rho l}{\kappa} \\
&\quad \times \exp\Big\{2\rho\Big(8\rho(l_1^2 + 2l_2^2) + 2[32 l_2^2\rho(1+\vartheta) + \bar{\sigma}^2]\Big)\Big\} = 1, \qquad (23)
\end{aligned}$$

with $\vartheta = 3(1 - \check{o})^{-1}$, $\check{o} = 6\check{w}^2 l_2^2 + 9\check{w}(2\check{w}l_1^2 + \bar{\sigma}^2)(1 + 2\check{w}^2\, l_2^2)\exp\{3\check{w}(2\check{w}l_1^2 + \bar{\sigma}^2)\}$, $\rho > \frac{\ln(l)}{\kappa} > 0$.

Proof: For convenience, we write $x(t; t_0, x_0) \equiv x(t)$ and $y(t; t_0, y_0) \equiv y(t)$. From (16) and (17), together with Lemma 2, for any $t \ge t_0 \ge 0$, we have

$$\begin{aligned}
&E\|y(t) - x(t)\|^2 \\
&= E\Big\|\int_{t_0}^{t}[f(y(s), y(\beta(s)), s) - f(x(s), x(s), s)]\mathrm{d}s \\
&\quad + \int_{t_0}^{t}\sigma y(s)\mathrm{d}B(s)\Big\|^2 \\
&\le 2E\Big\|\int_{t_0}^{t}[f(y(s), y(\beta(s)), s) - f(x(s), x(s), s)]\mathrm{d}s\Big\|^2 \\
&\quad + 2E\Big\|\int_{t_0}^{t}\sigma y(s)\mathrm{d}B(s)\Big\|^2 \\
&\le 4(t - t_0)\int_{t_0}^{t}[l_1^2E\|y(s) - x(s)\|^2 + l_2^2E\|y(\beta(s)) - x(s)\|^2]\mathrm{d}s\|^2 \\
&\quad + 2\sigma^2\int_{t_0}^{t}E\|y(s)\|^2\mathrm{d}s \\
&\le 4(t - t_0)\int_{t_0}^{t}(l_1^2 + 2l_2^2)E\|y(s) - x(s)\|^2\mathrm{d}s \\
&\quad + 16\, l_2^2(t - t_0)\int_{t_0}^{t}E\|y(\beta(s))\|^2\mathrm{d}s \\
&\quad + 16\, l_2^2(t - t_0)\int_{t_0}^{t}E\|y(s)\|^2\mathrm{d}s \\
&\quad + 2\sigma^2\int_{t_0}^{t}E\|y(s)\|^2\mathrm{d}s \\
&\le 4(t - t_0)\int_{t_0}^{t}(l_1^2 + 2l_2^2)E\|y(s) - x(s)\|^2\mathrm{d}s \\
&\quad + \Big[16 l_2^2(t - t_0)(1 + \varpi) + 2\sigma^2\Big]\int_{t_0}^{t}E\|y(s)\|^2\mathrm{d}s \\
&\le \Big\{4(t - t_0)(l_1^2 + 2l_2^2) + 2\Big[16 l_2^2(t - t_0)(1 + \varpi) + 2\sigma^2\Big]\Big\} \\
&\quad \times \int_{t_0}^{t}E\|y(s) - x(s)\|^2\mathrm{d}s \\
&\quad + 2\Big[16 l_2^2(t - t_0)(1 + \varpi) + 2\sigma^2\Big]\frac{l}{\kappa}\|y_0\|^2(t - t_0). \qquad (24)
\end{aligned}$$

Applied Gronwall-Bellman lemma to (24), we obtain for $t_0+\alpha \le t \le t_0+2\rho$,

$$\begin{aligned}&E\|y(t)-x(t)\|^2\\&\le 2[32l_2^2\rho(1+\varpi)+2\sigma^2]\frac{2\rho l}{\kappa}\|y_0\|^2\\&\quad\times\exp\left\{2\rho\left(8\rho[l_1^2+2l_2^2]+2[32l_2^2\rho(1+\varpi)+2\sigma^2]\right)\right\}.\end{aligned}$$

Therefore, for $t_0+\alpha \le t \le t_0+2\rho$,

$$\begin{aligned}&E\|y(t)\|^2\\&\le 2E\|x(t)\|^2+2E\|y(t)-x(t)\|^2\\&\le 2l\|y_0\|^2\exp\{-\kappa(t-t_0)\}\\&\quad+4[32l_2^2\rho(1+\varpi)+2\sigma^2]\frac{2\rho l}{\kappa}\|y_0\|^2\\&\quad\times\exp\left\{2\rho\left(8\rho(l_1^2+2l_2^2)+2[32l_2^2\rho(1+\varpi)+2\sigma^2]\right)\right\}.\end{aligned}\tag{25}$$

Thus, for $t_0-\alpha+\rho \le t \le t_0-\alpha+2\rho$,

$$\begin{aligned}&E\|y(t)\|^2\\&\le 2l\|y_0\|^2\exp\{-\kappa(\rho-\alpha)\}\\&\quad+4[32l_2^2\rho(1+\varpi)+2\sigma^2]\frac{2\rho l}{\kappa}\|y_0\|^2\\&\quad\times\exp\left\{2\rho\left(8\rho(l_1^2+2l_2^2)+2[32l_2^2\rho(1+\varpi)+2\sigma^2]\right)\right\}\\&=\tilde{o}\|y_0\|^2,\end{aligned}$$

where

$$\begin{aligned}\tilde{o}&=2l\exp\{-\kappa(\rho-\alpha)\}+4[32l_2^2\rho(1+\varpi)+2\sigma^2]\frac{2\rho l}{\kappa}\\&\quad\times\exp\left\{2\rho\left(8\rho(l_1^2+2l_2^2)+2[32l_2^2\rho(1+\varpi)+2\sigma^2]\right)\right\}.\end{aligned}$$

From (22) and (23), we see $\tilde{o}<1$ when $\alpha<\min(\frac{\rho}{2},\bar{\alpha})$, $|\sigma|<\frac{\bar{\sigma}}{\sqrt{2}}$. Setting $\upsilon=-\frac{\ln(\tilde{o})}{\rho}$, we obtain

$$E\|y(t)\|^2 \le \exp\{-\rho\upsilon\}\|y_0\|^2.\tag{26}$$

From the uniqueness of solution of system (16),

$$y(t;t_0,y_0)=y\big(t;t_0+(m-1)\rho,y(t_0+(m-1)\rho;t_0,y_0)\big),\tag{27}$$

where m is positive integer. Therefore, from (26) and (27), for $t\ge t_0-\alpha+m\rho$,

$$\begin{aligned}&E\|y(t;t_0,y_0)\|^2\\&=E\|y\big(t;t_0+(m-1)\rho,y(t_0+(m-1)\rho;t_0,y_0)\big)\|^2\\&\le\exp\{-\rho\upsilon\}\|y(t_0+(m-1)\rho;t_0,y_0)\|^2\\&=\exp\{-\rho\upsilon\}\|y\big(t;t_0+(m-2)\rho,y(t_0+(m-2)\rho;t_0,y_0)\big)\|^2\\&\le\exp\{-m\rho\upsilon\}\|y_0\|^2.\end{aligned}$$

Thus, for any $t>t_0-\alpha+\rho$, there is a positive integer m such that $t_0-\alpha+(m-1)\rho\le t\le t_0-\alpha+m\rho$,

$$E\|y(t;t_0,y_0)\|^2\le\exp\{-\upsilon(t-t_0)\}\exp\{\upsilon(\rho-\alpha)\}\|y_0\|^2.\tag{28}$$

Clearly, (28) also holds for $t_0\le t\le t_0-\alpha+\eta$. So system (16) is mean square exponentially stable. According to Remark 7, system (16) is also almost surely exponentially stable.

Remark 8: If system (17) is globally exponentially stable, then the corresponding perturbed system (16) can remain to be mean square exponentially stable and also almost surely exponentially stable when the length of interval of deviating function $\beta(t)$ and the noise intensity σ are lower than the given bounds $\min(\frac{\rho}{2},\bar{\alpha})$ and $\frac{\bar{\sigma}}{\sqrt{2}}$, respectively. That is, Theorem 2 gives a calculation numerical result on the robustness of global exponential stability of perturbed nonlinear systems in the presence of deviating argument and stochastic disturbance.

Remark 9: The bounds of the length of interval of deviating function $\beta(t)$ and the noise intensity σ in Theorem 2 can be easily calculated. By using MATLAB, transcendental equations (22) and (23) can be solved numerically for $\bar{\sigma}$ and $\bar{\alpha}$, respectively, where any other parameters are known.

Remark 10: Huang et al. [24] have analyzed the robust stability of uncertain neurodynamic systems with stochastic disturbance by using the Ito formula, Lyapunov function, and Halanay inequality. In this paper, the robustness results of global exponential stability of nonlinear systems in the presence of deviating argument and stochastic disturbance are established by applying the idea of stochastic analysis theory and inequality technique. The derived criteria possess less conservatism. In fact, it is possible to generalize the main results here to other complex systems, such as multi-agent systems in the presence of deviating argument and stochastic disturbance, sensor networks in the presence of deviating argument and stochastic disturbance, etc. The relevant results will be carried out in the near future.

IV. NUMERICAL EXAMPLES

In this section, two examples are given to verify the obtained theoretical results.

Example 1: Consider a two-dimensional nonlinear system with deviating argument

$$\begin{cases}\dot{y}_1(t)=-y_1(t)-0.01\sin^2(y_1(\beta(t)))+0.01\sin^2(y_2(\beta(t))),\\\dot{y}_2(t)=-y_2(t)+0.01\sin^2(y_1(\beta(t)))+0.01\sin^2(y_2(\beta(t))),\end{cases}\tag{29}$$

where $\{\alpha_k\}=\left\{\frac{k}{4}\right\}$, $\{\eta_k\}=\left\{\frac{2k+1}{8}\right\}$, $k\in N$. The deviating function $\beta(t)=\eta_k$, if $t\in[\alpha_k,\alpha_{k+1})$, $k\in N$.

Consider the undisturbed system of (29) as follows:

$$\begin{cases}\dot{x}_1(t)=-x_1(t)-0.01\sin^2(x_1(t))+0.01\sin^2(x_2(t)),\\\dot{x}_2(t)=-x_2(t)+0.01\sin^2(x_1(t))+0.01\sin^2(x_2(t)).\end{cases}\tag{30}$$

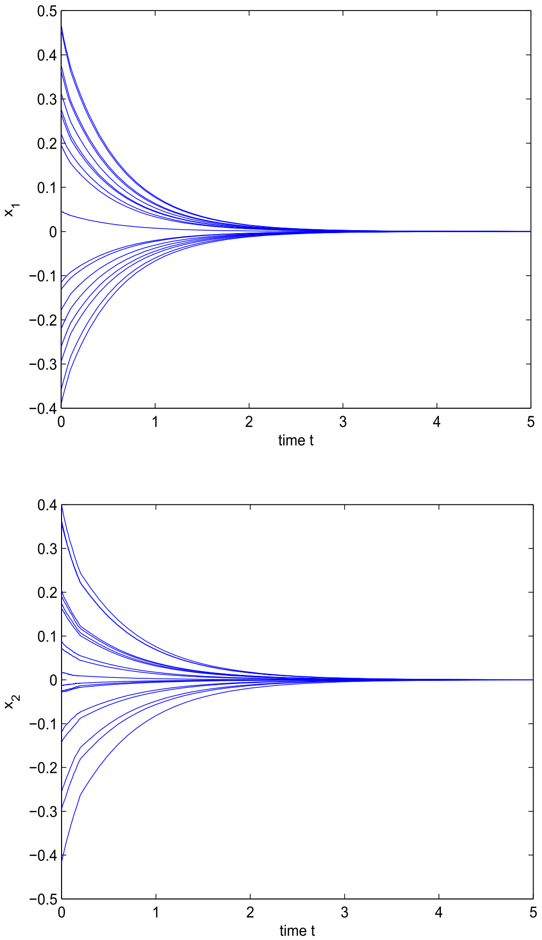

FIGURE 1. The convergent behavior of system (30).

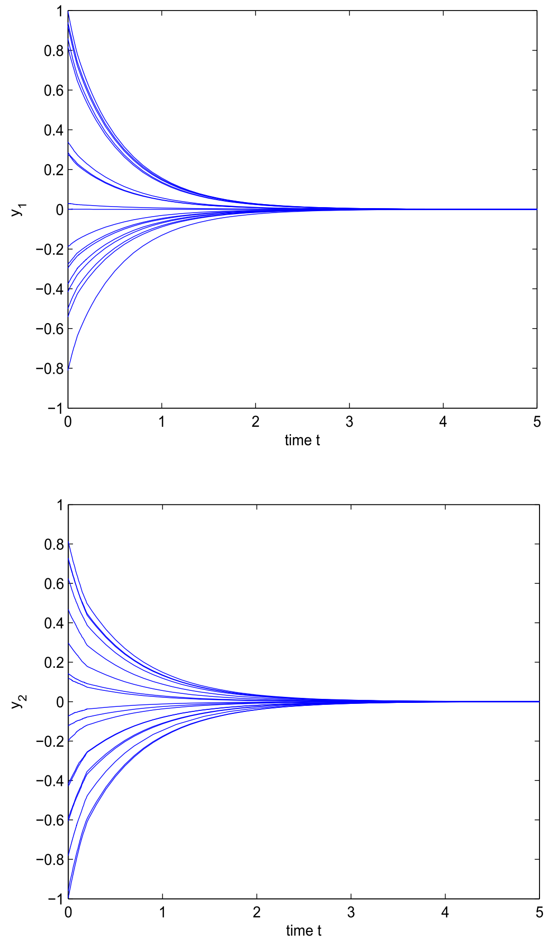

FIGURE 2. The evolvement behavior of system (29).

By many of the existing criteria, we can easily obtain that system (30) is globally exponentially stable with $l = 1.2$ and $\kappa = 0.9$. Figure 1 describes the convergent behavior of system (30). Let $\eta = 1 > \frac{\ln(1.2)}{0.9} = 0.2026$, $l_1 = 0.02$, $l_2 = 0.03$, substituting them into (8) and (9), then

$$\hat{x}\left[0.02(1+0.03\hat{x})\exp\left\{0.02\hat{x}\right\}+0.03\right]=1,$$

$$\begin{aligned}
&1.2\exp\left\{-0.9(1-\check{x})\right\}\\
&\quad+0.03\left[1+\left(1-\check{x}[0.02(1+0.03\check{x})\right.\right.\\
&\quad\left.\left.\times\exp\left\{0.02\check{x}\right\}+0.03]\right)\right]/0.9\\
&\quad\times\exp\left\{2\left[0.08+0.03\left(1-\check{x}[0.02(1+0.03\check{x})\right.\right.\right.\\
&\quad\left.\left.\left.\times\exp\left\{0.02\check{x}\right\}+0.03]\right)\right]\right\}\\
&=1.
\end{aligned}$$

By solving the transcendental equations above, we get $\bar{\alpha} = 14.5$ and $\bar{\bar{\alpha}} = 0.67$. Therefore, according to Theorem 1, when $\alpha < \min(\frac{\eta}{2}, \bar{\alpha}, \bar{\bar{\alpha}})$, that is, $\alpha < 0.5$, so system (29) still is globally exponentially stable. Figure 2 depicts the evolvement behavior of system (29).

Example 2: Consider a one-dimensional neurodynamic system

$$\dot{x}(t) = -3.1x(t) + 0.1\tanh(x(t)). \tag{31}$$

By many of the existing criteria, we can easily obtain that system (31) is globally exponentially stable with $l = 1.1$ and $\kappa = 3$. Figure 3 describes the convergent behavior of system (31).

When the deviating argument and stochastic disturbance are exerted to (31), then we can get a class of disturbed systems of (31) as follows:

$$\begin{aligned}
\mathrm{d}y(t) = \Big[&-3.1y(t) + 0.099\tanh(y(t))\\
&+0.001\tanh\big(y(\beta(t))\big)\Big]\mathrm{d}t + \sigma y(t)\mathrm{d}B(t),
\end{aligned} \tag{32}$$

where the deviating function $\beta(t) = \eta_k$, if $t \in [\alpha_k, \alpha_{k+1})$, $k \in N$, $t \in \Re^+$, σ is the noise intensity, $B(t)$ is a one-dimensional Brownian motion on the compete space.

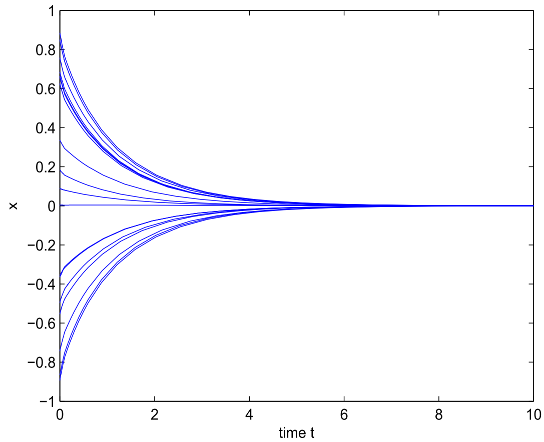

FIGURE 3. The convergent behavior of system (31).

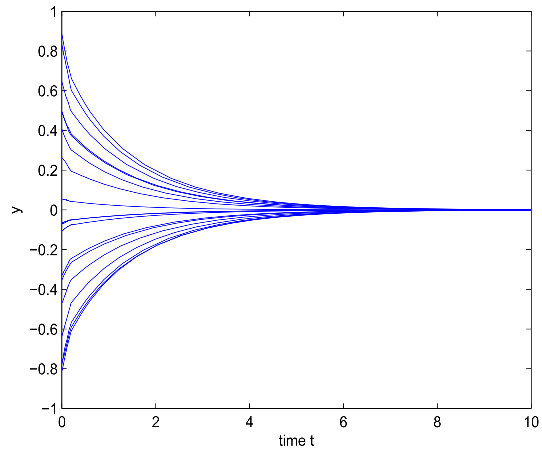

FIGURE 4. The evolvement behavior of system (32) with $\sigma = 0.01$, $\{\alpha_k\} = \left\{\frac{k}{100}\right\}$, $\{\eta_k\} = \left\{\frac{2k+1}{200}\right\}$, $k \in N$.

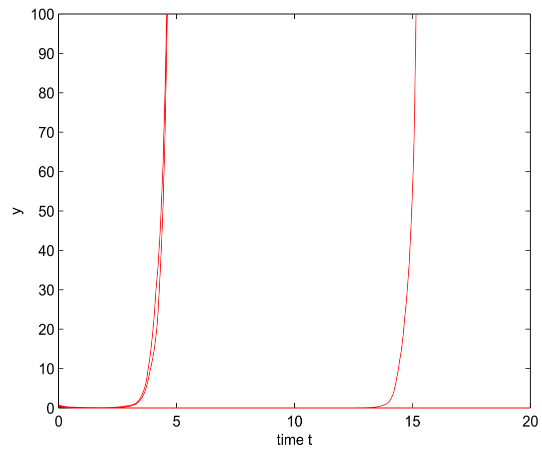

FIGURE 5. The instability behavior of system (32) with $\sigma = 1$, $\{\alpha_k\} = \left\{\frac{k}{100}\right\}$, $\{\eta_k\} = \left\{\frac{2k+1}{200}\right\}$, $k \in N$.

Substituting the computing parameters into (22), it yields

$$(5.76 \times 0.00001 + 4\hat{w}^2) \times \exp\left\{3.3246 + 0.96\hat{w}^2\right\}/3$$
$$+2\exp\{-0.72\} = 1.$$

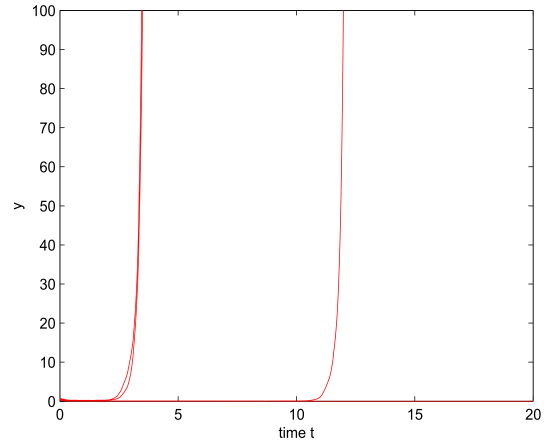

FIGURE 6. The instability behavior of system (32) with $\sigma = 0.01$, $\{\alpha_k\} = \left\{\frac{k}{2}\right\}$, $\{\eta_k\} = \left\{\frac{2k+1}{4}\right\}$, $k \in N$.

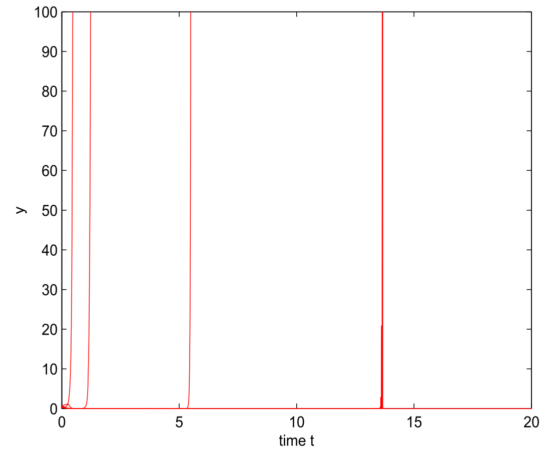

FIGURE 7. The instability behavior of system (32) with $\sigma = 1$, $\{\alpha_k\} = \left\{\frac{k}{2}\right\}$, $\{\eta_k\} = \left\{\frac{2k+1}{4}\right\}$, $k \in N$.

Thus, we have $\bar{\sigma} = 0.0265$. Note that $|\sigma| < \frac{\bar{\sigma}}{\sqrt{2}}$, it derives $|\sigma| < 0.0187$.

Combined with $\bar{\sigma} = 0.0265$, substituting the other computing parameters into (23), then

$$(4.701 \times 0.0001 + 3.84 \times 0.000001\check{w})$$
$$\times \exp\left\{3.3249 + 2.765 \times 0.000001\check{w}\right\}$$
$$+2\exp\{-0.72\} = 1.$$

Hence, it is relatively easy to obtain $\bar{\alpha} = 125.7645$. Recalling that $\alpha < \min(\frac{\rho}{2}, \bar{\alpha})$, therefore, $\alpha < 0.0159$.

In (32), select $\sigma = 0.01$, $\{\alpha_k\} = \left\{\frac{k}{100}\right\}$, $\{\eta_k\} = \left\{\frac{2k+1}{200}\right\}$, $k \in N$. The deviating function $\beta(t) = \eta_k$, if $t \in [\alpha_k, \alpha_{k+1})$, $k \in N$. Then the conditions in Theorem 2 are all satisfied. Accordingly, in this case, system (32) is mean square exponentially stable and also almost surely exponentially stable. Figure 4 depicts the evolvement behavior of system (32) with $\sigma = 0.01$, $\{\alpha_k\} = \left\{\frac{k}{100}\right\}$, $\{\eta_k\} = \left\{\frac{2k+1}{200}\right\}$, $k \in N$.

Figure 5 shows the instability behavior of system (32) with $\sigma = 1$, $\{\alpha_k\} = \left\{\frac{k}{100}\right\}$, $\{\eta_k\} = \left\{\frac{2k+1}{200}\right\}$, $k \in N$.

Actually, in this case, such parameters are again not suitable for the conditions of Theorem 2.

Figure 6 shows the instability behavior of system (32) with $\sigma = 0.01$, $\{\alpha_k\} = \left\{\frac{k}{2}\right\}$, $\{\eta_k\} = \left\{\frac{2k+1}{4}\right\}$, $k \in N$. Actually, in this case, such parameters are again not suitable for the conditions of Theorem 2.

Figure 7 shows the instability behavior of system (32) with $\sigma = 1$, $\{\alpha_k\} = \left\{\frac{k}{2}\right\}$, $\{\eta_k\} = \left\{\frac{2k+1}{4}\right\}$, $k \in N$. Actually, in this case, such parameters are again not suitable for the conditions of Theorem 2.

V. CONCLUDING REMARKS

This paper aims to investigate the robustness of global exponential stability of nonlinear systems evoked by deviating argument and stochastic disturbance. The results derived here show that an exponentially stable nonlinear system perturbed by deviating argument, stochastic disturbance, or both is able to sustain exponential stability provided that the length of interval of deviating function and the noise intensity are lower than the upper bounds derived herein. Analysis and design method in this paper is available for ever more complex control systems.

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