

Received June 1, 2017, accepted June 20, 2017, date of publication July 4, 2017, date of current version July 24, 2017.

Digital Object Identifier 10.1109/ACCESS.2017.2720578

# Precoding-Based Blind Separation of MIMO FIR Mixtures

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This work was supported in part by the National Natural Science Foundation of China under Grant 61432012, Grant U1435213, and Grant 61172180, and in part by the Guangzhou Education Bureau Grant under Project 1201431118.

**ABSTRACT** This paper focuses on the problem of blind separation of sources mixed by multi-input multi-output finite impulse response channels, which is also called convolutive blind source separation (BSS) in short. This problem has been intensively studied in the context that the sources possess certain favourable properties, such as independence and sparsity. However, these properties may not exist in some practical applications. In this paper, we propose a precoding-based convolutive BSS method, which can deal with mutually correlated sources without requiring the sources to be sparse. It is also applicable to mutually independent sources. In the proposed method, the sources are preprocessed in transmitters prior to transmission by order-one precoders. At the receiving side, the second-order statistics of the sources and the Z-domain features of the precoders are exploited to estimate the coded signals, from which the sources are recovered. Simulation results demonstrate the effectiveness of the new convolutive BSS method.

**INDEX TERMS** Blind source separation, correlated sources, MIMO FIR system, Z-domain precoding.

## I. INTRODUCTION

Blind source separation (BSS) aims to recover multiple source signals mixed by an unknown system only from their mixtures. It requires no or little prior knowledge of the sources and the mixing system. It is a fundamental problem arising from a wide range of applications such as digital communications, speech identification, biomedical image processing and remote sensing [1]–[3]. Convolutive BSS is a type of BSS problem where the mixing system is with reflections [4]. In this paper, we limit our attention to the multi-input multi-output (MIMO) finite impulse response (FIR) system, which is a widely used system model in wireless communications.

Most existing convolutive BSS methods are designed under the framework of independent component analysis (ICA), aiming to recover the independent components from the observed mixtures. In [5] and [6], the convolutive BSS problem is treated as a joint-approximate-diagonalization (JAD) problem in frequency domain by exploiting the independency and quasi-stationarity of the source signals. Specially, it is assumed that the second-order statistics (SOS) of each source varies slowly within a specific time length (an “epoch”) such that over an epoch,

the source can be considered approximately stationary, but non-stationary in different epochs. In [7], the sources are required to be mutually statistically independent and sufficiently sparse in frequency domain, namely, the supports of all sources in frequency domain are mutually disjoint [8]. In [9], a convex geometry-based method combined with the technique of non-negative matrix factorization (NMF) is introduced to deal with both linear and convolutive BSS problems. This method not only requires the sources to be uncorrelated and quasi-stationary, but also their SOS features to be local dominant, i.e., there exist some time instants at which the SOS features of all sources are dominated by only one source.

In spite of the fact that the condition of independent or uncorrelated sources holds in many applications, spatially correlated sources are also encountered in practice. For example, in a wireless sensor network, in order to provide high reliability in face of the failure of individual sensors, and/or facilitate superior spatial localization of objects of interests, some wireless sensors could be densely deployed [10], [11]. As a result, signals observed by these spatially proximal sensors are highly correlated and their cross-correlations are unknown. Besides, if each sensor is

equipped with a video camera, the wireless sensor network becomes a wireless video surveillance system. It is known that images which look irrelevant are often mutually correlated [12]–[14]. The scenario of mutually correlated signals can also be found in MIMO wireless relay systems [15]. Unfortunately, most traditional convolutive BSS methods are unable to separate mutually correlated sources and this problem has not been fully resolved yet.

Recently, the hypothesis of mutual independence of the sources in ICA has been replaced with a weaker domain separability assumption [16]. It is shown that given a set of nondegenerate sources with bounded support, the convex hull of the support of the joint density of sources can be written as the Cartesian product of the convex hull of the supports of the individual source marginals. Note that this domain separability assumption is a necessary but not sufficient condition of ICA, which means ICA can be treated as a special case for bounded sources [17]. Based on this framework, which is called bounded component analysis (BCA), several algorithms have been proposed for linear blind extraction of bounded sources [18], [19]. More recently, some BCA-based algorithms for convolutive BSS have been developed [20], [21], where the sources can be independent or dependent (even correlated) but must satisfy the domain separability assumption.

Different from the methods in [20] and [21], we propose to tackle convolutive BSS for both spatially independent and correlated sources via precoding. The concept of precoding-based BSS was first proposed in [22], where the sources were assumed to be mixed linearly and simultaneously. In [22], a set of precoders are applied to the correlated source signals in such a way that the coded signals had zero cross-correlations at some time lags. Then the time-domain features of the coded signals were exploited at the receiving side to separate the coded signals from the measured mixtures. However, the order of the precoders used in [22] is almost four times as big as the number of the sources. The method in [15] reduces the order of precoders to two, regardless of the number of sources. This leads to considerable delay reduction in data transmission and simplifies the implementation of the precoders in practical systems. Nevertheless, both [15] and [22] can only deal with instantaneous BSS.

In this paper, we propose a precoding-based method to separate sources mixed by MIMO FIR channels. In the proposed method, each source is filtered by an order-1 precoder prior to transmission and then the coded signals are transmitted over the MIMO FIR channel. At the receiving end, the SOS features of the sources and the Z-domain characteristics of the precoders are utilized to separate the coded signals and then recover the original sources. Since this method is irrelevant to the spatial relationship between the sources, it can deal with both mutually independent and correlated sources. The novel contributions of this new method include the ability of separating both independent and correlated sources, lower computational cost due to the avoidance of calculating of many convolutions encountered in most time-domain

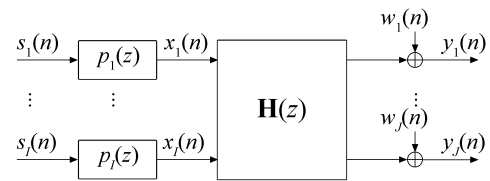


FIGURE 1. Block diagram of precoding-based MIMO FIR system model.

methods [23], [24] and no permutation alignment problem suffered by most frequency-domain methods [5], [25].

The remainder of this paper is organized as follows. Section II formulates the problem of precoding-based MIMO FIR transmission together with a set of assumptions. The new precoding-based convolutive BSS method is presented in Section III. Its performance is illustrated in Section IV by numerical simulations in comparison with other existing methods. Some conclusions are drawn in Section V.

## II. PROBLEM FORMULATION AND ASSUMPTIONS

The precoding-based MIMO FIR system is shown in Fig. 1. The source signals are denoted as \$s\_1(n), s\_2(n), \dots, s\_I(n)\$, which are mutually correlated. Prior to transmission, they are filtered by \$I\$ precoders \$p\_1(z), p\_2(z), \dots, p\_I(z)\$, respectively. The coded signals \$x\_1(n), x\_2(n), \dots, x\_I(n)\$ are transmitted over a \$J \times I\$ MIMO FIR channel. Based on these notations, the discrete-time channel system can be expressed as

$$\begin{cases} \mathbf{y}(n) = \mathbf{H}(z)\mathbf{x}(n) + \mathbf{w}(n) \\ \mathbf{H}(z) = \sum_{l=0}^L \mathbf{H}(l)z^{-l} \end{cases} \quad (1)$$

where \$\mathbf{x}(n) = [x\_1(n), x\_2(n), \dots, x\_I(n)]^T\$ is the coded signal vector (or channel input vector), \$\mathbf{y}(n) = [y\_1(n), y\_2(n), \dots, y\_J(n)]^T\$ is the channel output vector, \$\mathbf{H}(l)\$ is the \$J \times I\$ impulse response matrix of the MIMO FIR system with \$l = 0, 1, \dots, L\$, \$\mathbf{H}(z)\$ is the channel matrix, \$\mathbf{w}(n) = [w\_1(n), w\_2(n), \dots, w\_J(n)]^T\$ is the additive noise vector, and the superscript \$T\$ denotes transpose. Denote the degree of the \$i\$th column of \$\mathbf{H}(z)\$ by \$L\_i\$, \$i = 1, 2, \dots, I\$, and \$\mathbf{L} = [L\_1, L\_2, \dots, L\_I]\$. Clearly, the degree of the channel matrix is \$L = \max\{L\_1, L\_2, \dots, L\_I\}\$.

Define the \$i\$th precoder of order-1 by

$$p_i(z) = 1 - r_i z^{-1} \quad (2)$$

where \$r\_i\$ is the zero of the \$i\$th precoder \$p\_i(z)\$. Then the \$i\$th coded signal can be written as

$$x_i(n) = p_i(z)s_i(n) = s_i(n) - r_i s_i(n-1). \quad (3)$$

Letting \$\mathbf{s}(n) = [s\_1(n), s\_2(n), \dots, s\_I(n)]^T\$, the covariance matrix of \$\mathbf{s}(n)\$ at time lag \$\tau\$ is

$$\mathbf{R}_s(\tau) = E(\mathbf{s}(n)\mathbf{s}(n-\tau)^H)$$

where \$E(\cdot)\$ denotes mathematical expectation and the superscript \$H\$ stands for complex conjugate transpose. We assume in the sequel that

- 1) There are more channel output signals than input signals, i.e.,  $J > I \geq 2$ , and the channel matrix  $\mathbf{H}(z)$  is irreducible and column-reduced [26].
- 2) The source signals  $s_1(n), s_2(n), \dots, s_I(n)$  are zero-mean, temporally white and mutually correlated, namely, for any  $i, j = 1, 2, \dots, I$ ,

$$\begin{cases} r_{ij}^s \triangleq E \{s_i(n)s_j^*(n)\} \neq 0, \\ E \{s_i(n)s_j^*(n-\tau)\} = 0 \quad \text{with } \tau \neq 0. \end{cases} \quad (4)$$

- 3) The noise signals  $w_1(n), w_2(n), \dots, w_J(n)$  are zero-mean, temporally and spatially white, of equal variance  $\sigma_w^2$ , and independent of the source signals.
- 4) All the precoder zeros are distinct and satisfy  $0 < |r_i| < 1, i = 1, 2, \dots, I$ . They are known at the receiver.

It is worth noting 4 implies that the precoders are reversible by stable filters, thus the sources  $\mathbf{s}(n)$  can be recovered from the coded signals  $\mathbf{x}(n)$ . The ultimate objective of BSS is to obtain  $x_1(n), x_2(n), \dots, x_I(n)$  from  $y_1(n), y_2(n), \dots, y_J(n)$ . Then, based on the estimate of  $x_i(n)$  and (3), one can recover  $s_i(n)$  by

$$s_i(n) = x_i(n) + r_i s_i(n-1), \quad i = 1, 2, \dots, I. \quad (5)$$

### III. PROPOSED CONVOLUTIVE BSS METHOD

In this section, we will show how to utilize the SOS properties of the sources and the Z-domain properties of the precoders given in (2) to perform precoding-based convolutive BSS.

#### A. SYSTEM MODEL

In this subsection, the MIMO FIR model in (1) is at first transformed into an instantaneous linear mixing model by stacking the mixtures in the time domain as follows. Based on the  $j$ th channel output  $y_j(n)$ , we define

$$\tilde{\mathbf{y}}_j(n) = [y_j(n), y_j(n-1), \dots, y_j(n-W+1)]^T \quad (6)$$

where the slide-window width  $W$  is chosen to satisfy

$$W > \bar{L} \triangleq \sum_{i=1}^I L_i. \quad (7)$$

Denote the  $(j, i)$ th entry of  $\mathbf{H}(l)$  by  $H_{j,i}(l)$ . From (1) and (6), it follows

$$\tilde{\mathbf{y}}_j(n) = \sum_{i=1}^I \mathcal{H}_{j,i} \tilde{\mathbf{x}}_i(n) + \tilde{\mathbf{w}}_j(n), \quad j = 1, 2, \dots, J \quad (8)$$

with

$$\begin{aligned} \tilde{\mathbf{w}}_j(n) &= [w_j(n), w_j(n-1), \dots, w_j(n-W+1)]^T \\ \tilde{\mathbf{x}}_i(n) &= \begin{bmatrix} x_i(n) \\ x_i(n-1) \\ \vdots \\ x_i(n-\gamma_i+1) \end{bmatrix} \end{aligned} \quad (9)$$

where  $\gamma_i \triangleq W + L_i$ , and  $\mathcal{H}_{j,i}$  is of size  $W \times \gamma_i$  and given by (10), as shown at the bottom of this page, with  $i = 1, 2, \dots, I, j = 1, 2, \dots, J$ .

Denote

$$\tilde{\mathbf{y}}(n) = [\tilde{\mathbf{y}}_1^T(n), \tilde{\mathbf{y}}_2^T(n), \dots, \tilde{\mathbf{y}}_J^T(n)]^T \quad (11)$$

then

$$\tilde{\mathbf{y}}(n) = \mathcal{H} \tilde{\mathbf{x}}(n) + \tilde{\mathbf{w}}(n) \quad (12)$$

with

$$\tilde{\mathbf{x}}(n) = [\tilde{\mathbf{x}}_1^T(n), \tilde{\mathbf{x}}_2^T(n), \dots, \tilde{\mathbf{x}}_I^T(n)]^T \quad (13)$$

$$\tilde{\mathbf{w}}(n) = [\tilde{\mathbf{w}}_1^T(n), \tilde{\mathbf{w}}_2^T(n), \dots, \tilde{\mathbf{w}}_J^T(n)]^T$$

and

$$\mathcal{H} = \begin{bmatrix} \mathcal{H}_{1,1} & \mathcal{H}_{1,2} & \dots & \mathcal{H}_{1,I} \\ \mathcal{H}_{2,1} & \mathcal{H}_{2,2} & \dots & \mathcal{H}_{2,I} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{H}_{J,1} & \mathcal{H}_{J,2} & \dots & \mathcal{H}_{J,I} \end{bmatrix}.$$

Under assumption 1 and according to (7), it is shown in [27] that the matrix  $\mathcal{H}$  has full column rank  $IW + \bar{L}$ . To estimate  $x_i(n)$  from  $\tilde{\mathbf{y}}(n)$ , one needs to find a  $JW \times 1$  separation vector  $\mathbf{u}_i$  that makes all elements of the row vector  $\mathbf{u}_i^H \mathcal{H}$  zero except the  $k(i) \triangleq \left(\sum_{l=1}^{i-1} \gamma_l + 1\right)$ th element, i.e.,

$$\mathbf{u}_i^H \mathcal{H} = \underbrace{[0, 0, \dots, 0, c_i, 0, 0, \dots, 0]}_{\sum_{l=1}^{i-1} \gamma_l} \quad \underbrace{[0, 0, \dots, 0]}_{\sum_{l=i}^I \gamma_l - 1}$$

where  $c_i \neq 0$  and  $i = 1, 2, \dots, I$ . Then

$$\hat{x}_i(n) = c_i x_i(n)$$

namely,  $x_i(n)$  is estimated up to an unknown scalar. Once  $\hat{x}_i(n)$  is obtained, the corresponding source signal  $s_i(n)$  can be recovered by (5).

$$\mathcal{H}_{j,i} = \begin{bmatrix} H_{j,i}(0) & H_{j,i}(1) & \dots & H_{j,i}(L_i) & 0 & \dots & 0 & 0 \\ 0 & H_{j,i}(0) & \dots & H_{j,i}(L_i-1) & H_{j,i}(L_i) & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & H_{j,i}(L_i) & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & H_{j,i}(L_i-1) & H_{j,i}(L_i) \end{bmatrix}. \quad (10)$$

**B. ALGORITHM DEVELOPMENT**

To proceed, let us consider the covariance properties of the stacked observation  $\tilde{\mathbf{y}}(n)$  and its Z-transform. We start from the additive noise  $\tilde{\mathbf{w}}(n)$  in (12). Let  $\mathbf{I}_I$  be an  $I \times I$  identity matrix,  $\mathbf{J}_I$  be an  $I \times I$  Jordan matrix with the following form

$$\mathbf{J}_I = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & 0 \\ 0 & \cdots & 1 & 0 \end{bmatrix}$$

and  $\text{DIAG}(\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_K)$  stand for a block diagonal matrix, where  $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_K$  are the matrices along the diagonal. Denote

$$\mathbf{J}_I^k = \underbrace{\mathbf{J}_I \times \mathbf{J}_I \times \cdots \times \mathbf{J}_I}_k$$

where  $k$  is a positive integer. According to assumption 3, the covariance matrix of  $\tilde{\mathbf{w}}(n)$  at time lag  $\tau$ , defined as  $\mathbf{R}_{\tilde{\mathbf{w}}}(\tau) = \text{E}(\tilde{\mathbf{w}}(n)\tilde{\mathbf{w}}(n - \tau)^H)$ , can be written as

$$\begin{cases} \mathbf{R}_{\tilde{\mathbf{w}}}(0) = \sigma_w^2 \mathbf{I}_{JW}, & (14a) \\ \mathbf{R}_{\tilde{\mathbf{w}}}(\tau) = \sigma_w^2 \cdot \text{DIAG}(\underbrace{\mathbf{J}_W^\tau, \mathbf{J}_W^\tau, \dots, \mathbf{J}_W^\tau}_J), & (14b) \\ \mathbf{R}_{\tilde{\mathbf{w}}}(\tau) = \mathbf{0}, & \text{if } |\tau| \geq W. \end{cases} \quad (14c)$$

Then the denoised covariance matrices of  $\tilde{\mathbf{y}}(n)$  can be obtained as follows. From (12) and assumption 3, the covariance matrix of  $\tilde{\mathbf{y}}(n)$  at time lag  $\tau$  can be expressed as

$$\begin{aligned} \mathbf{R}_{\tilde{\mathbf{y}}}(\tau) &= \text{E}(\tilde{\mathbf{y}}(n)\tilde{\mathbf{y}}(n - \tau)^H) \\ &= \mathcal{H}\mathbf{R}_{\tilde{\mathbf{x}}}(\tau)\mathcal{H}^H + \mathbf{R}_{\tilde{\mathbf{w}}}(\tau) \end{aligned} \quad (15)$$

where  $\mathbf{R}_{\tilde{\mathbf{x}}}(\tau)$  is the covariance matrix of  $\tilde{\mathbf{x}}(n)$  at time lag  $\tau$ . Substituting (14a) into (15), it follows

$$\mathbf{R}_{\tilde{\mathbf{y}}}(0) = \mathcal{H}\mathbf{R}_{\tilde{\mathbf{x}}}(0)\mathcal{H}^H + \sigma_w^2 \mathbf{I}_{JW}.$$

Since  $J > I$  and  $W > \bar{L}$ , it is clear that  $\mathcal{H}$  is a tall matrix. This implies that  $\sigma_w^2$  is the smallest eigenvalue of  $\mathbf{R}_{\tilde{\mathbf{y}}}(0)$  and thus can be computed from  $\mathbf{R}_{\tilde{\mathbf{y}}}(0)$ . As a result, one can construct  $\mathbf{R}_{\tilde{\mathbf{w}}}(\tau)$  according to (14) and then remove the noise term from  $\mathbf{R}_{\tilde{\mathbf{y}}}(\tau)$  to obtain

$$\bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(\tau) = \mathbf{R}_{\tilde{\mathbf{y}}}(\tau) - \mathbf{R}_{\tilde{\mathbf{w}}}(\tau) = \mathcal{H}\mathbf{R}_{\tilde{\mathbf{x}}}(\tau)\mathcal{H}^H. \quad (16)$$

Based on (16), the Z-transform of  $\bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(\tau)$  can be obtained as

$$\bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(z) = \sum_{k=-\infty}^{\infty} \bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(k)z^{-k} = \mathcal{H}\mathbf{Q}_{\tilde{\mathbf{x}}}(z)\mathcal{H}^H \quad (17)$$

where  $\mathbf{Q}_{\tilde{\mathbf{x}}}(z)$  is the Z-transform of  $\mathbf{R}_{\tilde{\mathbf{x}}}(\tau)$ .

In order to exploit the Z-domain features of the precoders and the SOS properties of the sources, the mixing system in (12) needs to be transformed into Z-domain. To proceed, we define

$$\tilde{\mathbf{s}}_i(n) = [s_i(n), s_i(n - 1), \dots, s_i(n - \gamma_i + 1)]^T \quad (18)$$

and

$$\tilde{\mathbf{s}}(n) = [\tilde{\mathbf{s}}_1^T(n), \tilde{\mathbf{s}}_2^T(n), \dots, \tilde{\mathbf{s}}_I^T(n)]^T. \quad (19)$$

Denote the covariance matrix of  $\tilde{\mathbf{s}}(n)$  at time lag  $\tau$  by  $\mathbf{R}_{\tilde{\mathbf{s}}}(\tau)$ , and the Z-transform of  $\mathbf{R}_{\tilde{\mathbf{s}}}(\tau)$  by  $\mathbf{Q}_{\tilde{\mathbf{s}}}(z)$ . Let

$$\mathbf{P}_i(z) = \text{diag}(\underbrace{p_i(z), p_i(z), \dots, p_i(z)}_{\gamma_i}), \quad i = 1, 2, \dots, I$$

as a  $\gamma_i \times \gamma_i$  diagonal matrix that has equal diagonal entries, and

$$\mathbf{P}(z) = \text{DIAG}(\mathbf{P}_1(z), \mathbf{P}_2(z), \dots, \mathbf{P}_I(z))$$

then it holds that

$$\mathbf{Q}_{\tilde{\mathbf{x}}}(z) = \mathbf{P}(z)\mathbf{Q}_{\tilde{\mathbf{s}}}(z)\mathbf{P}((z^*)^{-1})^H. \quad (20)$$

Substituting (20) into (17) yields

$$\bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(z) = \mathcal{H}\mathbf{P}(z)\mathbf{Q}_{\tilde{\mathbf{s}}}(z)\mathbf{P}((z^*)^{-1})^H\mathcal{H}^H.$$

Therefore, we have

$$\bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(r_i) = \mathcal{H}\mathbf{P}(r_i)\mathbf{Q}_{\tilde{\mathbf{s}}}(r_i)\mathbf{P}((r_i^*)^{-1})^H\mathcal{H}^H \quad (21)$$

with  $i = 1, 2, \dots, I$ . According to assumption 4, it is clear that the  $i$ th diagonal sub-matrix  $\mathbf{P}_i(r_i)$  in  $\mathbf{P}(r_i)$  is a zero matrix, while all the other diagonal entries of  $\mathbf{P}(r_i)$  are nonzero. Besides, no matter how  $i$  varies among  $\{1, 2, \dots, I\}$ , all diagonal entries of  $\mathbf{P}((r_i^*)^{-1})$  are nonzero.

Next we shall show that the separation vector  $\mathbf{u}_i$  can be obtained by exploiting the properties of  $\bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(r_i)$ ,  $i = 1, 2, \dots, I$ ,  $\bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(2)$  and  $\bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(0)$ , respectively. First of all, let us consider the feature of  $\bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(r_i)$ . Since it is originated from  $\mathbf{Q}_{\tilde{\mathbf{s}}}(r_i)$ , we propose the following lemma about  $\mathbf{Q}_{\tilde{\mathbf{s}}}(r_i)$ .

*Lemma 1:* The rank of  $\mathbf{Q}_{\tilde{\mathbf{s}}}(r_i)$  equals  $I$  with probability one, for any  $i = 1, 2, \dots, I$ .

*Proof:* See Appendix A. ■

Second, to find the property of  $\bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(2)$ , the structure of  $\mathbf{R}_{\tilde{\mathbf{x}}}(2)$  needs to be investigated. According to (3), for any  $i, j \in \{1, 2, \dots, I\}$ ,

$$\begin{aligned} r_{ij}^x(\tau) &\triangleq \text{E}\{x_i(n)x_j^*(n - \tau)\} \\ &= \text{E}\{[s_i(n) - r_i s_i(n - 1)][s_j(n - \tau) - r_j s_j(n - \tau - 1)]^*\} \\ &= (1 + r_i r_j^*)r_{ij}^s(\tau) - r_i r_{ij}^s(\tau - 1) - r_j^* r_{ij}^s(\tau + 1). \end{aligned}$$

Based on (4) in assumption 2, together with assumption 4, it holds that for any  $i, j \in \{1, 2, \dots, I\}$ ,

$$\begin{cases} r_{ij}^x(0) = (1 + r_i r_j^*)r_{ij}^s \neq 0, & (22a) \\ r_{ij}^x(1) = -r_i r_{ij}^s \neq 0, & (22b) \\ r_{ij}^x(-1) = -r_j^* r_{ij}^s \neq 0, & (22c) \\ r_{ij}^x(\tau) = 0, \quad |\tau| \geq 2. & (22d) \end{cases}$$

Rewrite  $\mathbf{R}_{\tilde{\mathbf{x}}}(2)$  in the form of block-matrix as

$$\mathbf{R}_{\tilde{\mathbf{x}}}(2) = \text{E}\{\tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^H(n - 2)\} \triangleq \begin{bmatrix} \mathbf{R}_{\tilde{\mathbf{x}}}(2)^{(11)} \\ \mathbf{R}_{\tilde{\mathbf{x}}}(2)^{(12)} \\ \vdots \\ \mathbf{R}_{\tilde{\mathbf{x}}}(2)^{(I1)} \end{bmatrix} \quad (23)$$

where the  $i$ th sub-matrix of  $\mathbf{R}_{\tilde{\mathbf{x}}}(2)$ , i.e.,  $\mathbf{R}_{\tilde{\mathbf{x}}}(2)^{(i)}$ , is expressed as

$$\begin{aligned} \mathbf{R}_{\tilde{\mathbf{x}}}(2)^{(i)} &\triangleq [\mathbf{R}_{\tilde{\mathbf{x}}_{i1}}(2), \mathbf{R}_{\tilde{\mathbf{x}}_{i2}}(2), \dots, \mathbf{R}_{\tilde{\mathbf{x}}_{iI}}(2)] \\ &= \begin{bmatrix} \mathbf{E} \{ \tilde{\mathbf{x}}_i(n) \tilde{\mathbf{x}}_1^H(n-2) \}^T \\ \mathbf{E} \{ \tilde{\mathbf{x}}_i(n) \tilde{\mathbf{x}}_2^H(n-2) \}^T \\ \vdots \\ \mathbf{E} \{ \tilde{\mathbf{x}}_i(n) \tilde{\mathbf{x}}_I^H(n-2) \}^T \end{bmatrix}^T. \end{aligned} \quad (24)$$

Then we have the following lemma with regard to  $\mathbf{R}_{\tilde{\mathbf{x}}}(2)$ .

**Lemma 2:** The covariance matrix  $\mathbf{R}_{\tilde{\mathbf{x}}}(2)$  satisfies  
c1) its  $k(i)$ th row vector is all-zero,  $i = 1, 2, \dots, I$ ;  
c2) its remaining row vectors are linearly independent.

*Proof:* See appendix B. ■

Note that according to the proof of conclusion c2) in Lemma 2, the condition number of the matrix made up by the remaining row vectors of  $\mathbf{R}_{\tilde{\mathbf{x}}}(2)$ , increases when either the condition number of the covariance matrix of  $\mathbf{s}(n)$  increases or the zero of any precoder gets very close to the origin. Therefore, the selection of the precoder zeros may affect the performance of the proposed method, which will be further discussed later in the simulation section.

Finally, based on the results in Lemma 1 and Lemma 2, we propose the following source separation criterion.

**Theorem 1:** While the precoder zeros are distinct,  $\mathbf{u}_i$  is a  $JW \times 1$  separation vector ensuring

$$\mathbf{u}_i^H \mathcal{H} = \underbrace{[0, 0, \dots, 0, c_i, 0, 0, \dots, 0]}_{\sum_{l=1}^{i-1} \gamma_l} \underbrace{[0, 0, \dots, 0]}_{\sum_{l=i}^I \gamma_l} \triangleq c_i \mathbf{e}_{k(i)}^H \quad (25)$$

if and only if

$$\begin{cases} \mathbf{u}_i^H \bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(2) = \mathbf{0} & (26a) \\ \mathbf{u}_i^H \bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(r_i) = \mathbf{0} & (26b) \\ \mathbf{u}_i^H \bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(0) \mathbf{u}_i \neq 0 & (26c) \end{cases}$$

where  $c_i \neq 0$ ,  $\mathbf{e}_{k(i)}$  is an  $(IW + \bar{L}) \times 1$  unit vector with the  $k(i)$ th element being one and the others zero,  $i = 1, 2, \dots, I$ .

*Proof:* See appendix C. ■

Theorem 1 verifies that  $\mathbf{u}_i$  is the separation vector of the  $i$ th coded signal if and only if  $\mathbf{u}_i$  satisfies (26). Note that unlike the traditional frequency-domain convolutive BSS algorithms, the proposed method does not lead to permutation indeterminacy among the recovered sources in different frequency bins. Denote  $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_I]$ , then one can use  $\mathbf{W} = \mathbf{U}^H \mathcal{H}$  to evaluate the performance of the proposed method.

**Remark 1:** According to Lemma 2, the rank of  $\mathbf{R}_{\tilde{\mathbf{x}}}(2)$  is  $IW + \bar{L} - I$ . Since  $\mathcal{H}$  is of full column rank, the rank of  $\bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(2) = \mathcal{H} \mathbf{R}_{\tilde{\mathbf{x}}}(2) \mathcal{H}^H$  is  $IW + \bar{L} - I$ . Therefore, there are  $JW - (IW + \bar{L} - I) = (J - I)W - \bar{L} + I$  orthogonal vectors that satisfy (26a).

**Remark 2:** Obviously, the rank of  $\bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(r_i)$  is equal to the rank of  $\mathbf{P}(r_i) \mathbf{Q}_{\tilde{\mathbf{s}}}(r_i)$ . According to Lemma 1 and its proof, the rank of  $\mathbf{P}(r_i) \mathbf{Q}_{\tilde{\mathbf{s}}}(r_i)$  equals  $I - 1$ . Thus, there are  $((J - I)W - \bar{L} + I) - (I - 1) = (J - I)$

$W - \bar{L} + 1$  orthogonal vectors that satisfy (26a) and (26b) at the same time.

**Remark 3:** It is important to notice that, in the subspace spanned by the above  $(J - I)W - \bar{L} + 1$  orthogonal vectors, there exists at least one nonzero vector that satisfies (26c). This is due to the fact that  $\mathcal{H}$  is a  $JW \times (IW + \bar{L})$  matrix and of full column rank, which implies that there are at most  $(J - I) \times W - \bar{L}$  orthogonal vectors  $\mathbf{v}$  that make  $\mathbf{v}^H \mathcal{H} = \mathbf{0}$ .

**Remark 4:** Although the sources are assumed to be mutually correlated in assumption 2, Lemma 1, Lemma 2 and Theorem 1 still hold when the sources are independent of each other. Hence, the proposed method can be used to deal with independent sources as well.

Based on the above lemmas and theorem, the proposed method is summarized as follows:

*Step 1:* Obtain  $\tilde{\mathbf{y}}_i(n)$  by (6) and  $\tilde{\mathbf{y}}(n)$  by (11).

*Step 2:* Compute

$$\mathbf{R}_{\tilde{\mathbf{y}}}(\tau) \approx \frac{1}{N} \sum_{n=0}^{N-1} \tilde{\mathbf{y}}(n) \tilde{\mathbf{y}}^H(n - \tau)$$

where  $N$  is the sample number of observations and the time lag  $|\tau| = 0, 1, \dots, W + L$ .

*Step 3:* Estimate the noise variance  $\sigma_w^2$  from  $\mathbf{R}_{\tilde{\mathbf{y}}}(0)$ , construct  $\bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(\tau)$  by (14), and obtain  $\bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(\tau)$  from (16).

*Step 4:* For each  $i = 1, 2, \dots, I$ ,

i) Find all the left singular vectors  $\xi_m$  corresponding to the zero singular value of  $\bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(2)$ . According to Remark 1,  $m = 1, 2, \dots, (J - I)W - \bar{L} + I$ .

ii) Compute  $\bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(r_i)$  by

$$\bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(r_i) = \sum_{\tau=-W-L}^{W+L} \bar{\mathbf{R}}_{\tilde{\mathbf{y}}}(\tau) r_i^{-\tau}.$$

iii) From the subspace spanned by  $\{\xi_m\}$ , find all of the orthogonal vectors  $\eta_n$  that satisfy (26b),<sup>1</sup> i.e.,  $\eta_n^H \bar{\mathbf{Q}}_{\tilde{\mathbf{y}}}(r_i) = \mathbf{0}$ . On the basis of Remark 2,  $n = 1, 2, \dots, (J - I)W - \bar{L} + 1$ .

iv) Among the subspace spanned by  $\{\eta_n\}$ , choose the vectors that also satisfy (26c) as  $\mathbf{u}_i$ .

*Step 5:* Estimate the coded signals by

$$\hat{x}_i = \mathbf{u}_i^H \tilde{\mathbf{y}}(n), \quad i = 1, 2, \dots, I.$$

*Step 6:* Recover the source signals  $s_1(n), \dots, s_I(n)$  from  $\hat{x}_1, \hat{x}_2, \dots, \hat{x}_I$  by (5), respectively.

#### IV. SIMULATIONS

This section presents experimental results to demonstrate the effectiveness of the proposed method in comparison with four existing algorithms, including two frequency-domain

<sup>1</sup>This can be done by following the approach in [22].

algorithms and two geometry-based algorithms. The precoder zeros are chosen as

$$r_i = \eta \exp^{j\theta_i}, \quad i = 1, 2, \dots, I \quad (27)$$

where  $\eta \in (0, 1)$ ,  $j = \sqrt{-1}$  and  $\theta_i$  is a random angle generated from uniform distribution  $\mathcal{U}[0, 2\pi]$ . Clearly, the selection of precoder zeros satisfies assumption A4).

The spatially correlated sources  $\mathbf{s}(n)$  are generated as follows. Firstly, let  $\epsilon_1(n), \epsilon_2(n), \dots, \epsilon_I(n)$  be  $I$  temporally white sequences that are generated randomly from normal distribution  $\mathcal{N}(0, 1)$ . It is clear that these  $I$  signals are independent of each other. Denote  $\mathbf{E}(n) = [\epsilon_1(n), \epsilon_2(n), \dots, \epsilon_I(n)]^T$ . Secondly, let  $\mathbf{C}$  be an  $I \times I$  symmetric matrix. Its diagonal entries are of value 1 and the absolute values of its off-diagonal entries are less than 1. Denote the eigenvalue decomposition of  $\mathbf{C}$  by  $[\mathbf{V}, \mathbf{D}] = \text{eig}(\mathbf{C})$ . Then the spatially correlated source signals can be obtained by

$$\mathbf{s}(n) = [s_1(n), s_2(n), \dots, s_I(n)]^T = (\mathbf{V}\mathbf{D}^{0.5}\mathbf{V}^H)\mathbf{E}(n).$$

Obviously, the covariance matrix  $\mathbf{R}_s$  of  $\mathbf{s}(n)$  equals  $\mathbf{C}$  in theory. In practice,  $\mathbf{R}_s$  can be computed by  $\mathbf{R}_s \approx \mathbf{s}(n)\mathbf{s}(n)^H/T$ , where  $T$  is the sample size of  $\mathbf{s}(n)$  and  $\mathbf{R}_s$  approximately equals  $\mathbf{C}$  if  $T$  is large enough. Clearly, the off-diagonal entries of  $\mathbf{R}_s$  (or  $\mathbf{C}$ ) indicate the degree of correlations among  $s_1(n), s_2(n), \dots, s_I(n)$ .

### A. PERFORMANCE OF CODED SIGNAL SEPARATION

As mentioned in Section III, the performance of separating the coded signals can be measured by analysing the structure of  $\mathbf{W} = \mathbf{U}^H\mathcal{H}$ . Here we use the mean interference rejection level (MIRL) defined as follows [28], [29]

$$\text{MIRL}_{\text{dB}} = 10\log_{10} \left( \frac{1}{I(I-1)} \sum_{i=1}^I \sum_{\substack{j=1 \\ j \neq k(i)}}^{I+L} \frac{E|(\mathbf{W}_{ij})|^2}{E|(\mathbf{W}_{ik(i)})|^2} \right).$$

In this paper, the MIRL is computed and averaged over 150 independent runs, each one with an independent realization of the channel and the sources.

Fig. 2 shows MIRL of the proposed method versus the signal-to-noise ratio (SNR) while four kinds of MIMO FIR systems are considered. In the first three systems, the channel has  $I = 2$  inputs and  $J = 3$  outputs, and the channel order is  $\mathbf{L} = [1, 2]$ ,  $\mathbf{L} = [2, 2]$ , and  $\mathbf{L} = [2, 3]$ , respectively. In the last system,  $I = 3, J = 4$  and  $\mathbf{L} = [1, 1, 2]$ . The slide-window width  $W$  is chosen as  $\bar{L} + 1$ , i.e.,  $\sum_{i=1}^I L_i + 1$ , the sample size is  $N = 50000$  and all the channel matrices are generated randomly from  $\mathcal{N}(0, 1)$ . In addition, the amplitude of precoder zeros in (27) are selected as  $\eta = 0.7$ . Furthermore, the sources transmitted in these four systems are mutually correlated with their correlation coefficients satisfying

$$\begin{cases} r_{ii}^s = 1, & i = 1, 2, \dots, I, \\ r_{ij}^s = 0.5, & i, j = 1, 2, \dots, I, i \neq j. \end{cases}$$

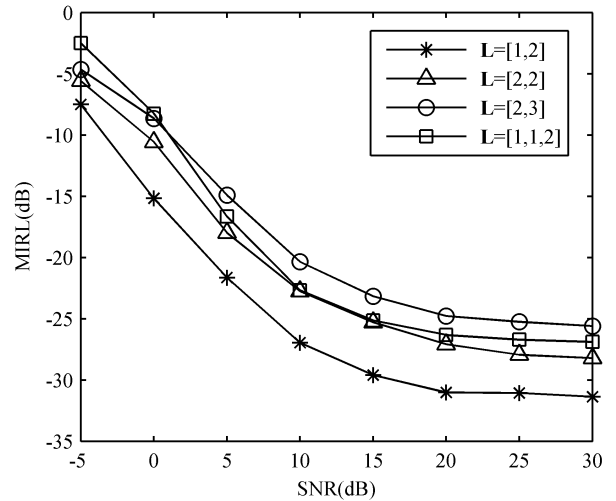


FIGURE 2. MIRL versus SNR when the sources are correlated.

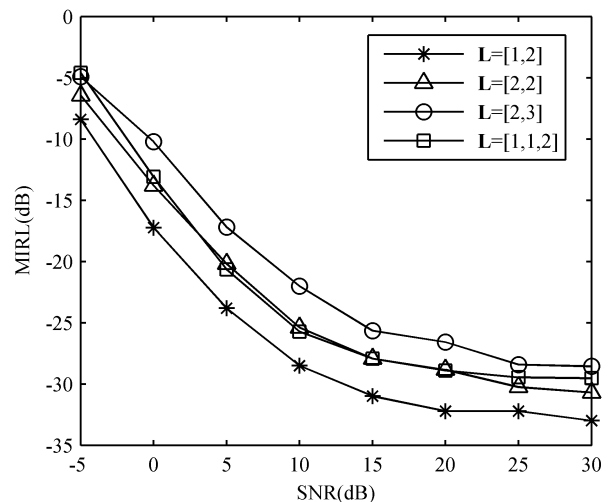


FIGURE 3. MIRL versus SNR when the sources are independent of each other.

From Fig. 2, it can be seen that a higher SNR yields a lower MIRL, i.e., better separation performance. Besides, satisfactory MIRL can be obtained even at low SNRs.

Fig. 3 illustrates the MIRL of the proposed method versus SNR under the same settings as in Fig. 2, except that the sources are independent of each other. Comparing Fig. 3 with Fig. 2, it is clear that the proposed method can completely deal with both spatially independent and correlated sources.

### B. PERFORMANCE OF SOURCE RECOVERY

In this subsection, we evaluate the source recovery performance of the proposed method, together with four existing algorithms. These algorithms are the JAD algorithm combined with alternating least squares optimization (abbreviated to ‘‘Alcobs’’) in [5], the parallel factor analysis (PARAFAC) based algorithm in [6], the projected successive projection algorithm (ProSPA) in [9] and the BCA algorithm in [21]. It should be noted that ‘‘Alcobs’’, ‘‘ProSPA’’ and

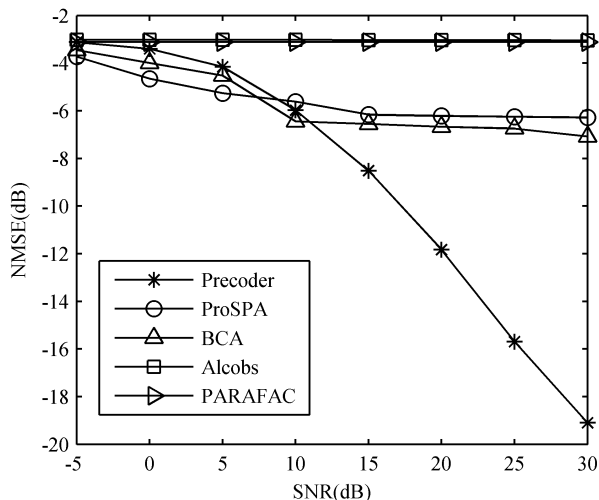


FIGURE 4. NMSE versus SNR when the sources are independent of each other.

“PARAFAC” require the sources to be uncorrelated or independent, while “BCA” is designed for both independent and dependent source separation but requires the sources to satisfy the weaker domain separability assumption.

The accuracy of the source recovery is evaluated by the normalized mean squared error (NMSE) defined as [30]

$$NMSE_{dB} = 10 \log_{10} \left( \frac{\min_{\Lambda_r} \sum_{n=0}^{T-1} \|\Lambda_r \hat{\mathbf{s}}(n) - \mathbf{s}(n)\|^2}{\sum_{n=0}^{T-1} \|\mathbf{s}(n)\|^2} \right)$$

where  $\hat{\mathbf{s}}(n) = [\hat{s}_1(n), \hat{s}_2(n), \dots, \hat{s}_I(n)]^T$  is the estimation of the source  $\mathbf{s}(n)$ ,  $\Lambda_r$  is a diagonal matrix obtained by mean square error. According to Theorem 1, the proposed method does not introduce permutation indeterminacy among the estimated sources, so only scaling indeterminacy needs to be taken into account in this NMSE criterion. In this paper, NMSE is computed and averaged over 150 independent realizations as well.

Fig. 4 shows the source recovery performance comparison of the proposed precoding-based method, labelled as “Precoder”, and the other four algorithms: “Alcobs”, “ProSPA”, “PARAFAC” and “BCA”. Here, we consider  $I = 2$  mutually independent sources and  $J = 3$  channel outputs. The channel is generated randomly from  $\mathcal{N}(0, 1)$  with channel order  $\mathbf{L} = [1, 2]$ . The slide-window width  $W = \bar{L} + 1$  and sample size  $N = 50000$ . The amplitude of precoder zeros in the proposed method is chosen as  $\eta = 0.7$  and SNR varies from  $-5$  dB to  $30$  dB. From Fig. 4, it can be seen that the proposed method outperforms the other four algorithms. First, “Alcobs” and “PARAFAC” almost do not work, and “ProSPA” does not perform well. This is partly because they are designed for quasi-stationary sources such as speeches, but the signals dealt here are communication signals which are usually supposed to be stationary. Second, the performance of “BCA” is not satisfactory. Although the sources are independent of each other and their ranges are bounded, i.e.,  $s_i(n) \in [\alpha_i, \beta_i]$ , where  $\alpha_i, \beta_i \in \mathbb{R}, \beta_i > \alpha_i$ , for

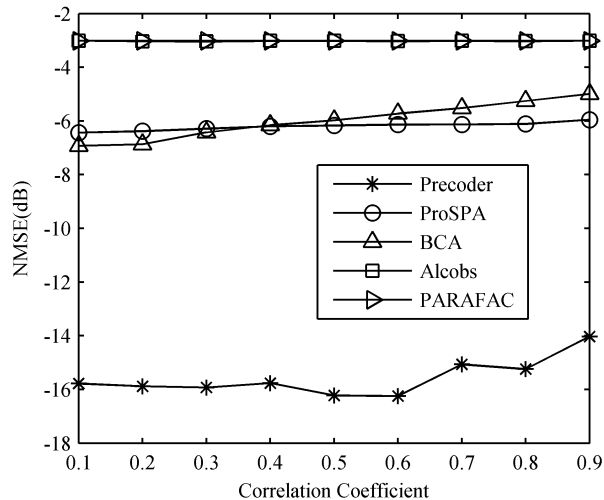


FIGURE 5. NMSE versus correlation coefficient for SNR= 25 dB.

$i = 1, 2, \dots, I$ , “BCA” further requires that a set made up by the samples of the sources should contain the vertices of its bounding hyper-rectangle [17]. Therefore, “BCA” is better at dealing with sources generated from uniform distribution rather than normal distribution.

To further investigate the source separation performances of the five algorithms in separating mutually correlated sources, two sources are considered with their correlation coefficient varying from 0.1 to 0.9, while SNR is chosen as 25 dB and other parameters are kept unchanged as in Fig. 4. The results are illustrated in Fig.5. One can see from Fig. 5 that as expected, “Alcobs”, “PARAFAC” and “ProSPA” perform poorly as they need the sources to be spatially uncorrelated or independent. “BCA” does not perform well either since the correlated sources might not meet the domain separability assumption required by “BCA”. In contrast, the proposed “Precoder” method performs much better. Furthermore, as the correlation coefficient approaches one, the performances of all the five algorithms deteriorate due to the similarity and co-linearity of the sources.

### C. DISCUSSION OF SELECTING SUITABLE PRECODER ZEROS

Now with the definitions of MIRL and NMSE, let us explore the impact of the selection of precoder zeros  $r_i = \eta \exp^{j\theta_i}$  on the performance of the proposed method. Table 1 shows the MIRL and NMSE versus  $\eta$  while  $\theta_i$  is generated randomly from  $\mathcal{U}[0, 2\pi]$ ,  $i = 1, 2, \dots, I$ . The numbers of input and output signals are  $I = 2$  and  $J = 3$ , respectively. The channel is generated randomly from  $\mathcal{N}(0, 1)$  with channel order  $\mathbf{L} = [1, 2]$ . The slide-window width  $W = \bar{L} + 1$ , the sample size  $N = 50000$ , SNR = 25 dB, and the correlation coefficient of the two sources is set as 0.5.

From Table 1, it can be seen that MIRL and NMSE are not varying linearly with  $\eta$ . On one hand, MIRL is not satisfactory when  $\eta < 0.5$ . There are two reasons. First, the reciprocals of the precoder zeros are involved while the

TABLE 1. MIRL and NMSE versus  $\eta$  of the precoders.

$\eta$	MIRL(dB)	NMSE(dB)
0.1	-1.2666	-4.6151
0.2	-1.3246	-4.6694
0.3	-2.3920	-4.7279
0.4	-4.7650	-6.0919
0.5	-13.6117	-10.5582
0.6	-23.4845	-14.9021
0.7	-27.8956	-15.7186
0.8	-30.4435	-14.9440
0.9	-31.0848	-13.0966

Z-transform of  $\tilde{\mathbf{R}}_{\tilde{\mathbf{y}}}(\tau)$  is computed in the process of coded signal separation. When the observations are contaminated by noise, the noise will be cumulated and enlarged, and the smaller the absolute value of  $r_i$  is, the severer the noise enlargement. Second, as mentioned before, when zero of any precoder gets very close to the origin, the condition number of the matrix made up by the row vectors of  $\mathbf{R}_{\tilde{\mathbf{x}}}(2)$  except its  $k(i)$ th row vectors,  $i = 1, 2, \dots, I$ , becomes greater, which will deteriorate the accuracy of the solution to (26a), and consequently make the MIRL unsatisfactory. On the other hand, the NMSE of source separation does not vary in consistent with MIRL while  $\eta$  is larger than 0.7. This is because the sources are recovered according to (5) where the noise is multiplied by the precoder zeros. Therefore, the larger the absolute values of the precoder zeros are, the more serious the noise is. In addition, since the precoder zeros are important markers to identify different sources, these zeros should not be too closed to each other. In this paper, we suggest the Euclidean distance between each two zeros be not less than 0.1.

V. CONCLUSION

In this paper, we propose a new precoding-based BSS method to separate mutually correlated sources mixed by MIMO FIR channels. In the proposed method, each source is preprocessed by an order-1 precoder prior to transmission. At the receiving end, the zeros of the precoders are used to mark and extract the corresponding sources, combined with the Z-domain features of the precoders and the SOS properties of the sources. It can deal with both independent and correlated source separation, which makes it a widely applicable approach to achieve convolutive BSS. Besides, the new method has lower computation complexity compared with traditional frequency-domain and time-domain convolutive

BSS algorithms. Simulation results show its superior performance over the existing algorithms.

Appendix A  
PROOF OF LEMMA 1

From (18) and (19),  $\mathbf{R}_{\tilde{\mathbf{s}}}(\tau)$  can be written as

$$\begin{bmatrix} E \{ \tilde{\mathbf{s}}_1(n) \tilde{\mathbf{s}}_1^H(n - \tau) \} & \cdots & E \{ \tilde{\mathbf{s}}_1(n) \tilde{\mathbf{s}}_I^H(n - \tau) \} \\ \vdots & \ddots & \vdots \\ E \{ \tilde{\mathbf{s}}_I(n) \tilde{\mathbf{s}}_1^H(n - \tau) \} & \cdots & E \{ \tilde{\mathbf{s}}_I(n) \tilde{\mathbf{s}}_I^H(n - \tau) \} \end{bmatrix}$$

Besides, for any  $i, j \in \{1, 2, \dots, I\}$ , the  $(i, j)$ th sub-matrix of  $\mathbf{R}_{\tilde{\mathbf{s}}}(\tau)$ , denoted as  $\mathbf{R}_{\tilde{\mathbf{s}}_{ij}}(\tau)$ , can be written in the form of (28), as shown at the bottom of this page.

Denoting  $\gamma_{ij} = W + \max\{L_i, L_j\}$ , it holds from (4) in assumption 2 that if  $|\tau| > \gamma_{ij} - 1$ , then  $\mathbf{R}_{\tilde{\mathbf{s}}_{ij}}(\tau) = \mathbf{0}$ . Therefore, it can be inferred that the Z-transform of  $\mathbf{R}_{\tilde{\mathbf{s}}_{ij}}(\tau)$  has the following form

$$\begin{aligned} \mathbf{Q}_{\tilde{\mathbf{s}}_{ij}}(z) &= \sum_{\tau = -\gamma_{ij} + 1}^{\gamma_{ij} - 1} \mathbf{R}_{\tilde{\mathbf{s}}_{ij}}(\tau) z^{-\tau} \\ &= \begin{bmatrix} r_{ij}^s & r_{ij}^s z & \cdots & r_{ij}^s z^{\gamma_{ij} - 1} \\ r_{ij}^s z^{-1} & r_{ij}^s & \cdots & r_{ij}^s z^{\gamma_{ij} - 2} \\ \vdots & \vdots & \ddots & \vdots \\ r_{ij}^s z^{-(\gamma_i - 1)} & r_{ij}^s z^{-(\gamma_i - 2)} & \cdots & r_{ij}^s z^{-(L_i - L_j)} \end{bmatrix}. \end{aligned} \tag{29}$$

Let

$$\begin{aligned} \mathbf{Q}_{\tilde{\mathbf{s}}_i}(z) &= [\mathbf{Q}_{\tilde{\mathbf{s}}_{i1}}(z), \mathbf{Q}_{\tilde{\mathbf{s}}_{i2}}(z), \dots, \mathbf{Q}_{\tilde{\mathbf{s}}_{iI}}(z)], \\ \mathbf{Q}_{\tilde{\mathbf{s}}}(\tau) &= [\mathbf{Q}_{\tilde{\mathbf{s}}_1}(z)^T, \mathbf{Q}_{\tilde{\mathbf{s}}_2}(z)^T, \dots, \mathbf{Q}_{\tilde{\mathbf{s}}_I}(z)^T]^T. \end{aligned}$$

Obviously, according to (29), the last  $\gamma_i - 1$  row vectors of  $\mathbf{Q}_{\tilde{\mathbf{s}}_i}(z)$  are linearly proportional to its first row vector, and the coefficients of proportionality are  $z^{-1}, z^{-2}, \dots, z^{-(\gamma_i - 1)}$ , respectively.

Denote

$$\begin{aligned} \mathbf{Q}_{\tilde{\mathbf{s}}_i}(z) &= [\mathbf{Q}_{\tilde{\mathbf{s}}_{i1}}(z)^T, \mathbf{Q}_{\tilde{\mathbf{s}}_{i2}}(z)^T, \dots, \mathbf{Q}_{\tilde{\mathbf{s}}_{iI}}(z)^T]^T, \\ \mathbf{Q}_{\tilde{\mathbf{s}}}(z) &= [\mathbf{Q}_{\tilde{\mathbf{s}}_1}(z), \mathbf{Q}_{\tilde{\mathbf{s}}_2}(z), \dots, \mathbf{Q}_{\tilde{\mathbf{s}}_I}(z)]. \end{aligned}$$

Similarly, the last  $\gamma_j - 1$  column vectors of  $\mathbf{Q}_{\tilde{\mathbf{s}}_j}(z)$  are linearly proportional to its first column vector, and the coefficients of proportionality are  $z, z^2, \dots, z^{\gamma_j - 1}$ , respectively.

Recall that  $k(i) = \sum_{l=0}^{i-1} \gamma_l + 1$ , therefore, the rank of  $\mathbf{Q}_{\tilde{\mathbf{s}}}(z)$  equals the rank of a matrix composed of its

$$\mathbf{R}_{\tilde{\mathbf{s}}_{ij}}(\tau) \triangleq E \{ \tilde{\mathbf{s}}_i(n) \tilde{\mathbf{s}}_j^H(n - \tau) \} = \begin{bmatrix} E \{ s_i(n) s_j^*(n - \tau) \} & \cdots & E \{ s_i(n) s_j^*(n - \gamma_j + 1 - \tau) \} \\ E \{ s_i(n - 1) s_j^*(n - \tau) \} & \cdots & E \{ s_i(n - 1) s_j^*(n - \gamma_j + 1 - \tau) \} \\ \vdots & \ddots & \vdots \\ E \{ s_i(n - \gamma_i + 1) s_j^*(n - \tau) \} & \cdots & E \{ s_i(n - \gamma_i + 1) s_j^*(n - \gamma_j + 1 - \tau) \} \end{bmatrix}. \tag{28}$$





$$\mathbf{c}_i^H \mathbf{P}(r_i) = [\underbrace{\varepsilon_1^{(1)}, 0, \dots, 0}_{\gamma_1-1}, \dots, \varepsilon_{i-1}^{(1)}, \underbrace{0, \dots, 0}_{\gamma_{i-1}-1}, \underbrace{0, 0, \dots, 0}_{\gamma_i}, \varepsilon_{i+1}^{(1)}, \underbrace{0, \dots, 0}_{\gamma_{i+1}-1}, \dots, \varepsilon_I^{(1)}, \underbrace{0, \dots, 0}_{\gamma_I-1}]. \quad (32)$$

“Sufficiency”: Denote  $\mathbf{u}_i^H \mathcal{H} = \mathbf{c}_i^H$ , where  $\mathbf{c}_i$  is an  $(IW + \bar{L}) \times 1$  vector. Since  $\mathbf{u}_i^H \bar{\mathbf{R}}_{\bar{\mathbf{y}}}(2) = \mathbf{0}$ , it holds from (16) that

$$\mathbf{u}_i^H \mathcal{H} \mathbf{R}_{\bar{\mathbf{x}}}(2) \mathcal{H}^H = \mathbf{0}.$$

Because  $\mathcal{H}$  is of full column rank, it has

$$\mathbf{u}_i^H \mathcal{H} \mathbf{R}_{\bar{\mathbf{x}}}(2) = \mathbf{c}_i^H \mathbf{R}_{\bar{\mathbf{x}}}(2) = \mathbf{0}. \quad (30)$$

According to c2) in Lemma 2, in order to satisfy (30),  $\mathbf{c}_i$  must have the following structure:

$$\mathbf{c}_i = [\varepsilon_1, \underbrace{0, \dots, 0}_{\gamma_1-1}, \varepsilon_2, \underbrace{0, \dots, 0}_{\gamma_2-1}, \dots, \varepsilon_I, \underbrace{0, \dots, 0}_{\gamma_I-1}]^T \quad (31)$$

where  $\varepsilon_i$  ( $i = 1, 2, \dots, I$ ) stands for some unknown value.

Moreover, when  $\mathbf{u}_i^H \bar{\mathbf{Q}}_{\bar{\mathbf{y}}}(r_i) = \mathbf{0}$ , according to (21),

$$\mathbf{u}_i^H \bar{\mathbf{Q}}_{\bar{\mathbf{y}}}(r_i) = \mathbf{u}_i^H \mathcal{H} \mathbf{P}(r_i) \mathbf{Q}_{\bar{\mathbf{s}}}(r_i) \mathbf{P}((r_i^*)^{-1})^H \mathcal{H}^H = \mathbf{0}.$$

As previously mentioned,  $\mathbf{P}((r_i^*)^{-1})$  is a diagonal matrix with all diagonal entries being nonzero, thus the above equation leads to,

$$\mathbf{u}_i^H \mathcal{H} \mathbf{P}(r_i) \mathbf{Q}_{\bar{\mathbf{s}}}(r_i) = \mathbf{c}_i^H \mathbf{P}(r_i) \mathbf{Q}_{\bar{\mathbf{s}}}(r_i) = \mathbf{0}.$$

Because the  $i$ th diagonal sub-matrix  $\mathbf{P}_i(r_i)$  in  $\mathbf{P}(r_i)$  is a zero matrix and the remaining diagonal entries of  $\mathbf{P}(r_i)$  are nonzero, based on (31),  $\mathbf{c}_i^H \mathbf{P}(r_i)$  can be written in the form of (32), as shown at the top of this page, in which  $\varepsilon_l^{(1)}$  ( $l = 1, \dots, i-1, i+1, \dots, I$ ) stands for some unknown value. Besides, if  $\varepsilon_l$  in  $\mathbf{c}_i$  is nonzero, then  $\varepsilon_l^{(1)}$  in  $\mathbf{c}_i^H \mathbf{P}(r_i)$  is nonzero, and vice versa.

As shown in the proof of Lemma 1, all  $k(l)$ th row vectors of  $\mathbf{Q}_{\bar{\mathbf{s}}}(r_i)$ ,  $l = 1, 2, \dots, I$ , compose a full row rank matrix. Thus in order to make  $\mathbf{c}_i^H \mathbf{P}(r_i) \mathbf{Q}_{\bar{\mathbf{s}}}(r_i) = \mathbf{0}$ , all the unknown  $\varepsilon_l^{(1)}$  ( $l = 1, \dots, i-1, i+1, \dots, I$ ) must equal zero, which means  $\varepsilon_l = 0$  when  $l \neq i$ . Therefore,  $\mathbf{c}_i$  can be further written as

$$\mathbf{c}_i = [\underbrace{0, \dots, 0}_{\gamma_1}, \dots, \varepsilon_i, \underbrace{0, \dots, 0}_{\gamma_i-1}, \dots, \underbrace{0, \dots, 0}_{\gamma_I}]^T.$$

That is to say, except the  $k(i)$ th element, all other elements in  $\mathbf{c}_i$  are zero.

Furthermore, when  $\mathbf{u}_i^H \bar{\mathbf{R}}_{\bar{\mathbf{y}}}(0) \mathbf{u}_i \neq 0$ , substituting (16) into this inequality yields

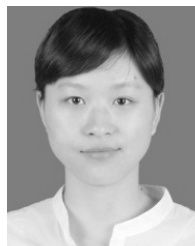
$$\begin{aligned} \mathbf{u}_i^H \bar{\mathbf{R}}_{\bar{\mathbf{y}}}(0) \mathbf{u}_i &= \mathbf{u}_i^H \mathcal{H} \mathbf{R}_{\bar{\mathbf{x}}}(0) \mathcal{H}^H \mathbf{u}_i \\ &= \mathbf{c}_i^H \mathbf{R}_{\bar{\mathbf{x}}}(0) \mathbf{c}_i \\ &= \varepsilon_i \times \varepsilon_i^* \times r_{ii}^x(0) \\ &= |\varepsilon_i|^2 \times r_{ii}^x(0) \neq 0. \end{aligned}$$

According to (22a),  $r_{ii}^x(0) \neq 0$ . Thus  $\varepsilon_i \neq 0$ , which means  $\mathbf{c}_i = \varepsilon_i \mathbf{e}_{k(i)}$ , i.e.,  $\mathbf{u}_i^H \mathcal{H} = \varepsilon_i^H \mathbf{e}_{k(i)}^H$ . The proof of Theorem 1 is completed.

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