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The Ghost Operator and Its Applications to Reveal the Physical Meaning of Reactive Power for Electrical and Mechanical Systems and Others

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ABSTRACT In science and engineering, many concepts are introduced without a clear physical meaning, e.g., imaginary numbers in mathematics and reactive power in electrical engineering. In this paper, a new operator, coined *the ghost operator g*, is introduced to physically construct the ghost of a system. It satisfies $g^2 = -1$ but is different from the imaginary operator. With the help of the port-Hamiltonian systems theory, it is proved that the ghost of a system behaves exactly in the opposite way as the original system. This brings the ghost of a system into reality and paves the way to reveal the physical meaning of some imaginary concepts. Two applications are given as an example. One is to reveal the physical meaning of reactive power in electrical systems: it is the (real) power of the ghost system, which leads to a significantly simplified instantaneous power theory called the ghost power theory. The other is to define the reactive power for mechanical systems to complete the electrical–mechanical analogy. As a matter of fact, the resulting instantaneous power theory is generic and applicable to any dynamic system that can be described by the port-Hamiltonian model.

INDEX TERMS Ghost operator, imaginary operator, reactive power, power factor, instantaneous power theory, ghost power theory, electrical-mechanical analogy, mechanical systems, port-Hamiltonian systems, physical meaning, quantum mechanics.

I. INTRODUCTION

In electrical engineering, real power and reactive power are two well-known concepts. The physical meaning of the real power is very clear: it represents the real power (energy) that is consumed. But what is the physical meaning of the reactive power? It is a mathematical concept and has different interpretations; see e.g. [1]–[4] and the references therein. It reflects the quantity of power due to the phase mismatch between the current and the voltage caused by reactive elements like capacitors and inductors. The reactive power is conventionally defined as a component of the instantaneous (real) power. It is sometimes regarded as the power oscillating in the system. However, for balanced three-phase systems there is no oscillating power; for single-phase resistive systems there is no reactive power although the power pulsates. Many attempts have been made to reveal the physical meaning of reactive power, e.g., by using the vector product [1], the Clarke transformation [2], the Poynting theorem [3], and the newly-introduced *mno* transformation [4]. To the best knowledge of the author, the physical meaning of reactive power is still not well established. It is imaginary.

Actually, this phenomenon is quite common in science and engineering: many imaginary concepts are introduced without a physical meaning. Reactive power is just one of them. Another famous example is the imaginary number, as the name itself indicates. An imaginary number is a complex number that can be written as a real number multiplied by the imaginary unit j, which is defined by $j^2 = -1$. Imaginary numbers do not exist physically but have been playing a fundamental role in science and engineering. The searching for the physical meaning of these imaginary concepts has been ongoing for years. The objective of this paper is to reveal the physical meaning of the reactive power, not just for electrical systems but also for mechanical systems.

In this paper, a new mathematical operator called *the ghost operator*, in short the g-operator, is introduced to shift the phase of a sine or cosine function by 90° leading. The ghost operator is similar to the imaginary operator because it satisfies $g^2 = -1$ but it does not return imaginary numbers. Here, by $g^2 = -1$, it means applying the ghost operator g twice to a sine or cosine function returns the opposite of the sine or cosine function. The ghost operator is then applied to *physically construct* the ghost of a signal and furthermore the ghost of a system. Interestingly and surprisingly, by adopting the port-Hamiltonian systems theory [5], it is proven that the ghost of a system behaves exactly in the opposite way as the original system if the input to the ghost is the ghost of the input to the original system. A significant property of the g-operator is that the operator $\begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$ transforms the signal and its ghost into themselves. The ghost operator is then applied to reveal the physical meaning of reactive power: the instantaneous reactive power of a system is the instantaneous (real) power of its ghost system. This leads to an instantaneous power theory, called the ghost power theory for the sake of ease reference. It is potentially very useful for power electronics-enabled future power systems [6].

It is well known that electrical systems and mechanical systems are dual to each other [7], [8]. While the role and importance of reactive power is well recognized for electrical systems, it is rarely mentioned for mechanical systems. There are only very limited attempts in the literature, e.g. [9], [10], trying to understand the role of reactive power and power factor in mechanical systems. Following the understanding of the physical meaning of reactive power for electrical systems, the electrical-mechanical analogy is reviewed and a missing term, the reactive power, is identified. The ghost operator is then applied to define the reactive power for mechanical systems, completing the electrical-mechanical analogy.

The rest of the paper is organized as follows. In Section II, the ghost operator is introduced at first, followed by the physical construction of the ghost of a signal and the ghost of a system. The behavior of the ghost of a system is then characterized according to the port-Hamiltonian systems theory. In Section III, the ghost operator is applied to reveal the physical meaning of the reactive power of electrical systems, leading to a simple instantaneous power theory. In Section IV, the electrical-mechanical analogy is reviewed and then the ghost operator is applied to define the reactive power for mechanical systems. Conclusions are made in Section V, together with discussions on other potential applications of the ghost operator. It is pointed out that the instantaneous power theory is actually generic and applicable to any dynamic system that can be described by the port-Hamiltonian model.

II. PHYSICAL CONSTRUCTION OF GHOST SYSTEMS

A. THE GHOST OPERATOR

Definition 1: The ghost operator, *in short the g-operator, is coined to describe the operator that shifts the phase of a sine or cosine function by $\frac{\pi}{2}$ rad leading.*

Lemma 2: The ghost operator satisfies $g \sin\theta = \cos\theta$, $g\cos\theta = -\sin\theta$ and $g^2 = -1$.

Proof: It is straightforward to see that

$$g\sin\theta = \sin(\theta + \frac{\pi}{2}) = \cos\theta,$$
$$g\cos\theta = \cos(\theta + \frac{\pi}{2}) = -\sin\theta.$$

Applying the g-operator once more, then

$$g^2\sin\theta = g\cos\theta = \cos(\theta + \frac{\pi}{2}) = -\sin\theta,$$
$$g^2\cos\theta = -g\sin\theta = -\sin(\theta + \frac{\pi}{2}) = -\cos\theta.$$

Hence, $g^2 = -1$. This completes the proof. ∎

Note that by $g\sin\theta$ it means "applying" the ghost operator g to the function $\sin\theta$, instead of "multiplying" it with $\sin\theta$, and by $g^2 = -1$ it means applying the ghost operator g to a sine or cosine function twice returns the opposite of the sine or cosine function. Other notation involving the ghost operator g is similar. Apparently, $g^3 = -g$ and $g^4 = 1$. It seems that the ghost operator is very similar to the commonly-used imaginary operator j, which satisfies $j^2 = -1$, but they are actually very different. The ghost operator is applicable to a sine or cosine function but the imaginary operator is applicable to any (complex) number. Moreover, applying the g-operator to a sine or cosine function always returns a real function (value) but applying the imaginary operator to a real number returns an imaginary number. Fig. 1 illustrates the operation of the two operators. When both operators are applied to $\cos\theta$, respectively, the imaginary operator returns $j\cos\theta$ but the ghost operator returns $g\cos\theta = -\sin\theta$. When both are applied to $\sin\theta$, respectively, the imaginary operator returns $j\sin\theta$ but the ghost operator returns $g\sin\theta = \cos\theta$.

B. THE GHOST SIGNAL

Without loss of generality, for a sinusoidal signal

$$e = E\sin(\omega t + \phi),$$

its ghost signal is defined as

$$e_g = ge = Eg\sin(\omega t + \phi) = E\cos(\omega t + \phi).$$

It leads the signal e by 90°.

Lemma 3: The signal e and its ghost e_g satisfy

$$\begin{bmatrix} e_g \\ e \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} e_g \\ e \end{bmatrix} \quad or \quad \begin{bmatrix} e \\ e_g \end{bmatrix} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \begin{bmatrix} e \\ e_g \end{bmatrix}.$$

Proof: It is straightforward to show these. ∎

In other words, $\begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$ are identity operators, which transform the signal e and its ghost e_g into themselves without any change. This is a significant property because the eigenvalues of a skew-symmetric matrix always appear in pairs $\pm\lambda$ (plus an unpaired 0 eigenvalue in the odd-dimensional case) and the nonzero eigenvalues of a real skew-symmetric matrix are all purely imaginary. The fact that the eigenvalues of the skew-symmetric operators $\begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$ and

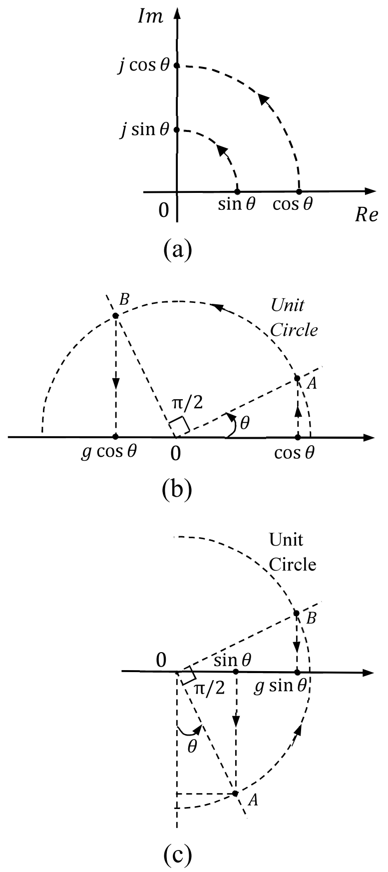

FIGURE 1. Illustrations of the imaginary operator and the ghost operator. (a) The imaginary operator applied to cos θ and sin θ. (b) The ghost operator applied to cos θ. (c) The ghost operator applied to sin θ.

$\begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$ are all equal to 1 may open a door for many new applications and deserve to be further explored.

C. THE GHOST SYSTEM

Systems take signals in the form of inputs and generate signals as outputs. There are many ways to describe a system. In this paper, the port-Hamiltonian framework is adopted.

Port-Hamiltonian systems theory [5] offers a systematic mathematical framework for structural modeling, analysis and control of complex networked multi-physics systems with lumped and/or distributed parameters. It combines the historical Hamiltonian modeling approach in geometric mechanics [11]–[13] and the port-based network modeling approach in electrical engineering [14]–[16], via geometrically associating the interconnected network with a Dirac structure [5], which is power-conserving. The Hamiltonian dynamics is defined with respect to the Dirac structure *and* the Hamiltonian representing the total stored energy. Port-Hamiltonian systems are open dynamical systems and interact/interconnect with their environment through ports.

For a dynamical system Z, if (i) there are no algebraic constraints between the state variables, (ii) the interconnection port power variables can be split into input and output variables, and (iii) the resistive structure is linear and of the input-output form, then the system Z can be described in the usual input u-state x-output y format [17] as

$$\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + G(x)u, \tag{1}$$

$$y = G^T(x)\frac{\partial H}{\partial x}(x), \tag{2}$$

where $x \in \mathcal{R}^{n\times 1}$ is the state vector, and u and $y \in \mathcal{R}^{m\times 1}$ are the input and the output, $H(x)$ is the Hamiltonian representing the total energy of the system and $\frac{\partial H(x)}{\partial x} \in \mathcal{R}^{n\times 1}$ is its gradient, $J(x) = -J^T(x)$ is a skew-symmetric matrix representing the network structure, $R(x)$ is a positive semi-definite symmetric matrix representing the resistive elements of the system, and $G(x)$ is the input matrix. All these matrices depend smoothly on the state x. Note that, the Hamiltonian $H(x)$ is not necessarily non-negative nor bounded from below.

In general, the input to a system can be arbitrary but in this paper it is assumed that the input u is periodic and hence can be described by the sum of a series of sinusoidal signals. This covers a wide range of engineering systems.

The time-derivative of the Hamiltonian $H(x)$ is

$$\frac{dH}{dt}(x(t)) = -\frac{\partial^T H}{\partial x}(x)R(x)\frac{\partial H}{\partial x}(x) + y^T u, \tag{3}$$

which characterizes the power conservation/balance property of port-Hamiltonian systems. The product $y^T u$ is called the supply rate and has the unit of power. As a result, the Hamiltonian always satisfies

$$H(x(t)) \leq H(x(0)) + \int_0^t y^T u dt \tag{4}$$

because of the dissipated energy associated with $R(x)$. Moreover, if the Hamiltonian $H(x)$ is bounded from below by $C > -\infty$, then the system is passive [18] with the non-negative storage function being $H_s(x) = H(x) - C$; the system is lossless if $\frac{dH}{dt}(x(t)) = y^T u = 0$ [5].

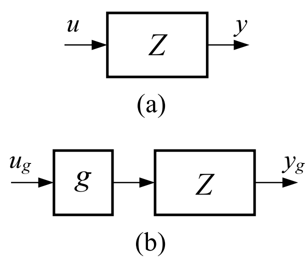

FIGURE 2. The system pair that consists of the original system and its ghost. (a) the original system Z. (b) the ghost system Z_g.

Definition 4: The ghost of a system is an exact copy of the system cascaded with the ghost operator at its input.

The system Z described in (1)-(2) and its corresponding ghost Z_g are illustrated in Fig. 2; both together form a system pair (Z, Z_g). Denote the state of the ghost system Z_g as x_g and the output as y_g. Then the following result holds.

Lemma 5: If the input to the ghost system Z_g is the ghost u_g of the input u to the original system Z, then the ghost state x_g is exactly the opposite of the system state x, i.e., $x_g = -x$, and the ghost output is exactly the opposite of the system output, i.e., $y_g = -y$. Moreover, if the system Z is passive, then its ghost Z_g is passive as well.

Proof: Since the ghost contains an exact copy of the system Z, its Hamiltonian can be chosen the same as H. The state equation of the ghost system satisfies

$$\begin{aligned}\dot{x}_g &= (J-R)\frac{\partial H}{\partial x_g} + Ggu_g \\ &= (J-R)\frac{\partial H}{\partial x_g} - Gu, \end{aligned} \tag{5}$$

where the property $g^2 = -1$ from Lemma 2 is invoked. Multiplying -1 to both sides, then

$$-\dot{x}_g = (J-R)\frac{\partial H}{\partial(-x_g)} + Gu.$$

Substituting $x_g = -x$ recovers the state equation of the system given in (1). Note that the Hamiltonian H satisfies $H(x_g) = H(-x) = H(x)$ so there is no need to differentiate it for the system and its ghost.

Substituting $x_g = -x$ into the output equation of the ghost

$$y_g = G^T\frac{\partial H}{\partial x_g},$$

then

$$y_g = -G^T\frac{\partial H}{\partial(-x_g)} = -y. \tag{6}$$

If the system Z is passive, then H can also be chosen as the storage function of the ghost system as well. According to (5) and (6), the time derivative of the storage function is

$$\begin{aligned}\dot{H} &= \frac{\partial^T H}{\partial x_g}\dot{x}_g \\ &= -\frac{\partial^T H}{\partial x_g}R\frac{\partial H}{\partial x_g} + \frac{\partial^T H}{\partial x_g}Ggu_g \\ &= -\frac{\partial^T H}{\partial x_g}R\frac{\partial H}{\partial x_g} + y_g^T(-u) \\ &\le -y_g^T u = y^T u,\end{aligned}$$

because R is positive semi-definite and $\frac{\partial^T H}{\partial x_g}J\frac{\partial H}{\partial x_g} = 0$. According to the definition of passive systems, the ghost system is passive as well. It has the same supply rate as the system Z. This completes the proof. ■

Remark 6: The ghost system behaves symmetrically (or oppositely) as the original system with respect to the origin but does not exist in reality and, hence, the name.

III. APPLICATION TO REVEAL THE PHYSICAL MEANING OF REACTIVE POWER IN ELECTRICAL SYSTEMS

Assume that the system Z described in (1)-(2) is an AC electrical system with the input voltage u and the output current i, i.e., $y = i$. Then the supply rate to the system Z is the instantaneous power

$$p = y^T u = i^T u. \tag{7}$$

For a single-phase system with

$$u = \sqrt{2}U\sin(\omega t), \qquad i = \sqrt{2}I\sin(\omega t - \phi),$$

where U and I represent the root-mean-square (rms) values of the voltage and current, respectively, there is

$$p = UI\cos\phi - UI\cos(2\omega t - \phi), \tag{8}$$

which can be rewritten as

$$p = UI\cos\phi(1-\cos(2\omega t)) - UI\sin\phi\sin(2\omega t).$$

The first term $UI\cos\phi(1-\cos(2\omega t))$ is non-negative and the second term $UI\sin\phi\sin(2\omega t)$ has a zero mean over the period $\tau = \frac{2\pi}{\omega}$, both oscillating at twice the frequency. The average value of p over one period τ is the same as the average value of the first term over one period, which is known as the active power P, i.e.,

$$P = \frac{1}{\tau}\int_0^\tau pdt = \frac{1}{\tau}\int_0^\tau i^T udt = UI\cos\phi.$$

It is also often called the real power or the average power. The maximum of the second term $UI\sin\phi\sin(2\omega t)$ is defined as the reactive power Q, i.e,

$$Q = UI\sin\phi.$$

The physical meaning of the real power has been very clear: it is the power that is actually consumed or does the real work. However, the physical meaning of the reactive power is not clear and it is just a mathematical formulation. It is known as an unwanted but unavoidable part of AC electric circuits, which is often regarded as the oscillating power in the system. However, this is not generally true [2]. For example, for balanced three-phase systems, the (real) power p is not oscillating.

Now, the puzzle is solved. The reactive power is the (real) power of the ghost system Z_g!

Indeed, the instantaneous power of the ghost system is the product of its voltage and current given by

$$q = u_g^T y_g = u_g^T i_g = i_g^T u_g = -i^T u_g, \tag{9}$$

which is

$$\begin{aligned} q &= -i^T u_g \\ &= -\sqrt{2}I\sin(\omega t - \phi) \times \sqrt{2}U\cos(\omega t) \\ &= UI\sin\phi - UI\sin(2\omega t - \phi) \end{aligned} \tag{10}$$

Its average over one period τ is the average reactive power

$$Q = \frac{1}{\tau}\int_0^\tau qdt = -\frac{1}{\tau}\int_0^\tau i^T u_g dt = UI\sin\phi.$$

For balanced three-phase systems, the oscillating terms in (8) and (10) all disappear, and the instantaneous real and reactive power are all equal to the average real and reactive power, respectively.

Comparing (8) and (10), it is easy to see that

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}.$$

Moreover, there is

$$\begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix}.$$

This is a very interesting result: $P = gQ$ and $Q = -gP$. Once again, it is shown that $\begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$ is an identity operator.

Actually, the reactive power is widely known as the imaginary power. The ghost system does not exist in reality so its power is of course imaginary. Since the definition of reactive power is now dual to the real power, it is applicable to systems under different scenarios like the real power: with any number of phases and any number of harmonics, and without any intermediate transformation.

Assume that, for a general system Z, the voltage and current are, respectively,

$$u = \Sigma_n \sqrt{2} U_n \sin(n\omega t - \alpha_n),$$
$$i = \Sigma_m \sqrt{2} I_m \sin(m\omega t - \beta_m).$$

The corresponding ghost voltage is

$$u_g = gu = \Sigma_n \sqrt{2} U_n \cos(n\omega t - \alpha_n).$$

Then, the instantaneous real power and reactive power are, respectively,

$$\begin{aligned} p &= i^T u \\ &= \Sigma_m \sqrt{2} I_m \sin(m\omega t - \beta_m) \cdot \Sigma_n \sqrt{2} U_n \sin(n\omega t - \alpha_n), \\ q &= -i^T u_g \\ &= -\Sigma_m \sqrt{2} I_m \sin(m\omega t - \beta_m) \cdot \Sigma_n \sqrt{2} U_n \cos(n\omega t - \alpha_n). \end{aligned}$$

Integrating both over one period, then the average real power and reactive power can be obtained as

$$P = \Sigma_n U_n I_n \cos\phi_n,$$
$$Q = \Sigma_n U_n I_n \sin\phi_n,$$

where $\phi_n = \beta_n - \alpha_n$, because the integration of the cross-frequency terms with $m \neq n$ over one period is 0. This is consistent with the conventional definition for reactive power (and real power) [19]. The apparent power S is defined to reflect the capacity of the system as

$$S = UI,$$

where $U = \sqrt{\Sigma_n U_n^2}$ and $I = \sqrt{\Sigma_m I_m^2}$ are the rms values of the voltage and the current, respectively. The difference between S^2 and P^2+Q^2 is then characterized by the distortive power D, also called the harmonic power, via

$$S^2 = P^2 + Q^2 + D^2.$$

Finally, the power factor of the system is defined as $\frac{P}{S}$ to reflect the utilization of the capacity.

What is described above is actually an instantaneous power theory for AC electrical systems that is significantly simpler than existing power theories, e.g. the commonly-used instantaneous $p - q$ power theory [2]. Simplicity is beauty. For the convenience of future references, this power theory is called the ghost power theory. Since the control of power electronic converters heavily relies on the accurate calculation of real and reactive power [4], [20], [21], the ghost power theory is expected to play an important role in future power electronics-enabled autonomous power systems [6], [22].

IV. APPLICATION TO COMPLETE THE ELECTRICAL-MECHANICAL ANALOGY

Voltage, current, flux linkage and charge are fundamental concepts in electrical systems. The voltage is defined as the change of the flux linkage and the current is defined as the change of the charge. The other relationships among these four concepts are then characterized by the four basic electric elements: resistor (voltage ∼ current), inductor (flux linkage ∼ current), capacitor (charge ∼ voltage) and memristor (flux linkage ∼ charge) [23]. The term impedance is defined in the frequency domain as the ratio between the voltage and the current, with its inverse called admittance. The term power is defined as the product of the voltage and the current in the time domain, with its average called the real power. For AC electrical systems, there is also the term reactive power as discussed in the previous section.

TABLE 1. The Electrical-Mechanical Analogy.

Electrical Systems	Mechanical Systems	
	Translational	Rotational
Resistor	Damper	
Inductor	Spring	
Capacitor	Inerter (mass)	Moment of inertia
Flux linkage	Displacement	Angle
Voltage	Velocity	Angular velocity
Current	Force	Torque
Charge	Momentum	Angular momentum
Impedance	Impedance	
Admittance	Admittance	
Real Power	(Real) Power	
Reactive Power	? *(Reactive Power)*	

It is well known that electrical systems and mechanical systems are analogous [7], [8], [24]. The fundamental concepts in both fields are summarized in Table 1 to demonstrate this duality. Here, the force-current analogy, which has led to the discovery of the two-terminal mechanical device (called the inerter) corresponding to the capacitor [25], instead of the commonly-used force-voltage analogy is adopted. Most of the dual concepts in Table 1 are well known, except the charge-momentum and flux linkage-displacement pairs. According to the Newton's second law, the rate of change of the momentum of a particle is proportional to the force acting on it, which makes the momentum dual to the charge in electric systems. For rotational mechanical systems, the corresponding term is the angular momentum. As to the flux linkage-displacement analogy, it is clear from the fact that the derivative of the flux linkage is the voltage and the derivative of the displacement is the velocity. For rotational systems, the corresponding term is the angle. Note that the fourth electric element, the memristor that describes the relationship between the charge and the flux linkage, is not included in the table. As a side note, for the sake of completeness of the electrical-mechanical analogy, there should exist a mechanical element that describes the relationship between the momentum and the displacement, corresponding to the memristor. However, no known mechanical device exists and efforts should be made to identify this device. Anyway, this is not the focus of the paper so it is not discussed further. The term impedance is defined as the ratio of velocity to force for translational systems and as the ratio of angular velocity to torque for rotational systems. Note that this is consistent with [24] and [25] but is not with those adopting the force–voltage analogy. The inverse of impedance is called admittance. As a result, a mechanical system can be analyzed or synthesized in the same way as its dual electrical system, if one more term — the reactive power — is properly defined.

For mechanical systems, the term of work, which is the integral of power, is used more often than the term power. For translational systems, the work done is the product of the force with the displacement; for rotational systems, the work done is the product of the torque with the angle. This leads to the power defined as the product of the force and velocity for translational systems and as the product of the torque and the angular velocity for rotational systems. Apparently, the power defined in this way represents the power actually consumed, i.e., the real power.

For AC electrical systems, there are real power and reactive power as described in the previous section. However, for mechanical systems, there does not exist a well-accepted way to define the reactive power. Actually, there are only very limited attempts trying to understand the role of reactive power and power factor in mechanical systems, e.g. [9], [10]. Here, the reactive power in mechanical systems can be defined dually as the real power, following the electrical-mechanical analogy and the newly-introduced ghost operator.

One barrier for this might be due to the fact that mechanical motions are caused by forces and torques, which are dual to current sources, but electrical motions are often caused by voltage sources. In order to better understand reactive power in mechanical systems, consider the case of an inductor L driven by a current source $i = -\sqrt{2}I\sin(\omega t - \frac{\pi}{2}) = \sqrt{2}I\cos(\omega t)$ at first. The voltage induced is

$$u = L\frac{di}{dt} = -\sqrt{2}I\omega L\sin(\omega t) = -\sqrt{2}\omega LI\sin(\omega t),$$

and its ghost is

$$u_g = gu = -\sqrt{2}\omega LI\cos(\omega t).$$

According to (9), the (instantaneous) reactive power of the system is the power of its ghost given by

$$\begin{aligned} q &= i_g^T u_g = -i^T u_g \\ &= \sqrt{2}I\cos(\omega t)\times\sqrt{2}\omega LI\cos(\omega t) \\ &= \omega LI^2 - \omega LI^2\sin(2\omega t), \end{aligned} \tag{11}$$

with the average reactive power being

$$Q = \omega LI^2.$$

This is consistent with the value obtained from the conventional way.

Now, consider a rigid body Z with the moment of inertia J_m driven by the torque $y = \sqrt{2}T\cos(\omega t)$, as described in Fig. 2(a). The resulting angular velocity is $u = \sqrt{2}\frac{T}{\omega J_m}\sin(\omega t)$, with its ghost angular velocity being $u_g = gu = \sqrt{2}\frac{T}{\omega J_m}\cos(\omega t)$. As a result, dual to (9), the instantaneous reactive power is the power of the ghost illustrated in Fig. 2(b), given by the product of $y_g = -y$ and the ghost angular velocity u_g as

$$\begin{aligned} q &= y_g^T u_g = -y^T u_g \\ &= -\sqrt{2}T\cos(\omega t)\times\sqrt{2}\frac{T}{\omega J_m}\cos(\omega t) \\ &= -\frac{T^2}{\omega J_m} + \frac{T^2}{\omega J_m}\sin(2\omega t), \end{aligned}$$

with its average reactive power being

$$Q = -\frac{T^2}{\omega J_m}.$$

The negative sign indicates that the rigid body "generates" reactive power, similar to the case of a capacitor. The second example to be considered is a spring with stiffness K subject to the force $y = \sqrt{2}F\cos(\omega t)$. The resulting velocity is $u = -\sqrt{2}\frac{\omega F}{K}\sin(\omega t)$, with its ghost velocity being $u_g = gu = -\sqrt{2}\frac{\omega F}{K}\cos(\omega t)$. As a result, dual to (9), the instantaneous reactive power is the power of the ghost given by the product of $y_g = -y$ and the ghost velocity u_g as

$$\begin{aligned} q &= y_g^T u_g = -y^T u_g \\ &= -\sqrt{2}F\cos(\omega t)\times(-\sqrt{2}\frac{\omega F}{K}\cos(\omega t)) \\ &= \frac{\omega F^2}{K} - \frac{\omega F^2}{K}\sin(2\omega t), \end{aligned}$$

with its average reactive power being

$$Q = \frac{\omega F^2}{K}.$$

The positive sign indicates that the spring "consumes" reactive power, similar to the case of an inductor.

Because of the electrical-mechanical analogy, the two examples illustrated above can be generalized to any mechanical system. That is, the reactive power of a mechanical system is the power of its ghost system, i.e.,

$$q = y_g^T u_g = -y^T u_g,$$

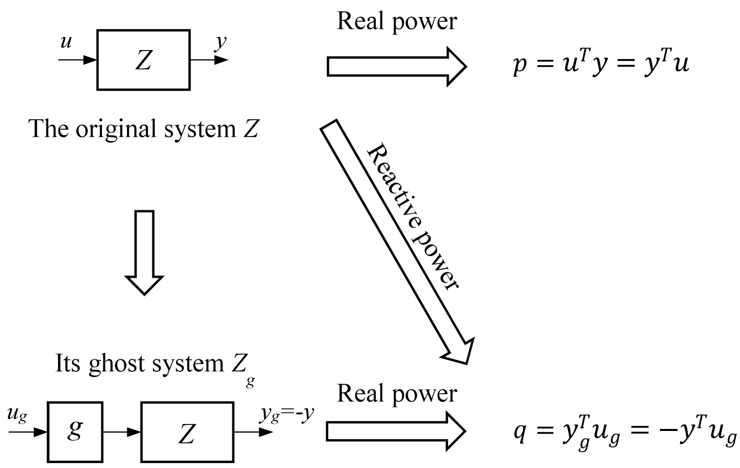

FIGURE 3. The ghost power theory that physically interprets the instantaneous reactive power of a system as the (real) power of its ghost system, where g is the ghost operator.

where y_g and u_g are, respectively, the output and input of the ghost. For translational systems, they are related to the force and velocity of the system; for rotational systems, they are related to the torque and angular velocity of the system.

Note that for the real power defined in the normal way as

$$p = u^T y = y^T u$$

there are

$$q = -gp \qquad \text{and} \qquad p = gq.$$

Furthermore, the average reactive power and the power factor can be defined for mechanical systems as well in a similar way as for electrical systems. This completes the electrical-mechanical analogy, as shown in Table 1.

It is well known that reactive power plays a critical role in electrical systems. However, this has not been well recognized for mechanical systems. The definition introduced here is expected to help understand the role of reactive power in mechanical systems and opens a new door for the analysis and synthesis of mechanical systems.

V. GENERALIZATION, CONCLUSIONS AND DISCUSSIONS

A new mathematical operator called the ghost operator g has been introduced in this paper for sine or cosine functions, followed by the physical construction of the ghost for a signal and the ghost for a system. The operator satisfies $g^2 = -1$ but it is different from the well-known imaginary operator j. The g-operator does not return an imaginary number but the imaginary operator j does when both are applied to $\cos\theta$ or $\sin\theta$ with θ being a real number. The ghost of a system behaves exactly in the opposite way as the original system when its input is the ghost of the input to the original system, showing perfect symmetry with respect to the origin. This has then been applied to reveal the physical meaning of reactive power: it is the (real) power of the ghost system. Moreover, it has been shown that it can be applied to introduce reactive power for mechanical systems, providing a missing concept in the electrical-mechanical analogy.

As a matter of fact, the instantaneous power theory described above can be generalized to any dynamical system Z that is described by the port-Hamiltonian model (1)-(2) with a periodic input u. Its instantaneous real power and reactive power are

$$p = u^T y = y^T u,$$

and

$$q = y_g^T u_g = -y^T u_g,$$

where $u_g = gu$ is the ghost of the input u to the original system Z and $y_g = -y$ is the output of the ghost. The average real power and reactive power can be easily obtained by taking the averages of p and q over one period. Since the port-Hamiltonian system (1)-(2) can be applied to model complex networked multi-physics systems with lumped and/or distributed parameters [5], the above interpretation/definition for instantaneous real power and reactive power is very generic and can be applied to other systems involving energy or power conversion as well, including fluid, thermal, magnetic, and chemical systems, in addition to the electrical and mechanical systems illustrated above. This ghost power theory can be summarized as illustrated in Fig. 3.

The Chinese yin-yang philosophy describes how seemingly opposite or contrary objects may actually be complementary, interconnected, and interdependent, and how they may give rise to each other as they interrelate to one another. The ghost of a system and the original system actually form a perfect pair of yin-yang. Some interesting results may emerge after further research on this.

Another possible application of the ghost operator is in quantum mechanics, where the imaginary operator and complex numbers play a critical role. However, does quantum mechanics have to be based on complex numbers [26]? The ghost operator introduced in this paper may shed some new light on this.

REFERENCES

[1] F. Z. Peng and J.-S. Lai, "Generalized instantaneous reactive power theory for three-phase power systems," *IEEE Trans. Instrum. Meas.*, vol. 45, no. 1, pp. 293–297, Feb. 1996.

[2] H. Akagi, E. Watanabe, and M. Aredes, *Instantaneous Power Theory and Applications to Power Conditioning*. Hoboken, NJ,USA: Wiley, 2007.

[3] F. de Leon, L. Qaseer, and J. Cohen, "AC power theory from Poynting theorem: Identification of the power components of magnetic saturating and hysteretic circuits," *IEEE Trans. Power Del.*, vol. 27, no. 3, pp. 1548–1556, Jul. 2012.

[4] A. A. Montanari and A. M. Gole, "Enhanced instantaneous power theory for control of grid connected voltage sourced converters under unbalanced conditions," *IEEE Trans. Power Electron.*, vol. 32, no. 8, pp. 6652–6660, Aug. 2017.

[5] A. van der Schaft and D. Jeltsema, "Port-Hamiltonian systems theory: An introductory overview," *Found. Trends Syst. Control*, vol. 1, nos. 2–3, pp. 173–378, 2014. [Online]. Available: http://dx.doi.org/10.1561/2600000002

[6] Q.-C. Zhong, "Power electronics-enabled autonomous power systems: Architecture and technical routes," *IEEE Trans. Ind. Electron.*, vol. 64, no. 7, pp. 5907–5918, Jul. 2017.

[7] J. Z. Jiang and M. C. Smith, "Regular positive-real functions and five-element network synthesis for electrical and mechanical networks," *IEEE Trans. Autom. Control*, vol. 56, no. 6, pp. 1275–1290, Jun. 2011.

[8] S. Evangelou, D. J. N. Limebeer, R. S. Sharp, and M. C. Smith, "Control of motorcycle steering instabilities," *IEEE Control Syst.*, vol. 26, no. 5, pp. 78–88, Oct. 2006.

[9] C. Rengifo, B. Kaddar, Y. Aoustin, and C. Chevallereau, "Reactive power compensation in mechanical systems," in *Proc. 2nd Joint Int. Conf. Multibody Syst. Dyn.*, Stuttgart, Germany, May 2012, pp. 576–581.

[10] T. Mizoguchi, T. Nozaki, and K. Ohnishi, "The power factor in mechanical system," in *Proc. IEEE Int. Conf. Mechatronics (ICM)*, Feb. 2013, pp. 576–581.

[11] D. D. Holm, *Geometric Mechanics Part I: Dynamics and Symmetry*. London, U.K.: Imperial College Press, Jul. 2011.

[12] D. D. Holm, *Geometric Mechanics Part II: Rotating, Translating and Rolling*. London, U.K.: Imperial College Press, Oct. 2011.

[13] D. D. Holm, T. Schmah, and C. Stoica, *Geometric Mechanics and Symmetry: From Finite to Infinite Dimensions*. Oxford, U.K: Oxford Univ. Press, Oct. 2009.

[14] B. Tellegen, "A general network theorem, with applications," *Philips Res. Rep.*, vol. 7, pp. 259–269, Mar. 1952.

[15] H. J. Carlin, "Network theory without circuit elements," *Proc. IEEE*, vol. 55, no. 4, pp. 482–497, Apr. 1967.

[16] H. J. Carlin, "Correction to 'network theory without circuit elements,'" *Proc. IEEE*, vol. 57, no. 6, p. 1171, Jun. 1969.

[17] B. Maschke, R. Ortega, and A. J. V. D. Schaft, "Energy-based Lyapunov functions for forced Hamiltonian systems with dissipation," *IEEE Trans. Autom. Control*, vol. 45, no. 8, pp. 1498–1502, Aug. 2000.

[18] C. I. Byrnes, A. Isidori, and J. C. Willems, "Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems," *IEEE Trans. Autom. Control*, vol. 36, no. 11, pp. 1228–1240, Nov. 1991.

[19] M. S. Erlicki and A. Emanuel-Eigeles, "New aspects of power factor improvement part I—Theoretical basis," *IEEE Trans. Ind. General Appl.*, vols. IGA–4, no. 4, pp. 441–446, Jul. 1968.

[20] Q.-C. Zhong and T. Hornik, *Control of Power Inverters in Renewable Energy and Smart Grid Integration*. Hoboken, NJ, USA: Wiley, 2013.

[21] E. Rezaei, M. Ebrahimi, and A. Tabesh, "Control of DFIG wind power generators in unbalanced microgrids based on instantaneous power theory," *IEEE Trans. Smart Grid*, to be published.

[22] Q.-C. Zhong, *Power Electronics-Enabled Autonomous Power Systems: Next Generation Smart Grids*. Hoboken, NJ, USA: Wiley, 2017.

[23] L. O. Chua, "Memristor-the missing circuit element," *IEEE Trans. Circuit Theory*, vol. 18, no. 5, pp. 507–519, Sep. 1971.

[24] F. A. Firestone, "A new analogy between mechanical and electrical systems," *J. Acoust. Soc. Amer.*, vol. 4, no. 3, pp. 249–267, 1933.

[25] M. C. Smith, "Synthesis of mechanical networks: The inerter," *IEEE Trans. Autom. Control*, vol. 47, no. 10, pp. 1648–1662, Oct. 2002.

[26] F. M. Lev, "Why is quantum physics based on complex numbers?" *Finite Fields Their Appl.*, vol. 12, no. 3, pp. 336–356, 2006.

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