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# Robust Passivity Control for 2-D Uncertain Markovian Jump Linear Discrete-Time Systems

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**ABSTRACT** This paper discusses the problem of robust controller design for two-dimensional (2-D) Markovian jump linear systems. The problem is demonstrated using Fornasini–Marchesini local state-space models, which are affected by uncertainties. The transition-mode probability matrix is homogenous and known. It is assumed that the mode information is available for the controller design and implementation. Then, a mode-dependent state-feedback controller is proposed. By substituting the controller into the 2-D system, a stochastic closed-loop system is obtained, because the stochastic variable, external disturbance, and uncertainties are all included in the closed-loop system. Based on the analysis results, an approach to design the controller and its gains is proposed, and the gains are calculated by solving linear matrix inequalities. In section V, a 2-D case is used to verify the performance of the controller.

**INDEX TERMS** Two-dimensional digital system, Markovian jump system, passivity analysis, linear matrix inequality (LMI), dissipation analysis.

## I. INTRODUCTION

During the past decades, much attention has been focused on multi-dimensional systems, mainly because these systems have a wide variety of applications, including multidimensional signal processing, image processing, and physical and chemical analysis [1]–[4]. On the theoretical side, much effort has been devoted to enhancing the applications of multi-dimensional systems. Liu [5] presented a stability analysis for two-dimensional (2-D) linear systems. A stability analysis was performed by Chen [6] for 2-D systems with interval time-varying delays and saturation nonlinearities. Peng and Guan [7] studied the output feedback in the design problem for 2-D state-delayed systems. The delay-dependent stabilizability of 2-D delayed continuous systems, in which the input is constrained and the state is subject to delay, was investigated by Mohamed et al. [8]. The nonlinear case was studied by Liang et al. [9], and the switched case was studied in [10]-[12]. Zhang et al. [13] aimed to study the observer design problem for polytopic linear parameter varying systems with uncertain measurements of scheduling variables. The filtering problem, which is an important aspect of control, has been examined in various studies of 2-D systems, such as [14]–[20] and the references therein.

The study of Markovian jump systems has attracted significant attention because, according to a review by Zhang and Boukas [21], their applications include diverse fields, such as economics, fault diagnosis, biomedicine, and communication networks. Markovian jump systems have been reported to better capture systems that are subject to abrupt changes such as those in structures or parameters induced by external causes. The external causes could include sudden environmental changes or component failures according to Zhang et al. [22]. Due to their significant application prospects, much work on Markovian jump systems has been performed and presented in the literature. Souza et al. [23] presented a design for mode-independent filters for Markovian jump linear systems, and the filter gains were calculated via solving a set of linear matrix inequalities (LMIs). The mode-reduction problem, in which a low-order system can replace a high-order system, was studied by Zhang et al. [24]. Costa and Marques [25] investigated the controller-design problem. de Souza [26] exploited the robust stability and

stabilization of uncertain discrete-time Markovian jump linear systems. The performances of both  $H_2$  and  $H_\infty$  were considered simultaneously. hang et al. [27] developed a mixed  $H/H_\infty$  fault detector design method. Xiong and Lam [28] demonstrated that the stabilization problem for discrete-time Markovian jump linear systems can be solved via timedelayed controllers. Other works include [29]–[36] and the references therein.

If a system is subject to external noise, an optimal controller or filter should have the capacity to attenuate the effect of noise. In the literature, various approaches have been used to achieve specific targets, such as Kalman filtering and  $H_2$  and  $H_\infty$  filtering/control. Another method termed passive control has gained relatively less attention. Passivity not only offers a useful physical interpretation of system stability/stabilization but also attenuates the noise. Zhang and Wang [37] developed an observer gain tuning method based on the stability analysis of the estimation error system. Thus, passive control has played important roles in many areas, including circuit systems and mechanical systems [38]-[40]. The investigation of passive control for different types of setups can be seen in [41]-[49]. Despite the extensive research on robust control, Markovian jump systems, and 2-D systems, most studies are focused on one or two aspects. Little progress has been made for passivity analysis of 2-D Markovian jump systems. Because passive systems have played an important role in many practical applications [50], it is worth examining robust passive control for 2-D Markovian jump linear systems.

Based on the application perspectives of the AGV (Automated Guided Vehicle) carrier platform and its sensors (such as image sensors and 2-D laser sensors), in this work, we investigate the robust passive controller design problem for 2-D Markovian jump linear systems. The Markovian jump mode is employed to improve the controller design, that is, the controller is dependent on the mode. To design a robust passive controller, the stability and passivity were analysed simultaneously for the closed-loop systems by assuming that the controller gains are given. A set of matrix inequalities was obtained for the stability and passivity analysis. According to the obtained condition, the mode-dependent controller gains could be calculated by solving a set of LMIs. The proposed controller design method is validated via a numerical example.

#### **II. PROBLEM FORMULATION AND PRELIMINARIES**

Consider the following 2-D uncertain Markovian jump linear discrete-time systems

$$\begin{aligned} x(i+1,j+1) &= A_1(r_k)x(i,j+1) + A_2(r_k)x(i+1,j) \\ &+ B_1(r_k)\omega(i,j+1) + B_2(r_k)\omega(i+1,j) \\ &+ B_3(r_k)u(i,j+1) + B_4(r_k)u(i+1,j) \\ &+ B_5(r_k)p(i,j+1) + B_6(r_k)p(i+1,j) \end{aligned}$$

$$z(i,j) &= C_1(r_k)x(i,j) + D_1(r_k)\omega(i,j) + D_2(r_k)u(i,j) \\ q(i,j) &= C_2(r_k)x(i,j) \\ p(i,j) &= \Delta(i,j)q(i,j), \|\Delta(i,j)\| < I, \end{aligned}$$

where  $x(i, j) \in \mathbb{R}^n$  is the state vector;  $u(i, j) \in \mathbb{R}^o$  is the control input;  $\omega(i, j) \in \mathbb{R}^s$  is the external disturbance;  $z(i, j) \in \mathbb{R}^l$  is the controlled output; q(i, j) and p(i, j) are used to denote the uncertain structures of the systems  $i, j \in \mathbb{Z}^+$ ; and  $\Delta(i, j)$  is the uncertain element. In addition,  $A_1(r_k), A_2(r_k), B_1(r_k), B_2(r_k)$ ,  $B_3(r_k), B_4(r_k), B_5(r_k), B_6(r_k), C_1(r_k), C_2(r_k), D_1(r_k)$  and  $D_2(r_k)$  are mode-dependent real matrices with compatible dimensions;  $r_k$  with k = i + j represents a discrete-time, discrete-state Markovian chain that uses values in a finite set  $\Phi = \{1 \ 2 \cdots N\}$ ; and the transition-mode probability matrix is denoted by  $\Lambda = [\lambda_{lm}]$ . The element  $\lambda_{lm}$  in the transition-mode probability matrix  $\Lambda$  denotes the probability of jumping from the *l*th mode to the *m*th mode, which can be represented by

$$\lambda_{lm} = \Pr(r_{k+1} = m | r_k = l) \tag{2}$$

According to the mode property,  $\lambda_{lm}$ ,  $\forall l, m \in N$  is a non-negative scalar and  $\sum_{m=1}^{N} \lambda_{lm} = 1$ . In this study, the transition-mode probability matrix  $\Lambda$  is assumed to be known a priori. To simplify the notation, when  $r_k$  has a value of,  $A_1(r_k)$ ,  $A_2(r_k)$ ,  $B_1(r_k)$ ,  $B_2(r_k)$ ,  $B_3(r_k)$ ,  $B_4(r_k)$ ,  $B_5(r_k)$ ,  $x(t) \in \mathbb{R}^n$ ,  $C_1(r_k)$ ,  $C_2(r_k)$ ,  $D_1(r_k)$  and  $D_2(r_k)$  are represented by  $A_{1,l}$ ,  $A_{2,l}$ ,  $B_{1,l}$ ,  $B_{2,l}$ ,  $B_{3,l}$ ,  $B_{4,l}$ ,  $B_{5,l}$ ,  $B_{6,l}$ ,  $C_{1,l}$ ,  $C_{2,l}$ ,  $D_{1,l}$ and  $D_{2,l}$ . Moreover, it is assumed that the mode information is available for the controller to be designed.

Because the mode information is available for the controller design, the controller in this work uses a mode-dependent state-feedback controller expressed by the following:

$$u(i,j) = K(r_k)x(i,j)$$
(3)

where  $K(r_k)$  is the mode-dependent gain to be determined. By substituting the control law in (3) into the system dynamics in (1), the closed-loop 2-D system becomes

$$\begin{aligned} x(i+1,j+1) &= A_1(r_k)x(i,j+1) + A_2(r_k)x(i+1,j) \\ &+ B_1(r_k)\omega(i,j+1) \\ &+ B_2(r_k)\omega(i+1,j) + B_5(r_k)p(i,j+1) \\ &+ B_6(r_k)p(i+1,j) \\ z(i,j) &= \bar{C}_1(r_k)x(i,j) + D_1(r_k)\omega(i,j) \end{aligned}$$
(4)

where

$$\bar{A}_1(r_k) = A_1(r_k) + B_3(r_k)K(r_k), \bar{A}_2(r_k)$$
  
=  $A_2(r_k) + B_4(r_k)K(r_k),$   
 $\bar{C}_1(r_k) = C_1(r_k) + D_2(r_k)K(r_k).$ 

Expression (4) describes a random system because  $r_k$  is a variable in the Markovian jump system. Therefore, the closed-loop control system defined in (4) is not suitable for the classical asymptotic-stability method. The feedback-control strategy is designed based on the relationship between passivity and stability (mean-square asymptotic stability). Definition 1: The closed-loop system in (4) with  $\omega$  (i, j) = 0 is termed mean-square asymptotically stable with any initialization boundary conditions if the following condition is satisfied:

$$\lim_{i+j\to\infty} E\left\{|x(i,j)|^2\right\} = 0$$
(5)

Definition 2 [33]: If the closed-loop system specified in (4) with an initial condition of zero satisfies the inequality in (6), it is termed mean-square passive.

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \mathbf{E}\left\{\bar{\omega}^{\mathrm{T}}(i,j)\bar{z}(i,j)\right\} \ge 0$$
(6)

where

$$\bar{\omega}(i,j) = \begin{bmatrix} \omega(i,j+1) \\ \omega(i+1,j) \end{bmatrix}, \ \bar{z}(i,j) = \begin{bmatrix} z(i,j+1) \\ z(i+1,j) \end{bmatrix}$$

Definition 3: If the closed-loop system in (4) with an initial condition of zero satisfies the dissipation inequality in (7), it is termed mean-square passive with dissipation  $\eta$ .

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \mathbb{E}\left\{\bar{\omega}^{\mathrm{T}}(i,j)z(i,j) - \eta\bar{\omega}^{\mathrm{T}}(i,j)\bar{\omega}(i,j)\right\} \ge 0 \qquad (7)$$

For the closed-loop system in (4), we now state the passivity control problem of 2-D Markovian jump linear systems as follows: the mode-dependent controller defined by (3) is determined such that the closed-loop system in (4) is meansquare asymptotically stable and passive with a prescribed dissipation rate  $\eta$ . Before proceeding, we introduce a useful lemma for robust control.

Lemma1 [50]: Let X and Y be real matrices with appropriate dimensions, and let Z be a symmetric matrix. Then, the condition

$$Z + X\Delta Y + Y^{\mathrm{T}}\Delta X^{\mathrm{T}} < 0 \tag{8}$$

is satisfied for all  $\Delta$  with  $\Delta^{T}\Delta \leq I$  if and only if a positive scalar  $\varepsilon$  exists such that

$$\begin{bmatrix} Z & X & \varepsilon Y^{\mathrm{T}} \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0$$
(9)

#### **III. ANALYSIS OF PASSIVITY AND STABILITY**

We will analyse the closed-loop system in (4) in terms of the two aspects of stability and passivity by assuming that the mode-dependent controller gains are given.

Theorem 1. Suppose that the mode-dependent gain  $K(r_k)$  is given. As a closed-loop system in (4) with zero input, the system is mean-square asymptotically stable if matrices  $P_l = P_l^{\rm T} > 0$  and  $Q = Q^{\rm T} > 0$  exist for  $\forall l \in \Phi$  that follow the condition

$$\begin{bmatrix} \Psi_1 & \Psi_2 & \Psi_3 \\ * & Q - P_l & 0 \\ * & * & -Q \end{bmatrix} < 0$$
(10)

where

$$\Psi_{1} = \operatorname{diag} \left\{ -P_{1} - P_{2} \cdots -P_{N} \right\}$$

$$\Psi_{2} = \begin{bmatrix} \sqrt{\lambda_{l1}}P_{1}\hat{A}_{1,l} \\ \sqrt{\lambda_{l2}}P_{2}\hat{A}_{1,l} \\ \vdots \\ \sqrt{\lambda_{lN}}P_{N}\hat{A}_{1,l} \end{bmatrix},$$

$$\Psi_{3} = \begin{bmatrix} \sqrt{\lambda_{l1}}P_{1}\hat{A}_{2,l} \\ \sqrt{\lambda_{l2}}P_{2}\hat{A}_{2,l} \\ \vdots \\ \sqrt{\lambda_{lN}}P_{N}\hat{A}_{2,l} \end{bmatrix}$$

$$\hat{A}_{1,l} = \bar{A}_{1,l} + B_{5,l}\Delta(i,j)C_{2,l},$$

$$\hat{A}_{2,l} = \bar{A}_{2,l} + B_{6,l}\Delta(i,j)C_{2,l}$$

Proof: Select the mode-dependent Lyapunov function candidate(11) for the closed-loop system in (4) with zero external input:

$$V(i, j, r_k) = x_{i_1 j_1}^{\mathrm{T}}(i, j) W_{i_1 j_1}(r_k) x_{i_1 j_1}(i, j)$$
(11)

where

$$x_{i_1j_1}(i,j) = x(i+i_1,j+j_1), W_{i_1j_1}(r_k) = W_{i_1j_1}^{\mathrm{T}}(r_k) > 0,$$
  
$$i_1 \in \mathbb{Z}, j_1 \in \mathbb{Z}.$$

If  $r_k = l$ , the Lyapunov function candidate expected difference can be described as

$$\Delta V(i,j) = \mathbb{E} \{ V_{11}(i,j) - V_{10}(i,j) - V_{01}(i,j) | r_k = l \}, \quad (12)$$

where

$$V_{11}(i,j) = x_{11}^{\mathrm{T}}(i,j) W_{11}(r_{k+1})x_{11}(i,j)$$
  

$$V_{10}(i,j) = x_{10}^{\mathrm{T}}(i,j) W_{10}(l)x_{10}(i,j),$$
  

$$V_{01}(i,j) = x_{01}^{\mathrm{T}}(i,j) W_{01}(l)x_{01}(i,j).$$

With zero inputs and considering the dynamics of the closed-loop system in (4), the expected difference is computed using equation(13) as follows:

$$\begin{split} &\Delta V(i,j) \\ &= \mathbb{E} \left\{ \begin{cases} \bar{A}_{1,l} x(i,j+1) + \bar{A}_{2,l} x(i+1,j) + \\ B_{5,l} \Delta(i,j) C_{2,l} x(i,j+1) + B_{6,l} \Delta(i,j) C_{2,l} x(i+1,j) \end{cases} \right\}^{\mathrm{T}} \\ &\times W_{11}(r_{k+1}) \\ &\times \left\{ \bar{A}_{1,l} x(i,j+1) + \bar{A}_{2,l} x(i+1,j) + \\ B_{5,l} \Delta(i,j) C_{2,l} x(i,j+1) + B_{6,l} \Delta(i,j) C_{2,l} x(i+1,j) \right\} \right\} \\ &- x^{\mathrm{T}}(i+1,j) W_{10,l} x(i+1,j) - x^{\mathrm{T}}(i,j+1) W_{01,l} x(i,j+1) \\ &= \mathbb{E} \left\{ \left\{ \hat{A}_{1,l} x(i,j+1) + \hat{A}_{2,l} x(i+1,j) \right\}^{\mathrm{T}} W_{11}(r_{k+1}) \\ &\times \left\{ \hat{A}_{1,l} x(i,j+1) + \hat{A}_{2,l} x(i+1,j) \right\} \right\} \\ &- x^{\mathrm{T}}(i+1,j) W_{10,l} x(i+1,j) - x^{\mathrm{T}}(i,j+1) W_{01,l} x(i,j+1) \\ &= \sum_{m=1}^{N} \lambda_{lm} x^{\mathrm{T}}(i,j+1) \hat{A}_{1,l}^{\mathrm{T}} W_{11,m} \hat{A}_{1,l} x(i,j+1) \end{split}$$

$$+\sum_{m=1}^{N} \lambda_{lm} x^{\mathrm{T}}(i, j+1) \hat{A}_{1,l}^{\mathrm{T}} W_{11,m} \hat{A}_{2,l} x(i+1,j) \\ +\sum_{m=1}^{N} \lambda_{lm} x^{\mathrm{T}}(i+1,j) \hat{A}_{2,l}^{\mathrm{T}} W_{11,m} \hat{A}_{1,l} x(i, j+1) \\ +\sum_{m=1}^{N} \lambda_{lm} x^{\mathrm{T}}(i+1,j) \hat{A}_{2,l}^{\mathrm{T}} W_{11,m} \hat{A}_{2,l} x(i+1,j) \\ -x^{\mathrm{T}}(i+1,j) W_{10,l} x(i+1,j) - x^{\mathrm{T}}(i, j+1) W_{01,l} x(i, j+1) \\ = \begin{bmatrix} x \ (i, j+1) \\ x \ (i+1,j) \end{bmatrix}^{\mathrm{T}} \\ \times \begin{bmatrix} \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} \hat{A}_{1,l} - W_{01} & \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} \hat{A}_{2,l} \\ & & \hat{A}_{2,l}^{\mathrm{T}} \hat{W}_{11,l} \hat{A}_{2,l} - W_{10} \end{bmatrix} \\ \begin{bmatrix} x \ (i, j+1) \\ x \ (i+1, j) \end{bmatrix}$$
(13)

With  $\hat{W}_{11,l} = \sum_{m=1}^{N} \lambda_{lm} W_{11,m}$ , suppose that the Lyapunov matrices have the following representatives:  $W_{11,l} = P_l, W_{10,l} = Q$ , and  $W_{01,l} = P_l - Q$ . According to the Schur complement, if the condition in (10) is satisfied,  $\Delta V(i, j)$  is negative. Using similar steps as in [20], we can conclude that the system is mean-square asymptotically stable.

It is necessary to note that the system uncertainties are restricted by condition (10). Using Lemma 1, the uncertainties can be eliminated, and the following theorem is obtained.

Theorem 2. Suppose that the mode-dependent gains  $K(r_k)$  are given. Under the same conditions as Theorem 1, the following expression is feasible if matrices  $P_l = P_l^T > 0$  and  $Q = Q^T > 0$  exist for  $\forall l \in \Phi$ :

$$\begin{bmatrix} \Psi_1 & \bar{\Psi}_2 & \bar{\Psi}_3 & \Psi_4 \\ * & Q - P_l & 0 & \Psi_5 \\ * & * & -Q & \Psi_6 \\ * & * & * & \Psi_7 \end{bmatrix} < 0$$
(14)

where

$$\begin{split} \bar{\Psi}_{2} &= \begin{bmatrix} \sqrt{\lambda_{l1}}P_{1}\bar{A}_{1,l} \\ \sqrt{\lambda_{l2}}P_{2}\bar{A}_{1,l} \\ \vdots \\ \sqrt{\lambda_{lN}}P_{N}\bar{A}_{1,l} \end{bmatrix}, \ \bar{\Psi}_{3} = \begin{bmatrix} \sqrt{\lambda_{l1}}P_{1}\bar{A}_{2,l} \\ \sqrt{\lambda_{l2}}P_{2}\bar{A}_{2,l} \\ \vdots \\ \sqrt{\lambda_{lN}}P_{N}\bar{A}_{1,l} \end{bmatrix}, \\ \Psi_{4} &= \begin{bmatrix} \sqrt{\lambda_{l1}}\varepsilon_{1}P_{1}B_{5,l} & 0 & \sqrt{\lambda_{l1}}\varepsilon_{2}P_{1}B_{6,l} & 0 \\ \sqrt{\lambda_{l2}}\varepsilon_{1}P_{2}B_{5,l} & 0 & \sqrt{\lambda_{l2}}\varepsilon_{2}P_{2}B_{6,l} & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \sqrt{\lambda_{lN}}\varepsilon_{1}P_{N}B_{5,l} & 0 & \sqrt{\lambda_{lN}}\varepsilon_{2}P_{N}B_{6,l} & 0 \end{bmatrix}, \\ \Psi_{5} &= \begin{bmatrix} 0 & C_{2,l}^{T} & 0 & 0 \end{bmatrix}, \Psi_{6} &= \begin{bmatrix} 0 & 0 & 0 & C_{2,l}^{T} \end{bmatrix}, \\ \Psi_{7} &= \begin{bmatrix} -\varepsilon_{1}I & 0 & 0 & 0 \\ * & -\varepsilon_{1}I & 0 & 0 \\ * & * & * & -\varepsilon_{2}I & 0 \\ * & * & * & -\varepsilon_{2}I \end{bmatrix}. \end{split}$$

Proof: The condition in (10) can be rewritten as:

$$\begin{bmatrix} \Psi_{1} & \bar{\Psi}_{2} & \bar{\Psi}_{3} \\ * & Q - P_{l} & 0 \\ * & * & -Q \end{bmatrix} + \begin{bmatrix} \bar{\Psi}_{4} \\ 0 \\ 0 \end{bmatrix} \Delta (i, j) \begin{bmatrix} 0 & C_{2,l} & 0 \end{bmatrix}$$
$$+ \left( \begin{bmatrix} \bar{\Psi}_{4} \\ 0 \\ 0 \end{bmatrix} \Delta (i, j) \begin{bmatrix} 0 & C_{2,l} & 0 \end{bmatrix} \right)^{\mathrm{T}}$$
$$+ \begin{bmatrix} \bar{\Psi}_{5} \\ 0 \\ 0 \end{bmatrix} \Delta (i, j) \begin{bmatrix} 0 & 0 & C_{2,l} \end{bmatrix}$$
$$+ \left( \begin{bmatrix} \bar{\Psi}_{5} \\ 0 \\ 0 \end{bmatrix} \Delta (i, j) \begin{bmatrix} 0 & 0 & C_{2,l} \end{bmatrix} \right)^{\mathrm{T}} < 0 \qquad (15)$$

where

$$\bar{\Psi}_{4} = \begin{bmatrix} \sqrt{\lambda_{l1}} P_{1} B_{5,l} & 0\\ \sqrt{\lambda_{l2}} P_{2} B_{5,l} & 0\\ \vdots & \vdots\\ \sqrt{\lambda_{lN}} P_{N} B_{5,l} & 0 \end{bmatrix}, \quad \bar{\Psi}_{5} = \begin{bmatrix} \sqrt{\lambda_{l1}} P_{1} B_{6,l} & 0\\ \sqrt{\lambda_{l2}} P_{2} B_{6,l} & 0\\ \vdots\\ \sqrt{\lambda_{lN}} P_{N} B_{6,l} & 0 \end{bmatrix}$$

By applying the lemma twice, we can obtain the condition in (14).

Theorem 1 and Theorem 2 provide the stability condition for the closed-loop system without any external input. In the following, we will study the passivity with dissipation when the system is subject to external inputs.

Theorem 3. Suppose that the mode-dependent gains  $\Omega$  are given. Under the same conditions as Theorem 1, then (16), as shown at the top of this page is feasible if matrices  $P_l = P_l^{\rm T} > 0$  and  $Q = Q^{\rm T} > 0$  exist for  $\forall l \in \Phi$  where

$$\Psi_{8} = \begin{bmatrix} \sqrt{\lambda_{l1}} P_{1} B_{1,l} \\ \sqrt{\lambda_{l2}} P_{2} B_{1,l} \\ \vdots \\ \sqrt{\lambda_{lN}} P_{N} B_{1,l} \end{bmatrix}, \Psi_{9} = \begin{bmatrix} \sqrt{\lambda_{l1}} P_{1} B_{2,l} \\ \sqrt{\lambda_{l2}} P_{2} B_{2,l} \\ \vdots \\ \sqrt{\lambda_{lN}} P_{N} B_{2,l} \end{bmatrix}$$

Proof: The expected difference of the Lyapunov function can be recalculated according to equation (17). If interference is present, then

$$\begin{split} \Delta V(i,j) &= \mathbb{E} \left\{ \left\{ \hat{A}_{1,l} x(i,j+1) + \hat{A}_{2,l} x(i+1,j) \\ &+ B_{1,l} \omega(i,j+1) + B_{2,l} \omega(i+1,j) \right\}^{\mathrm{T}} W_{11}(r_{k+1}) \\ &\times \left\{ \hat{A}_{1,l} x(i,j+1) + \hat{A}_{2,l} x(i+1,j) + B_{1,l} \omega(i,j+1) \\ &+ B_{2,l} \omega(i+1,j) \right\} \right\} \\ &- x^{\mathrm{T}}(i+1,j) W_{10,l} x(i+1,j) - x^{\mathrm{T}}(i,j+1) W_{01,l} x(i,j+1) \\ &= x^{\mathrm{T}}(i,j+1) \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} \hat{A}_{1,l} x(i,j+1) \\ &+ x^{\mathrm{T}}(i,j+1) \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} \hat{A}_{2,l} x(i+1,j) \\ &+ x^{\mathrm{T}}(i,j+1) \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} B_{1,l} \omega(i,j+1) \\ &+ x^{\mathrm{T}}(i,j+1) \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} B_{2,l} x(i+1,j) \\ &+ x^{\mathrm{T}}(i,j+1) \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} \hat{A}_{1,l} x(i,j+1) \end{split}$$

$$\begin{bmatrix} \Psi_{1} & \bar{\Psi}_{2} & \bar{\Psi}_{3} & \Psi_{4} & \Psi_{8} & \Psi_{9} \\ * & Q - P_{l} & 0 & \Psi_{5} & -\bar{C}_{1,l}^{\mathrm{T}} & 0 \\ * & * & -Q & \Psi_{6} & 0 & -\bar{C}_{1,l}^{\mathrm{T}} \\ * & * & * & \Psi_{7} & 0 & 0 \\ * & * & * & * & -D_{1,l} - D_{1,l}^{\mathrm{T}} + 2\eta I & 0 \\ * & * & * & * & * & -D_{1,l} - D_{1,l}^{\mathrm{T}} + 2\eta I \end{bmatrix} < 0$$
(16)

$$+ x^{\mathrm{T}}(i+1,j)\hat{A}_{2,l}^{\mathrm{T}}\hat{W}_{11,l}\hat{A}_{2,l}x(i+1,j) + x(i+1,j)\hat{A}_{2,l}^{\mathrm{T}}\hat{W}_{11,l}B_{1,l}x(i,j+1) + x(i+1,j)\hat{A}_{2,l}^{\mathrm{T}}\hat{W}_{11,l}B_{2,l}x(i+1,j) + \omega^{\mathrm{T}}(i,j+1)B_{1,l}^{\mathrm{T}}\hat{W}_{11,l}\hat{A}_{1,l}x(i,j+1) + \omega^{\mathrm{T}}(i,j+1)B_{1,l}^{\mathrm{T}}\hat{W}_{11,l}\hat{A}_{2,l}x(i+1,j) + \omega^{\mathrm{T}}(i,j+1)B_{1,l}^{\mathrm{T}}\hat{W}_{11,l}B_{1,l}\omega(i,j+1) + \omega^{\mathrm{T}}(i,j+1)B_{1,l}^{\mathrm{T}}\hat{W}_{11,l}B_{2,l}\omega(i+1,j) + \omega^{\mathrm{T}}(i+1,j)B_{2,l}^{\mathrm{T}}\hat{W}_{11,l}\hat{A}_{2,l}x(i+1,j) + \omega^{\mathrm{T}}(i+1,j)B_{2,l}^{\mathrm{T}}\hat{W}_{11,l}\hat{A}_{2,l}x(i+1,j) + \omega^{\mathrm{T}}(i+1,j)B_{2,l}^{\mathrm{T}}\hat{W}_{11,l}B_{1,l}x(i,j+1) + \omega^{\mathrm{T}}(i+1,j)B_{2,l}^{\mathrm{T}}\hat{W}_{11,l}B_{1,l}x(i,j+1) + \omega^{\mathrm{T}}(i+1,j)B_{2,l}^{\mathrm{T}}\hat{W}_{11,l}B_{1,l}x(i,j+1) + \omega^{\mathrm{T}}(i+1,j)B_{2,l}^{\mathrm{T}}\hat{W}_{11,l}B_{2,l}\omega(i+1,j) - x^{\mathrm{T}}(i+1,j)W_{10,l}x(i+1,j) - x^{\mathrm{T}}(i,j+1)W_{01,l}x(i,j+1).$$
(17)

To study the passivity with a dissipation rate  $\eta$ , the following cost function is considered:

$$J = \Delta V (i, j) + E \left\{ -\bar{\omega}^{T} (i, j) \bar{z} (i, j) - \bar{z}^{T} (i, j) \bar{\omega} (i, j) \right\} + 2\eta \bar{\omega}^{T} (i, j) \bar{\omega} (i, j) = \Delta V (i, j) + E \left\{ -\omega^{T} (i + 1, j) z (i + 1, j) - \omega^{T} (i, j + 1) z (i, j + 1) \right\} + E \left\{ -z^{T} (i + 1, j) \omega (i + 1, j) - z^{T} (i, j + 1) \omega (i, j + 1) \right\} + 2\eta \omega^{T} (i + 1, j) \omega (i + 1, j) + 2\eta \omega^{T} (i, j + 1) \omega (i, j + 1).$$
(18)

The difference in equation (17) is substituted into the composite cost function (18)

$$J = \zeta^{\mathrm{T}}(i, j) \,\Omega\zeta(i, j) \,, \tag{19}$$

with

$$\begin{split} \zeta \left( i,j \right) &= \begin{bmatrix} x \left( i,j+1 \right) \\ x \left( i+1,j \right) \\ \omega \left( i,j+1 \right) \\ \omega \left( i+1,j \right) \end{bmatrix}, \ \Omega = \begin{bmatrix} \Omega_{11} \ \Omega_{12} \ \Omega_{13} \ \Omega_{14} \\ * \ \Omega_{22} \ \Omega_{23} \ \Omega_{24} \\ * \ * \ \Omega_{33} \ \Omega_{34} \\ * \ * \ \Omega_{44} \end{bmatrix}, \\ \Omega_{11} &= \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} \hat{A}_{1,l} - W_{01,l}, \ \Omega_{12} &= \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} \hat{A}_{2,l}, \\ \Omega_{13} &= \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} B_{1,l} - \bar{C}_{1}^{\mathrm{T}}, \\ \Omega_{14} &= \hat{A}_{1,l}^{\mathrm{T}} \hat{W}_{11,l} B_{2,l} \Omega_{22} = \hat{A}_{2,l}^{\mathrm{T}} \hat{W}_{11,l} \hat{A}_{1,l} - W_{10,l}, \\ \Omega_{23} &= \hat{A}_{2,l}^{\mathrm{T}} \hat{W}_{11,l} B_{1,l}, \end{split}$$

$$\begin{split} \Omega_{24} &= \hat{A}_{2,l}^{\mathrm{T}} \hat{W}_{11,l} B_{2,l} - \bar{C}_{1}^{\mathrm{T}}, \\ \Omega_{33} &= B_{1,l}^{\mathrm{T}} W_{11,l} B_{1,l} - D_{1} - D_{1}^{\mathrm{T}} + 2\eta I, \\ \Omega_{34} &= B_{1,l}^{\mathrm{T}} W_{11,l} B_{2,l}, \\ \Omega_{44} &= B_{2,l}^{\mathrm{T}} W_{11,l} B_{2,l} - D_{1} - D_{1}^{\mathrm{T}} + 2\eta I \end{split}$$

The matrix inequality(16) indicates that matrix  $\Omega$  is a negative-definite matrix; using the Schur complement, the value of the cost function is observed to be negative. According to the methods used in [20], with positive integers *p* and *q*, we obtain

$$E\left\{ \sum_{i=0}^{p} \sum_{j=0}^{q} \left\{ -\bar{\omega}^{T}(i,j) \,\bar{z}(i,j) - \bar{z}^{T}(i,j) \,\bar{\omega}(i,j) + 2\eta \bar{\omega}^{T}(i,j) \,\bar{\omega}(i,j) \right\} \right\} < 0.$$
 (20)

When p and q approach infinity,

$$\mathsf{E}\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left\{\bar{\omega}^{\mathrm{T}}\left(i,j\right)\bar{z}\left(i,j\right)-\eta\bar{\omega}^{\mathrm{T}}\left(i,j\right)\bar{\omega}\left(i,j\right)\right\}\right\}>0.$$
(21)

Suppose that  $W_{11,l} = P_l$ ,  $W_{10,l} = Q$ , and  $W_{01,l} = P_l - Q$ . According to Definition 3, with the dissipation rate  $\eta$ , the closed-loop system in (4) is mean-square asymptotically stable and passive.

## **IV. CONTROLLER DESIGN**

Theorem 3 provides the robust mean-square stability and passivity conditions for the closed-loop system by assuming that the controller gain is given. In this section, the controller design method will be provided according to the conditions in Theorem 3.

Theorem 4. Suppose that with dissipation rate  $\eta$ , the closed-loop system in (4) is mean-square asymptotically stable and passive. Condition (22), as shown at the bottom of the next page is feasible if matrices  $\bar{K}_l$ ,  $R_l = R_l^T > 0$  and  $M_l = M_l^T > 0$  exist for  $\forall l \in \Phi$ .

In addition, the gains of the controller can be computed as:

$$K_l = \bar{K}_l S_l^{-1} \tag{24}$$

Proof: This theorem can be proven by performing a congruence transformation with diag{ $P_1^{-1}P_2^{-1}\cdots P_N^{-1}P_l^{-1}P_l^{-1}III$ } using the condition in (16) and defining new variables as  $S_l = P_l^{-1}, M_l = P_l^{-1}QP_l^{-1}$ , and  $\bar{K}_l = K_lS_l$ .



FIGURE 1. Markovian jumping modes during the simulation.

## **V. RESULTS AND DISCUSSION**

In this section, we provide a set of case data to verify the validity of the controller in Theorem 4. In the first mode, the system parameters are as follows:

$$A_{1,1} = \begin{bmatrix} 0.5 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, A_{2,1} = \begin{bmatrix} 0.9 & 0.2 \\ 0.3 & 0.3 \end{bmatrix}, B_{1,1} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, B_{2,1} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, B_{3,1} = \begin{bmatrix} 0.7 & 0.4 \\ 0.5 & 0.3 \end{bmatrix}, B_{4,1} = \begin{bmatrix} 0.5 & 0.1 \\ 0.9 & 0.2 \end{bmatrix}, B_{5,1} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, B_{6,1} = \begin{bmatrix} 0.15 & 0.15 \\ 0.15 & 0.15 \end{bmatrix},$$



FIGURE 2. The first state trajectories when the system is not controlled.

$$C_{1,1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, C_{2,1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$D_{1,1} = \begin{bmatrix} 0.8 & 0.3 \\ 0.3 & 0.3 \end{bmatrix}, D_{2,1} = \begin{bmatrix} 0.6 & 0.6 \\ 0.1 & 0.2 \end{bmatrix},$$
$$\eta = 0.2, \Delta (i, j) = \sin (i + j).$$

In the second mode, the system parameters are given as follows:

$$A_{1,2} = \begin{bmatrix} 0.4 & 0.2 \\ 0.3 & 0.2 \end{bmatrix}, A_{2,2} = \begin{bmatrix} 0.3 & 0.2 \\ 0.3 & 0.9 \end{bmatrix}, B_{1,2} = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, B_{2,2} = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.3 \end{bmatrix},$$

$$\begin{bmatrix} \hat{\Psi}_{1} & \hat{\Psi}_{2} & \hat{\Psi}_{3} & \hat{\Psi}_{4} & \hat{\Psi}_{8} & \hat{\Psi}_{9} \\ * & M_{l} - R_{l} & 0 & \hat{\Psi}_{5} & -S_{l}C_{1,l}^{\mathrm{T}} - \bar{K}_{l}^{\mathrm{T}}D_{2,l}^{\mathrm{T}} & 0 \\ * & * & -M_{l} & \hat{\Psi}_{6} & 0 & -S_{l}C_{1,l}^{\mathrm{T}} - \bar{K}_{l}^{\mathrm{T}}D_{2,l}^{\mathrm{T}} \\ * & * & * & \hat{\Psi}_{7} & 0 & 0 \\ * & * & * & * & -D_{1,l} - D_{1,l}^{\mathrm{T}} + 2\eta I & 0 \\ * & * & * & * & * & -D_{1,l} - D_{1,l}^{\mathrm{T}} + 2\eta I \end{bmatrix} < 0$$
(22)

where

$$\hat{\Psi}_{1} = \operatorname{diag}\left\{-R_{1} - R_{2} \cdots - R_{N}\right\}, \quad \hat{\Psi}_{2} = \begin{bmatrix} \sqrt{\lambda_{l1}} (A_{1,l}S_{1} + B_{3,l}\bar{K}_{l}) \\ \sqrt{\lambda_{l2}} (A_{1,l}S_{2} + B_{3,l}\bar{K}_{l}) \\ \vdots \\ \sqrt{\lambda_{lN}} (A_{1,l}S_{N} + B_{3,l}\bar{K}_{l}) \end{bmatrix}, \quad \hat{\Psi}_{3} = \begin{bmatrix} \sqrt{\lambda_{l1}} (A_{2,l}S_{1} + B_{4,l}\bar{K}_{l}) \\ \sqrt{\lambda_{l2}} (A_{2,l}S_{2} + B_{4,l}\bar{K}_{l}) \\ \vdots \\ \sqrt{\lambda_{lN}} (A_{2,l}S_{N} + B_{4,l}\bar{K}_{l}) \end{bmatrix}$$

$$\hat{\Psi}_{4} = \begin{bmatrix} \sqrt{\lambda_{l1}} \varepsilon_{1}B_{5,l} & 0 & \sqrt{\lambda_{l1}} \varepsilon_{2}B_{6,l} & 0 \\ \sqrt{\lambda_{l2}} \varepsilon_{1}B_{5,l} & 0 & \sqrt{\lambda_{l2}} \varepsilon_{2}B_{6,l} & 0 \\ \vdots & \vdots & \vdots \\ \sqrt{\lambda_{lN}} \varepsilon_{1}B_{5,l} & 0 & \sqrt{\lambda_{lN}} \varepsilon_{2}B_{6,l} & 0 \end{bmatrix}, \quad \hat{\Psi}_{5} = \begin{bmatrix} 0 & S_{l}C_{2,l}^{T} & 0 & 0 \end{bmatrix}, \quad \hat{\Psi}_{6} = \begin{bmatrix} 0 & 0 & 0 & S_{l}C_{2,l}^{T} \end{bmatrix}, \\
\hat{\Psi}_{7} = \begin{bmatrix} -\varepsilon_{1}I & 0 & 0 & 0 \\ * & -\varepsilon_{1}I & 0 & 0 \\ * & -\varepsilon_{2}I & 0 \\ * & * & -\varepsilon_{2}I & 0 \\ * & * & -\varepsilon_{2}I & 0 \\ * & * & -\varepsilon_{2}I & 0 \end{bmatrix}, \quad \hat{\Psi}_{8} = \begin{bmatrix} \sqrt{\lambda_{l1}}B_{1,l} \\ \sqrt{\lambda_{l2}}B_{1,l} \\ \vdots \\ \sqrt{\lambda_{lN}}B_{1,l} \end{bmatrix}, \quad \hat{\Psi}_{9} = \begin{bmatrix} \sqrt{\lambda_{l1}}B_{2,l} \\ \sqrt{\lambda_{l2}}B_{2,l} \\ \vdots \\ \sqrt{\lambda_{lN}}B_{2,l} \end{bmatrix}$$
(23)



**FIGURE 3.** The second state trajectories when the system is not controlled.



FIGURE 4. The first state trajectories when the system is controlled.



FIGURE 5. The second state trajectories when the system is controlled.

$$B_{3,2} = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.7 \end{bmatrix}, B_{4,2} = \begin{bmatrix} 0.2 & 0.1 \\ 0.9 & 0.5 \end{bmatrix},$$
  

$$B_{5,2} = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.07 \end{bmatrix}, B_{6,2} = \begin{bmatrix} 0.02 & 0.01 \\ 0.1 & 0.05 \end{bmatrix},$$
  

$$C_{1,2} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, C_{2,2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},$$
  

$$D_{1,2} = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.8 \end{bmatrix}, D_{2,2} = \begin{bmatrix} 0.2 & 0.6 \\ 0.1 & 0.8 \end{bmatrix},$$
  

$$\eta = 0.2, \Delta (i,j) = \sin (i+j).$$

The transition-mode probability matrix is

$$\Lambda = \begin{bmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \end{bmatrix}$$

According to Theorem 4, the calculated controller gains are

$$K_1 = \begin{bmatrix} -0.3985 & 0.4224 \\ 0.0774 & -0.7818 \end{bmatrix}, K_2 = \begin{bmatrix} -0.5249 & -0.4458 \\ -0.0974 & 0.2722 \end{bmatrix}.$$

To illustrate the performance of the designed controller, it is assumed that the external disturbance is

$$\boldsymbol{\omega} = \left[ \begin{array}{c} \frac{1}{i+j+1} & \frac{1}{2(i+j+1)} \end{array} \right]^T$$

Moreover,  $x(0, 0) = x(0, 1) = x(1, 0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ . The Markovian jumping modes during the simulation are shown in Fig.1 in which  $r_k = 1$  indicates that the system is in the first mode and  $r_k = 2$  indicates that the system is in the second mode. With the mode information and the initial values, a simulation can be run when the system is not controlled by the designed controller. Fig.2 and Fig.3 illustrate the state trajectories when the system is not controlled. It is obvious that the system is unstable. Fig.4 and Fig.5 depict the state trajectories when the system is controlled by the designed controller. We can see that the states converge to zero asymptotically, that is, the designed controller successfully stabilizes the unstable 2-D system.

## **VI. CONCLUSIONS**

We have investigated robust passivity control for 2-D uncertain Markovian jump linear discrete-time systems. It was assumed that the 2-D system parameters are subject to homogenous Markovian jumps and the transition-mode probability matrix is known beforehand. A mode-dependent state-feedback controller was proposed, and a stochastic closed-loop system was obtained. Both the passivity and the dissipation of the closed-loop systemwere investigated, and the gains were calculated. The performance of the controller was verifiedusing a2-D case.

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