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New Closed-Form Results on Ordered Statistics of Partial Sums of Gamma Random Variables and Its Application to Performance Evaluation in the Presence of Nakagami Fading

SUNG SIK NAM[1], (Member, IEEE), YOUNG-CHAI KO[1], (Senior Member, IEEE), AND MOHAMED-SLIM ALOUINI[2], (Fellow, IEEE)
[1]School of Electrical Engineering, Korea University, Seoul 02841, South Korea
[2]Computer, Electrical, and Mathematical Sciences and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia

Corresponding author: Young-Chai Ko (koyc@korea.ac.kr)

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ABSTRACT Complex wireless transmission systems require multi-dimensional joint statistical techniques for performance evaluation. Here, we first present the exact closed-form results on order statistics of any arbitrary partial sums of gamma random variables with the closed-form results of core functions specialized for independent and identically distributed Nakagami-*m* fading channels based on a moment generating function-based unified analytical framework. These both exact closed-form results have never been published in the literature. In addition, as a feasible application example in which our new offered derived closed-form results can be applied is presented. In particular, we analyze the outage performance of the finger replacement schemes over Nakagami fading channels as an application of our method. Note that these analysis results are directly applicable to several applications, such as millimeter-wave communication systems in which an antenna diversity scheme operates using a finger replacement schemes-like combining scheme, and other fading scenarios. Note also that the statistical results can provide potential solutions for ordered statistics in any other research topics based on gamma distributions or other advanced wireless communications research topics in the presence of Nakagami fading.

INDEX TERMS Fading channels, outage performance, order statistics, partial sums, Nakagami-*m* fading.

I. INTRODUCTION

Order statistics have played a critical role in the design and analysis of many emerging wireless transmission techniques, such as advanced diversity combining, channel adaptive transmission, and multiuser scheduling [2]–[17]. Previous order statistics results in [5]–[9] were obtained based on conventional or slightly modified statistical theories (e.g., simple one or two dimensional joint statistics). Later, with the advent of complex transmission systems, more complicated multi-dimensional joint statistical techniques became necessary [10]–[17]. Some previous results have been helpful in the accurate quantification of performance versus complexity among different transmission design options. Other results, however, such as the joint distribution functions of linear functions of ordered random variables (RVs) are not helpful due to their high complexity. Comprehensive analysis of how both conventional and new order statistics results help in obtaining the desired statistics of the received output signal-to-noise ratio (SNR) in wireless transmission systems has not yet been reported.

Recently, [18], [19] introduced new results to determine the joint statistics of partial sums of ordered exponential RVs. In [19], a successive conditioning approach was used to convert dependent ordered RVs into independent unordered RVs. Obtaining distribution functions, including the probability density function (PDF), the cumulative distribution function (CDF), and the moment generating function (MGF), is now possible with this framework and related results. However, this approach requires some case-specific manipulations, which may not always be generalizable. In [18], we introduced a unified analytical framework to determine the joint statistics of partial sums of ordered RVs using an MGF-based approach. With our proposed approach, the joint statistics of any arbitrary partial sums of ordered statistics in terms of MGF and PDF, especially in the presence of Rayleigh fading, can be derived systematically.

On another front, the Nakagami-m distribution often gives the best fit to urban [20] and indoor [21] multipath propagation of wireless transmission. Most importantly, Nakagami fading captures a wide range of multipath channels via the fading parameter, m, including the Rayleigh distribution ($m = 1$) as a special case [22]. In addition, when $m > 1$, the Nakagami-m distribution closely approximates the Rice distribution [22] by one-to-one mapping between the Rician factor and the Nakagami fading parameter. Some analytical results on Nakagami-m fading assumptions based on order statistics can be found in [6] and [23]–[25]. However, in most cases, fundamental one- or trivial two-dimensional joint statistical results are provided. These results do not lend themselves to more sophisticated performance evaluation. Thus far, no exact closed-form results, even simplified results, of complicated multi-dimensional joint statistics under Nakagami fading conditions are available in the literature. The primary goal of this paper is thus to provide new exact closed-form results on the order statistics of any arbitrary partial sums of Gamma random variables, we present a feasible performance evaluation example, in which we apply closed-form results under independent and identically distributed (i.i.d.) Nakagami-m fading conditions to the MGF-based unified approach in [18].

A. MAIN CONTRIBUTIONS

The main contributions and points of difference between the previous works and this work are briefly summarized as follows:

- In [18], some closed-form results for Rayleigh fading assumptions were provided using a unified MGF-based approach. Especially, with the newly provided MGF-based unified framework and related core functions specialized for Rayleigh fading, the joint statistic closed-form results of any arbitrary partial sums of ordered statistics were derived systematically. Although [18] provides new useful closed-form results on ordered statistics, deriving the closed-form results over Nakagami fading channel is another challenge. Therefore, in this paper, we provide some new closed-form results of core functions specialized for Nakagami fading and then with these results, some exact closed-form results on ordered statistics of partial sums of Gamma random variables are newly provided. These both exact closed-form results have never been published in the literature and may stimulate researchers to find new results in the general order statistics theory.
- As a feasible application example in which our derived joint statistic closed-form results can be applied, we consider the outage performance analysis of the finger replacement schemes (FRS) proposed in [26] by extending channel model to Nakagami-m fading channels. It is very noticeable that the FRS in [26] can also apply to the new "trendy" applications such as millimeter-wave (mmWave) communication systems in which an antenna diversity scheme operates using an FRS-like combining scheme. In mmWave systems, with an increase of the number of Rake fingers, a significant improvement is expected because the channel impulse response is completely decayed in a very short time period compared with the typical RAKE receiver based systems (i.e., carrier frequencies below 10 GHz) [27], [28]. Therefore, a larger number of fingers are required while there exist the limited number of fingers in the mobile unit. This can point to very clear conclusion that it is more necessary to apply the low complexity and low power consumption finger management schemes with a minimal amount of additional network resources for RAKE reception in the SHO region with multiple base stations (BSs) to achieve the required performance. Here, for mmWave communication systems, Nakagami assumption is more proper than Rayleigh assumption because it is not always possible to satisfy Rayleigh criterion [29], [30]. However, in [26], the author has investigated and analyzed the performance over i.i.d. Rayleigh fading environments with multiple BSs based on the statistical derivation approach used in [19]. In [19], the required joint statistics of ordered RVs were obtained by applying the conditional PDF based approach proposed. However, this approach is limited to when assuming Rayleigh fading from path to path and does not allow for similar simplifications for Nakagami case. Therefore, we address this mathematical issue by providing a general comprehensive analysis framework for outage performance analysis in the presence of Nakagami fading by adopting the MGF-based unified approach in [18] instead of [19]. More specifically, we slightly modify the performance analysis framework used in [26] to make it suitable for these newly derived joint statistical results.

Note that the slightly modified analytical framework suitable for the derived statistical results can also be configured to be directly applicable to other various fading scenarios while the analytical framework in [26], the conditional PDF based approach, and related results were limited only to i.i.d. Rayleigh fading assumptions. Note also that our derived statistical results are much simpler than the original multiple-fold integral forms based on the conventional MGF based approaches.

II. SYSTEM MODELS AND STATISTICAL ANALYSIS OF THE OUTAGE PERFORMANCE OF APPLICATION EXAMPLE

Here, we consider the full scanning method in [26] in the presence of Nakagami fading. Using the system model assumptions in [26], we assume that L base stations (BSs) are active and that there are a total of $N_{(L)}$ resolvable paths which is defined as $N_{(L)} = \sum_{n=1}^{L} N_n$ where N_n is the number of resolvable paths from the n-th BS. [26] assumed that in the soft handover (SHO) region, for RAKE reception, only N_c out of $N_{(n)}$ $(1 \leq n \leq L)$ paths are used. Without loss of generality, N_1 is defined as the number of resolvable paths from the serving BS while $N_2, N_3, \cdots, N_L$ are defined as those from the target BSs. In the SHO region, the receiver is assumed at first to rely only on N_1 resolvable paths and, as such, to start with N_c/N_1-generalized selection combining (GSC) [5] which combines the strongest N_c resolvable paths among the N_1 available ones. These schemes are based on the comparison of blocks consisting of $N_s\,(< N_c < N_n)$ paths from each BS.

If we let Y be the sum of the $N_c - N_s$ strongest paths from the serving BS, $Y = \sum_{i=1}^{N_c-N_s} \gamma_{i:N_1}$, and W_n be the sum of the N_s smallest paths from the serving BS for $n = 1$ and be the sum of the N_s strongest paths from the target BS for $n = 2, \cdots, L$, $W_n = \sum_{i=N_c-N_s+1}^{N_c} \gamma_{i:N_n}$ for $n = 1$ and $W_n = \sum_{i=1}^{N_s} \gamma_{i:N_n}$ for $n = 2, \cdots, L$, then after GSC, the received output SNR becomes $Y + W_1$, where $\gamma_{i:N_n}$ $(i = 1, 2, \cdots, N_n)$ is the i-th order statistics out of N_n SNRs of paths from the n-th BS by arranging N_n nonnegative i.i.d. RVs, $\{\gamma_j\}_{j=1}^{N_n}$, where γ_j is the SNR of the j-th path from the n-th BS, such that $\gamma_{i:N_1} \geq \gamma_{i:N_2} \geq \cdots \geq \gamma_{i:N_n}$. Based on [26], the receiver compares the output SNR, $Y + W_1$, with a certain target SNR at the beginning of every time slot. Then, if the sum of the $N_c - N_s$ strongest paths from the serving BS and the N_s smallest paths from the serving BS, $Y + W_1$ is greater than or equal to the target SNR, a one-way SHO is used and no finger replacement is needed. On the other hand (i.e., $Y + W_1$ falls below the target SNR), the receiver attempts a two-way SHO by starting to scan additional paths from other target BSs.

To show the validity of our derivations, we consider outage performance. We modify the mathematical analysis framework in [26] to make it suitable for our newly derived joint statistical results. This framework to determine outage performance is configured to be directly applicable to other fading scenarios with the help of the unified MGF-based approach in [18] rather than the approaches in [26] and [19]. Based on the mode of operation in [26, Sec. II-B], an overall outage probability is declared when the final combined SNR, γ_F, falls below a predetermined threshold, x. Based on it, we can define the outage probability as $F_{\gamma_F}(x) = \Pr[\gamma_F < x]$, where $\gamma_F = Y + W_1$ for $Y + W_1 \geq \gamma_T$ and $\gamma_F = Y + \max\{W_1, W_2, \cdots, W_L\}$ for $Y + W_1 < \gamma_T$. Then, by separately considering two cases i) when the combined SNR falls below the target SNR (i.e., $0 < x < \gamma_T$) and ii) when the combined SNR is greater than or equal to the target SNR, γ_T, (i.e., $x \geq \gamma_T$), the outage probability can be rewritten as

$$F_{\gamma_F}(x) = \begin{cases} \Pr[Y + \max\{W_1, W_2, \cdots, W_L\} < x], & 0 < x < \gamma_T; \\ \Pr[\gamma_T \leq Y + W_1 < x] \\ \quad + \Pr[Y + W_1 < \gamma_T, \gamma_T \leq Y \\ \quad + \max\{W_1, W_2, \cdots, W_L\} < x], & x \geq \gamma_T. \end{cases} \tag{1}$$

Here, Y and W_n for $n = 1$ are correlated while Y and W_n $(n = 2, \cdots, L)$ are independent. Thus, by adopting the proposed mathematical approach in [18] instead of applying [19], we can evaluate key statistics in (1) as

$$\Pr[\gamma_T \leq Y + W_1 < x] = F_{Y+W_1}(x) - F_{Y+W_1}(\gamma_T), \tag{2}$$

$$\Pr[Y + \max\{W_1, W_2, \cdots, W_L\} < x] = \int_0^x \int_0^{x-y} f_{Y,W_1}(y, w_1) \prod_{n=2}^{L} F_{W_n}(x-y)\, dw_1 dy, \tag{3}$$

and

$$\Pr[Y + W_1 < \gamma_T, \gamma_T \leq Y + \max\{W_1, W_2, \cdots, W_L\} < x] = \int_0^{\gamma_T} \int_0^{\gamma_T - y} f_{Y,W_1}(y, w_1) \prod_{n=2}^{L} F_{W_n}(x-y)\, dw_1 dy. \tag{4}$$

III. JOINT STATISTICS OF PARTIAL SUMS OF ORDERED RANDOM VARIABLE OVER I.I.D. NAKAGAMI-*M* FADING

A. MAIN APPROACH

For the Nakagami-m fading case, the instantaneous SNR, γ, has the PDF given by [2, eq. (2.55)]

$$p(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\bar{\gamma}}\gamma\right), \quad \gamma \geq 0, \tag{5}$$

where $\Gamma(\cdot)$ denotes the gamma function [31, eq. (8.310.1)] and $\bar{\gamma}$ is the common average faded SNR. Note that the major difficulty lies in deriving the required joint statistics of ordered RVs. In [26], by applying the conditional PDF based approach proposed in [19], the required joint statistics were obtained, especially with an assumption of i.i.d. Rayleigh fading. However, our concern is Nakagami-m fading which includes a wide range of multipath channels via the fading parameter, m, [22]. In this setting, we cannot directly adopt the proposed method in [19]. Hence, we borrow the concept of the unified MGF-based systematical framework proposed in [18].

B. COMMON CORE FUNCTIONS AND RELATIONS

For mathematical tractability, let us consider integer-order fading parameters (i.e., m takes positive integer values). Even with integer fading parameter values, closed-form results of partial sums of ordered RV over Nakagami-m fading remain an open problem. Many previous studies [32]–[39]

$$[c(\gamma,-s_i)]^n = \left(\frac{m}{\bar{\gamma}}\right)^{n\cdot m}\left(\frac{m}{\bar{\gamma}}+s_i\right)^{-n\cdot m}\sum_{k=0}^{n}\binom{n}{k}(-1)^k\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_i\right)k\cdot\gamma\right)$$
$$\times\sum_{\substack{n_1,n_2,\cdots,n_m\ge 0\\ n_1+n_2+\cdots+n_m=k}}\binom{k}{n_1,n_2,\cdots,n_m}\frac{\gamma^{N(m)}}{\prod_{l=0}^{m-1}(l!)^{n_{l+1}}}\sum_{j=0}^{N(m)}\binom{N(m)}{j}\left(\frac{m}{\bar{\gamma}}\right)^{N(m)-j}s_i^j, \tag{6}$$

$$[e(\gamma,-s_i)]^n = \left(\frac{m}{\bar{\gamma}}\right)^{n\cdot m}\left(\frac{m}{\bar{\gamma}}+s_i\right)^{-n\cdot m}\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_i\right)n\cdot\gamma\right)$$
$$\times\sum_{\substack{n_1,n_2,\cdots,n_m\ge 0\\ n_1+n_2+\cdots+n_m=n}}\binom{n}{n_1,n_2,\cdots,n_m}\frac{\gamma^{N(m)}}{\prod_{l=0}^{m-1}(l!)^{n_{l+1}}}\sum_{j=0}^{N(m)}\binom{N(m)}{j}\left(\frac{m}{\bar{\gamma}}\right)^{N(m)-j}s_i^j, \tag{7}$$

and

$$[\mu(\gamma_a,\gamma_b,-s_i)]^n = \left(\frac{m}{\bar{\gamma}}\right)^{n\cdot m}\left(\frac{m}{\bar{\gamma}}+s_i\right)^{-n\cdot m}\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_i\right)\gamma_a\cdot n\right)\sum_{h=0}^{n}\binom{n}{h}(-1)^{n-h}$$
$$\times\sum_{\substack{n_1,n_2,\cdots,n_m\ge 0\\ n_1+n_2+\cdots+n_m=h}}\sum_{\substack{n'_1,n'_2,\cdots,n'_m\ge 0\\ n'_1+n'_2+\cdots+n'_m=n-h}}\binom{h}{n_1,n_1,\cdots,n_m}\binom{n-h}{n'_1,n'_2,\cdots,n'_m}$$
$$\times\frac{{\gamma_a}^{N(m)}}{\prod_{l=0}^{m-1}(l!)^{n_{l+1}}}\cdot\frac{{\gamma_b}^{M(m)}}{\prod_{k=0}^{m-1}(k!)^{n'_{k+1}}}\sum_{j=0}^{N(m)}\sum_{q=0}^{M(m)}\binom{N(m)}{j}\binom{M(m)}{q}\left(\frac{m}{\bar{\gamma}}\right)^{N(m)+M(m)-j-q}s_i^{j+q}, \tag{8}$$

where $N(m)=\sum_{l=0}^{m-1} l\cdot n_{l+1}$ and $M(m)=\sum_{k=0}^{m-1} k\cdot m_{k+1}$.

focused on performance analysis over Nakagami fading channels with the integer fading parameter. These works showed that the integer fading parameter is sufficient to model a wide range of fading conditions and can cover most cases of interest in practice (e.g., for many practical channels, $1\le m\le 15$, [32]).

Here, we first observe three common core functions of i.i.d. Nakagami distributions: i) a mixture of a CDF and an MGF, $c(\gamma,\lambda)=\int_0^{\gamma}dx\,p(x)\exp(\lambda x)$, ii) a mixture of an exceedance distribution function (EDF) and an MGF, $e(\gamma,\lambda)=\int_{\gamma}^{\infty}dx\,p(x)\exp(\lambda x)$, and iii) an interval MGF, $\mu(\gamma,\lambda)=\int_{\gamma_a}^{\gamma_b}dx\,p(x)\exp(\lambda x)$, where γ is real and λ can be complex [18, Sec. III-A]. We further consider the n-th power of these common core functions for arbitrary n, such as $[c(\gamma,\lambda)]^n$, $[e(\gamma,\lambda)]^n$, and $[\mu(\gamma,\lambda)]^n$. The closed-form results of these functions will play a important role to simplify the derivation of joint MGFs in later sections.

As shown in Appendix A, each function can be expressed in a finite summation form, enabling us to apply an inverse Laplace transform (LT) with the MGF expressions in deriving the closed-form expressions of the final PDF. The resulting n-th power of common core functions are as shown in top of this page.

In the special case of the Rayleigh fading channel ($m=1$), the results are given in [18]. With (6)-(8), as shown at the top of this page, and the unified framework for Rayleigh fading assumptions in [18], we can obtain the generic MGF expressions in a compact form as well as the desired PDF expressions through an inverse LT (see Appendix B).

In what follows, we show how our results can be greatly simplified. Let $Z_1=\sum_{i=1}^{n}\gamma_{i:N}$ and $Z_2=\sum_{i=n+1}^{N}\gamma_{i:N}$ for example. Then, the original second-order MGF expression of $Z=[Z_1,Z_2]$ can be written as an N-fold integral expression

$$\begin{aligned}
&MGF_Z(\lambda_1,\lambda_2)\\
&\quad= E\{\exp(\lambda_1 Z_1+\lambda_2 Z_2)\}\\
&\quad=\frac{N!}{(N-n)!\,(n-1)!}\int_0^{\infty}d\gamma_{1:N}p(\gamma_{1:N})\exp(\lambda_1\gamma_{1:N})\\
&\qquad\cdots\int_0^{\gamma_{n-1:N}}d\gamma_{n:N}p(\gamma_{n:N})\exp(\lambda_1\gamma_{n:N})\\
&\qquad\times\int_0^{\gamma_{n:N}}d\gamma_{n+1:N}p(\gamma_{n+1:N})\exp(\lambda_2\gamma_{n+1:N})\\
&\qquad\cdots\int_0^{\gamma_{K-1:N}}d\gamma_{N:N}p(\gamma_{N:N})\exp(\lambda_2\gamma_{N:N}).
\end{aligned}\tag{9}$$

Following from (9) and simplifying the N-fold integral expression with the help of the interchange of multiple integrals technique and simplified results given in [18, eqs. (10) and (12)], the 2-dimensional joint PDF of $Z = [Z_1, Z_2]$ can be expressed, more specifically by applying the PDF of the RV of interest, specifically (5), into the simplified form [18, eq. (25)], as

$$\begin{aligned} f_Z(z_1, z_2) &= \mathcal{L}^{-1}_{S_1,S_2}\{MGF_Z(-S_1, -S_2)\} \\ &= \frac{N!}{(N-n)!\,(n-1)!} \\ &\quad\times \int_0^\infty d\gamma_{n:N} \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma_{n:N}{}^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\bar{\gamma}}\cdot\gamma_{n:N}\right) \\ &\quad\times \mathcal{L}^{-1}_{s_1}\left\{\exp(-s_1\gamma_{n:N})\left[e(\gamma_{n:N}, -s_1)\right]^{n-1}\right\} \\ &\quad\times \mathcal{L}^{-1}_{s_2}\left\{\left[c(\gamma_{n:N}, -s_2)\right]^{N-n}\right\}. \end{aligned} \tag{10}$$

Then, adapting (7) to (10) yields the first inverse LT term as

$$\begin{aligned} &\mathcal{L}^{-1}_{s_1}\left\{\exp(-s_1\gamma_{n:N})\left[e(\gamma_{n:N}, -s_1)\right]^{n-1}\right\} \\ &= \sum_{\substack{n'_1,n'_2,\cdots,n'_m\ge 0\\ n'_1+n'_2+\cdots+n'_m=n-1}} \binom{n-1}{n'_1, n'_2, \cdots, n'_m}\left(\frac{m}{\bar{\gamma}}\right)^{(n-1)\cdot m} \\ &\quad\times \exp\left(-\frac{m}{\bar{\gamma}}\cdot\gamma_{n:N}\right)\frac{\gamma_{n:N}{}^{N'(m)}}{\prod_{l'=0}^{m-1}(l'!)^{n'_{l'+1}}} \\ &\quad\times \sum_{j'=0}^{N'(m)}\binom{N'(m)}{j'}\left(\frac{m}{\bar{\gamma}}\right)^{N'(m)-j'} \\ &\quad\times \mathcal{L}^{-1}_{s_1}\left\{s_1{}^{j'}\left(\frac{m}{\bar{\gamma}}+s_1\right)^{-(n-1)\cdot m} \right. \\ &\qquad\qquad \left. \times\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_1\right)\cdot n\cdot\gamma_{n:N}\right)\right\}, \end{aligned} \tag{11}$$

where $N'(m) = \sum_{\ell'=0}^{m-1}\ell'\cdot n'_{\ell'+1}$ and, with the help of (38) in Appendix B,

$$\begin{aligned} &\mathcal{L}^{-1}_{s_1}\left\{s_1{}^{j'}\left(\frac{m}{\bar{\gamma}}+s_1\right)^{-(n-1)\cdot m}\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_1\right)\cdot n\cdot\gamma_{n:N}\right)\right\} \\ &= \begin{cases} \exp\left(-\frac{m}{\bar{\gamma}}\cdot n\cdot\gamma_{n:N}\right)\frac{(z_1-n\cdot\gamma_{n:N})^{(n-1)\cdot m-1}}{((n-1)\cdot m)!} \\ \quad\times\exp\left(-\frac{m}{\bar{\gamma}}\cdot(z_1-n\cdot\gamma_{n:N})\right)U(z_1-n\cdot\gamma_{n:N}), \\ \quad\text{for } j'=0 \\ \exp\left(-\frac{m}{\bar{\gamma}}\cdot n\cdot\gamma_{n:N}\right)\left[\frac{d^{j'}g(z_1-n\cdot\gamma_{n:N})}{dz_1{}^{j'}}\right. \\ \quad\left.+\sum_{k'=0}^{j'-1}g^{(k')}(0)\,\delta^{(j'-k'-1)}(z_1-n\cdot\gamma_{n:N})\right] \\ \quad\times U(z_1-n\cdot\gamma_{n:N}),\quad\text{for } j'>0. \end{cases} \end{aligned} \tag{12}$$

where

$$g(t) = \frac{t^{(n-1)\cdot m-1}\exp(-at)}{((n-1)\cdot m-1)!}, \tag{13}$$

or equivalently (12) can be also simplified when $n > m$ as

$$\begin{aligned} &\mathcal{L}^{-1}_{s_1}\left\{s_1{}^{j'}\left(\frac{m}{\bar{\gamma}}+s_1\right)^{-(n-1)\cdot m}\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_1\right)\cdot n\cdot\gamma_{n:N}\right)\right\} \\ &= \exp\left(-\frac{m}{\bar{\gamma}}\cdot n\cdot\gamma_{n:N}\right)\frac{(z_1-n\cdot\gamma_{n:N})^{(n-1)\cdot m-j'-1}}{((n-1)\,m)!} \\ &\quad\times {}_1\tilde{F}_1\left((n-1)\,m, (n-1)\,m-j', -\frac{m}{\bar{\gamma}}(z_1-n\cdot\gamma_{n:N})\right) \\ &\quad\times U(z_1-n\cdot\gamma_{n:N}). \end{aligned} \tag{14}$$

Similarly, with (6), the second inverse LT term in (10) can also be written as

$$\begin{aligned} &\mathcal{L}_{s_2}{}^{-1}\left\{\left[c(\gamma_{n:N}, -s_2)\right]^{N-n}\right\} \\ &= \sum_{k=0}^{N-n}\sum_{\substack{n_1,n_2,\cdots,n_m\ge 0\\ n_1+n_2+\cdots+n_m=k}}\binom{N-n}{k}\binom{k}{n_1, n_2, \cdots, n_m}(-1)^k \\ &\quad\times\left(\frac{m}{\bar{\gamma}}\right)^{(N-n)\cdot m}\sum_{j=0}^{N(m)}\binom{N(m)}{j}\left(\frac{m}{\bar{\gamma}}\right)^{N(m)-j} \\ &\quad\times\mathcal{L}_{s_2}{}^{-1}\left\{s_2{}^{j}\left(\frac{m}{\bar{\gamma}}+s_2\right)^{-(N-n)\cdot m}\right. \\ &\qquad\qquad\left.\times\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_2\right)\cdot k\cdot\gamma_{n:N}\right)\right\}, \end{aligned} \tag{15}$$

and in (15) the inverse LT term, $\mathcal{L}_{s_2}{}^{-1}\{\cdot\}$, can be obtained as

$$\begin{aligned} &\mathcal{L}^{-1}_{s_2}\left\{s_2{}^{j}\left(\frac{m}{\bar{\gamma}}+s_2\right)^{-(N-n)\cdot m}\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_2\right)\cdot k\cdot\gamma_{n:N}\right)\right\} \\ &= \begin{cases} \exp\left(-\frac{m}{\bar{\gamma}}\cdot k\cdot\gamma_{n:N}\right)\frac{(z_2-k\cdot\gamma_{n:N})^{(N-n)\cdot m-1}}{((N-n)\cdot m-1)!} \\ \times\exp\left(-\frac{m}{\bar{\gamma}}\cdot(z_2-k\cdot\gamma_{n:N})\right)U(z_2-k\cdot\gamma_{n:N}), \\ \quad\text{for } j=0 \\ \exp\left(-\frac{m}{\bar{\gamma}}\cdot k\cdot\gamma_{n:N}\right)\left[\frac{d^{j}g(z_2-k\cdot\gamma_{n:N})}{dz_2{}^{j}}\right. \\ \quad\left.+\sum_{k'=0}^{j-1}g^{(k')}(0)\,\delta^{(j-k'-1)}(z_2-k\cdot\gamma_{n:N})\right] \\ \quad\times U(z_2-k\cdot\gamma_{n:N}),\quad\text{for } j>0 \end{cases} \end{aligned} \tag{16}$$

where

$$g(t) = \frac{t^{(N-n)\cdot m-1}\exp(-at)}{((N-n)\cdot m-1)!}, \tag{17}$$

or equivalently for $n > m$, the inverse LT term in (15) can be obtained as

$$\begin{aligned} &\mathcal{L}^{-1}_{s_2}\left\{s_2{}^{j}\left(\frac{m}{\bar{\gamma}}+s_2\right)^{-(N-n)\cdot m}\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_2\right)\cdot k\cdot\gamma_{n:N}\right)\right\} \\ &= \exp\left(-\frac{m}{\bar{\gamma}}\cdot k\cdot\gamma_{n:N}\right)(z_2-k\cdot\gamma_{n:N})^{(N-n)\cdot m-j-1} \\ &\quad\times {}_1\tilde{F}_1\left((N-n)\,m, (N-n)\,m-j, -\frac{m}{\bar{\gamma}}(z_2-k\cdot\gamma_{n:N})\right) \\ &\quad\times U(z_2-k\cdot\gamma_{n:N}). \end{aligned} \tag{18}$$

IV. CLOSED-FORM EXPRESSIONS FOR KEY JOINT STATISTICS FOR FRS

In this section, we investigate the following key joint statistics for outage performance evaluation: $f_{Y,W_1}(\cdot,\cdot)$, $F_{Y+W_1}(\cdot)$, and $F_{W_n}(\cdot)$ for $2 \le n \le L$ in Sec. II.

A. TWO-DIMENSIONAL JOINT PDF OF TWO ADJACENT PARTIAL SUMS OF ORDERED RVs, $f_{Y,W_1}(x,y)$

In this case, the target 2-dimensional joint PDF of $Y = \sum_{i=1}^{N_c-N_s} \gamma_{i:N_1}$ and $W_1 = \sum_{i=N_c-N_s+1}^{N_c} \gamma_{i:N_1}$ can be obtained with the 4-dimensional joint PDF of $Z_1 = \sum_{i=1}^{N_c-N_s-1} \gamma_{i:N_1}$, $Z_2 = \gamma_{N_c-N_s:N_1}$, $Z_3 = \sum_{i=N_c-N_s+1}^{N_c-1} \gamma_{i:N_1}$, and $Z_4 = \gamma_{N_c:N_1}$, where the order statistics of N_1 resolvable paths can be viewed as

$$\overbrace{\underbrace{\gamma_{1:N_1},\cdots,\gamma_{N_c-N_s-1:N_1}}_{Z_1},\underbrace{\gamma_{N_c-N_s:N_1}}_{Z_2}}^{Y},$$
$$\overbrace{\underbrace{\gamma_{N_c-N_s+1:N_1},\cdots,\gamma_{N_c-1:N_1}}_{Z_3},\underbrace{\gamma_{N_c:N_1}}_{Z_4}}^{W_1},\gamma_{N_c+1:N_1},\cdots,\gamma_{N_1:N_1}. \tag{19}$$

In (19), Z_1, Z_2, Z_3, and Z_4 have the following conditions, such that $Z_4 < Z_2$, $Z_1 > (N_c - N_s - 1)Z_2$ and $(N_s-1)Z_4 < Z_3 < (N_s-1)Z_2$. Based on these conditions and with the help of a function of a marginal PDF, the joint PDF of Y and W_1, f_{Y,W_1}, can be obtained by integrating out z_2 and z_4 as

$$f_{Y,W_1}(x,y) = \int_0^{\frac{y}{N_s}} \int_{\frac{y}{N_s}}^{\frac{x}{N_c-N_s}} f_{Z_1,Z_2,Z_3,Z_4}(x-z_2,z_2,y-z_4,z_4)\,dz_2 dz_4. \tag{20}$$

Here, by adopting the unified MGF approach proposed in [18], we obtain the 4-dimensional joint PDF in (20), $f_{Z_1,Z_2,Z_3,Z_4}(z_1,z_2,z_3,z_4)$, for i.i.d. Nakagami-$m$ fading assumption after applying (6), (7), (8), and (5) to the generic 2-dimensional PDF form in [18, eq. (42)] as given in (21) at the top of the next page. Then, we substitute (21) into (20) and then, after re-arranging and some simplification, (20) can be expressed as shown in (22).

Following the detailed derivations in Appendix C, we can obtain the closed-form expressions of the double integral term in (22) as follows,

i) For $h = N_s - 1$,

$$\left(\frac{m}{\bar{\gamma}}(N_s-1)\right)^{-\alpha-1}\left(\frac{m}{\bar{\gamma}}k\right)^{-\beta-1}\gamma\left(\beta+1,\frac{m}{\bar{\gamma}}\cdot\frac{k}{N_s}y\right)\times\left[\gamma\left(\alpha+1,\frac{m}{\bar{\gamma}}\cdot\frac{N_s-1}{N_c-N_s}x\right)-\gamma\left(\alpha+1,\frac{m}{\bar{\gamma}}\cdot\frac{N_s-1}{N_s}y\right)\right]. \tag{23}$$

ii) For $0 \le h \le N_s - 2$,

$$\begin{aligned}
&\left(\frac{m}{\bar{\gamma}}(N_s-1)\right)^{-\alpha-1}\left(\frac{m}{\bar{\gamma}}k\right)^{-\beta-1}\\
&\times\left[\left\{1-U\left(\frac{x}{N_c-N_s}-\frac{y}{N_s-(h+1)}\right)\right\}\right.\\
&\times\gamma\left(\beta+1,\frac{m}{\bar{\gamma}}ka\right)\left\{\gamma\left(\alpha+1,\frac{m}{\bar{\gamma}}\cdot\frac{N_s-1}{N_c-N_s}x\right)-1\right\}\\
&\left.-\gamma\left(\beta+1,\frac{m}{\bar{\gamma}}\cdot\frac{k}{N_s}y\right)\left\{\gamma\left(\alpha+1,\frac{m}{\bar{\gamma}}\cdot\frac{N_s-1}{N_s}y\right)-1\right\}\right]\\
&+\sum_{t_1=0}^{\alpha}\sum_{t_2=0}^{t_1}\binom{t_1}{t_2}\frac{(-1)^{t_1-t_2}\alpha!}{t_1!}\left(\frac{m}{\bar{\gamma}}(N_s-1)\right)^{t_1-\alpha-1}\\
&\times\frac{y^{t_2}(h+1)^{t_1-t_2}}{(N_s-(h+1))^{t_1}}\exp\left(-\frac{m}{\bar{\gamma}}\cdot\frac{N_s-1}{N_s-(h+1)}y\right)\\
&\times\left\{\frac{m}{\bar{\gamma}}\left(k-\frac{(N_s-1)(h+1)}{N_s-(h+1)}\right)\right\}^{-\beta-t_1+t_2-1}\\
&\times\left[\left\{1-U\left(\frac{x}{N_c-N_s}-\frac{y}{N_s-(h+1)}\right)\right\}\right.\\
&\times\gamma\left(\beta+t_1-t_2+1,\frac{m}{\bar{\gamma}}\left(k-\frac{(N_s-1)(h+1)}{N_s-(h+1)}\right)a\right)\\
&\left.-\gamma\left(\beta+t_1-t_2+1,\frac{m}{\bar{\gamma}}\cdot\frac{1}{N_s}\left(k-\frac{(N_s-1)(h+1)}{N_s-(h+1)}\right)y\right)\right],
\end{aligned} \tag{24}$$

where $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function [40, eq. (8.352.1)].

B. ONE-DIMENSIONAL CDF OF THE N_c/N_1-GSC OUTPUT SNR, $F_{Y+W_1}(x)$

For convenience, we let $Z' = Y + W_1$. Then, the target CDF of $Z' = \sum_{i=1}^{N_c} \gamma_{i:N_1}$ with the 2-dimensional joint PDF of $Z_1 = \sum_{i=1}^{N_c-1} \gamma_{i:N_1}$ and $Z_2 = \gamma_{N_c:N_1}$ can be obtained as

$$F_{Y+W_1}(x) = \int_0^x \int_0^{\frac{z}{N_c}} f_{Z_1,Z_2}(z-z_2,z_2)\,dz_2 dz. \tag{28}$$

Here, by applying a similar approach used in (11) and (21), and adopting the generic form in [18, eq. (44)] with (5) and the related common core functions given in (6) and (7), we can obtain the target 2-dimensional joint PDF, $f_{Z_1,Z_2}(z_1,z_2)$, in (26) at the top of the next page for the i.i.d. Nakagami-m fading assumption.

Substituting (26) into (28) and then, with the help of [31, eqs. (8.352.6), (3.381.1)] and then after some re-arranging and some mathematical simplification, (28) can be expressed as provided in (27).

$$
\begin{aligned}
&f_{Z_1,Z_2,Z_3,Z_4}(z_1,z_2,z_3,z_4)\\
&= \sum_{k=0}^{N_1-N_c} \sum_{\substack{n_1,n_2,\cdots,n_m\geq 0\\ n_1+n_2+\cdots+n_m=k}} \sum_{\substack{n'_1,\cdots,n'_m\geq 0\\ n'_1+\cdots+n'_m=N_c-N_s-1}} \sum_{j=0}^{N'(m)} \sum_{h=0}^{N_s-1} \sum_{\substack{n''_1,\cdots,n''_m\geq 0\\ n''_1+\cdots+n''_m=h}} \sum_{\substack{n'''_1,\cdots,n'''_m\geq 0\\ n'''_1+\cdots+n'''_m=N_s-1-h}} \sum_{j'=0}^{N''(m)} \sum_{q=0}^{M(m)}\\
&\quad\times \binom{N_1-N_c}{k}\binom{k}{n_1,n_2,\cdots,n_m}\binom{N_c-N_s-1}{n'_1,\cdots,n'_m}\binom{N'(m)}{j}\binom{N_s-1}{h}\binom{h}{n''_1,\cdots,n''_m}\binom{N_s-1-h}{n''_1,\cdots,n'''_m}\binom{N''(m)}{j'}\\
&\quad\times \binom{M(m)}{q}\frac{F(-1)^{k+N_s-1-h}\left(\frac{m}{\bar\gamma}\right)^{N(m)+N'(m)+N''(m)+M(m)+N_c\cdot m-j-j'-q}}{(N_c-N_s-1)!\,(N_s-1)!\Gamma(m)^2\prod_{\ell_1=0}^{m-1}(\ell_1!)^{n_{\ell_1+1}}\prod_{\ell_2=0}^{m-1}(\ell_2!)^{n'_{\ell_2+1}}\prod_{\ell_3=0}^{m-1}(\ell_3!)^{n''_{\ell_3+1}}\prod_{\ell_4=0}^{m-1}(\ell_4!)^{n'''_{\ell_4+1}}}\\
&\quad\times z_2^{N'(m)+M(m)+m-1}\exp\left(-\frac{m}{\bar\gamma}z_2\right)z_4^{M(m)+N''(m)+m-1}\exp\left(-\frac{m}{\bar\gamma}(1+k)z_4\right)U(z_2-z_4)\\
&\quad\times \mathcal{L}_{s_1}^{-1}\left\{\frac{s_1^j}{\left(\frac{m}{\bar\gamma}+s_1\right)^{(N_c-N_s-1)m}}\exp\left(-\left(\frac{m}{\bar\gamma}+s_1\right)(N_c-N_s-1)\,z_2\right)\right\}\\
&\quad\times \mathcal{L}_{s_3}^{-1}\left\{\frac{s_3^{j'+q}}{\left(\frac{m}{\bar\gamma}+s_3\right)^{(N_s-1)m}}\exp\left(-\left(\frac{m}{\bar\gamma}+s_3\right)(h\cdot z_4+(N_s-1-h)\,z_2)\right)\right\},
\end{aligned}
\tag{21}
$$

where $F=\frac{N_1!}{(N_1-N_c)!}$, $N''(m)=\sum_{\ell''=0}^{m-1}\ell''\cdot n''_{\ell''+1}$,

$$
\begin{aligned}
&\mathcal{L}_{s_1}^{-1}\left\{\frac{s_1^j}{\left(\frac{m}{\bar\gamma}+s_1\right)^{(N_c-N_s-1)m}}\exp\left(-\left(\frac{m}{\bar\gamma}+s_1\right)(N_c-N_s-1)\,z_2\right)\right\}\\
&\quad= \sum_{k'=0}^{j}\binom{j}{k'}\left(\frac{m}{\bar\gamma}\right)^{j-k'}\frac{(-1)^{j+k'}\prod_{k_1=0}^{k'}((N_c-N_s-1)\,m-k_1)}{((N_c-N_s-1)\,m)!}(z_1-(N_c-N_s-1)\,z_2)^{(N_c-N_s-1)m-1-k'}\\
&\qquad\times\exp\left(-\frac{m}{\bar\gamma}z_1\right)\exp\left(-\frac{m}{\bar\gamma}(N_c-N_s-1)\,z_2\right)U(z_1-(N_c-N_s-1)\,z_2),
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathcal{L}_{s_3}^{-1}\left\{\left(\frac{m}{\bar\gamma}+s_3\right)^{-(N_s-1)\cdot m}(s_3)^{j'+q}\exp\left(-\left(\frac{m}{\bar\gamma}+s_3\right)(h\cdot z_4+(N_c-N_s-1)\,z_2)\right)\right\}\\
&\quad= \sum_{k''=0}^{j'+q}\binom{j'+q}{k''}\left(\frac{m}{\bar\gamma}\right)^{j'+q-k''}\frac{(-1)^{j'+q+k''}\prod_{k_2=0}^{k''}((N_s-1)\cdot m-k_2)}{((N_s-1)\cdot m)!}\\
&\qquad\times(z_3-(h\cdot z_4+(N_s-1-h)\,z_2))^{(N_s-1)\cdot m-1-k''}\exp\left(-\frac{m}{\bar\gamma}z_3\right)U(z_3-(h\cdot z_4+(N_s-1-h)\,z_2)).
\end{aligned}
$$

C. ONE-DIMENSIONAL CDF OF THE SUMS OF THE N_s STRONGEST PATHS FROM EACH TARGET BS, $F_{W_n}(x)$

In this case, by applying a function of a marginal PDF with the 2-dimensional joint PDF of $Z'_1=\sum_{i=1}^{N_s-1}\gamma_{i:N_n}$ and $Z'_2=\gamma_{N_s:N_n}$, the target one-dimensional CDF of $W_n=\sum_{i=1}^{N_s}\gamma_{i:N_n}$ can be derived as

$$
F_{W_n}(x)=\int_0^x\int_0^{\frac{z}{N_s}} f_{Z'_1,Z'_2}\left(z-z'_2,z'_2\right)dz'_2dz. \tag{28}
$$

The closed-form expression of (28) can be easily obtained by replacing N_c and N_1 with N_s and N_n in (27), respectively. The closed-form result of the integral form in (22) can be

$$
\begin{aligned}
&f_{Y,W_1}(x,y) \\
&= \sum_{k=0}^{N_1-N_c} \sum_{\substack{n_1,n_2,\cdots,n_m\geq 0\\ n_1+\cdots+n_m=k}} \sum_{\substack{n'_1,\cdots,n'_m\geq 0\\ n'_1+\cdots+n'_m=N_c-N_s-1}} \sum_{j=0}^{N'(m)} \sum_{h=0}^{N_s-1} \sum_{\substack{n''_1,\cdots,n''_m\geq 0\\ n''_1+\cdots+n''_m=h}} \sum_{\substack{n'''_1,\cdots,n'''_m\geq 0\\ n'''_1+\cdots+n'''_m=N_s-1-h}} \sum_{j'=0}^{N''(m)} \sum_{q=0}^{M(m)} \sum_{k'=0}^{j} \sum_{k''=0}^{j'+q} \\
&\times \sum_{p_1=0}^{(N_c-N_s-1)m-1-k'} \sum_{p_2=0}^{(N_s-1)m-1-k''} \sum_{p_3=0}^{(N_s-1)m-1-k''-p_2} \binom{N_1-N_c}{k} \binom{k}{n_1,\cdots,n_m} \binom{N_c-N_s-1}{n'_1,\cdots,n'_m} \binom{N'(m)}{j} \binom{N_s-1}{h} \binom{h}{n''_1,\cdots,n''_m} \\
&\times \binom{N_s-1-h}{n'''_1,\cdots,n'''_m} \binom{N''(m)}{j'} \binom{M(m)}{q} \binom{j}{k'} \binom{j'+q}{k''} \binom{(N_c-N_s-1)m-1-k'}{p_1} \\
&\times \binom{(N_s-1)m-1-k''}{p_2} \binom{(N_s-1)m-1-k''-p_2}{p_3} \\
&\times \frac{F \prod_{k_1=0}^{k'}((N_c-N_s-1)m-k_1) \prod_{k_2=0}^{k''}((N_s-1)m-k_2)(N_c-N_s-1)^{p_1} h^{p_3} (N_s-1-h)^{(N_s-1)m-1-k''-p_2-p_3}}{\prod_{l_1=0}^{m-1}(l_1!)^{n_{l_1+1}} \prod_{l_2=0}^{m-1}(l_2!)^{n'_{l_2+1}} \prod_{l_3=0}^{m-1}(l_3!)^{n''_{l_3+1}} \prod_{l_4=0}^{m-1}(l_4!)^{n'''_{l_4+1}} (N_c-N_s-1)!(N_s-1)!\Gamma(m)^2 ((N_s-1)m)!\,((N_c-N_s-1)m)!} \\
&\times \left(\frac{m}{\bar{\gamma}}\right)^{\substack{N(m)+N'(m)+N''(m)\\+M(m)+N_c m-k'-k''}} \sum_{p_4=0}^{(N_c-N_s-1)m-1-k'-p_1} \sum_{p_5=0}^{p_2} \binom{(N_c-N_s-1)m-1-k'-p_1}{p_4} \binom{p_2}{p_5} x^{p_4} y^{p_5} (-1)^{\substack{k+(N_s-1-h)+j+k'\\+j'+q+k''-p_4-p_5}} \exp\left(-\frac{m}{\bar{\gamma}}(x+y)\right) \\
&\times \int_0^{\frac{y}{N_s}} \int_{\frac{y}{N_s}}^{\frac{x}{N_c-N_s}} {z_2}^{\alpha} {z_4}^{\beta} \exp\left(-\frac{m}{\bar{\gamma}}(N_s-1)z_2\right) \exp\left(-\frac{m}{\bar{\gamma}} k z_4\right) U(z_2-z_4)\, U(x-(N_c-N_s)z_2) \\
&\times U(y-((h+1)z_4+(N_s-(h+1))z_2))\, dz_2 dz_4, \qquad (22)
\end{aligned}
$$

where $\alpha = N'(m) + M(m) + (N_c-1)m - k' - k'' - 3 - p_2 - p_3 - p_4$ and $\beta = N(m) + N''(m) + m - 1 + p_3 + p_2 - p_4$.

$$
\begin{aligned}
f_{Z_1,Z_2}(z_1,z_2) &= \sum_{k=0}^{N_1-N_c} \sum_{\substack{n'_1,n'_2,\cdots,n'_m\geq 0\\ n'_1+n'_2+\cdots+n'_m=k}} \sum_{\substack{n_1,n_2,\cdots,n_m\geq 0\\ n_1+n_2+\cdots+n_m=N_c-1}} \sum_{j=0}^{N(m)} \binom{N_1-N_c}{k} \binom{k}{n'_1,n'_2,\cdots,n'_m} \binom{N(m)}{j} \\
&\times \frac{F(-1)^k \left(\frac{m}{\bar{\gamma}}\right)^{N'(m)+N(m)+N_c\cdot m-j}}{(N_c-1)!\,(m-1)! \left(\prod_{\ell'=0}^{m-1}(\ell'!)^{n'_{\ell'+1}}\right)\left(\prod_{\ell=0}^{m-1}(\ell!)^{n_{\ell+1}}\right)} z_2^{N'(m)+N(m)+m-1} \exp\left(-\frac{m}{\bar{\gamma}}(k+1)z_2\right) \\
&\times \mathcal{L}_{s_1}^{-1}\left\{\left(\frac{m}{\bar{\gamma}}+s_1\right)^{-(N_c-1)\cdot m} (s_1)^j \exp\left(-\left(\frac{m}{\bar{\gamma}}+s_1\right)(N_c-1)z_2\right)\right\}, \qquad (26)
\end{aligned}
$$

where $F = N_c!$ and

$$
\begin{aligned}
&\mathcal{L}_{s_1}^{-1}\left\{\left(\frac{m}{\bar{\gamma}}+s_1\right)^{-(N_c-1)\cdot m} (s_1)^j \exp\left(-\left(\frac{m}{\bar{\gamma}}+s_1\right)(N_c-1)z_2\right)\right\} \\
&= \sum_{k'=0}^{j} \binom{j}{k'} (-1)^{j+k'} \left(\frac{m}{\bar{\gamma}}\right)^{j-k'} \frac{\prod_{\ell=0}^{k'}((N_c-1)\cdot m-\ell)}{((N_c-1)\cdot m)!} (z_1-(N_c-1)z_2)^{(N_c-1)\cdot m-1-k'} \exp\left(-\frac{m}{\bar{\gamma}}z_1\right) U(z_1-(N_c-1)z_2).
\end{aligned}
$$

obtained by separately considering i) $h = N_s - 1$ and ii) $0 \leq h \leq N_s - 2$ as shown in (23) and (24), respectively.

V. DISCUSSIONS AND CONCLUSIONS

In this work, we provided new exact closed-form order statistics of partial sums of Gamma random variables by deriving the closed-form results of common core functions specialized for Nakagami fading. In addition, we analyzed the outage performance of FRS proposed in [26] operating over Nakagami-m fading channels as a feasible application example.

$$
\begin{aligned}
&F_{Y+W_1}(x) \\
&= \sum_{k=0}^{N_1-N_c} \sum_{\substack{n'_1,\cdots,n'_m\geq 0 \\ n'_1+\cdots+n'_m=k}} \sum_{\substack{n_1,\cdots,n_m\geq 0 \\ n_1+\cdots+n_m=N_c-1}} \sum_{j=0}^{N(m)} \sum_{k'=0}^{j} \sum_{p=0}^{(N_c-1)\cdot m+k'} \sum_{q=0}^{\substack{(N_c-1)m-1 \\ -k'-p}} \binom{N_1-N_c}{k}\binom{k}{n'_1,\cdots,n'_m} \\
&\quad\times \binom{N_c-1}{n_1,\cdots,n_m}\binom{N(m)}{j}\binom{j}{k'}\binom{(N_c-1)m-1+k'}{p}\binom{(N_c-1)m-1-k'-p}{q} \\
&\quad\times \frac{F(-1)^{k+j+(N_c-1)m-q-1}\prod_{\ell=0}^{k'}((N_c-1)m-\ell)(N_c-1)^p}{(N_c-1)!\,((N_c-1)m)!\,\Gamma(m)\left(\sum_{\ell'=0}^{m-1}(\ell'!)^{n'_{\ell'+1}}\right)\left(\prod_{\ell=0}^{m-1}(\ell!)^{n_{\ell+1}}\right)}\left(N'(m)+N(m)+m\cdot N_c-k'-q-2\right)! \\
&\quad\times\left[\gamma\left(q+1,\frac{m}{\bar{\gamma}}\cdot x\right)-\left\{\sum_{t=0}^{\substack{N'(m)+N(m)+mN_c \\ -k'-q-2}}\frac{1}{t!}(k)^{\substack{t-N'(m)-N(m) \\ -mN_c+k'+q+1}}\left(\frac{1}{N_c}\right)^t\left(1+\frac{k}{N_c}\right)^{-q-t-1}\gamma\left(q+t+1,\frac{m}{\bar{\gamma}}\left(1+\frac{k}{N_c}\right)x\right)\right\}\right].
\end{aligned}
\tag{27}
$$

If the closed-form results of the joint statistics, especially in order statistics, are not available, the numerical estimation of multi-fold integral expressions (e.g., the N-fold integrals given in (9)) are required. However, estimating them accurately as N increases is difficult even with conventional mathematical tools. When the N is large, estimating the analytical results is almost impossible. However, with closed-form results derived here, probabilistic analysis is numerically possible with conventional mathematical tools. Note that the closed-form expressions seems to be complicated and they can be further summarized in a functionalized shape. However, to demonstrate how we obtained the derived results and the feasibility of applying the derived results to an application, we maintain them in a minimally simplified form. With these results, the user can directly change/apply the obtained results in the form desired by the user. The closed-form expressions may appear to be complicated, but the numerical results can be easily obtained. On the other hand, it is almost impossible to obtain numerical results physically with conventional mathematical tools due to estimation difficulties.

Further, as a validation of our analytical formula for the outage probability, in Fig. 1, we cross-verified the analytical results and the simulation results obtained via Monte-Carlo simulation. Fig. 1 showed that the derived analytical results match the simulation results. As a result, we believe that we can accurately predict the performance with them.

Note that closed-form results on ordered statistics of partial sums of ordered random variables over Nakagami fading remained in an open problem, even with integer fading parameter values. Note also that although derived closed-form results limited to integer fading parameter values, they can still covers most cases of interest in practice. Therefore, in the view of contribution to ordered statistics, these new statistical results can provide the potential solution of both other ordered statistics in the presence of Nakagami fading in advanced wireless communications research and any other research topics based on Gamma distributions.

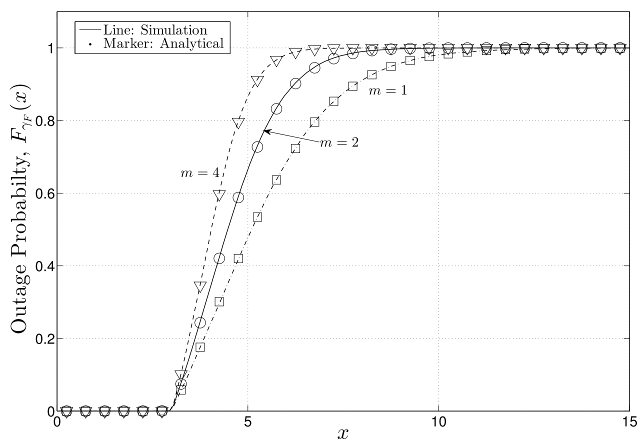

FIGURE 1. **Outage probability of finger replacement schemes for RAKE reception in the soft handover region over i.i.d. Nakagami-*m* fading channels when $L = 4$, $N_1 = \cdots = N_4 = 5$, $N_c = 3$, $N_s = 2$, $\gamma_T = 3$, and $\bar{\gamma} = 1$.**

APPENDIX A
DERIVATION OF CLOSED-FORM EXPRESSIONS OF THREE COMMON CORE FUNCTIONS

In Sec. III, with (5), we can write the mixture of a CDF and an MGF for the i.i.d. Nakagami fading assumption as

$$
c(\gamma,-s_i)=\left(\frac{m}{\bar{\gamma}}\right)^m\frac{1}{\Gamma(m)}\int_0^{\gamma}\gamma^{m-1}\exp\left(-\left(\frac{m}{\bar{\gamma}}+s_i\right)x\right)dx.
\tag{29}
$$

Then, with the help of [31, eq. (3.381.1) and eq. (8.352.1)], the closed-form result of (29) can be obtained as

$$\begin{aligned} &c\left(\gamma,-s_i\right)\\ &=\left(\frac{m}{\bar\gamma}\right)^m\frac{1}{\Gamma\left(m\right)}\left(\frac{m}{\bar\gamma}+s_i\right)^{-m}\gamma\left(m,\left(\frac{m}{\bar\gamma}+s_i\right)\gamma\right)\\ &\overset{\text{or}}{=}\left(\frac{m}{\bar\gamma}\right)^m\left(\frac{m}{\bar\gamma}+s_i\right)^{-m}\\ &\quad\times\left[1-\exp\left(-\left(\frac{m}{\bar\gamma}+s_i\right)\gamma\right)\sum_{l=0}^{m-1}\frac{\left[\left(\frac{m}{\bar\gamma}+s_i\right)\gamma\right]^l}{l!}\right]. \end{aligned}\tag{30}$$

With the summand expression in (30), by applying the binomial theorem, the n-th power of $c\left(\gamma,-s_i\right)$ for arbitrary n can be obtained as

$$\begin{aligned} \left[c\left(\gamma,-s_i\right)\right]^n=&\left(\frac{m}{\bar\gamma}\right)^{nm}\left(\frac{m}{\bar\gamma}+s_i\right)^{-nm}\\ &\times\sum_{k=0}^{n}\binom{n}{k}(-1)^k\exp\left(-\left(\frac{m}{\bar\gamma}+s_i\right)k\gamma\right)\\ &\times\left[\sum_{l=0}^{m-1}\frac{\left(\frac{m}{\bar\gamma}+s_i\right)^l\gamma^l}{l!}\right]^k. \end{aligned}\tag{31}$$

Here, with the help of the multinomial theorem, we obtain the following relationship:

$$\begin{aligned} &\left[\sum_{l=0}^{m-1}\frac{\left(\frac{m}{\bar\gamma}+s_i\right)^l\gamma^l}{l!}\right]^k\\ &=\sum_{\substack{n_1,n_1,\cdots,n_m\geq0\\ n_1+n_1+\cdots+n_m=k}}\binom{k}{n_1,n_1,\cdots,n_m}M_0^{n_1}M_1^{n_2}\cdots M_{m-1}^{n_m}, \end{aligned}\tag{32}$$

where $M_l=\left[\left(\frac{m}{\bar\gamma}+s_i\right)\gamma\right]^l/l!$ and some mathematical manipulations give us

$$\begin{aligned} &M_0^{n_1}M_1^{n_2}\cdots M_{m-1}^{n_m}\\ &\quad=\frac{\gamma^{N(m)}}{\prod_{l=0}^{m-1}(l!)^{n_{l+1}}}\sum_{j=0}^{N(m)}\binom{N\left(m\right)}{j}\left(\frac{m}{\bar\gamma}\right)^{N(m)-j}s_i^j, \end{aligned}\tag{33}$$

where $N\left(m\right)=\sum_{l=0}^{m-1}l\cdot n_{l+1}$. Thus, after successive substitution from (33) to (31), we can get (6).

Similarly, with the help of [31, eq. (3.381.3) and eq. (8.352.2)] and then by applying the binomial theorem, the closed-form expressions of the n-th power of $e\left(\gamma,-s_i\right)$ and $\mu\left(\gamma_a,\gamma_b,-s_i\right)$ for arbitrary n can be obtained as shown in (7) and (8), respectively.

APPENDIX B
INVERSE LT PAIR AND RELATED USEFUL FUNCTION

The following inverse LT is useful for the derivation of final PDF closed-form expressions from MGF expressions in Sec. III

$$\mathcal{L}_s^{-1}\left\{\frac{s^m}{\left(a+s\right)^n}\exp\left(-b\left(a+s\right)\right)\right\}.\tag{34}$$

Here, let $F\left(s\right)=\frac{s^m}{(a+s)^n}$, then, $\mathcal{L}_s^{-1}\left\{F\left(s\right)\right\}=f\left(t\right)$ and we can obtain the following inverse LT pair for $b>0$:

$$\begin{aligned} &\mathcal{L}_s^{-1}\left\{F\left(s\right)\exp\left(-b\left(a+s\right)\right)\right\}\\ &\qquad\overset{L.T}{\longleftrightarrow}\exp\left(-ba\right)f\left(t-b\right)U\left(t-b\right). \end{aligned}\tag{35}$$

In (35), let $G\left(s\right)=\frac{1}{(a+s)^n}$ and $\mathcal{L}_s^{-1}\left\{G\left(s\right)\right\}=g\left(t\right)$, then $F\left(s\right)=s^mG\left(s\right)$ and we can obtain the inverse LT pair of $F\left(s\right)$ by applying classical inverse LT pairs and properties as

$$\begin{aligned} F\left(s\right)=s^mG\left(s\right)&\overset{L.T}{\longleftrightarrow}f\left(t\right)\\ &=\frac{d^mg\left(t\right)}{dt^m}+\sum_{k=0}^{m-1}g^{(k)}\left(0\right)\delta^{(m-k-1)}\left(t\right), \end{aligned}\tag{36}$$

where $g\left(t\right)=\frac{t^{n-1}\exp(-at)}{(n-1)!}$. Therefore, the inverse LT pair in (34) can be obtained as

$$\begin{aligned} &\mathcal{L}_s^{-1}\left\{s^mG\left(s\right)\exp\left(-b\left(a+s\right)\right)\right\}\\ &\overset{L.T}{\longleftrightarrow}\exp(-ba)\left[\frac{d^mg(t-b)}{dt^m}+\sum_{k=0}^{m-1}g^{(k)}(0)\delta^{(m-k-1)}(t-b)\right]\\ &\quad\times U\left(t-b\right). \end{aligned}\tag{37}$$

Here, for $m>0$, $\sum_{k=0}^{m-1}g^{(k)}\left(0\right)\delta^{(m-k-1)}\left(t-b\right)U\left(t-b\right)\approx0$. Thus, (37) can be finally simplified as

$$\begin{aligned} &\mathcal{L}_s^{-1}\left\{s^mG\left(s\right)\exp\left(-b\left(a+s\right)\right)\right\}\\ &\qquad\overset{L.T}{\longleftrightarrow}\exp\left(-ba\right)\frac{d^mg\left(t-b\right)}{dt^m}U\left(t-b\right). \end{aligned}\tag{38}$$

With (38), we still need to derive the m-th derivative of $g\left(t\right)$. In this paper, we assume that $g(t)=\frac{\exp(-a\cdot t)t^{n-1}}{(n-1)!}$ and we derive the derivative of $g\left(t\right)$ for a special case and then we can extend this result to the general case. More specifically, after i) differentiating $g(t)$ based on the product rule one time, two times, and three times and then ii) rearranging and simplifying them, the first, second, and third derivative of $g\left(t\right)$ can be written, respectively, as

$$g'\left(t\right)=\frac{\exp\left(-a\cdot t\right)}{\left(n-1\right)!}\left(-1\right)\left(a\cdot t^{n-1}-\left(n-1\right)t^{n-2}\right),\tag{39}$$

$$\begin{aligned} g''\left(t\right)=&\frac{\exp\left(-a\cdot t\right)}{\left(n-1\right)!}\left(-1\right)^2\left(a^2\cdot t^{n-1}-2a\left(n-1\right)t^{n-2}\right.\\ &\left.+\left(n-1\right)\left(n-2\right)t^{n-3}\right), \end{aligned}\tag{40}$$

and

$$\begin{aligned} g'''\left(t\right)=&\frac{\exp\left(-a\cdot t\right)}{\left(n-1\right)!}\left(-1\right)^3\left(a^3\cdot t^{n-1}-3a^2(n-1)t^{n-2}\right.\\ &\left.+3a(n-1)(n-2)t^{n-3}-(n-1)(n-2)(n-3)t^{n-4}\right). \end{aligned}\tag{41}$$

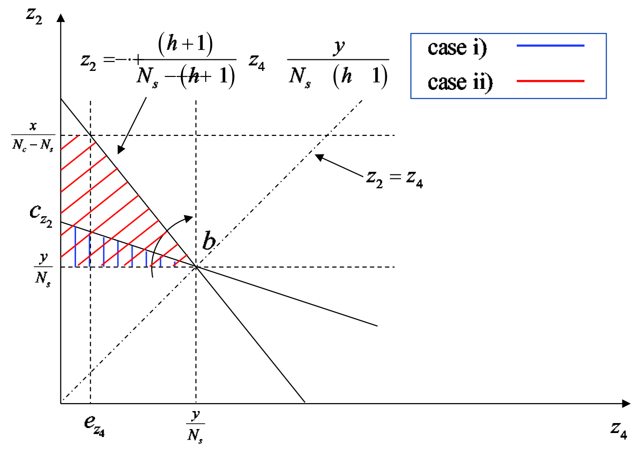

FIGURE 2. Integral regions for Eq. (43).

As a result, from (39)-(41), we can now obtain the m-th derivative of $g(t)$ for arbitrary m as

$$g^{(m)}(t) = \sum_{k=0}^{m} (-1)^{k+m} \binom{m}{k} a^{m-k} \frac{\prod_{l=0}^{k}(n-l)}{n!} t^{n-1-k} \times \exp(-at). \quad (42)$$

APPENDIX C
DERIVATION OF THE CLOSED-FORM EXPRESSION OF (22)

To obtain the closed-form expression of (22), we need to calculate the following double integral term

$$\int_0^{\frac{y}{N_s}} \int_{\frac{y}{N_s}}^{\frac{x}{N_c-N_s}} {z_2}^{\alpha} {z_4}^{\beta} \times \exp\left(-\frac{m}{\bar{\gamma}}(N_s-1)z_2\right) \exp\left(-\frac{m}{\bar{\gamma}}kz_4\right) U(z_2-z_4) \times U(x-(N_c-N_s)z_2) \times U(y-((h+1)z_4+(N_s-(h+1))z_2)) dz_2 dz_4. \quad (43)$$

Based on the given conditions associated with parameters z_2 and z_4, we need to consider two different shaded regions shown in Fig. 2. More specifically, the overall intersection region depends on the intersection point between $z_2 = -\frac{h+1}{N_s-(h+1)} \cdot z_4 + \frac{y}{N_s-(h+1)}$ (for $0 \le h \le N_s - 1$) and the z_2-axis. For case i), the z_2-coordinate term for the intersection point of $z_2 = -\frac{h+1}{N_s-(h+1)} \cdot z_4 + \frac{y}{N_s-(h+1)}$ and z_2-axis (i.e., $c_{z_2} = \frac{y}{N_s-(h+1)}$, where c_{z_2} represents the z_2-axis value of the intersection.) is located between $\frac{y}{N_s}$ and $\frac{x}{N_c-N_s}$. Therefore, the intersection becomes the shaded region filled with a blue line under $z_2 = -\frac{h+1}{N_s-(h+1)} \cdot z_4 + \frac{y}{N_s-(h+1)}$. For case ii), the z_2-coordinate term for the intersection point is located over $\frac{x}{N_c-N_s}$ on the z_2-axis. Therefore, the intersection region becomes the shaded region filled with a red line under both $z_2 = \frac{x}{N_c-N_s}$ and $z_2 = -\frac{h+1}{N_s-(h+1)} \cdot z_4 + \frac{y}{N_s-(h+1)}$. Specifically, for $z_4 < e_{z_4}$ (where e_{z_4} represents the z_4-axis value of the intersection point between $z_2 = \frac{x}{N_c-N_s}$ and $z_2 = -\frac{h+1}{N_s-(h+1)} \cdot z_4 + \frac{y}{N_s-(h+1)}$ and $e_{z_4} = \frac{y}{(h+1)} - \frac{N_s-(h+1)}{(N_c-N_s)}x$), the intersection becomes the shaded region under $z_2 = \frac{x}{N_c-N_s}$. Otherwise, the intersection becomes the shaded region under $z_2 = -\frac{h+1}{N_s-(h+1)} \cdot z_4 + \frac{y}{N_s-(h+1)}$. Note that these two cases are dominated by the relationship among parameters (N_c and N_s).

Based on the above observations, the valid integration for case i) is $0 \le z_4 \le \frac{y}{N_s}$ and $\frac{y}{N_s} \le z_2 \le -\frac{h+1}{N_s-(h+1)} \cdot z_4 + \frac{y}{N_s-(h+1)}$. For case ii), we need to consider the following two cases: a) $h = N_s - 1$ and b) $0 \le h \le N_s - 2$. As a result, for case ii)-a), the valid integration is $0 \le z_4 \le \frac{y}{N_s}$ and $\frac{y}{N_s} \le z_2 \le \frac{x}{N_c-N_s}$. Otherwise, based on the above observations, we need to consider two regions separately. More specifically, for the shaded region under $z_2 = \frac{x}{N_c-N_s}$, the valid integration region is $0 \le z_4 \le e_{z_4}$ and $\frac{y}{N_s} \le z_2 \le \frac{x}{N_c-N_s}$. Otherwise, the valid integration is $e_{z_4} \le z_4 \le \frac{y}{N_s}$ and $\frac{y}{N_s} \le z_2 \le -\frac{h+1}{N_s-(h+1)} \cdot z_4 + \frac{y}{N_s-(h+1)}$.

As a result, we can rewrite (43) as

a) For $h = N_s - 1$,

$$\int_0^{\frac{y}{N_s}} \int_{\frac{y}{N_s}}^{\frac{x}{N_c-N_s}} {z_2}^{\alpha} {z_4}^{\beta} \times \exp\left(-\frac{m}{\bar{\gamma}}(N_s-1)z_2\right) \exp\left(-\frac{m}{\bar{\gamma}}kz_4\right) dz_2 dz_4. \quad (44)$$

b) For $0 \le h < N_s - 1$,

$$\begin{aligned} &U\left(\frac{x}{N_c-N_s} - \frac{y}{N_s-(h+1)}\right) \\ &\times \int_0^{\frac{y}{N_s}} \int_{\frac{y}{N_s}}^{-\frac{h+1}{N_s-(h+1)} \cdot z_4 + \frac{y}{N_s-(h+1)}} {z_2}^{\alpha} {z_4}^{\beta} \\ &\times \exp\left(-\frac{m}{\bar{\gamma}}(N_s-1)z_2\right) \exp\left(-\frac{m}{\bar{\gamma}}kz_4\right) dz_2 dz_4 \\ &+ \left[1 - U\left(\frac{x}{N_c-N_s} - \frac{y}{N_s-(h+1)}\right)\right] \\ &\times \left\{ \int_0^{e_{z_4}} \int_{\frac{y}{N_s}}^{\frac{x}{N_c-N_s}} {z_2}^{\alpha} {z_4}^{\beta} \right. \\ &\times \exp\left(-\frac{m}{\bar{\gamma}}(N_s-1)z_2\right) \exp\left(-\frac{m}{\bar{\gamma}}kz_4\right) dz_2 dz_4 \\ &+ \int_{e_{z_4}}^{\frac{y}{N_s}} \int_{\frac{y}{N_s}}^{-\frac{h+1}{N_s-(h+1)} z_4 + \frac{y}{N_s-(h+1)}} {z_2}^{\alpha} {z_4}^{\beta} \exp\left(-\frac{m}{\bar{\gamma}}(N_s-1)z_2\right) \\ &\left. \times \exp\left(-\frac{m}{\bar{\gamma}}kz_4\right) dz_2 dz_4 \right\}. \end{aligned} \quad (45)$$

With (44) and (45), we need to determine four double-integral terms over z_2 and z_4. For the double-integral term in (44) and the second double-integral term in (45), we can simply obtain the closed-form expression by simply adopting [31, eq. (3.381.1)] for each integration over z_2 and z_4 separately as shown in (23) and (24). For the first and third double-integral terms in (45), we can obtain the closed-form expression of the inner integral term as the function of the exponential and the incomplete Gamma function, $\gamma(\cdot,\cdot)$ [31, eq. (3.381.1)]. By rearranging the incomplete Gamma function as the summation form with the help of [31, eq. (8.352.6)] and then applying [31, eq. (3.381.1)], we can obtain the closed-form expression as given in (24).

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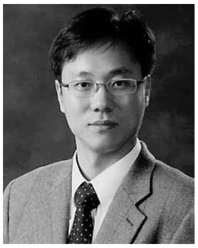
SUNG SIK NAM (S'05–M'09) received the B.S. and M.S. degrees in electronic engineering from Hanyang University, South Korea, in 1998 and 2000, respectively, the M.S. degree in electrical engineering from the University of Southern California, USA, in 2003, and the Ph.D. degree from Texas A&M University, College Station, USA, in 2009. From 1998 to 1999, he was a Researcher with ETRI, South Korea. From 2003 to 2004, he was a Manager with Korea Telecom Corporation, South Korea. From 2009 to 2010 and from 2011 to 2013, he was with Hanyang University and Sungkyunkwan University, South Korea, respectively. From 2013 to 2016, he was with Hanyang University. Since 2017, he has been with Korea University, South Korea. His research interests include the design and performance analysis of wireless communication system, diversity techniques, power control, multiuser scheduling, cooperative communications, energy harvesting, and wireless optical communication.



YOUNG-CHAI KO (S'97–M'01–SM'06) received the B.Sc. degree in electrical and telecommunication engineering from Hanyang University, Seoul, South Korea, and the M.S.E.E. and Ph.D. degrees in electrical engineering from the University of Minnesota, Minneapolis, MN, in 1999 and 2001, respectively. He was with Novatel Wireless as a Research Scientist in 2001. In 2001, he joined the Wireless Center, Texas Instruments, Inc., San Diego, CA, as a Senior Engineer. He is currently with the School of Electrical Engineering, Korea University, as a Professor. His current research interests include the design and evaluations of multi-user cellular system, MODEM architecture, and mm-wave and tera Hz wireless systems.

MOHAMED-SLIM ALOUINI (S'94–M'98–SM'03–F'09) was born in Tunis, Tunisia. He received the Ph.D. degree in electrical engineering from the California Institute of Technology, Pasadena, CA, USA, in 1998. He was with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN, USA, then with the Electrical and Computer Engineering Program at Texas A&M University at Qatar, Doha, Qatar. Since 2009, he has been a Professor of Electrical Engineering with the Division of Physical Sciences and Engineering, KAUST, Saudi Arabia, where his current research interests include the design and performance analysis of wireless communication systems.